Extending a Constrained Hybrid Dynamics Solver for Energy-Optimal Robot Motions in the Presence of Static Friction

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Motivation

While walking, **humans** take advantage of **gravity** to make motions:

→ e.g. natural swing phase of legs!

But traditional **robot control** approaches do not, and thus **waste energy**:

→ e.g. robot control on kinematic level!

Figure 1: Humanoid robot Asimo charging battery [1]

Motivation

Optimization of robot's motions, in terms of **energy consumption** is important for:

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How?

Popov-Vereshchagin hybrid dynamics solver gives us tool to do that!

Background: Constrained Hybrid Dynamics Solver

Popov-Vereshchagin solver is derived from **Gauss principle of least constraint** [3]:

$$\min_{\ddot{q}} \quad Z(\ddot{q}) = \overbrace{\sum_{i=0}^{N}\{\frac{1}{2}\ddot{X}_i{}^T I_i \ddot{X}_i + F_{bias,i}^T \ddot{X}_i\} + \sum_{j=1}^{N}\{\frac{1}{2} d_j \ddot{q}_j{}^2 - \tau_j \ddot{q}_j\}}^{Acceleration\ Energy} \tag{1}$$

$$subject\ to: \quad \left.\begin{aligned} \ddot{X}_{i+1} &= {}^{i+1}X_i \ddot{X}_i + \ddot{q}_{i+1} S_{i+1} + \ddot{X}_{bias,i+1} \\ A_N^T\ \ddot{X}_N &= b_N \end{aligned}\right\} \begin{array}{c} Motion \\ Constraints \end{array}$$

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Properties

- Gauss function is **quadratic** & constraints are **linear**!
- Defines **quadratic programming** problem!
- Enables an efficient solver → **Linear O(n) complexity**!

What about static friction?

Example with **KUKA youBot**

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Problems

- Include static friction effects in **equations** of motion!
- For making motion, we need to **overcome** friction!
- If we want to stand-still, how to **make use** of static friction?

Problem Formulation

Static friction effects are not included in **original** computations.

$\Longrightarrow$ We want to give **more knowledge** to the solver!

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Procedure for modelling static friction

Briefly outlined by Vereshchagin in the paper [4]:

The required representation obviously is

⋮

$$\sum_{j\in\Theta_0} f_j|\ddot{q}_j| = \max_{\mu\in\Lambda}\left\{\sum_{j\in\Theta_0}\mu_j\ddot{q}_j\right\}, \text{ where } \Lambda = \{\mu_j : |\mu_j| \leqslant f_j,\ j\in\Theta_0\}.$$

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[4] Anatolii Fedorovich Vereshchagin, "Modelling and control of motion of manipulation robots", Soviet Journal of Computer and Systems Sciences, vol. 27, pp. 29-38, 1989.

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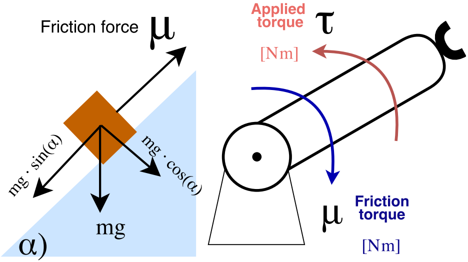
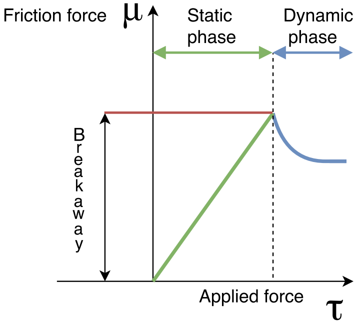
Our contribution

- Detailed **derivation** of the approach
- **Interpretation** & **evaluation** to show its feasibility

Approach: Procedure for Extending the Solver





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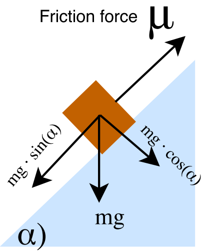

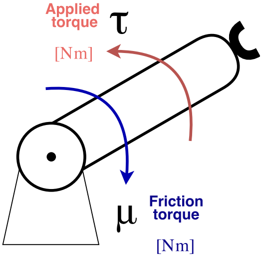

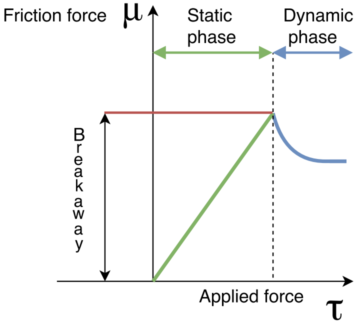

Modelling **static friction** force using **principle of maximum dissipation**:

$$Z_1 = \max_{\mu \in \Lambda} \{\mu^T W \ddot{q}\}$$

$$\Lambda = \{-breakaway \leq \ \mu \ \leq breakaway\}$$

$$W = diag(w_j), \ w_j = \begin{cases} 0, & moving \\ 1, & not \ moving \end{cases}$$

Definition

For a pair of bodies that are in contact the **true reaction force** is the one that produces **maximum rate** of **energy dissipation** [5].

Approach: Procedure for Extending the Solver

Combine **Gauss** and **maximum dissipation** principle! [6]

Integration of **static friction** requires:

- **Extended** Gauss function
- **Extended** optimization problem

$$\overbrace{\mu^*}^{\text{True friction torques}} = arg \overbrace{\max_{\mu}}^{\text{Friction extension}} \underbrace{\min_{\ddot{q}}\{ \overbrace{Z(\ddot{q})}^{\text{Original Gauss Function}} + \overbrace{\mu^T W \ddot{q}}^{\text{Additional Acceleration Energy}} \}}_{\text{Solution computed using Popov-Vereshchagin solver}} \quad (2)$$

With **optimum** friction torques μ^*:

"*The accelerations* $\ddot{q}(\mu^*)$ *will be true accelerations in the system*" [4].

Approach: Properties of the Optimization Problem

After **extension**, the Gauss function:

- Loses its **quadratic** property
- But continues being **convex** [4]

Convexity ensures existence of **unique solution**! [7]

$$\max_{\mu} \min_{\ddot{q}} \{Z(\ddot{q}) + \mu^T W \ddot{q}\}$$

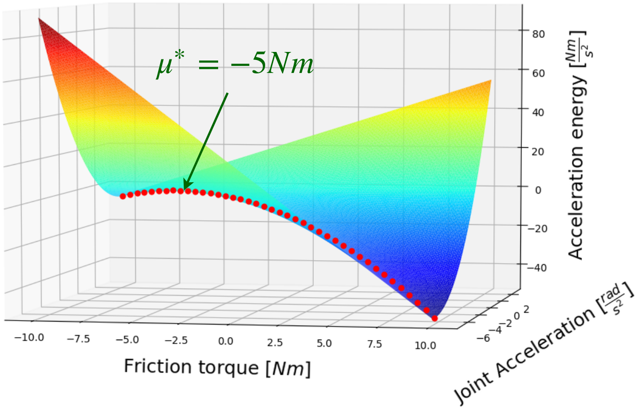

Figure 2: Extended Gauss function for case of 1 *DOF* robot.

- Red points $\implies$ **minimums** of the extended Gauss function
- What we want to do $\implies$ find **maximum** over those points

Evaluation: Setup

All experiments in **simulation**.

Artificially generated values.

Breakaway value of joint friction force is **10Nm**:

$$\Longrightarrow -10Nm < \mu < 10Nm$$

Why this type of setup?

Necessity for:

- Simple **interpretation**
- Straightforward **prediction** of correct results

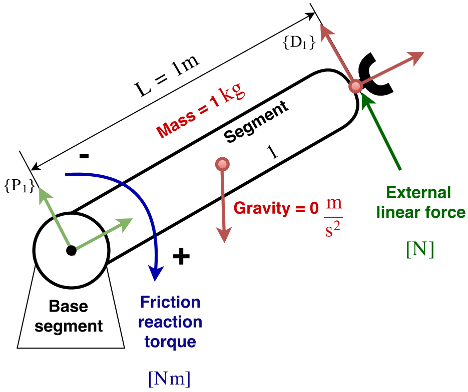

Figure 3: Model of 1 *DOF* robot arm and forces influencing its motion.

Evaluation: Scenario 1

- **External linear** force of $-\mathbf{5N}$ is applied on the robot's **end-effector**.
- It will produce $-\mathbf{5Nm}$ of **torque** in the robots **joint**.
- **Join torque** magnitude $<$ **breakaway** value of **static friction** force.
- Expected result: External force will **not** move the arm!

Evaluation 1: Acceleration Energy

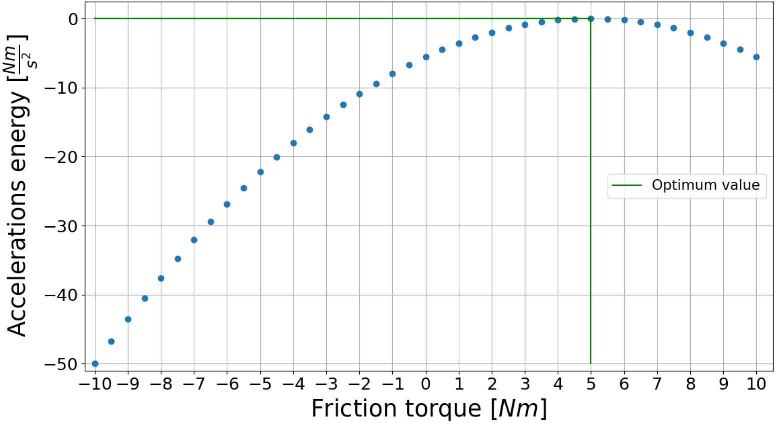

Figure 4: Acceleration energy when external force $= -5N$.

Maximum acceleration energy is induced by **friction torque** of **5 Nm**.

Evaluation 1: Resulting Joint Torque

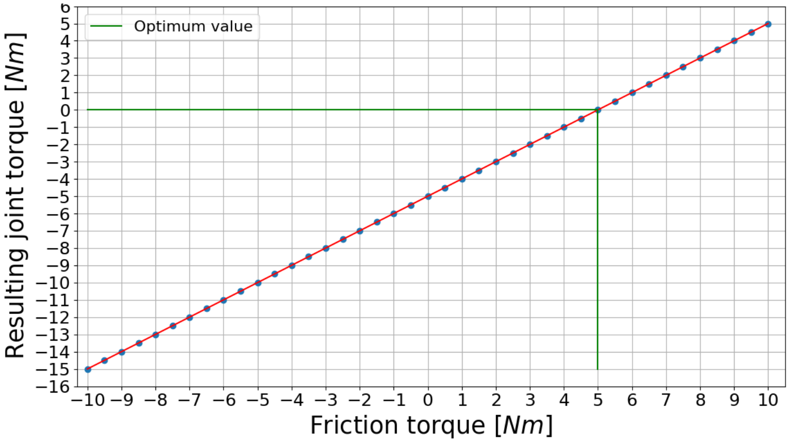

Figure 5: Resulting joint torques when external force $= -5N$.

Evaluation 1: Resulting Joint Acceleration

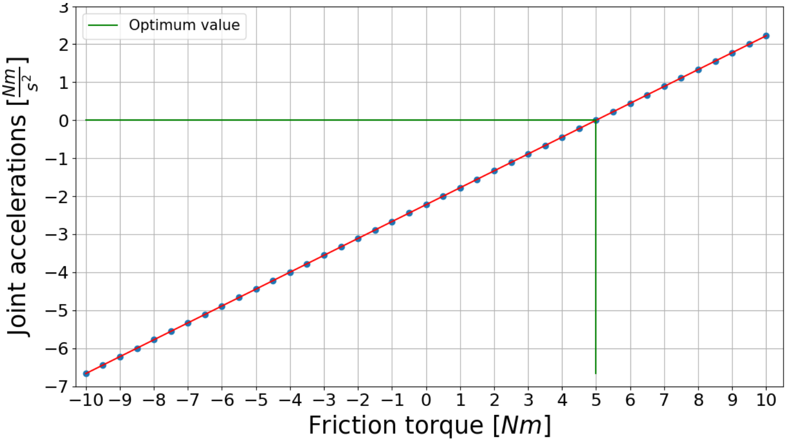

Figure 6: Joint accelerations when external force $= -5N$.

Evaluation: Scenario 2

- **External force** increased to $-$**11N**.
- It will produce $-$**11Nm** of **torque** in the robots **joint**.
- **Join torque** magnitude $>$ **breakaway** value of **static friction** force.
- Expected result: Robot **moving**!

Evaluation 2: Acceleration Energy

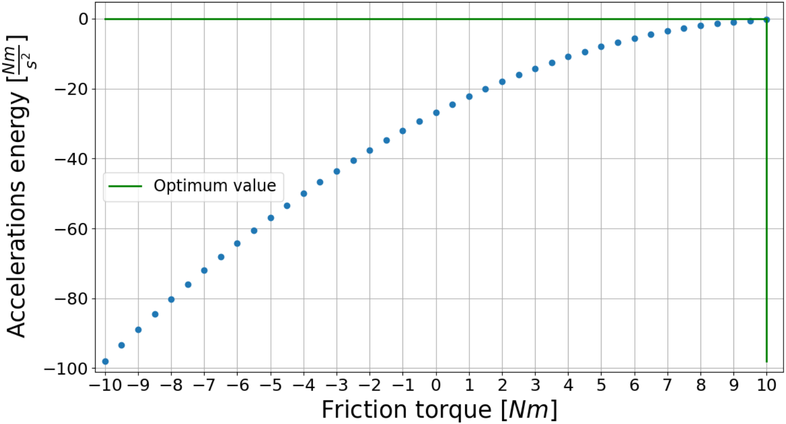

Figure 7: Acceleration energy when external force $= -11N$.

Maximum acceleration energy is induced by **friction torque** of **10 Nm**.

Evaluation 2: Resulting Joint Torque

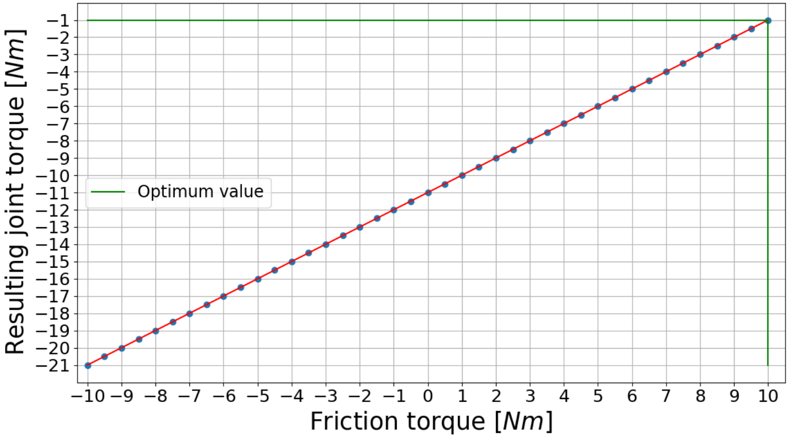

Figure 8: Resulting joint torques when external force $= -11N$.

Evaluation 2: Resulting Joint Acceleration

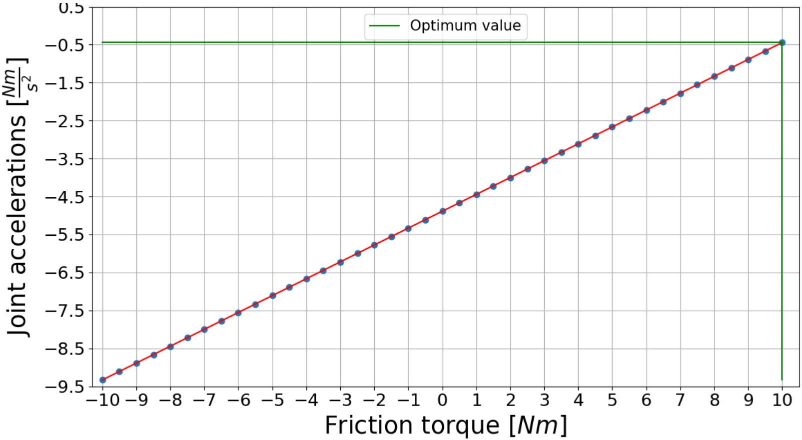

Figure 9: Joint accelerations when external force $= -11N$.

Bad News

No analytical solution to the **maximization** problem!

For finding the optimum, **Vereshchagin** proposed the **convex simplex method** [8]:

- It is a **numerical method**!
- Required **non-constant** number of iterations
- May **not** be usable for **real-time** application

Further Extensions: Improving Efficiency

Problem with extension

For each step in the “outer” **maximization** problem:
Computations of whole **hybrid dynamics** are repeated.

$\Longrightarrow$ Computationally **expensive**!

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Method of solution

Vereshchagin **mentioned** idea for improving **run-time** of approach:

"*This relation is linear and it can be obtained analytically if we determine beforehand all characteristics of the equations of dynamics or the Gauss function*" [4]. $\implies$ No additional information!

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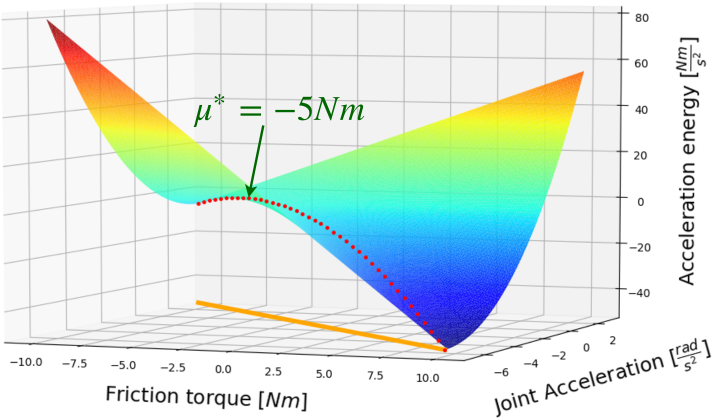
Our contribution

Derivation & **Interpretation** of the method!

Further Extensions: Improving Efficiency - Interpretation

Figure 10: Gauss function and characteristic curve for case of 1 *DOF* robot.



Orange line represent **characteristic curve** of the Gauss function:

$$\ddot{q}^*(\mu) = \ddot{q}^*(0) + \frac{\partial \ddot{q}}{\partial \mu}\mu$$

Further Extensions: Improving Efficiency - Interpretation

Properties of the Gauss function allow us to find **characteristic equation**:

- An ordinary differential equation (ODE)

Characteristic curve defines how joint accelerations change w.r.t friction torques:

- $\ddot{q}^*(\mu) = \ddot{q}^*(0) + \frac{\partial \ddot{q}}{\partial \mu}\mu$

This method allows us to compute $Z(\ddot{q})$ more efficiently:

- By using values of $\ddot{q}$ computed directly from **characteristic equation**
- Without computing **whole hybrid dynamics** for each considered friction torque!

Based on **computed motion**, Gauss function can be evaluated!

Use the original P.V. solver **only once** while maximizing Gauss function!

Future Work

- Implementation of the **convex simplex method**
- Investigation of **real-time feasibility** of the complete approach
- Implementation of **derived method** for improving **run-time efficiency**
- Same approach works for including **joint limits**:
  - Future work: Implementation of this extension!
- Integration of the extended solver on a **real robot**

Conclusions

Approach for including **static friction** effects:

- Has been **completely derived**
- Has been **proof of concept** evaluated
- Its **feasibility** has been shown

Method for improving **run-time efficiency** has been derived.

Solver has more knowledge to compute **energy optimal** motions.

Thank you for your attention!

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