

# 2.151 Final Project

Lucille Hosford, Chris Welch, Brian Wilcox,  
Manjinder Singh

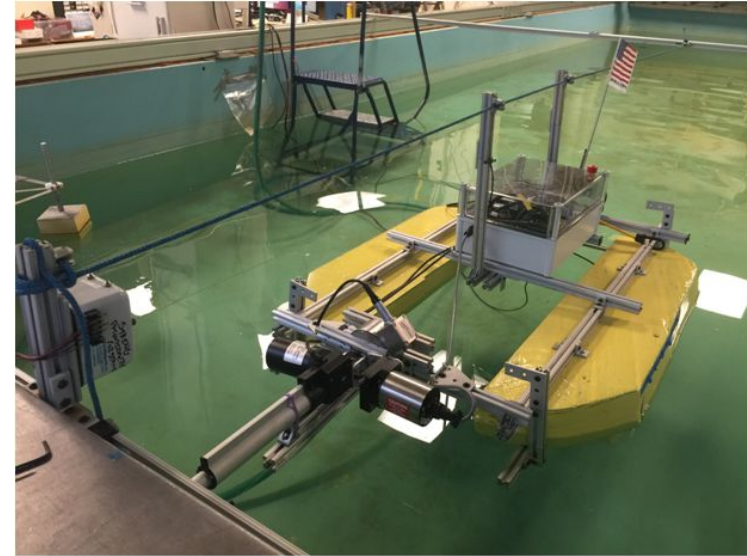
# Impedance Control in the Oil and Gas Industry

- **Decommissioning** is the dismantling of offshore structures
- Current state of practices uses ROVs and divers
  - ROVs = Very expensive operational costs
  - Divers = Very significant safety risks
- **Replace with autonomy?**
  - Autonomous decommissioning vehicles must demonstrate comparable ability to **sense** and **manipulate**
- **Key tasks** include scrubbing biofouling and testing valve functionality
  - **Suggests impedance control!**

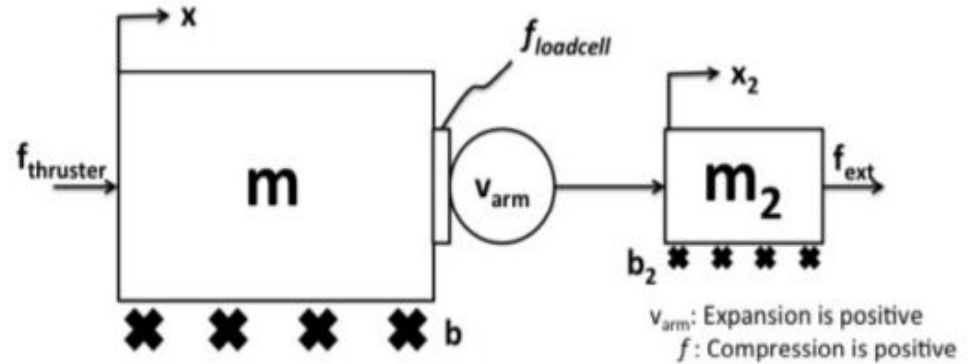
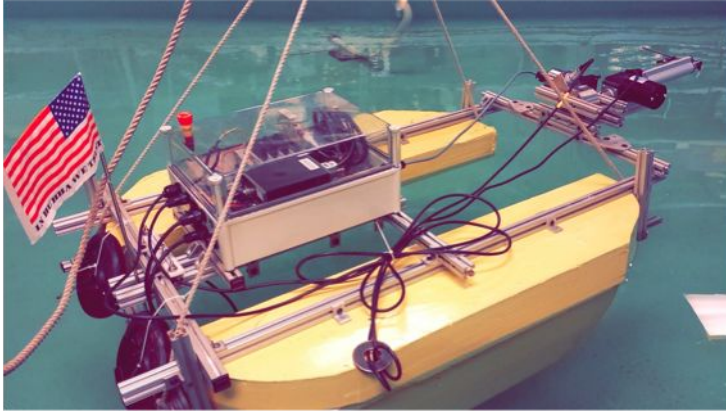


# Baxter and Dexter

- **Long term goal:** Dexter affords ability to investigate free-flight collisions, but Baxter's manipulator more realistically exemplifies desired dexterity of a light intervention AUV
  - Use **Dexter's** linear actuator to **find range of impedances** for acceptable collision characteristics
  - Demonstrate that **Baxter's** multi dof arm can **reproduce those impedances**



# Dexter State Space Representation: Open Loop Model

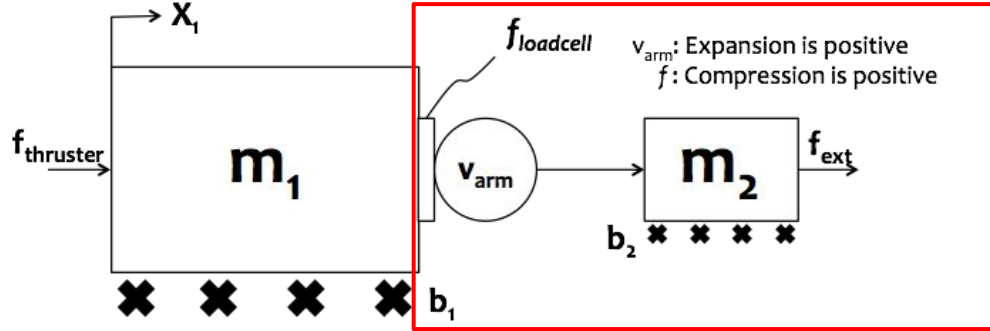


## State Space Representation

$$\begin{bmatrix} \frac{d(x_1)}{dt} \\ \frac{d^2(x_1)}{dt^2} \\ \frac{d(x_2)}{dt} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & \frac{-(b_1+b_2)}{m_1+m_2} & 0 \\ 0 & 0 & 0 \end{bmatrix} * \begin{bmatrix} x_1 \\ \frac{d(x_1)}{dt} \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ \frac{-b_2}{m_1+m_2} & \frac{-m_2}{m_1+m_2} & \frac{1}{m_1+m_2} \\ 1 & 0 & 0 \end{bmatrix} * \begin{bmatrix} \frac{d(x_2)}{dt} \\ \frac{d^2(x_2)}{dt^2} \\ F_t \end{bmatrix}$$

# Dexter Equations of Motion: Closed Loop Model

What We Have:

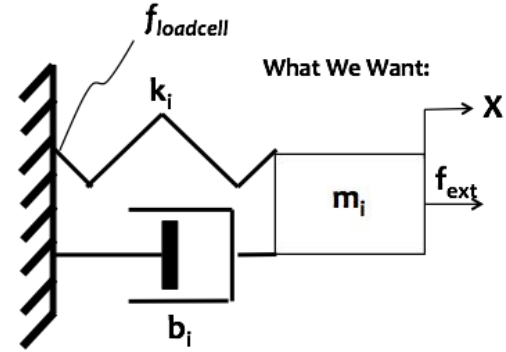


$$m_1 \ddot{x}_1 = f_t - f_l - b_1 \dot{x}_1$$

$$m_2 \ddot{x}_2 = f_l + f_e - b_2 \dot{x}_2$$

$$x_2 = x_1 + x_{arm}$$

What We Want:



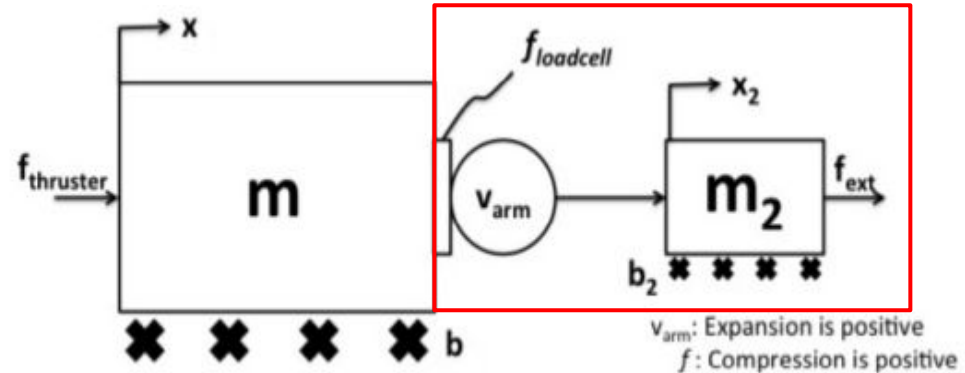
$$\ddot{x} = -\frac{b_i}{m_i} \dot{x} - \frac{k_i}{m_i} x + \frac{1}{m_i} f_l$$

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$$\frac{d}{dt} \begin{bmatrix} x_1 \\ \dot{x}_1 \\ \dot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ k_i & \frac{-(b_2-b_i)}{m_2-m_i} & \frac{-(b_2-b_i)}{m_2-m_i} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x}_1 \\ \dot{x}_a \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ \frac{1}{m_2-m_i} & \frac{k_i}{m_2-m_i} & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -k_i \\ -b_i \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} [F_l]$$

$$\begin{bmatrix} F_t \\ \ddot{x}_a \end{bmatrix} = \begin{bmatrix} \frac{m_1 k_i}{m_2-m_i} & b_1 - \frac{m_1(b_2-b_i)}{m_2-m_i} & \frac{m_1(b_2-b_i)}{m_2-m_i} \\ \frac{k_i}{b_2-b_i} & \frac{m_2-m_i}{m_1(b_2-b_i)} - \frac{b_2}{b_2-b_i} + \frac{b_i}{b_2-b_i} & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \dot{x}_a \end{bmatrix} + \begin{bmatrix} \frac{m_1+m_2-m_i}{m_2-m_i} & \frac{m_1 k_i}{m_2-m_i} & -m_1 \\ \frac{m_1+m_2-m_i}{m_1(b_2-b_i)} & \frac{k_i}{b_2-b_i} & \frac{-(m_2-m_i)}{b_2-b_i} \\ \frac{-m_1 k_i}{m_2-m_i} & \frac{-m_1 b_i}{m_2-m_i} & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} F \\ \ddot{x}_a \end{bmatrix}$$

- The above state space model describes necessary actuation with respect to sensor inputs in order to achieve a desirable, user-defined endpoint impedance
- However, **focus for Baxter** is on the **actuator's impedance characteristics**



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# Impedance Control in the Oil and Gas Industry

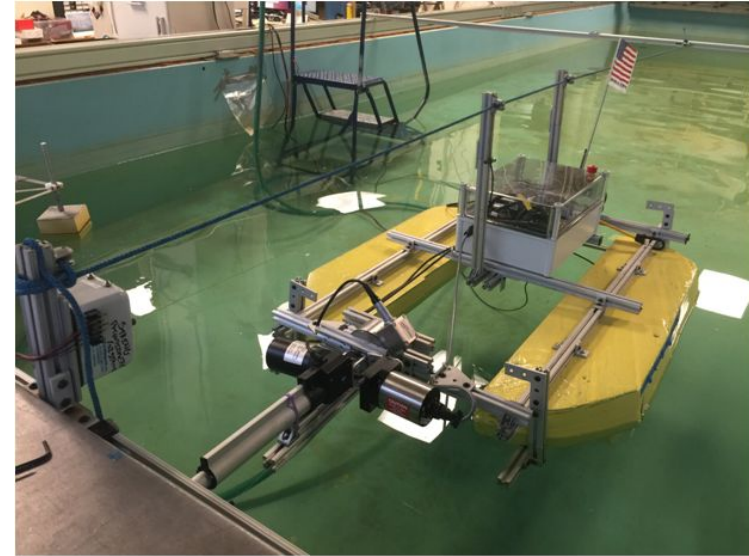
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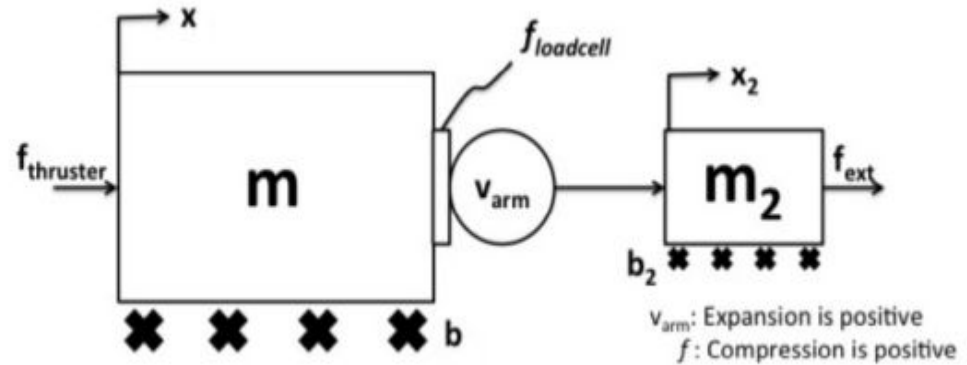
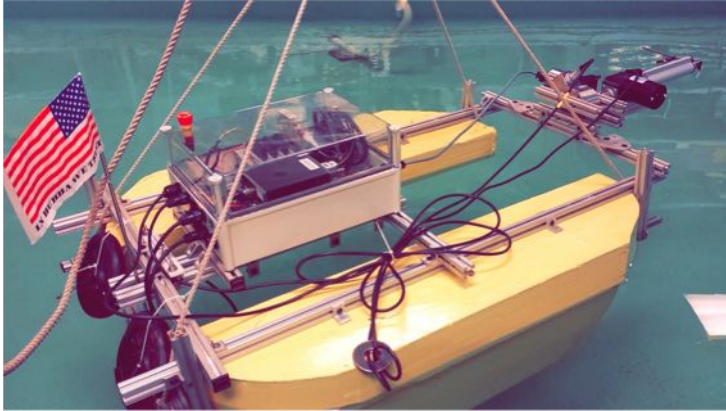


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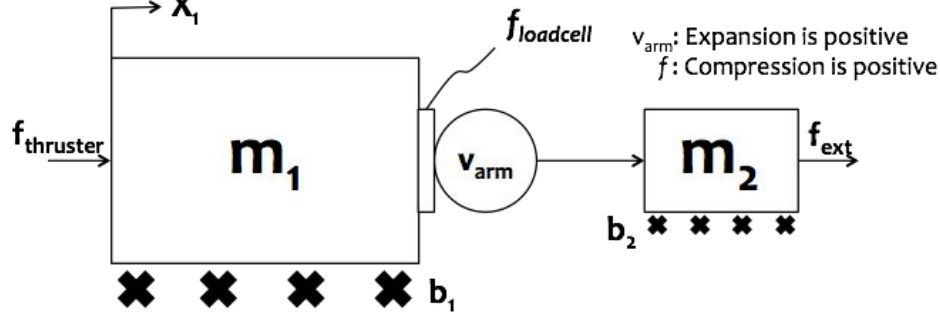


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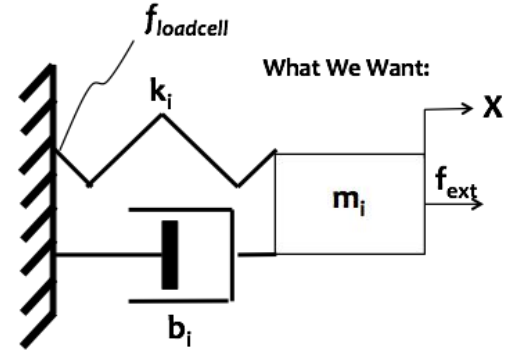


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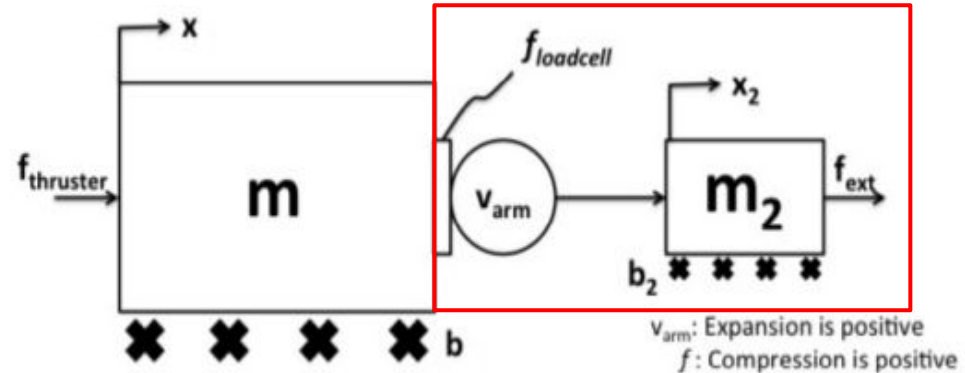
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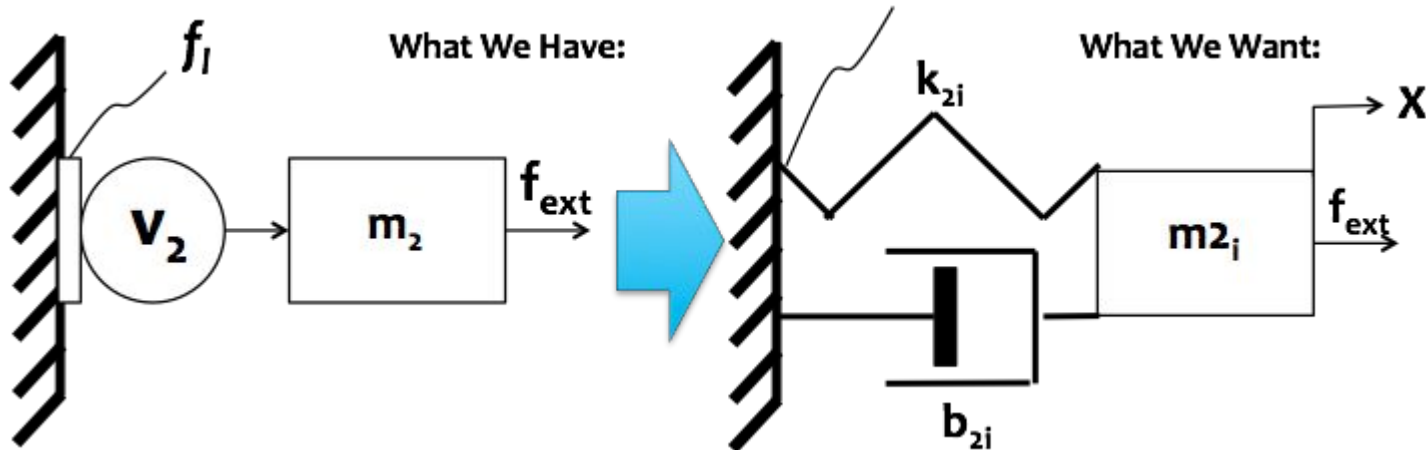
$$\begin{bmatrix} F_t \\ \ddot{x}_a \end{bmatrix} = \begin{bmatrix} \frac{m_1 k_i}{m_2-m_i} & b_1 - \frac{m_1(b_2-b_i)}{m_2-m_i} & \frac{m_1(b_2-b_i)}{m_2-m_i} \\ \frac{k_i}{b_2-b_i} & \frac{m_2-m_i}{m_1(b_2-b_i)} - \frac{b_2}{b_2-b_i} + \frac{b_i}{b_2-b_i} & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \ddot{x}_a \end{bmatrix} + \begin{bmatrix} \frac{m_1+m_2-m_i}{m_2-m_i} & \frac{m_1 k_i}{m_2-m_i} & -m_1 \\ \frac{m_1+m_2-m_i}{m_1(b_2-b_i)} & \frac{k_i}{b_2-b_i} & \frac{-(m_2-m_i)}{b_2-b_i} \\ \frac{-m_1 k_i}{m_2-m_i} & \frac{-m_1 b_i}{m_2-m_i} & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x}_1 \\ \ddot{x}_a \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{-k_i}{b_2-b_i} & \frac{b_i}{b_2-b_i} & \frac{-(m_2-m_i)}{m_1(b_2-b_i)} \end{bmatrix} \begin{bmatrix} F \\ \ddot{x}_a \end{bmatrix}$$

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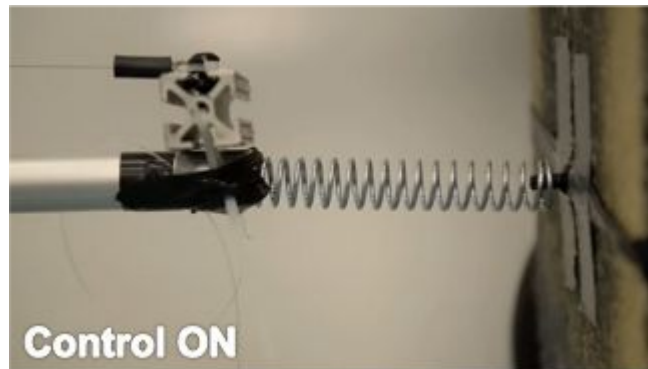
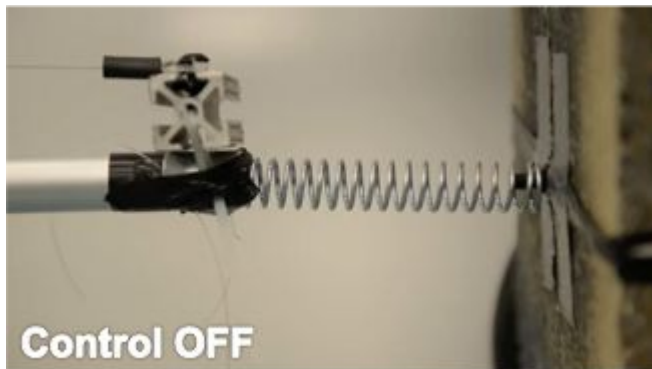
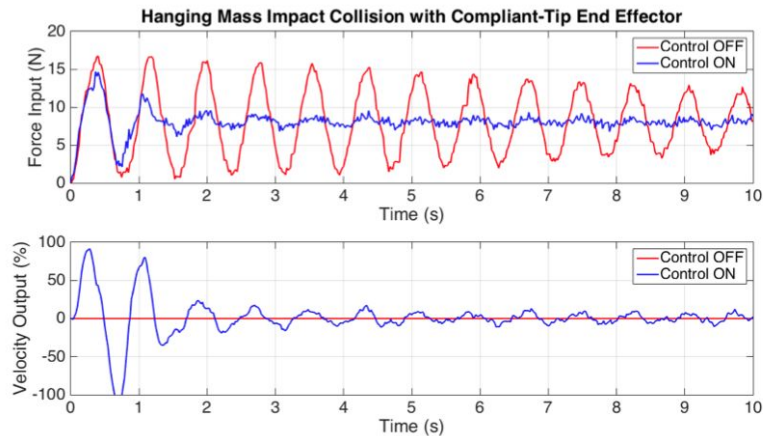
# Turning a Non-Backdrivable Linear Actuator Into a Passive System

- Creating a controller for the linear actuator based desired impedance parameters allows for:
  - A closed-loop system that promotes robust and stable interactions due to passivity
  - Achievable design constraints (e.g., Peak impact force, Settling time)
  - Definitive impedance values for Baxter to reproduce

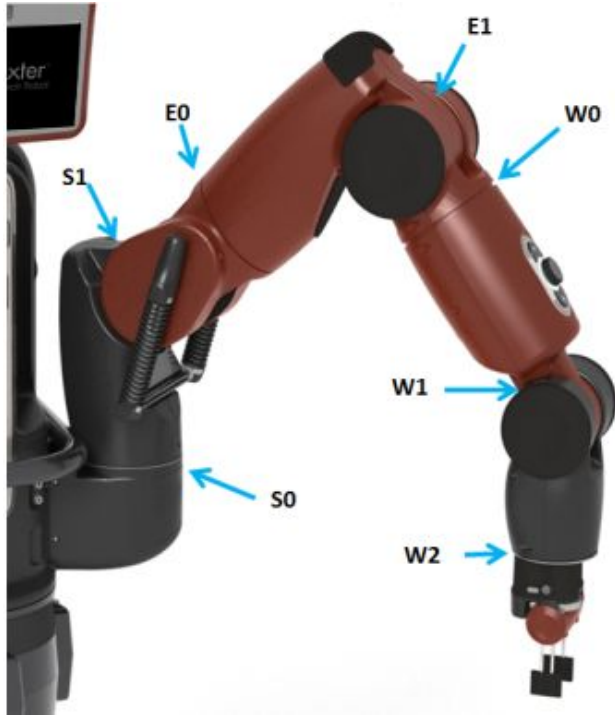


# Determining Desirable Impedance Characteristics

- A **reduction in peak force** provides a buffer for a controller to react to any unforeseen collisions
  - Alleviates significant risks for valuable equipment on hard-to-reach AUVs
- A **sharp reduction in settling time** suggests a smaller influence by external disturbances, such as underwater currents
  - Minimizes unpredictable disturbances during any on going contact tasks
- **Feed impedance values to Baxter!**



# Baxter Research Robot



For the purposes of the experiment, Baxter was constrained to planar motion by only allowing joints S1, E1, and W1 to move.



# Baxter Research Robot: Open loop model

Neglecting SEA dynamics, Equations of Motion

$$(M(q) + I_r)\ddot{q} + C(\dot{q}, q) + D\dot{q} + g(q) = \tau_{motors}$$

Simplifies to

$$(M(q) + I_r)\ddot{q} + D\dot{q} = \tau_{motors}$$

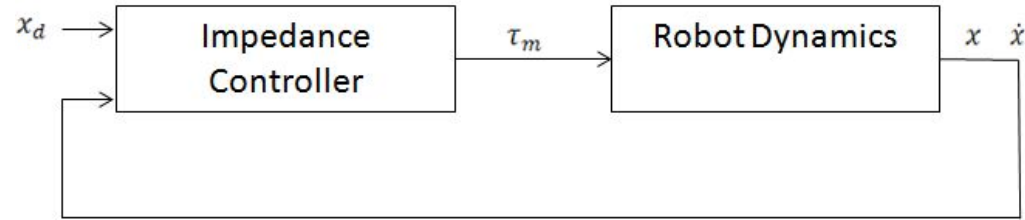
State space representation

$$\frac{d}{dt} \begin{bmatrix} q \\ \dot{q} \end{bmatrix} = \begin{bmatrix} 0 & I \\ 0 & -(M(q) + I_r)^{-1}D \end{bmatrix} \begin{bmatrix} q \\ \dot{q} \end{bmatrix} + \begin{bmatrix} 0 \\ (M(q) + I_r)^{-1} \end{bmatrix} \tau_{motors}$$

$$\dot{x} = [0 \ I] \begin{bmatrix} q \\ \dot{q} \end{bmatrix} + [0] \tau_{motors}$$



# Baxter Research Robot: Closed loop model

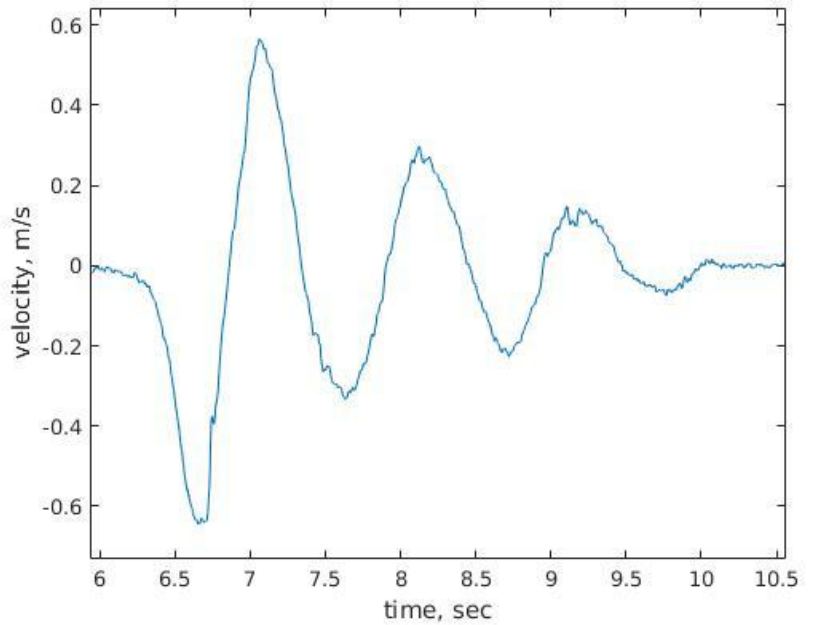
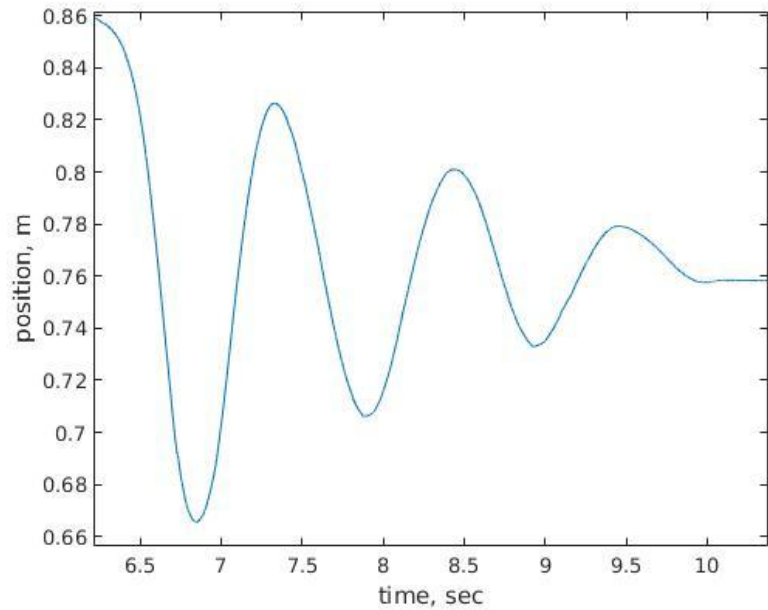


$$F_{motor} = K(x_d - x) - B\dot{x} \quad \tau_{motor} = J^T(F_{motor})$$

$$\frac{d}{dt} \begin{bmatrix} q \\ \dot{q} \end{bmatrix} = \begin{bmatrix} 0 & I \\ -(M(q) + I_r)^{-1} J^T K J & -(M(q) + I_r)^{-1} (D + J^T B J) \end{bmatrix} \begin{bmatrix} q \\ \dot{q} \end{bmatrix} + \begin{bmatrix} 0 \\ (M(q) + I_r)^{-1} J^T K J \end{bmatrix} q_d$$

$$\dot{x} = [0 \ J] \begin{bmatrix} q \\ \dot{q} \end{bmatrix} + [0] q_d$$

# Baxter Research Robot: Real Data



# Baxter Research Robot: LQR Design

- What if we want to specify performance criteria for the states?
  - Max actuator force
  - Stroke Length
- Use LQR to optimize performance and control effort

$$\min_{\mathbf{u}(t)} V = \int_{\tau=t}^{\tau=T} (\mathbf{x}^t \mathbf{Q} \mathbf{x} + \mathbf{u}^t \mathbf{R} \mathbf{u}) d\tau$$

# Baxter Research Robot: LQR Design

- Bryson's Rule

$$Q = \begin{bmatrix} \frac{\alpha_1^2}{(x_1)_{\max}^2} & & & \\ & \frac{\alpha_2^2}{(x_2)_{\max}^2} & & \\ & & \ddots & \\ & & & \frac{\alpha_n^2}{(x_n)_{\max}^2} \end{bmatrix}$$

$$R = \rho \begin{bmatrix} \frac{\beta_1^2}{(u_1)_{\max}^2} & & & \\ & \frac{\beta_2^2}{(u_2)_{\max}^2} & & \\ & & \ddots & \\ & & & \frac{\beta_m^2}{(u_m)_{\max}^2} \end{bmatrix}$$

- Baxter system criteria:
  - $x_{1\max}$  (x position) = 0.5214 m
  - $u_{1\max}$  (shoulder torque) = 50 Nm
  - $u_{2\max}$  (elbow torque) = 15 Nm
  - $u_{3\max}$  (wrist torque) = 15 Nm

# Baxter Research Robot: LQR Design

Q =

3.6787	0	0	0	0	0
0	0.0100	0	0	0	0
0	0	0.0100	0	0	0
0	0	0	0.0100	0	0
0	0	0	0	0.0100	0
0	0	0	0	0	0.0100

## Expensive vs Cheap Control

$p = 1$  (equal weighting)

$p = 1e6$  (expensive control)

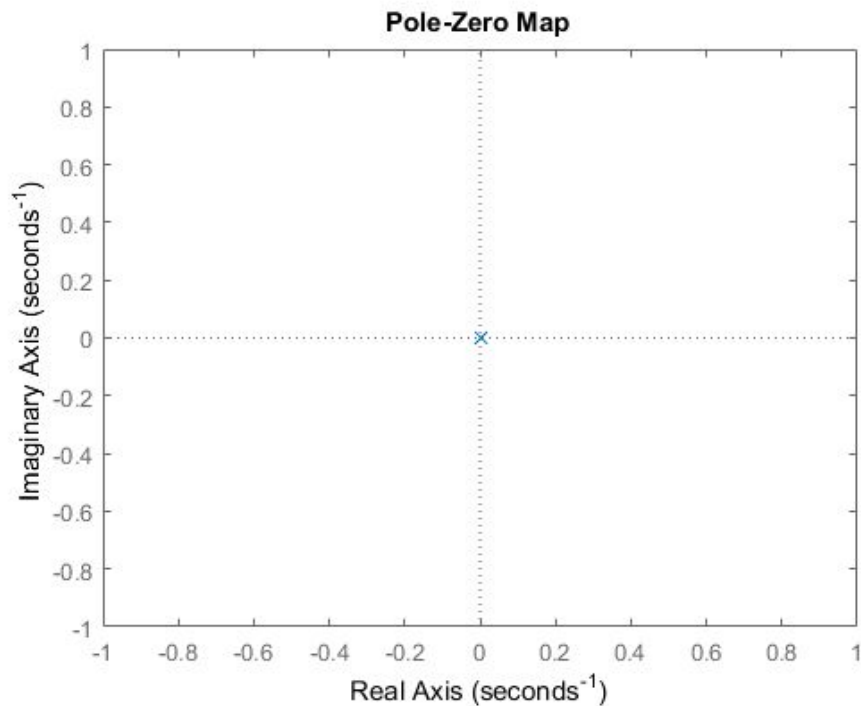
$p = 1e-6$  (cheap control)

R =

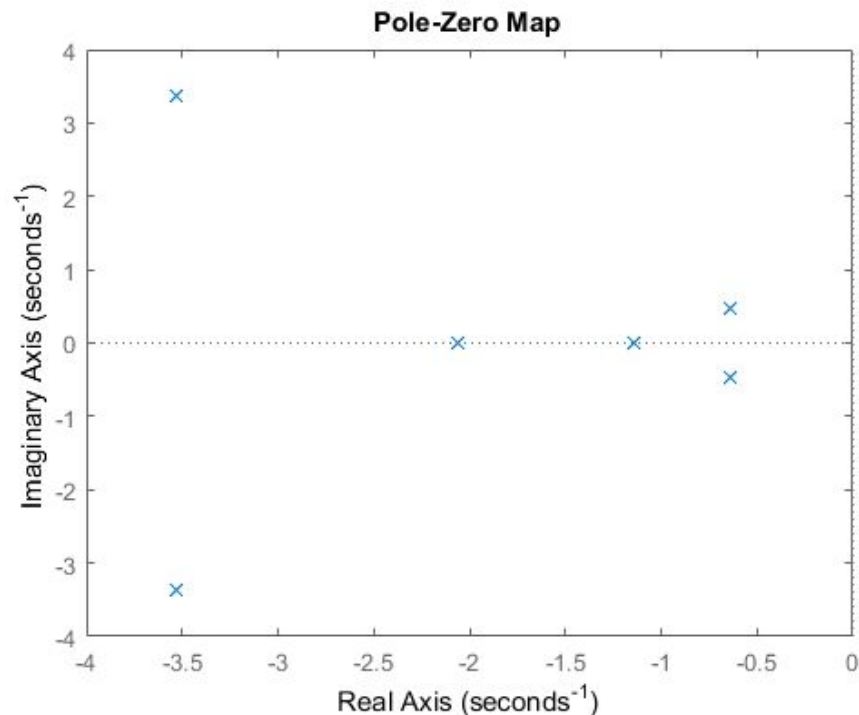
0.0004	0	0
0	0.0044	0
0	0	0.0044

# Baxter Research Robot: LQR Design

Open-Loop



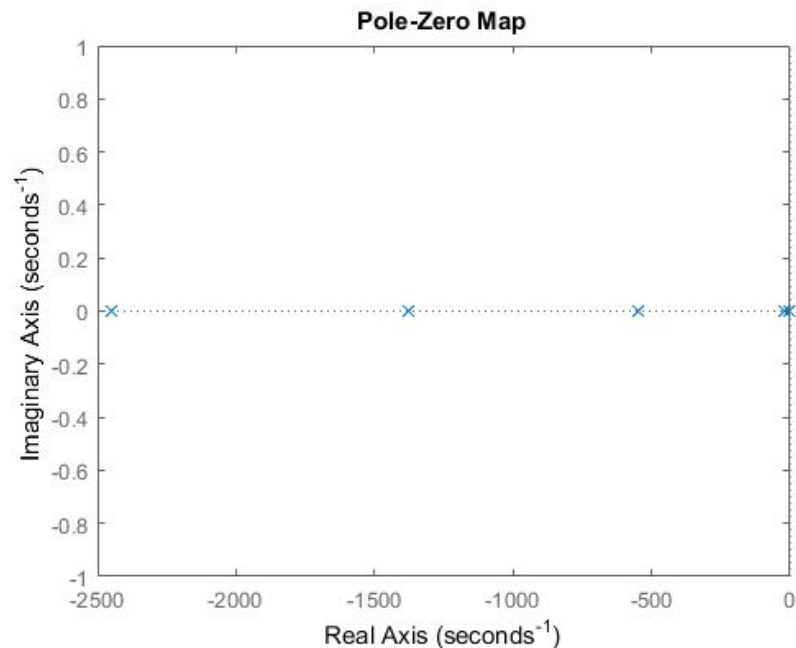
Closed Loop: Equal Weighting



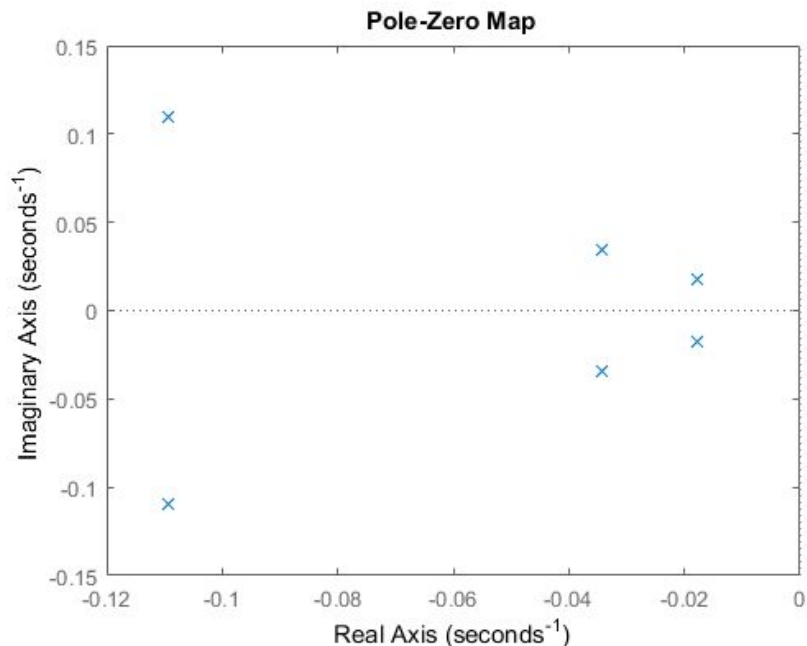


# Baxter Research Robot: LQR Design

Closed Loop: Cheap



Closed Loop: Expensive

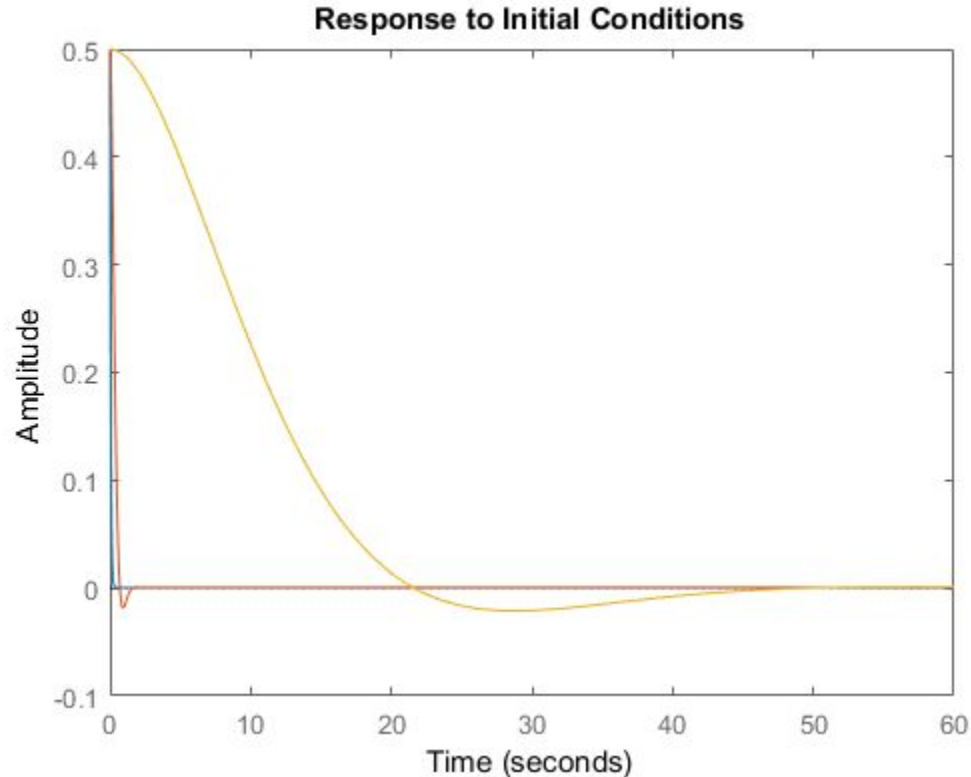


# Baxter Research Robot: LQR Design

blue - cheap

orange - expensive

yellow - equal



# Baxter Research Robot: LQR Discussion

- Expensive control is realistic given good performance results and less control (actuator effort)
- Choosing relative weighting of states/inputs may be significant
- Can we design performance criteria around impact collision response?
- Can we combine impedance into cost function for LQR design?

# Dexter LQR Design

Criteria:

Max Thrust=10 Nm

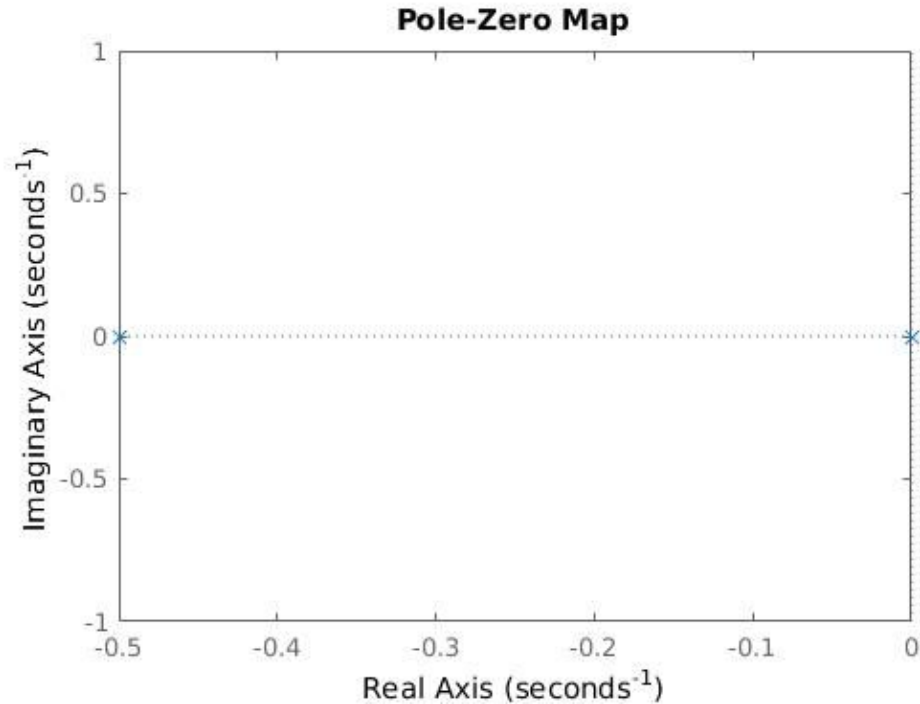
Max Stroke length=10 in

Different Designs:

- Cheap
- Expensive
- Equal
- Weighting  $x_a$  vs  $x_1$

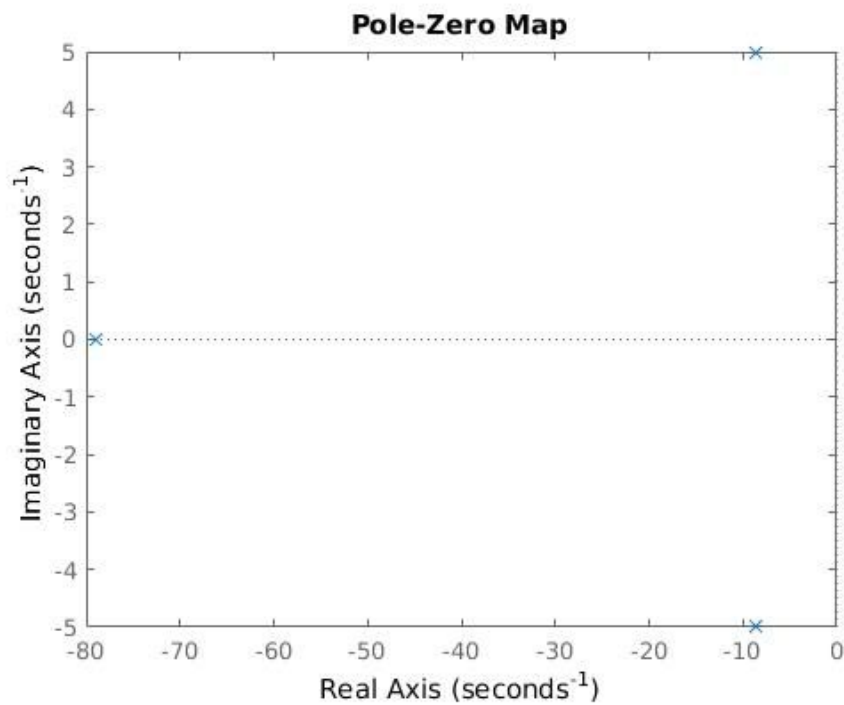
# Dexter LQR Design

Open-Loop

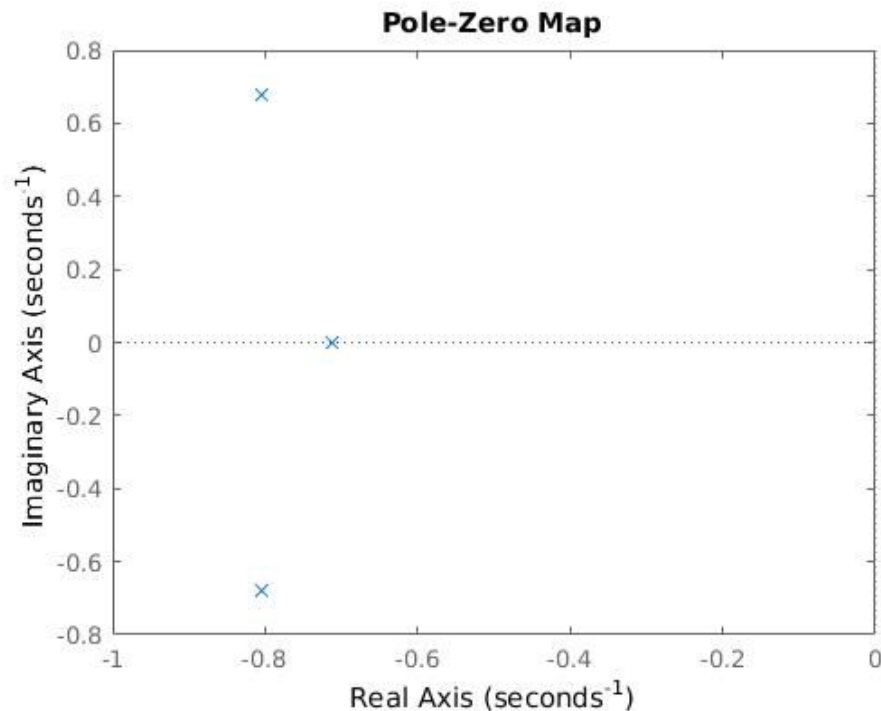


# Dexter LQR Design

Cheap

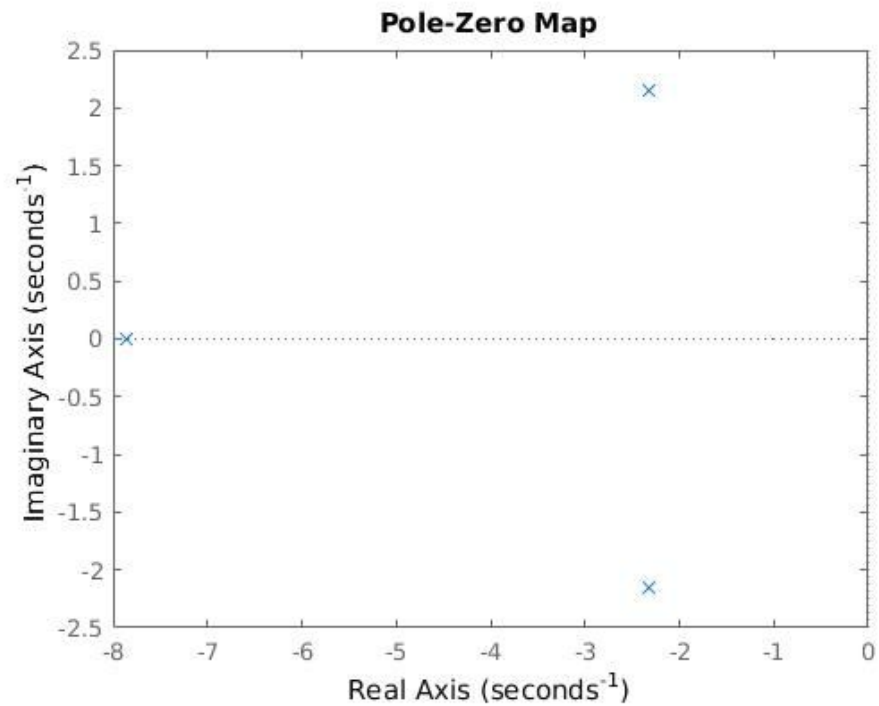


Expensive

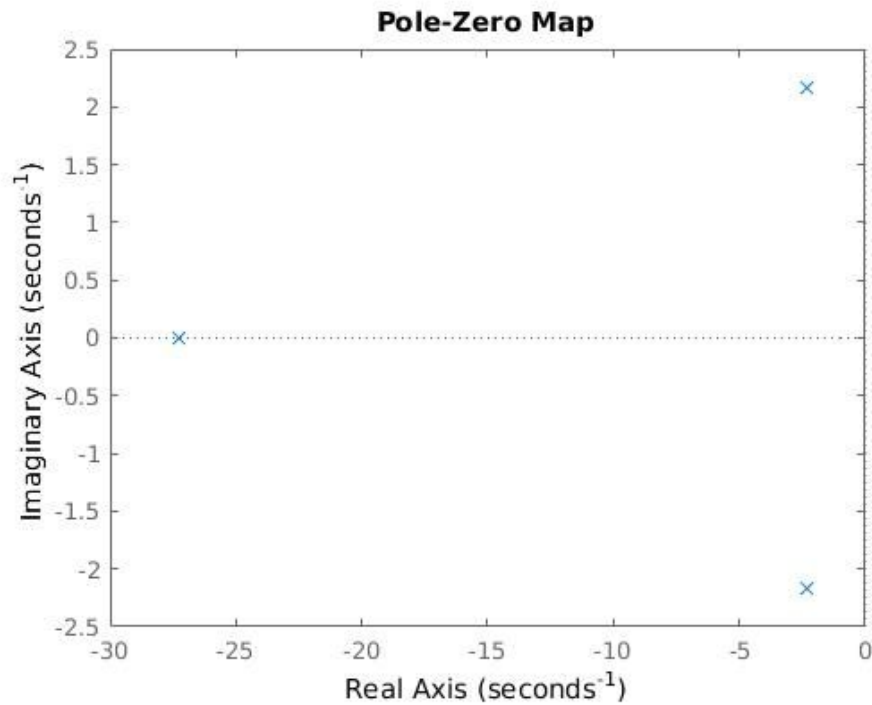


# Dexter LQR Design

Equal



Diff Weights





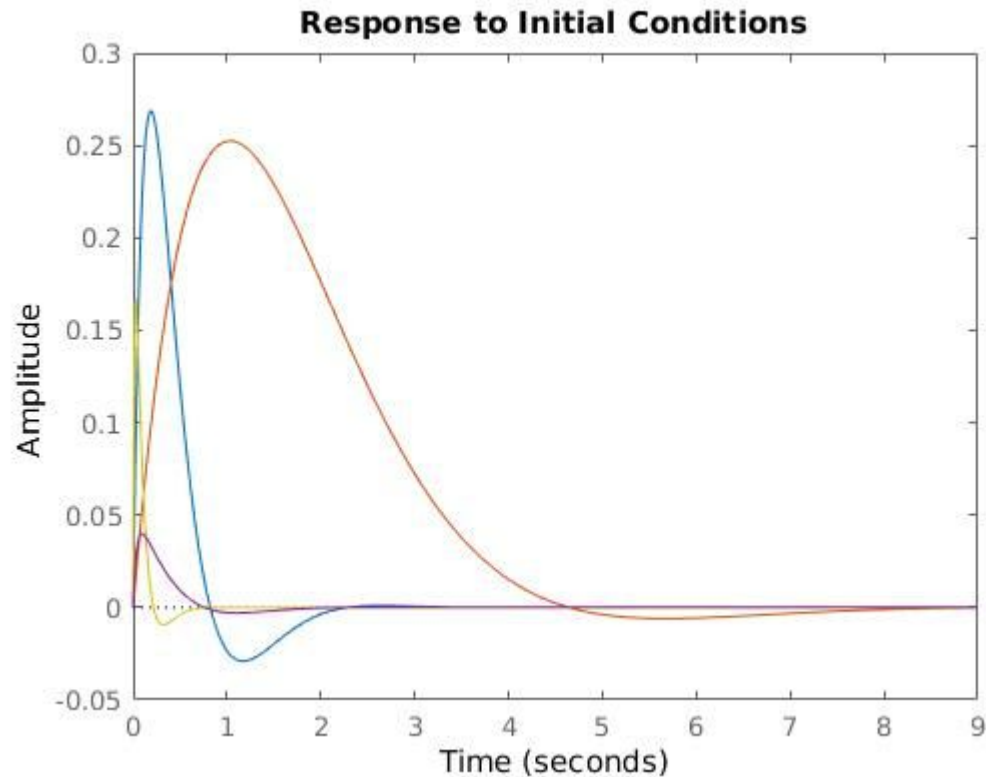
# Dexter LQR Design

yellow - cheap

red - expensive

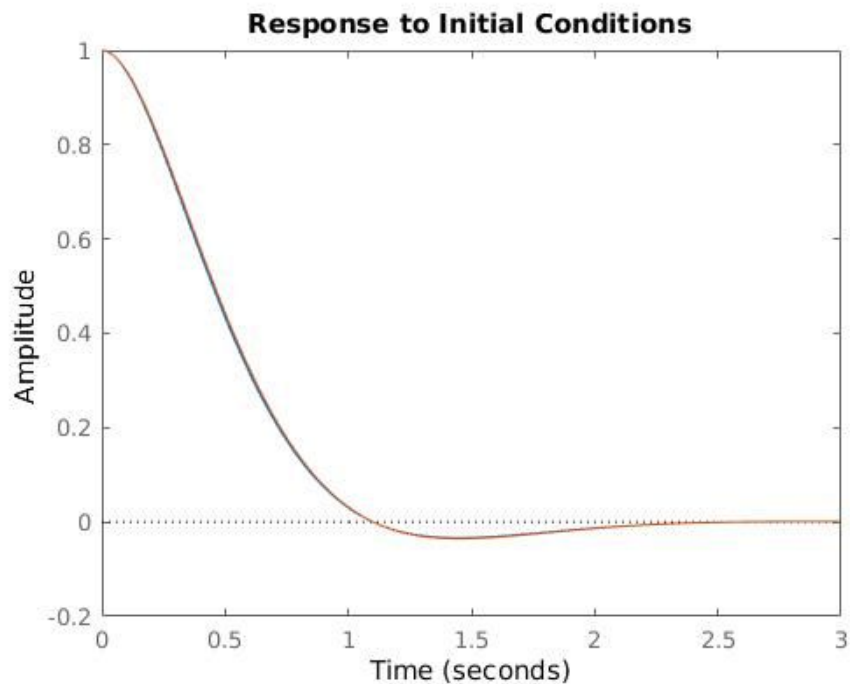
blue - equal

blue- different weighting for  
state\_variables

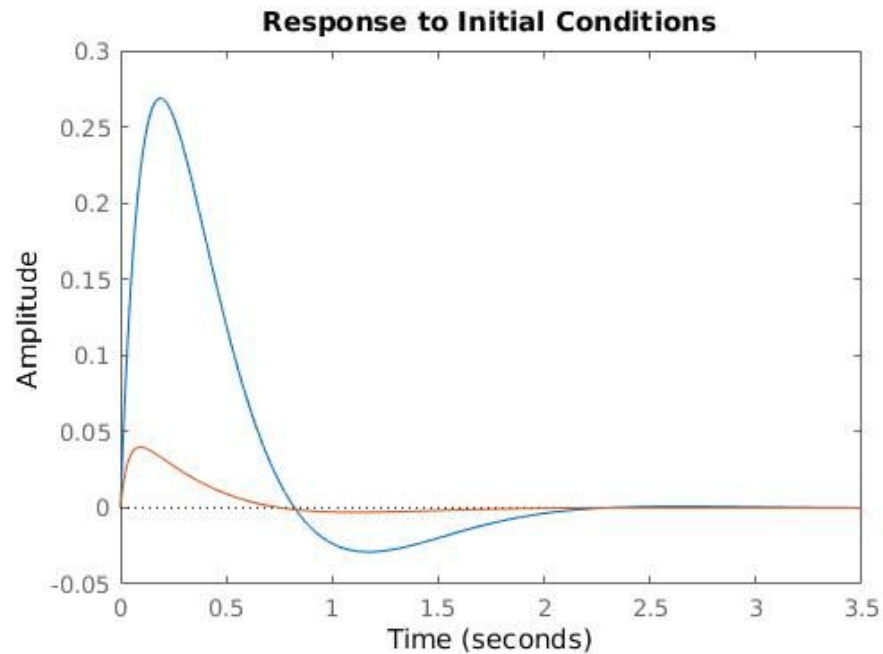


# Dexter LQR Design

**x1**



**xa**



# Future Work

## Dexter

- Implement the MIMO system on the raft. Determine how robust it is to the nonlinear damping of the surrounding fluid

## Baxter

- Determine how delays in system affect achievable range of impedances.

# Appendix: Series Elastic Actuators: Open loop model

Equations of motion

$$M(q)\ddot{q} + C\dot{q} = \tau_{measured} + \tau_{ext}$$

$$I_r\ddot{\theta} + D\dot{\theta} = \tau_{motor} - \tau_{measured}$$

$$\tau_{measured} = K_{SEA}(\theta - q)$$

State space representation

$$\frac{d}{dt} \begin{bmatrix} q \\ \dot{q} \\ \theta \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & I & 0 & 0 \\ -M^{-1}K_{SEA} & -M^{-1}C & M^{-1}K_{SEA} & 0 \\ 0 & 0 & 0 & I \\ I_r^{-1}K_{SEA} & 0 & -I_r^{-1}K_{SEA} & -I_r^{-1}D \end{bmatrix} \begin{bmatrix} q \\ \dot{q} \\ \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ I_r^{-1} \end{bmatrix} \tau_{motor}$$

$$\dot{x} = \begin{bmatrix} 0 & I & 0 & 0 \end{bmatrix} \begin{bmatrix} q \\ \dot{q} \\ \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} \tau_{motor}$$

# Appendix: Baxter Research Robot: Real System

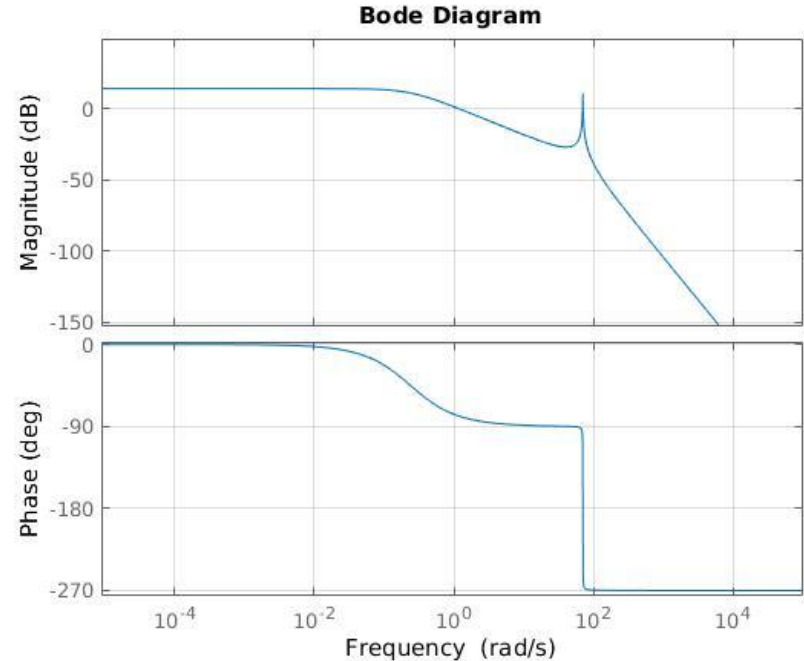
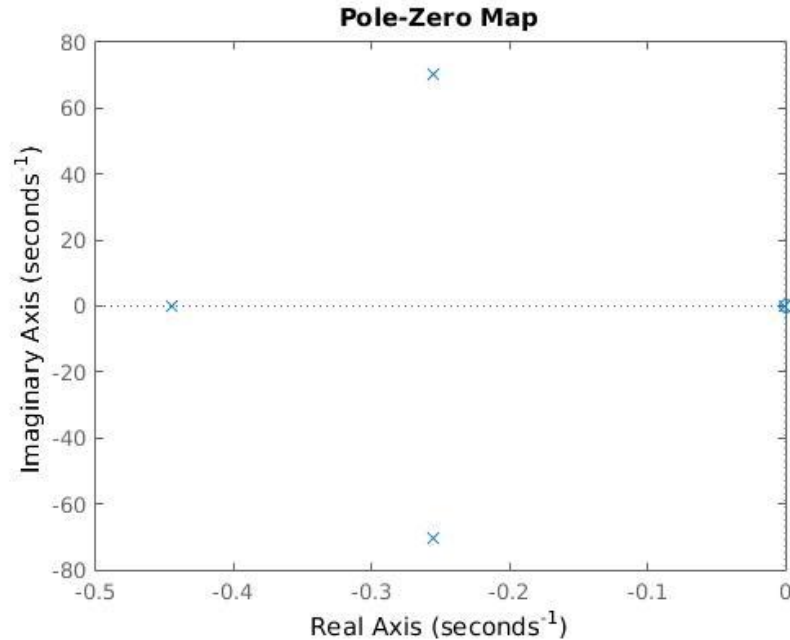
Can achieve a fairly broad range of impedances using this approach.

There is a limit to how high the gains can be turned up that is not accounted for in this “simplified” model.

two likely reasons:

1. Delay in the system→ not a “state space” problem
2. The model does not account for all the dynamics of the system

# Appendix: Single SEA



- Acts like a second order model until about 100 rad/s (far above Baxter's operating frequency)
- The second pair of poles not accounted for in the simplified model can still be driven into the RHP at high gains

# Appendix: Series Elastic Actuators: Closed loop model

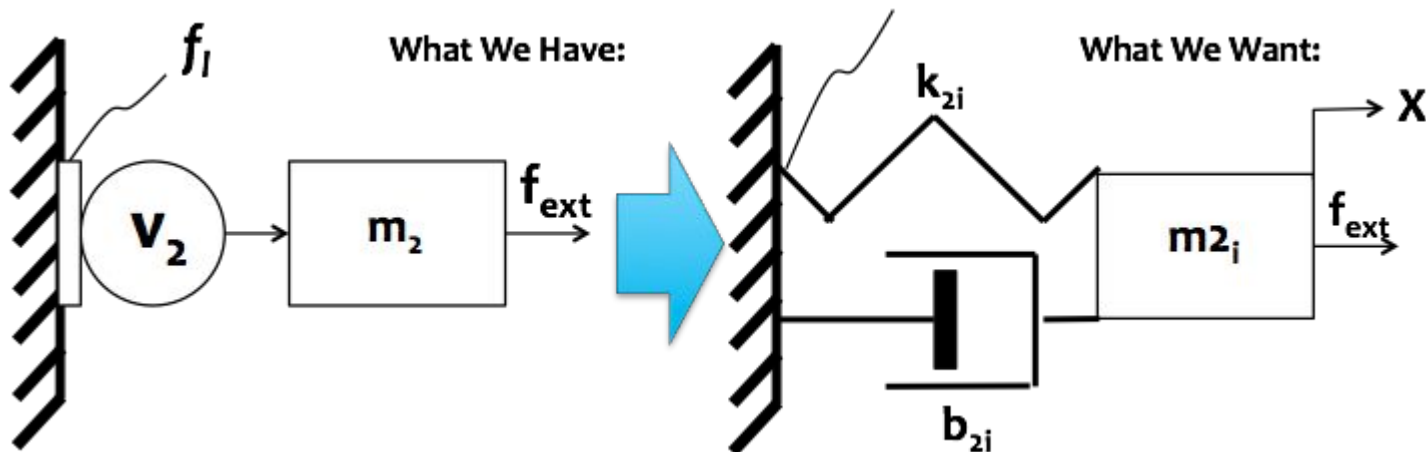
$$\frac{d}{dt} \begin{bmatrix} q \\ \dot{q} \\ \theta \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & I & 0 & 0 \\ -M^{-1}K_{SEA} & -M^{-1}C & M^{-1}K_{SEA} & 0 \\ 0 & 0 & 0 & I \\ I_r^{-1}K_{SEA} - I_r^{-1}J^TKJ & -I_r^{-1}J^TBJ & -I_r^{-1}K_{SEA} & -I_r^{-1}D \end{bmatrix} \begin{bmatrix} q \\ \dot{q} \\ \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ I_r^{-1}J^TKJ \end{bmatrix} q_d$$

$$\dot{x} = \begin{bmatrix} 0 & J & 0 & 0 \end{bmatrix} \begin{bmatrix} q \\ \dot{q} \\ \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} q_d$$



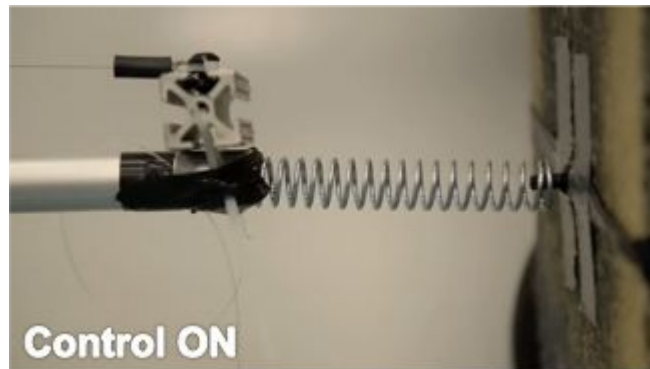
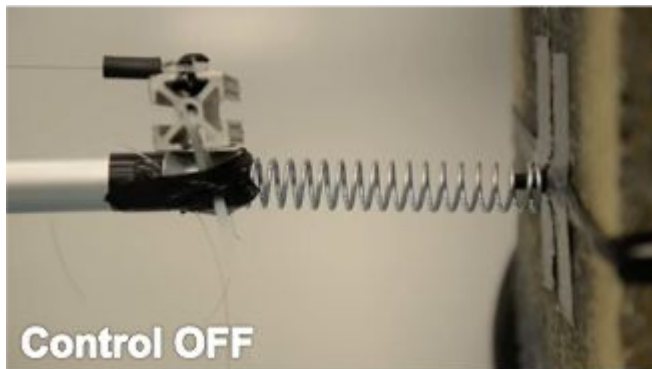
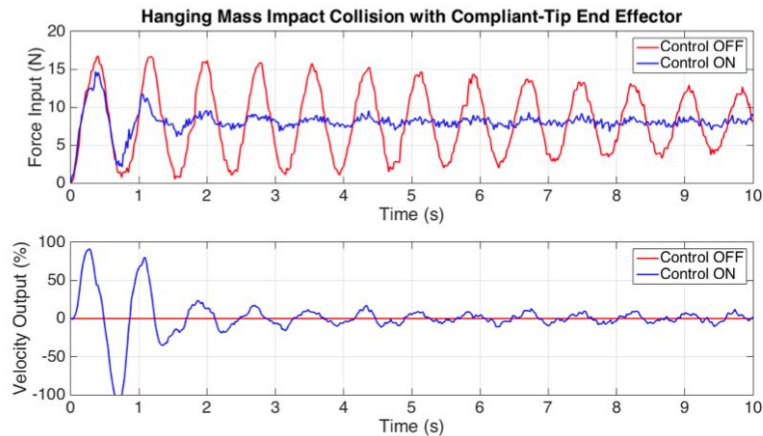
# Turning a Non-Backdrivable Linear Actuator Into a Passive System

- Creating a controller for the linear actuator based desired impedance parameters allows for:
  - A closed-loop system that promotes robust and stable interactions due to passivity
  - Achievable design constraints (e.g., Peak impact force, Settling time)
  - Definitive impedance values for Baxter to reproduce

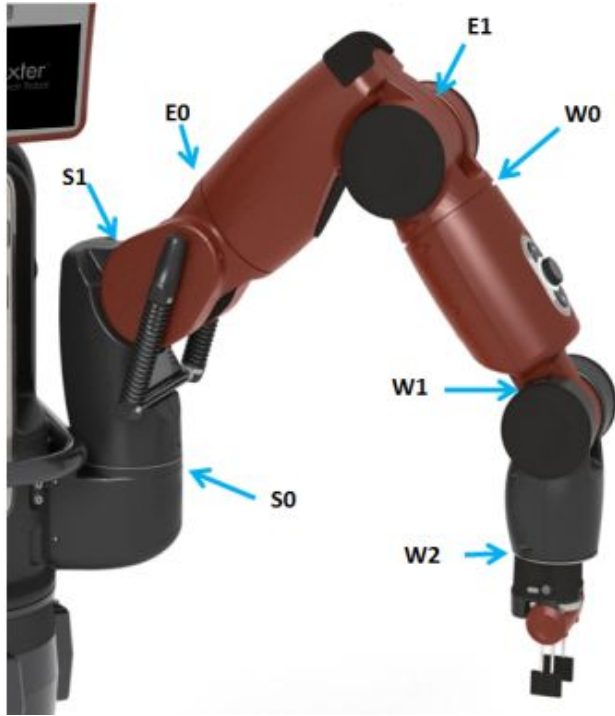


# Determining Desirable Impedance Characteristics

- A **reduction in peak force** provides a buffer for a controller to react to any unforeseen collisions
  - Alleviates significant risks for valuable equipment on hard-to-reach AUVs
- A **sharp reduction in settling time** suggests a smaller influence by external disturbances, such as underwater currents
  - Minimizes unpredictable disturbances during any on going contact tasks
- **Feed impedance values to Baxter!**



# Baxter Research Robot



For the purposes of the experiment, Baxter was constrained to planar motion by only allowing joints S1, E1, and W1 to move.

# Baxter Research Robot: Open loop model

Neglecting SEA dynamics, Equations of Motion

$$(M(q) + I_r)\ddot{q} + C(\dot{q}, q) + D\dot{q} + g(q) = \tau_{motors}$$

Simplifies to

$$(M(q) + I_r)\ddot{q} + D\dot{q} = \tau_{motors}$$

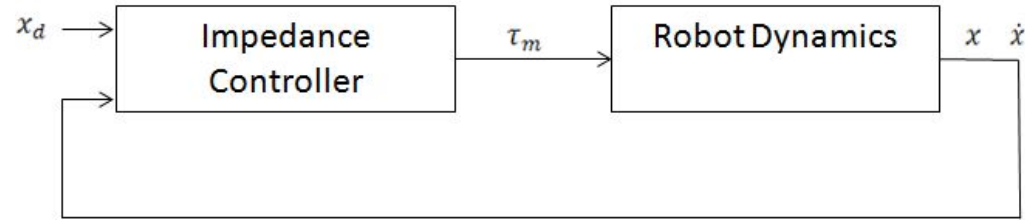
State space representation

$$\frac{d}{dt} \begin{bmatrix} q \\ \dot{q} \end{bmatrix} = \begin{bmatrix} 0 & I \\ 0 & -(M(q) + I_r)^{-1}D \end{bmatrix} \begin{bmatrix} q \\ \dot{q} \end{bmatrix} + \begin{bmatrix} 0 \\ (M(q) + I_r)^{-1} \end{bmatrix} \tau_{motors}$$

$$\dot{x} = [0 \ I] \begin{bmatrix} q \\ \dot{q} \end{bmatrix} + [0] \tau_{motors}$$



# Baxter Research Robot: Closed loop model

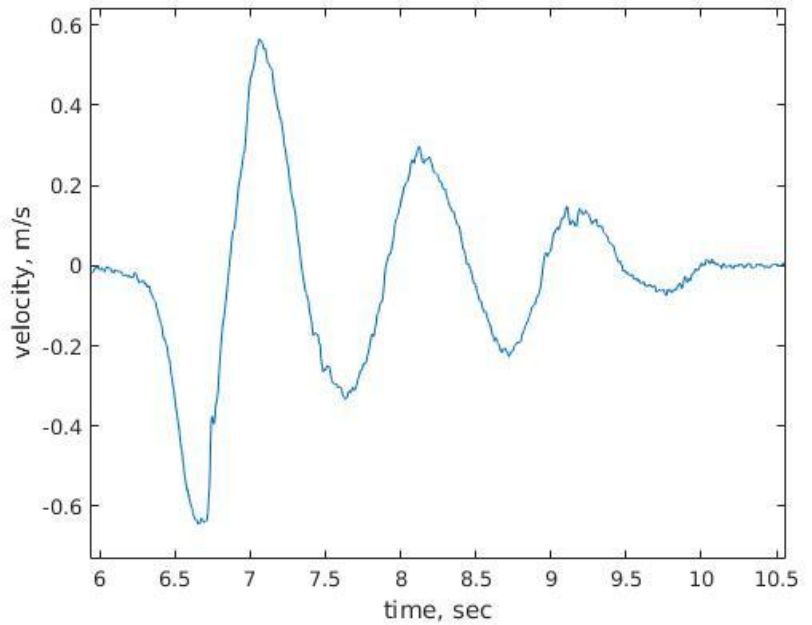
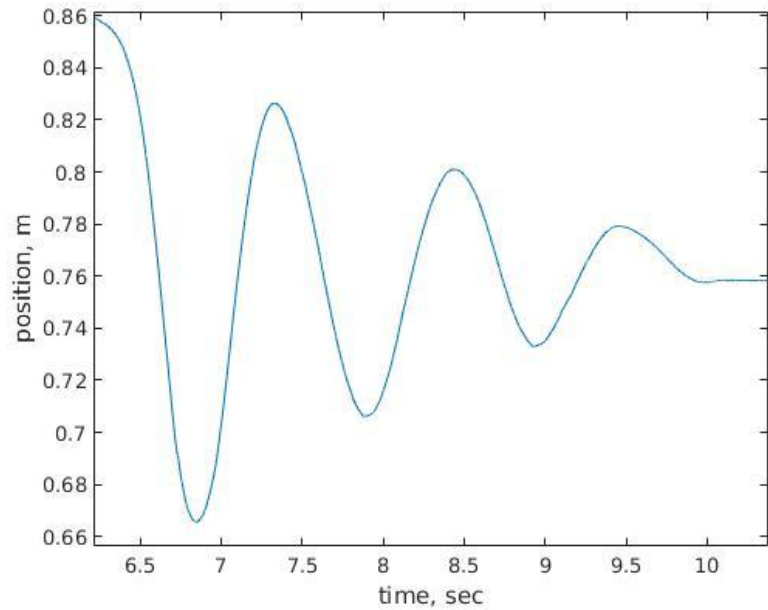


$$F_{motor} = K(x_d - x) - B\dot{x} \quad \tau_{motor} = J^T(F_{motor})$$

$$\frac{d}{dt} \begin{bmatrix} q \\ \dot{q} \end{bmatrix} = \begin{bmatrix} 0 & I \\ -(M(q) + I_r)^{-1} J^T K J & -(M(q) + I_r)^{-1} (D + J^T B J) \end{bmatrix} \begin{bmatrix} q \\ \dot{q} \end{bmatrix} + \begin{bmatrix} 0 \\ (M(q) + I_r)^{-1} J^T K J \end{bmatrix} q_d$$

$$\dot{x} = [0 \ J] \begin{bmatrix} q \\ \dot{q} \end{bmatrix} + [0] q_d$$

# Baxter Research Robot: Real Data



# Baxter Research Robot: LQR Design

- What if we want to specify performance criteria for the states?
  - Max actuator force
  - Stroke Length
- Use LQR to optimize performance and control effort

$$\min_{\mathbf{u}(t)} V = \int_{\tau=t}^{\tau=T} (\mathbf{x}^t \mathbf{Q} \mathbf{x} + \mathbf{u}^t \mathbf{R} \mathbf{u}) d\tau$$

# Baxter Research Robot: LQR Design

- Bryson's Rule

$$Q = \begin{bmatrix} \frac{\alpha_1^2}{(x_1)_{\max}^2} & & & \\ & \frac{\alpha_2^2}{(x_2)_{\max}^2} & & \\ & & \ddots & \\ & & & \frac{\alpha_n^2}{(x_n)_{\max}^2} \end{bmatrix}$$

$$R = \rho \begin{bmatrix} \frac{\beta_1^2}{(u_1)_{\max}^2} & & & \\ & \frac{\beta_2^2}{(u_2)_{\max}^2} & & \\ & & \ddots & \\ & & & \frac{\beta_m^2}{(u_m)_{\max}^2} \end{bmatrix}$$

- Baxter system criteria:
  - $x_{1\max}$  (x position) = 0.5214 m
  - $u_{1\max}$  (shoulder torque) = 50 Nm
  - $u_{2\max}$  (elbow torque) = 15 Nm
  - $u_{3\max}$  (wrist torque) = 15 Nm



# Baxter Research Robot: LQR Design

Q =

3.6787	0	0	0	0	0
0	0.0100	0	0	0	0
0	0	0.0100	0	0	0
0	0	0	0.0100	0	0
0	0	0	0	0.0100	0
0	0	0	0	0	0.0100

## Expensive vs Cheap Control

$p = 1$  (equal weighting)

$p = 1e6$  (expensive control)

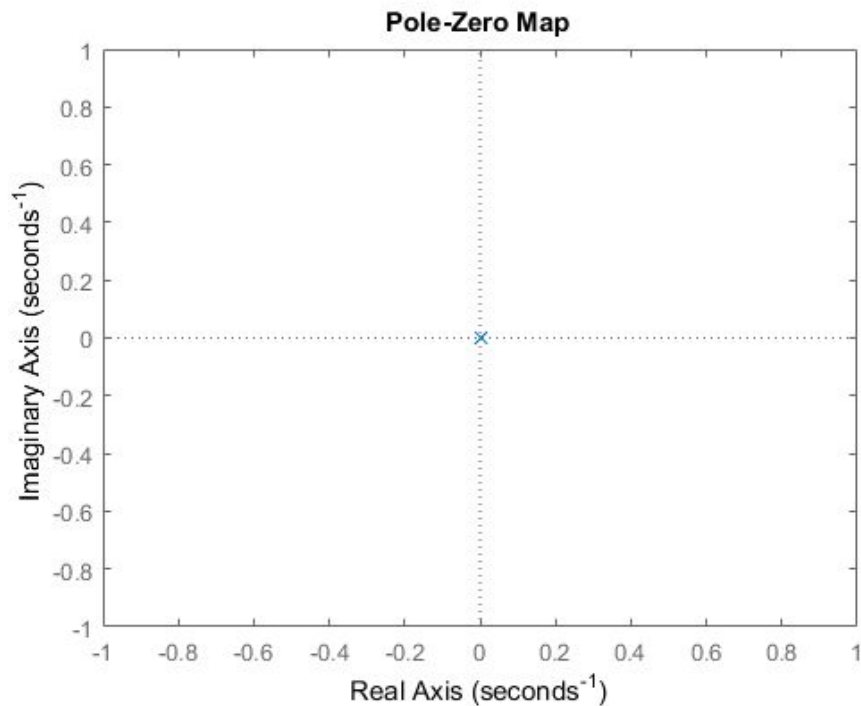
$p = 1e-6$  (cheap control)

R =

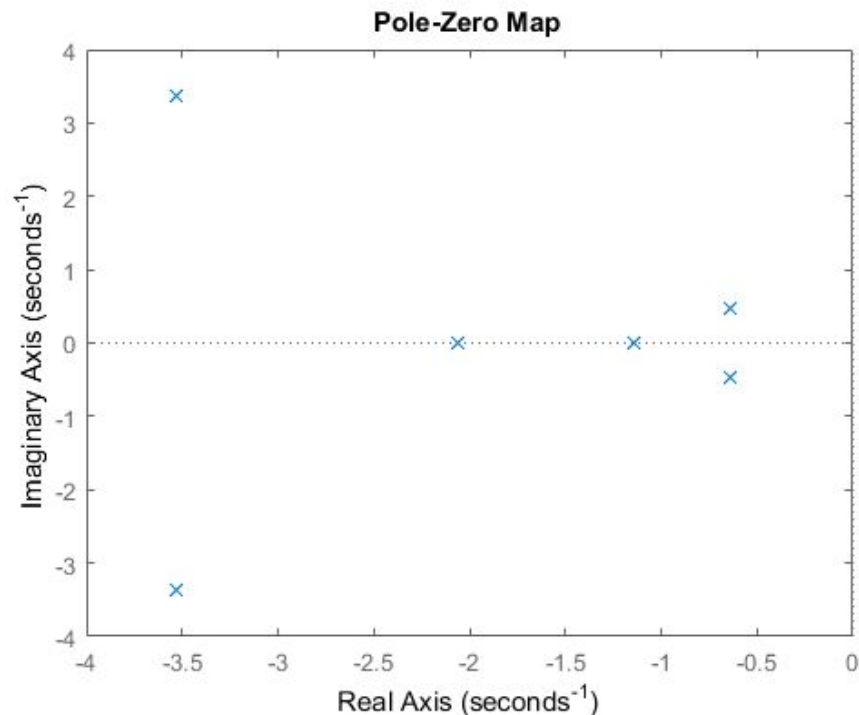
0.0004	0	0
0	0.0044	0
0	0	0.0044

# Baxter Research Robot: LQR Design

Open-Loop

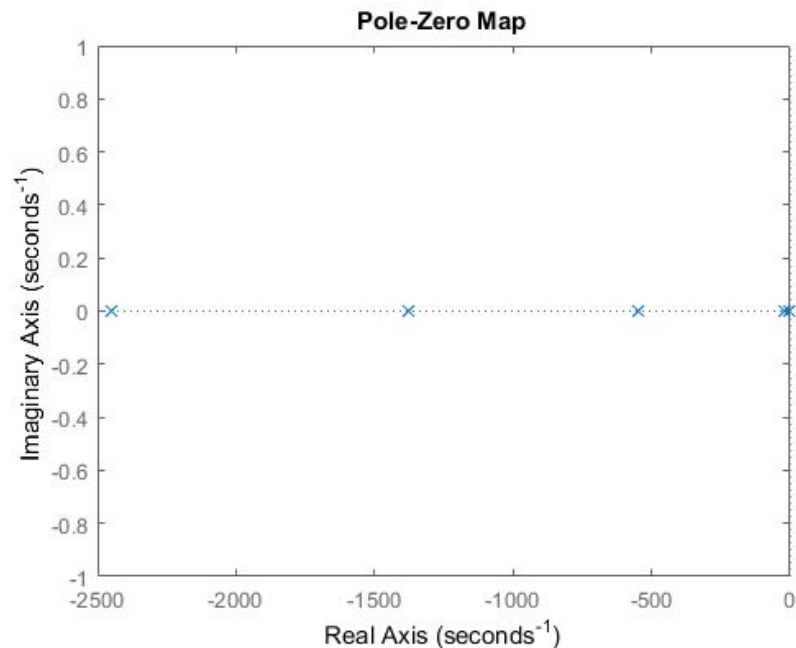


Closed Loop: Equal Weighting

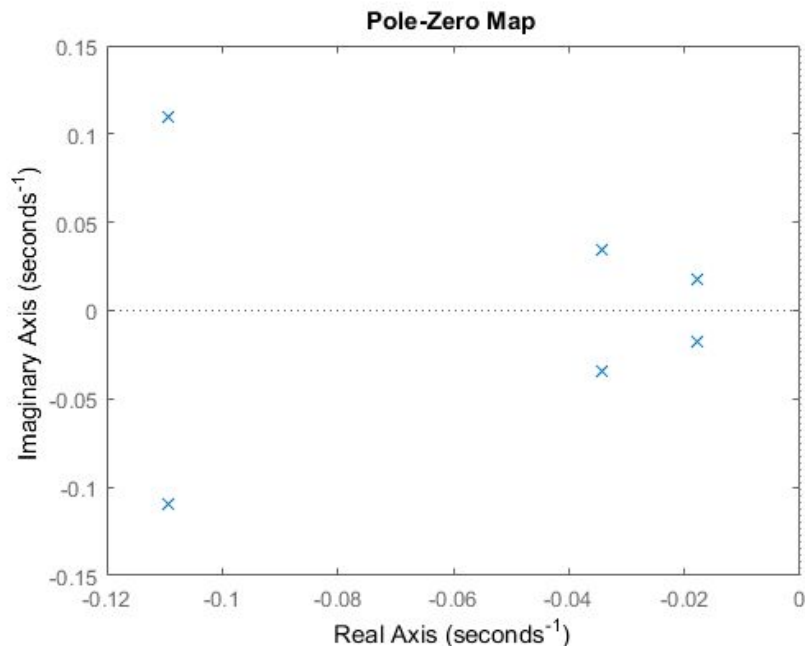


# Baxter Research Robot: LQR Design

Closed Loop: Cheap



Closed Loop: Expensive

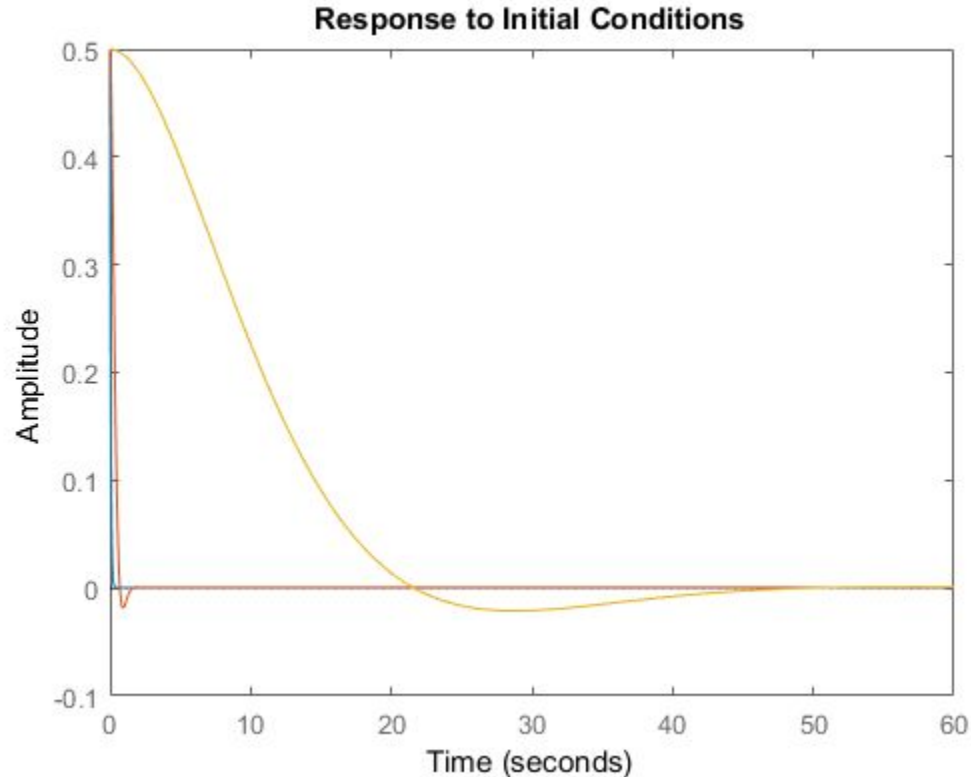


# Baxter Research Robot: LQR Design

blue - cheap

orange - expensive

yellow - equal



# Baxter Research Robot: LQR Discussion

- Expensive control is realistic given good performance results and less control (actuator effort)
- Choosing relative weighting of states/inputs may be significant
- Can we design performance criteria around impact collision response?
- Can we combine impedance into cost function for LQR design?

# Dexter LQR Design

Criteria:

Max Thrust=10 Nm

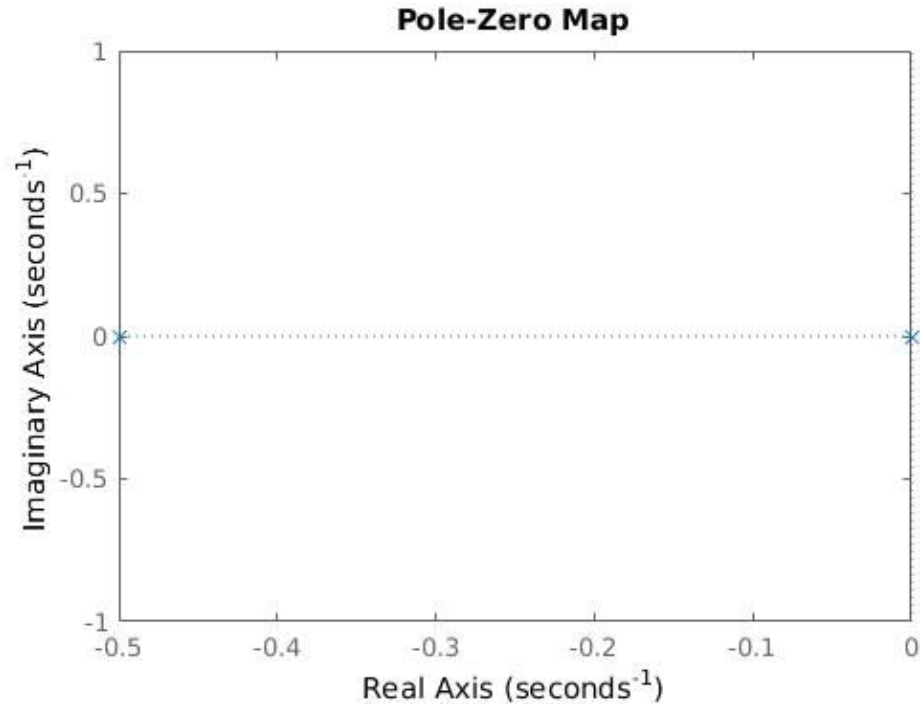
Max Stroke length=10 in

Different Designs:

- Cheap
- Expensive
- Equal
- Weighting  $x_a$  vs  $x_1$

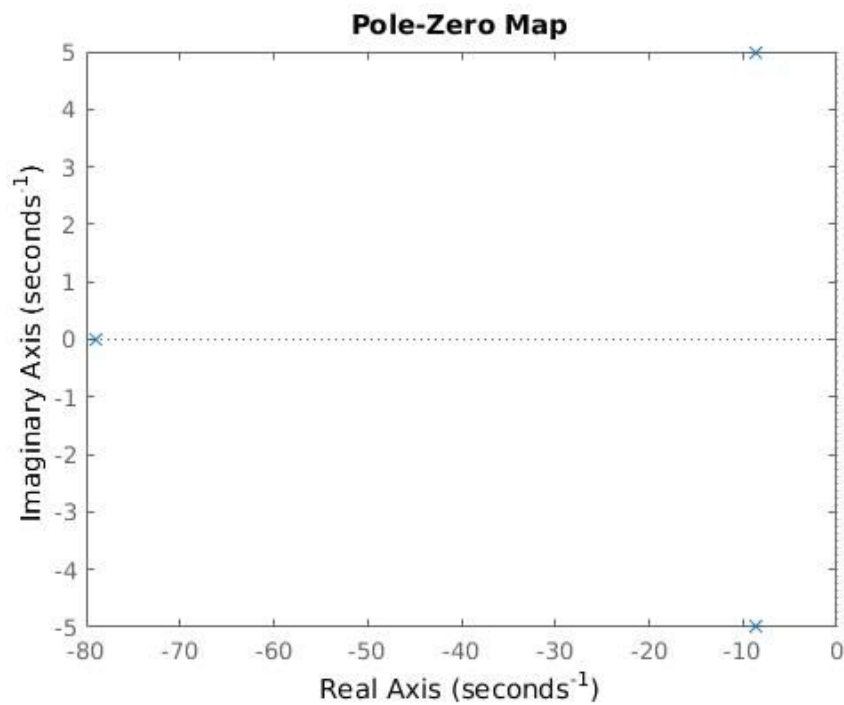
# Dexter LQR Design

Open-Loop

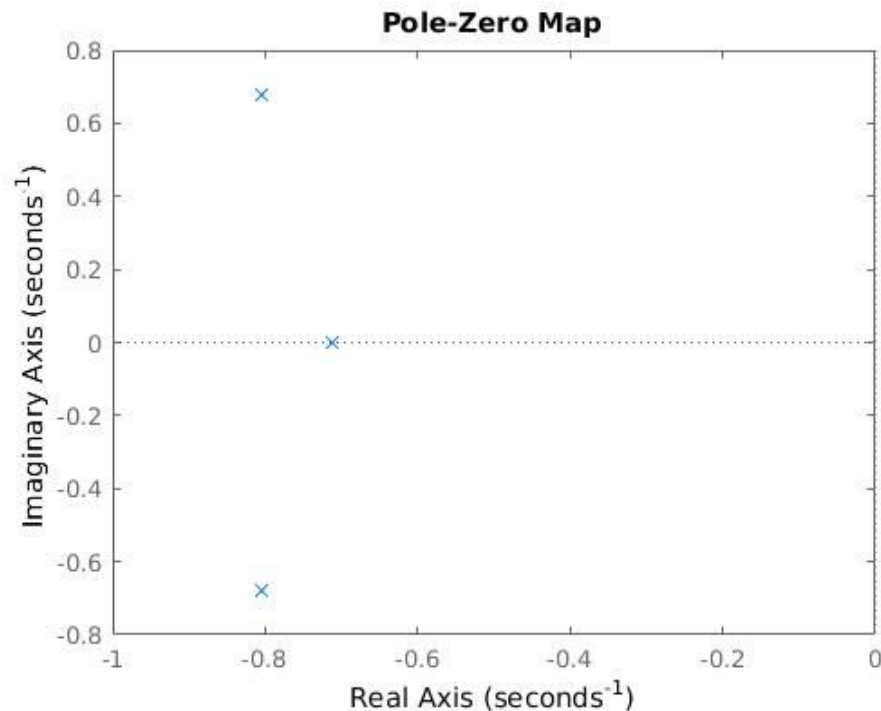


# Dexter LQR Design

Cheap



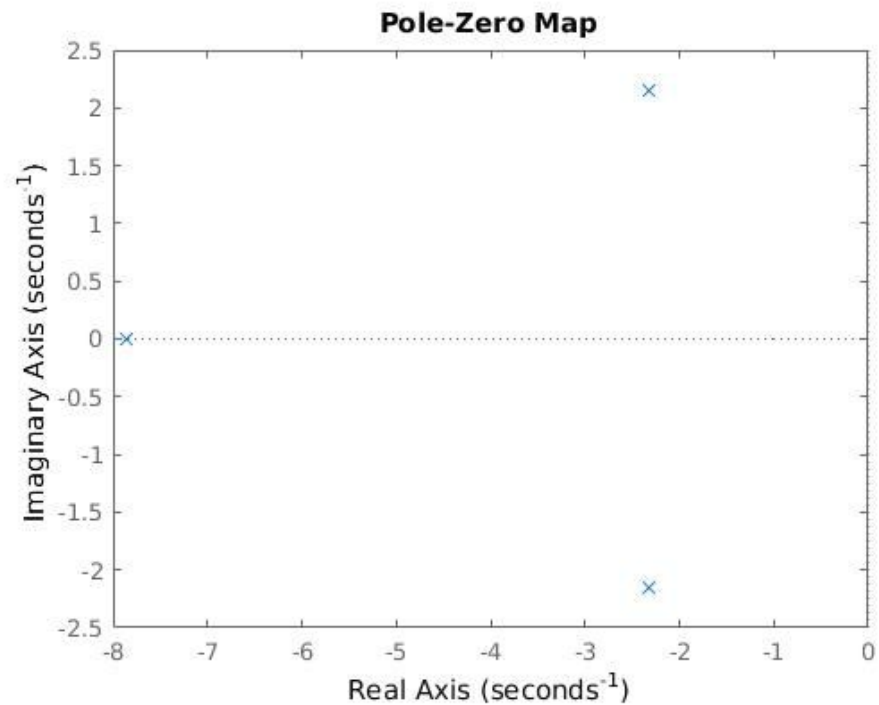
Expensive



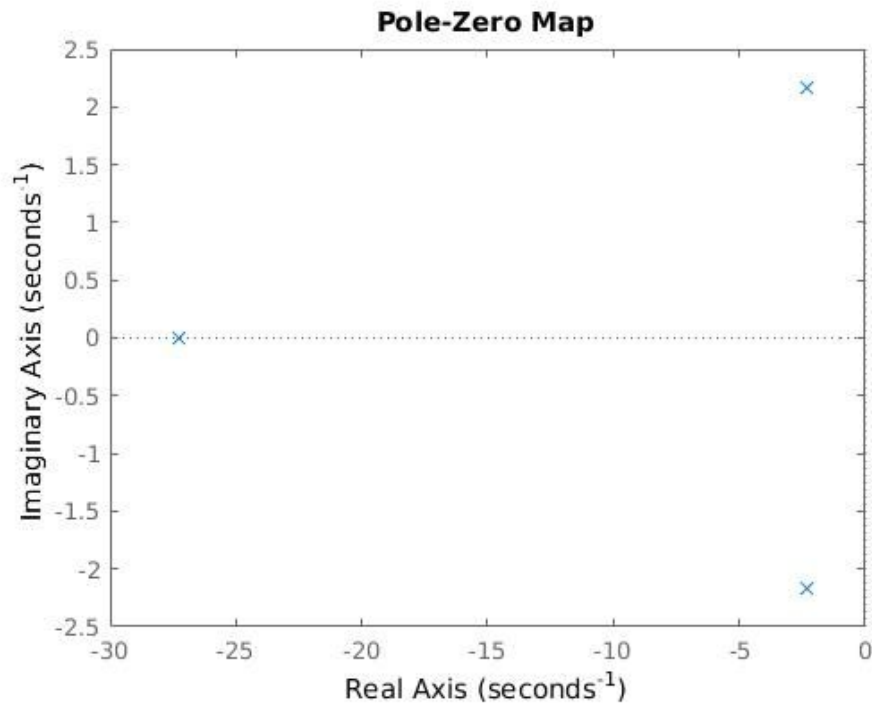


# Dexter LQR Design

Equal



Diff Weights



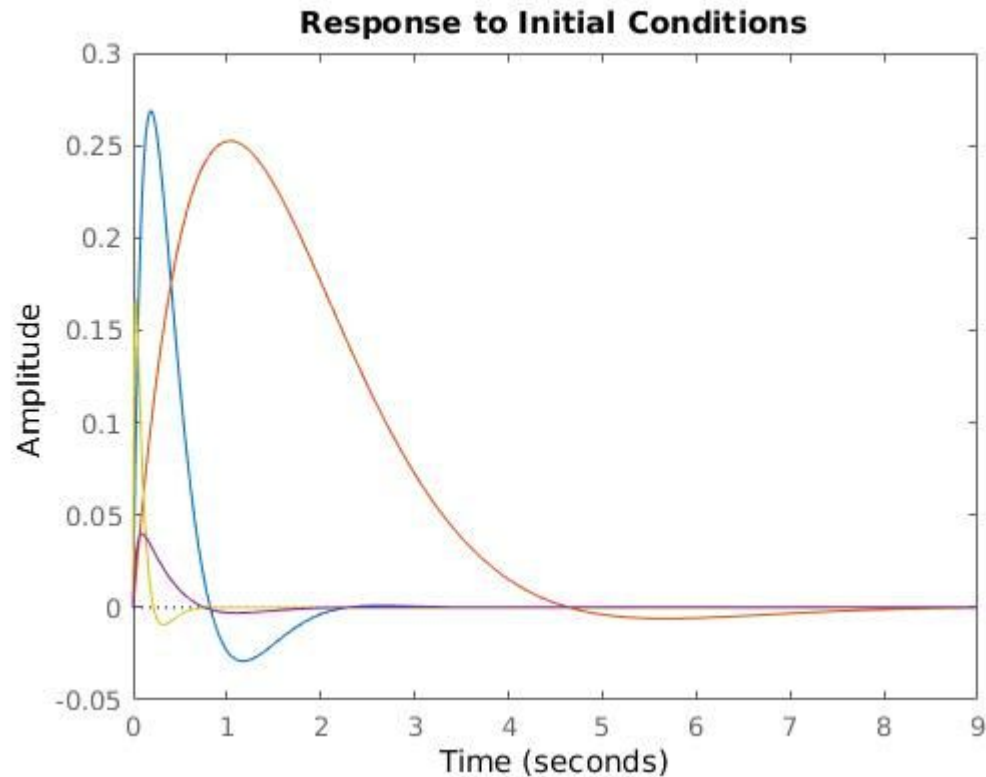
# Dexter LQR Design

yellow - cheap

red - expensive

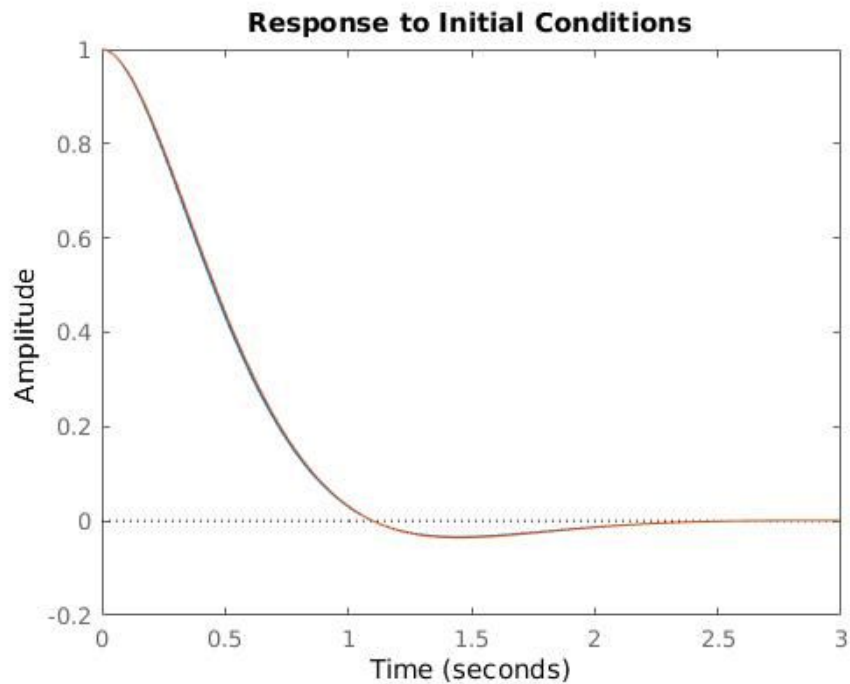
blue - equal

blue- different weighting for  
state\_variables

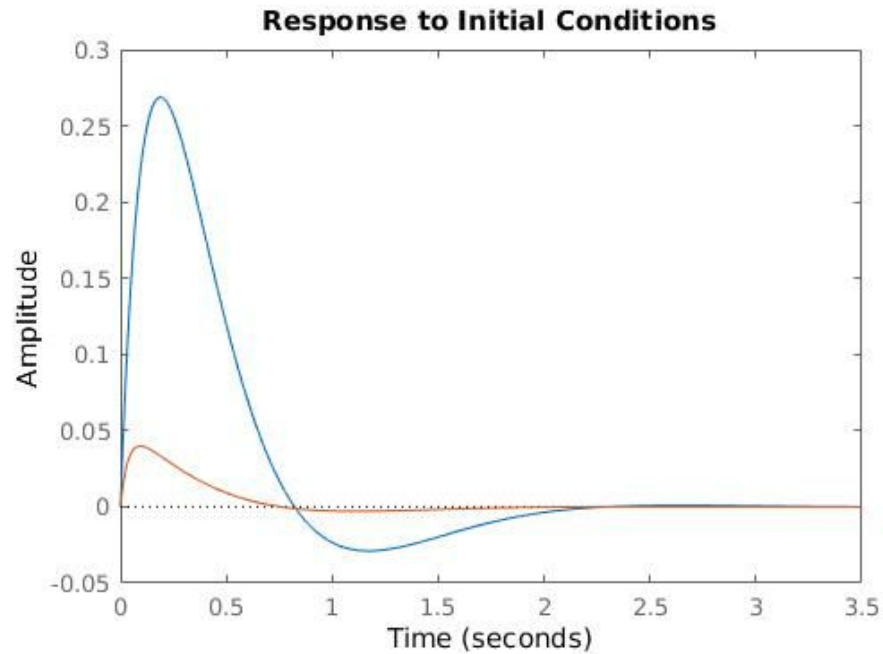


# Dexter LQR Design

**x1**



**xa**



# Future Work

## Dexter

- Implement the MIMO system on the raft. Determine how robust it is to the nonlinear damping of the surrounding fluid

## Baxter

- Determine how delays in system affect achievable range of impedances.

# Appendix: Series Elastic Actuators: Open loop model

Equations of motion

$$M(q)\ddot{q} + C\dot{q} = \tau_{measured} + \tau_{ext}$$

$$I_r\ddot{\theta} + D\dot{\theta} = \tau_{motor} - \tau_{measured}$$

$$\tau_{measured} = K_{SEA}(\theta - q)$$

State space representation

$$\frac{d}{dt} \begin{bmatrix} q \\ \dot{q} \\ \theta \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & I & 0 & 0 \\ -M^{-1}K_{SEA} & -M^{-1}C & M^{-1}K_{SEA} & 0 \\ 0 & 0 & 0 & I \\ I_r^{-1}K_{SEA} & 0 & -I_r^{-1}K_{SEA} & -I_r^{-1}D \end{bmatrix} \begin{bmatrix} q \\ \dot{q} \\ \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ I_r^{-1} \end{bmatrix} \tau_{motor}$$

$$\dot{x} = \begin{bmatrix} 0 & I & 0 & 0 \end{bmatrix} \begin{bmatrix} q \\ \dot{q} \\ \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} \tau_{motor}$$

# Appendix: Baxter Research Robot: Real System

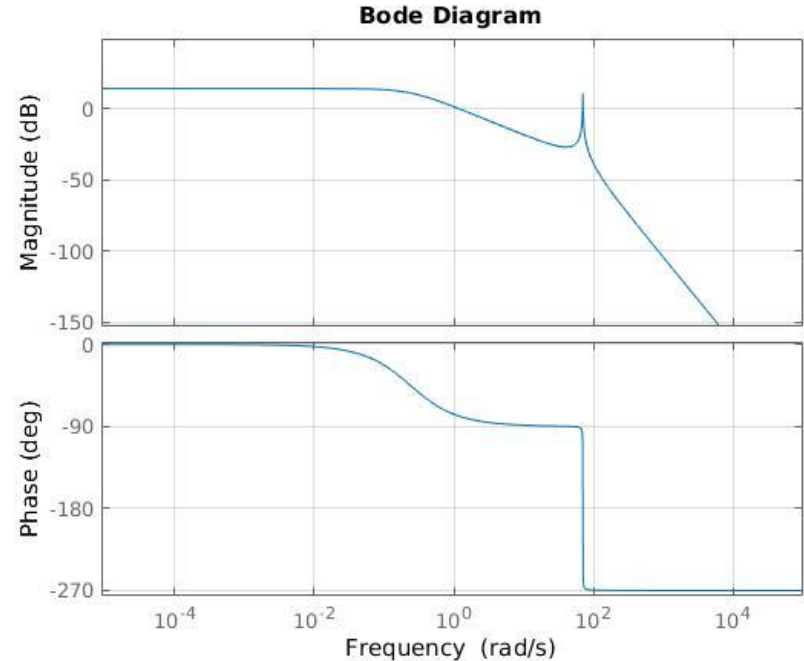
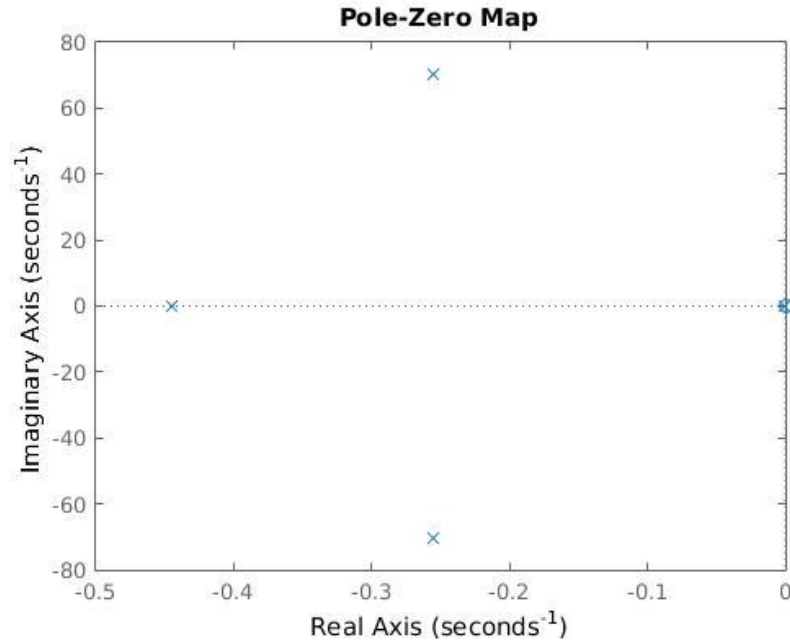
Can achieve a fairly broad range of impedances using this approach.

There is a limit to how high the gains can be turned up that is not accounted for in this “simplified” model.

two likely reasons:

1. Delay in the system → not a “state space” problem
2. The model does not account for all the dynamics of the system

# Appendix: Single SEA



- Acts like a second order model until about 100 rad/s (far above Baxter's operating frequency)
- The second pair of poles not accounted for in the simplified model can still be driven into the RHP at high gains

# Appendix: Series Elastic Actuators: Closed loop model

$$\frac{d}{dt} \begin{bmatrix} q \\ \dot{q} \\ \theta \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & I & 0 & 0 \\ -M^{-1}K_{SEA} & -M^{-1}C & M^{-1}K_{SEA} & 0 \\ 0 & 0 & 0 & I \\ I_r^{-1}K_{SEA} - I_r^{-1}J^TKJ & -I_r^{-1}J^TBJ & -I_r^{-1}K_{SEA} & -I_r^{-1}D \end{bmatrix} \begin{bmatrix} q \\ \dot{q} \\ \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ I_r^{-1}J^TKJ \end{bmatrix} q_d$$

$$\dot{x} = \begin{bmatrix} 0 & J & 0 & 0 \end{bmatrix} \begin{bmatrix} q \\ \dot{q} \\ \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} q_d$$