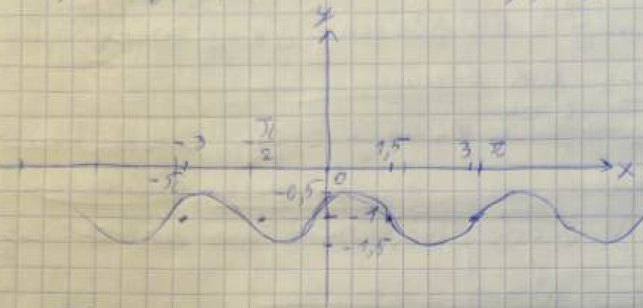
Дз-2 на баллы Казаченко Даниил

Вариант 14

№1

а) $y = 0,5\sin(3-2x) - 1$ $D(y) = (-\infty; +\infty)$

$y = 0,5\sin(-2(x-1,5)) - 1$ $E(y) = [-1,5; -0,5]$



Графиком функции $y = f(x)$ является график функции $y = \sin x$, смещенный на 1 вниз и на 1,5 влево, далее отзеркаленный относительно оси OY. Сжатый к прямой $y = -1$ в 2 раза (т.к. коэфф при $\sin = 0,5$) и к прямой $x = 1,5$ в ~~к. коэф~~ 2 раза (т.к. коэфф. при $x = 2$)

б)

$$y = -\left|\frac{3-|x-1|}{\sqrt{(3-3x)^2}-2}\right|$$

$$y = -\left|\frac{3-|x-1|}{3|x-1|-2}\right|$$

$$y = \begin{cases} -\left|\frac{3-x+1}{3x-3-2}\right|, & x \geq 1 \\ -\left|\frac{3-1+x}{3-3x-2}\right|, & x < 1 \end{cases}$$

$$y = \begin{cases} -\left|\frac{4-x}{3x-5}\right|, & x \geq 1 \\ -\left|\frac{2+x}{1-3x}\right|, & x < 1 \end{cases}$$

$$\begin{array}{r|l} -x+4 & 3x-5 \\ \hline -x+\frac{5}{3} & -\frac{1}{3} \\ \hline \frac{7}{3} & \end{array}$$

$$\begin{array}{r|l} x+2 & -3x+1 \\ \hline x-\frac{1}{3} & -\frac{1}{3} \\ \hline \frac{7}{3} & \end{array}$$

$D(y) = \left(-\infty; \frac{1}{3}\right) \cup \left(\frac{1}{3}; \frac{5}{3}\right) \cup \left(\frac{5}{3}; +\infty\right)$

$E(y) = (-\infty; 0]$

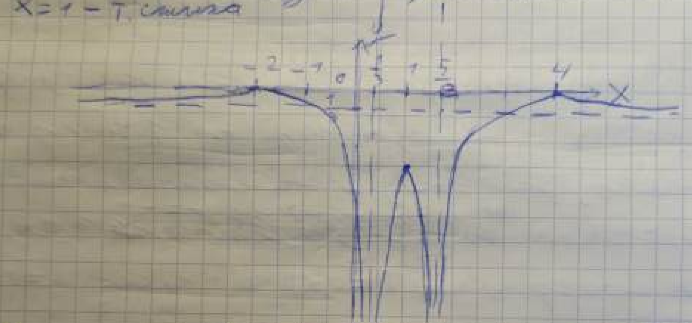
$$y = \begin{cases} -\left|-\frac{1}{3} + \frac{7}{9\left(x-\frac{5}{3}\right)}\right|, & x \geq 1 \quad (1) \\ -\left|-\frac{1}{3} + \frac{7}{9\left(\frac{1}{3}-x\right)}\right|, & x < 1 \quad (2) \end{cases}$$

· Графиком функции $f(x)$ будет график фун-ии (1) для $x \geq 1$ и (2) для $x < 1$.

· График (1) - график функции $y = \frac{1}{9x}$, смещённый вниз на $\frac{1}{3}$ и вправо на $\frac{5}{3}$ и отражённый относительно оси OX вниз.

· График (2) - график ф-ии $y = \frac{7}{3x}$, смещённый вниз на $\frac{7}{3}$ и влево на $\frac{1}{3}$, отзеркаленный относительно оси Oy и отражённый отн-но оси OX вниз.

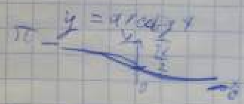
$x = 1$ — т. стыка



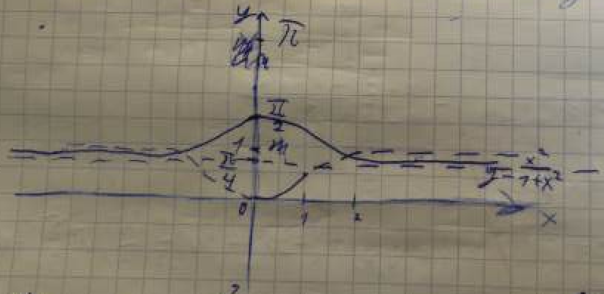
в) $y = \operatorname{arcctg} \frac{x^2}{1+x^2}$

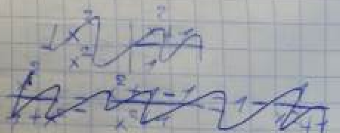
$D(y) = (-\infty; +\infty)$

$E(y) = (\frac{\pi}{4}; \frac{\pi}{2}]$



$y(x) = y(-x) \Rightarrow$ фун-я чётная $\Rightarrow$ ∂ ф-ию для $x \geq 0$ и симм отразим относительно оси Oy.





· Фун-я $y = \frac{x^2}{1+x^2}$ · монотонно ~~убыв~~ возраст. при $x \geq 0$.

· Построим график искомой ф-ии, ~~[illegible]~~ через график ф-ии $y = \operatorname{arcctg} x$, но вместо x возьмём значения ф-ии $y = \frac{x^2}{1+x^2}$ в соотв. точках

№ 2.

$$\lim_{x \to \frac{5}{2}} \frac{2x^2-9x+10}{2x-5} = \frac{1}{2} \Leftrightarrow \forall \varepsilon > 0 \ \exists \delta = \delta(\varepsilon): |x-a| < \delta \Rightarrow |f(x)-A| < \varepsilon$$

$$\cancel{\lim_{x \to a} f(x) = a \varepsilon \exists}$$

$$\left|\frac{2x^2-9x+10}{2x-5} - \frac{1}{2}\right| < \varepsilon$$

$$\begin{array}{r|l} 2x^2-9x+10 & 2x-5 \\ \hline -\ 2x^2-5x & x-2 \\ -\ -4x+10 & \\ -4x+10 & \\ \hline 0 & \end{array}$$

$$\begin{cases} x \neq \frac{5}{2} \\ \left|x-2-\frac{1}{2}\right| < \varepsilon \end{cases}$$

$$\left|x-\frac{5}{2}\right| < \varepsilon = \delta$$

$$\cancel{|x - \frac{5}{2}| < \varepsilon} \quad \delta = \varepsilon \Rightarrow \cancel{\forall \varepsilon} \ \forall \varepsilon > 0 \ \exists \delta = \delta(\varepsilon): \left|x-\frac{5}{2}\right| < \delta$$

$$\Downarrow$$

$$\left|f(x) - \frac{1}{2}\right| < \varepsilon$$

ч.т.д.

№ 3.

$$\lim_{x \to 0} ctg\left(\frac{1}{x^2}\right) \qquad f(x) = ctg\frac{1}{x^2}$$

$$\text{Ǝ } x_n = \sqrt{\frac{1}{\pi n + \frac{\pi}{4}}} \qquad x_n' = \sqrt{\frac{1}{\pi n + \frac{\pi}{3}}}$$

$$\{x_n\} \to 0 \qquad f(x_n) = ctg\left(\pi n + \frac{\pi}{4}\right) \qquad \lim_{n \to \infty} ctg\left(\pi n + \frac{\pi}{4}\right) = 1$$

$$\{x_n'\} \to 0 \qquad f(x_n') = ctg\left(\pi n + \frac{\pi}{3}\right) \qquad \lim_{n \to \infty} ctg\left(\pi n + \frac{\pi}{3}\right) = \frac{\sqrt{3}}{3}$$

$$\lim_{n \to \infty} f(x_n) \neq \lim_{n \to \infty} f(x_n') \Rightarrow \lim_{x \to 0} f(x) \ \nexists$$

ч.т.д.

№ 1

$$1)\ \lim_{x \to 3} \frac{x^3-4x^2-3x+18}{x^3-5x^2+3x+9} = \lim_{x \to 3} \frac{(x-3)(x^2-x-6)}{(x-3)(x^2-2x-3)} = \lim_{x \to 3} \frac{\cancel{(x-3)}(x+2)}{\cancel{(x-3)}(x+1)} = \frac{5}{4}$$

	1	−4	−3	18
3	1	−1	−6	0
3	1	2	0	

	1	−5	3	9
3	1	−2	−3	0
3	1	1	0	

№ 2

$$2)\ \lim_{x \to 8} \frac{\sqrt{9+2x}-5}{\sqrt[3]{x^2}-4} = \lim_{x \to 8} \frac{2x-16}{(\sqrt[3]{x^2}-4)(\sqrt{9+2x}+5)} = \lim_{x \to 8} \frac{2(x-8)\left((\sqrt[3]{x^2})^2+4\sqrt[3]{x^2}+16\right)}{(x^2-64)(\sqrt{9+2x}+5)} =$$

$$= \lim_{x\to 8} \frac{2\left(\sqrt[3]{x^4} + 4\sqrt[3]{x^2} + 16\right)}{(x+8)\left(\sqrt{9+2x}+5\right)} = \frac{2(16+16+16)}{16 \cdot 10} = 0{,}6$$

$$3)\ \lim_{x\to 1} \frac{\sqrt{x^2-3x+3}-1}{\sin \pi x} = \left[\frac{0}{0}\right] = \lim_{x\to 1} \frac{2x-3}{2\sqrt{x^2-3x+3}\cdot \pi\cos(\pi x)} =$$

$$= \frac{-1}{2\cdot\pi(-1)} = \frac{1}{2\pi}$$

$$3)\ \lim_{x\to -1} \frac{tg(x+1)}{e^{\sqrt[3]{x^3-4x^2+6}}-e} = \lim_{t\to 0} \frac{tg(t)}{e^{\sqrt[3]{(t-1)^3-4(t-1)^2+6}}-e} =$$

$x=-1+t$

$t=x+1$

$t\to 0$

$$= \lim_{t\to 0} \frac{t}{e^{\sqrt[3]{t^3-3t^2+3t-1-4t^2+8t-4+6}}-e} =$$

$$= \lim_{t\to 0} \frac{t}{e^{\sqrt[3]{t^3-7t^2+11t+1}}-e} = \lim_{t\to 0}$$

$= e$

$$4)\ \lim_{x\to -1} \frac{tg(x+1)}{e^{\sqrt[3]{x^3-4x^2+6}}-e} = \lim_{x\to -1} \frac{3(x^3-4x^2+6)^{\frac{2}{3}}}{\cos^2(x+1)\left(e^{\sqrt[3]{x^3-4x^2+6}}\right)\cdot(3x^2-8x)} =$$

$$\left(\sqrt[3]{x^3-4x^2+6}\right)' = \frac{3x^2-8x}{3(x^3-4x^2+6)^{\frac{2}{3}}}$$

$$\operatorname{cosec} x = \frac{1}{\sin x}$$

$$= \lim_{x\to -1} \frac{3}{e\cdot 11} = \frac{3}{11e}$$

$$5)\ \lim_{x\to 0} (3-2\cos x)^{\operatorname{cosec}^2 x} = [1^\infty] = \lim_{x\to 0} (3-2\cos x)^{\frac{1}{\sin^2 x}} =$$

$$= \lim_{x\to 0} \left(1+(2-2\cos x)\right)^{\frac{1}{2-2\cos x}\left(\frac{2-2\cos x}{\sin^2 x}\right)} = e^{\lim\limits_{x\to 0} -\frac{2(\cos x+1)}{\sin^2 x}} =$$

$$= e^{\lim\limits_{x\to 0} -\frac{2x^2}{2x^2}} = e^{-1} = \frac{1}{e}$$

$$6)\ \lim_{x\to 1+0} \left(3e^{x-1}-1\right)^{x/(x-1)} = +\infty;\quad \lim_{x\to 1-0} \left(3e^{x-1}-1\right)^{\frac{x}{x-1}} = 0$$

$$\lim_{x\to 1} \left(3e^{x-1}-1\right) = 2 > 1 \quad \lim_{x\to 1+0} \frac{x}{x-1} = +\infty \qquad \lim_{x\to 1-0} \frac{x}{x-1} = -\infty$$

$$7)\ \lim_{x\to 1+0} \left(\frac{\cos(2\pi x)}{2+\left(e^{\sqrt{x-1}}-1\right)\sin\frac{x+2}{x-1}}\right) = \lim_{x\to 1+0} \frac{1}{2+0\cdot\left(\sin\frac{x+2}{x-1}\right)} = \frac{1}{2}$$

$$8)\ \lim_{x\to 1} \frac{6\arcsin\left(\frac{x^2}{2}\right)-\pi}{x^2+4x-5} = \lim_{x\to 1} \left(\frac{6\cdot x}{\sqrt{1-\left(\frac{x^2}{2}\right)^2}(2x+4)}\right) = \frac{6\cdot 2}{\sqrt{3}\cdot 6} = \frac{2\sqrt{3}}{3}$$

№5

а)

$$x_t = \frac{2t}{1-t^2}, \quad y_t = \frac{t^2}{1-t^2}$$

$$y'_x = \frac{y'_t}{x'_t} = \frac{\left(2t(1-t^2)-(-2t)t^2\right)(1-t^2)^2}{(1-t)^2\cdot\left(2(1-t^2)-(-2t)\cdot 2t\right)} = \frac{2t}{2t^2+2} = \frac{t}{t^2+1}$$

$2+2t^2+4t^3$

крит. т. $t=0$

$$\underset{}{\overset{-\quad\quad +}{\longrightarrow}}\ t \qquad (0)$$

$$y''_{xx} = \frac{(y'_x)'_t}{x'_t} = \frac{t^2+1-t(2t)}{(t^2+1)^2\cdot\left(2(1-t^2)-2t(-2t)\right)} = \frac{-t^2+1}{(t^2+1)^2\cdot 2(t^2+1)} = \frac{1-t^2}{2(t^2+1)^3}$$

$2t^2+2$

$t = \pm 1$

$-\quad +\quad -$; -1, 1 $\to t$

t	$(-\infty;-1)$	-1	$(-1;0)$	0	$(0;1)$	1	$(1;+\infty)$
x	$(0;+\infty)$	–	$(-\infty;0)$	0	$(0;+\infty)$	–	$(-\infty;0)$
y	$(-1;-\infty)$	–	$(+\infty;0)$	0	$(0;+\infty)$	–	$(-\infty;-1)$
y'_x	–		–		+		+
y''_{xx}	–		+		+		–
$y(x)$	↘ ∩		↘ ∪		↗ ∪		↗ ∩

верт. асимптот нет.

~~Верх.~~ Накл. асимптоты: $y = kx+b$

~~$k = \lim_{t\to\infty} \frac{y(t)}{x} = \lim_{t\to\infty} \frac{t^2(1-t^2)}{(1-t^2)(2t)} = \lim_{t\to\infty} \frac{t}{2} = \infty \Rightarrow$ накл. асимпт. нет~~

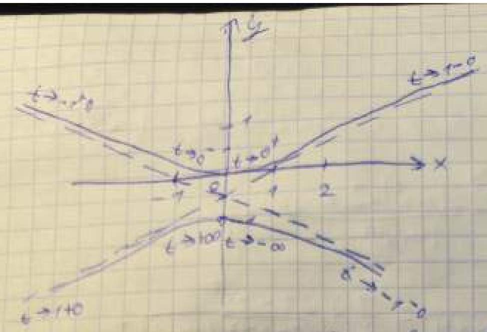
$$k_1 = \lim_{t\to -1} \frac{y(t)}{x} = \lim_{t\to -1} \frac{t^2(1-t^2)}{2t(1-t^2)} = \lim_{t\to -1} \frac{t}{2} = -\frac{1}{2};$$

$$k_2 = \lim_{t\to 1} \frac{y(t)}{x} = \frac{1}{2};$$

$$b_1 = \lim_{t\to -1}\left(y(t) - kx(t)\right) = \lim_{t\to -1}\left(\frac{t^2}{1-t^2} + \frac{1\cdot 2t}{2(1-t^2)}\right) =$$

$$= \lim_{t\to -1} \frac{2t(t+1)}{2(1-t)(1+t)} = -\frac{1}{2} \qquad y = -\frac{1}{2}x - \frac{1}{2} \text{ - 1 накл. ас.}$$

$$b_2 = \lim_{t\to 1}\left(y(t) - kx(t)\right) = \lim_{t\to 1} \frac{2t(t-1)}{2(1-t)(t+1)} = -\frac{1}{2} \quad y = \frac{1}{2}x - \frac{1}{2} \text{ - накл. асим.}$$



б) $y=(x+1)^3x^{\frac{2}{3}}$

нули: $x=-1,\ x=0$

$y'=3(x+1)^2x^{\frac{2}{3}}+\frac{2(x+1)^3x^{-\frac{1}{3}}}{3}=$

$=(x+1)^2\left(3x^{\frac{2}{3}}+\frac{2(x+1)}{3x^{\frac{1}{3}}}\right)=(x+1)^2\left(\frac{9x^{\frac{2}{3}}x^{\frac{1}{3}}+2x+2}{3x^{\frac{1}{3}}}\right)=\frac{(x+1)^2(11x+2)}{3x^{\frac{1}{3}}}$

Крит. т. $\cancel{x\neq 0}$

$x=-\frac{2}{11}$

$y'(x)$: $+$ $\ -1\ $ $+$ $\ -\frac{2}{11}\ $ $-$ $\ 0\ $ $+$ $\to x$

$y(x)$: $\nearrow$ $\ -1\ $ $\nearrow$ $\ -\frac{2}{11}\ $ $\searrow$ $\ 0\ $ $\nearrow$

$y''=\frac{3x^{\frac{1}{3}}(2(x+1)(11x+2)+11(x+1)^2)-(x+1)^2(11x+2)x^{-\frac{2}{3}}}{9x^{\frac{2}{3}}}=$

$=\frac{3x^{\frac{1}{3}}(x+1)(22x+4+11x+11)-(x+1)^2(11x+2)x^{-\frac{2}{3}}}{9x^{\frac{2}{3}}}=$

$=\frac{x^{\frac{1}{3}}(x+1)(3(33x+15)-(x+1)(11x+2)x^{-1})}{9x^{\frac{2}{3}}}=$

$=\frac{(x+1)(99x+45-11x-13-2x^{-1})}{9x^{\frac{2}{3}}}=\frac{(x+1)(88x^2+32x-2)}{9x^{\frac{2}{3}}}$

Т. перегиба: $44x^2+16x-1=0$

$x=-1$

$x=0$

$\frac{D}{4}=64+44=108$

$x=\frac{-8\pm 6\sqrt{3}}{44}=\frac{-4\pm 3\sqrt{3}}{22}$

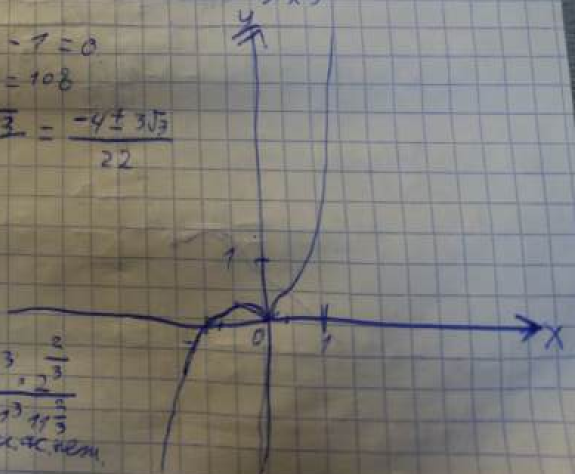
$y''(x)$: $-$ $\ -1\ $ $+$ $\ \frac{-4-3\sqrt{3}}{22}\ $ $-$ $\ 0\ $ $-$ $\ \frac{-4+3\sqrt{3}}{22}\ $ $+$ $\to x$

$y(x)$: $\frown$ $\smile$ $\frown$ $\frown$ $\smile$

$D(y)=\mathbb{R}$ $\qquad y_{min}=y(0)=0$

$E(y)=\mathbb{R}$ $\qquad y_{max}=y\left(-\frac{2}{11}\right)=\frac{9^3\cdot 2^{\frac{2}{3}}}{11^3\cdot 11^{\frac{2}{3}}}$

$\lim\limits_{x\to\infty}f(x)=\infty \Rightarrow$ накл. ас. нет



в) $y = \frac{x}{2} - \arccos\frac{2x}{1+x^2}$

$$\begin{cases} \frac{2x}{1+x^2} \le 1 \\ \frac{2x}{1+x^2} \ge -1 \end{cases} \qquad \begin{cases} \frac{x^2-2x+1}{x^2+1} \ge 0 \\ \frac{x^2+2x+1}{x^2+1} \ge 0 \end{cases}$$

$\cancel{y' = \frac{1}{2} + \frac{1}{\sqrt{2x}}}$

$$y' = \frac{1}{2} + \frac{1}{\sqrt{1-\left(\frac{2x}{1+x^2}\right)^2}} \cdot \frac{(2(1+x^2) - 2x \cdot 2x)}{(1+x^2)^2} =$$

$$= \frac{1}{2} + \frac{2-2x^2}{\sqrt{(1+x^2)^2 - 4x^2}\,(1+x^2)} = \frac{1}{2} + \frac{2-2x^2}{\sqrt{(1-x^2)^2}\,(1+x^2)} =$$

$$= \frac{(1+x^2)\sqrt{(1-x^2)^2} + 4(1-x^2)}{2(1+x^2)\sqrt{(1-x^2)^2}} = \cancel{\frac{1+x^2+4}{2(1+x^2)}} = \cancel{\frac{x^2+5}{2x^2+2}}$$

$$\left[\begin{array}{l} \begin{cases} x^2 \le 1 \\ \frac{5+x^2}{2+2x^2} = 0 \end{cases} \\ \begin{cases} x^2 \ge 1 \\ \frac{x^2-3}{2+2x^2} = 0 \end{cases} \end{array}\right. \iff x = \pm\sqrt{3}$$

$y'(x)$: $+$ $-\sqrt{3}$ $-$ -1 $+$ 1 $-$ $\sqrt{3}$ $+$ → x

$y(x)$: ↗ ↘ ↗ ↘ ↗

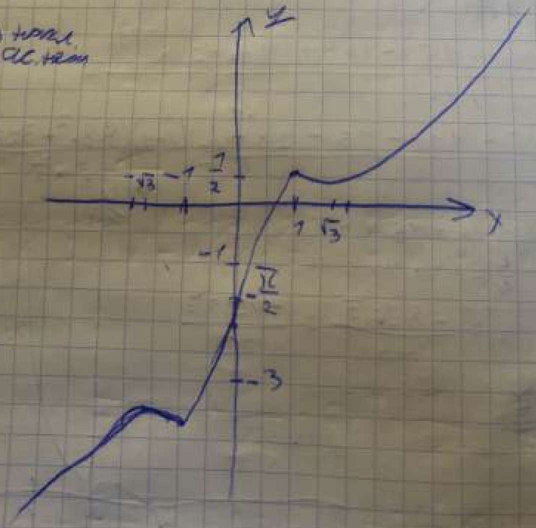
$$y'' = \left(\frac{(2x)(1-x^2) + (1+x^2)(-2x) + (-8x)}{4(1+x^2)^2\left(\sqrt{(1-x^2)^2}\right)^2}\right) =$$

$$= \frac{4(x^3-x)}{(x^2+1)\sqrt{x^4-2x^2+1}\,\sqrt{x^4+2x^2+1}}$$

y'': $-$ -1 $+$ 0 $-$ 1 $+$ → x

y: ∩ ∪ ∩ ∪

$\lim\limits_{x\to\infty} f(x) = \infty \Rightarrow$ няма ас.



№6 

$d=a$

$h-?$ $S-max$

$h^2=\frac{l^2}{1}-\frac{x^2}{4}$

$l^2=h^2+\frac{x^2}{4}$

$S=\frac{l^2\cdot x\cdot 2}{4a}=\frac{l^2 x}{2a}=\frac{xh}{2}$

$\frac{l^2}{a}=h$

$x=\sqrt{4l^2-4h^2}=2\sqrt{l^2-h^2}$

$R^2\sin 2\lambda$

$S=\frac{l^2\sin 2\lambda}{2}=l^2\cdot\frac{x}{2l}\cdot\frac{h}{l}=\frac{xh}{2}$

Докажем, что S-max, если Δ-правильный.

$S(\lambda)=\frac{l^2\sin 2\lambda}{2}$ $S(\beta)=lx\sin\beta$

$S'(\lambda)=l^2\cos 2\lambda$ $S'(\beta)=lx\cos\beta$

$S=\frac{l^2\sin 2\lambda}{2}=$

$r=\frac{a}{2\sqrt3}$ $R=\frac{a}{\sqrt3}$ $R\sqrt3=2r\sqrt3$

$h=R+r=\frac{a}{2}+\frac{a}{4}=\frac{3a}{4}$ Ответ: $\frac{3a}{4}$

№7 а) $\lim\limits_{x\to0}\frac{\sin(x\cos x)+x\ln(1+\frac{2x^2}{3})-x}{\sqrt{1+x^5}-1}=\lim\limits_{x\to0}\frac{x-\frac{2x^3}{3}+\frac{2x^3}{3}-x+o(x^3)}{\frac{x^5}{2}+o(x^5)}=\frac{7}{45}$

$\sqrt{1+x^5}-1=\frac{x^5}{2}+o(x^5)$

$\ln(1+\frac{2x^2}{3})=\frac{2x^2}{3}+o(x^3)$ $x\ln(1+\frac{2x^2}{3})=\frac{2x^3}{3}+o(x^3)$

$\cos x=1-\frac{x^2}{2}+o(x^3)$

$\sin(x\cos x)=\sin(x-\frac{x^3}{2}+xo(x^3))=x-\frac{x^3}{2}+xo(x^3)-\frac{(x-\frac{x^3}{2}+xo(x^3))^3}{6}+$

$+o(x-\frac{x^3}{2}+xo(x^3))=$

$=x-\frac{2x^3}{3}+\frac{3x^5}{10}+o(x^5)$

б) $\lim\limits_{x\to0}(\cos(\sin x)+\frac{1}{2}\operatorname{arctg}x^2)^{1/\sin x^4}=$

$=\lim\limits_{x\to0}\left(e^{\frac{1}{\sin x^4}\{\ln(\cos(\sin x)+\frac{1}{2}\operatorname{arctg}x^2)\}}\right)=$

$$= \lim_{x\to 0}\left(e^{\frac{1}{x^4}\ln\left(\cos(\sin x)+\frac{1}{2}\operatorname{arctg} x^2\right)}\right) = \lim_{x\to 0}\left(e^{\frac{5x^4+o(x^4)}{24x^4}}\right) = e^{\frac{5}{24}}$$

$$\cos(\sin x) = 1-\frac{x^2}{2}+\cancel{o(x^3)}+\frac{5x^4}{24}+o(x^4)$$

$$\frac{1}{2}\operatorname{arctg} x^2 = \frac{x^2}{2}+o(x^4)$$

$$\ln\left(\cos(\sin x)+\frac{1}{2}\operatorname{arctg} x^2\right) = \frac{5x^4}{24}+o(x^4)$$

$= \left| \frac{(2x'^3-1)x''^2 - (2x''^3-1)x'^2}{x'^2 x''^2} \right| \le \left| \cancel{x''^2 - x'^2} \right| \le \left| \cancel{(x''-x')(x''+x'')} \right| \ge$

$\ge 2|x''-x'| \ge |x'-x''| < \left| 2(x'-x'') - \frac{1}{x'^2} + \frac{1}{x''^2} \right| \le$

$\le \left| 2(x'-x'') + (x'^2 - x''^2) \right| = \left| (x'-x'')(2+x'+x'') \right| <$ $\quad -1 < 2x-1 < 1$

$x'^2 - x''^2 \le x_1^2 - x_2$ $\quad 2x'^3 - 1$ $\quad \frac{2x'^3-1}{x} < \frac{1}{x}$

$(x-x)(2+\frac{x+x}{xx})$ $\quad 0<x<1$

№8 $\quad f(x) = 2x - \frac{1}{x^2}$

Ф-я равном. непрер., если $\forall \varepsilon > 0 \ \exists \delta = \delta(\varepsilon) > 0$:

$\forall x', x'' \in X: |x'-x''| < \delta \Rightarrow |f(x') - f(x'')| < \varepsilon$

d) $X = [1; +\infty)$

$|f(x') - f(x'')| = \left| 2x' - \frac{1}{x'^2} - 2x'' + \frac{1}{x''^2} \right| \le \left| \cancel{2(x'-x'')} + \cancel{\frac{x'^2 - x''^2}{x'^2 x''^2}} \right| \ge$

$\le \left| 2(x'-x'') + \frac{1}{x''} - \frac{1}{x'} \right| = \left| 2(x'-x'') + \frac{x'-x''}{x'x''} \right| \le 2|x'-x''| + |x'-x''| \le 3|x'-x''| <$

$< \delta = \frac{\varepsilon}{3}$

$\cancel{f(x)} \Rightarrow f(x)$ непрер. на $X \in [1; +\infty)$

8) $X = (0;1)$

$$|f(x') - f(x'')| = \left|2x' - \frac{1}{x'^2} - 2x'' + \frac{1}{x''^2}\right| = \left|2(x' - x'') + \left(\frac{1}{x''^2} - \frac{1}{x'^2}\right)\right| \le$$

$\exists\ x'_n = \frac{1}{2n}$, $x''_n = \frac{1}{n}$, тогда $|f(x'_n) - f(x''_n)| = \left|2\left(\frac{1}{2n} - \frac{1}{n}\right) + \frac{n^2}{1} - \frac{4n^2}{1}\right| = \left|\frac{3n^2}{4} + \frac{2}{n}\right| = \frac{3n^2}{4} + \frac{2}{n}$

$$= \left|\frac{1}{n} - \frac{1}{2n} + n^2 - 4n^2\right| = 3n^2 + \frac{1}{n}$$

$\Rightarrow$ при $n \ge 1$ $3n^2 + \frac{1}{n} \ge 4 \Rightarrow |f(x') - f(x'')| \ge 4 = \varepsilon \Rightarrow$

$\Rightarrow$ ф-я не явл. неразр.