Chapter 1

- Signals: Analog signals & Digital signals (continuous-time and discrete-time), Deterministic & Random Signals [There's analog, digital, mixed processing]
- Analog representation: uses a voltage or a current
- Digital representation: uses an ON/OFF pulses corresponding to the digit of a binary number.
- Deterministic signal: (completely predictable) period signals, sinusoidal signals, sinc function, etc.
- Random signal: noise signal, stock price, etc.
- (ASP) Analog signal processing: conversion of analog signal into electrical signal and their processing by analog device or circuits.
- (DSP) Digital signal processing: conversion of $\underbrace{\text{continuous-time}}_{\text{CTS}}$ signal into $\underbrace{\text{discrete-time}}_{\text{DTS}}$ signals, the transformation of DTS through digital computation, and into analog signa

Chapter 2

- **Length**: $L_x = n_2 - n_1 + 1$
- **Energy**: $E_x = \sum_{n=-\infty}^{\infty} |x[n]|^2$
- **Power**: $P_x \triangleq \lim_{L\to\infty} \frac{1}{2L+1} \sum_{n=-L}^{L} |x[n]|^2$

1. Unit Sample Sequence: $\delta[n] = \begin{cases} 1, & n=0 \\ 0, & n\neq 0 \end{cases}$
2. Unit Step Sequence: $u[n] = \begin{cases} 1, & n\geq 0 \\ 0, & n<0 \end{cases}$
3. Sinusoidal Sequence: $x[n] = A\cos(\omega_0 n + \phi)$ $\quad -\infty < n < \infty$
4. Exponential Sequence: $x[n] \triangleq A a^n \quad -\infty<n<\infty$
5. Complex Exponential Sequence: $x[n] \triangleq Ae^{j\omega n} = A\cos(\omega n) + jA\sin(\omega$
6. Periodic Sequence: $x[n] = x[n+N]$, for all n

(1) Time-reversal or Folding: $y[n] = x[-n]$
- Even symmetry: $x[n] = x[-n]$
- Odd symmetry: $x[n] = -x[-n]$

(2) Time-shifting: $y[n] = x[n-n_0]$
- If $n_0 > 0$, $x[n]$ shifted RIGHT (time delay)
- If $n_0 < 0$, $x[n]$ shifted LEFT (time advance)

⚠ shifting & folding are not commutative operation
[Shift → fold ≠ fold → shift]

*** Causality**: (required for systems that operate in real-time), the issue with the real time (online) processing
- A system is causal if the present value of output doesn't depend on the future value of input. Or not depend on the input value of earlier sample with reference to present sample.
- LTI system with impulse response $h[n]$ is causal if $h[n] = 0$, $n<0$
- $y[n] = x[n] + x[n-1]$ ⇒ Causal
- $y[n] = x[n+1] + x[n-1]$ ⇒ Non-causal

*** Stability**: (should be satisfied by practical system)

* FIR is always stable
* IIR may/may not be stable
* transient response dies out if system is stable.

- A system is stable if every bounded input signal results in a bounded output (BIBO)
- LTI system w/ impulse response $h[n]$ is stable if impulse is abs. summable
  ↳ $\sum_{n=-\infty}^{\infty} |h[n]| < M_h < \infty$
- $y[n] = \frac{1}{3}(x[n] + x[n-1] + x[n-2])$ ⇒ Stable
- $y[n] = \sum_{k=0}^{\infty} x[n-k]$ ⇒ Unstable

*** Linearity**:
- A system is called linear if it supports superposition: $H\{a_1x_1[n] + a_2x_2[n]\} = a_1 H\{x_1[n]\} + a_2 \cdot H\{x_2[n]\}$

(means that a linear combination of input signals produces the same linear combination of outputs of individual input signals.)

*** Time-Invariant**:
- A system is called time-invariant if it supports: $y[n] = H\{x[n]\} \Rightarrow y[n-n_0] = H\{x[n-n_0]\}$

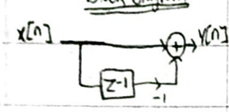
* Continuous LTI system is causal & stable if its impulse response satisfies the condition
① $h(t) = 0$, $t<0$ ② $\int_{-\infty}^{\infty} |h(\tau)|\, d\tau < \infty$

- $(N+M-1) \times M$ matrix (Toeplitz)

- $y[n] = x[n] * h[n] = \underbrace{\sum_{k=-\infty}^{\infty} x[k]h[n-k]}_{\text{Conventional convolution}} = \underbrace{\sum_{k=-\infty}^{\infty} h[k]\,x[n-k]}_{\text{Convolution as a superposition of scaled \& shifted replicas}}$

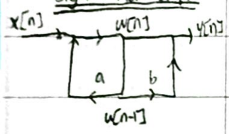
Block Diagrams & Signal Flow Graphs

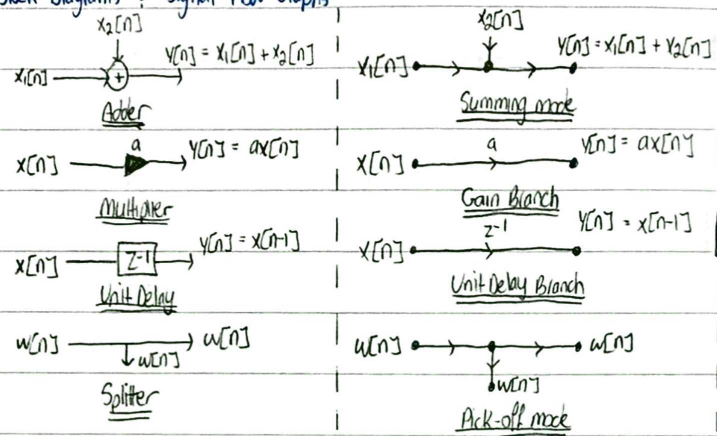
Block Diagram



$\Rightarrow y[n] = x[n] - x[n-1]$

Signal Flow Graph





Type of Response	Input Sequence	Output Sequence
Impulse	$x[n] = \delta[n] \xmapsto{H}$	$y[n] = h[n]$
Step	$x[n] = u[n] \xmapsto{H}$	$y[n] = s[n] = \sum_{k=-\infty}^{n} h[k]$
Exponential	$x[n] = a^n$, all $n \xmapsto{H}$	$y[n] = H(a)a^n$, all n
Complex sinusoidal	$x[n] = e^{j\omega n}$, all $n \xmapsto{H}$	$y[n] = H(e^{j\omega})e^{j\omega n}$, all n

Partial overlap (left) : $y[n] = \sum_{k=N_1}^{n-M_1} x[k]h[n-k]$, for $N_1 + M_1 \le n < N_1 + M_2$

Full overlap : $y[n] = \sum_{k=n-M_2}^{n-M_1} x[k]h[n-k]$, for $N_1 + M_2 \le n \le N_2 + M_1$

Partial overlap (right) : $y[n] = \sum_{k=n-M_2}^{N_2} x[k]h[n-k]$, for $N_2 + M_1 < n \le N_2 + M_2$

Properties of LTI (convolution) Systems

- Convolution operation in LTI system is commutative : $h[n] * x[n] = x[n] * h[n]$
- Cascade interconnection of two LTI systems : $h[n] = h_1[n] * h_2[n]$
- Parallel " " " : $h[n] = h_1[n] + h_2[n]$

Property	Formula
identity	$x[n] * \delta[n] = x[n]$
Delay	$x[n] * \delta[n-n_0] = x[n-n_0]$
Commutative	$x[n] * h[n] = h[n] * x[n]$
Associative	$(x[n] * h_1[n]) * h_2[n] = x[n] * (h_1[n] * h_2[n])$
Distributive	$x[n] * (h_1[n] + h_2[n]) = x[n] * h_1[n] + x[n] * h_2[n]$

Examples (system properties)

System	Linear	TI	Causal	Stable	Comments
$y[n] = x^2[n]$	✗	✓	✓	✓	Square-law
$y[n] = x[-n]$	✓	✗	✗	✓	Time-reversal
$y[n] = \sum_{k=-\infty}^{n} x[k]$	✓	✓	✓	✗	Accumulator
$y[n] = x[n] - x[n-1]$	✓	✓	✓	✓	First-difference
$y[n] = 1/L \sum_{k=0}^{L-1} x[n-k]$	✓	✓	✓	✓	Moving average
$y[n] = \text{round}\{x[n]\}$	✗	✓	✓	✓	Quantizer

FIR Spatial Filters (popular & useful in digital image processing)

- 2D filters to take care of 2D digital image processing (extension of 1D filters)
- We can use convolution sum to calculate the output of FIR systems.
- We cannot use convolution sum for the IIR system because of its infinite # of impulse responses.

Zero-input & Zero-state Response

$y_{ss}[n] \neq y_{zs}[n]$

$y_{tr}[n] \neq y_{zi}[n]$

- $y[n] = \left(\sum_{k=0}^{n} h[k]x[n-k]\right)u[n]$ // convolution sum
- $y[n] = a^{n+1}y[-1] + h[n]x[0] + h[n-1]x[1] + \ldots + bx[n]$
- $y_{zi}[n] = a^{n+1}y[-1]$, $n \geq 0$ // zero-input response
- $y_{zs}[n] = \sum_{k=0}^{n} h[k]x[n-k]$ // zero-state response

$\Rightarrow y[n] = a^{n+1}y[-1] + \sum_{k=0}^{n} h[k]x[n-k] = y_{zi}[n] + y_{zs}[n]$

More Formulas, etc.

sampling property

- $\int_{-\infty}^{\infty} x(t)\delta(t-t_0)\,dt = x(t_0)\int_{-\infty}^{\infty} \delta(t-t_0)\,dt = x(t_0)$

* Processing time to generate an output sample should be less than a symbol period (or sampling period of input signal) to make it real-time processing.

* In real time processing, system generate the output sample before the next input sample arrives at the input of the system.

▲ Discrete-time signal : $y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau$