UNIDAD I

Transformada Discreta de Fourier

REPRESENTACIÓN

$$X[k] = R[k] + jI[k]$$

- Magnitud

$$|X[k]| = \sqrt{R^2[k] + \mathrm{Im}^2[k]}$$

- Fase

$$\phi[k] = \tan^{-1}\frac{\mathrm{Im}[k]}{R[k]}$$

TRANSFORMADA DE FOURIER DE TIEMPO DISCRETO

- Representación en el dominio de la frecuencia de las señales de tiempo discreto
  - Señales discretas representadas con espectros periódicos

$$X_p(F) = \sum_{k=-\infty}^{\infty} x[k]e^{-j2\pi kF}$$

- Inversa

$$x[n] = \int_{-\frac{1}{2}}^{\frac{1}{2}} X_p(F)\, e^{j2\pi nF} dF$$

TRANSFORMADA DISCRETA DE FOURIER

- Señales periódicas y discretas en un dominio son periódicas y discretas en el otro

$$X_{DFT}[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi nk}{N}} \qquad k = 0,1,2, \dots N-1$$

- Inversa

$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X_{DFT}[k] e^{j\frac{2\pi nk}{N}} \qquad n = 0,1,2, \cdots, N-1$$

TRANSFORMADA RÁPIDA DE FOURIER

- Considerando

$$X_{DFT}[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi nk}{N}} \quad k = 0,1,2,\ldots N-1$$

- Entonces

$$W_N = e^{-\frac{j2\pi}{N}}$$

$$X[k] = \sum_{n=0}^{N-1} x[n] W_N^{kn} \qquad k = 0,1,2,\cdots,N-1$$

RELACIONES DE W N

$$W_N = e^{-\frac{j2\pi}{N}}$$

$$W_N^2 = \left(e^{-j\frac{2\pi}{N}}\right)^2 = e^{-j\frac{2\pi 2}{N}} = e^{-j\frac{2\pi}{\left(\frac{N}{2}\right)}} = W_{\frac{N}{2}}$$

$$W_N^{\left(k+\frac{N}{2}\right)} = W_N^k W_N^{\frac{N}{2}} = W_N^k e^{-j\left(\frac{2\pi}{N}\right)^{\left(\frac{N}{2}\right)}} = W_N^k e^{-j\pi} = -W_N^k$$

TRANSFORMADA RÁPIDA DE FOURIER

$$X[k] = \sum_{n=0}^{N-1} x[n] W_N^{kn} \qquad k = 0,1,2,\cdots,N-1$$

$$X_1[k] = \underbrace{\sum_{n=0}^{N/2-1} x_{2n} W_N^{2nk}}_{Par} + \underbrace{\sum_{n=0}^{N/2-1} x_{2n+1} W_N^{(2n+1)k}}_{Impar}$$

$$X[k] = \sum_{n=0}^{\frac{N}{2}-1} x_{2n} W_N^{2nk} + W_N^k \sum_{n=0}^{\frac{N}{2}-1} x_{2n+1} W_N^{2nk}, \quad k = 0,1,2,\cdots,N-1$$

TRANSFORMADA RÁPIDA DE FOURIER

$$X[k] = \sum_{n=0}^{\frac{N}{2}-1} x_{2n} W_{\frac{N}{2}}^{nk} + W_N^k \sum_{n=0}^{\frac{N}{2}-1} x_{2n+1} W_{\frac{N}{2}}^{nk}, \quad k = 0,1,2,\cdots,N-1$$

$$X_1[k] = \underbrace{X_{11}[k]}_{Par} + \underbrace{W_N^k X_{12}[k]}_{Impar}\ , \qquad k = 0,1,2,\ldots,N-1$$

FFT DE 8 PUNTOS

<table>
<tr><th></th><th></th><th colspan="7"></th><th>k</th><th>n</th></tr>
<tr><td>Datos</td><td>A_0</td><td colspan="7">$x_0 x_1 x_2 x_3 x_4 x_5 x_6 x_7$</td><td></td><td>0…7</td></tr>
<tr><td>DFT 8 puntos</td><td colspan="8">$X_1[k] = X_{11}[k] + W_N^k X_{12}[k]$</td><td>0…N-1
0…7</td><td>0…7</td></tr>
<tr><td>Dos secuencias</td><td>A_1</td><td colspan="3">$x_0 x_2 x_4 x_6$</td><td>A_2</td><td colspan="3">$x_1 x_3 x_5 x_7$</td><td></td><td>0…3</td></tr>
<tr><td>DFT 4 puntos</td><td colspan="4">$X_{11}[k] = X_{21}[k] + W_{\frac{N}{2}}^k X_{22}[k]$</td><td colspan="4">$X_{12}[k] = X_{23}[k] + W_{\frac{N}{2}}^k X_{24}[k]$</td><td>0…N/2-1
0…3</td><td>0…3</td></tr>
<tr><td>4 secuencias</td><td>A_3</td><td>$x_0 x_4$</td><td>A_4</td><td>$x_2 x_6$</td><td>A_5</td><td>$x_1 x_5$</td><td>A_6</td><td>$x_3 x_7$</td><td></td><td>0…1</td></tr>
<tr><td>DFT 2 puntos</td><td colspan="2">$X_{21}[k] = x_0 + W_{\frac{N}{4}}^k x_4$</td><td colspan="2">$X_{22}[k] = x_2 + W_{\frac{N}{4}}^k x_6$</td><td colspan="2">$X_{23}[k] = x_1 + W_{\frac{N}{4}}^k x_5$</td><td colspan="2">$X_{24}[k] = x_3 + W_{\frac{N}{4}}^k x_7$</td><td>0…N/4-1
0…1</td><td>0…1</td></tr>
</table>

CÁLCULO FFT

$$X_{21}[k] = x_0 \quad + W_{\frac{N}{4}}^{k} x_4$$

$$X_{21}[0] = x_0 + x_4$$

$$X_{21}[1] = x_0 + W_2 x_4 = x_0 + e^{-j\frac{2\pi}{2}} x_4 = x_0 + e^{-j\pi} x_4 = x_0 - x_4$$

$$X_{21}[0] = x_0 + x_4 \qquad X_{21}[1] = x_0 - x_4$$

$$X_{22}[0] = x_2 + x_6 \qquad X_{22}[1] = x_2 - x_6$$

$$X_{23}[0] = x_1 + x_5 \qquad X_{23}[1] = x_1 - x_5$$

$$X_{24}[0] = x_3 + x_7 \qquad X_{24}[1] = x_3 - x_7$$

CÁLCULO FFT

$$X_{11}[k] = X_{21}[k] \quad + W_{\frac{N}{2}}^{k} X_{22}[k]$$

$$X_{11}[0] = X_{21}[0] + W_{N/2}^{0} X_{22}[0] = X_{21}[0] + X_{22}[0]$$

$$X_{11}[1] = X_{21}[1] \quad + W_{\frac{N}{2}}^{1} X_{22}[1] = X_{21}[1] \quad + e^{-j\frac{\pi}{2}} X_{22}[1] = X_{21}[1] \quad - jX_{22}[1]$$

$$X_{11}[2] = X_{21}[2] \quad + W_{\frac{N}{2}}^{2} X_{22}[2] = X_{21}[2] \quad + e^{-j\left(\frac{2\pi}{8}\right)(2)(2)} X_{22}[2]$$

$$X_{11}[2] = X_{21}[2] \quad + e^{-j\pi} X_{22}[2] = X_{21}[2] \quad - X_{22}[2]$$

$$X_{21}[2] = x_0 + W_{\frac{N}{4}}^{2} x_4 = x_0 + W_2^2 x_4 = x_0 + x_4 = X_{21}[0]$$

$$X_{22}[2] = x_2 + W_{\frac{N}{4}}^{2} x_6 = x_2 + W_2^2 x_6 = x_2 + x_6 = X_{22}[0]$$

$$\longrightarrow \quad X_{11}[2] = X_{21}[0] \quad - X_{22}[0]$$

CÁLCULO FFT

$$X_{11}[3] = X_{21}[3] \quad + W_{\frac{N}{2}}^{3} X_{22}[3] = X_{21}[3] \quad + e^{-j\left(\frac{2\pi}{4}\right)(3)} X_{22}[3]$$

$$X_{11}[3] = X_{21}[3] \quad + e^{-j\frac{3\pi}{2}} X_{22}[3] = X_{21}[3] \quad + jX_{22}[3]$$

$$X_{21}[3] = x_0 + W_{\frac{N}{4}}^{3} x_4 = x_0 + e^{-j\left(\frac{2\pi}{2}\right)(3)} x_4 = x_0 + e^{-j3\pi} x_4 = x_0 - x_4 = X_{21}[1]$$

$$X_{22}[3] = x_2 + W_{\frac{N}{4}}^{3} x_6 = x_2 + W_{2}^{3} x_6 = x_2 - x_6 = X_{22}[1]$$

$$\longrightarrow \quad X_{11}[3] = X_{21}[1] \quad + jX_{22}[1]$$

CÁLCULO FFT

$$X_{11}[0] = X_{21}[0] + W_8^0 X_{22}[0] = X_{21}[0] + X_{22}[0]$$

$$X_{11}[2] = X_{21}[0] - W_8^0 X_{22}[0] = X_{21}[0] - X_{22}[0]$$

$$X_{11}[1] = X_{21}[1] + W_8^2 X_{22}[1] = X_{21}[1] - \mathrm{j}X_{22}[1]$$

$$X_{11}[3] = X_{21}[1] - W_8^2 X_{22}[1] = X_{21}[1] + \mathrm{j}X_{22}[1]$$

MARIPOSA (FFT)