

BSc Econometrics and Operations Research
FEB23006-25 Quantitative Methods for Finance
Assignment 1 - Volatility models and risk management

INSTRUCTIONS

This assignment should be made by teams consisting of (max) three students.

Your solutions for this assignment should be provided in a report of max. 10 pages (not including a possible appendix). The report should provide a detailed and careful description of your analysis, the results and their interpretation. Make the report self-contained, so that it can be read without reading these instructions first.¹ All relevant results should be included in the report (preferably in tables and graphs). Do not copy-paste ‘raw’ output from the software you use into the main text of the report (but make proper Tables and/or Figures).

Reports should be submitted via Canvas (under “Assignments” on the course pages). Deadline for submission is **Friday January 30, 2026 at 16:59h**. Submit your report as a PDF file with the name “QMF25-A1-GroupXX.pdf”, where XX is your group number (01,02,...,75). Include names and student numbers of all team members in (preferably the top-right corner of) the first page of the report.

Note 1: Motivate your answers! For example, the statement “Yes, the returns exhibit the three stylized facts” at question A1.1) will not be rewarded any points. It needs to be substantiated by further ‘evidence’.

Note 2: Some questions in the assignment are not specified up to the last detail and require you to make some choices yourself. Explain and motivate your choices in your report.

Note 3: Feel free to use any software [EViews, Matlab, Python, R, etc] that you are comfortable with. Mention which software was used in the report, as this may impact the results to some extent.

¹While the report should explain in detail how you addressed the different questions, you do not need to take this to the extreme. For example, at question A1.4, it is sufficient to state that “We estimated the parameters in a GARCH(1,1) model using Maximum Likelihood assuming a normal distribution for the standardized unexpected returns”. You do not need to provide the expression of the likelihood function, you do not need to mention which numerical optimization algorithm was used, or which starting values for the parameters were used [unless of course this impacted the results in some way].

Note 4: Avoid plagiarism and other forms of fraud! It is not allowed to cooperate with other teams, or to copy their results. Copy-pasting text from the course's exercises (or any other course material) is also considered plagiarism and will be punished accordingly.

Questions related to this assignment can be posted on Canvas.

Good luck!!

DATA

The Excel file `JNJ1423.xlsx` contains (continuously compounded) daily returns (in percentage points) and realized variance (based on 5-minute intraday returns) for the [Johnson & Johnson](#) stock (ticker symbol: `JNJ`) over the period January 2, 2014 - December 29, 2023 (2516 observations).² Returns and realised variance are included in columns B and C, headed R and RV. Column A contains the calendar date.

QUESTIONS

A1.1) [STYLED FACTS OF RETURNS AND REALIZED VARIANCE]

★ Using the complete sample period, analyze the statistical properties of the daily returns r_t . In particular, examine whether they exhibit the three “stylized facts” of (daily) asset returns:

1. Distribution of returns is not normal;
2. (Almost) no significant autocorrelations in returns;
3. Small, but slowly declining autocorrelations in squared and absolute returns.

Also examine (and compare) the properties of the standardized returns computed as $r_t/\sqrt{RV_t}$, where RV_t is the realized variance.

★ Analyze the properties of the realized variance RV_t (consider distributional characteristics such as mean, variance, skewness, kurtosis, as well as dynamic properties such as autocorrelations. You may also want to consider transformations such as the natural logarithm of this series.) Compare this with the properties of the squared daily returns.

²Given that the stock market is closed on some days (such as Christmas Day and other US federal holidays), the time series is not ‘continuous’, in the sense that it does not contain observations for every calendar day. We ignore this feature for the purpose of this assignment, and consider the time series as a ‘continuous’ sequence.

A1.2) [HISTORICAL VOLATILITY]

- ★ Compute the ‘historical volatility’ of the daily returns based on a moving window of $T = 63, 126$, and 252 days. Do this for the complete sample period. You may ignore the sample mean return, and simply compute the historical variance at day t as the mean of squared returns on the previous T days, that is, $\frac{1}{T} \sum_{i=1}^T r_{t-i}^2$.
- ★ Describe the properties of these historical volatility estimates, and interpret/explain the differences in the volatility estimates for the different values of T .

A1.3) [RISKMETRICS]

- ★ Compute the ‘Riskmetrics’ volatility of the daily returns with $\lambda = 0.94$, for the complete sample period. Again you may ignore the sample mean return, and compute the Riskmetrics variance at day t as $\sigma_{t,RM}^2 = \lambda\sigma_{t-1,RM}^2 + (1 - \lambda)r_{t-1}^2$. Use the first available historical volatility estimate with $T = 63$ to initialize the recursion.
- ★ Describe the properties of the Riskmetrics volatility estimates and compare these with the properties of the historical volatility estimates.

A1.4) [GARCH - ESTIMATION AND EVALUATION]

- ★ Estimate a GARCH(1,1) model for the daily returns using the sample period January 2, 2014 - December 31, 2018 (1258 observations). Specify the conditional mean of r_t as a constant μ and use a normal distribution for the standardized unexpected returns $z_t = (r_t - \mu)/\sigma_t$, where σ_t denotes the GARCH(1,1) conditional standard deviation.
- ★ Examine the conditional volatility estimates that result from the GARCH(1,1) model and compare this with the historical volatility and Riskmetrics estimates.
- ★ Check the properties of the standardized residuals $\hat{z}_t = (r_t - \hat{\mu})/\hat{\sigma}_t$. In particular, consider its kurtosis, and the autocorrelation function of \hat{z}_t^2 . Is the assumption of normality of the standardized unexpected returns appropriate? Does the GARCH(1,1) model adequately capture the conditional heteroskedasticity in the daily returns?

A1.5) [ARCH - ESTIMATION AND EVALUATION]

- ★ Repeat question A1.4 using an ARCH(1) model. Is this simpler model sufficient to describe the characteristics of the returns and their volatility, or do we really need the ‘G’ of the GARCH model?

A1.6) [TGARCH AND GARCHX - ESTIMATION AND COMPARISON]

★ Estimate the parameters in (i) Threshold GARCH(1,1) and (ii) GARCHX(1,1) models using the sample period January 2, 2014 - December 31, 2018 (1258 observations).³ As before, specify the conditional mean of r_t as a constant μ and use a normal distribution for the standardized unexpected returns $z_t = (r_t - \mu)/\sigma_t$. A GARCHX(1,1) model is defined as a GARCH(1,1) model with the lagged realized variance as an additional explanatory variable in the specification of the conditional variance, that is, $\sigma_t^2 = \omega + \alpha(r_{t-1} - \mu)^2 + \beta\sigma_{t-1}^2 + \delta RV_{t-1}$.⁴

★ What do the parameter estimates in the Threshold GARCH model suggest about the effects of positive and negative unexpected returns on volatility?

★ What do the parameter estimates in the GARCHX model suggest about the (relative) importance of the lagged daily squared unexpected returns, the previous conditional variance, and the lagged realized variance for the volatility dynamics?

★ The largest positive return during the estimation period occurred on Tuesday January 26, 2016.⁵ Examine the behavior of the **conditional variances** implied by GARCH, Threshold GARCH and GARCHX around this date. Discuss similarities and differences across these variances, also relating them to parameter estimates in the different models. Also do this for the day with the largest negative return, i.e. Friday December 14, 2018.⁶

A1.7) [VOLATILITY FORECASTS - COMPUTATION AND EVALUATION]

★ Use the remaining part of the sample, i.e. January 2, 2019 to December 31, 2023, to compute the Mean Squared Prediction Error [MSPE] of the **variance** forecasts using the squared daily return as proxy for the true conditional variance, and use these to rank the volatility forecasts. Consider the following models: (i) historical volatility based on 126 days, (ii) Riskmetrics, (iii) the ARCH model, (iv) the GARCH model, (v) the Threshold GARCH model, and (vi) the GARCHX model.

★ Repeat the forecast evaluation but now using the realized variance as proxy for the true variance instead of the squared daily return. Do you find any differences in the results?

³You do not need to evaluate these models in as much detail as the GARCH(1,1) and ARCH(1) models in the previous questions.

⁴This corresponds with equation (5) in Shephard and Sheppard (2010, *Journal of Applied Econometrics* **25**, 197–231) <https://doi.org/10.1002/jae.1158>.

⁵This followed the release of [results for Q4 2015](#).

⁶This followed the release of a Reuters report about possible [asbestos in JNJ's baby powder](#).

- ★ Finally, plot the **variance** forecasts obtained with the GARCH, Threshold GARCH and GARCHX models between January 9 and February 17, 2023. Do they adequately describe the actual variance (as proxied by the realized variance), in particular during the days/weeks following January 24, 2023?⁷ Also, can you explain the (similarities and differences in the) patterns in these forecasts as given by the different models?
- ★ Repeat the analysis before, this time considering the period from July 3 and August 11, 2023. What happen after the day when the stock attains its highest return over this period?

A1.8) [VALUE-AT-RISK - COMPUTATION AND EVALUATION]

- ★ Construct 1-day ahead Value-at-Risk estimates at 90%, 95% and 99% for the period January 2, 2019 - December 31, 2023 using the volatility forecasts obtained with (i) historical volatility based on 126 days, (ii) Riskmetrics, (iii) the ARCH model, (iv) the GARCH model, (v) the Threshold GARCH model, and (vi) the GARCHX model. Evaluate their properties by applying the Likelihood Ratio tests of correct unconditional coverage and independence of Christoffersen (1998).

⁷Perhaps it might be also useful/insightful to examine whether any important economic and/or financial events occurred on this date!