Multiagent decision making and Coordination

Outline

- **Introduction to multiagent coordination**
- Coordination games
- Social conventions
- Social conventions with communication
- Coordination games + Social Conventions (with 3 agents)
- Roles
- Roles with communication
- Coordination Graphs

Multiagent Coordination

- **What is coordination in multiagent systems?**
  - *Coordination* is managing the interdependencies between activities
  - For example:
    - A non-sharable resource in the environment
    - Agents need to coordinate to use this resource

Multiagent Coordination

- **What is coordination in multiagent systems?**
  - *Coordination* is managing **robustly and efficiently** the interdependencies between activities
    - For example:
      - Two teammate soccer robots must coordinate their actions when deciding who should go for the ball

Multiagent Coordination

- **Multiagent coordination** is relevant for and **multiagent cooperation**
  - *Cooperation* is working together as a **team** to achieve a **shared goal**
  - **But we should use coordination mechanisms** within teamwork settings so that teams can efficiently achieve their goals

Multiagent Coordination

- For example, in collaborative/cooperative agents, coordination mechanisms ensure that:
  - Agents **do not obstruct** each other when taking actions
  - Agents **efficiently** perform joint actions
  - These actions serve the **common goal of the team**

Multiagent Coordination

- **Informally:**
  - **Coordination** can be regarded as the process where every agent's individual decision leads to **a good joint action for the group**

Outline

- Introduction to multiagent coordination
- **Coordination games**
- Social conventions
- Social conventions with communication
- Coordination games + Social Conventions (with 3 agents)
- Roles
- Roles with communication
- Coordination Graphs

Coordination Games

- We can model a coordination problem as a **coordination game** using the tools from game theory

  - We can use the **normal-form representation**
    - Actions sets and payoffs

  - Solve it using some **solution concept**
    - Nash equilibrium

Coordination Games

- Consider the **stag hunt game** with two hunters:
  - If they both hunt hares, they each capture half of the hares in the range

  - If one hunts the stag and the other hunts hares, the stag hunter goes home empty-handed while the hare hunter captures all the hares

  - Finally, if both hunt the stag, then each of their shares of the stag is greater than the value of all the hares

Coordination Games

- The payoff matrix of the **stag hunt game**:

		Hunter 2	
		Stag	***Hare***
Hunter 1	***Stag***	3, 3	0, 2
	Hare	2, 0	1, 1

Coordination Games

- What is the NE in the **stag hunt game?**

Hunter 2

Hunter 1		***Stag***	***Hare***
	Stag	$\underline{3}, \underline{3}$	0, 2
	Hare	2, 0	$\underline{1}, \underline{1}$

Nash equilibria

Which equilibrium should the hunters choose?

Coordination Games

- **No coordination** in the stag hunt game:

  - The action to hunt a ***Stag*** is a **risk-taking strategy**
    - If **both hunters choose to hunt the *Stag***, then both get the **highest payoff**
    - If **only one hunter chooses to hunt the *Stag*** then he gets the **lowest payoff**

  - The action to hunt a ***Hare*** is a **conservative strategy**
    - The hunter will **never get the highest payoff or lowest payoff**
    - The hunter will **never go home empty-handed**

Coordination Games

- **With coordination** in the stag hunt game:
  - What if both hunters enter into some kind of **agreement** (in advance) and both **trust** each other?
    - One agreement could be to choose the joint action that is **Pareto optimal** (or strictly Pareto efficient)

Coordination Games

- **Definition:** *A joint action* a **Pareto dominates** *joint action* a' *if for all* $i \in N$, $u_i(a) \geq u_i(a')$, *and there exists some* $j \in N$ *for which* $u_j(a) > u_j(a')$

- **Definition:** *A joint action* a *is* **Pareto optimal**, *or* strictly Pareto efficient, *if there does not exist another joint action* $a' \in A$ *that Pareto dominates* a

Coordination Games

- What is the NE in the **stag hunt game?**

Hunter 2

Hunter 1		***Stag***	***Hare***
	Stag	$\underline{3}$, $\underline{3}$	0, 2
	Hare	2, 0	$\underline{1}$, $\underline{1}$

Nash equilibria

Pareto optimal

Coordination Games

- We can now formally define **coordination**:
  - ***The process in which a group of agents choose a single Pareto optimal Nash equilibrium in a game***

Coordination Games

- Example of two autonomous vehicles at a crossroad

Coordination Games

- Example:
  - Each agent wants to cross first
  - But if they both cross they will crash
- **How can we represent this game with the normal-form representation and solve it?**

Coordination Games

- The payoff matrix of the **car coordination game**:

		Car 2	
		Cross	***Stop***
Car 1	***Cross***	-1, -1	1, 0
	Stop	0, 1	0, 0

Coordination Games

- How do we solve this coordination game?
  - Pareto optimal Nash equilibria

Car 2

Car 1		*Cross*	*Stop*
	Cross	-1, -1	<u>1</u>, <u>0</u>
	Stop	<u>0</u>, <u>1</u>	0, 0

Which equilibrium should the agents choose?

Coordination Games

Coordination Games

- Robot soccer example (within a **teamwork** or **collaborative** setting):
  - Two robots (of the same team) want to run to get the ball
  - But if they both try to get the ball at the same time, they will crash
- **How can we represent this game with the normal-form representation and solve it?**

Coordination Games

		Robot 2	
		Run	***Stay***
Robot 1	***Run***	-1, -1	1, 1
	Stay	1, 1	0, 0

- In the case of n **collaborative agents**, all agents in the team share the same payoff function:

$$u_1(a) = \cdots = u_n(a) \equiv u(a)$$

- These game are known as **pure coordination games** or **team games**

Coordination Games

- How do we solve this coordination game?
  - Pareto optimal Nash equilibria

Robot 2

Robot 1		**_Run_**	**_Stay_**
	Run	-1, -1	1, 1
	Stay	1, 1	0, 0

Which equilibrium should the agents choose?

Outline

- Introduction to multiagent coordination
- Coordination games
- **Social conventions**
- Social conventions with communication
- Coordination games + Social Conventions (with 3 agents)
- Roles
- Roles with communication
- Coordination Graphs

Social Conventions

- To solve a coordination problem:
  - A group of agents are faced with the problem of **how to choose their actions** in order to select the **same Nash equilibrium**
  - Unfortunately, **there is not a single recipe** that can be used for all games
  - Nevertheless, we can **devise recipes that will instruct the agents** on how to choose a single equilibrium for a given game

Social Conventions

- A **social convention** (or **social law**) is such a recipe
  - It places **constraints on the possible action choices** of the agents
  - It can be regarded as a **rule that dictates how the agents should choose their actions in a coordination game** in order to reach an equilibrium
  - Moreover, **given that the convention has been established** and is common knowledge among agents, **no agent can benefit from not abiding by it**

Social Conventions

- **Boutilier (1996)** proposed a **general convention** that achieves coordination in a large class of systems and **is very easy to implement:**
  - The convention **assumes a unique ordering scheme** of joint actions that is common knowledge among agents
  - In a particular game, **each agent first computes all equilibria** of the game, and **then selects the first equilibrium according to this ordering scheme**

Boutilier, C. (1996). Planning, learning and coordination in multiagent decision processes.
In *Proc. Conf. on Theoretical Aspects of Rationality and Knowledge*, Renesse, The Netherlands.

Social Conventions

- Two agents who want to go to the movies together but not alone

		Agent 2	
		Thriller	***Comedy***
Agent 1	***Thriller***	1, 1	0, 0
	Comedy	0, 0	1, 1

Social Conventions

- **Each agent first computes all equilibria** of the game

		Agent 2	
		Thriller	***Comedy***
Agent 1	***Thriller***	$\underline{1}, \underline{1}$	0, 0
	Comedy	0, 0	$\underline{1}, \underline{1}$

Social Conventions

- **Then each agent selects the first equilibrium according to an ordering scheme:**

  - **Order the agents** by $1 \succ 2$
    - meaning that agent 1 has 'priority' over agent 2

  - **Order the actions** by $\mathit{Thriller} \succ \mathit{Comedy}$

- ***What action should each agent choose?***

Social Conventions

- **Following the ordering scheme:**

Agent 1 \ Agent 2	***Thriller***	***Comedy***
Thriller	$\underline{1}, \underline{1}$	0, 0
Comedy	0, 0	$\underline{1}, \underline{1}$

Social Conventions

- Hence, the first equilibrium in the resulting ordering of joint actions is:

  (***Thriller*, *Thriller***)

- And this will be the **unanimous choice of the agents**

- **Now each agent can choose his individual action**

Social Conventions

- Recall the **car coordination game**:

		Car 2	
		Cross	***Stop***
Car 1	***Cross***	-1, -1	1, 0
	Stop	0, 1	0, 0

Social Conventions

- **Each agent first computes all equilibria** of the game

		Car 2	
		Cross	***Stop***
Car 1	***Cross***	-1, -1	$\underline{1}$, $\underline{0}$
	Stop	$\underline{0}$, $\underline{1}$	0, 0

Social Conventions

- **Then each agent selects the first equilibrium according to an ordering scheme:**
  - **The driver coming from the right will always have priority over the other driver**
    - Note that agents need to have **more information about the environment and/or other agents** (and not just the payoffs and action sets)
    - Let us assume that Car 1 sees Car 2 coming from the right
      - Then Car 2 has ‘priority’ over Car 1
  - **Order the actions** by *Cross* $\succ$ *Stop*
- ***What action should each agent choose?***

Social Conventions

- **Following the ordering scheme:**

Car 2

Car 1		***Cross***	***Stop***
	Cross	-1, -1	$\underline{1}$, $\underline{0}$
	Stop	$\underline{0}$, $\underline{1}$	0, 0

Social Conventions

- Hence, the first equilibrium in the resulting ordering of joint actions is:

  (***Stop*, *Cross***)

- And this will be the **unanimous choice of the agents**

- **Now each agent can choose his individual action**

Exercise

- **Battle of the sexes:**

  - A man and woman want to get together for an evening of entertainment, but they have no means of communication

  - They can either go to the ballet or the fight
    - The man prefers going to the fight
    - The woman prefers going to the ballet
    - But they both prefer being together than being alone

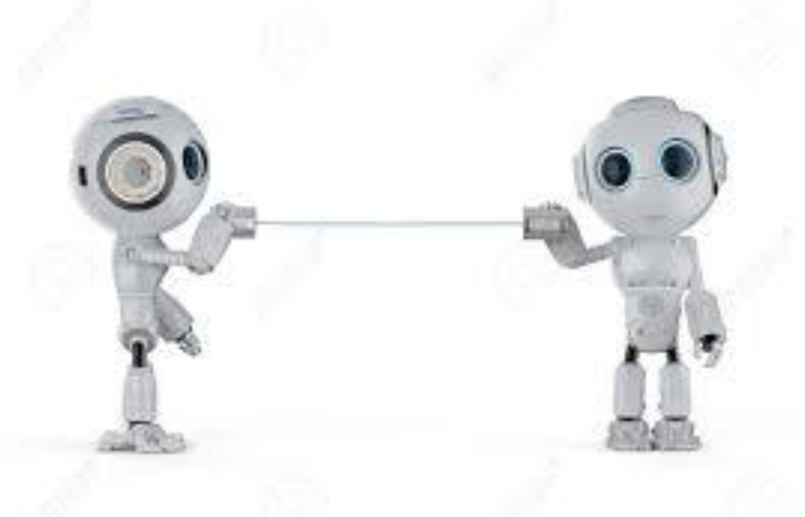
- **Create an ordering scheme and solve the coordination problem**

Outline

- Introduction to multiagent coordination
- Coordination games
- Social conventions
- **Social conventions with communication**
- Coordination games + Social Conventions (with 3 agents)
- Roles
- Roles with communication
- Coordination Graphs

Social Conventions with Communication

- When **communication** is available:
  - **We only need to impose an ordering of the agents**
    - And this ordering is **common knowledge**



Social Conventions with Communication

- We first assume an ordering $i = 1, \ldots, n$ of agents

- Coordination can now be achieved by the following algorithm:

1. Each agent i (except agent 1) waits until all previous agents $1, \ldots, i-1$ in the ordering have broadcast their chosen actions

2. agent i computes its component a_i^* of an equilibrium that is consistent with the choices of the previous agents

3. agent i broadcasts a_i^* to all agents that have not chosen an action yet

Social Conventions with Communication

- Note that:
  - The **fixed ordering of the agents** together with the **wait/send primitives** induce a **synchronized sequential execution order of the coordination algorithm**

Social Conventions with Communication

- Recall the **car coordination game**:

Car 2

Car 1		*Cross*	*Stop*
	Cross	-1, -1	1, 0
	Stop	0, 1	0, 0

Social Conventions with Communication

- Let us assume the following ordering of agents:

(Car 1, Car2)

- Car 1 chooses action a_1^*= *Cross* of an equilibrium

		Car 2: ***Cross***	Car 2: ***Stop***
Car 1	***Cross***	-1, -1	<u>1</u>, <u>0</u>
	Stop	<u>0</u>, <u>1</u>	0, 0

- Car 1 broadcasts a_1^* = *Cross* to all agents that have not chosen an action yet
  - i.e., sends a message to Car 2

Social Conventions with Communication

- Car 2 waits until all previous agents in the ordering have broadcast their chosen actions
  - Thus, it waits a message from Car 1: a_1^* = *Cross*

- Car 2 computes its component a_2^* of an equilibrium that is consistent with the choices of the previous agents
  - Thus, a_2^* = *Stop*

		Car 2	
		Cross	***Stop***
Car 1	***Cross***	-1, -1	$\underline{1}$, $\underline{0}$
	~~***Stop***~~	~~$\underline{0}$, $\underline{1}$~~	~~0, 0~~

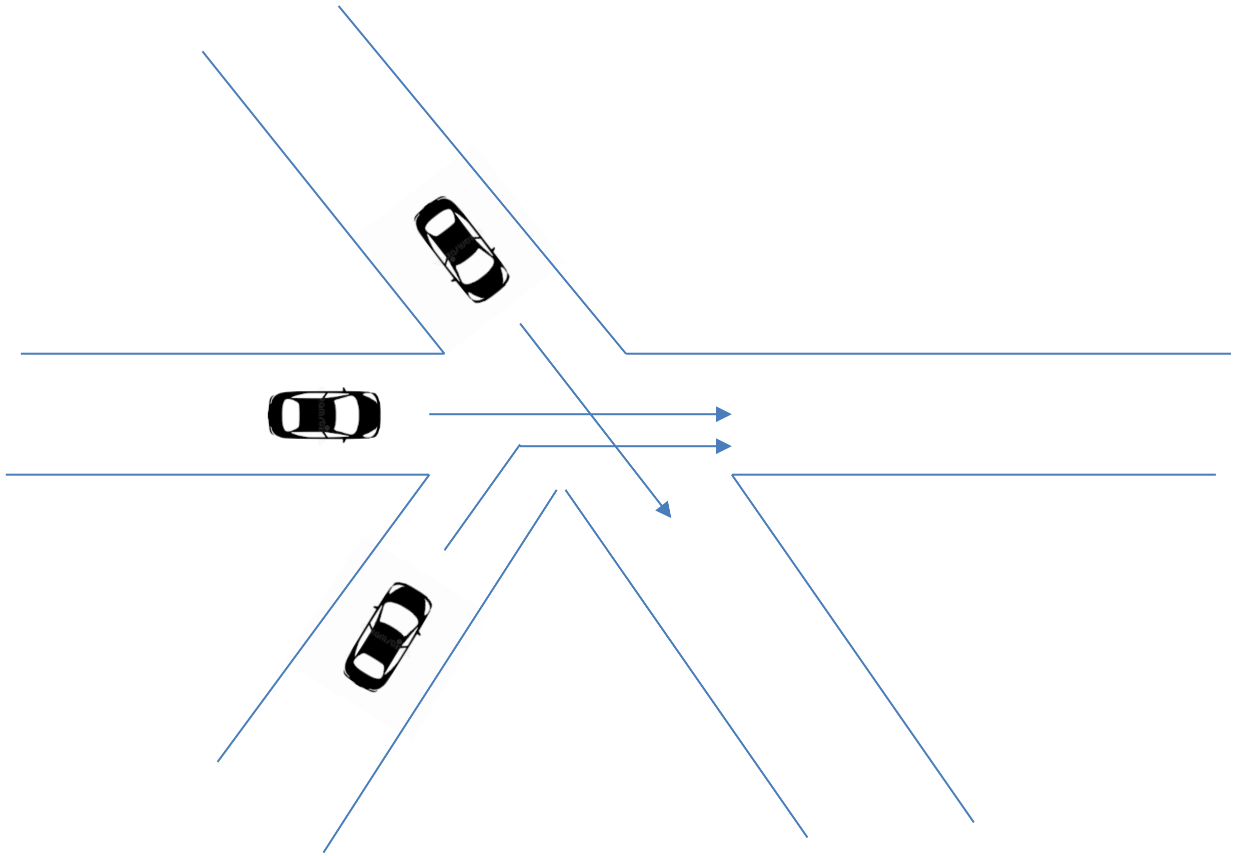
Outline

- Introduction to multiagent coordination
- Coordination games
- Social conventions
- Social conventions with communication
- **Coordination games + Social Conventions (with 3 agents)**
- Roles
- Roles with communication
- Coordination Graphs

Coordination Games

- Example of **THREE** autonomous vehicles at a crossroad

Coordination Games



Coordination Games

- Example:
    - Each agent/car wants to cross first
    - But if they two cross they will crash
- **How can we represent this game with the normal-form representation and solve it?**

Coordination Games

Joint actions (a)	$u_1(a)$	$u_2(a)$	$u_3(a)$
(cross, stop, stop)	1	0	0
(cross, cross, stop)	-1	-1	0
(cross, stop, cross)	-1	0	-1
(cross, cross, cross)	-1	-1	-1
(stop, stop, stop)	0	0	0
(stop, cross, stop)	0	1	0
(stop, stop, cross)	0	0	1
(stop, cross, cross)	0	-1	-1

Coordination Games

- Let us calculate the best response functions:

  - $B_1(a_2 = stop, a_3 = stop) = ?$
  - $B_1(a_2 = cross, a_3 = stop) = ?$
  - $B_1(a_2 = stop, a_3 = cross) = ?$
  - $B_1(a_2 = cross\ a_3 = cross) = ?$

  - $B_2(a_1 = stop, a_3 = stop) = ?$
  - $B_2(a_1 = cross, a_3 = stop) = ?$
  - $B_2(a_1 = stop, a_3 = cross) = ?$
  - $B_2(a_1 = cross, a_3 = cross) = ?$

  - $B_3(a_1 = stop, a_2 = stop) = ?$
  - $B_3(a_1 = cross, a_2 = stop) = ?$
  - $B_3(a_1 = stop, a_2 = cross) = ?$
  - $B_3(a_1 = cross\ a_2 = cross) = ?$

Coordination Games

Joint actions (a)	$u_1(a)$	$u_2(a)$	$u_3(a)$
(<u>cross</u>, stop, stop)	1	0	0
(stop, stop, stop)	0	0	0

	$stop, stop$
<u>cross</u>	1, 0, 0
stop	0, 0, 0

$$B_1(a_2 = stop, a_3 = stop) = \ cross$$

Coordination Games

Joint actions (a)	$u_1(a)$	$u_2(a)$	$u_3(a)$
(cross, cross, stop)	-1	-1	0
(stop, cross, stop)	0	1	0

	$cross, stop$
cross	-1, -1, 0
stop	0, 1, 0

$$B_1(a_2 = cross, a_3 = stop) = stop$$

Coordination Games

Joint actions (a)	$u_1(a)$	$u_2(a)$	$u_3(a)$
(cross, stop, cross)	-1	0	-1
(stop, stop, cross)	0	0	1

	$stop, cross$
cross	-1, 0, -1
stop	0, 0, 1

$$B_1(a_2 = stop, a_3 = cross) = stop$$

Coordination Games

Joint actions (a)	$u_1(a)$	$u_2(a)$	$u_3(a)$
(cross, cross, cross)	-1	-1	-1
(<u>stop</u>, cross, cross)	0	-1	-1

	$cross, cross$
cross	-1, -1, -1
<u>stop</u>	0, -1, 1

$$B_1(a_2 = cross, a_3 = cross) = stop$$

Coordination Games

Joint actions (a)	$u_1(a)$	$u_2(a)$	$u_3(a)$
(cross, stop, stop)	1	0	0
(cross, cross, stop)	-1	-1	0
(cross, stop, cross)	-1	0	-1
(cross, cross, cross)	-1	-1	-1
(stop, stop, stop)	0	0	0
(stop, cross, stop)	0	1	0
(stop, stop, cross)	0	0	1
(stop, cross, cross)	0	-1	-1

Coordination Games

Joint actions (a)	$u_1(a)$	$u_2(a)$	$u_3(a)$
(stop, stop, stop)	0	0	0
(<u>stop</u>, <u>cross</u>, stop)	0	1	0

	$stop, stop$
stop	0, 0, 0
<u>cross</u>	0, 1, 0

$$B_2(a_1 = stop, a_3 = stop) = cross$$

Coordination Games

Joint actions (a)	$u_1(a)$	$u_2(a)$	$u_3(a)$
(<u>cross</u>, <u>stop</u>, stop)	1	0	0
(cross, cross, stop)	-1	-1	0

	$cross, stop$
<u>stop</u>	1, 0, 0
cross	-1, -1, 0

$$B_2(a_1 = cross, a_3 = stop) = stop$$

Coordination Games

Joint actions (a)	$u_1(a)$	$u_2(a)$	$u_3(a)$
(<u>stop</u>, <u>stop</u>, cross)	0	0	1
(<u>stop</u>, cross, cross)	0	-1	-1

	$stop, cross$
<u>stop</u>	0, 0, 1
cross	0, -1, -1

$$B_2(a_1 = stop, a_3 = cross) = stop$$

Coordination Games

Joint actions (a)	$u_1(a)$	$u_2(a)$	$u_3(a)$
(cross, stop, cross)	-1	0	-1
(cross, cross, cross)	-1	-1	-1

	$cross, cross$
stop	-1, 0, -1
cross	-1, -1, -1

$$B_2(a_1 = cross, a_3 = cross) = stop$$

Coordination Games

Joint actions (a)	$u_1(a)$	$u_2(a)$	$u_3(a)$
(cross, stop, stop)	1	0	0
(cross, cross, stop)	-1	-1	0
(cross, stop, cross)	-1	0	-1
(cross, cross, cross)	-1	-1	-1
(stop, stop, stop)	0	0	0
(stop, cross, stop)	0	1	0
(stop, stop, cross)	0	0	1
(stop, cross, cross)	0	-1	-1

Coordination Games

Joint actions (a)	$u_1(a)$	$u_2(a)$	$u_3(a)$
(stop, stop, stop)	0	0	0
(<u>stop</u>, <u>stop</u>, <u>cross</u>)	0	0	1

	$stop, stop$
stop	0, 0, 0
<u>cross</u>	0, 0, 1

$$B_3(a_1 = stop, a_2 = stop) = cross$$

Coordination Games

Joint actions (a)	$u_1(a)$	$u_2(a)$	$u_3(a)$
(<u>cross</u>, <u>stop</u>, <u>stop</u>)	1	0	0
(cross, <u>stop</u>, cross)	-1	0	-1

	$cross, stop$
<u>stop</u>	1, 0, 0
cross	-1, 0, -1

$$B_3(a_1 = cross, a_2 = stop) = stop$$

Coordination Games

Joint actions (a)	$u_1(a)$	$u_2(a)$	$u_3(a)$
(stop, cross, stop)	0	1	0
(stop, cross, cross)	0	-1	-1

	$stop, cross$
stop	0, 1, 0
cross	0, -1, -1

$$B_3(a_1 = stop, a_2 = cross) = stop$$

Coordination Games

Joint actions (a)	$u_1(a)$	$u_2(a)$	$u_3(a)$
(cross, cross, stop)	-1	-1	0
(cross, cross, cross)	-1	-1	-1

	$cross, cross$
stop	-1, -1, 0
cross	-1, -1, -1

$$B_3(a_1 = cross, a_2 = cross) = stop$$

Coordination Games

Joint actions (a)	$u_1(a)$	$u_2(a)$	$u_3(a)$
(cross, stop, stop)	1	0	0
(cross, cross, stop)	-1	-1	0
(cross, stop, cross)	-1	0	-1
(cross, cross, cross)	-1	-1	-1
(stop, stop, stop)	0	0	0
(stop, cross, stop)	0	1	0
(stop, stop, cross)	0	0	1
(stop, cross, cross)	0	-1	-1

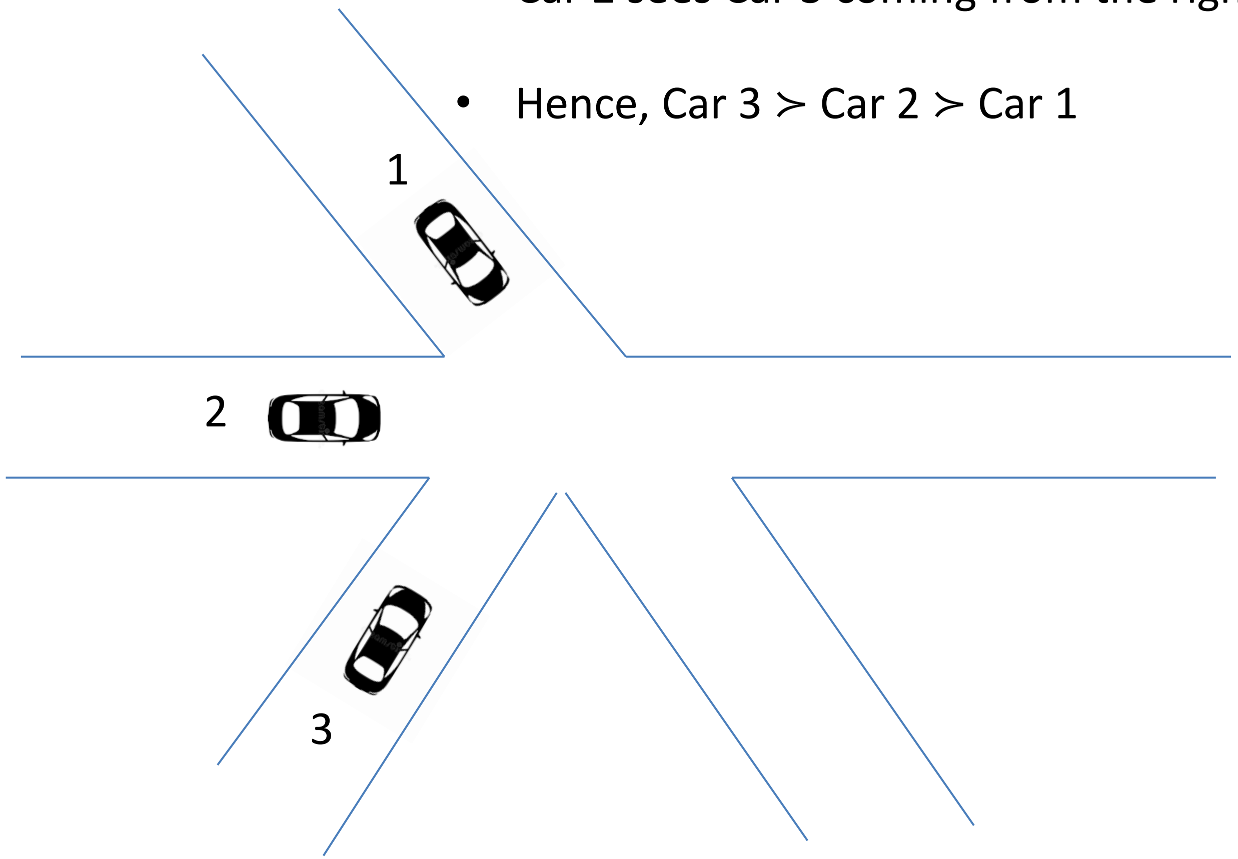
Pareto optimal Nash equilibria

Coordination Games & Social Conventions

- **Then each agent selects the first equilibrium according to an ordering scheme:**
  - **The driver coming from the right will always have priority over the other driver**
  - **Order the actions** by *Cross* $\succ$ *Stop*
- ***What action should each agent choose?***

Coordination Games & Social Conventions

- Car 1 sees Car 2 and Car 3 coming from the right
- Car 2 sees Car 3 coming from the right
- Hence, Car 3 $\succ$ Car 2 $\succ$ Car 1



Coordination Games & Social Conventions

Joint actions (a)	$u_1(a)$	$u_2(a)$	$u_3(a)$
(cross, stop, stop)	1	0	0
(cross, cross, stop)	-1	-1	0
(cross, stop, cross)	-1	0	-1
(cross, cross, cross)	-1	-1	-1
(stop, stop, stop)	0	0	0
(stop, cross, stop)	0	1	0
(stop, stop, cross)	0	0	1
(stop, cross, cross)	0	-1	-1

Now each agent can choose his individual action

Outline

- Introduction to multiagent coordination
- Coordination games
- Social conventions
- Social conventions with communication
- Coordination games + Social Conventions (with 3 agents)
- **Roles**
- Roles with communication
- Coordinaiton Graphs

Roles

- Coordination by **social conventions** relies on the following assumption:
  - An **agent can compute all equilibria in a game before choosing a single one**
- However, **computing equilibria can be expensive** when the agents’ action sets are large

Roles

- Hence, it makes sense **to start trying to reduce the size of the action sets**
- Such a reduction can lead to:
  - computational advantages in terms of **speed**
  - simplify the **equilibrium selection problem**

Roles

- A **natural way to reduce the agents' action sets** is to assign **roles** to the agents
- Formally:
  - **Given a particular state,** a role can be viewed as a **masking operator on the agent's action set**

Roles

- In practical terms:
  - if an **agent is assigned a role** at a particular state, then some of the **agent's actions are deactivated** at this state

For instance, a **robot soccer agent** that is currently with a **defender role cannot attempt to *Score***

Roles

- **Example**: two agents who want to go to the movies together but not alone

- if **Agent 2** is assigned a role that **forbids him to select the action *Thriller***
  - Because **Agent 2 is under 12**

		Agent 2	
		Thriller	***Comedy***
Agent 1	***Thriller***	$\underline{1}, \underline{1}$	0, 0
	Comedy	0, 0	$\underline{1}, \underline{1}$

Roles

So how can we assign a role to an agent?

- Suppose that:
  - there are n available roles (not necessarily distinct)
  - the state is fully observable to the agents

Roles

- Suppose that the following **facts are common knowledge** among agents:

  - There is a **fixed ordering** $\{1,2,\ldots,n\}$ **of the roles**
    - In other words, role 1 must be assigned first, followed by role 2, etc.

Roles

- Suppose that the **following facts are common knowledge** among agents:
  - For each role there is a **function that assigns to each agent a 'potential'**
    - The potential reflects **how appropriate that agent is for the specific role**, given the current state
  - For example, in robot soccer scenario:
    - the **potential** of a robot for the **role attacker** can be given by its **negative Euclidean distance to the ball**

Roles

- Suppose that the **following facts are common knowledge** among agents:
  - Each agent can be assigned only one role

Roles

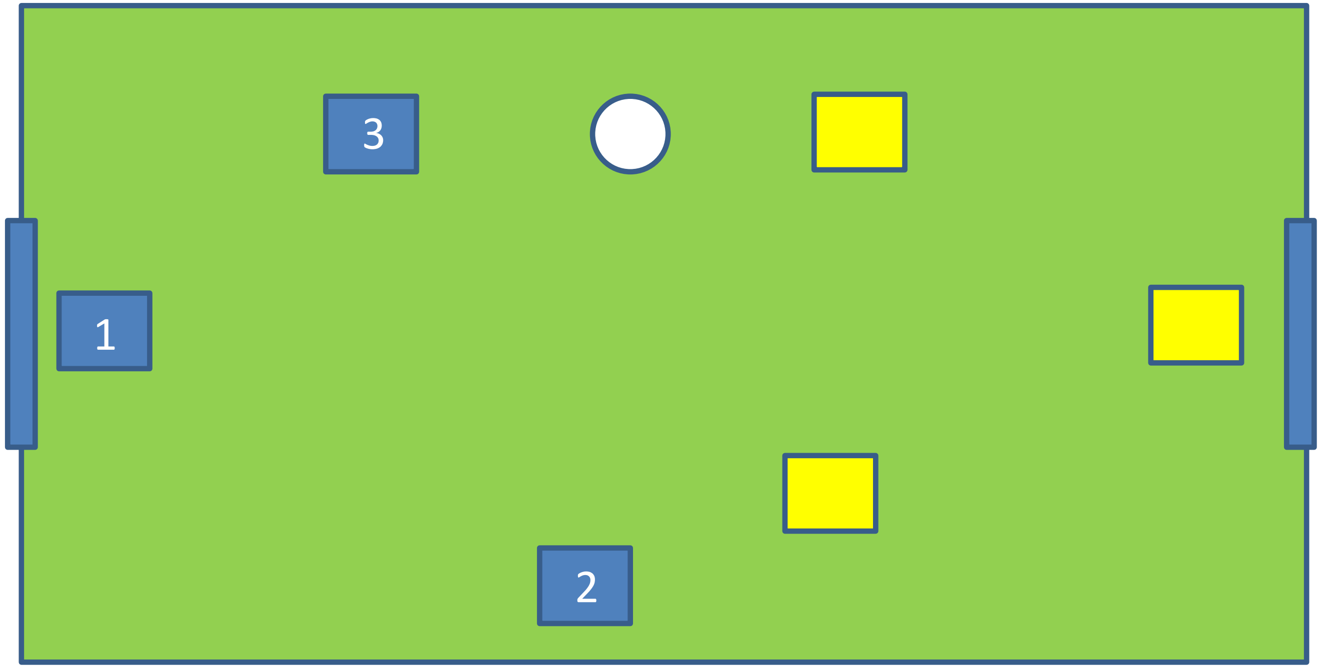
- Then **role assignment** can be carried out, for instance, by a **greedy algorithm**:
  - each role (starting from role 1) is assigned to the agent that has the highest potential for that role
  - and so on until all agents have been assigned a role

Roles

- Example: Robot soccer

Roles

- 3 robots: 1, 2, and 3
- Action sets: $A_1, A_2, A_3 = \{run, stay\}$



Roles

Joint actions (a)	$u_1(a)$	$u_2(a)$	$u_3(a)$
(run, run, run)	-1	-1	-1
(run, run, stay)	-1	-1	-1
(run, stay, run)	-1	-1	-1
(run, stay, stay)	1	1	1
(stay, run, run)	-1	-1	-1
(stay, run, stay)	1	1	1
(stay, stay, run)	1	1	1
(stay, stay, stay)	0	0	0

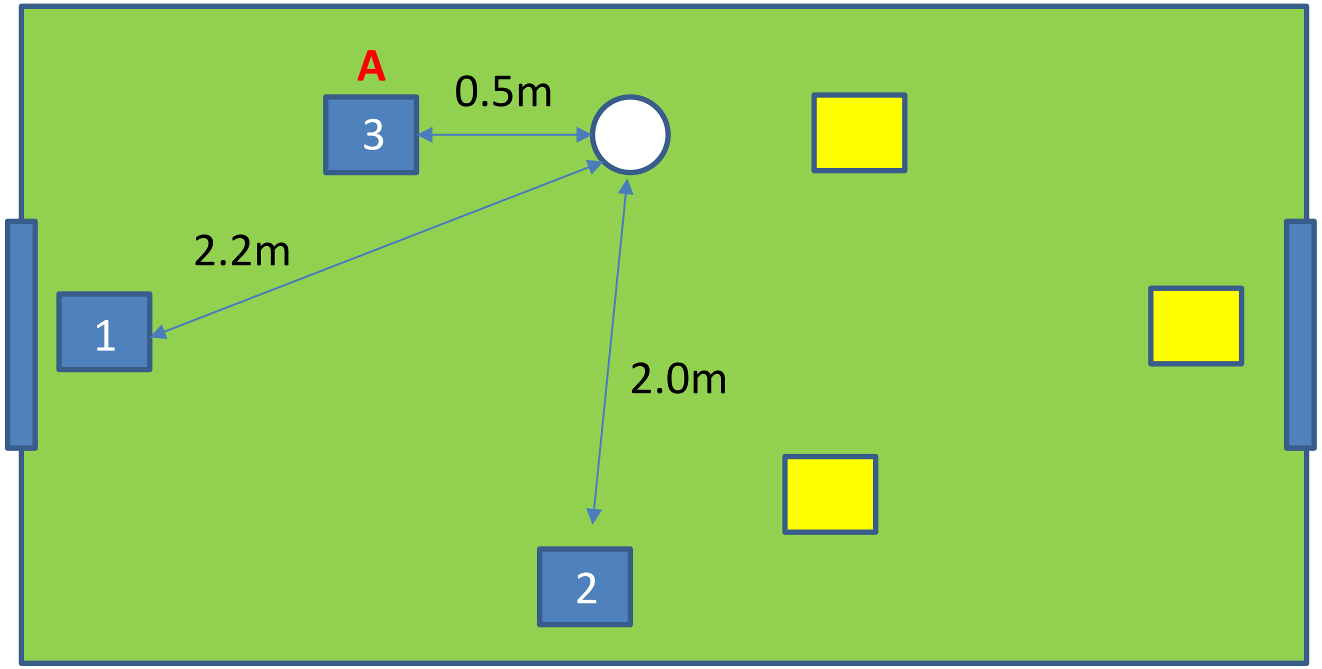
Pareto optimal Nash equilibria

Roles

- Fixed ordering of the roles = {Attacker, Goalkeeper, Defender}
- **Potential** for role **Attacker**: negative Euclidean distance to the ball
- **Potential** for role **Goalkeeper**: negative Euclidean distance to its own team's goal
- **Potential** for role **Defender**: positive Euclidean distance to the ball

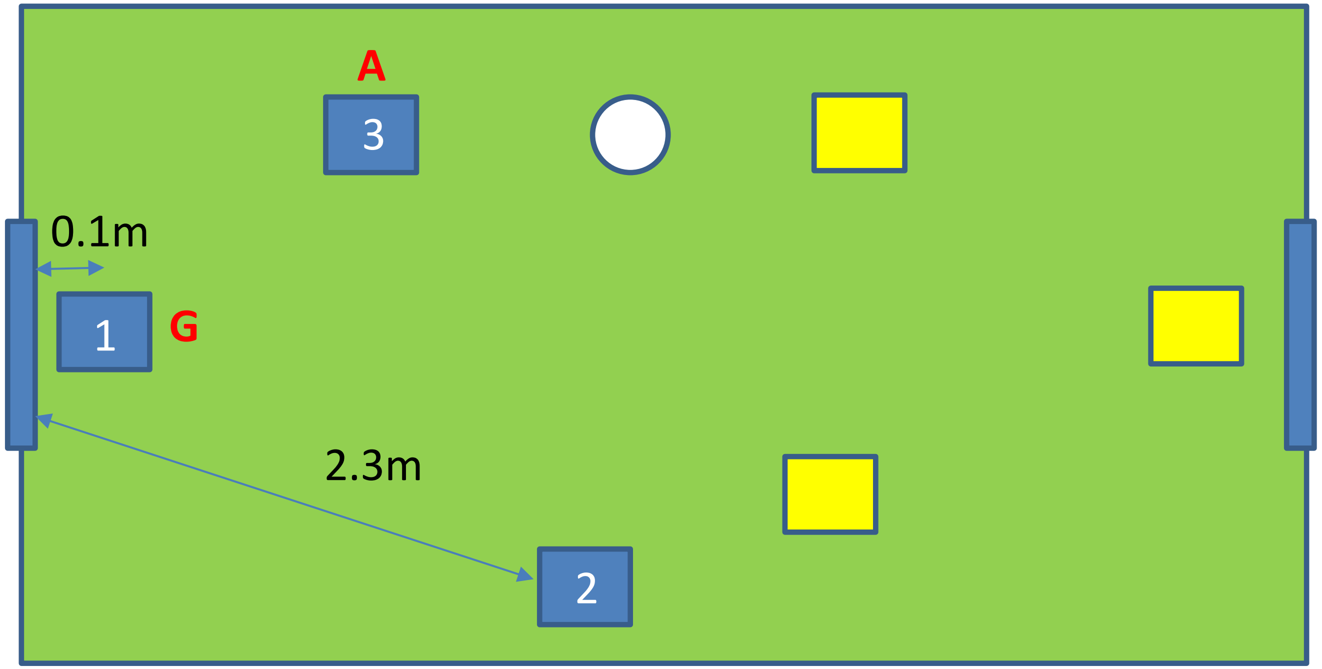
Roles

- Assigning the **Attacker** role (**A**)
  - Potential is the negative Euclidean distance to the ball



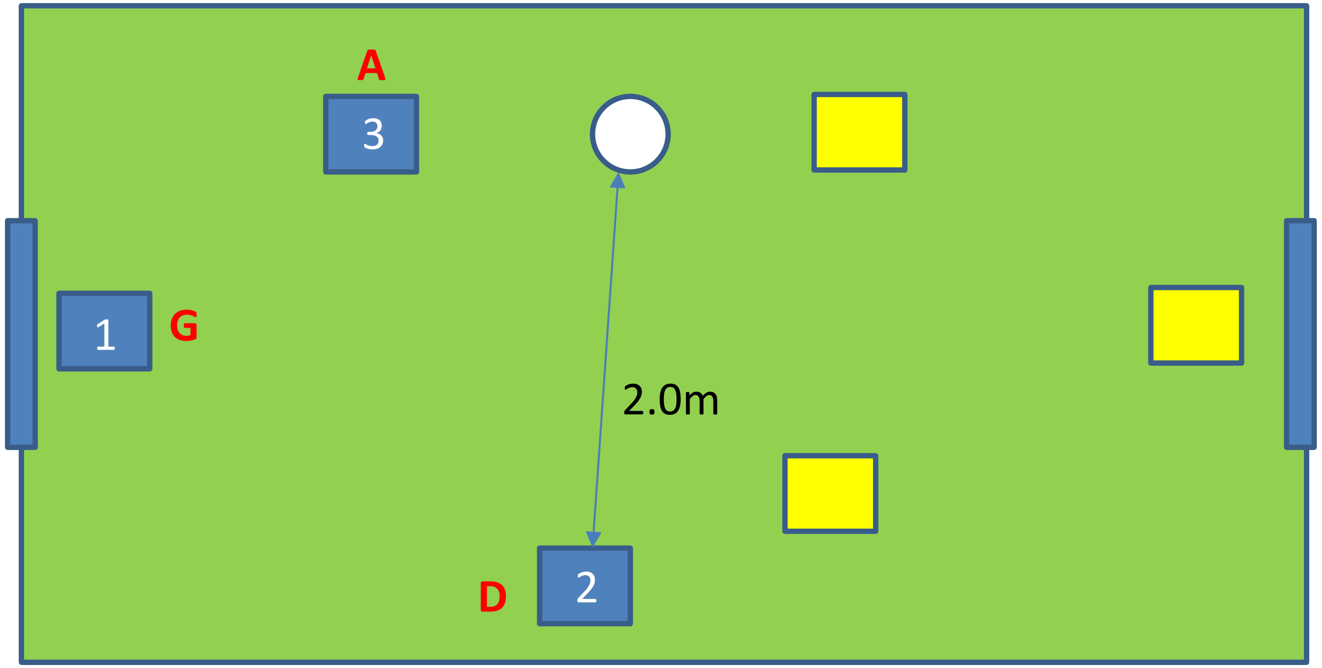
Roles

- Assigning the **Goalkeeper** role (**G**)
  - Potential is the negative Euclidean distance to its own team's goal



Roles

- Assigning the **Defender** role (D)
  - Potential is the positive Euclidean distance to the ball



Roles

- Hence, if an **agent is assigned a role** at a particular state then some of the **agent's actions are deactivated** at this state
  - The role **Defender** and **Goalkeeper** cannot choose the action $\boldsymbol{run}$ (so they can only choose $\boldsymbol{stay}$)
  - The role **Attacker** can either choose the action $\boldsymbol{run}$ or $\boldsymbol{stay}$

Joint actions (a)	$u_1(a)$	$u_2(a)$	$u_3(a)$
(<u>stay</u>, <u>stay</u>, <u>run</u>)	1	1	1
(stay, stay, stay)	0	0	0

Outline

- Introduction to multiagent coordination
- Coordination games
- Social conventions
- Social conventions with communication
- Coordination games + Social Conventions (with 3 agents)
- Roles
- **Roles with communication**
- Coordination Graphs

Roles with communication

- When **communication is not available:**
  - **Each agent can run** the previous algorithm identically and **in parallel**
    - Assuming that each agent can compute the **potential of each other agent**

Roles with communication

- When **communication is available:**
  - An agent only needs to compute its own **potentials** for the set of roles
  - Then broadcast them to the rest of the agents.
  - Next it can wait for all other potentials to arrive in order to compute the assignment of roles to agents as the previous algorithm

Roles with communication

<u>For</u> each agent i in parallel

  $I = \{\}$.

  <u>For</u> each role $j = 1, \ldots, n$

    Compute the potential r_{ij} of agent i for role j.

    Broadcast r_{ij} to all agents.

  <u>End</u>

  Wait until all $r_{i'j}$, for $j = 1, \ldots, n$, are received.

  <u>For</u> each role $j = 1, \ldots, n$

    Assign role j to agent $i^* = \arg\max_{i' \notin I}\{r_{i'j}\}$.

    Add i^* to I.

  <u>End</u>

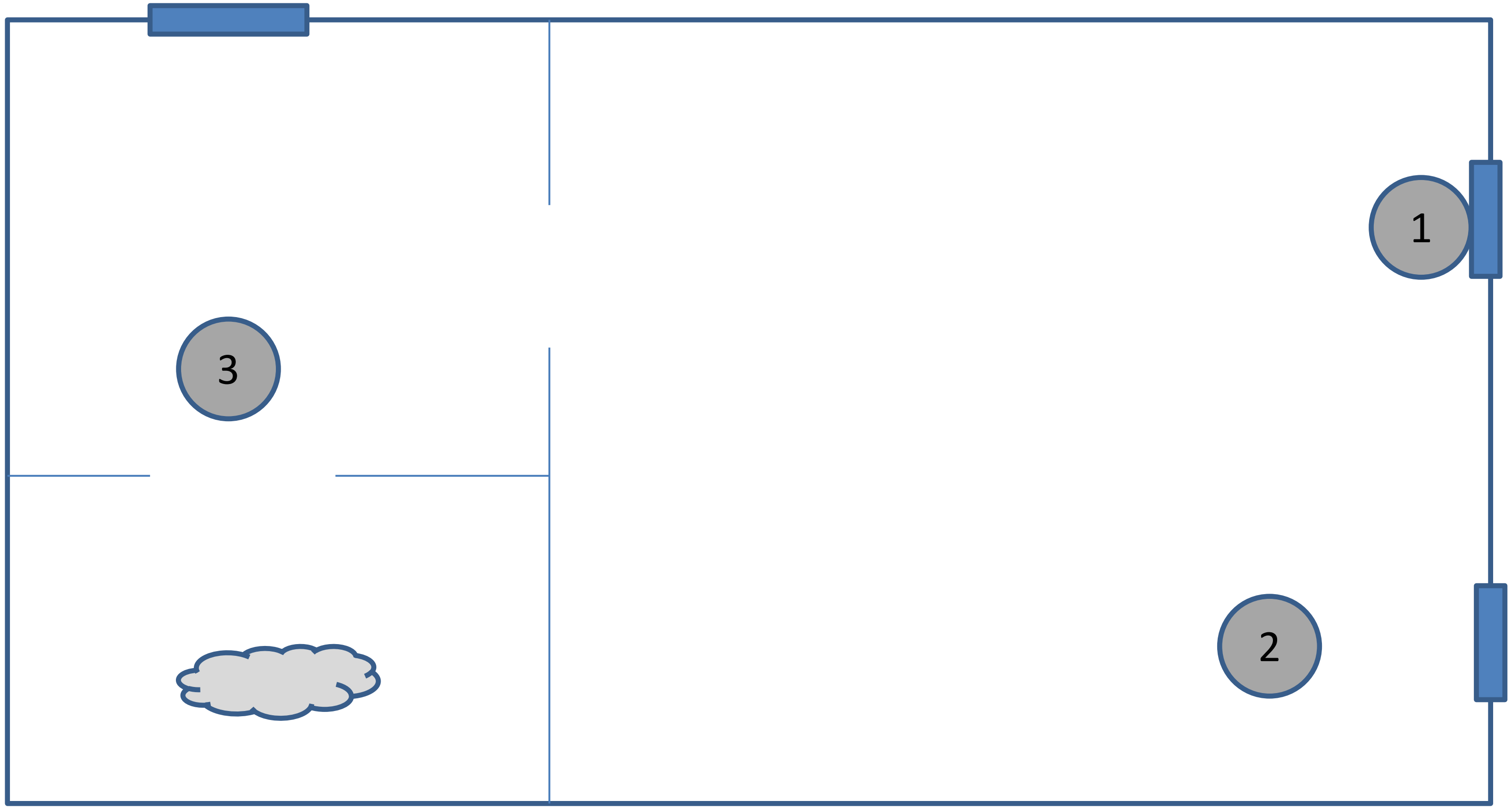
<u>End</u>

Final remarks

- In the **communication-free algorithm**
  - The greedy algorithm has a complexity of $O(n^2)$
- In the **communication-based algorithm**
  - Each agent needs to compute $O(n)$ potentials (its own)
  - However, this is added to the total number $O(n^2)$ of potentials that need to be broadcast and processed by the agents

Exercise

Cleaning robot coordination problem

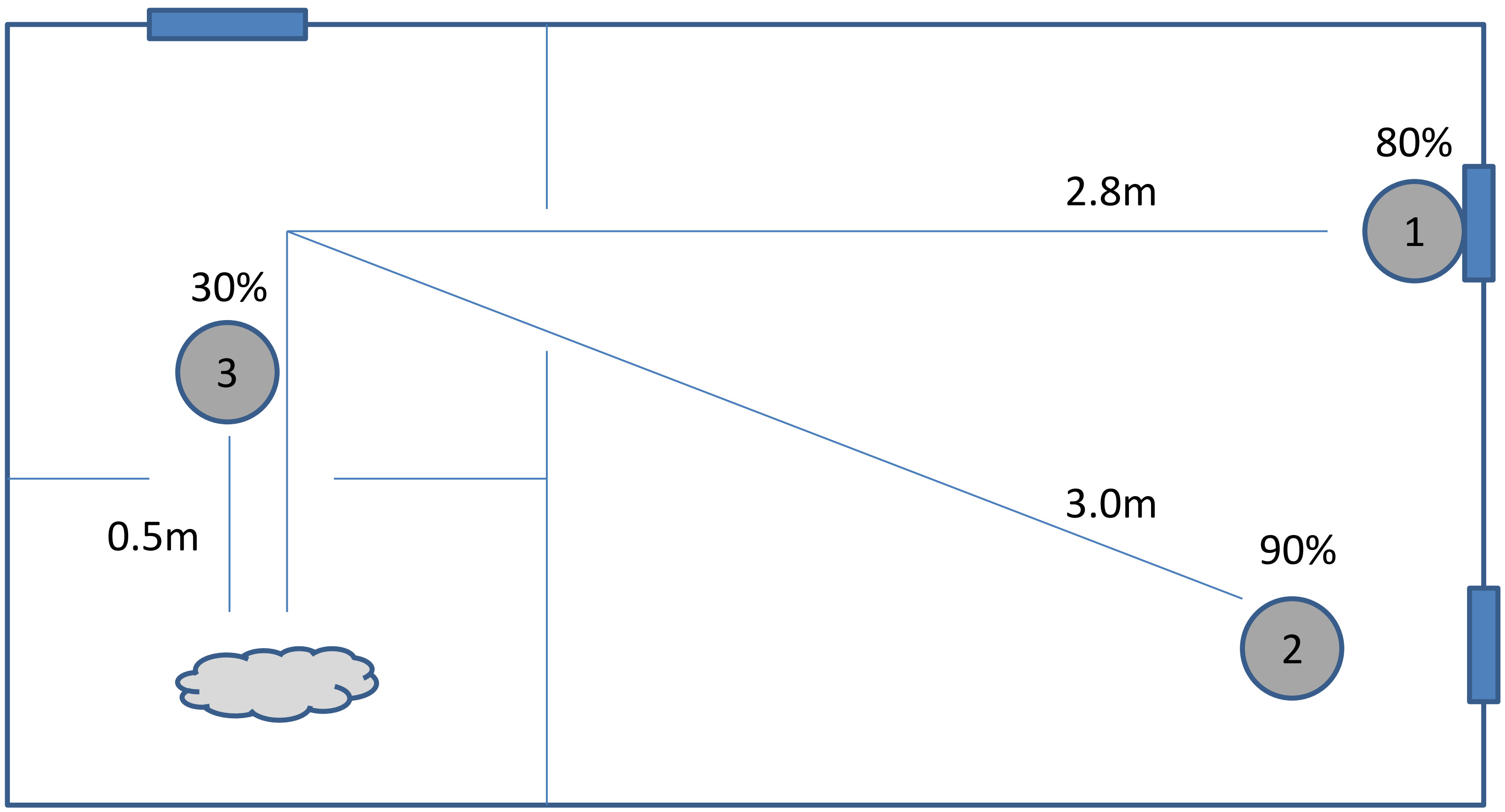


Exercise

- 3 cleaning robots: 1, 2, and 3
- Action sets: $A_1, A_2, A_3 = \{clean, charge, stay\}$
- Ordering of roles = {Need Charging (NC), Go Clean (GC), Wait for Work (WfW)}
- Potential for NC: negative of the battery level
- Potential for GC: negative distance to dirt
- Potential for WfW: positive distance to dirt

Exercise

Current State



Exercise

- Deactivated actions for each role:
  - Need Charging (NC): cannot clean and cannot stay
  - Go Clean (GC): cannot stay and cannot charge
  - Wait for Work (WfW): cannot clean
- **Use the communication-based algorithm to assign the roles and solve the coordination problem**

Exercise

- Role assignment
  - Need Charging (NC): 3
  - Go Clean (GC): 1
  - Wait for Work (WfW): 3
- **Can we improve this?**

Exercise

- **Change the ordering?**
- **Role admissibility / activation conditions**
  - Add a function that tests if the role is needed in the current state

```
NC.available(A)
        If A.battery_level < 15%
```

Outline

- Introduction to multiagent coordination
- Coordination games
- Social conventions
- Social conventions with communication
- Coordination games + Social Conventions (with 3 agents)
- Roles
- Roles with communication
- **Coordination Graphs**

Roles

- **Roles** can help us solve coordination game by **reducing the agents' action sets** prior to **computing the equilibria**

- However, computing equilibria in a subgame can still be a **difficult task** when the **number of agents is large**

- Recall that the number of joint actions is exponential in the number of agents:

$$m^n$$

where m is the number of the agents' actions and n is the number of agents

Coordination Graphs

- We also **need a method** that **reduces the number of agents** in a coordination game
- **Coordination graph** (Guestrin et al., 2002) is a framework for **solving large-scale coordination problems**

Coordination Graphs

- A coordination graph **decomposes a coordination game** into several smaller **subgames that are easier to solve**
  - In **roles**, we reduce the action sets and form a **single subgame**
  - In **coordination graphs**, we form **various subgames,** each involving a smaller number of agents

Coordination Graphs

- **Main assumption** in coordination graphs:

  - the **global payoff function** $u(a)$ can be written **as a linear combination of $\boldsymbol{k}$ local payoff functions** f_j, for $j = 1, \ldots, k$, each involving fewer agents

- For instance, a **coordination problem** with $\boldsymbol{n = 4}$ **agents** and $\boldsymbol{k = 3}$ **local payoff functions**, each involving two agents, could have the following decomposition:

$$u(a) = f_1(a_1, a_2) + f_2(a_1, a_3) + f_3(a_3, a_4)$$

Coordination Graphs

- Example of the decomposition of the global payoff function:

$$u(a) = f_1(a_1, a_2) + f_2(a_1, a_3) + f_3(a_3, a_4)$$

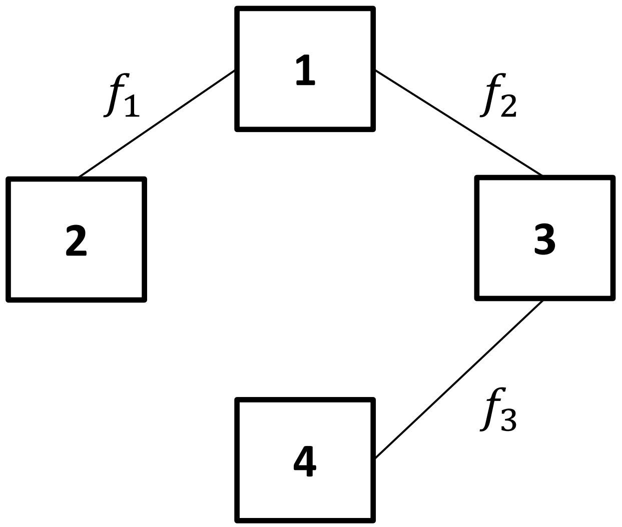
- Note:
  - $f_1(a_1, a_2)$ involves only agents 1 and 2, with their actions a_1 and a_2
  - $f_2(a_1, a_3)$ involves only agents 1 and 3, with their actions a_1 and a_3
  - $f_3(a_3, a_4)$ involves only agents 3 and 4, with their actions a_3 and a_4

Coordination Graphs

- Example of the decomposition of the global payoff function:

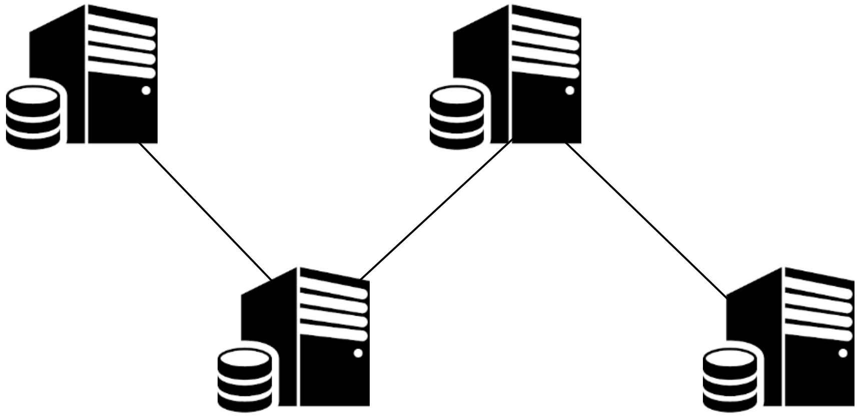
$$u(a) = f_1(a_1, a_2) + f_2(a_1, a_3) + f_3(a_3, a_4)$$

- The decomposition can be represented with the following graph:



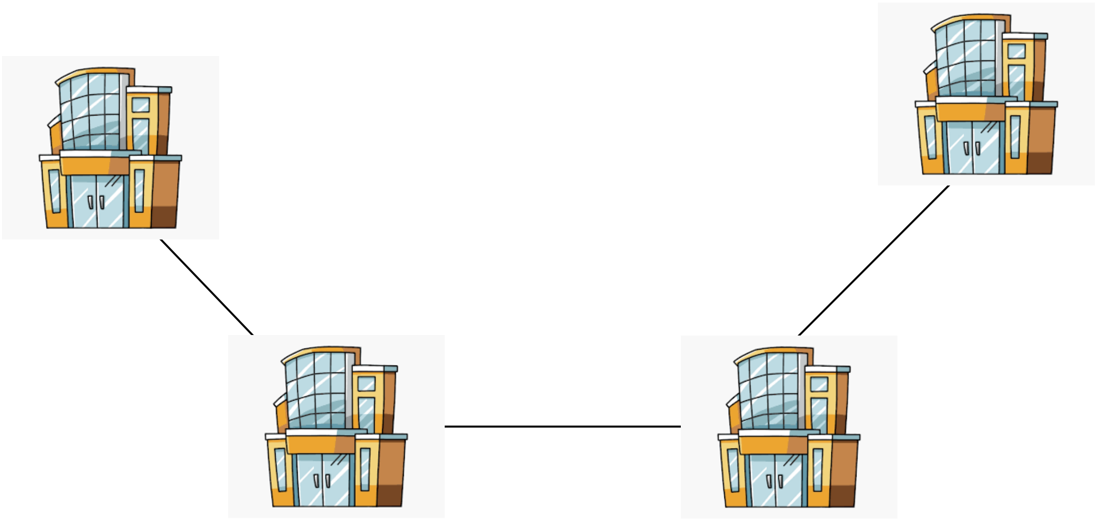
Coordination Graphs

- **Practical examples** that could use coordination graphs:
  - **Servers in a computer network** that need to coordinate in order to **optimize the overall network traffic**



Coordination Graphs

- **Practical examples** that could use coordination graphs:
  - Within a corporation, **offices in nearby cities** may need to coordinate in order to **maximize global sales**

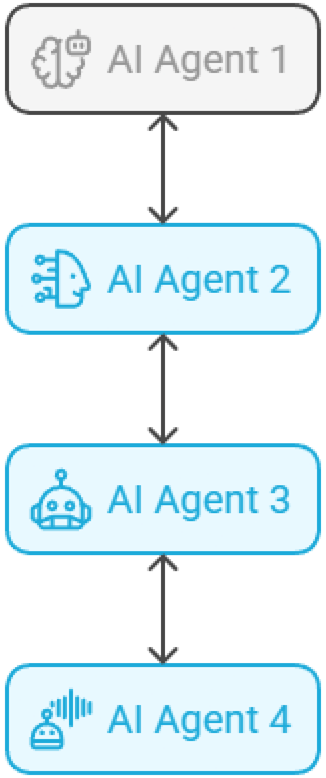


Coordination Graphs

- **Practical examples** that could use coordination graphs:
  - In a soccer team, **nearby players** may need to coordinate in order to **improve team performance**

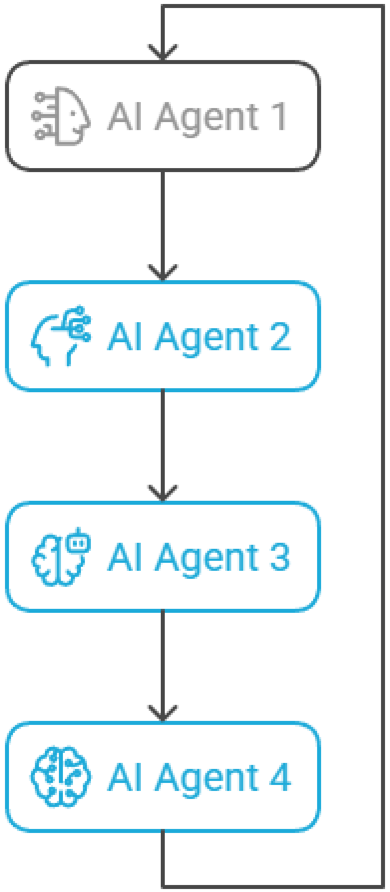
Coordination Graphs

- Linear coordination



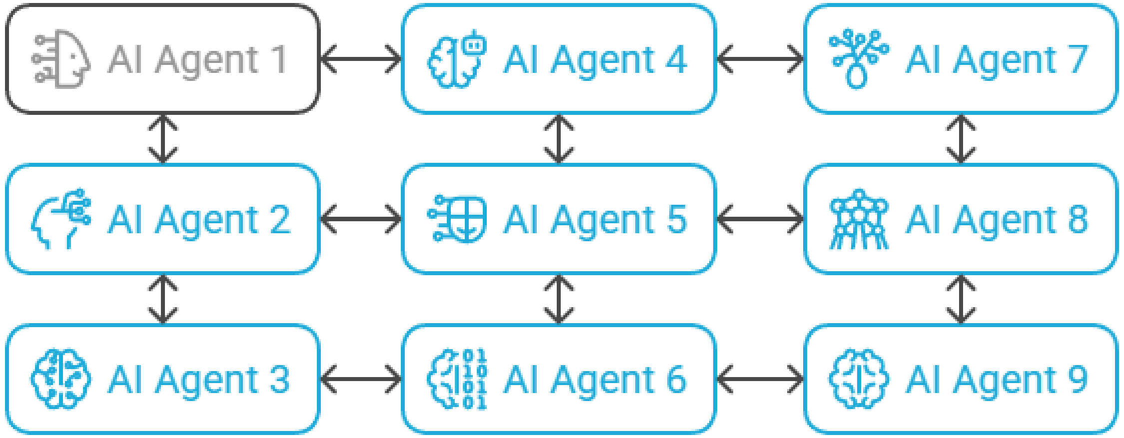
Coordination Graphs

- Cycle/ring coordination



Coordination Graphs

- Grid coordination

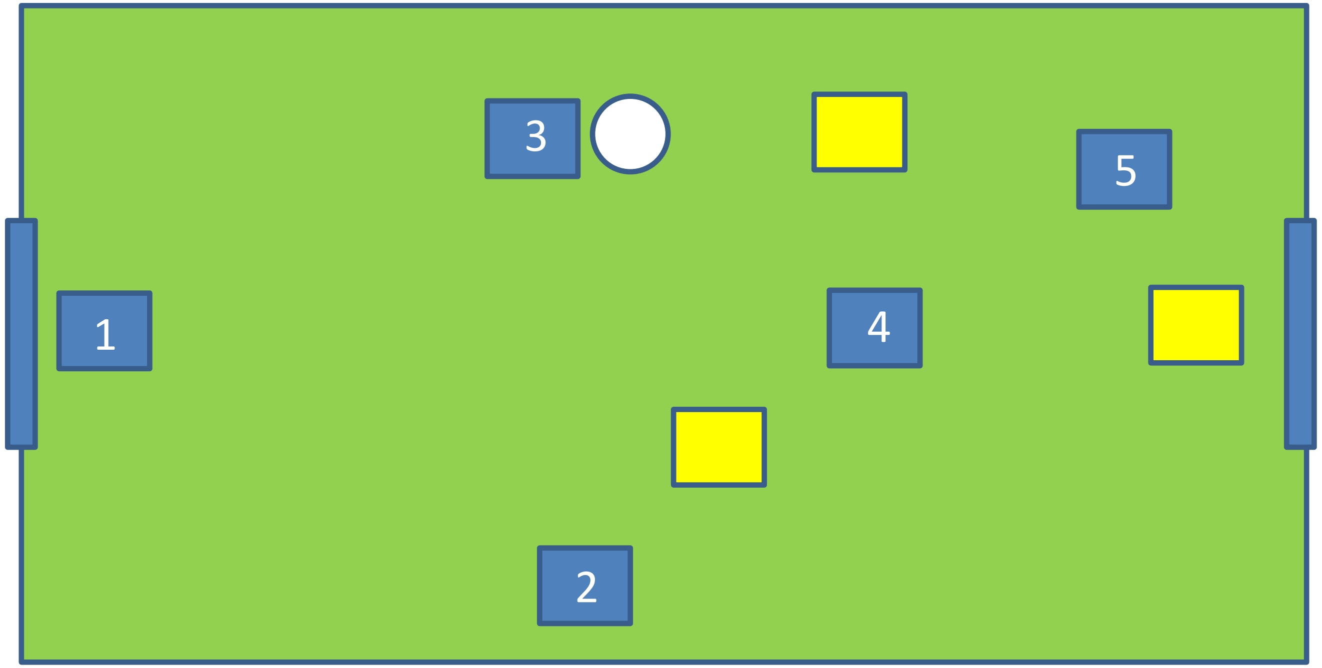


Coordination Graphs

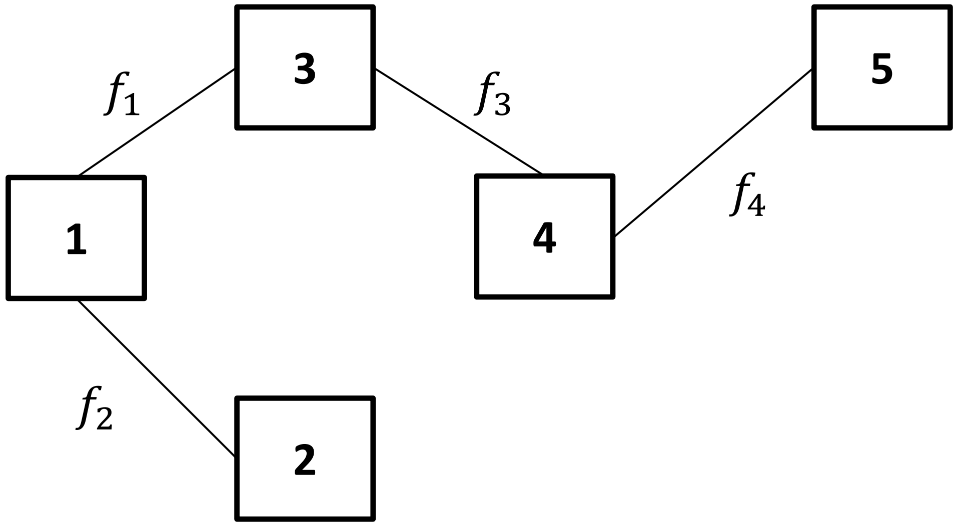
- Example: Robot soccer

Coordination Graphs

- 5 robots: 1, 2, 3, 4 and 5
- Action sets: $A_1, A_2, A_3, A_4, A_5 = \{stay, pass\}$



Coordination Graphs



$$u(a) = f_1(a_1, a_3) + f_2(a_1, a_2) + f_3(a_3, a_4) + f_4(a_4, a_5)$$

Thank You

rui.prada@tecnico.ulisboa.pt