

Combinatorics: An Introduction

Understanding Permutations and Combinations

- ▶ Combinatorics is a branch of mathematics that deals with counting, arranging, and choosing objects.
- ▶ Two fundamental concepts are permutations and combinations.

Permutations

- ▶ A permutation is an ordered arrangement of objects.
- ▶ The number of permutations of n distinct objects taken r at a time is denoted by $P(n, r)$ and given by:

$$P(n, r) = \frac{n!}{(n - r)!}$$

where $n!$ is the factorial of n .

Permutations Example

- ▶ Example: Permutations of 3 elements taken 2 at a time.

R Code

```
library(gtools)
```

```
permutations <- permutations(3, 2)
```

Combinations

- ▶ A combination is an unordered selection of objects.
- ▶ The number of combinations of n distinct objects taken r at a time is denoted by $C(n, r)$ and given by:

$$C(n, r) = \frac{n!}{r!(n-r)!}$$

Combinations Example

- ▶ Example: Combinations of 4 elements taken 2 at a time.

R Code

```
library(gtools)
```

```
combinations <- combinations(4, 2)
```

Combining Elements in R

Using the Base Package

- ▶ Introduction to combining vectors, lists, and data frames in R.

Combining Vectors

- Use the `c()` function to combine vectors.

Example

```
vector1 <- c(1, 2, 3)
```

```
vector2 <- c(4, 5, 6)
```

```
combined_vector <- c(vector1, vector2)
```

Combining Lists

- ▶ Use the `list()` function to create lists.
- ▶ Use the `c()` function to combine lists.

Example

```
list1 <- list(a = 1, b = 2)
list2 <- list(c = 3, d = 4)
combined_list <- c(list1, list2)
```


Combining Data Frames

- ▶ Use the `data.frame()` function to create data frames.
- ▶ Use functions like `rbind()` or `cbind()` to combine data frames.

Example

```
df1 <- data.frame(ID = 1:3, Value = c("A", "B", "C"))  
df2 <- data.frame(ID = 4:6, Value = c("D", "E", "F"))  
combined_df <- rbind(df1, df2)  # Combine rows
```

Conclusion

- ▶ Combining elements is essential for data manipulation in R.
- ▶ The base package provides functions like `c()`, `list()`, `data.frame()`, `rbind()`, and `cbind()` for combining vectors, lists, and data frames.