Likelihood: Outline

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1 Probability in Biology: Hobbs 4.1

1.1 Introduction

- Example: Tadpole observation in a pond.

1.2 Example: Tadpole Observation

Scenario:

- Collecting data on the number of tadpoles per volume of water in a pond.
- Observed 14 tadpoles in a 1 L sample.
- **TRUE** average number of tadpoles per liter of water in the pond is 23.

First Observation:

- It is Poisson
- Probability of observing 14 tadpoles: $P(y_1 = 14|\lambda = 23) = \text{Poisson }(y_1 = 14|\lambda = 23) = 0.0136$.

Second Observation:

- Probability of observing 34 tadpoles: $P(y_2 = 34|\lambda = 23) = \text{Poisson }(y_2 = 34|\lambda = 23) = 0.0069$.

Joint Probability:

- Assuming independence: Joint probability $= 0.0136 \times 0.0069 = 9.38 \times 10^{-5}$.

1.3 Independence of Observations

- Independence assumption: Knowledge of one observation tells us nothing about the other.
- Joint probability calculation extended to any number of independent observations.

1.4 Remarks

- Probability calculations provide insights into the likelihood of observations given a fixed average.
- Independence assumption crucial for joint probability calculations.
- The Poisson distribution to model catching probabilities.

2 Probability in Biology: Hobbs 4.2

2.1 Introduction

- Investigating decomposition of leaf litter over time.
- Using a simple model of exponential decay: $\mu_t = e^{-kt}$.
- Data: y_t - observed proportions, **modeled with a beta distribution**.
- Parameters: k (decay rate) and σ^2 (variance).

2.2 Beta Distribution for y_t

- Model the probability density of y_t with a beta distribution
- Moment matching for α_t and β_t:

2.3 Conditional on Decay Rate k and σ^2

- Conditional on known, fixed decay rate $k = 0.01\, day^{-1}$ and known, fixed $\sigma^2 = 6 \times 10^{-4}$.
- Calculate parameters for the beta distribution on day 30: $\alpha_{30} = 236.33$ $\beta_{30} = 82.68$

2.4 Probability Density Calculation

- Given $y_{30} = 0.7$, calculate the probability density:

  f($\mathrm{y}_{30} = 0.7$) = 4.040

- Interpretation: The probability that 70% of the mass remains at time $t = 30$ is 4.040.

2.5 Remarks

- The beta distribution to model decay over time.
- Moment matching provides a method for estimating distribution parameters.

3 Introduction to Likelihood

3.1 Introduction

- Likelihood measures the support provided by the observed data for different values of the parameter in a statistical model.
- The likelihood function is the foundation of maximum likelihood estimation (MLE).

3.2 Likelihood Function

- The likelihood function, denoted as $L(\theta; \mathbf{x})$, represents the probability of observing the given data $\mathbf{x}$ for various parameter values θ in the model.
- The likelihood function is not a probability distribution but provides a basis for estimating parameters.

3.3 Likelihood Example: Coin Toss

- Consider a simple example: coin toss.
- Let θ be the probability of getting heads ($\theta \in [0, 1]$).
- If we

  observe k heads in n tosses, the likelihood function is given by the binomial distribution:

  $$\mathrm{L}(\theta; k, n) = nk\theta^k(1 - \theta)^{n-k}$$

3.4 Interpretation of Likelihood

- Likelihood is not a probability, but it measures the compatibility of the observed data with different parameter values.
- Larger likelihood values indicate a better fit of the model to the observed data.
- The goal is to find the