Probability and Statistics: Outline

DRME

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Understanding the Basics

Overview of the Concept: The Set of All Possible Events

- Fundamental concept in probability theory.

- Represents all potential outcomes in an experiment.

- Significance in defining sample space and event probabilities.

Examples: Coin Toss and Rolling a Die

- Coin Toss: {Heads, Tails}

- Die Roll: {1, 2, 3, 4, 5, 6}

Coin Flips: Why You Should Care

- Illustrate it using coin flips.

- Linking the concept to real-world scenarios.

Probability: Outside or Inside the Head

- Outside the head: Long-run relative frequency.

- Inside the head: Subjective belief.

- Probabilities assign numbers to possibilities.

Understanding Probabilities: Coin Toss, Die Roll, Card Drawing

- Coin Toss: 0.5 for both Heads and Tails.

- Die Roll: $\frac{1}{6}$ for each number.

- Card Drawing: $\frac{4}{52}$ for Aces, $\frac{26}{52}$ for red cards.

Probability Distributions: Discrete and Continuous

- Overview of probability distributions.
- Discrete: PMF (Bernoulli, Binomial, Poisson).
- Continuous: PDF (Uniform, Exponential, Normal).

Properties of PDFs and Focus on Normal Distribution

- Emphasis on normalization and non-negativity.
- Special focus on the normal distribution.

Mean, Variance, and Highest Density Interval (HDI)

- Mean as minimized variance.
- Definition and importance of HDI.

Two-Way Distributions: Conditional Probability and Independence

- Conditional Probability: $P(A|B) = \frac{P(A \cap B)}{P(B)}$
- Independence: $P(A \cap B) = P(A) \cdot P(B)$

Joint Probability and Applications

- Definition and interpretation.
- Applications in finance, medicine, and weather.

Rules of Factoring Joint Probabilities

1. Product Rule

$$P(A \cap B) = P(A|B) \cdot P(B)$$

- Genetic inheritance and species coexistence examples.

2. Chain Rule

$$P(A \cap B \cap C) = P(A|B \cap C) \cdot P(B|C) \cdot P(C)$$

- Application in gene expression and ecosystem dynamics.

3. Marginalization

$$P(A) = \sum_{B} P(A \cap B)$$

- Example with a joint probability distribution table.