

source/linear_least_squares.hpp

```
1  #pragma once
2
3  #include "qr_factorization.hpp"
4
5  // Backwards gaussian elimination.  $O(N^2)$  complexity.
6  //
7  // Assumes 'R' to be upper-triangular matrix.
8  //
9  inline Vector backwards_gaussian_elimination(const Matrix& R, Vector rhs) {
10     for (Idx i = R.rows() - 1; i >= 0; --i) {
11         for (Idx j = i + 1; j < R.cols(); ++j) rhs(i) -= R(i, j) * rhs(j);
12         rhs(i) /= R(i, i);
13     }
14
15     return rhs;
16 }
17
18 // Linear Least Squares problem.  $O(N^3)$  complexity.
19 //
20 // LLS has a following solution:
21 //      $x = A^+ b$ 
22 //     where  $A^+ = R^{-1} * Q^T$ 
23 //
24 // We can rewrite it as a SLAE:
25 //      $R x = Q^T b$ 
26 //
27 // since 'R' is upper-triangular, we only need to do the backwards gaussian
28 // elimination, which is  $O(N^2)$ .
29 //
30 Vector linear_least_squares(const Matrix& A, const Matrix& b) {
31     // Computing QR the usual way
32     // const auto [Q, R] = qr_factorize(A);
33     // const auto x      = backwards_gaussian_elimination(R, Q.transpose() * b);
34
35     // Computing QR with  $(Q^T * b)$  directly
36     const auto [QTb, R] = qr_factorize_lls(A, b);
37     const auto x        = backwards_gaussian_elimination(R, QTb);
38
39     return x;
40 }
```