Short report on module 3 exam:

1) Outline of numerical procedure

Euler - Maruyama method was used as a numerical procedure. Equation for price is a basic SDE:

$$dS_t = S_t\mu dt + S_t\sigma dW_t.$$

, where t is time, sigma is volatility, S is a stock price, mu is a drift rate, dW are independent random variables with distribution N(0,dt).

The exact solution is:

$$S_t = S_0 e^{\sigma W_t + (\mu - \frac{1}{2}\sigma^2)t}.$$

So we can model Stock price. Than we use results to get the Payoff (formulas for payoff are below)

Formulas for lookback option payoff with floating strike:

Call:

$$LC_{float} = \max(S_T - S_{min}, 0) = S_T - S_{min}$$

Put:

$$LP_{float} = \max(S_{max} - S_T, 0) = S_{max} - S_T$$

Formulas for lookback option payoff with fixed strike:

Call:

$$LC_{fix} = \max(S_{max} - K, 0)$$

Put:

$$LP_{fix} = \max(K - S_{min}, 0)$$

To get the the value of the option we should multiply Payoff by discount factor.

For Binary call

$$\text{Binary Call Option Payoff}$$
$$= \begin{cases} 1 & , \quad \text{Underlying's Price} \geq \text{Exercise Price} \\ 0 & , \quad \text{Exercise Price} < \text{Underlying's Price} \end{cases}$$

Binary put is the opposite to call. Besides there were two measurement of maximum: discrete and continuous. The difference is quite obvious: there is a fixed timestep to determine maximum in a discrete case.
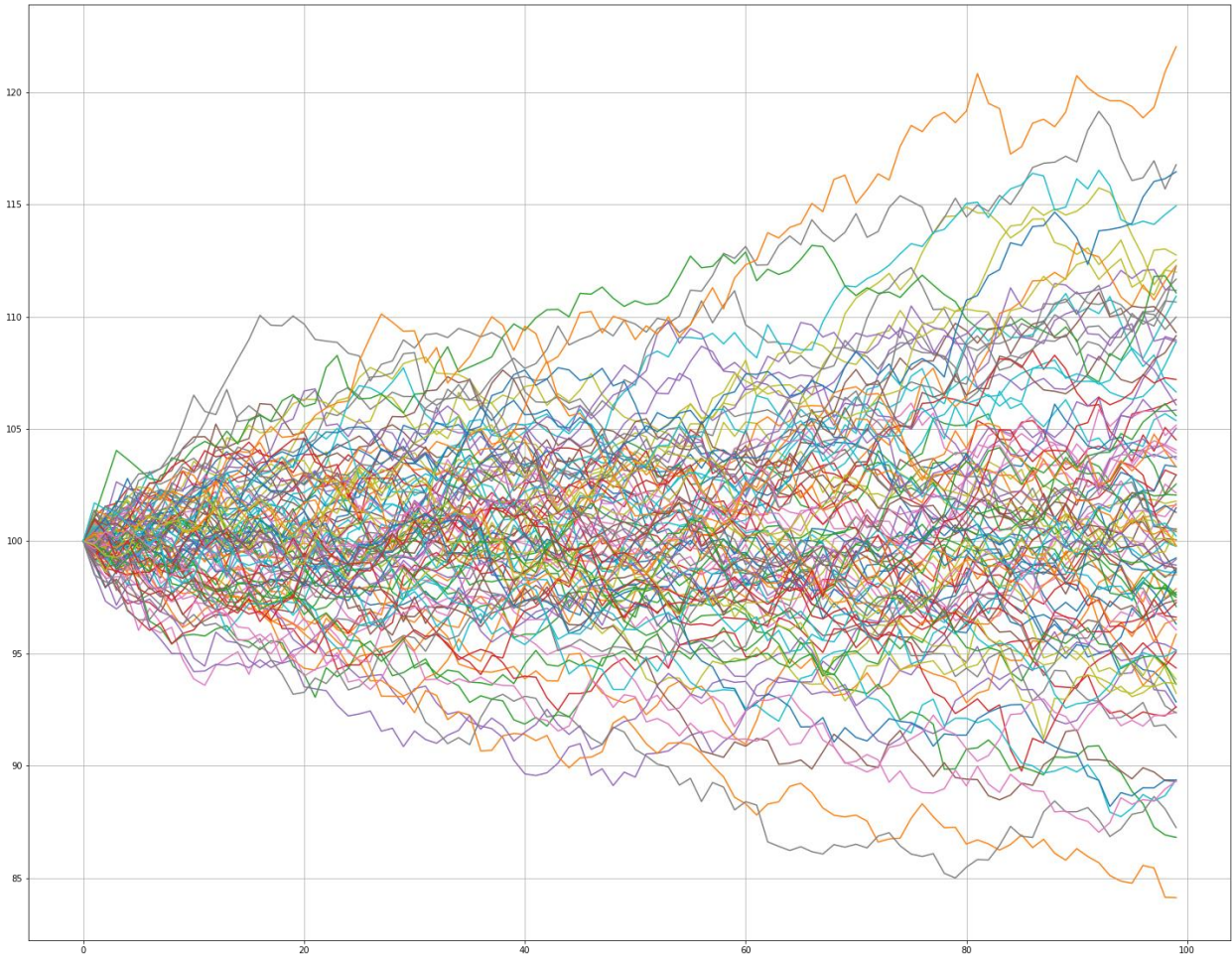
2) Results for different numbers of paths:

For n=1000
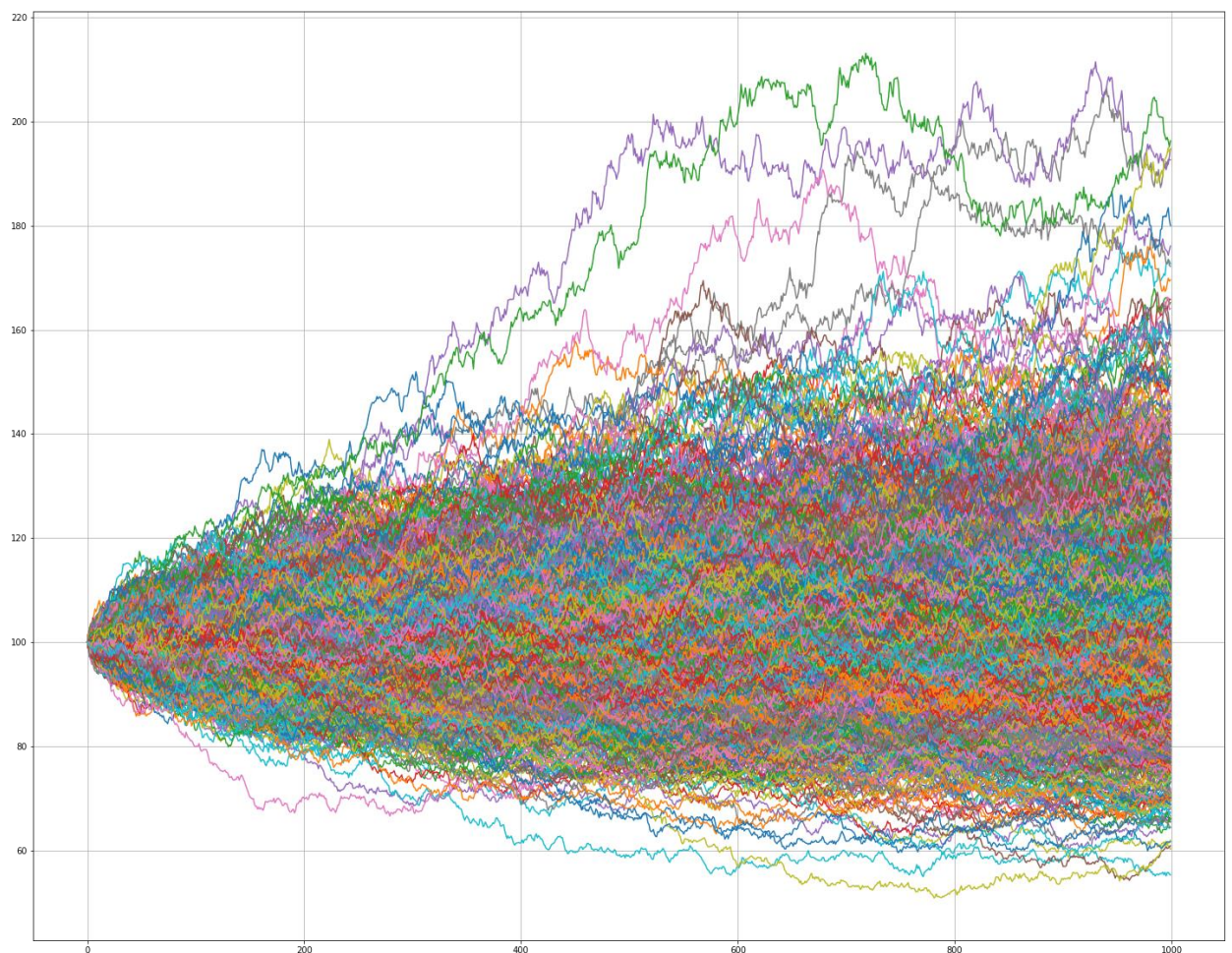
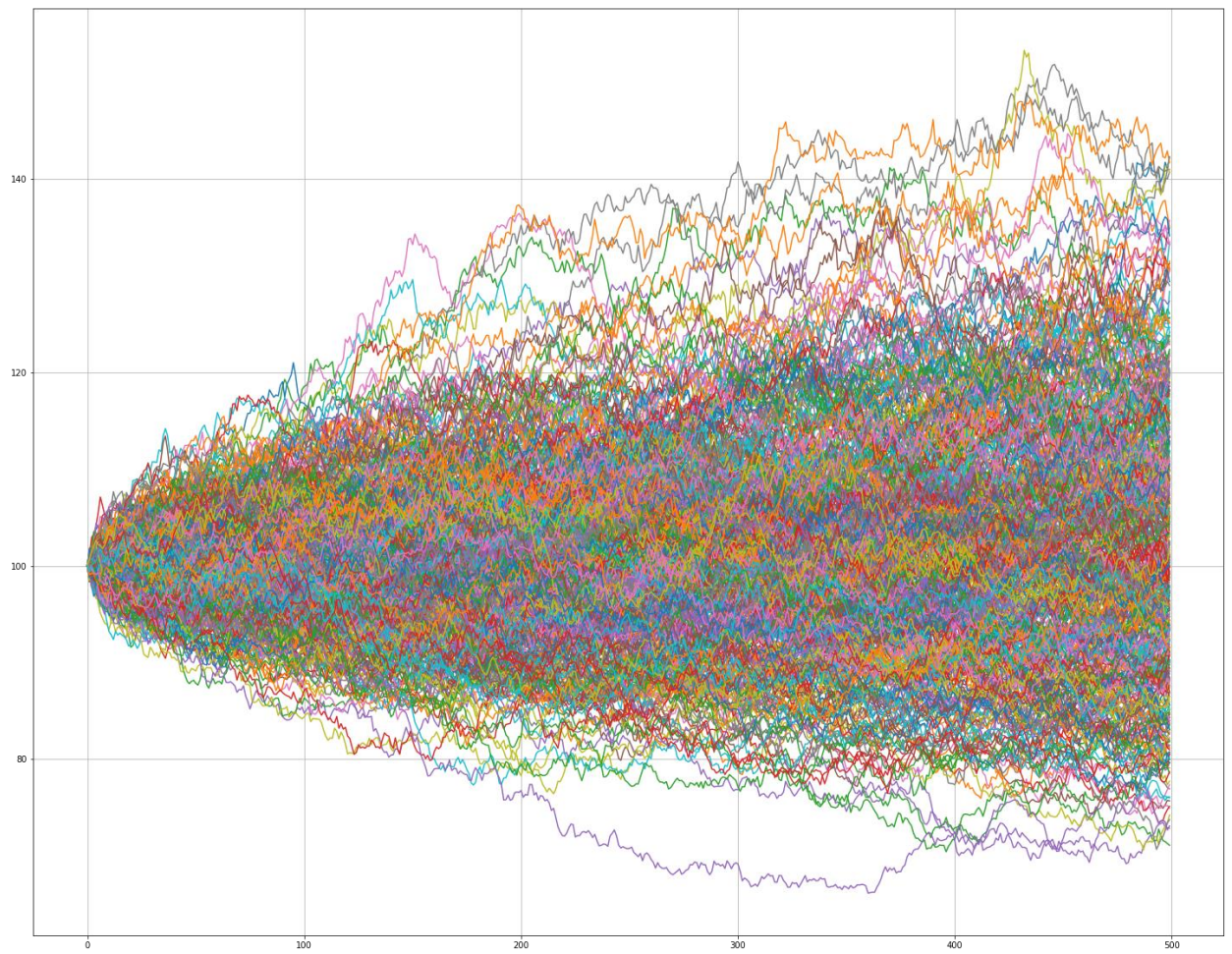| Binary_call	Binary_put	Call_Floating	Put_Floating	Call_Fixed	Put_Fixed	Call_Floating_discrete	Put_Floating_discrete	Call_Fixed_discrete	Put_Fixed_discrete
0.52603	0.4252	16.91324	13.872898	18.637139	12.148999	16.282498	12.999716	17.672867	11.476363

For n=100

Binary_call	Binary_put	Call_Floating	Put_Floating	Call_Fixed	Put_Fixed	Call_Floating_discrete	Put_Floating_discrete	Call_Fixed_discrete	Put_Fixed_discrete
0.504152	0.447078	5.271355	4.132638	5.262695	4.141297	4.601699	3.527115	4.319624	3.238606

For n=500

Result

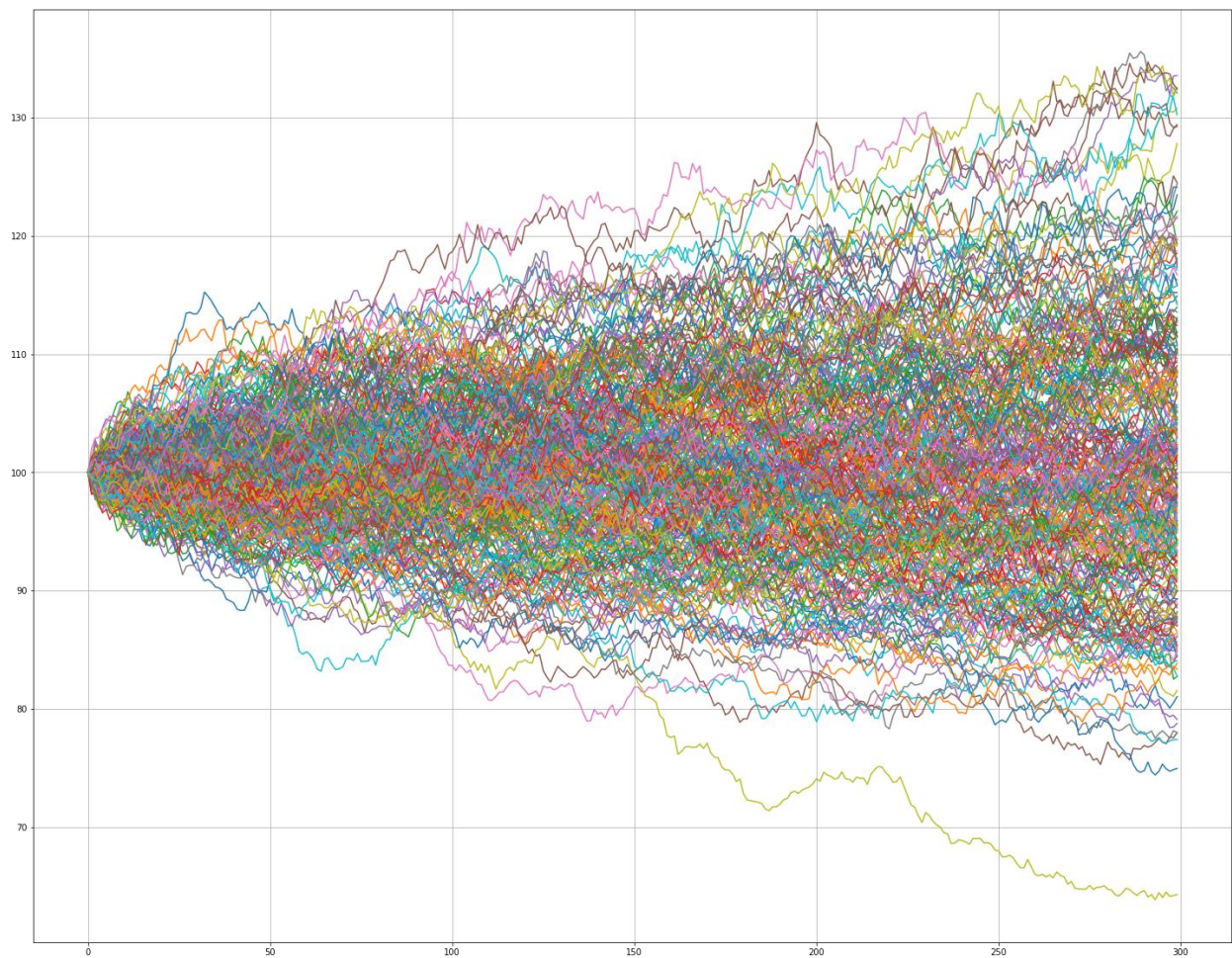| Binary_call	Binary_put	Call_Floating	Put_Floating	Call_Fixed	Put_Fixed	Call_Floating_discrete	Put_Floating_discrete	Call_Fixed_discrete	Put_Fixed_discrete
0.517469	0.433761	11.23279	9.716642	12.133925	8.815507	10.582973	8.901506	11.211236	8.114104

For n=300

Binary_call	Binary_put	Call_Floating	Put_Floating	Call_Fixed	Put_Fixed	Call_Floating_discrete	Put_Floating_discrete	Call_Fixed_discrete	Put_Fixed_discrete
0.459761	0.491469	8.247322	8.074538	9.065211	7.256649	7.582351	7.275734	8.122183	6.492578

Changing volatility to 0.4

For n=300

Result

Binary_call	Binary_put	Call_Floating	Put_Floating	Call_Fixed	Put_Fixed	Call_Floating_discrete	Put_Floating_discrete	Call_Fixed_discrete	Put_Fixed_discrete
0.469273	0.481956	16.123063	15.517685	18.114092	13.526657	14.86156	13.950908	16.213066	12.018514

For n=100

Result

Binary_call	Binary_put	Call_Floating	Put_Floating	Call_Fixed	Put_Fixed	Call_Floating_discrete	Put_Floating_discrete	Call_Fixed_discrete	Put_Fixed_discrete
0.447078	0.504152	9.473365	9.649243	10.234602	8.888006	8.195483	8.138369	8.208152	6.955356

For n=500

Result

Binary_call	Binary_put	Call_Floating	Put_Floating	Call_Fixed	Put_Fixed	Call_Floating_discrete	Put_Floating_discrete	Call_Fixed_discrete	Put_Fixed_discrete
0.468005	0.483225	20.530731	21.510562	23.90306	18.138234	19.383737	19.656949	21.720071	16.850974

Changing strike to 105

For n=100

Binary_call	Binary_put	Call_Floating	Put_Floating	Call_Fixed	Put_Fixed	Call_Floating_discrete	Put_Floating_discrete	Call_Fixed_discrete	Put_Fixed_discrete
0.294881	0.656348	5.566759	3.617876	1.9711	8.63398	4.860145	2.949105	1.388836	7.786087

For n=300

Binary_call	Binary_put	Call_Floating	Put_Floating	Call_Fixed	Put_Fixed	Call_Floating_discrete	Put_Floating_discrete	Call_Fixed_discrete	Put_Fixed_discrete
0.180734	0.770496	4.749723	4.046734	1.636613	9.035766	4.109069	3.412955	1.241745	8.274365

3)   Changing number of paths we can see very clear thing, that option Price slowly increasing with the number of paths. Other thing that I notice is that I can tune volatility to make the same option price, as for option with more time to expiry (results for pricing options n=300 with volatility 0.4 are close to results with n=1000 and vol=0.2) . And I can see that changing volatility, price is changing much faster. For different strike rate results does not change very fast. Speed of changing is much slower than in two Upper cases