Case 1

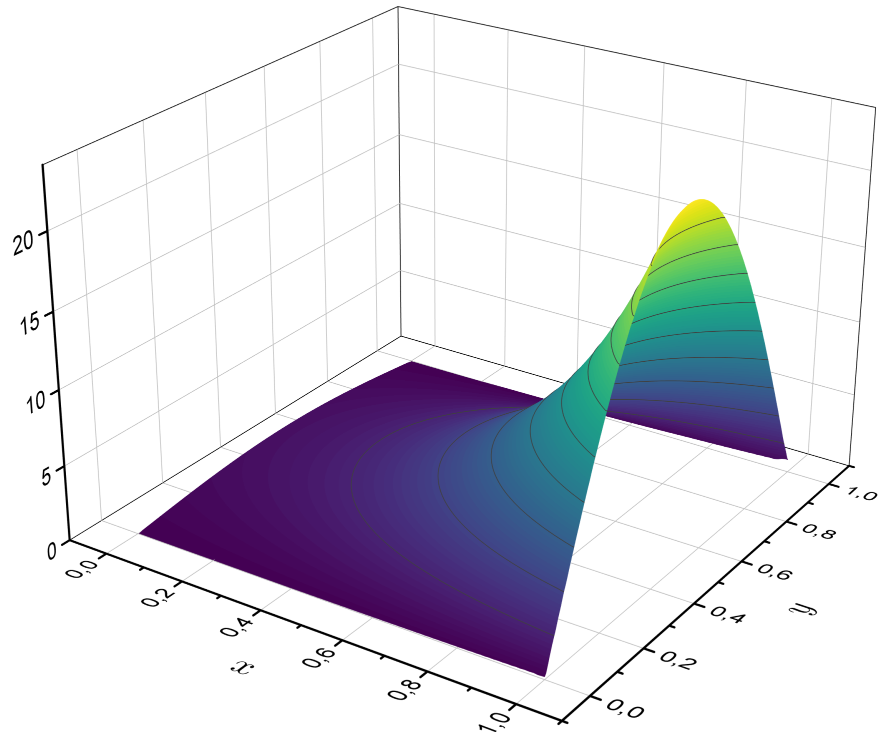

Figure 1: Numerical solution of the Poisson problem in Case 1.

Solving the equation

$$\phi_{xx} + \phi_{yy} = x^2 + y^2$$

in a square domain

$$x \in [0,1], \;\; y \in [0,1]$$

with Dirichlet boundary conditions

$$\phi(x,0) = 0,$$

$$\phi(x,1) = x^2/2,$$

$$\phi(0,y) = \sin(\pi y),$$

$$\phi(1,y) = \exp(\pi)\sin(\pi y) + y^2/2.$$

The analytical solution is

$$\phi(x,y) = \exp(\pi x)\sin(\pi y) + (xy)^2/2.$$

Case 2

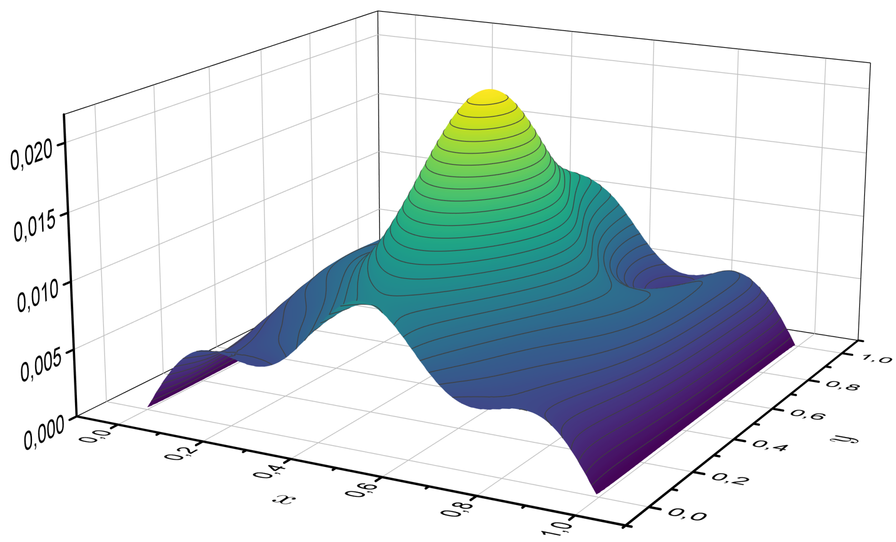

Figure 2: Numerical solution of the Poisson problem in Case 2.

Solving the equation

$$\phi_{xx} + \phi_{yy} = y\,\sin(5\pi x) + \exp\left(-\frac{(x-0.5)^2 + (y-0.5)^2}{0.02}\right)$$

in a square domain

$$x \in [0,1],\ \ y \in [0,1]$$

with Dirichlet boundary conditions

$$\phi(x,0) = 0,$$

$$\phi(x,1) = 0,$$

and a periodic boundary along y.

Case 3

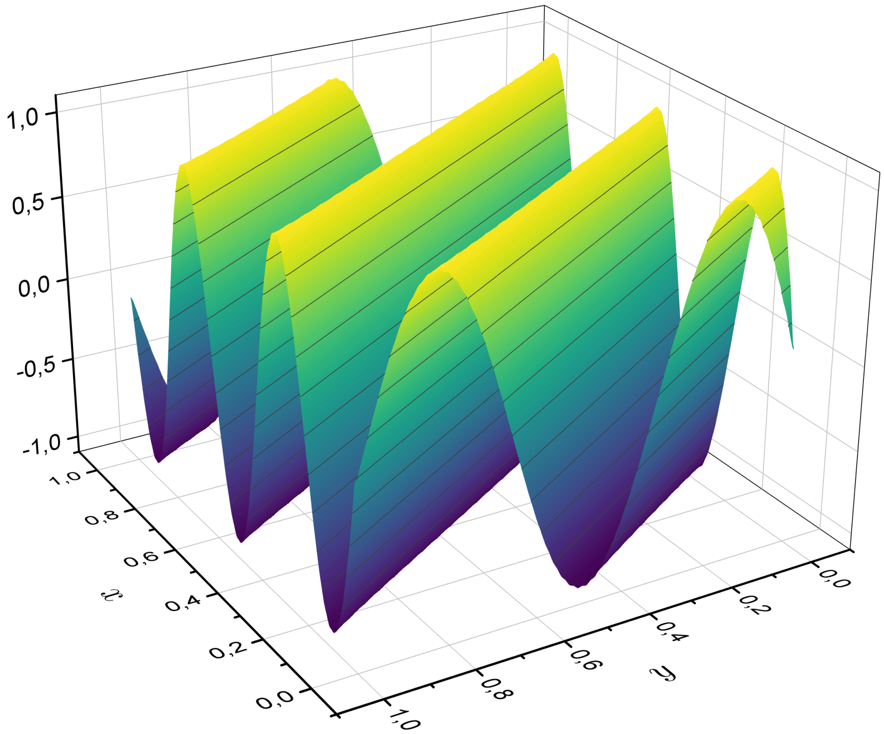

Figure 3: Numerical solution of the Poisson problem in Case 3.

Solving the equation

$$\phi_{xx} + \phi_{yy} = -34\,\pi^2 \sin(5\pi x + 3\pi y)$$

in a square domain

$$x \in [0, 1],\;\; y \in [0, 1]$$

with Dirichlet boundary conditions

$$\phi(x, 0) = \sin(3\pi y),$$

$$\phi(x, 1) = \sin(5\pi + 3\pi y),$$

$$\phi(0, y) = \sin(5\pi x),$$

$$\phi(1, y) = \sin(5\pi x + 3\pi).$$

The analytical solution is

$$\phi(x, y) = \sin(5\pi x + 3\pi y).$$