

.\*VSTU.\*

Team Reference Document

01/12/2018  
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1. CODE TEMPLATES

1.1. Basic Configuration.

```
1.1.1. .vimrc.
set cin nu ts=2 sw=2 sts=2 mouse=a
syn on

function! Compile()
    :!g++ -std=gnu++11 -g % -o %<.exe
endfunction

function! Run()
    :!time ./%<.exe
endfunction

map <F4> :call Compile()<cr>
map <F5> :call Run()<cr>
map <C-A> ggVG" +y

1.1.2. stress and template.
// g++ -std=c++11 main.cpp -o main -D"_DEBUG_TEMICH_"
```

```
#include <algorithm>
#include <cmath>
#include <functional>
#include <iostream>
#include <map>
#include <queue>
#include <set>
#include <sstream>
#include <string>
#include <vector>
```

```
using namespace std;

using LL = long long;
using pii = pair<int, int>;
```

```
#define X first
#define Y second
```

```
template<typename T>
ostream& operator<<(ostream& out, const vector<T>& v);

template<typename U, typename V>
ostream& operator<<(ostream& out, const map<U, V>& v);

template<typename U, typename V>
ostream& operator<<(ostream& out, const pair<U, V>& v);

template<typename U, typename V>
ostream& operator<<(ostream& out, const pair<U, V>& v) {
    return out << "(" << v.first << ", " << v.second << ")";
}

template<typename U, typename V>
```

```
ostream& operator<<(ostream& out, const map<U, V>& v) {
    out << "{";
    bool f = false;
    for (const auto& p : v) {
        out << (!f ? "" : ", ") << p;
        f = true;
    }
    return out << "}";
}

template<typename T>
ostream& operator<<(ostream& out, const vector<T>& v) {
    out << "{";
    for (int i = 0; i < int(v.size()); ++i)
        out << (i == 0 ? "" : ", ") << v[i];
    return out << "}";
}

void cerr_printer(bool start) {}
template<typename T, typename ... Args>
void cerr_printer(bool start, const T& x, const Args& ... args) {
    if (!start) cerr << ", ";
    cerr << x;
    cerr_printer(false, args...);
}

template<typename ... Args>
void dbg(const char * name, int line, const Args& ... args) {
    cerr << "[" << line << "] (" << name << ") = (";
    cerr_printer(true, args...);
    cerr << ")" << endl;
}

#define DBG(...) { dbg(#__VA_ARGS__, __LINE__, __VA_ARGS__); }

struct Solver {
    void solve(istream& cin, ostream& cout) {
        int a, b;
        cin >> a >> b;
        cout << a + b << endl;
    }
};

struct Brute {
    void solve(istream& cin, ostream& cout) {
        int a, b;
        cin >> a >> b;
        while (b--) ++a;
        cout << a << endl;
    }
};

template <typename Solution>
struct SolutionStr {
    string solve(string input) {
        istringstream is(input);
```

```
        istringstream os;
        Solution().solve(is, os);
        return os.str();
    }
};

string gen_input(int it) {
    (void)it;
    return "10 20";
}

void stress() {
    for (int it = 0; it < 1000; ++it) {
        auto input = gen_input(it);
        auto brute_out = SolutionStr<Solver>().solve(input);
        auto sol_out = SolutionStr<Brute>().solve(input);
        if (sol_out != brute_out) {
            cerr << "WA #" << it << endl;
            cerr << "input: " << endl;
            cerr << input << endl;
            cerr << "expected: " << brute_out << endl;
            cerr << "got: " << sol_out << endl;
            exit(1);
        }
    }

    cerr << "OK" << endl;
}

int main() {
    #ifdef _DEBUG_TEMICH_
        stress();
    #endif
    Solver().solve(cin, cout);
}

1.2. Vector.
struct Vec {
    LL x, y;
    explicit Vec(LL x = 0, LL y = 0) : x(x), y(y) {}
    Vec operator+(const Vec& o) const {
        return Vec(x + o.x, y + o.y);
    }
    Vec operator-(const Vec& o) const {
        return Vec(x - o.x, y - o.y);
    }
    Vec operator*(const LL p) const {
        return Vec(x * p, y * p);
    }
    double len() const { return sqrt(x * x + y * y); }
    LL cross(const Vec& o) const { return x * o.y - y * o.x; }
    LL dot(const Vec& o) const { return x * o.x + y * o.y; }
    static Vec read(istream& cin) {
        LL x, y;
        cin >> x >> y;
        return Vec(x, y);
    }
};

bool cmp(Vec a, Vec b) {
```

```

    return a.x < b.x || (a.x == b.x && a.y < b.y);
}

bool cw(Vec a, Vec b, Vec c) {
    return (b - a).cross(c - b) < 0;
}

bool ccw(Vec a, Vec b, Vec c) {
    return (b - a).cross(c - b) > 0;
}

void convex_hull(vector<Vec> & a) {
    if (a.size() == 1) return;
    sort(a.begin(), a.end(), &cmp);
    Vec p1 = a[0], p2 = a.back();
    vector<Vec> up, down;
    up.push_back(p1);
    down.push_back(p2);
    for (size_t i=1; i<a.size(); ++i) {
        if (i==a.size()-1 || cw(p1, a[i], p2)) {
            while (up.size()>=2
                && !cw(up[up.size()-2], up[up.size()-1], a[i]))
                up.pop_back();
            up.push_back(a[i]);
        }
        if (i == a.size()-1 || ccw(p1, a[i], p2)) {
            while (down.size()>=2
                && !ccw(down[down.size()-2],
                    down[down.size()-1], a[i]))
                down.pop_back();
            down.push_back(a[i]);
        }
    }
    a.clear();
    for (size_t i=0; i<up.size(); ++i)
        a.push_back(up[i]);
    for (size_t i=down.size()-2; i>0; --i)
        a.push_back(down[i]);
}

```

### 1.3. FFT.

```

struct Complex {
    long double re, im;
    explicit Complex(long double re = 0,
        long double im = 0) : re(re), im(im) {}
    Complex operator+(const Complex& o) const {
        return Complex(re + o.re, im + o.im);
    }
    Complex operator-(const Complex& o) const {
        return Complex(re - o.re, im - o.im);
    }
    Complex operator*(const Complex& o) const {
        return Complex(re * o.re - im * o.im, re * o.im + im * o.re);
    }
};

const int MAX_SHIFT = 22;
const int MAX_N = 1 << MAX_SHIFT;

const double Pi = acos(-1);

Complex roots[MAX_N / 2];

```

```

int bit_reverse[MAX_N];

void prep() {
    bit_reverse[0] = 0;
    for (int i = 1; i < MAX_N; ++i)
        bit_reverse[i] = (bit_reverse[i >> 1]
            | ((i & 1) << MAX_SHIFT)) >> 1;

    for (int i = 0; i + i < MAX_N; ++i) {
        double angle = 2 * i * Pi / MAX_N;
        roots[i] = Complex(cos(angle), sin(angle));
    }
}

Complex arr[MAX_N];
void fft(int k) {
    assert(k <= MAX_SHIFT);

    const int n = 1 << k;
    for (int i = 0; i < n; ++i) {
        int rv = bit_reverse[i] >> (MAX_SHIFT - k);
        if (rv < i) swap(arr[i], arr[rv]);
    }

    for (int bs = 2; bs <= n; bs *= 2) {
        const int hbs = bs / 2;
        const int factor = (MAX_N / 2) / hbs;
        for (int i = 0; i < n; i += bs) {
            for (int j = 0; j < hbs; ++j) {
                auto a = arr[i + j];
                auto b = arr[i + j + hbs] * roots[factor * j];
                arr[i + j] = a + b;
                arr[i + j + hbs] = a - b;
            }
        }
    }

    const int Base = 100;

    void square(vector<int>& number) {
        int sz = number.size() * 2;
        int k = 1;
        {
            int rsz = 2;
            while (rsz < sz) {
                rsz *= 2;
                ++k;
            }

            sz = rsz;
        }

        assert(sz <= MAX_N);

        for (int i = 0; i < sz; ++i)

```

```

        arr[i] = Complex(i < number.size() ? number[i] : 0);

        fft(k);
        for (int i = 0; i < sz; ++i)
            arr[i] = arr[i] * arr[i];
        fft(k);
        reverse(arr + 1, arr + sz);

        number.resize(sz);
        int cr = 0;
        for (int i = 0; i < sz; ++i) {
            number[i] = cr + int(arr[i].re / sz + 0.5);
            cr = number[i] / Base;
            number[i] %= Base;
        }

        while (number.back() == 0) number.pop_back();
    }
}

```

### 1.4. Matrix.

```

struct Matrix {
    ULL vals[N][N];
    Matrix() {
        for (int i = 0; i < N; ++i)
            fill(vals[i], vals[i] + N, 0);
    }

    ULL* operator[](const int idx) {
        return vals[idx];
    }

    const ULL* operator[](const int idx) const {
        return vals[idx];
    }

    static Matrix Ident() {
        Matrix res;
        for (int i = 0; i < N; ++i)
            res[i][i] = 1;

        return res;
    }

    Matrix operator*(const Matrix& o) const {
        Matrix res;

        for (int i = 0; i < N; ++i) {
            for (int j = 0; j < N; ++j) {
                for (int k = 0; k < N; ++k) {
                    res[i][j] += vals[i][k] * o[k][j];
                    if (k == 7)
                        res[i][j] %= MOD;
                }
                res[i][j] %= MOD;
            }
        }
    }
}

```

```
    return res;
}
};

1.5. SegmTree.
class SegmTreeSum {
    vector<int> tree;
    int n;

    int get(int v, int l, int r, int L, int R) const {
        if (L > R) return 0;
        if (l == L && r == R) return tree[v];

        int mid = (l + r) / 2;

        int a = get(2 * v + 1, l, mid, L, min(R, mid));
        int b = get(2 * v + 2, mid + 1, r, max(L, mid + 1), R);

        return a + b;
    }

    void set(int v, int l, int r, int pos, int val) {
        if (l == r) {
            tree[pos] = val;
            return;
        }

        int mid = (l + r) / 2;

        if (pos <= mid) set(2 * v + 1, l, mid, pos, val);
        else set(2 * v + 2, mid + 1, r, pos, val);

        tree[v] = tree[2 * v + 1] + tree[2 * v + 2];
    }

public:
    void init(int n_) {
        n = n_;
        tree.assign(4 * n, 0);
    }

    int get(int l, int r) const {
        return get(0, 0, n - 1, l, r);
    }

    void set(int pos, int val) {
        set(0, 0, n - 1, pos, val);
    }
};
```

```
class SegmTreeMax {
    vector<Pair> tree;
    vector<int> psh;
    int n;
```

```
void build(int v, int l, int r, const vector<int>& dp) {
    if (l == r) {
        tree[v] = Pair(dp[l], l);
        return;
    }

    int mid = (l + r) / 2;

    build(2 * v + 1, l, mid, dp);
    build(2 * v + 2, mid + 1, r, dp);

    tree[v] = max(tree[2 * v + 1], tree[2 * v + 2]);
}

void push(int v, int l, int r) {
    if (l != r) {
        psh[2 * v + 1] += psh[v];
        psh[2 * v + 2] += psh[v];
    }
    tree[v].X += psh[v];
    psh[v] = 0;
}

Pair getMax(int v, int l, int r, int L, int R) {
    push(v, l, r);
    if (L > R) return Pair(-INF, -INF);

    if (l == L && r == R)
        return tree[v];

    int mid = (l + r) / 2;

    Pair a = getMax(2 * v + 1, l, mid, L, min(R, mid));
    Pair b = getMax(2 * v + 2, mid + 1, r, max(L, mid + 1), R);

    return max(a, b);
}

void add(int v, int l, int r, int L, int R, int val) {
    push(v, l, r);

    if (L > R) return;
    if (l == L && r == R) {
        psh[v] += val;
        push(v, l, r);
        return;
    }

    int mid = (l + r) / 2;

    add(2 * v + 1, l, mid, L, min(R, mid), val);
    add(2 * v + 2, mid + 1, r, max(L, mid + 1), R, val);

    tree[v] = max(tree[2 * v + 1], tree[2 * v + 2]);
}
```

```
public:
    void init(const vector<int>& dp) {
        n = dp.size();
        tree.resize(4 * n);
        psh.assign(4 * n, 0);

        build(0, 0, n - 1, dp);
    }

    Pair getMax(int l, int r) {
        return getMax(0, 0, n - 1, l, r);
    }

    void add(int l, int r, int val) {
        add(0, 0, n - 1, l, r, val);
    }
};

1.6. Aho.
struct Matcher {
    static const int LETTERS_COUNT = 'z' - 'a' + 1;
    struct Next {
        int nxt[LETTERS_COUNT];
        Next() { fill(nxt, nxt + LETTERS_COUNT, -1); }
        int& operator[](char c) { return nxt[c - 'a']; }
    };

    vector<Next> next;
    vector<int> link;
    vector<char> p_char;
    vector<int> p;
    vector<int> id;

    void build(const set<string>& strings) {
        int total_size = 0;
        for (const auto& s : strings)
            total_size += s.size();
        next.reserve(total_size);
        link.reserve(total_size);
        p_char.reserve(total_size);
        p.reserve(total_size);

        push();

        int _id = 0;
        for (const auto& s : strings) {
            add(s, _id);
            ++_id;
        }
    }

    void push() {
        next.push_back(Next());
        link.push_back(-1);
        p_char.push_back('#');
        p.push_back(-1);
    }
};
```

```
    id.push_back(-1);
}

void add(const string& s, int _id) {
    int state = 0;

    for (char c : s) {
        int next_state = next[state][c];
        if (next_state == -1) {
            push();
            p_char.back() = c;
            p.back() = state;
            next_state = p.size() - 1;
            next[state][c] = next_state;
        }

        state = next_state;
    }

    id[state] = _id;
}

int get_next(int state, char c) {
    int x = _get_next(state, c);
    // cerr << "get next " << state << " " << c << " = " << x << endl;
    return x;
}

int _get_next(int state, char c) {
    if (next[state][c] == -1 && state == 0)
        return 0;
    if (next[state][c] == -1)
        next[state][c] = get_next(get_link(state), c);
    return next[state][c];
}

int get_link(int state) { int x = _get_link(state);
    // cerr << "get link " << state << " = " << x << endl;
    return x;
}

int _get_link(int state) {
    if (state == 0)
        return 0;
    if (p[state] == 0)
        return 0;

    int& l = link[state];
    if (l == -1)
        l = get_next(get_link(p[state]), p_char[state]);
    return l;
}

int get_id(int state) { return id[state]; }
};
```

2. Misc

2.1. Debugging Tips.

- Stack overflow? Recursive DFS on tree that is actually a long path?
- Floating-point numbers
  - Getting NaN? Make sure `acos` etc. are not getting values out of their range (perhaps `1+eps`).
  - Rounding negative numbers?
  - Outputting in scientific notation?
- Wrong Answer?
  - Read the problem statement again!
  - Are multiple test cases being handled correctly? Try repeating the same test case many times.
  - Integer overflow?
  - Think very carefully about boundaries of all input parameters
  - Try out possible edge cases:
    - \*  $n = 0, n = -1, n = 1, n = 2^{31} - 1$  or  $n = -2^{31}$
    - \* List is empty, or contains a single element
    - \*  $n$  is even,  $n$  is odd
    - \* Graph is empty, or contains a single vertex
    - \* Graph is a multigraph (loops or multiple edges)
    - \* Polygon is concave or non-simple
  - Is initial condition wrong for small cases?
  - Are you sure the algorithm is correct?
  - Explain your solution to someone.
  - Are you using any functions that you don't completely understand? Maybe STL functions?
  - Maybe you (or someone else) should rewrite the solution?
  - Can the input line be empty?
- Run-Time Error?
  - Is it actually Memory Limit Exceeded?

2.2. Solution Ideas.

- Dynamic Programming
  - Parsing CFGs: CYK Algorithm
  - Drop a parameter, recover from others
  - Swap answer and a parameter
  - When grouping: try splitting in two
  - $2^k$  trick
  - When optimizing
    - \* Convex hull optimization
      - $dp[i] = \min_{j < i} \{dp[j] + b[j] \times a[i]\}$
      - $b[j] \geq b[j + 1]$
      - optionally  $a[i] \leq a[i + 1]$
      - $O(n^2)$  to  $O(n)$
    - \* Divide and conquer optimization
      - $dp[i][j] = \min_{k < j} \{dp[i - 1][k] + C[k][j]\}$
      - $A[i][j] \leq A[i][j + 1]$
      - $O(kn^2)$  to  $O(kn \log n)$
      - sufficient:  $C[a][c] + C[b][d] \leq C[a][d] + C[b][c], a \leq b \leq c \leq d$  (QI)
    - \* Knuth optimization
      - $dp[i][j] = \min_{i < k < j} \{dp[i][k] + dp[k][j] + C[i][j]\}$
      - $A[i][j - 1] \leq A[i][j] \leq A[i + 1][j]$
      - $O(n^3)$  to  $O(n^2)$
      - sufficient: QI and  $C[b][c] \leq C[a][d], a \leq b \leq c \leq d$
  - Greedy

- Randomized
- Optimizations
  - Use bitset (`/64`)
  - Switch order of loops (cache locality)
- Process queries offline
  - Mo's algorithm
- Square-root decomposition
- Precomputation
- Efficient simulation
  - Mo's algorithm
  - Sqrt decomposition
  - Store  $2^k$  jump pointers
- Data structure techniques
  - Sqrt buckets
  - Store  $2^k$  jump pointers
  - $2^k$  merging trick
- Counting
  - Inclusion-exclusion principle
  - Generating functions
- Graphs
  - Can we model the problem as a graph?
  - Can we use any properties of the graph?
  - Strongly connected components
  - Cycles (or odd cycles)
  - Bipartite (no odd cycles)
    - \* Bipartite matching
    - \* Hall's marriage theorem
    - \* Stable Marriage
  - Cut vertex/bridge
  - Biconnected components
  - Degrees of vertices (odd/even)
  - Trees
    - \* Heavy-light decomposition
    - \* Centroid decomposition
    - \* Least common ancestor
    - \* Centers of the tree
  - Eulerian path/circuit
  - Chinese postman problem
  - Topological sort
  - (Min-Cost) Max Flow
  - Min Cut
    - \* Maximum Density Subgraph
  - Huffman Coding
  - Min-Cost Arborescence
  - Steiner Tree
  - Kirchhoff's matrix tree theorem
  - Prüfer sequences
  - Lovász Toggle
  - Look at the DFS tree (which has no cross-edges)
  - Is the graph a DFA or NFA?
    - \* Is it the Synchronizing word problem?
- Mathematics
  - Is the function multiplicative?
  - Look for a pattern
  - Permutations
    - \* Consider the cycles of the permutation

- Functions
  - \* Sum of piecewise-linear functions is a piecewise-linear function
  - \* Sum of convex (concave) functions is convex (concave)
- Modular arithmetic
  - \* Chinese Remainder Theorem
  - \* Linear Congruence
- Sieve
- System of linear equations
- Values too big to represent?
  - \* Compute using the logarithm
  - \* Divide everything by some large value
- Linear programming
  - \* Is the dual problem easier to solve?
- Can the problem be modeled as a different combinatorial problem? Does that simplify calculations?
- Logic
  - 2-SAT
  - XOR-SAT (Gauss elimination or Bipartite matching)
- Meet in the middle
- Only work with the smaller half ( $\log(n)$ )
- Strings
  - Trie (maybe over something weird, like bits)
  - Suffix array
  - Suffix automaton (+DP?)
  - Aho-Corasick
  - `eerTree`
  - Work with  $S + S$
- Hashing
- Euler tour, tree to array
- Segment trees
  - Lazy propagation
  - Persistent
  - Implicit
  - Segment tree of X
- Geometry
  - Minkowski sum (of convex sets)
  - Rotating calipers
  - Sweep line (horizontally or vertically?)
  - Sweep angle
  - Convex hull
- Fix a parameter (possibly the answer).
- Are there few distinct values?
- Binary search
- Sliding Window (+ Monotonic Queue)
- Computing a Convolution? Fast Fourier Transform
- Computing a 2D Convolution? FFT on each row, and then on each column
- Exact Cover (+ Algorithm X)
- Cycle-Finding
- What is the smallest set of values that identify the solution? The cycle structure of the permutation? The powers of primes in the factorization?
- Look at the complement problem
  - Minimize something instead of maximizing

- Immediately enforce necessary conditions. (All values greater than 0? Initialize them all to 1)
- Add large constant to negative numbers to make them positive
- Counting/Bucket sort

PRACTICE CONTEST CHECKLIST

- How many operations per second? Compare to local machine.
- What is the stack size?
- How to use printf/scanf with long long/long double?
- Are `__int128` and `__float128` available?
- Does MLE give RTE or MLE as a verdict? What about stack overflow?
- What is `RAND_MAX`?
- How does the judge handle extra spaces (or missing newlines) in the output?
- Look at documentation for programming languages.
- Try different programming languages: C++, Java and Python.
- Try the submit script.
- Try local programs: `i?python[23]`, `factor`.
- Try submitting with `assert(false)` and `assert(true)`.
- Return-value from `main`.
- Look for directory with sample test cases.
- Make sure printing works.
- Remove this page from the notebook.