

.*VSTU.*

Team Reference Document

01/12/2018

CONTENTS

1.	Code Templates	2
1.1.	Basic Configuration	2
1.2.	Vector	2
1.3.	FFT	3
1.4.	Matrix	3
1.5.	SegmTree	4
1.6.	Aho	4
1.7.	Suffix Automaton	5
1.8.	Stoer Wagner	5
1.9.	Flow	5
1.10.	Prefix function	6
1.11.	BPWS	6
2.	Misc	8
2.1.	Debugging Tips	8
2.2.	Solution Ideas	8
2.3.	The Twelfefold Way	9
	Practice Contest Checklist	10

1. CODE TEMPLATES

1.1. Basic Configuration.

1.1.1. .vimrc.

```
set cin nu ts=2 sw=2 sts=2 mouse=a
syn on
```

```
function! Compile()
    :!g++ -std=gnu++11 -g % -o %<.exe
endfunction
```

```
function! Run()
    :!time ./%<.exe
endfunction
```

```
map <F4> :call Compile()<cr>
map <F5> :call Run()<cr>
map <C-A> ggVG"+y
```

1.1.2. stress and template.

```
// g++ -std=c++11 main.cpp -o main -D"_DEBUG_TEMICH_"
// -Wall -Wextra -pedantic -std=c++11
// -O2 -Wshadow -Wformat=2 -Wfloat-equal
// -Wconversion -Wlogical-op -Wshift-overflow=2
// -Wduplicated-cond -Wcast-qual -Wcast-align
// -D_GLIBCXX_DEBUG -D_GLIBCXX_DEBUG_PEDANTIC
// -D_FORTIFY_SOURCE=2 -fsanitize=address
// -fsanitize=undefined -fno-sanitize-recover
// -fstack-protector
#pragma GCC optimize("O3")
#pragma GCC target(
    "sse,sse2,sse3,ssse3,sse4,popcnt,abm,mmx")
```

```
#include <algorithm>
#include <cmath>
#include <functional>
#include <iostream>
#include <map>
#include <queue>
#include <set>
#include <sstream>
#include <string>
#include <vector>
```

```
using namespace std;
```

```
using LL = long long;
using pii = pair<int, int>;
```

```
#define X first
#define Y second
```

```
template<typename T>
ostream& operator<<(ostream& out, const vector<T>& v);
```

```
template<typename U, typename V>
```

```
ostream& operator<<(ostream& out, const map<U, V>& v);
```

```
template<typename U, typename V>
ostream& operator<<(ostream& out, const pair<U, V>& v);
```

```
template<typename U, typename V>
ostream& operator<<(ostream& out, const pair<U, V>& v) {
    return out << "(" << v.first << ", " << v.second << ")";
}
```

```
template<typename U, typename V>
ostream& operator<<(ostream& out, const map<U, V>& v) {
    out << "{";
    bool f = false;
    for (const auto& p : v) {
        out << (!f ? "" : ", ") << p;
        f = true;
    }
    return out << "}";
}
```

```
template<typename T>
ostream& operator<<(ostream& out, const vector<T>& v) {
    out << "{";
    for (int i = 0; i < int(v.size()); ++i)
        out << (i == 0 ? "" : ", ") << v[i];
    return out << "}";
}
```

```
void cerr_printer(bool start) {}
template<typename T, typename ... Args>
void cerr_printer(bool start, const T& x, const Args& ... args) {
    if (!start) cerr << ", ";
    cerr << x;
    cerr_printer(false, args...);
}
```

```
template<typename ... Args>
void dbg(const char * name, int line, const Args& ... args) {
    cerr << "[" << line << "]" (" << name << ") = (";
    cerr_printer(true, args...);
    cerr << ")" << endl;
}
```

```
#define DBG(...) { dbg(#__VA_ARGS__, __LINE__, __VA_ARGS__); }
```

```
struct Solver {
    void solve(istream& cin, ostream& cout) {
        int a, b;
        cin >> a >> b;
        cout << a + b << endl;
    }
};
```

```
struct Brute {
    void solve(istream& cin, ostream& cout) {
```

```
int a, b;
cin >> a >> b;
while (b--) ++a;
cout << a << endl;
}
};
```

```
template <typename Solution>
struct SolutionStr {
    string solve(string input) {
        istringstream is(input);
        ostringstream os;
        Solution().solve(is, os);
        return os.str();
    }
};
```

```
string gen_input(int it) {
    (void)it;
    return "10 20";
}
```

```
void stress() {
    for (int it = 0; it < 1000; ++it) {
        auto input = gen_input(it);
        auto brute_out = SolutionStr<Solver>().solve(input);
        auto sol_out = SolutionStr<Brute>().solve(input);
        if (sol_out != brute_out) {
            cerr << "WA #" << it << endl;
            cerr << "input: " << endl;
            cerr << input << endl;
            cerr << "expected: " << brute_out << endl;
            cerr << "got: " << sol_out << endl;
            exit(1);
        }
    }
}
```

```
cerr << "OK" << endl;
}
```

```
int main() {
#ifdef _DEBUG_TEMICH_
    stress();
#endif
    Solver().solve(cin, cout);
}
```

1.2. Vector.

```
struct Vec {
    LL x, y;
    explicit Vec(LL x = 0, LL y = 0) : x(x), y(y) {}
    Vec operator+(const Vec& o) const {
        return Vec(x + o.x, y + o.y);
    }
    Vec operator-(const Vec& o) const {
        return Vec(x - o.x, y - o.y);
    }
    Vec operator*(const LL p) const {
```

```
        return Vec(x * p, y * p); }
double len() const { return sqrt(x * x + y * y); }
LL cross(const Vec& o) const { return x * o.y - y * o.x; }
LL dot(const Vec& o) const { return x * o.x + y * o.y; }
static Vec read(istream& cin) {
    LL x, y;
    cin >> x >> y;
    return Vec(x, y);
}
};
bool cmp(Vec a, Vec b) {
    return a.x < b.x || (a.x == b.x && a.y < b.y);
}
bool cw(Vec a, Vec b, Vec c) {
    return (b - a).cross(c - b) < 0;
}
bool ccw(Vec a, Vec b, Vec c) {
    return (b - a).cross(c - b) > 0;
}
void convex_hull(vector<Vec> & a) {
    if (a.size() == 1) return;
    sort(a.begin(), a.end(), &cmp);
    Vec p1 = a[0], p2 = a.back();
    vector<Vec> up, down;
    up.push_back(p1);
    down.push_back(p1);
    for (size_t i=1; i<a.size(); ++i) {
        if (i==a.size()-1 || cw(p1, a[i], p2)) {
            while (up.size())>=2
                && !cw(up[up.size()-2], up[up.size()-1], a[i]))
                up.pop_back();
            up.push_back (a[i]);
        }
        if (i == a.size()-1 || ccw(p1, a[i], p2)) {
            while (down.size())>=2
                && !ccw(down[down.size()-2],
                    down[down.size()-1], a[i]))
                down.pop_back();
            down.push_back(a[i]);
        }
    }
    a.clear();
    for (size_t i=0; i<up.size(); ++i)
        a.push_back(up[i]);
    for (size_t i=down.size()-2; i>0; --i)
        a.push_back(down[i]);
}
```

1.3. FFT.

```
struct Complex {
    long double re, im;
    explicit Complex(long double re = 0,
        long double im = 0) : re(re), im(im) {}
    Complex operator+(const Complex& o) const {
        return Complex(re + o.re, im + o.im); }
    Complex operator-(const Complex& o) const {
```

```
        return Complex(re - o.re, im - o.im); }
    Complex operator*(const Complex& o) const {
        return Complex(re * o.re - im * o.im, re * o.im + im * o.re); }
};

const int MAX_SHIFT = 22;
const int MAX_N = 1 << MAX_SHIFT;

const double Pi = acos(-1);

Complex roots[MAX_N / 2];
int bit_reverse[MAX_N];

void prep() {
    bit_reverse[0] = 0;
    for (int i = 1; i < MAX_N; ++i)
        bit_reverse[i] = (bit_reverse[i >> 1]
            | ((i & 1) << MAX_SHIFT)) >> 1;

    for (int i = 0; i + i < MAX_N; ++i) {
        double angle = 2 * i * Pi / MAX_N;
        roots[i] = Complex(cos(angle), sin(angle));
    }
}

Complex arr[MAX_N];
void fft(int k) {
    assert(k <= MAX_SHIFT);

    const int n = 1 << k;
    for (int i = 0; i < n; ++i) {
        int rv = bit_reverse[i] >> (MAX_SHIFT - k);
        if (rv < i) swap(arr[i], arr[rv]);
    }

    for (int bs = 2; bs <= n; bs *= 2) {
        const int hbs = bs / 2;
        const int factor = (MAX_N / 2) / hbs;
        for (int i = 0; i < n; i += bs) {
            for (int j = 0; j < hbs; ++j) {
                auto a = arr[i + j];
                auto b = arr[i + j + hbs] * roots[factor * j];
                arr[i + j] = a + b;
                arr[i + j + hbs] = a - b;
            }
        }
    }
}
```

const int Base = 100;

```
void square(vector<int>& number) {
    int sz = number.size() * 2;
    int k = 1;
    {
        int rsz = 2;
```

```
        while (rsz < sz) {
            rsz *= 2;
            ++k;
        }

        sz = rsz;
    }

    assert(sz <= MAX_N);

    for (int i = 0; i < sz; ++i)
        arr[i] = Complex(i < number.size() ? number[i] : 0);

    fft(k);
    for (int i = 0; i < sz; ++i)
        arr[i] = arr[i] * arr[i];
    fft(k);
    reverse(arr + 1, arr + sz);

    number.resize(sz);
    int cr = 0;
    for (int i = 0; i < sz; ++i) {
        number[i] = cr + int(arr[i].re / sz + 0.5);
        cr = number[i] / Base;
        number[i] %= Base;
    }

    while (number.back() == 0) number.pop_back();
}
```

1.4. Matrix.

```
struct Matrix {
    ULL vals[N][N];
    Matrix() {
        for (int i = 0; i < N; ++i)
            fill(vals[i], vals[i] + N, 0);
    }

    ULL* operator[](const int idx) {
        return vals[idx];
    }

    const ULL* operator[](const int idx) const {
        return vals[idx];
    }

    static Matrix Ident() {
        Matrix res;
        for (int i = 0; i < N; ++i)
            res[i][i] = 1;

        return res;
    }

    Matrix operator*(const Matrix& o) const {
        Matrix res;
```

```
for (int i = 0; i < N; ++i) {
    for (int j = 0; j < N; ++j) {
        for (int k = 0; k < N; ++k) {
            res[i][j] += vals[i][k] * o[k][j];
            if (k == 7)
                res[i][j] %= MOD;
        }
        res[i][j] %= MOD;
    }
}

return res;
};
```

1.5. SegmTree.

```
class SegmTreeSum {
    vector<int> tree;
    int n;

    int get(int v, int l, int r, int L, int R) const {
        if (L > R) return 0;
        if (l == L && r == R) return tree[v];

        int mid = (l + r) / 2;

        int a = get(2 * v + 1, l, mid, L, min(R, mid));
        int b = get(2 * v + 2, mid + 1, r, max(L, mid + 1), R);

        return a + b;
    }

    void set(int v, int l, int r, int pos, int val) {
        if (l == r) {
            tree[pos] = val;
            return;
        }

        int mid = (l + r) / 2;

        if (pos <= mid) set(2 * v + 1, l, mid, pos, val);
        else set(2 * v + 2, mid + 1, r, pos, val);

        tree[v] = tree[2 * v + 1] + tree[2 * v + 2];
    }

public:
    void init(int n_) {
        n = n_;
        tree.assign(4 * n, 0);
    }

    int get(int l, int r) const {
        return get(0, 0, n - 1, l, r);
    }
};
```

```
void set(int pos, int val) {
    set(0, 0, n - 1, pos, val);
}

class SegmTreeMax {
    vector<Pair> tree;
    vector<int> psh;
    int n;

    void build(int v, int l, int r, const vector<int>& dp) {
        if (l == r) {
            tree[v] = Pair(dp[l], l);
            return;
        }

        int mid = (l + r) / 2;

        build(2 * v + 1, l, mid, dp);
        build(2 * v + 2, mid + 1, r, dp);

        tree[v] = max(tree[2 * v + 1], tree[2 * v + 2]);
    }

    void push(int v, int l, int r) {
        if (l != r) {
            psh[2 * v + 1] += psh[v];
            psh[2 * v + 2] += psh[v];
        }
        tree[v].X += psh[v];
        psh[v] = 0;
    }

    Pair getMax(int v, int l, int r, int L, int R) {
        push(v, l, r);
        if (L > R) return Pair(-INF, -INF);

        if (l == L && r == R)
            return tree[v];

        int mid = (l + r) / 2;

        Pair a = getMax(2 * v + 1, l, mid, L, min(R, mid));
        Pair b = getMax(2 * v + 2, mid + 1, r, max(L, mid + 1), R);

        return max(a, b);
    }

    void add(int v, int l, int r, int L, int R, int val) {
        push(v, l, r);
        if (L > R) return;
        if (l == L && r == R) {
            psh[v] += val;
            push(v, l, r);
        }
    }
};
```

```
return;
}

int mid = (l + r) / 2;

add(2 * v + 1, l, mid, L, min(R, mid), val);
add(2 * v + 2, mid + 1, r, max(L, mid + 1), R, val);

tree[v] = max(tree[2 * v + 1], tree[2 * v + 2]);
}

public:
    void init(const vector<int>& dp) {
        n = dp.size();
        tree.resize(4 * n);
        psh.assign(4 * n, 0);

        build(0, 0, n - 1, dp);
    }

    Pair getMax(int l, int r) {
        return getMax(0, 0, n - 1, l, r);
    }

    void add(int l, int r, int val) {
        add(0, 0, n - 1, l, r, val);
    }
};

1.6. Aho.
struct Matcher {
    static const int LETTERS_COUNT = 'z' - 'a' + 1;
    struct Next {
        int nxt[LETTERS_COUNT];
        Next() { fill(nxt, nxt + LETTERS_COUNT, -1); }
        int& operator[](char c) { return nxt[c - 'a']; }
    };

    vector<Next> next;
    vector<int> link;
    vector<char> p_char;
    vector<int> p;
    vector<int> id;

    void build(const set<string>& strings) {
        int total_size = 0;
        for (const auto& s : strings)
            total_size += s.size();
        next.reserve(total_size);
        link.reserve(total_size);
        p_char.reserve(total_size);
        p.reserve(total_size);

        push();

        int _id = 0;
    }
};
```

```

    for (const auto& s : strings) {
        add(s, _id);
        ++_id;
    }
}

void push() {
    next.push_back(Next());
    link.push_back(-1);
    p_char.push_back('#');
    p.push_back(-1);
    id.push_back(-1);
}

void add(const string& s, int _id) {
    int state = 0;

    for (char c : s) {
        int next_state = next[state][c];
        if (next_state == -1) {
            push();
            p_char.back() = c;
            p.back() = state;
            next_state = p.size() - 1;
            next[state][c] = next_state;
        }

        state = next_state;
    }

    id[state] = _id;
}

int get_next(int state, char c) {
    int x = _get_next(state, c);
    // cerr << "get next " << state << " " << c << " = " << x << endl;
    return x;
}

int _get_next(int state, char c) {
    if (next[state][c] == -1 && state == 0)
        return 0;
    if (next[state][c] == -1)
        next[state][c] = get_next(get_link(state), c);
    return next[state][c];
}

int get_link(int state) { int x = _get_link(state);
    // cerr << "get link " << state << " = " << x << endl;
    return x;
}

int _get_link(int state) {
    if (state == 0)
        return 0;
    if (p[state] == 0)

```

```

        return 0;

        int& l = link[state];
        if (l == -1)
            l = get_next(get_link(p[state]), p_char[state]);
        return l;
    }

    int get_id(int state) { return id[state]; }
};

1.7. Suffix Automaton.

struct State {
    map<char, int> nxt;
    int link;
    int len;
    bool added;
    int cnt;
};

State st[N];
int lst;
int sz;

void init() {
    lst = 0;
    sz = 1;
    st[0].link = -1;
    st[0].len = 1;
}

void ext(char c) {
    // cerr << "ext : " << c << endl;
    int cur = sz++;
    st[cur].len = st[lst].len + 1;

    int p;
    for (p = lst; p != -1 && !st[p].nxt.count(c); p = st[p].link)
        st[p].nxt[c] = cur;

    if (p == -1) {
        st[cur].link = 0;
    } else {
        int q = st[p].nxt[c];
        if (st[p].len + 1 == st[q].len) {
            st[cur].link = q;
        } else {
            int clone = sz++;
            st[clone] = st[q];
            st[clone].len = st[p].len + 1;
            st[clone].cnt = st[st[clone].link].cnt;

            st[q].link = st[cur].link = clone;

            for (; p != -1 && st[p].nxt[c] == q; p = st[p].link)
                st[p].nxt[c] = clone;

```

```

        }
    }

    lst = cur;
    st[cur].cnt = st[st[cur].link].cnt;
}

1.8. Stoer Wagner.
const int MAXN = 500;
int n, g[MAXN][MAXN];
int best_cost = 1000000000;
vector<int> best_cut;

void mincut() {
    vector<int> v[MAXN];
    for (int i=0; i<n; ++i)
        v[i].assign(1, i);
    int w[MAXN];
    bool exist[MAXN], in_a[MAXN];
    memset(exist, true, sizeof exist);
    for (int ph=0; ph<n-1; ++ph) {
        memset(in_a, false, sizeof in_a);
        memset(w, 0, sizeof w);
        for (int it=0, prev; it<n-ph; ++it) {
            int sel = -1;
            for (int i=0; i<n; ++i)
                if (exist[i] && !in_a[i] && (sel == -1 || w[i] > w[sel]))
                    sel = i;
            if (it == n-ph-1) {
                if (w[sel] < best_cost)
                    best_cost = w[sel], best_cut = v[sel];
                v[prev].insert(v[prev].end(),
                    v[sel].begin(), v[sel].end());
                for (int i=0; i<n; ++i)
                    g[prev][i] = g[i][prev] += g[sel][i];
                exist[sel] = false;
            }
            else {
                in_a[sel] = true;
                for (int i=0; i<n; ++i)
                    w[i] += g[sel][i];
                prev = sel;
            }
        }
    }
}

1.9. Flow.
struct Edge {
    int u, v, flow, cap;
    Edge() : u(0), v(0), flow(0), cap(0) {}
    Edge(int u, int v, int c) : u(u), v(v), flow(0), cap(c) {}
};

const int N = 666;
const int T = 1111;

```

```
const int MAXN = N + 2 * T + 100;
```

```
vector<int> g[500000];
vector<Edge> edges;
```

```
int flow, s, t;
int start[MAXN], used[MAXN], dist[MAXN];
```

```
bool bfs() {
    memset(start, 0, sizeof(start));
    memset(dist, -1, sizeof(dist));
    dist[s] = 0;
```

```
queue<int> q;
q.push(s);
```

```
while (q.size()) {
    int u = q.front();
    q.pop();
```

```
    for (int id : g[u]) {
        Edge &e = edges[id];
        int v = e.v;

        if (dist[v] == -1 && e.flow < e.cap) {
            dist[v] = dist[u] + 1;
            q.push(v);
        }
    }
}
```

```
return dist[t] != -1;
```

```
int dfs(int u, int fl = -1) {
    if (fl == -1) memset(used, false, sizeof(used));
    used[u] = true;
```

```
if (u == t) return fl;
```

```
for (int &i = start[u]; i < g[u].size(); ++i) {
    int id = g[u][i];
    Edge &e = edges[id];
    int v = e.v;
    if (!used[v] && dist[v] == dist[u] + 1 && e.flow < e.cap) {
        int can = e.cap - e.flow;
        int df = dfs(v, fl == -1 ? can : min(fl, can));
        if (df > 0) {
            edges[id ^ 0].flow += df;
            edges[id ^ 1].flow -= df;
            return df;
        }
    }
}
return 0;
```

```
}
```

```
void add_edge(int u, int v, int c) {
    //cout << "add (" << u << " " << v << " " << c << ") " << endl;
    g[u].push_back(edges.size());
    edges.emplace_back(u, v, c);
    g[v].push_back(edges.size());
    edges.emplace_back(v, u, 0);
}
```

```
int calc(int ss, int tt) {
    //cout << "calc (" << ss << ", " << tt << ")" << endl;
    flow = 0, s = ss, t = tt;
    while (bfs()) {
        while (int add = dfs(ss)) {
            flow += add;
        }
    }
    return flow;
}
```

1.10. Prefix function.

```
vector<int> prefix_function (string s) {
    int n = (int) s.length();
    vector<int> pi (n);
    for (int i=1; i<n; ++i) {
        int j = pi[i-1];
        while (j > 0 && s[i] != s[j])
            j = pi[j-1];
        if (s[i] == s[j]) ++j;
        pi[i] = j;
    }
    return pi;
}
```

1.11. BPWS.

```
const int trivial_limit = 50;
int p[1000];
```

```
int gcd (int a, int b) {
    return a ? gcd (b%a, a) : b;
}
```

```
int powmod (int a, int b, int m) {
    int res = 1;
    while (b)
        if (b & 1)
            res = (res * 1ll * a) % m, --b;
        else
            a = (a * 1ll * a) % m, b >>= 1;
    return res;
}
```

```
bool miller_rabin (int n) {
    int b = 2;
    for (int g; (g = gcd (n, b)) != 1; ++b)
        if (n > g)
```

```
        return false;
    int p=0, q=n-1;
    while ((q & 1) == 0)
        ++p, q >>= 1;
    int rem = powmod (b, q, n);
    if (rem == 1 || rem == n-1)
        return true;
    for (int i=1; i<p; ++i) {
        rem = (rem * 1ll * rem) % n;
        if (rem == n-1) return true;
    }
    return false;
}
```

```
int jacobi (int a, int b)
{
    if (a == 0) return 0;
    if (a == 1) return 1;
    if (a < 0)
        if ((b & 2) == 0)
            return jacobi (-a, b);
        else
            return - jacobi (-a, b);
    int a1=a, e=0;
    while ((a1 & 1) == 0)
        a1 >>= 1, ++e;
    int s;
    if ((e & 1) == 0 || (b & 7) == 1 || (b & 7) == 7)
        s = 1;
    else
        s = -1;
    if ((b & 3) == 3 && (a1 & 3) == 3)
        s = -s;
    if (a1 == 1)
        return s;
    return s * jacobi (b % a1, a1);
}
```

```
bool bpsw (int n) {
    if ((int)sqrt(n+0.0) *
        (int)sqrt(n+0.0) == n) return false;

    int dd=5;
    for (;;) {
        int g = gcd (n, abs(dd));
        if (1<g && g<n) return false;
        if (jacobi (dd, n) == -1) break;
        dd = dd<0 ? -dd+2 : -dd-2;
    }
    int p=1, q=(p*p-dd)/4;
    int d=n+1, s=0;
    while ((d & 1) == 0)
        ++s, d >>= 1;
    long long u=1, v=p, u2m=1, v2m=p, qm=q, qm2=q*2, qkd=q;
    for (int mask=2; mask<=d; mask<=1) {
        u2m = (u2m * v2m) % n;
        v2m = (v2m * v2m) % n;
```

```
while (v2m < qm2)    v2m += n;    }
v2m -= qm2;    }
qm = (qm * qm) % n;
qm2 = qm * 2;
if (d & mask) {
    long long t1 = (u2m * v) % n,
    t2 = (v2m * u) % n,
    t3 = (v2m * v) % n,
    t4 = (((u2m * u) % n) * dd) % n;

    u = t1 + t2;
    if (u & 1) u += n;
    u = (u >> 1) % n;
    v = t3 + t4;
    if (v & 1) v += n;
    v = (v >> 1) % n;
    qkd = (qkd * qm) % n;
}
}
if (u==0 || v==0) return true;
long long qkd2 = qkd*2;
for (int r=1; r<s; ++r) {
    v = (v * v) % n - qkd2;
    if (v < 0) v += n;
    if (v < 0) v += n;
    if (v >= n) v -= n;
    if (v >= n) v -= n;
    if (v == 0) return true;
    if (r < s-1) {
        qkd = (qkd * 1ll * qkd) % n;
        qkd2 = qkd * 2;
    }
}
return false;
}

bool prime (int n) {
    // Call for prime check
    for (int i=0; i<trivial_limit && p[i]<n; ++i)
        if (n % p[i] == 0)
            return false;
    if (p[trivial_limit-1]*p[trivial_limit-1] >= n)
        return true;
    if (!miller_rabin (n))
        return false;
    return bpsw (n);
}

void prime_init() {
    // Call before prime check
    for (int i=2, j=0; j<trivial_limit; ++i) {
        bool pr = true;
        for (int k=2; k*k<=i; ++k)
            if (i % k == 0)
                pr = false;
        if (pr)
            p[j++] = i;
    }
```

2. Misc

2.1. Debugging Tips.

- Stack overflow? Recursive DFS on tree that is actually a long path?
- Floating-point numbers
 - Getting NaN? Make sure `acos` etc. are not getting values out of their range (perhaps `1+eps`).
 - Rounding negative numbers?
 - Outputting in scientific notation?
- Wrong Answer?
 - Read the problem statement again!
 - Are multiple test cases being handled correctly? Try repeating the same test case many times.
 - Integer overflow?
 - Think very carefully about boundaries of all input parameters
 - Try out possible edge cases:
 - * $n = 0, n = -1, n = 1, n = 2^{31} - 1$ or $n = -2^{31}$
 - * List is empty, or contains a single element
 - * n is even, n is odd
 - * Graph is empty, or contains a single vertex
 - * Graph is a multigraph (loops or multiple edges)
 - * Polygon is concave or non-simple
 - Is initial condition wrong for small cases?
 - Are you sure the algorithm is correct?
 - Explain your solution to someone.
 - Are you using any functions that you don't completely understand? Maybe STL functions?
 - Maybe you (or someone else) should rewrite the solution?
 - Can the input line be empty?
- Run-Time Error?
 - Is it actually Memory Limit Exceeded?

2.2. Solution Ideas.

- Dynamic Programming
 - Parsing CFGs: CYK Algorithm
 - Drop a parameter, recover from others
 - Swap answer and a parameter
 - When grouping: try splitting in two
 - 2^k trick
 - When optimizing
 - * Convex hull optimization
 - $dp[i] = \min_{j < i} \{dp[j] + b[j] \times a[i]\}$
 - $b[j] \geq b[j + 1]$
 - optionally $a[i] \leq a[i + 1]$
 - $O(n^2)$ to $O(n)$
 - * Divide and conquer optimization
 - $dp[i][j] = \min_{k < j} \{dp[i - 1][k] + C[k][j]\}$
 - $A[i][j] \leq A[i][j + 1]$
 - $O(kn^2)$ to $O(kn \log n)$
 - sufficient: $C[a][c] + C[b][d] \leq C[a][d] + C[b][c], a \leq b \leq c \leq d$ (QI)
 - * Knuth optimization
 - $dp[i][j] = \min_{i < k < j} \{dp[i][k] + dp[k][j] + C[i][j]\}$
 - $A[i][j - 1] \leq A[i][j] \leq A[i + 1][j]$
 - $O(n^3)$ to $O(n^2)$
 - sufficient: QI and $C[b][c] \leq C[a][d], a \leq b \leq c \leq d$
 - Greedy

- Randomized
- Optimizations
 - Use bitset (/64)
 - Switch order of loops (cache locality)
- Process queries offline
 - Mo's algorithm
- Square-root decomposition
- Precomputation
- Efficient simulation
 - Mo's algorithm
 - Sqrt decomposition
 - Store 2^k jump pointers
- Data structure techniques
 - Sqrt buckets
 - Store 2^k jump pointers
 - 2^k merging trick
- Counting
 - Inclusion-exclusion principle
 - Generating functions
- Graphs
 - Can we model the problem as a graph?
 - Can we use any properties of the graph?
 - Strongly connected components
 - Cycles (or odd cycles)
 - Bipartite (no odd cycles)
 - * Bipartite matching
 - * Hall's marriage theorem
 - * Stable Marriage
 - Cut vertex/bridge
 - Biconnected components
 - Degrees of vertices (odd/even)
 - Trees
 - * Heavy-light decomposition
 - * Centroid decomposition
 - * Least common ancestor
 - * Centers of the tree
 - Eulerian path/circuit
 - Chinese postman problem
 - Topological sort
 - (Min-Cost) Max Flow
 - Min Cut
 - * Maximum Density Subgraph
 - Huffman Coding
 - Min-Cost Arborescence
 - Steiner Tree
 - Kirchoff's matrix tree theorem
 - Prüfer sequences
 - Lovász Toggle
 - Look at the DFS tree (which has no cross-edges)
 - Is the graph a DFA or NFA?
 - * Is it the Synchronizing word problem?
- Mathematics
 - Is the function multiplicative?
 - Look for a pattern
 - Permutations
 - * Consider the cycles of the permutation

- Functions
 - * Sum of piecewise-linear functions is a piecewise-linear function
 - * Sum of convex (concave) functions is convex (concave)
- Modular arithmetic
 - * Chinese Remainder Theorem
 - * Linear Congruence
- Sieve
- System of linear equations
- Values too big to represent?
 - * Compute using the logarithm
 - * Divide everything by some large value
- Linear programming
 - * Is the dual problem easier to solve?
- Can the problem be modeled as a different combinatorial problem? Does that simplify calculations?
- Logic
 - 2-SAT
 - XOR-SAT (Gauss elimination or Bipartite matching)
- Meet in the middle
- Only work with the smaller half ($\log(n)$)
- Strings
 - Trie (maybe over something weird, like bits)
 - Suffix array
 - Suffix automaton (+DP?)
 - Aho-Corasick
 - `eerTree`
 - Work with $S + S$
- Hashing
- Euler tour, tree to array
- Segment trees
 - Lazy propagation
 - Persistent
 - Implicit
 - Segment tree of X
- Geometry
 - Minkowski sum (of convex sets)
 - Rotating calipers
 - Sweep line (horizontally or vertically?)
 - Sweep angle
 - Convex hull
- Fix a parameter (possibly the answer).
- Are there few distinct values?
- Binary search
- Sliding Window (+ Monotonic Queue)
- Computing a Convolution? Fast Fourier Transform
- Computing a 2D Convolution? FFT on each row, and then on each column
- Exact Cover (+ Algorithm X)
- Cycle-Finding
- What is the smallest set of values that identify the solution? The cycle structure of the permutation? The powers of primes in the factorization?
- Look at the complement problem
 - Minimize something instead of maximizing

- Immediately enforce necessary conditions. (All values greater than 0? Initialize them all to 1)
- Add large constant to negative numbers to make them positive
- Counting/Bucket sort

Catalan	$C_0 = 1, C_n = \frac{1}{n+1} \binom{2n}{n} = \sum_{i=0}^{n-1} C_i C_{n-i-1} = \frac{4n-2}{n+1} C_{n-1}$	
Stirling 1st kind	$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = 1, \begin{bmatrix} n \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ n \end{bmatrix} = 0, \begin{bmatrix} n \\ k \end{bmatrix} = (n-1) \begin{bmatrix} n-1 \\ k \end{bmatrix} + \begin{bmatrix} n-1 \\ k-1 \end{bmatrix}$	#perms of n objs with exactly k cycles
Stirling 2nd kind	$\left\{ \begin{smallmatrix} n \\ 1 \end{smallmatrix} \right\} = \left\{ \begin{smallmatrix} n \\ n \end{smallmatrix} \right\} = 1, \left\{ \begin{smallmatrix} n \\ k \end{smallmatrix} \right\} = k \left\{ \begin{smallmatrix} n-1 \\ k \end{smallmatrix} \right\} + \left\{ \begin{smallmatrix} n-1 \\ k-1 \end{smallmatrix} \right\}$	#ways to partition n objs into k nonempty sets
Euler	$\left\langle \begin{smallmatrix} n \\ 0 \end{smallmatrix} \right\rangle = \left\langle \begin{smallmatrix} n \\ n-1 \end{smallmatrix} \right\rangle = 1, \left\langle \begin{smallmatrix} n \\ k \end{smallmatrix} \right\rangle = (k+1) \left\langle \begin{smallmatrix} n-1 \\ k \end{smallmatrix} \right\rangle + (n-k) \left\langle \begin{smallmatrix} n-1 \\ k-1 \end{smallmatrix} \right\rangle$	#perms of n objs with exactly k ascents
Euler 2nd Order	$\left\langle\!\left\langle \begin{smallmatrix} n \\ k \end{smallmatrix} \right\rangle\!\right\rangle = (k+1) \left\langle\!\left\langle \begin{smallmatrix} n-1 \\ k \end{smallmatrix} \right\rangle\!\right\rangle + (2n-k-1) \left\langle\!\left\langle \begin{smallmatrix} n-1 \\ k-1 \end{smallmatrix} \right\rangle\!\right\rangle$	#perms of $1, 1, 2, 2, \dots, n, n$ with exactly k ascents
Bell	$B_1 = 1, B_n = \sum_{k=0}^{n-1} B_k \binom{n-1}{k} = \sum_{k=0}^n \left\{ \begin{smallmatrix} n \\ k \end{smallmatrix} \right\}$	#partitions of $1..n$ (Stirling 2nd, no limit on k)

#labeled rooted trees	n^{n-1}
#labeled unrooted trees	n^{n-2}
#forests of k rooted trees	$\frac{k}{n} \binom{n}{k} n^{n-k}$
$\sum_{i=1}^n i^2 = n(n+1)(2n+1)/6$	$\sum_{i=1}^n i^3 = n^2(n+1)^2/4$
$!n = n \times!(n-1) + (-1)^n$	$!n = (n-1)(!(n-1) +!(n-2))$
$\sum_{i=1}^n \binom{n}{i} F_i = F_{2n}$	$\sum_i \binom{n-i}{i} = F_{n+1}$
$\sum_{k=0}^n \binom{k}{m} = \binom{n+1}{m+1}$	$x^k = \sum_{i=0}^k i! \left\{ \begin{smallmatrix} k \\ i \end{smallmatrix} \right\} \binom{x}{i} = \sum_{i=0}^k \left\langle \begin{smallmatrix} k \\ i \end{smallmatrix} \right\rangle \binom{x+i}{k}$
$a \equiv b \pmod{x, y} \Rightarrow a \equiv b \pmod{\text{lcm}(x, y)}$	$\sum_{d n} \phi(d) = n$
$ac \equiv bc \pmod{m} \Rightarrow a \equiv b \pmod{\frac{m}{\text{gcd}(c, m)}}$	$(\sum_{d n} \sigma_0(d))^2 = \sum_{d n} \sigma_0(d)^3$
$p \text{ prime} \Leftrightarrow (p-1)! \equiv -1 \pmod{p}$	$\text{gcd}(n^a - 1, n^b - 1) = n^{\text{gcd}(a, b)} - 1$
$\sigma_x(n) = \prod_{i=0}^r \frac{p_i^{(a_i+1)x} - 1}{p_i^x - 1}$	$\sigma_0(n) = \prod_{i=0}^r (a_i + 1)$
$\sum_{k=0}^m (-1)^k \binom{n}{k} = (-1)^m \binom{n-1}{m}$	$\sum_{i=1}^n 2^{\omega(i)} = O(n \log n)$
$2^{\omega(n)} = O(\sqrt{n})$	$v_f^2 = v_i^2 + 2ad$
$d = v_i t + \frac{1}{2} a t^2$	$d = \frac{v_i + v_f}{2} t$
$v_f = v_i + at$	

2.3. The Twelvefold Way. Putting n balls into k boxes.

Balls	same	distinct	same	distinct	Remarks
Boxes	same	same	distinct	distinct	
-	$p_k(n)$	$\sum_{i=0}^k \left\{ \begin{smallmatrix} n \\ i \end{smallmatrix} \right\}$	$\binom{n+k-1}{k-1}$	k^n	$p_k(n)$: #partitions of n into $\leq k$ positive parts
size ≥ 1	$p(n, k)$	$\left\{ \begin{smallmatrix} n \\ k \end{smallmatrix} \right\}$	$\binom{n-1}{k-1}$	$k! \left\{ \begin{smallmatrix} n \\ k \end{smallmatrix} \right\}$	$p(n, k)$: #partitions of n into k positive parts
size ≤ 1	$[n \leq k]$	$[n \leq k]$	$\binom{k}{n}$	$n! \binom{k}{n}$	$[cond]$: 1 if $cond = true$, else 0

PRACTICE CONTEST CHECKLIST

- How many operations per second? Compare to local machine.
- What is the stack size?
- How to use printf/scanf with long long/long double?
- Are `__int128` and `__float128` available?
- Does MLE give RTE or MLE as a verdict? What about stack overflow?
- What is `RAND_MAX`?
- How does the judge handle extra spaces (or missing newlines) in the output?
- Look at documentation for programming languages.
- Try different programming languages: C++, Java and Python.
- Try the submit script.
- Try local programs: `i?python[23]`, `factor`.
- Try submitting with `assert(false)` and `assert(true)`.
- Return-value from `main`.
- Look for directory with sample test cases.
- Make sure printing works.
- Remove this page from the notebook.