Homework 1

FE621 Computational Finance

due 23:55, Sunday Feb 19, 2017

February 4, 2017

For all the problems in this assignment you need to design and use a computer program, output results and present the results in nicely formatted tables and figures. The computer program may be written in any programming language you want.

Problem 1 (60 points). Black–Scholes–Merton Pricing Formula. Implied Volatility.

a) Implement a function that computes the price of an European Call and Put Option. The function should receive as parameters the stock price S_0, the time to maturity $\tau = T - t$, the strike price K, the interest rate r, and the volatility σ. Calculate and output both the price of a call and a put for: $S_0 = K = 100$, time to expiry 1 month, i.e., $\tau = 30/252$, $r = 5\%$, and $\sigma = 20\%$. *Hint:* see Chapter 1.4 in [1].

b) Check that the Put-Call parity relation holds, please see Section 4 from [2]. Use the same parameters as above.

c) The implied volatility is, by definition, the value of σ for which the function

$$f(\sigma) = C_{BSM}(S_0, K, T, r; \sigma) - C_M(K, T)$$

is zero. Here, we denoted by $C_M(K, T)$ and $C_{BSM}(S_0, K, T; \sigma)$ the market price of an European Call Option and the Black–Scholes–Merton price, respectively. Typically, $C_M(K, T)$ is taken as the average of best bid and best ask quotes for the respective option from the financial market. In this problem please download option prices (you can use the Bloomberg Terminal, Yahoo! Finance, etc.) for an equity (any equity not an ETF or index), for 3 different maturities (nearest to 1 month, 2 months, and 6 months) and 20 strike prices. Compute the implied volatility using the bisection method for each of these options. Use a tolerance level $\varepsilon = 10^{-4}$ and present the results in a table.

d) Using the same data as in part c), calculate the implied volatilities using the Secant Method and compare the results with the ones in the previous part. What do you observe? Write a paragraph comparing the two methods. *Hint:* to compare the two algorithms, one can use two indicators: the time to execute the algorithm and the number of iterations necessary to reach convergence.

e) Consider the implied volatility values obtained in either of previous parts. Create a 2 dimensional plot of implied volatilities versus strike K for the closest to maturity options. What do you observe? Plot all implied volatilities for the three different maturities on the same plot, where you use a different color for each maturity. In total there should be 3 sets of points plotted with different color. (BONUS) Create a 3D plot of the same implied vols as a function of both maturity and strike, i.e.: $\sigma(\tau_i, K_j)$ where $i = 1, 2, 3$, and $j = 1, 2, \ldots, 20$.

f) (Greeks) Calculate the derivatives of the call option price with respect to S (Delta), and σ (Vega) and the second derivative with respect to S (Gamma). The parameters are as in part a). Approximate these derivatives using an approximation of the partial derivatives. Compare the numbers obtained by the two methods.

g) Apply the formulae developed in part f) to all the options you looked at in part c). To this end use the implied volatilities you previously calculated for each of the options.

Problem 2 (30 points). Numerical Integration of real–valued func-

tions. Consider the real–valued function

$$f(x) = \begin{cases} \dfrac{\sin(x)}{x}, & \text{for } x \neq 0, \\ \\ 1, & \text{for } x = 0. \end{cases}$$

We can calculate this integral as: $\int_{-\infty}^{\infty} f(x)\mathrm{d}x = \pi$.

a) Implement the trapezoidal and the Simpson's quadrature rules to numerically compute the definite integral above. The algorithms are presented in [3], please see Chapter 5. *Hint:* you can approximate the indefinite integral by considering a large interval $[-a, +a]$ (for example $a = 10^6$). Consider equidistant nodes $\{x_n\}_{n=0}^{N}$, i.e., $x_n = -a + n\frac{2a}{N}$, $n = 0, 1, \ldots, N$, where N is a large integer.

b) Compute the truncation error for the numerical algorithms implemented in a) for a particular $a \in \mathbb{R}$ and $N \in \mathbb{N}$. That is create a function of a and N that will output $I_N - \pi$, where $I_{N,a}$ is the numerical approximation of the integral. Study the changes in the approximation as N and a increase as well as the difference between the two quadrature approximations. Please write your observations.

c) In a typical scenario we do not know the true value of the integral. Thus, to ensure the convergence of the numerical algorithms we pick a small tolerance value ε and we check at every iteration $k = 1, 2, \ldots$ if the following condition holds:

$$|I_k - I_{k-1}| < \varepsilon,$$

where I_k is the value of the integral at step k. When the condition holds, the algorithm stops. Evaluate the number of steps until the algorithms from a) reach convergence for $\varepsilon = 10^{-4}$. What do you observe?

d) Consider

$$g(x) = 1 + e^{-x}\sin(8x^{2/3}).$$

Use the trapezoidal rule and Simpson's rule to approximate $\int_0^2 g(x)\mathrm{d}x$. Use a tolerance level of $\varepsilon = 10^{-4}$.

Problem 3 (10 points). The Heston Model and Stochastic Volatility. The numerical integration techniques in Problem 2 are used in practice to calculate the price of Options. Here we consider the Heston Model. The paper [4] provides a good implementation of the Heston Model and the corresponding pricing problem. Compute numerically the price of the European Call Option using the formula (1.4) in [4], with the parameters provided in the same paper in Section 6. Please note that in order to calculate the call option price $C(S_0, K, V_0, t, T)$ one needs to evaluate the integral in equation (1.5) in [4]. Use Simpson's Rule for this purpose, with tolerance $\varepsilon = 10^{-4}$. Please check your answers versus the Table 1 in the cited paper.

Submission specifications. Please submit a written report in the portable document format (pdf), where you detail your results and copy your code into an Appendix. Also submit a zip archive containing the code used properly commented.

References

[1] Clewlow, Les and Strickland, Chris. *Implementing Derivative Models (Wiley Series in Financial Engineering)*, John Wiley & Sons 1996.

[2] *Lecture1-ReviewBlackScholesmodel.pdf*, 2017.

[3] Rouah, F. D. *The Heston Model and Its Extensions in Matlab and C*, 2013, John Wiley & Sons.

[4] Mikhailov, Sergei and Nögel, Ulrich. "Heston's stochastic volatility model: Implementation, calibration and some extensions" *Wilmott Journal*, 2004.