Homework 2

FE621 Computational Methods in Finance
due 23:55ET, Sunday March 5, 2017

Instructions. For all the problems in this assignment you need to write computer programs and analyze data. All results need to be presented in clearly formatted tables and figures. You may use any programming language you want. Please submit an archive containing the code with comments. You will be graded based on the report you produce. The report should be in a pdf format and should contain relevant code as appropriate included in the Appendix and referenced throughout the text.

Problem 1 (40 points). The Binomial Tree.

(a) Construct code to calculate option values using *an additive binomial tree*. For this part you need for versions European and American as well as Call and Put. You may use the same tree construction for all options.

(b) Download Option prices (you can use the Bloomberg Terminal, Yahoo! Finance, etc.) for an equity, for 3 different maturities (1 month, 2 months, and 3 months) and 20 strike prices close to the value at the money. If 3 months does not exist use next one available. Please download the data DURING THE TRADING DAY (9:00am to 4:30pm ET). Otherwise your values will be way off. Do not forget to download the value of the underlying. For each strike price in the data, use the implied vol values in Homework 1 (see Problem 1c) and the current short-term interest rate (today the Feds Fund rate is $r = 0.75\%$). Calculate the option price (European Calls and Puts) using the binomial tree, and compare the results with the Black–Scholes price. Use at least 200 steps in your tree construction. Treat the options as American as well and plot these values side by side with the European and Black Scholes values. When you create the plot do not forget to plot the bid-ask values as well

(c) Comment of the table in the previous part.

(d) Consider $N \in \{10, 20, 30, 40, 50, 100, 150, 200, 250, 300, 350, 400\}$. Compute and plot the absolute error for the European Put ε_N for as a function of $N \in \mathbb{N}^*$ the number of steps in the tree:

$$\varepsilon_N = \left| P^{BSM}(S_0, K, T, r; \sigma) - P_N^{BinomTree}(S_0, K, T, r; \sigma) \right|,$$

where $C^{BSM}(S_0, K, T, r; \sigma)$ and $C_N^{BinomTree}(S_0, K, T, r; \sigma)$ are the Black–Scholes–Merton price and the price calculated using a binomial tree with N steps, respectively. What can you observe?

Bonus 5 points Using the binomial tree for American Calls and Puts, calculate the implied volatility corresponding to the data you have downloaded in part (b). You will need to use the bisection or Newton/secant method of finding roots with the respective binomial trees. Compare these values of the implied volatility with the volatilities calculated using the usual Black Scholes formula (as in Homework 1, Problem 1c). Write detailed observations.

Problem 2 (30 points). The Trinomial Tree.

(a) Implement a trinomial tree to price European, American Call and Put options. *Hint.* See Chapter 3 in [1].

(b) Consider $S_0 = 100, K = 100$, $T = 1$ year, $\sigma = 25\%, r = 6\%, \delta = 0.03$. Repeat the methods in problem 1 b) to d) with these parameters. Use at least $N = 200$ time steps and you do not need to download data. Make sure that convergence condition holds, i.e. $\Delta x \geq \sigma\sqrt{3\Delta t}$ (see Section 3.5 in [1]). Create a table containing all results and comment

Problem 3 (30 points). Pricing Exotic Options. We will use here a synthetic example to illustrate the power of the tree models. Please read Section 2.10 in [1] and Sections 1 and 5.1 in [2] (available here), and solve the following problems.

a) Construct a binomial tree to calculate the price of an European Up-and-Out call option. Use $S_0 = 10$, strike $K = 10$, maturity $T = 0.3$, volatility $\sigma = 0.2$, short rate $r = 0.01$, dividends $\delta = 0$, and barrier $H = 11$. Use as many steps in your tree as you think are necessary. *Hint.* Read the algorithm in the book [1] and try and figure out how to modify the code there to work with the new option.

b) For the European Up-and-Out Call option explicit formulas exist. For example, implement the formula (5.2) from [2] and compare your results with part (a). Use the same parameters as before. Are your results matching?

c) Price an European Up-and-In call option, using the same parameters as before. *Hint.* Two methods can be employed: the analytical solution in (5.1) or the In-Out parity. Use both methods in order to verify your results.

d) Calculate the price of an AMERICAN Up and In Put option

Bonus Problem (30 points). A two-dimensional tree for the Heston model. Beliaeva and Nawalkha (2010) have developed in [5] a path-independent two-dimensional tree for the Heston model. In their approach, separate trees for the stock price and for the variance are constructed independently of one another, and then recombined, please see Chapter 8 in [3] for a detailed presentation.

(a) Price an American Put Option using the Beliaeva and Nawalkha method. Please note that the code provided in the book is incomplete (marked with ". . . "). Consider the same numerical values for the parameters of interest as in [4].

(b) Price an European Call Option using the Beliaeva and Nawalkha method. Compare this result with the prices obtained in Homework 1, Problem 3 via the analytical formula. Consider the same numerical values as in [4]. What can you observe?

Bonus Problem (100 points). A multinomial recombining tree for general Stochastic Volatility models. We consider here an interesting method of option pricing under general assumptions, involving a multinomial recombining tree and particle filtering techniques. Please read the paper [6], and pay special attention to sections 3 and 4.1, 4.2.

(a) Using synthetic parameters, i.e., chosen by you, estimate the probability distribution for the volatility process Y_t at discrete time points $t_1, t_2, \ldots, t_n$. To this end, implement the particle filter described in Section 3 of [6]. You should store from this step the particles $\{\bar{Y}_1, \bar{Y}_2, \ldots, \bar{Y}_n\}$ together with their corresponding probabilities $\{p_1, p_2, \ldots, p_n\}$.

(b) Construct the successors in the multinomial tree with synthetic parameters, see Section 4.1. Please note that two separate cases were analyzed.

(c) You built in part (b) the one period model. Implement now the multi-period model described in Section 4.2. Price for illustrative purposes the European Call option and try to replicate the results in Section 6 of the paper.

(d) Build a multinomial tree as in [6] to price an American Put Option. Compare your results with Problem 4(a).

(e) The most important limitation here is that the two Brownian Motions W_t and Z_t in equation (1.1) are assumed independent, i.e.

$$\mathbb{E}[\underset{\cdot}{W}_t \underset{\cdot}{Z}_t] = 0.$$

To circumvent this, let $\sigma(y) = \sqrt{y}$, $\psi(y) = \gamma\sqrt{y}$, and $\mathbb{E}[\underset{\cdot}{W}_t \underset{\cdot}{Z}_t] = \rho$ in (1.1), i.e., the Heston Model, and consider the transformation

$$X_t = \ln S_t - \frac{\rho}{\gamma} Y_t - H_t, \text{ where } H_t = \left(\mu - \frac{\rho\alpha\nu}{\gamma}\right) t$$

Apply Ito's Lemma on X_t and show that $\underset{\cdot}{X}_t$ can be written as $\underset{\cdot}{X}_t = \mu_X(t)\underset{\cdot}{t} + \sigma_X(t)\underset{\cdot}{B}_t$, where B_t is uncorrelated with W_t, and μ_X, σ_X are real–valued functions. Build a multinomial tree as in [6] to price an American

Put Option. Compare your results with Problem 4(a) and part (d) of this problem.

References

[1] Clewlow, Les and Strickland, Chris. *Implementing Derivative Models (Wiley Series in Financial Engineering)*, John Wiley and Sons 1996.

[2] Niklas Westermark. *Barrier Option Pricing*, Degree Project in Mathematics, First Level. KTH Royal Institute of Technology, Stockholm, Sweden.

[3] Rouah, F. D. *The Heston Model and Its Extensions in Matlab and C*, 2013, John Wiley and Sons.

[4] Mikhailov, Sergei and Nögel, Ulrich. *Heston's stochastic volatility model: Implementation, calibration and some extensions* 2004, John Wiley and Sons.

[5] Beliaeva, Natalia A and Sanjay K, Nawalkha, *A simple approach to pricing American options under the Heston stochastic volatility model.* The Journal of Derivatives, Vol. 17, No. 4, pp. 25–43, 2010.

[6] Florescu, Ionuţ, Viens, Frederi G, *Stochastic volatility: option pricing using a multinomial recombining tree*, Applied Mathematical Finance, Vol. 15, No. 2, pp. 151–181, 2008, Taylor & Francis.