Question1

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Getting the data

```
df=read.csv(file="sample_data.csv",header=TRUE, sep=",")

head(df)
```

```
##      stock1    stock2    stock3    stock4   stock5
## 1 100.00000 100.00000 100.00000 100.00000 100.0000
## 2 100.02068  99.99415 100.04909  99.92849 100.1941
## 3 100.02660 100.01964  99.94416  99.92583 100.4359
## 4 100.00023 100.07849  99.94365  99.96918 100.4597
## 5 100.00754 100.13089  99.87517  99.93167 100.4614
## 6  99.96268 100.15621  99.91428  99.96727 100.3518
```

```
stock1=ts(df[1])
stock2=ts(df[2])
stock3=ts(df[3])
stock4=ts(df[4])
stock5=ts(df[5])
```

Defining the function to fit the 5 models defined below

Model 1 : • $dS_t = \theta_1 S_t dt + \theta 2 S_t^{\theta_3} dW_t$

Model 2 : • $dS_t = (\theta_1 + \theta_2 S_t)dt + \theta_3 S_t^{\theta_4} dW_t$

Model 3 : • $dS_t = (\theta_1 + \theta_2 S_t)dt + \theta_3 \sqrt{S_t} dW_t$

Model 4 : • $dS_t = \theta_1 dt + \theta_2 S_t^{\theta 3} dW t$

Model 5 : • $dS_t = \theta_1 S_t dt + (\theta_2 + \theta_3 S_t^{\theta_4} dW t)$

Diff SDEs

```
library(Sim.DiffProc)
library(Ecdat)
```

Defining the SDE’s

```
fx <-{}
gx <-{}
#model 1 drift and diffusion
fx[1] <- expression( theta[1]*x )
gx[1]<- expression( theta[2]*x^theta[3] )

#model 2 drift and diffusion
fx[2] <- expression( theta[1]+theta[2]*x )
gx[2]<- expression( theta[3]*x^theta[4] )

#model 3 drift and diffusion
fx[3] <- expression( theta[1]+theta[2]*x )
gx[3]<- expression( theta[3]*sqrt(x) )

#model 4 drift and diffusion
fx[4]<- expression( theta[1] )
gx[4] <- expression( theta[2]*x^theta[3] )

#model 5 drift and diffusion
fx[5] <- expression( theta[1]*x )
gx[5] <- expression(theta[2] + (theta[3]*x^theta[4]) )
```

Finding the best model

```
pmle=eval(formals(fitsde.default)$pmle)
print("We'll use euler method for our Maximum Likelyhood")
```

```
## [1] "We'll use euler method for our Maximum Likelyhood"
```

```
Best.fit<-function(data,pmle)
{
  #model1
  mod1 <- fitsde(data=data,drift=fx[1],diffusion=gx[1],start =
                  list(theta1=1, theta2=1,theta3=1),pmle=pmle)
  #model 2
  mod2 <- fitsde(data=data,drift=fx[2],diffusion=gx[2],start =
                  list(theta1=1, theta2=1,theta3=1,theta4=1),pmle=pmle)
  #model 3
  mod3 <- fitsde(data=data,drift=fx[3],diffusion=gx[3],start =
                  list(theta1=1, theta2=1,theta3=1),pmle=pmle)
  #model 4
  mod4 <- fitsde(data=data,drift=fx[4],diffusion=gx[4],start =
                  list(theta1=1, theta2=1,theta3=1),pmle=pmle)

  #model 5
  mod5 <- fitsde(data=data,drift=fx[5],diffusion=gx[5],start =
                  list(theta1=1, theta2=1,theta3=1, theta4=1),pmle=pmle)
  #Computes AIC
  AIC <- c(AIC(mod1),AIC(mod2),AIC(mod3),AIC(mod4),AIC(mod5))
  Test <- data.frame(AIC,row.names = c("Model 1","Model 2","Model 3", "Model 4","Model 5"))
  Test
  # Bestmod <- rownames(Test)[which.min(Test[,1])]
  Bestmod <- which.min(Test[,1])
  list('best.model'=Bestmod,'AIC.results'=Test)
}
```

Function to estimate parameter

```
Diff.mle <-function(fx,gx,data)
{
  pmle <- eval(formals(fitsde.default)$pmle)
  fitres <- lapply(1:4, function(i) fitsde(data=data,drift=fx,diffusion=gx,pmle=pmle[i],
                                   start = list(theta1=1,theta2=1,theta3=1,theta4=1)))
  Coef <- data.frame(do.call("cbind",lapply(1:4,function(i) coef(fitres[[i]]))))
  Info <- data.frame(do.call("rbind",lapply(1:4,function(i) AIC(fitres[[i]]))),
                   row.names=pmle)
  names(Coef) <- c(pmle)
  names(Info) <- c("AIC")
  list("Info"=Info,"Coef"=Coef)
}
```

For Stock 1

Part 1)

```
fit1=Best.fit(data =stock1,pmle = pmle[1])
print(paste("Best model = model ",fit1$best.model))
```

```
## [1] "Best model = model  2"
```

Part 2)

```
print("The parameter estimates are:")
```

```
## [1] "The parameter estimates are:"
```

```
ls1=Diff.mle(fx=fx[fit1$best.model],gx=gx[fit1$best.model],data = stock1)
print(ls1$Coef)
```

```
##                euler   kessler         ozaki         shoji
## theta1 3.504814e-05 1.0000448  0.0547352626 -5.415661e-04
## theta2 2.214008e-05 0.6862942 -0.0001325927  2.478136e-05
## theta3 7.536530e-03 0.8824043  0.0040184172  9.165665e-03
## theta4 3.903047e-01 0.4091350  0.5276808366  3.500519e-01
```

Part 3)

```
print(ls1$Info)
```

```
##                AIC
## euler   -252820.0
## kessler       8.0
## ozaki   -231969.0
## shoji   -251262.5
```

```
print(paste(rownames(ls1$Info)[which.min(ls1$Info[,1])]," method gives the best estimate"))
```

```
## [1] "euler  method gives the best estimate"
```

For Stock 2

Part 1)

```
fit2=Best.fit(data =stock2,pmle = pmle[1])
print(paste("Best model = model ",fit2$best.model))
```

```
## [1] "Best model = model  4"
```

Part 2)

```
print("The parameter estimates are:")
```

```
## [1] "The parameter estimates are:"
```

```
ls2=Diff.mle(fx=fx[fit2$best.model],gx=gx[fit2$best.model],data = stock2)
print(ls2$Coef)
```

```
##              euler     kessler ozaki shoji
## theta1 0.007636416 0.004397859     1     1
## theta2 0.006855019 0.007841422     1     1
## theta3 0.537076888 0.513984058     1     1
## theta4 1.000000000 1.000000000     1     1
```

Part 3)

```
print(ls2$Info)
```

```
##               AIC
## euler   -125662.2
## kessler -125065.5
## ozaki         8.0
## shoji         8.0
```

```
print(paste(rownames(ls2$Info)[which.min(ls2$Info[,1])]," method gives the best estimate"))
```

```
## [1] "euler  method gives the best estimate"
```

For Stock 3

Part 1)

```
fit3=Best.fit(data =stock3,pmle = pmle[1])
print(paste("Best model = model ",fit3$best.model))
```

```
## [1] "Best model = model  1"
```

Part 2)

```
print("The parameter estimates are:")
```

```
## [1] "The parameter estimates are:"
```

```
ls3=Diff.mle(fx=fx[fit3$best.model],gx=gx[fit3$best.model],data = stock3)
print(ls3$Coef)
```

```
##                euler   kessler         ozaki        shoji
## theta1  0.003498813 0.5498652  7.747055e-06 1.803336e-05
## theta2 -3.583787343 0.9359193 -2.858406e-03 5.676480e-03
## theta3  3.734064202 0.6267437  8.356426e-01 7.071774e-01
## theta4  1.000000000 1.0000000  1.000000e+00 1.000000e+00
```

Part 3)

```
print(ls3$Info)
```

```
##              AIC
## euler       8.00
## kessler     8.00
## ozaki   39763.07
## shoji   44836.87
```

```
print(paste(rownames(ls3$Info)[which.min(ls3$Info[,1])]," method gives the best estimate"))
```

```
## [1] "euler  method gives the best estimate"
```

For Stock 4

Part 1)

```
fit4=Best.fit(data =stock4,pmle = pmle[1])
print(paste("Best model = model ",fit4$best.model))
```

```
## [1] "Best model = model  2"
```

Part 2)

```
print("The parameter estimates are:")
```

```
## [1] "The parameter estimates are:"
```

```
ls4=Diff.mle(fx=fx[fit4$best.model],gx=gx[fit4$best.model],data = stock4)
print(ls4$Coef)
```

```
##                euler   kessler         ozaki        shoji
## theta1 -9.421015e-05 1.0072354  6.279268e-03 3.042835e-05
## theta2  9.618334e-06 0.4606634 -5.520604e-05 9.567708e-06
## theta3  1.471477e-02 0.6495840  2.751369e-02 1.455193e-02
## theta4  4.269734e-01 0.1907163  3.015314e-01 4.293345e-01
```

Part 3)

```
print(ls4$Info)
```

```
##               AIC
## euler   -128994.3
## kessler       8.0
## ozaki   -127672.7
## shoji   -129012.0
```

```
print(paste(rownames(ls4$Info)[which.min(ls4$Info[,1])]," method gives the best estimate"))
```

```
## [1] "shoji  method gives the best estimate"
```

For Stock 5

Part 1)

```
fit5=Best.fit(data =stock5,pmle = pmle[1])

print(paste("Best model = model ",fit5$best.model))
```

```
## [1] "Best model = model  4"
```

Part 2)

```
print("The parameter estimates are:")
```

```
## [1] "The parameter estimates are:"
```

```
ls5=Diff.mle(fx=fx[fit5$best.model],gx=gx[fit5$best.model],data = stock5)
print(ls5$Coef)
```

```
##              euler    kessler ozaki shoji
## theta1 0.003342042 0.01128962     1     1
## theta2 0.009936535 0.01075011     1     1
## theta3 0.530964394 0.51575952     1     1
## theta4 1.000000000 1.00000000     1     1
```

Part 3)

```
print(ls5$Info)
```

```
##                 AIC
## euler   -51791.13
## kessler -51313.00
## ozaki        8.00
## shoji        8.00
```

```
print(paste(rownames(ls5$Info)[which.min(ls5$Info[,1])]," method gives the best estimate"))
```

```
## [1] "euler  method gives the best estimate"
```