# Learning to rank

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#### learning to rank



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**Learning to rank** or machine-**learned ranking** (MLR) is the application of machine **learning**, typically supervised, semi-supervised or reinforcement **learning**, in the construction of **ranking** models for information retrieval systems.

### Методы обучения ранжированию (Learning to Rank)

machinelearning.ru > wiki/images/8/89/Voron...Ranking... v

Постановка задачи и приложения Основные подходы к ранжированию. Ранжирование в Яндексе. Методы обучения ранжированию (**Learning to Rank**).



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Winning entry in the recent Yahoo **Learning to Rank** competition used an ensemble of LambdaMART models.[22]. 2008.

#### W Learning to rank - WikiVisually

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Often a **learning-to-rank** problem is reformulated as an optimization problem with respect to one of these metrics. Examples of **ranking** quality measures

### & Ещё одна важная постановка задачи – learning to rank

logic.pdmi.ras.ru > Sergey Nikolenko > .../mlhse17/17-ranking.pdf ▼ развитие learning to rank. • Обычно подходы работают на pairwise-ошибке: • RankSVM: берём SVM с ошибкой ℓ(t) = max(0, 1 - t)...



#### Нашлось 67 млн результатов

23 показа в месяц

Дать объявление

# Relevancy

How to understand that document is relevant to some query? What different factors can we consider?

- Analyze content (text, images, links) of web page
- Analyze query and context

### Examples of content factors:

- Text representations
- Web-graph characteristics (the way to detects artificial structures)

### Examples of query/context factors:

- User age/location/gender
- User search story (preferences are important)

To increase the quality of search we probably should care about:

- diversity in top-k results
- spam/fraud/irrelevant porn pics filters
- speed

### **Problem**

### Given:

- X: documents (objects) from some universal set
- R: set of indexes pairs (i, j)
- y: "true relevance"

```
Objective: model a such that: a(X(i)) > a(X(j)) if and only if (i, j) is in R (or y(i) > y(j)).
```

Note that these two approaches differ from each other.

### Key point:

- structure we have to optimize is not continuous at all (need tricks to use common optimizations methods)
- where can we get data to optimize our algorithms?
   (some precomputed experts answers, user search history(!!!))

# Pointwise approach

- 1. look at every document
- 2. try to predict how relevant it is for the current query
- 3. sort the result list by model scores
- 4. profit? (not really)

### Loss Function:

sum of losses L(a(x) - y(x)) for every document x

### Good:

- + we already know enough methods
- + easy and fast in case we have "good" data to train

### Bad:

- we completely ignored initial problem :(
- extra work: for ranking system scores (1, 2) and (1.4, 1.6) lead to the same sort result

2007	QBRank 🔑	pairwise
2007	RankCosine 🔑	listwise
2007	RankGP <sup>[19]</sup>	listwise
2007	RankRLS 🔑	pairwise
2007	SVM <sup>map</sup>	listwise
2008	LambdaMART 🔼	pairwise/listwise
2008	ListMLE 🔼	listwise
2008	PermuRank 🔑	listwise
2008	SoftRank®	listwise
2008	Ranking Refinement [32]	pairwise
2008	SSRankBoost & [23]	pairwise
2008	SortNet [][24]	pairwise
2009	MPBoost 🔑	pairwise
2009	BoltzRank 🔑	listwise
2009	BayesRank 📜	listwise
2010	NDCG Boost [35]	listwise
2010	GBlend ₽	pairwise
2010	IntervalRank 🔑	pairwise & listwise
2010	CRR 🔑	pointwise & pairwise
2017	ES-Rank 🔑	listwise

# Pairwise approach

- 1. look at every pair (document, document)
- 2. try to predict the correct order for 2 documents
- 3. sort the result list by model scores
- 4. profit?

### Loss Function:

sum of losses L(a(x) - a(y)) for every pair of documents (x, y)

$$\sum_{(i,j)\in R} [a(x_j) - a(x_i) < 0] \leqslant \sum_{(i,j)\in R} L(a(x_j) - a(x_i))$$

#### Good:

+ seems reasonable & works well

### Bad:

- not as fast
- don't focus on the most relevant documents

2007	QBRank 🔑	pairwise
2007	RankCosine 🔑	listwise
2007	RankGP <sup>[19]</sup>	listwise
2007	RankRLS 🔑	pairwise
2007	SVM <sup>map</sup>	listwise
2008	LambdaMART 🔼	pairwise/listwise
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2009	BoltzRank 🔑	listwise
2009	BayesRank 🔑	listwise
2010	NDCG Boost [35]	listwise
2010	GBlend &	pairwise
2010	IntervalRank 🔑	pairwise & listwise
2010	CRR 🔑	pointwise & pairwise
2017	ES-Rank 🔑	listwise

# Listwise approach

- 1. look at all documents
- try to predict the correct permutation or some probability distribution z on all permutations

### Loss Function:

y is 'true relevancy' z in predicted permutations probabilities.

$$Q(y,z) = -\sum_{j=1}^{n_q} P_y(j) \log P_z(j). \quad P_z(j) = \frac{\varphi(z_j)}{\sum_{k=1}^n \varphi(s_k)} \quad P_z(\pi) = \prod_{j=1}^{n_q} \frac{\varphi(z_{\pi(j)})}{\sum_{k=j}^{n_q} \varphi(z_{\pi(k)})},$$

### Good:

+ we consider more aspects of initial problem

### Bad:

- probably not as fast
- not as easy

2007	QBRank 🔑	pairwise
2007	RankCosine 🔑	listwise
2007	RankGP <sup>[19]</sup>	listwise
2007	RankRLS	pairwise
2007	SVM <sup>map</sup>	listwise
2008	LambdaMART 🔼	pairwise/listwise
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2008	PermuRank 🔑	listwise
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2010	NDCG Boost [35]	listwise
2010	GBlendr	pairwise
2010	IntervalRank 🔑	pairwise & listwise
2010	CRR 🔎	pointwise & pairwise
2017	ES-Rank	listwise

# **Metrics**

For binary classification (relevant or not) for query/document pairs we could use simple metrics: F-score, AUC-ROC, etc.

We often focus on the quality of top-k relevant documents and don't really care about irrelevant documents ordering.

@k is the same as top-k.

### **Precision metrics**

In case relevancy can be represented as r^true - {0, 1} - variable:

precision at k:

$$p@K = \frac{\sum_{k=1}^{K} r^{true}(\pi^{-1}(k))}{K}$$

average precision at k:

$$ap@K = \frac{1}{K} \sum_{k=1}^{K} r^{true}(\pi^{-1}(k)) \cdot p@k.$$

# **Metrics**

In case relevancy can be represented as y - [0, 1] - variable:

DCG (discounted cumulative gain): g(y) represents metrics gain from including document d(i) represents discount based on document index

DCG@k(q) = 
$$\sum_{i=1}^{k} g(y_{(i)})d(i)$$
.

normalized DCG max is taken over all possible top-k documents

$$nDCG@k(q) = \frac{DCG@k(q)}{\max DCG@k(q)}.$$

# Cascade metrics (pFound)

Consider user who consequently looks at every document in top-k and guits randomly after every irrelevant document or in case he/she found needed information.

Let y(i) is probability to find needed information in i\_th document. Let p\_out is probability to quit after every failed attempt. Let p\_i is probability to look at i\_th document.

Obviously p\_1 = 1 and 
$$p_{i+1} = p_i (1-y_{(i)})(1-p_{\mathrm{out}})$$

Obviously p\_1 = 1 and  $p_{i+1}=p_i(1-y_{(i)})(1-p_{\mathrm{out}}),$  Then probability to find needed information  $\mathrm{pFound}@k(q)=\sum_{i=1}^k p_i y_{(i)}.$ 

### **TF-IDF**

### Déjà vu

### Variants of term frequency (TF) weight

weighting scheme	TF weight
binary	0,1
raw count	$f_{t,d}$
term frequency	$\left f_{t,d} \middle/ \sum_{t' \in d} f_{t',d}  ight $
log normalization	$1 + \log(f_{t,d})$
double normalization 0.5	$0.5 + 0.5 \cdot rac{f_{t,d}}{\max_{\{t' \in d\}} f_{t',d}}$
double normalization K	$K+(1-K)rac{f_{t,d}}{\max_{\{t'\in d\}}f_{t',d}}$

### Variants of inverse document frequency (IDF) weight

variante of involve accument frequency (151) weight			
weighting scheme	IDF weight ( $n_t =  \{d \in D: t \in d\} $ )		
unary	1		
inverse document frequency	$\log rac{N}{n_t} = -\log rac{n_t}{N}$		
inverse document frequency smooth	$\log \biggl(1 + \frac{N}{n_t}\biggr)$		
inverse document frequency max	$\log\!\left(rac{\max_{\{t'\in d\}} n_{t'}}{1+n_t} ight)$		
probabilistic inverse document frequency	$\log rac{N-n_t}{n_t}$		

BM25
$$(q, d) = \sum_{i=1}^{n} IDF(q_i) \frac{tf(q_i, d)(k_1 + 1)}{tf(q_i, d) + k_1 \left(1 - b + b \frac{|D|}{\bar{n}_d}\right)},$$

# **Online assessment**

We want both fast and smart system to navigate in changing Internet.

- Need recalculate our factors efficiently
- Need to store data efficiently (every extra byte is extra money to pay)
- Need to use new data from users to fit our models online

So here comes many algorithmical and engineering problems to solve.

- RankNet первая идея pairwise-подхода.
- Пусть у нас есть кое-какие прямые данные для обучения (т.е. про некоторые подмножества документов эксперт сказал, какие более релевантны, какие менее).
- Подход к решению: давайте обучать функцию, которая по данному вектору атрибутов  $\mathbf{x} \in \mathbb{R}^n$  выдаёт  $f(\mathbf{x})$  и ранжирует документы по значению  $f(\mathbf{x})$ .

• Итак, для тестовых примеров  $\mathbf{x}_i$  и  $\mathbf{x}_j$  модель считает  $s_i = f(\mathbf{x}_i)$  и  $s_j = f(\mathbf{x}_j)$ , а затем оценивает

$$p_{ij} = p(\mathbf{x}_i \succ \mathbf{x}_j) = \frac{1}{1 + e^{-\alpha(s_i - s_j)}}.$$

- А данные это на самом деле  $q(\mathbf{x}_i \succ \mathbf{x}_j)$ , либо точные из  $\{0,1\}$ , либо усреднённые по нескольким экспертам.
- Поэтому разумная функция ошибки кросс-энтропия

$$C = -q_{ij} \log p_{ij} - (1 - q_{ij}) \log(1 - p_{ij}).$$

- Ошибка:  $C = -q_{ij} \log p_{ij} (1 q_{ij}) \log (1 p_{ij})$ .
- Для самого частого случая, когда оценки релевантности точные, и  $q_{ij}=(1+S_{ij})/2$  для  $S_{ij}\in\{-1,0,+1\}$ , мы получаем

$$C = \frac{1}{2}(1-S_{ij})\alpha(s_i-s_j) + \log\left(1+e^{-\alpha(s_i-s_j)}\right),$$
 T.e.

$$C = \begin{cases} \log \left( 1 + e^{-\alpha(s_i - s_j)} \right), & \text{если } S_{ij} = 1, \\ \log \left( 1 + e^{-\alpha(s_j - s_i)} \right), & \text{если } S_{ij} = -1. \end{cases}$$

• Т.е. ошибка симметрична, что уже добрый знак.

- Ошибка:  $C = -q_{ij} \log p_{ij} (1 q_{ij}) \log (1 p_{ij})$ .
- Давайте подсчитаем градиент по  $s_i$ :

$$\frac{\partial C}{\partial s_i} = \alpha \left( \frac{1 - S_{ij}}{2} - \frac{1}{1 + e^{\alpha(s_i - s_j)}} \right) = -\frac{\partial C}{\partial s_i}.$$

 И теперь осталось использовать этот подсчёт для градиента по весам:

$$\frac{\partial C}{\partial w_k} = \sum_{i} \frac{\partial C}{\partial s_i} \frac{\partial s_i}{\partial w_k} + \sum_{i} \frac{\partial C}{\partial s_i} \frac{\partial s_j}{\partial w_k}.$$

$$\frac{\partial C}{\partial w_k} = \sum_i \frac{\partial C}{\partial s_i} \frac{\partial s_i}{\partial w_k} + \sum_i \frac{\partial C}{\partial s_j} \frac{\partial s_j}{\partial w_k} = \lambda_{ij} \left( \frac{\partial s_i}{\partial w_k} - \frac{\partial s_j}{\partial w_k} \right),$$

где

$$\lambda_{ij} = \frac{\partial C(s_i - s_j)}{\partial s_i} = \alpha \left( \frac{1 - S_{ij}}{2} - \frac{1}{1 + e^{\alpha(s_i - s_j)}} \right).$$

Переупорядочив пары так, чтобы всегда было  $\mathbf{x}_i \succ \mathbf{x}_j$  и  $S_{ij} = 1$ , получим

$$\lambda_{ij} = \frac{\partial C(s_i - s_j)}{\partial s_i} = -\alpha \frac{1}{1 + e^{\alpha(s_i - s_j)}}.$$

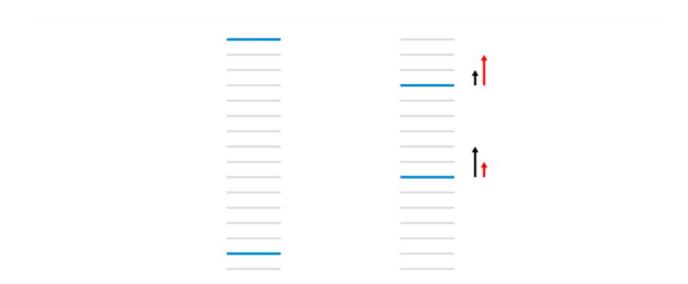
- $\lambda_{ij} = \frac{\partial C(s_i s_j)}{\partial s_i} = -\alpha \frac{1}{1 + e^{\alpha(s_i s_j)}}$ .
- Значит, если для данной выдачи есть множество пар I, в которых известно, что  $\mathbf{x}_i \succ \mathbf{x}_j$ ,  $(i,j) \in I$ , то суммарный апдейт для веса  $w_k$  будет

$$\Delta w_k = -\eta \left[ \sum_{(i,j) \in I} \lambda_{ij} \frac{\partial s_i}{\partial w_k} - \lambda_{ij} \frac{\partial s_j}{\partial w_k} \right] = -\eta \sum_i \lambda_i \frac{\partial s_i}{\partial w_k},$$

где 
$$\lambda_i = \sum_{j:(i,j)\in I} \lambda_{ij} - \sum_{j:(j,i)\in I} \lambda_{ij}.$$

# LambdaRank

- Проблема с RankNet в том, что оптимизируется число попарных ошибок, а это не всегда то, что нужно.
- Градиенты RankNet это не то же самое, что градиенты NDCG:



• Как оптимизировать, скажем, NDCG?

# LambdaRank

- Заметим, что нам сама ошибка не нужна, а нужны только градиенты  $\lambda$  (стрелочки).
- Давайте просто представим себе мифическую функцию ошибки C, у которой градиент

$$\lambda_{ij} = \frac{\partial C(s_i - s_j)}{\partial s_i} = \frac{-\alpha}{1 + e^{\alpha(s_i - s_j)}} \left| \Delta_{\text{NDCG}} \right|,$$

где  $\Delta_{\mathrm{NDCG}}$  – это то, на сколько NDCG изменится, если поменять i и j местами.

• То есть мы считаем градиенты уже после сортировки документов по оценкам, и градиенты как будто от NDCG.

# LambdaRank

· NDCG нужно максимизировать, так что берём

$$\Delta w_k = \eta \frac{\partial C}{\partial w_k}$$
, и тогда

$$\delta C = \frac{\partial C}{\partial w_k} \delta w_k = \eta \left( \frac{\partial C}{\partial w_k} \right)^2 > 0.$$

• Оказывается, что такой подход фактически напрямую оптимизирует NDCG (сглаженную версию).

# LambdaMART

- MART это бустинг, сделанный на регрессионных деревьях.
- Иначе говоря, окончательная модель будет, опять же, по  $\mathbf{x} \in \mathbb{R}^d$  искать  $y \in \mathbb{R}$ , и искать в виде

$$F_M(\mathbf{x}) = \sum_{m=1}^M \alpha_m f_m(\mathbf{x}),$$

где  $f_m(\mathbf{x})$  задаётся регрессионным деревом, а  $\alpha_m \in \mathbb{R}$  – веса бустинга, и в процессе обучения обучаются одновременно  $f_m$  и  $\alpha_m$ .

### LambdaMART

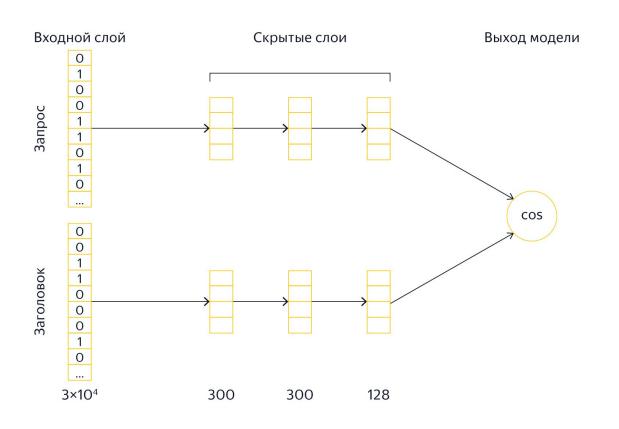
 Мы просто добавим в градиенты целевую метрику, например

$$\lambda_{ij} = S_{ij} \left| \Delta ext{NDCG} rac{\partial C_{ij}}{\partial o_{ij}} \right|, \quad o_{ij} = F(x_i) - F(x_j).$$

• Функция ошибки нам тоже уже известна:

$$C_{ij} = C(o_{ij}) = s_j - s_i + \log\left(1 + e^{s_i - s_j}\right), \ rac{\partial C_{ij}}{\partial o_{ij}} = rac{\partial C_{ij}}{\partial s_i} = -rac{1}{1 + e^{o_{ij}}}.$$

# **DSSM**



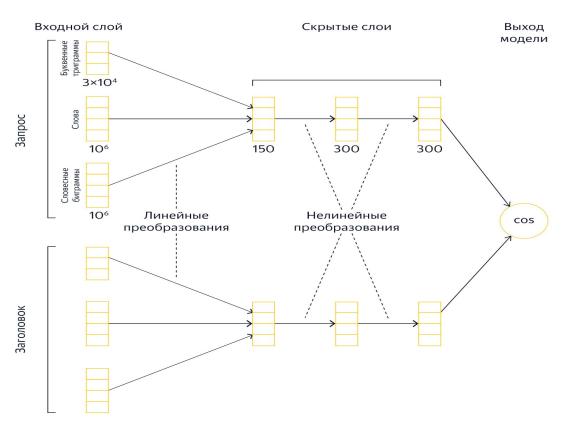
Word Hashing

$$R(Q, D) = \operatorname{cosine}(y_Q, y_D) = \frac{y_Q^T y_D}{\|y_Q\| \|y_D\|}$$

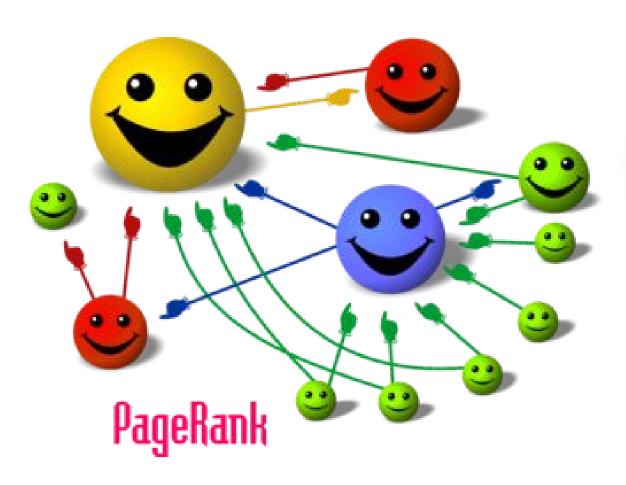
$$P(D|Q) = \frac{\exp(\gamma R(Q, D))}{\sum_{D' \in D} \exp(\gamma R(Q, D'))}$$

$$L(\Lambda) = -\log \prod_{(Q,D^+)} P(D^+|Q)$$

# **DSSM**



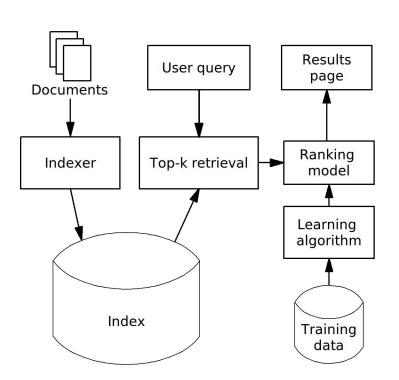
Hard negative mining



$$L = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} \longrightarrow P = \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}$$

$$PR(A) = rac{1-d}{N} + d\left(rac{PR(B)}{L(B)} + rac{PR(C)}{L(C)} + rac{PR(D)}{L(D)} + \cdots
ight).$$

### **Features**



- Query-independent or static features PageRank.
- Query-dependent or dynamic features — TF-IDF.
- Query level features or query features — the number of words in a query.

# **XGBoost**

### **Evaluation metrics**

- "ndcg": Normalized Discounted Cumulative Gain
- "map": Mean average precision
- "ndcg@n","map@n": n can be assigned as an integer to cut off the top positions in the lists for evaluation

### Additional info

- Objective rank:pairwise
- Algorithm LambdaRank

# Links

- https://everipedia.org/wiki/Learning\_to\_rank/
- https://www.microsoft.com/en-us/research/wp-content/uploads/2016/02/cikm2013 DSSM fullversion.pdf
- https://habrahabr.ru/company/yandex/blog/314222/
- https://logic.pdmi.ras.ru/~sergey/teaching/mlhse17/17-ranking.pdf
- https://logic.pdmi.ras.ru/~sergey/teaching/mlstc12/17-neural.pdf
- https://everipedia.org/wiki/PageRank/