

Theoretical Assignment

DeepBayes Summer School 2019 (deepbayes.ru)

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Problem 1

$$\begin{aligned}\xi &\sim P(\lambda), & p(\xi = k) &= e^{-\lambda} \frac{\lambda^k}{k!} \\ \gamma_n &\sim \text{Bin}(n, p), & p(\gamma_n = k) &= C_k^n p^k (1-p)^{n-k}\end{aligned}$$

η - number of successful outcomes of γ_k , where k is a realization of ξ .

$$\begin{aligned}p(\eta = k) &= \sum_{i=k}^{\infty} p(\xi = i) p(\gamma_i = k) = \sum_{i=k}^{\infty} \frac{\lambda^i}{i!} e^{-\lambda} C_k^i p^k (1-p)^{i-k} = \\ &= \sum_{j=0}^{\infty} \frac{\lambda^{k+j}}{(k+j)!} e^{-\lambda} \frac{(k+j)!}{k!j!} p^k (1-p)^j = \frac{(p\lambda)^k e^{-\lambda}}{k!} \sum_{j=0}^{\infty} \frac{\lambda^j (1-p)^j}{j!} = \\ &= \frac{(p\lambda)^k e^{-\lambda}}{k!} e^{\lambda(1-p)} = \frac{(p\lambda)^k e^{-\lambda p}}{k!}\end{aligned}$$

Problem 2

r - reviewer, $r \in \{\text{kind}, \text{strict}\}$

$t_1 \sim N(30, 10)$ - spend time if reviewer is strict.

$t_2 \sim N(20, 5)$ - spend time if reviewer is kind.

$p(r = \text{kind}) = p(r = \text{strict}) = 0.5$

Find $p(r = \text{kind} | t = 10)$.

$$\begin{aligned}p(r = \text{kind} | t = 10) &= \frac{p(t = 10 | r = \text{kind}) p(r = \text{kind})}{p(t = 10 | r = \text{kind}) p(r = \text{kind}) + p(t = 10 | r = \text{strict}) p(r = \text{strict})} = \\ &= \frac{p_1(10) * 0.5}{p_1(10) * 0.5 + p_2(10) * 0.5} = \frac{p_1(10)}{p_1(10) + p_2(10)} = \frac{1}{1 + \frac{5}{10} e^{-\frac{(10-30)^2}{200} + \frac{(10-20)^2}{50}}} = \\ &= \frac{1}{1 + 0.5 e^{-\frac{400}{200} + \frac{100}{50}}} = \frac{1}{1 + 0.5 e^0} = \frac{2}{3}\end{aligned}$$