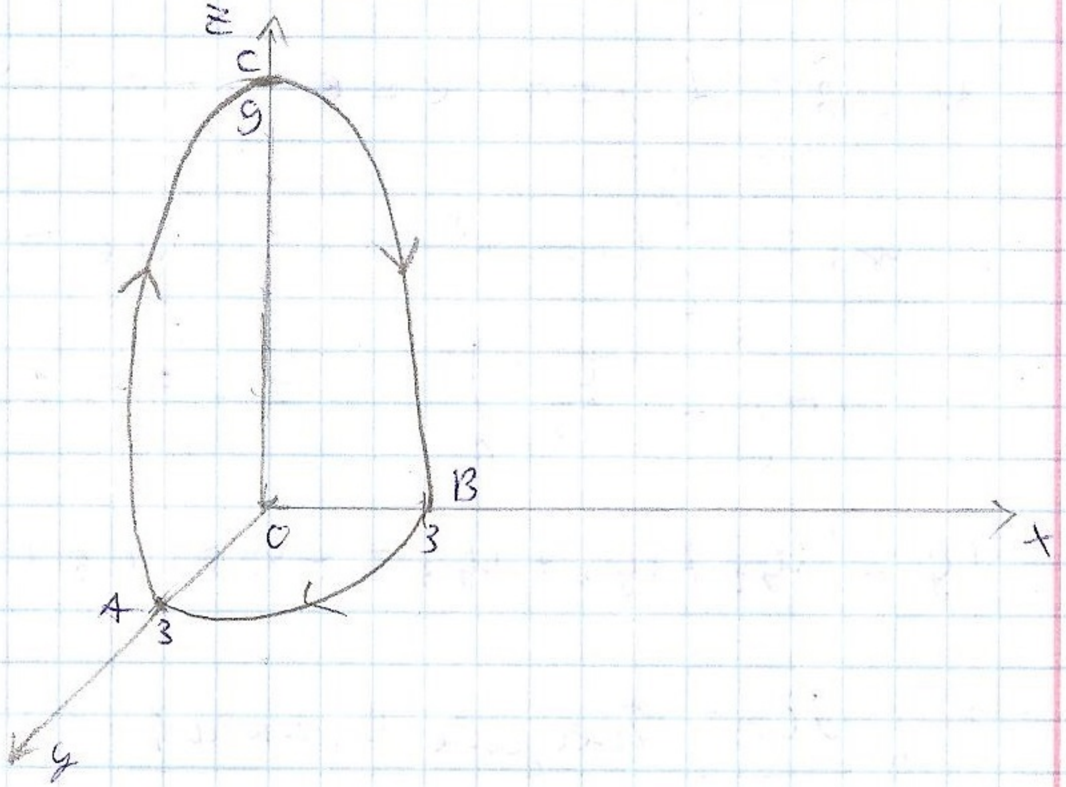
Задача № 10 из ТР (Вариант 1)

$a = z \cdot i - y \cdot j + y^2 \cdot k \qquad \bar{a} = \{ z, -y, y^2 \}$

$\Gamma = x^2 + y^2 = 9 - z \; ; \; x = 0, \; y = 0, \; z = 0$ (1 октант)



I. Непосредственно:

1) BA : $\{ z = 0; \; x = t; \; y = \sqrt{9 - t^2} \}$ $t \downarrow$

$Ц_1 = -\int_0^3 \left(0 - \sqrt{9-t^2} \cdot \frac{-2t}{2\sqrt{9-t^2}} + 0 \right) dt =$

$$= -\int_0^3 t\,dt = -\frac{t^2}{2}\Big|_0^3 = -\frac{9}{2} = -4,5$$

2) AC; $\{x=0,\ y=t,\ z=9-t^2\}$ $t\downarrow$

$$Ц_2 = -\int_0^3 \left(0 + 1\cdot(-t) + t^2\cdot(-2t)\right)dt =$$

$$= -\int_0^3 (-t-2t^3)\,dt = \frac{t^2}{2}\Big|_0^3 + \frac{t^4}{2}\Big|_0^3 =$$

$$= \frac{9}{2} + \frac{81}{2} = 45$$

3) CB; $\{y=0,\ x=t,\ z=9-t^2\}$ $t\uparrow$

$$Ц_3 = \int_0^3 \left((9-t^2)\cdot 1 + 0 + 0\right)dt = 9\int_0^3 dt - \int_0^3 t^2\,dt =$$

$$= 9t\Big|_0^3 - \frac{t^3}{3}\Big|_0^3 = 27 - 9 = 18$$

$$Ц = Ц_1 + Ц_2 + Ц_3 = -4,5 + 45 + 18 = \boxed{58,5}$$

II. <u>По теореме Стокса:</u>

$$\operatorname{rot}\bar{a} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ z & -y & y^2 \end{vmatrix} = \bar{i}\left(\frac{\partial(y^2)}{\partial y} + \frac{\partial y}{\partial z}\right) -$$

$$-\bar{j}\left(\frac{\partial(y^2)}{\partial x} - \frac{\partial z}{\partial z}\right) + \bar{k}\left(\frac{\partial(-y)}{\partial x} - \frac{\partial z}{\partial y}\right) =$$

$$= \bar{i}\,(2y+0) - \bar{j}\,(0-1) + \bar{k}\cdot 0 = \bar{i}\cdot 2y + \bar{j} =$$

$$= (2y;\ 1;\ 0).$$

$$z = -x^2 - y^2 + 9$$

$$F(x,y,z) = z - 9 + x^2 + y^2$$

$$\bar{n} = \operatorname{grad} F = (2x,\ 2y,\ 1)$$

$$\bar{n}_1 = \left(\frac{2x}{\sqrt{4x^2+4y^2+1}};\ \frac{2y}{\sqrt{4x^2+4y^2+1}};\ \frac{1}{\sqrt{4x^2+4y^2+1}}\right)$$

$$\iint_\sigma (\operatorname{rot}\bar{a}, \bar{n})\, d\sigma = \iint_\sigma \left(\frac{2y\cdot 2x}{\sqrt{4x^2+4y^2+1}} + \frac{2y\cdot 1}{\sqrt{4x^2+4y^2+1}} + 0\right)\cdot$$

$$\cdot\frac{\sqrt{4x^2+4y^2+1}}{1}\, dx\, dy = \iint_\sigma (4yx + 2y)\, dx\, dy =$$

$$= \int_0^3 dx \int_0^{\sqrt{9-x^2}} (4yx + 2y)\, dy = \int_0^3 (y^2 + 2xy^2)\Big|_0^{\sqrt{9-x^2}} dx =$$

$$= \int_0^3 \left(y^2(1+2x)\Big|_0^{\sqrt{9-x^2}}\right) dx = \int_0^3 (9-x^2)(1+2x)\, dx =$$

$$= \int_0^3 (9 + 18x - x^2 - 2x^3)\, dx = 9\int_0^3 dx + 18\int_0^3 x\, dx -$$

$$- \int_0^3 x^2 dx - 2\int_0^3 x^3 dx = 9x\Big|_0^3 + 9x^2\Big|_0^3 -$$

$$- \frac{x^3}{3}\Big|_0^3 - \frac{2x^4}{4}\Big|_0^3 = 81 + 27 - 9 - \frac{81}{2} =$$

$$= \boxed{58,5}$$