MatAnLab2(term 3)

Балакин Дмитрий M3235

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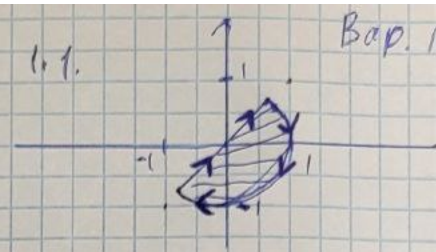
Аналитическая часть

Вар. 104

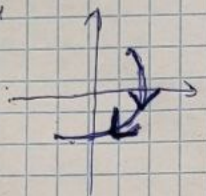
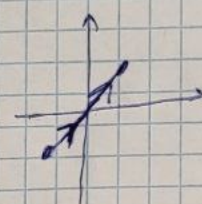
1.1.

$$\oint\limits_L (x^2+y^2)\,dx + 3xy\,dy$$



L – границы поверхности по часовой, $D: x^2+y^2 \le 1,\ x \ge y$

1.2. $$\int\limits_L (x^2+y^2)dx + 3xy\,dy = \int\limits_{L_1} (x^2+y^2)\,dx + 3xy\,dy + \int\limits_{L_2} (x^2+y^2)\,dx + 3xy\,dy$$

L_1:  L_2: 

L_1:

$$\begin{cases} x(t) = \cos t \\ y(t) = \sin t \end{cases} \quad t \in \left[-\frac{3\pi}{4};\ \frac{\pi}{4}\right] \Rightarrow \int\limits_{L_1} (x^2+y^2)\,dx + 5xy\,dy =$$

$$= \int\limits_{5\pi/4}^{-3\pi/4} d\cos t + 5\sin t\cos^2 t\,dt = -\int\limits_{-3\pi/4}^{\pi/4} (1-5\cos^2 t)\,d\cos t =$$

$$= -\left(\cos t - \cos^3 t\right)\Big|_{-\frac{3\pi}{4}}^{\pi/4} = -\left(\frac{1}{\sqrt2} - \frac{1}{2\sqrt2} + \frac{1}{\sqrt2} - \frac{1}{2\sqrt2}\right) = -\frac{1}{\sqrt2}$$

L_2:

$$\begin{cases} x = t \\ y = t \end{cases} \quad t \in \left[-1/\sqrt2;\ 1/\sqrt2\right] \Rightarrow \int\limits_{L_2} (x^2+y^2)\,dx + 5xy\,dy = \int\limits_{-1/\sqrt2}^{1/\sqrt2} 5t^2\,dt =$$

$$= \frac{5t^3}{3}\Big|_{-1/\sqrt2}^{1/\sqrt2} = \frac{5}{6\sqrt2} + \frac{5}{6\sqrt2} = \frac{5}{3\sqrt2}$$

$$\Rightarrow \int\limits_L (x^2+y^2)\,dx + 5xy\,dy = \frac{5}{3\sqrt2} - \frac{1}{\sqrt2} = \frac{2}{3\sqrt2}$$

1.3. $\oint (x^2+y^2)dx + 5xy\,dy = \iint\limits_D 5y - 2y \, dx\,dy = \iint\limits_D y\,dx\,dy$

$\begin{cases} x = r\cos\varphi \\ y = r\sin\varphi \end{cases}, \quad \begin{matrix} \varphi \in [-3\pi/4; \pi/4] \\ r \in [0; 1] \end{matrix} \Rightarrow |J| = r$

$\Rightarrow \iint\limits_D y\,dx\,dy = \int_{\pi/4}^{-\frac{3\pi}{4}} d\varphi \int_0^1 \cancel{r} r^2 \sin\varphi \, dr =$

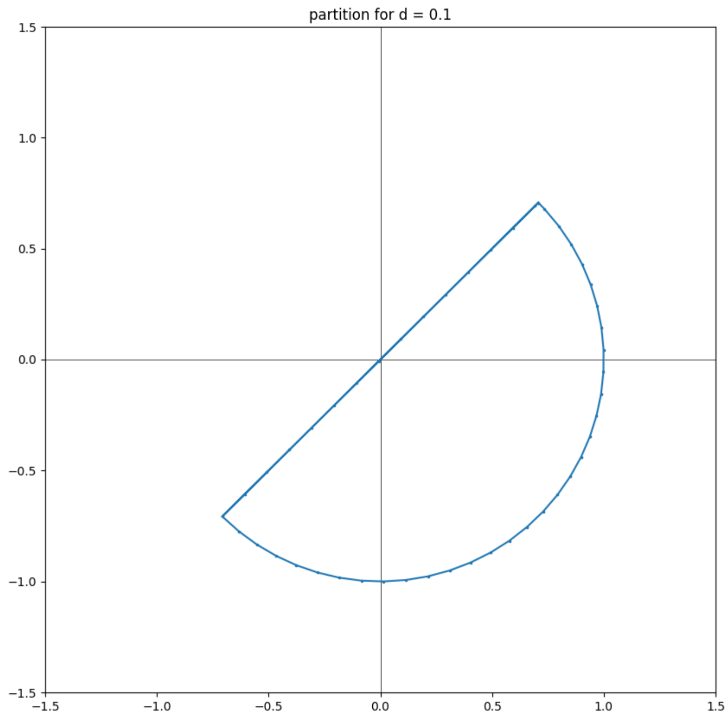
$= -\int_{-\frac{3\pi}{4}}^{\pi/4} \frac{r^3}{3} \sin\varphi \Big|_0^1 d\varphi = -\int_{-\frac{3\pi}{4}}^{\pi/4} \frac{\sin\varphi}{3} d\varphi = -\Big(-\frac{\cos}{3}$

$= -\left(-\frac{\sqrt{2}}{6} - \frac{\sqrt{2}}{6} \right) = \frac{1}{3\sqrt{2}}$

Численный метод

Результат работы программы

```
C:\programs\mathLab2Term3\venv\Scripts\pyt
----d = 0.1----
sum = 0.5097786117878471
the error = 0.03837409099681549
time =  0.0

int_sum = 0.4630580822896585
the error = 0.008346438501373121
time = 0.004998445510864258
max_sum - min_sum = 0.1837384566050133

----d = 0.01----
sum = 0.4749597375099491
the error = 0.0035552167189174533
time =  0.0

int_sum = 0.47190376335632594
the error = 0.0004992425652943222
time = 0.43839311599731445
max_sum - min_sum = 0.017694421420009543

----d = 0.001----
sum = 0.4717583381705453
the error = 0.00035381737951367676
time =  0.0031652450561523438

int_sum = 0.4714026463589509
the error = 1.8744320807173587e-06
time = 56.59248900413513
max_sum - min_sum = 0.0017866342851270978

time: 259.18475008010864
```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1.5
1.0
0.5
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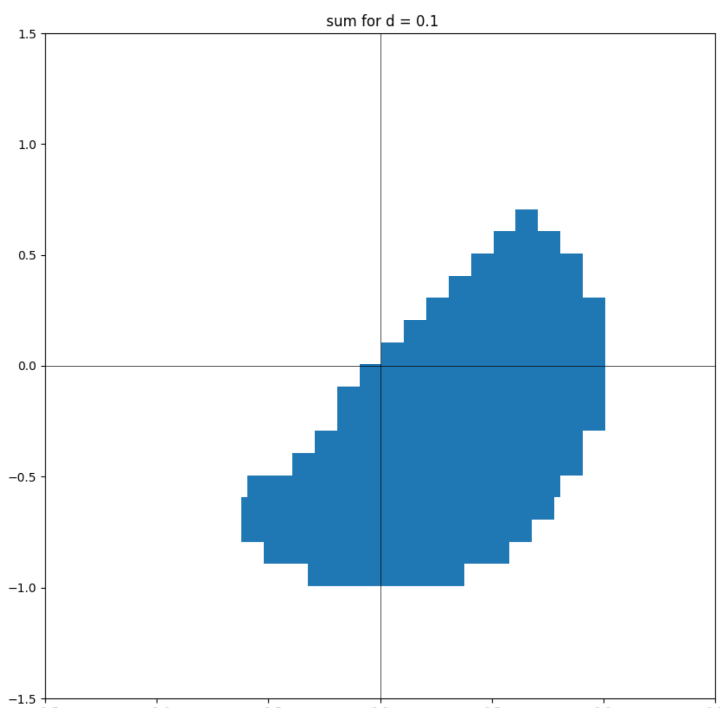
0.0
0.5
1.0
1.5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−0.5
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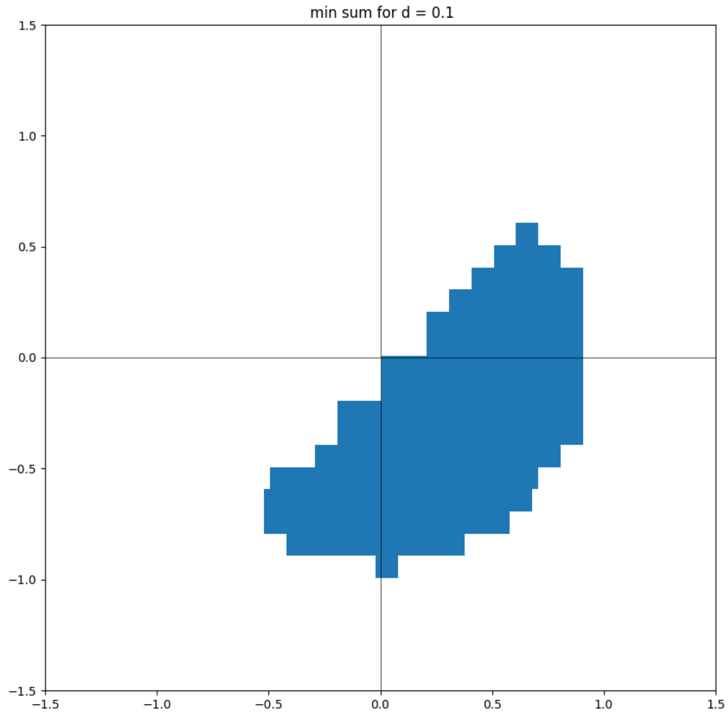
0.5
1.0
1.5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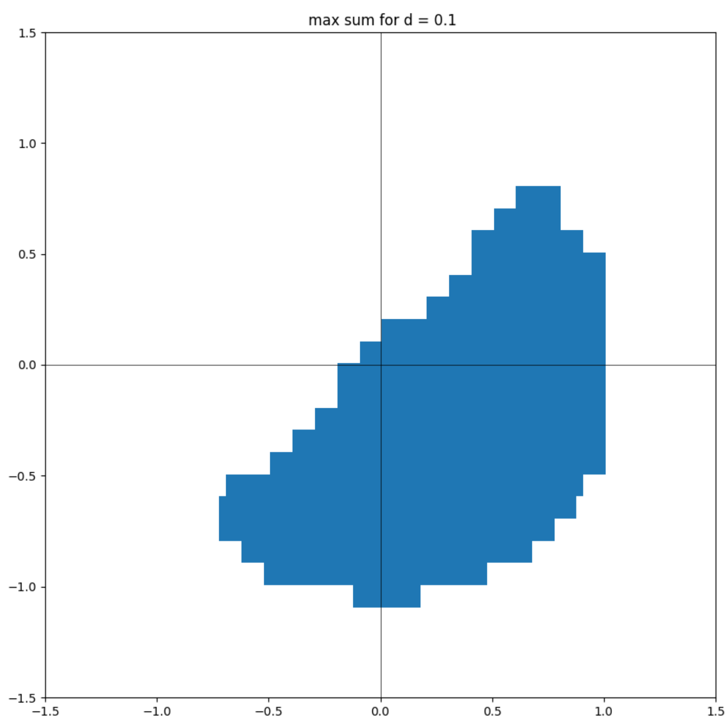
−0.5
0.0
0.5
1.0
1.5


max sum for d = 0.1
1.5
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0.5
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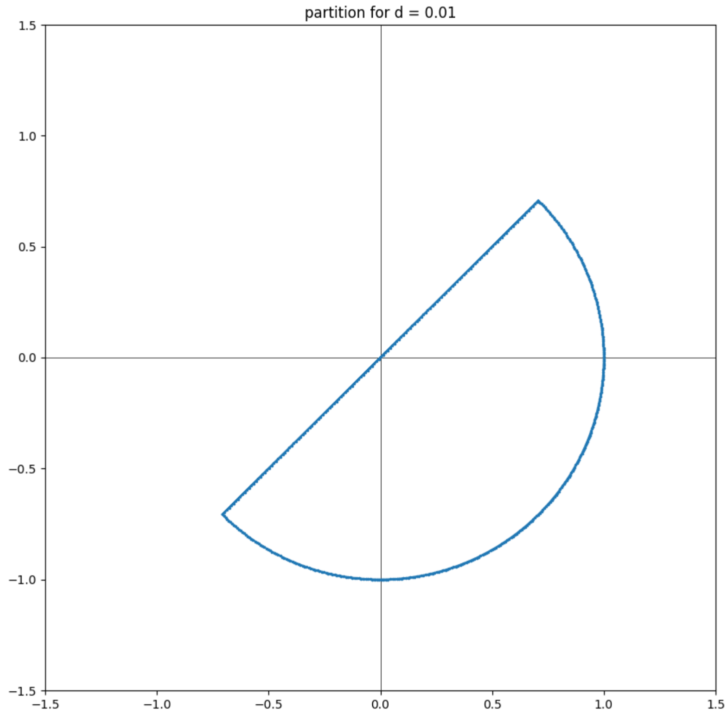
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5


partition for d = 0.01
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1.0
0.5
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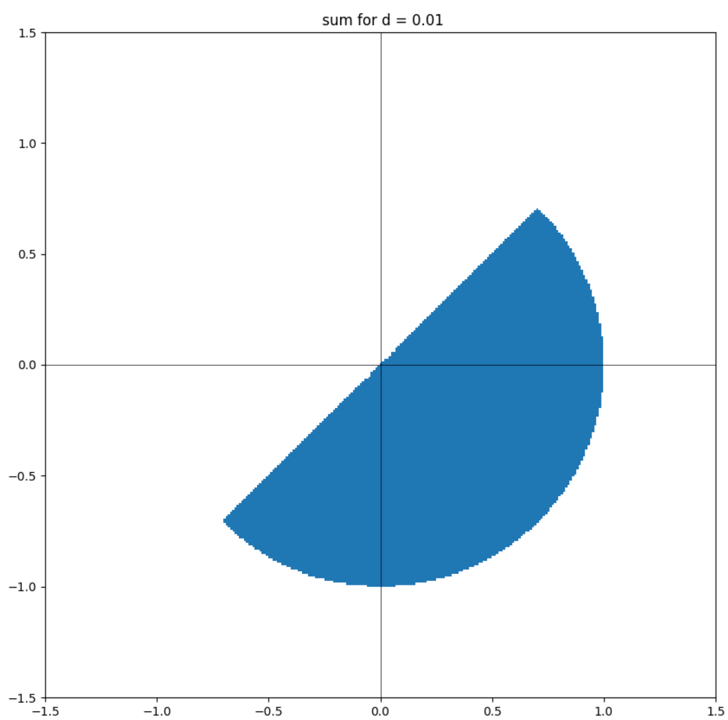
0.0
0.5
1.0
1.5


sum for d = 0.01
1.5
1.0
0.5
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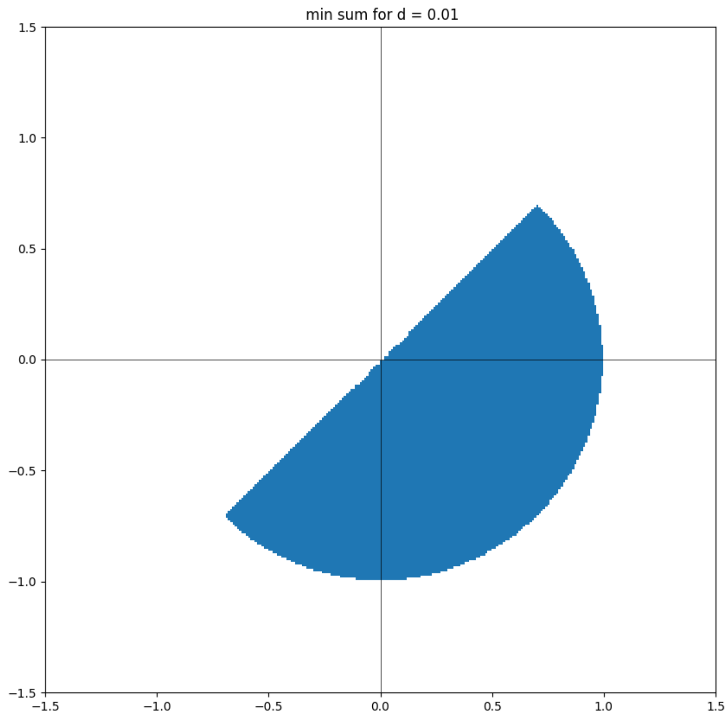
−1.5
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5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1.5
1.0
0.5
0.0
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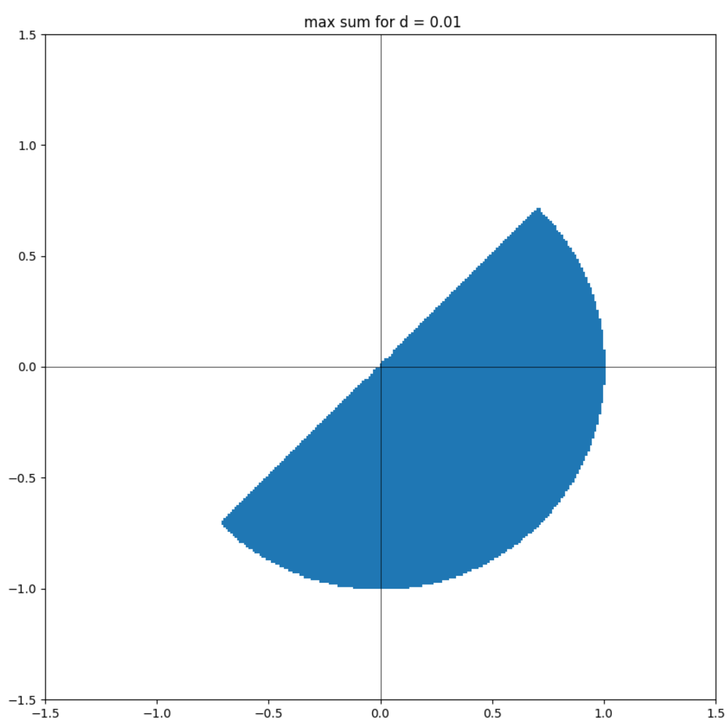
0.0
0.5
1.0
1.5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1.5
1.0
0.5
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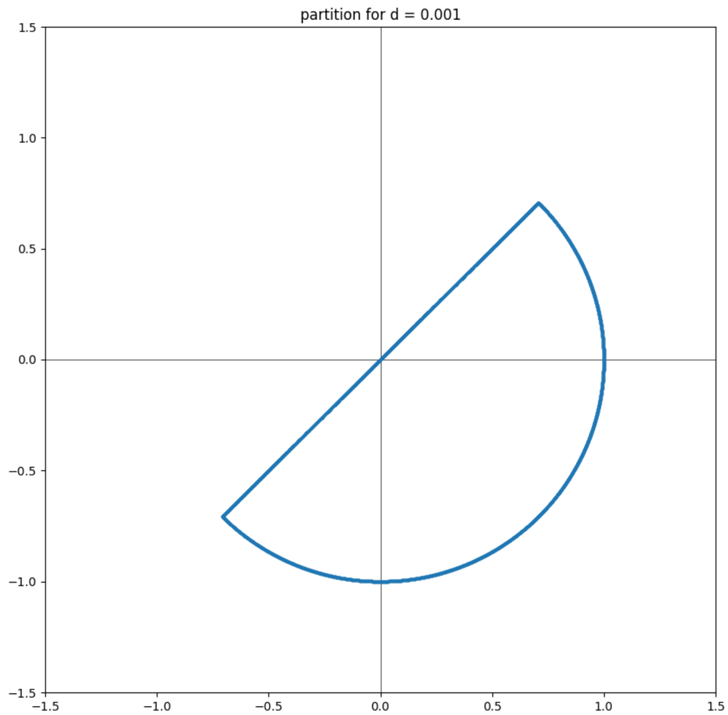
0.0
0.5
1.0
1.5


partition for d = 0.001
1.5
1.0
0.5
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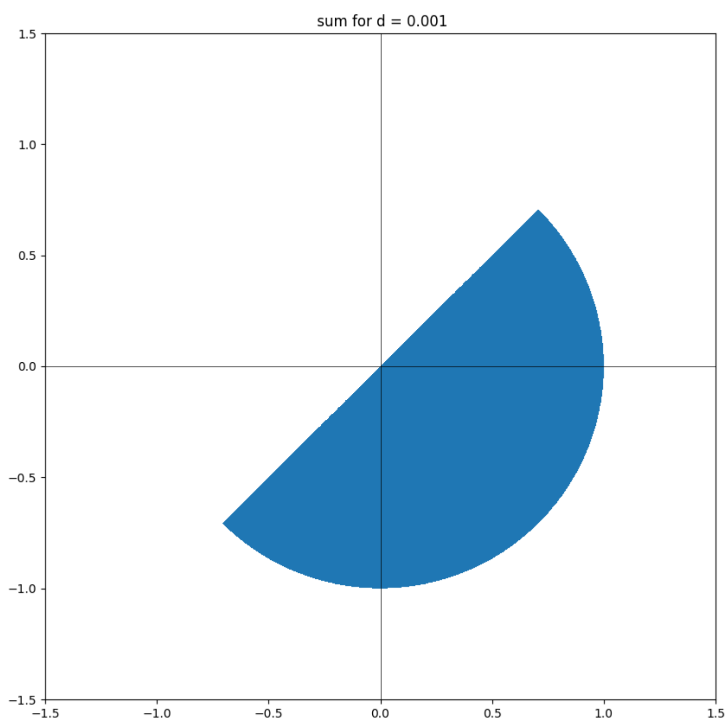
−1.0
−0.5
0.0
0.5
1.0
1.5


sum for d = 0.001
1.5
1.0
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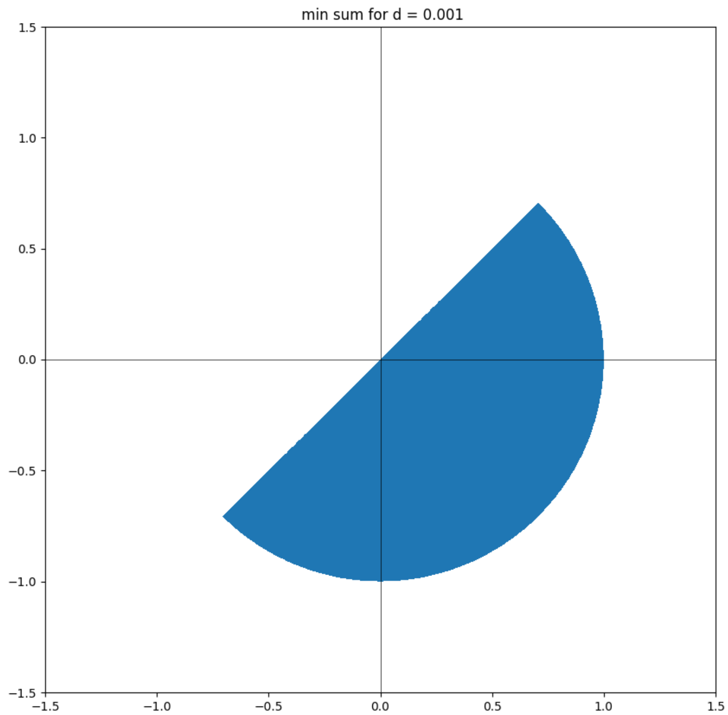
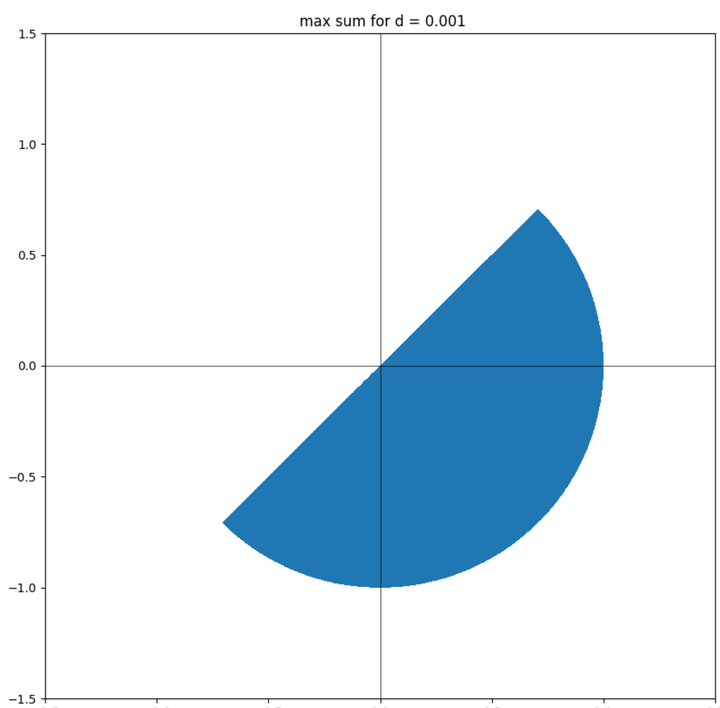
−0.5
0.0
0.5
1.0
1.5





l = d, вся остальная информация есть на первом скрине

Вывод: Видно, что при d -> 0 интегральная сумма стремиться к своему значению