

Introduction to Compiler construction

<http://cs143.stanford.edu>

Why Study Compilers?

- Build a **large, ambitious software system**.
- See theory **come to life**.
- Learn how to **build programming languages**.
- Learn **how programming languages work**.
- Learn **tradeoffs in language design**.

A Short History of Compilers

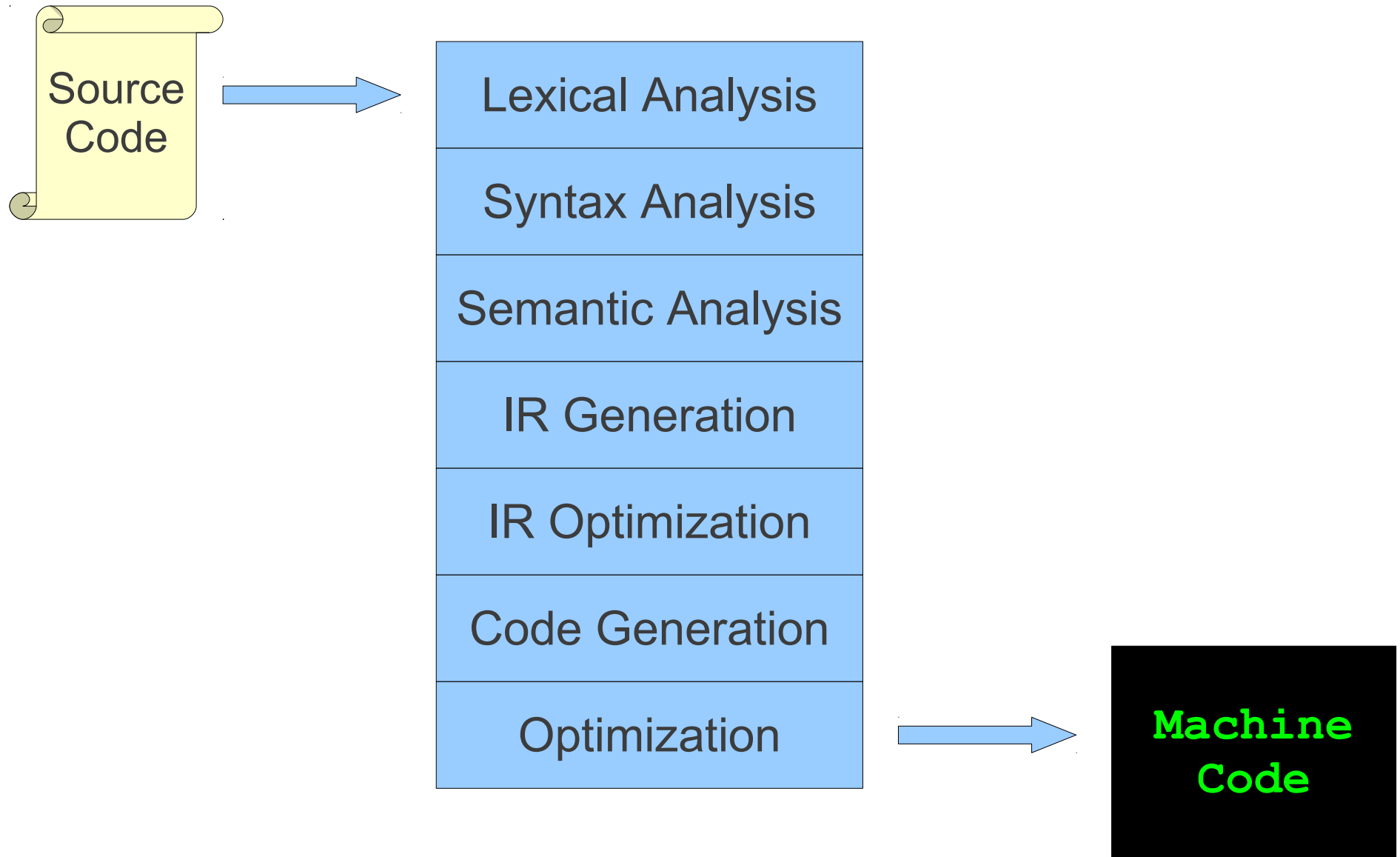
- First, there was nothing.
- Then, there was machine code.
- Then, there were assembly languages.
- Programming expensive; 50% of costs for machines went into programming.

How does a compiler work?

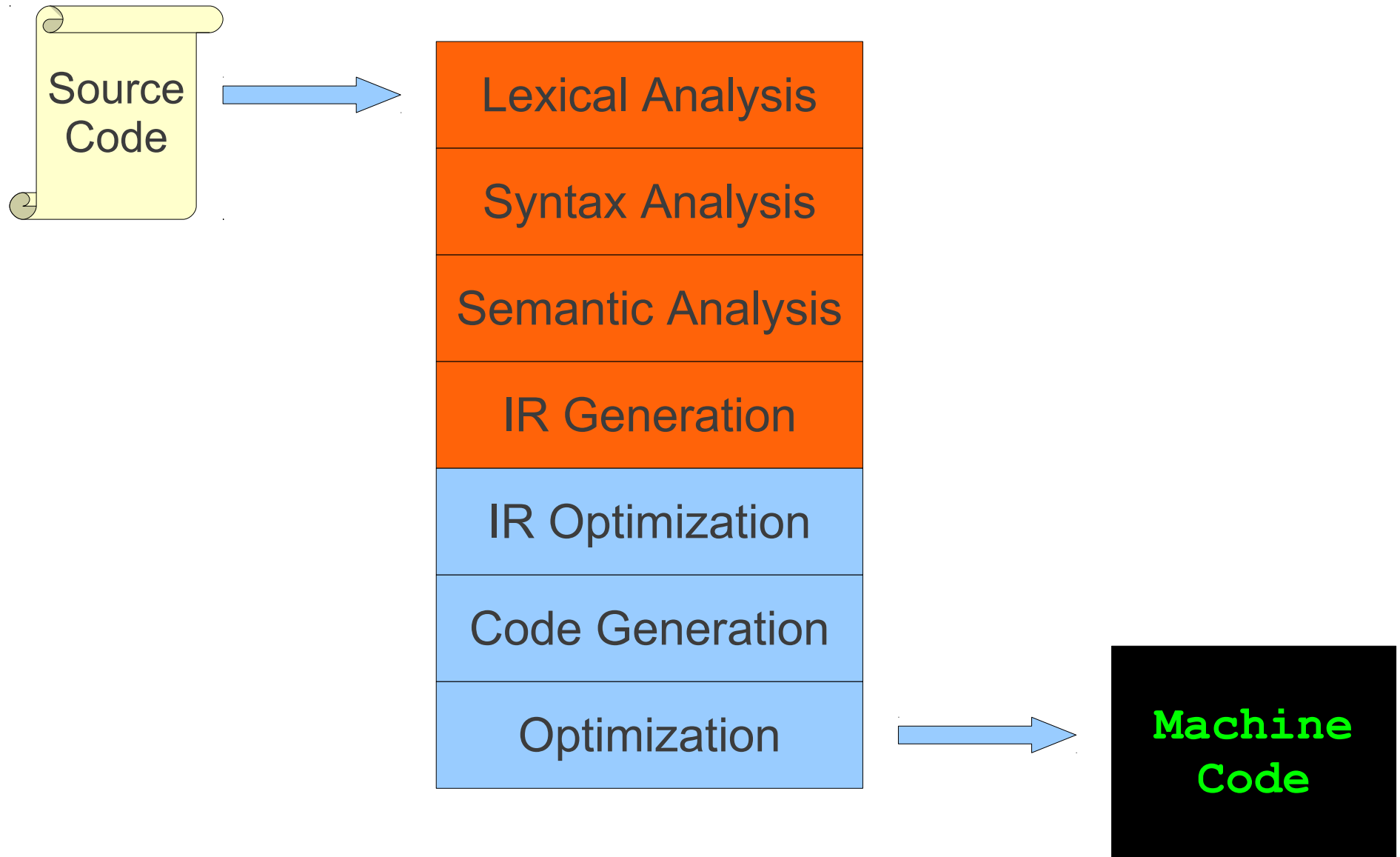
From Description to Implementation

- **Lexical analysis (Scanning):** Identify logical pieces of the description.
- **Syntax analysis (Parsing):** Identify how those pieces relate to each other.
- **Semantic analysis:** Identify the meaning of the overall structure.
- **IR Generation:** Design one possible structure.
- **IR Optimization:** Simplify the intended structure.
- **Generation:** Fabricate the structure.
- **Optimization:** Improve the resulting structure.

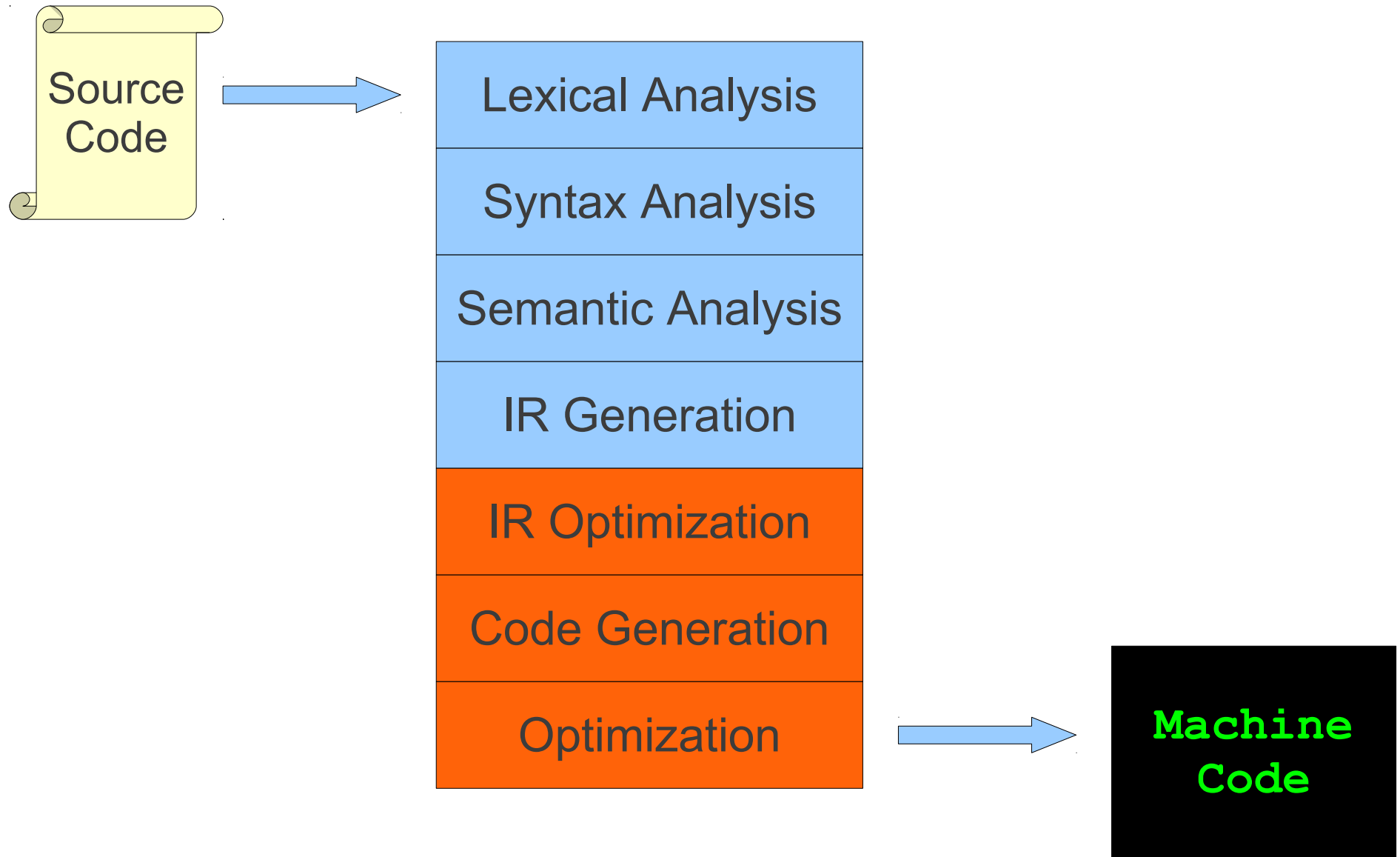
The Structure of a Modern Compiler



The Structure of a Modern Compiler



The Structure of a Modern Compiler



```
while (y < z) {  
    int x = a + b;  
    y += x;  
}
```

Lexical Analysis

Syntax Analysis

Semantic Analysis

IR Generation

IR Optimization

Code Generation

Optimization

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```
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T_Identifier y  
T_Less  
T_Identifier z  
T_RightParen  
T_OpenBrace  
T_Int  
T_Identifier x  
T_Assign  
T_Identifier a  
T_Plus  
T_Identifier b  
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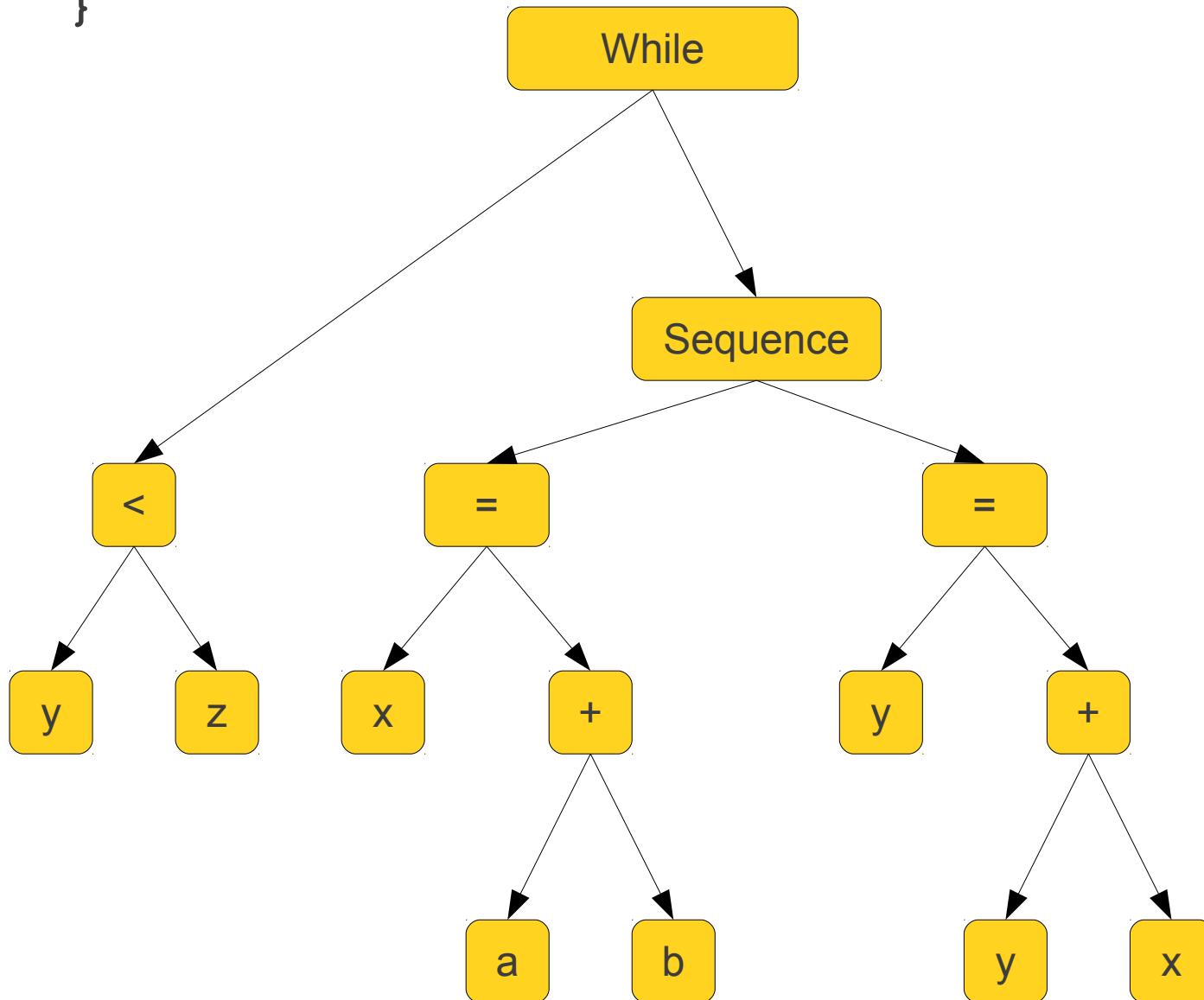
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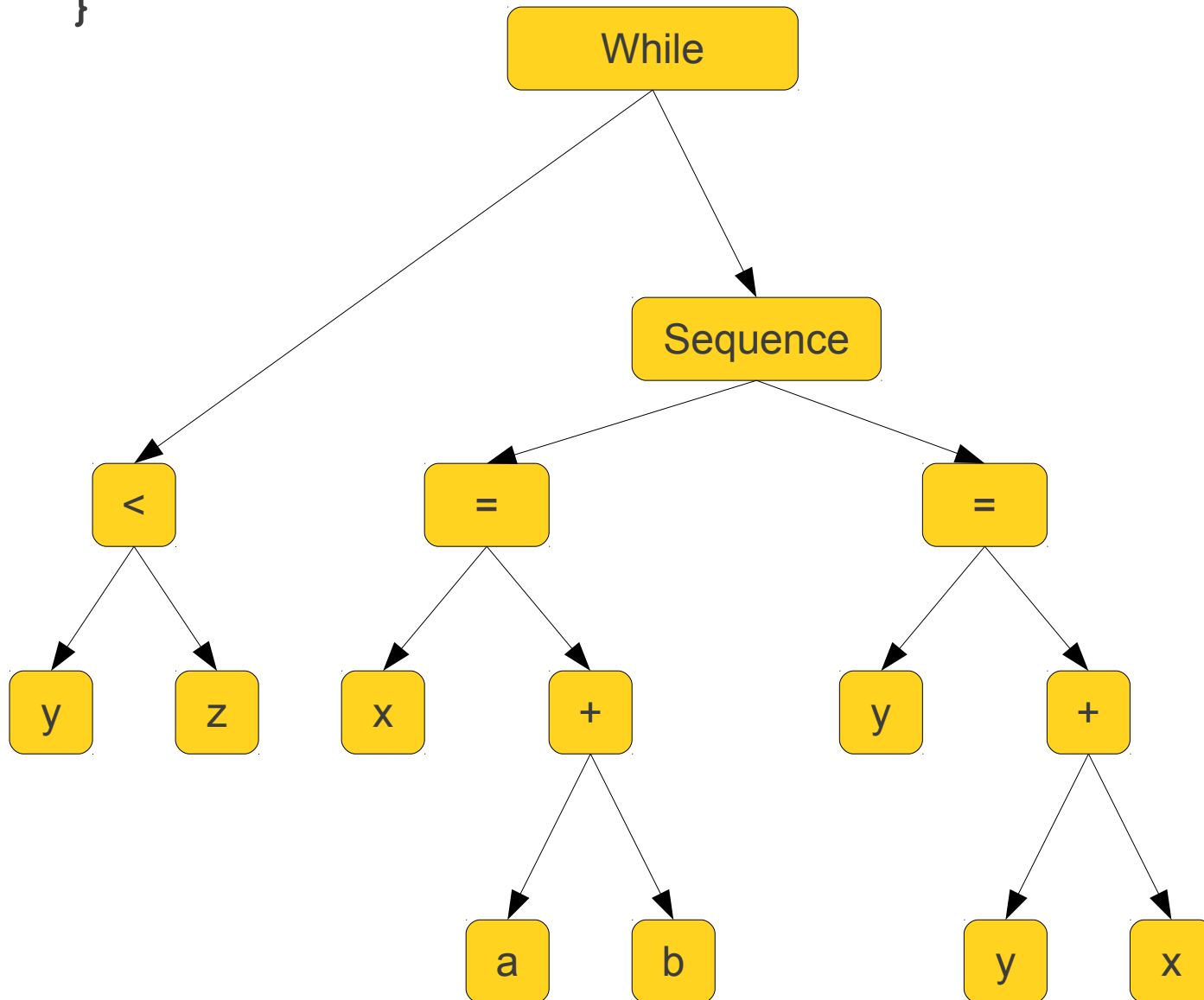
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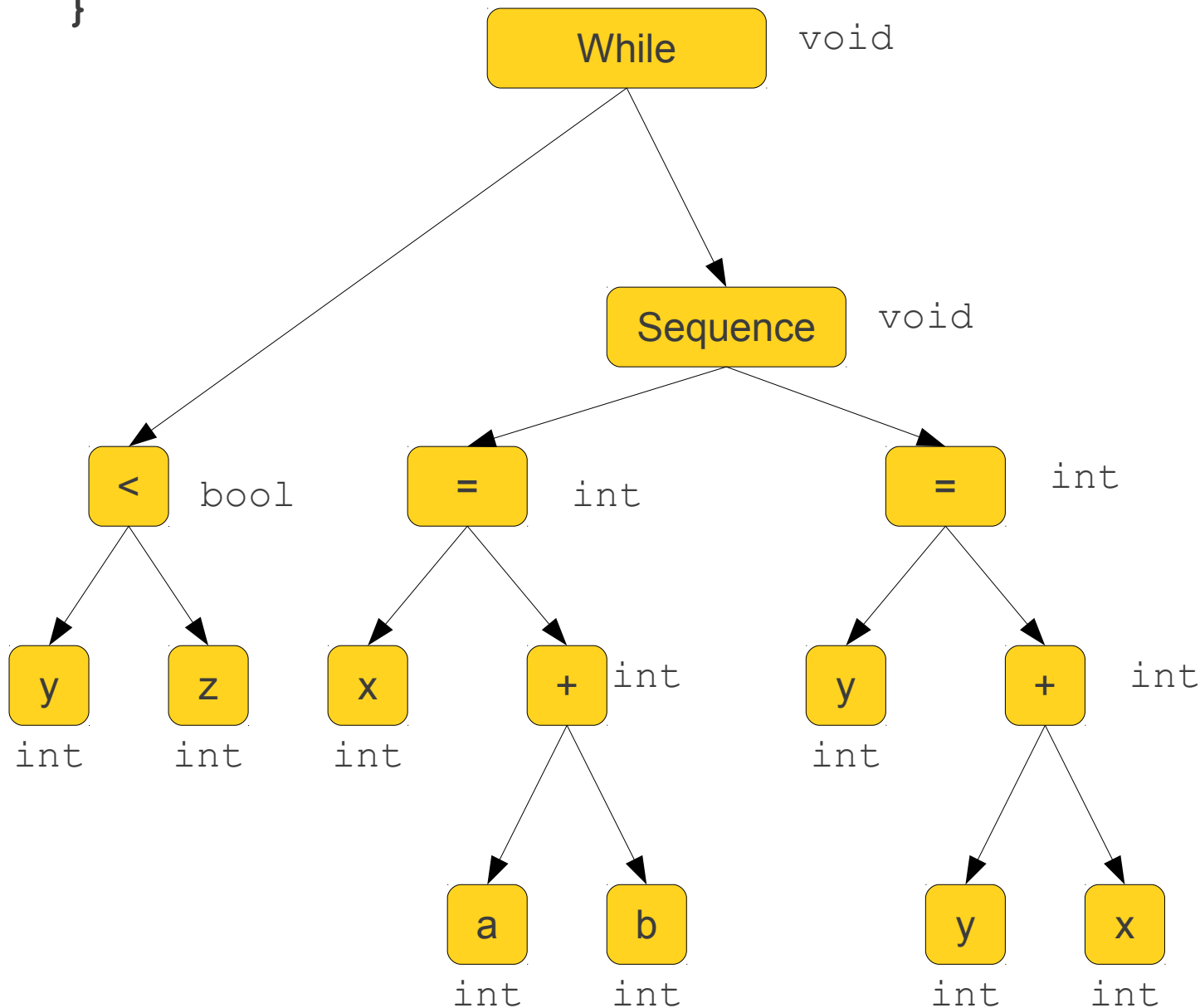
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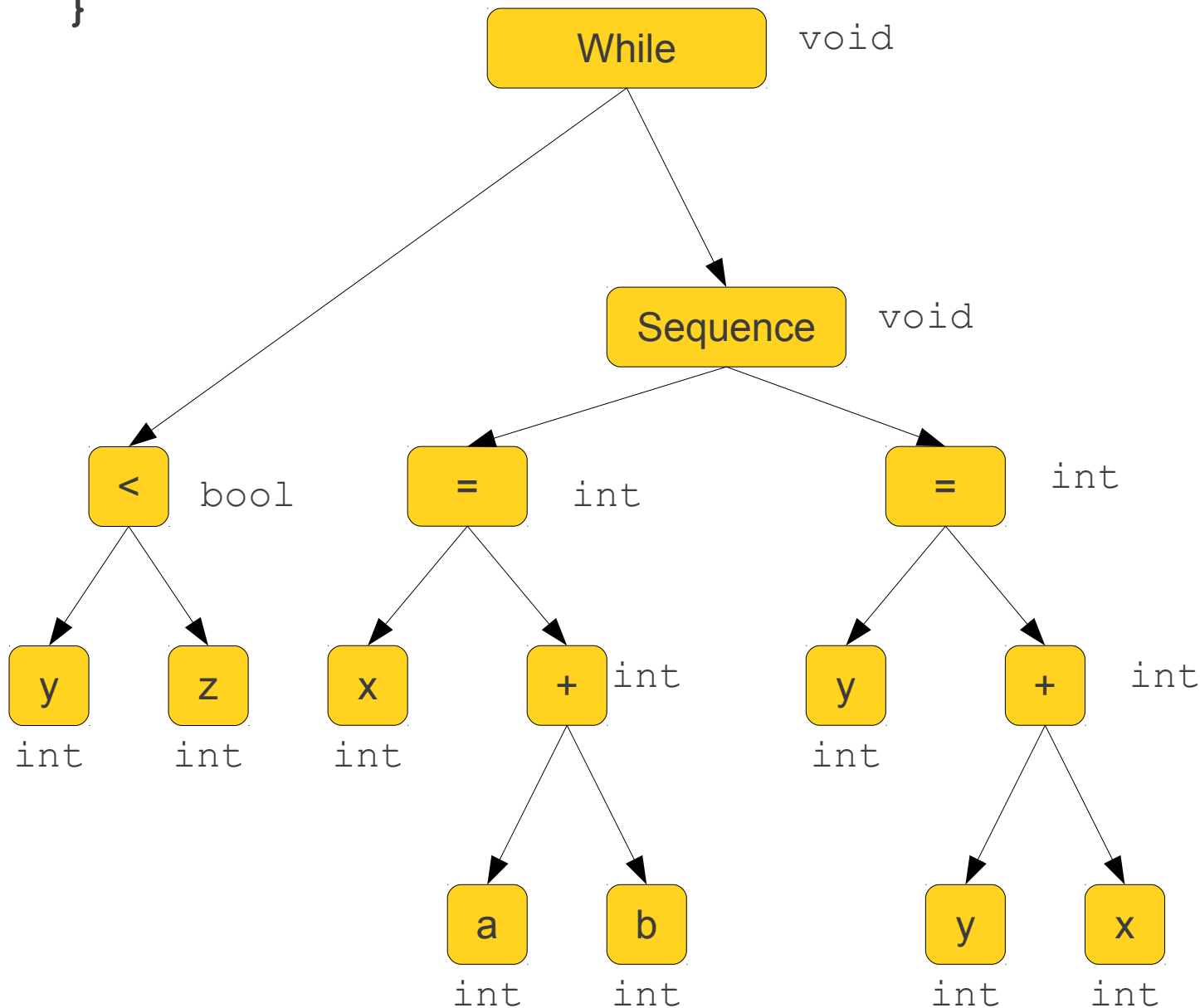
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```
Loop:  x      = a + b  
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      _t1    = y < z  
      if _t1 goto Loop
```

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```
while (y < z) {  
    int x = a + b;  
    y += x;  
}
```

```
          add $1, $2, $3  
Loop:    add $4, $1, $4  
          slt $6, $1, $5  
          beq $6, loop
```

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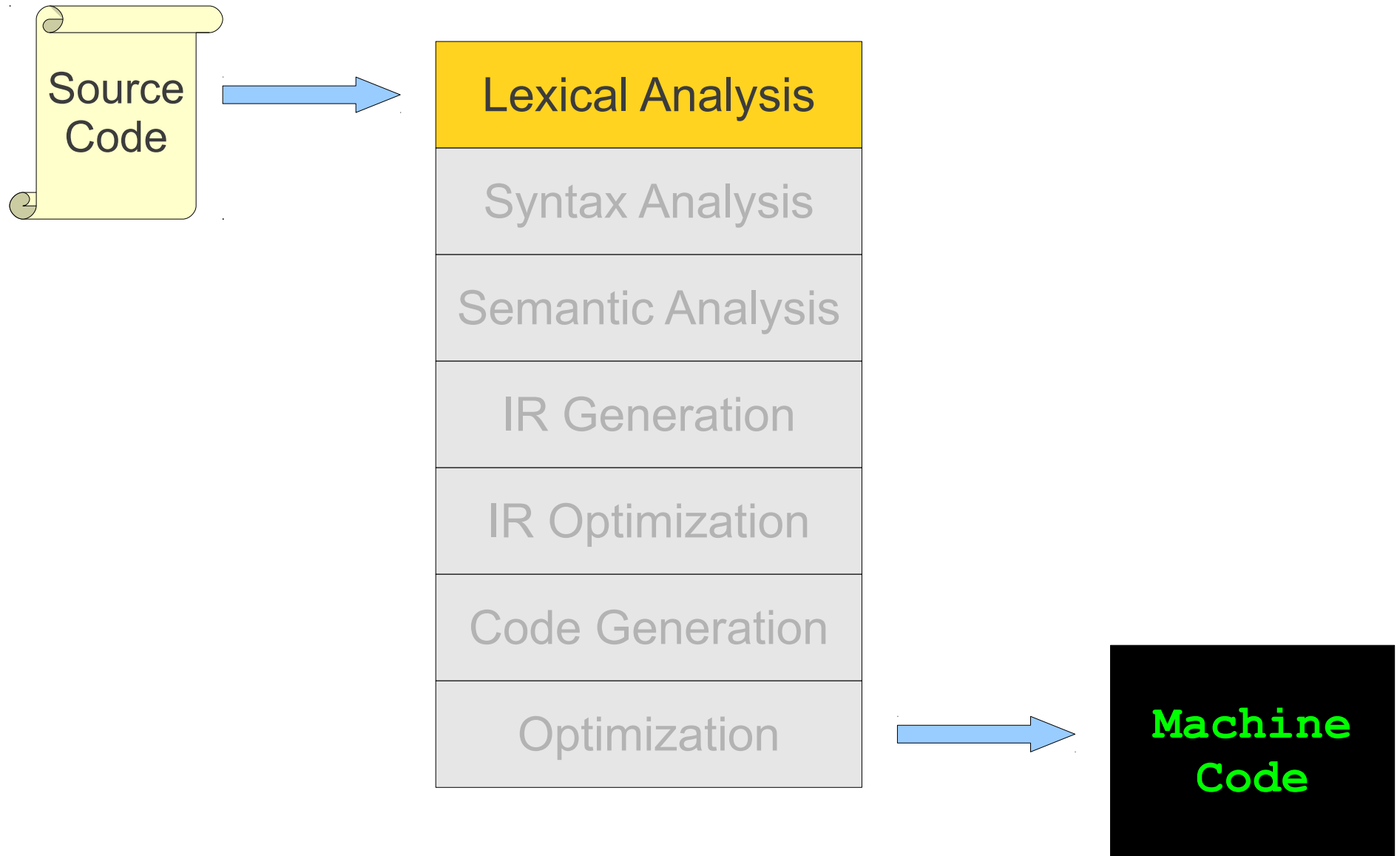
IR Optimization

Code Generation

Optimization

Lexical Analysis

Where We Are



Scanning a Source File

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T_While

Scanning a Source File

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The piece of the original program from which we made the token is called a **lexeme**.

T_While

This is called a **token**. You can think of it as an enumerated type representing what logical entity we read out of the source code.

Scanning a Source File

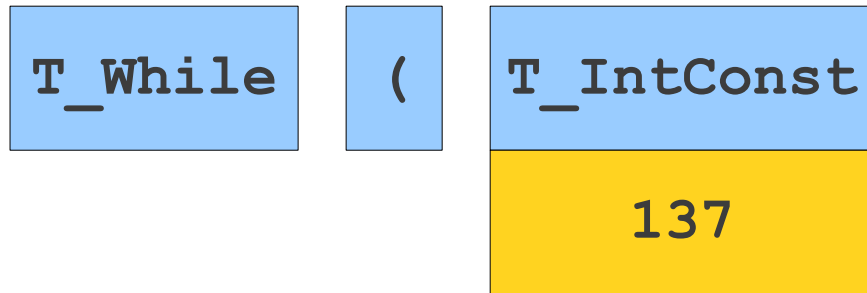
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T_While

Sometimes we will discard a lexeme rather than storing it for later use. Here, we ignore whitespace, since it has no bearing on the meaning of the program.

Scanning a Source File

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Some tokens can have **attributes** that store extra information about the token. Here we store which integer is represented.

Goals of Lexical Analysis

- Convert from physical description of a program into sequence of **tokens**.
 - Each token represents one logical piece of the source file – a keyword, the name of a variable, etc.
- Each token is associated with a **lexeme**.
 - The actual text of the token: “137,” “int,” etc.
- Each token may have optional **attributes**.
 - Extra information derived from the text – perhaps a numeric value.
- The token sequence will be used in the parser to recover the program structure.

Choosing Tokens

What Tokens are Useful Here?

```
for (int k = 0; k < myArray[5]; ++k) {  
    cout << k << endl;  
}
```


What Tokens are Useful Here?

```
for (int k = 0; k < myArray[5]; ++k) {  
    cout << k << endl;  
}
```

```
for      {  
int      }  
<<      ;  
=        <  
(        [  
)        ]  
++
```

Identifier

IntegerConstant

Choosing Good Tokens

- Very much dependent on the language.
- Typically:
 - Give keywords their own tokens.
 - Give different punctuation symbols their own tokens.
 - Group lexemes representing identifiers, numeric constants, strings, etc. into their own groups.
 - Discard irrelevant information (whitespace, comments)

Scanning is Hard

- FORTRAN: Whitespace is irrelevant

DO 5 I = 1,25

DO5I = 1.25

Scanning is Hard

- FORTRAN: Whitespace is irrelevant

DO 5 I = 1,25

DO5I = 1.25

- Can be difficult to tell when to partition input.

Challenges in Scanning

- How do we determine which lexemes are associated with each token?
- When there are multiple ways we could scan the input, how do we know which one to pick?
- How do we address these concerns efficiently?

Associating Lexemes with Tokens

Lexemes and Tokens

- Tokens give a way to categorize lexemes by what information they provide.
- Some tokens might be associated with only a single lexeme:
 - Tokens for keywords like **if** and **while** probably only match those lexemes exactly.
- Some tokens might be associated with lots of different lexemes:
 - All variable names, all possible numbers, all possible strings, etc.

Sets of Lexemes

- Idea: Associate a set of lexemes with each token.
- We might associate the “number” token with the set { 0, 1, 2, ..., 10, 11, 12, ... }
- We might associate the “string” token with the set { "", "a", "b", "c", ... }
- We might associate the token for the keyword **while** with the set { **while** }.

How do we describe which (potentially infinite) set of lexemes is associated with each token type?

Formal Languages

- A **formal language** is a set of strings.
- Many infinite languages have finite descriptions:
 - Define the language using an automaton.
 - Define the language using a grammar.
 - Define the language using a regular expression.
- We can use these compact descriptions of the language to define sets of strings.

Regular Expressions

- **Regular expressions** are a family of descriptions that can be used to capture certain languages (the *regular languages*).
- Often provide a compact and human-readable description of the language.
- Used as the basis for numerous software systems.

Atomic Regular Expressions

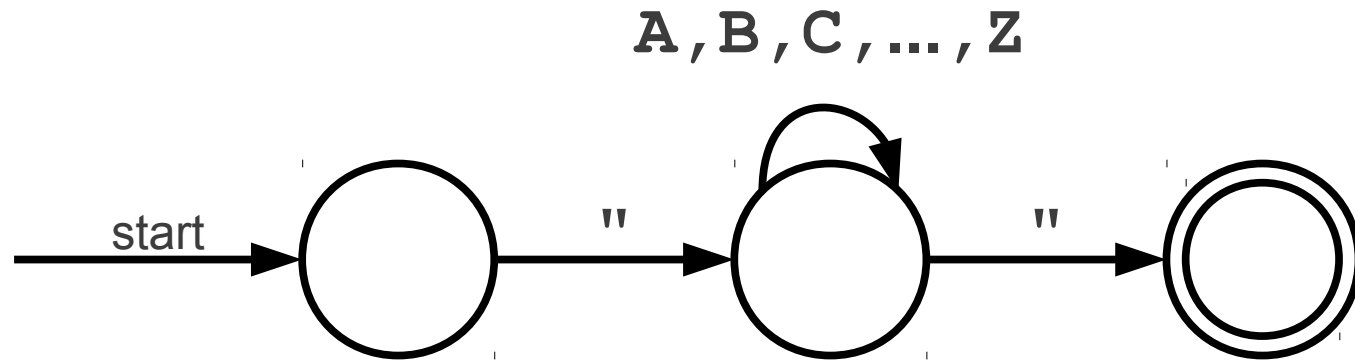
- The regular expressions we will use in this course begin with two simple building blocks.
- The symbol ϵ is a regular expression matches the empty string.
- For any symbol a , the symbol a is a regular expression that just matches a .

Matching Regular Expressions

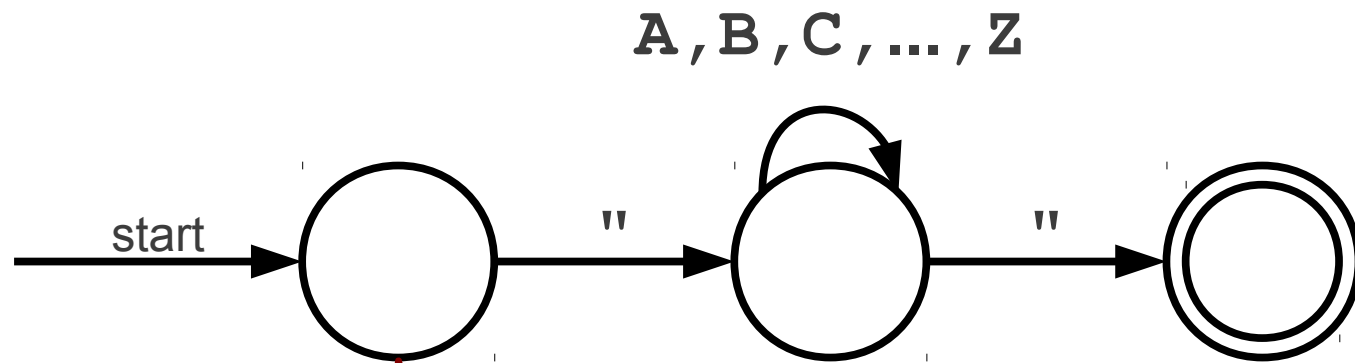
Implementing Regular Expressions

- Regular expressions can be implemented using **finite automata**.
- There are two main kinds of finite automata:
 - **NFAs** (**nondeterministic** finite automata), which we'll see in a second, and
 - **DFAs** (**deterministic** finite automata), which we'll see later.
- Automata are best explained by example...

A Simple Automaton

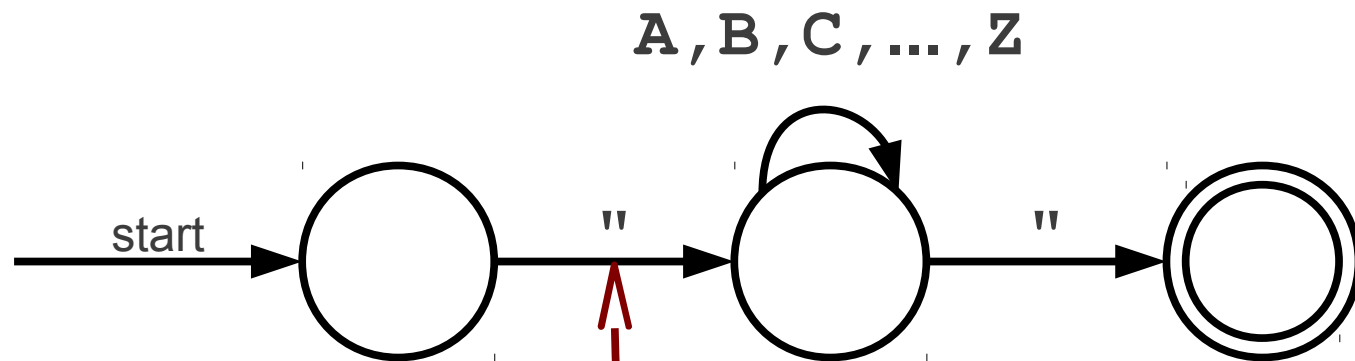


A Simple Automaton



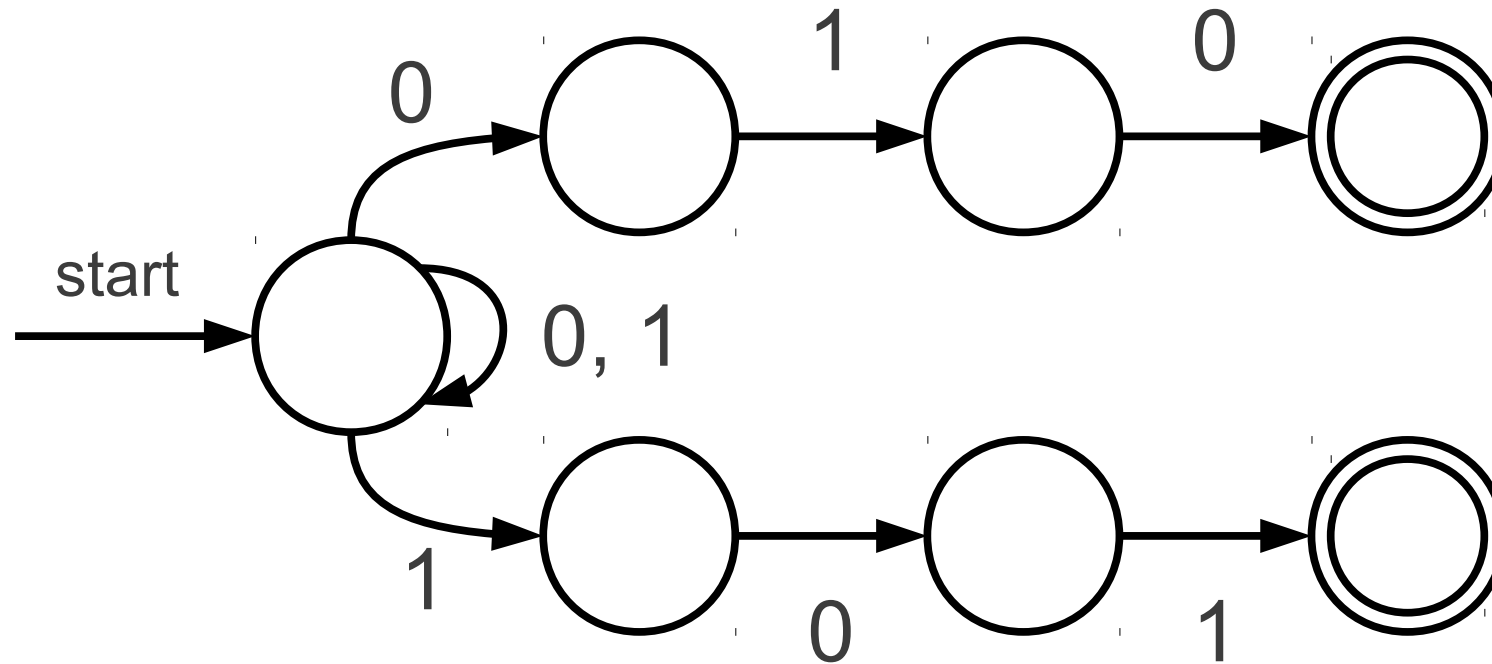
Each circle is a **state** of the automaton. The automaton's configuration is determined by what state(s) it is in.

A Simple Automaton

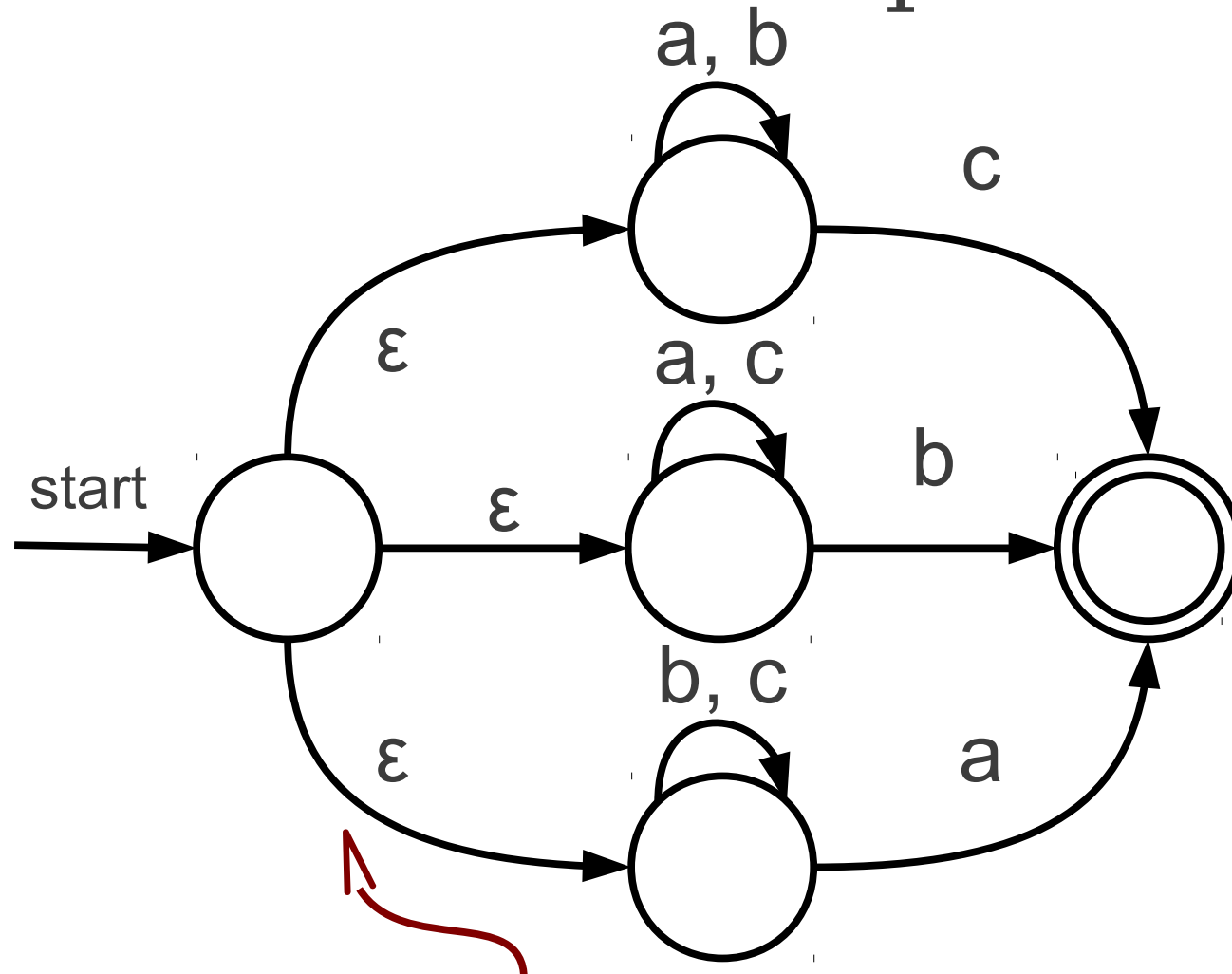


These arrows are called **transitions**. The automaton changes which state(s) it is in by following transitions.

A More Complex Automaton



An Even More Complex Automaton



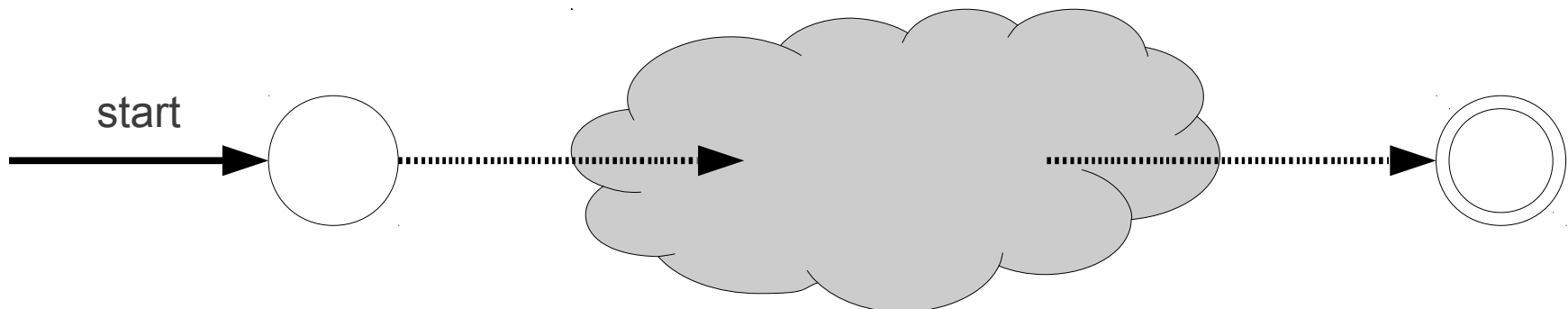
These are called ϵ -transitions. These transitions are followed automatically and without consuming any input.

Simulating an NFA

- Keep track of a set of states, initially the start state and everything reachable by ε -moves.
- For each character in the input:
 - Maintain a set of next states, initially empty.
 - For each current state:
 - Follow all transitions labeled with the current letter.
 - Add these states to the set of new states.
 - Add every state reachable by an ε -move to the set of next states.
- Complexity: $O(mn^2)$ for strings of length m and automata with n states.

From Regular Expressions to NFAs

- There is a (beautiful!) procedure from converting a regular expression to an NFA.
- Associate each regular expression with an NFA with the following properties:
 - There is exactly one accepting state.
 - There are no transitions out of the accepting state.
 - There are no transitions into the starting state.
- These restrictions are stronger than necessary, but make the construction easier.



Overall Result

- Any regular expression of length n can be converted into an NFA with $O(n)$ states.
- Can determine whether a string of length m matches a regular expression of length n in time $O(mn^2)$.
- We'll see how to make this $O(m)$ later (this is independent of the complexity of the regular expression!)

Challenges in Scanning

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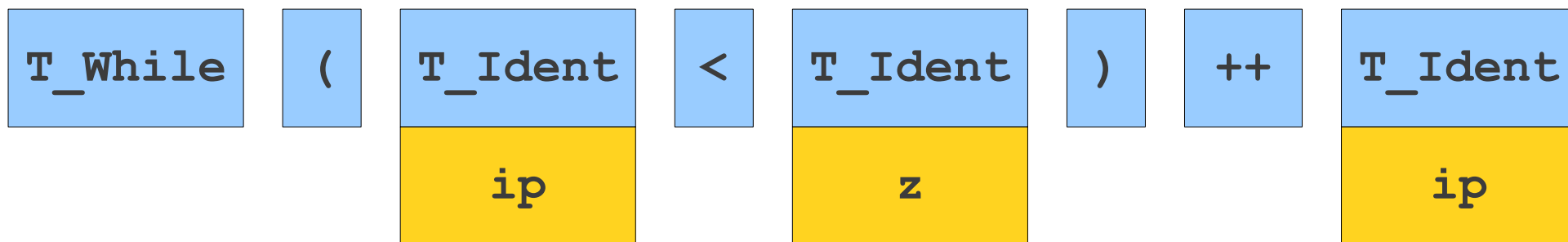
Summary

- Lexical analysis splits input text into **tokens** holding a **lexeme** and an **attribute**.
- Lexemes are sets of strings often defined with **regular expressions**.
- Regular expressions can be converted to **NFAs** and from there to **DFAs**.

Syntax Analysis

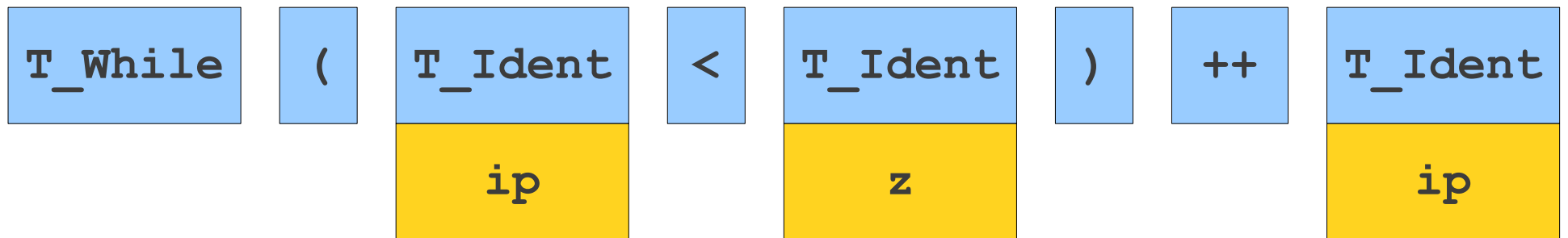
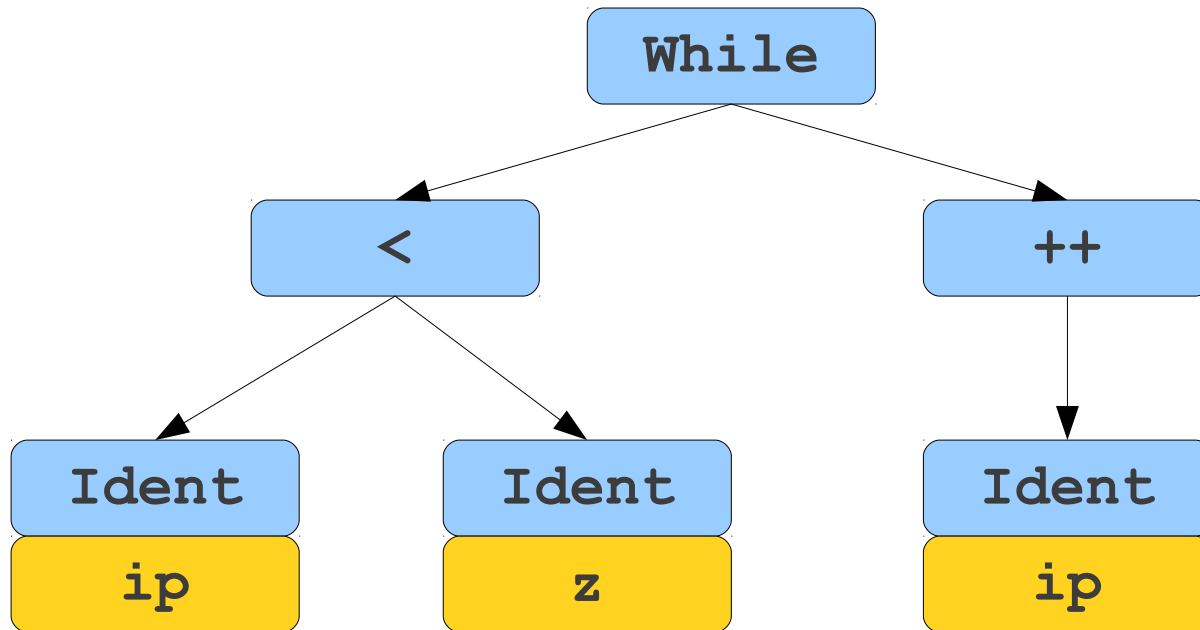
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|---|---|---|---|---|--|---|---|---|--|---|--|---|---|----|----|---|---|---|---|---|
| w | h | i | l | e | | (| i | p | | < | | z |) | \n | \t | + | + | i | p | ; |
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```
while (ip < z)
    ++ip;
```



| | | | | | | | | | | | | | | | | | | | | |
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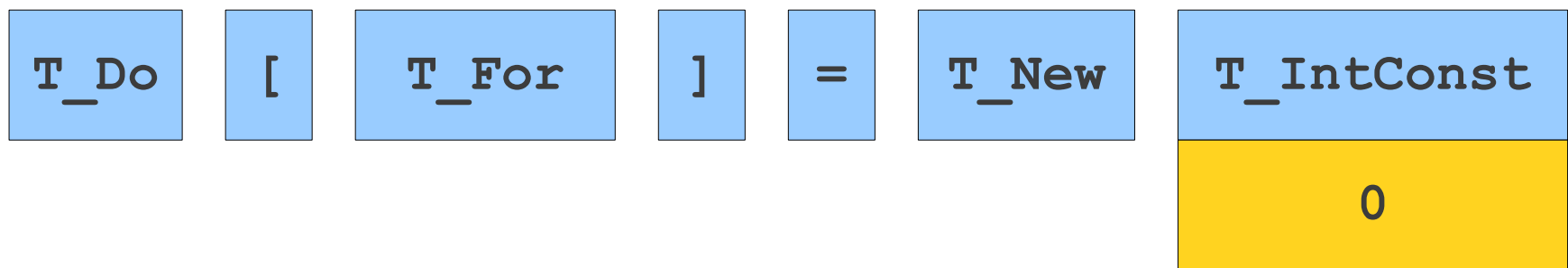


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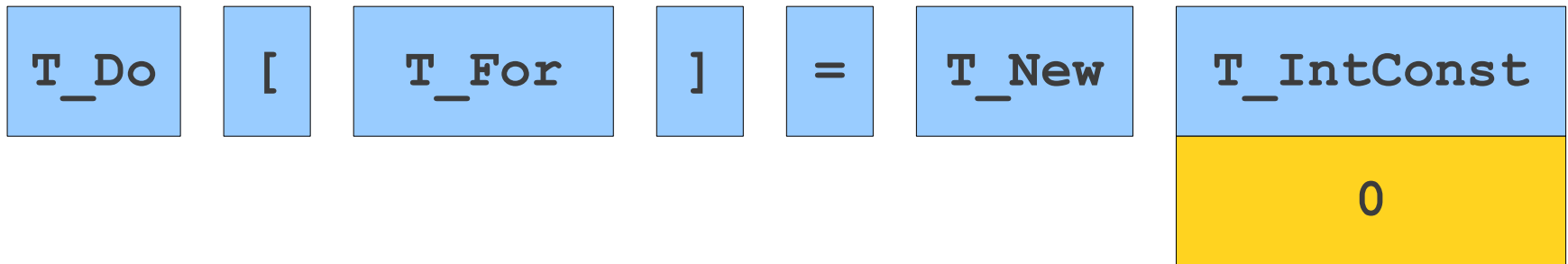
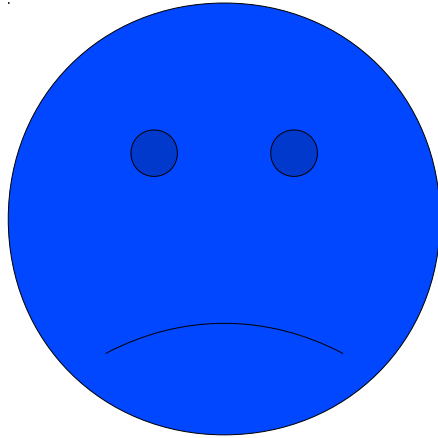
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| d | o | [| f | o | r |] | | = | | n | e | w | | 0 | ; |
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do[for] = new 0;



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`do[for] = new 0;`



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do[for] = new 0;

What is Syntax Analysis?

- After lexical analysis (scanning), we have a series of tokens.
- In **syntax analysis** (or **parsing**), we want to interpret what those tokens mean.
- Goal: Recover the *structure* described by that series of tokens.
- Goal: Report *errors* if those tokens do not properly encode a structure.

Formal Languages

- An **alphabet** is a set Σ of symbols that act as letters.
- A **language** over Σ is a set of strings made from symbols in Σ .
- When scanning, our alphabet was ASCII or Unicode characters. We produced tokens.
- When parsing, our alphabet is the set of tokens produced by the scanner.

The Limits of Regular Languages

- When scanning, we used regular expressions to define each token.
- Unfortunately, regular expressions are (usually) too weak to define programming languages.
 - Cannot define a regular expression matching all expressions with properly balanced parentheses.
 - Cannot define a regular expression matching all functions with properly nested block structure.
- We need a more powerful formalism.

Context-Free Grammars

- A **context-free grammar** (or **CFG**) is a formalism for defining languages.
- Can define the **context-free languages**, a strict superset of the regular languages.
- CFGs are best explained by example...

Arithmetic Expressions

- Suppose we want to describe all legal arithmetic expressions using addition, subtraction, multiplication, and division.
- Here is one possible CFG:

E → **int**

E → **E Op E**

E → **(E)**

Op → **+**

Op → **-**

Op → *****

Op → **/**

E
⇒ **E Op E**
⇒ **E Op (E)**
⇒ **E Op (E Op E)**
⇒ **E * (E Op E)**
⇒ **int * (E Op E)**
⇒ **int * (int Op E)**
⇒ **int * (int Op int)**
⇒ **int * (int + int)**

Arithmetic Expressions

- Suppose we want to describe all legal arithmetic expressions using addition, subtraction, multiplication, and division.
- Here is one possible CFG:

E → **int**

E → **E Op E**

E → **(E)**

Op → **+**

Op → **-**

Op → *****

Op → **/**

E

⇒ **E Op E**

⇒ **E Op int**

⇒ **int Op int**

⇒ **int / int**

Context-Free Grammars

- Formally, a context-free grammar is a collection of four objects:
 - A set of **nonterminal symbols** (or **variables**),
 - A set of **terminal symbols**,
 - A set of **production rules** saying how each nonterminal can be converted by a string of terminals and nonterminals, and
 - A **start symbol** that begins the derivation.

E → **int**

E → **E Op E**

E → **(E)**

Op → **+**

Op → **-**

Op → *****

Op → **/**

A Notational Shorthand

E \rightarrow **int** | **E Op E** | **(E)**

Op \rightarrow **+** | **-** | ***** | **/**

Some CFG Notation

- Capital letters at the beginning of the alphabet will represent nonterminals.
 - i.e. **A**, **B**, **C**, **D**
- Lowercase letters at the end of the alphabet will represent terminals.
 - i.e. **t**, **u**, **v**, **w**
- Lowercase Greek letters will represent arbitrary strings of terminals and nonterminals.
 - i.e. **α** , **γ** , **ω**

Examples

- We might write an arbitrary production as

$$\mathbf{A} \rightarrow \omega$$

- We might write a string of a nonterminal followed by a terminal as

$$\mathbf{A}t$$

- We might write an arbitrary production containing a nonterminal followed by a terminal as

$$\mathbf{B} \rightarrow \alpha \mathbf{A}t\omega$$

Derivations

E
 $\Rightarrow E \text{ Op } E$
 $\Rightarrow E \text{ Op } (E)$
 $\Rightarrow E \text{ Op } (E \text{ Op } E)$
 $\Rightarrow E * (E \text{ Op } E)$
 $\Rightarrow \text{int} * (E \text{ Op } E)$
 $\Rightarrow \text{int} * (\text{int} \text{ Op } E)$
 $\Rightarrow \text{int} * (\text{int} \text{ Op } \text{int})$
 $\Rightarrow \text{int} * (\text{int} + \text{int})$

- This sequence of steps is called a **derivation**.
- A string $\alpha A \omega$ **yields** string $\alpha \gamma \omega$ iff $A \rightarrow \gamma$ is a production.
- If α yields β , we write $\alpha \Rightarrow \beta$.
- We say that α **derives** β iff there is a sequence of strings where
$$\alpha \Rightarrow \alpha_1 \Rightarrow \alpha_2 \Rightarrow \dots \Rightarrow \beta$$
- If α derives β , we write $\alpha \Rightarrow^* \beta$.

Leftmost Derivations

| | | | |
|--------------|---|--|---|
| BLOCK | → | STMT { STMTS } | |
| STMTS | → | ε STMT STMTS | STMTS |
| STMT | → | EXPR; if (EXPR) BLOCK while (EXPR) BLOCK do BLOCK while (EXPR); BLOCK ... | ⇒ STMT STMTS ⇒ EXPR; STMTS ⇒ EXPR = EXPR; STMTS ⇒ id = EXPR; STMTS ⇒ id = EXPR + EXPR; STMTS |
| EXPR | → | identifier constant EXPR + EXPR EXPR - EXPR EXPR * EXPR EXPR = EXPR ... | ⇒ id = id + EXPR; STMTS ⇒ id = id + constant; STMTS ⇒ id = id + constant; |

Leftmost Derivations

- A **leftmost derivation** is a derivation in which each step expands the leftmost nonterminal.
- A **rightmost derivation** is a derivation in which each step expands the rightmost nonterminal.
- These will be of great importance when we talk about parsing.

Related Derivations

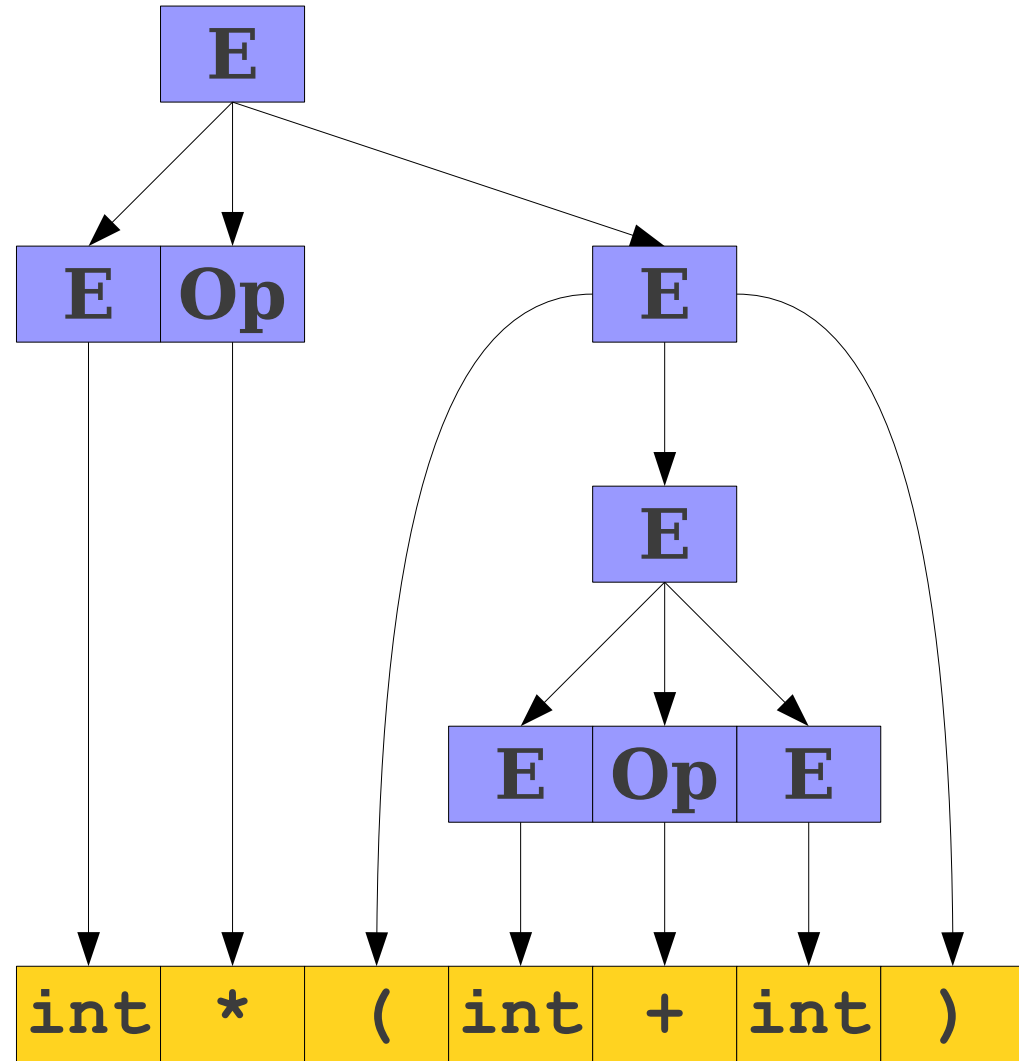
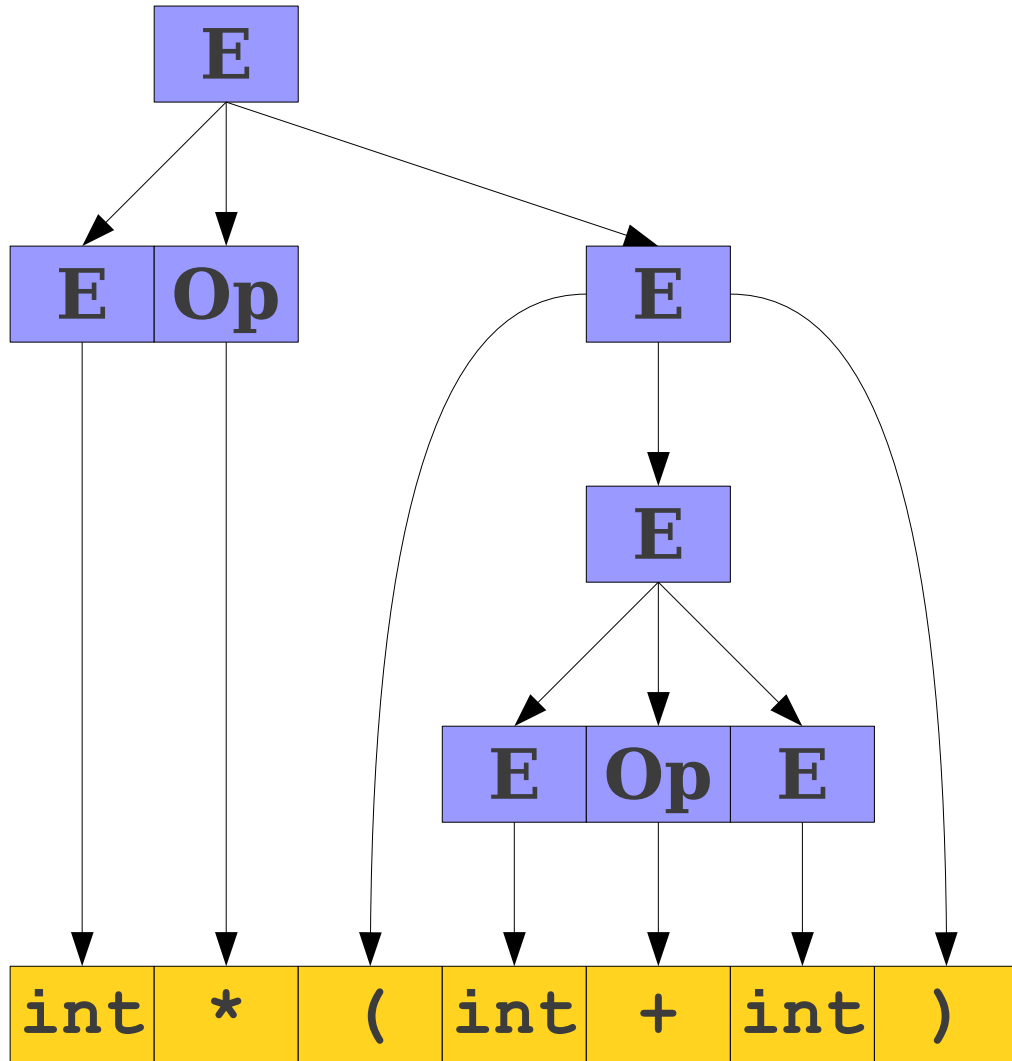
E
⇒ **E Op E**
⇒ int **Op E**
⇒ int * **E**
⇒ int * (**E**)
⇒ int * (**E Op E**)
⇒ int * (int **Op E**)
⇒ int * (int + **E**)
⇒ int * (int + int)

E
⇒ **E Op E**
⇒ **E Op (E)**
⇒ **E Op (E Op E)**
⇒ **E Op (E Op int)**
⇒ **E Op (E + int)**
⇒ **E Op (int + int)**
⇒ **E * (int + int)**
⇒ int * (int + int)

Derivations Revisited

- A derivation encodes two pieces of information:
 - What productions were applied produce the resulting string from the start symbol?
 - In what order were they applied?
- Multiple derivations might use the same productions, but apply them in a different order.

For Comparison



Parse Trees

- A **parse tree** is a tree encoding the steps in a derivation.
- Internal nodes represent nonterminal symbols used in the production.
- Inorder walk of the leaves contains the generated string.
- Encodes what productions are used, not the order in which those productions are applied.

The Goal of Parsing

- Goal of syntax analysis: Recover the **structure** described by a series of tokens.
- If language is described as a CFG, goal is to recover a parse tree for the the input string.
 - Usually we do some simplifications on the tree; more on that later.

Challenges in Parsing

Context-Free Grammars

- A regular expression can be
 - Any letter
 - ϵ
 - The concatenation of regular expressions.
 - The union of regular expressions.
 - The Kleene closure of a regular expression.
 - A parenthesized regular expression.

Context-Free Grammars

- This gives us the following CFG:

$$\mathbf{R} \rightarrow \mathbf{a} \mid \mathbf{b} \mid \mathbf{c} \mid \dots$$

$$\mathbf{R} \rightarrow \text{“}\epsilon\text{”}$$

$$\mathbf{R} \rightarrow \mathbf{RR}$$

$$\mathbf{R} \rightarrow \mathbf{R} \mid \mathbf{R}$$

$$\mathbf{R} \rightarrow \mathbf{R}^*$$

$$\mathbf{R} \rightarrow (\mathbf{R})$$

Abstract Syntax Trees (ASTs)

- A parse tree is a **concrete syntax tree**; it shows exactly how the text was derived.
- A more useful structure is an **abstract syntax tree**, which retains only the essential structure of the input.

Summary

- Syntax analysis (**parsing**) extracts the structure from the tokens produced by the scanner.
- Languages are usually specified by **context-free grammars (CFGs)**.
- A **parse tree** shows how a string can be **derived** from a grammar.
- A grammar is **ambiguous** if it can derive the same string multiple ways.
- There is no algorithm for eliminating ambiguity; it must be done by hand.
- **Abstract syntax trees (ASTs)** contain an abstract representation of a program's syntax.

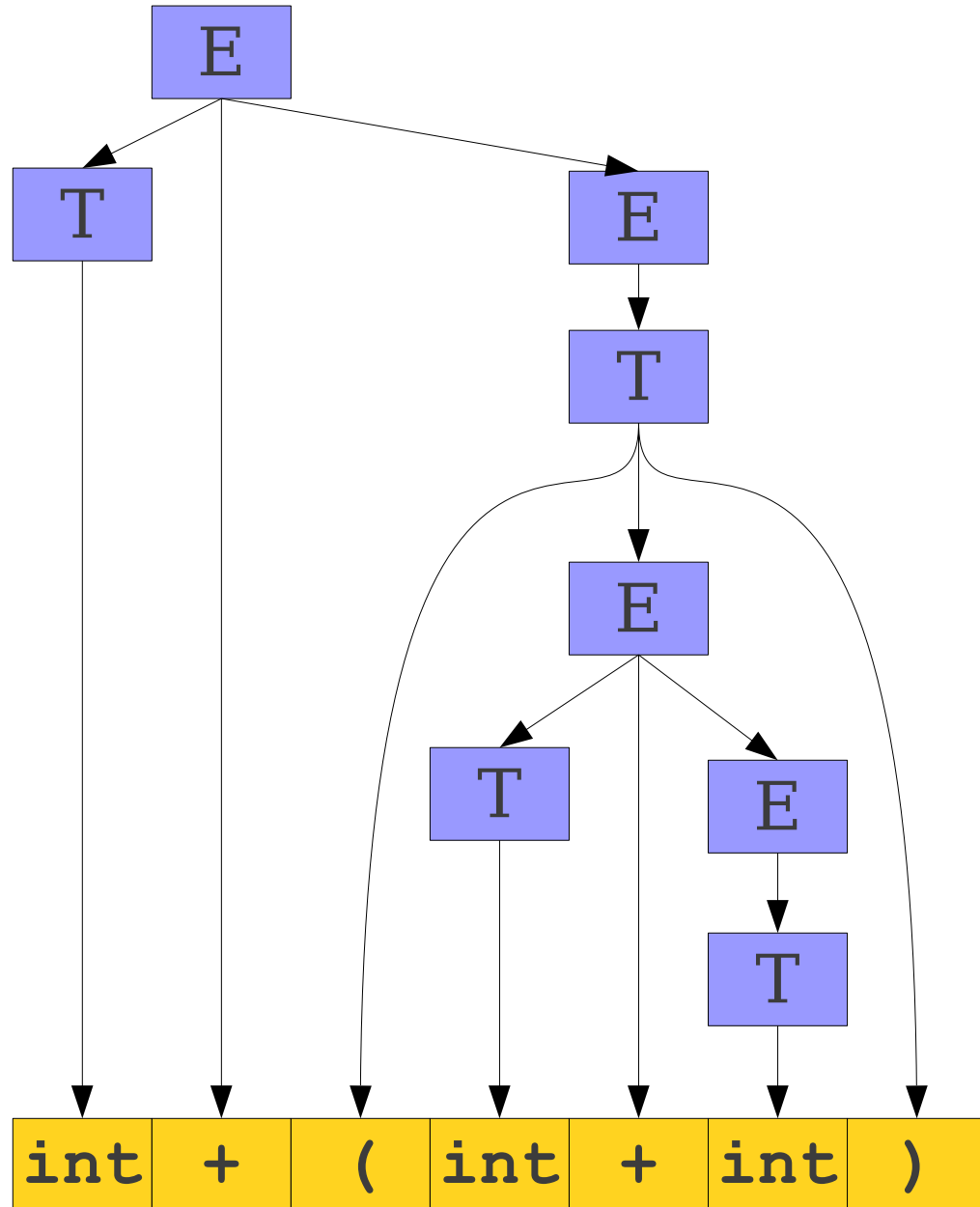
Top-Down Parsing

Different Types of Parsing

- **Top-Down Parsing** (Today)
 - Beginning with the start symbol, try to guess the productions to apply to end up at the user's program.
- **Bottom-Up Parsing**
 - Beginning with the user's program, try to apply productions in reverse to convert the program back into the start symbol.

Top-Down Parsing

$E \rightarrow T$
 $E \rightarrow T + E$
 $T \rightarrow \text{int}$
 $T \rightarrow (E)$



Challenges in Top-Down Parsing

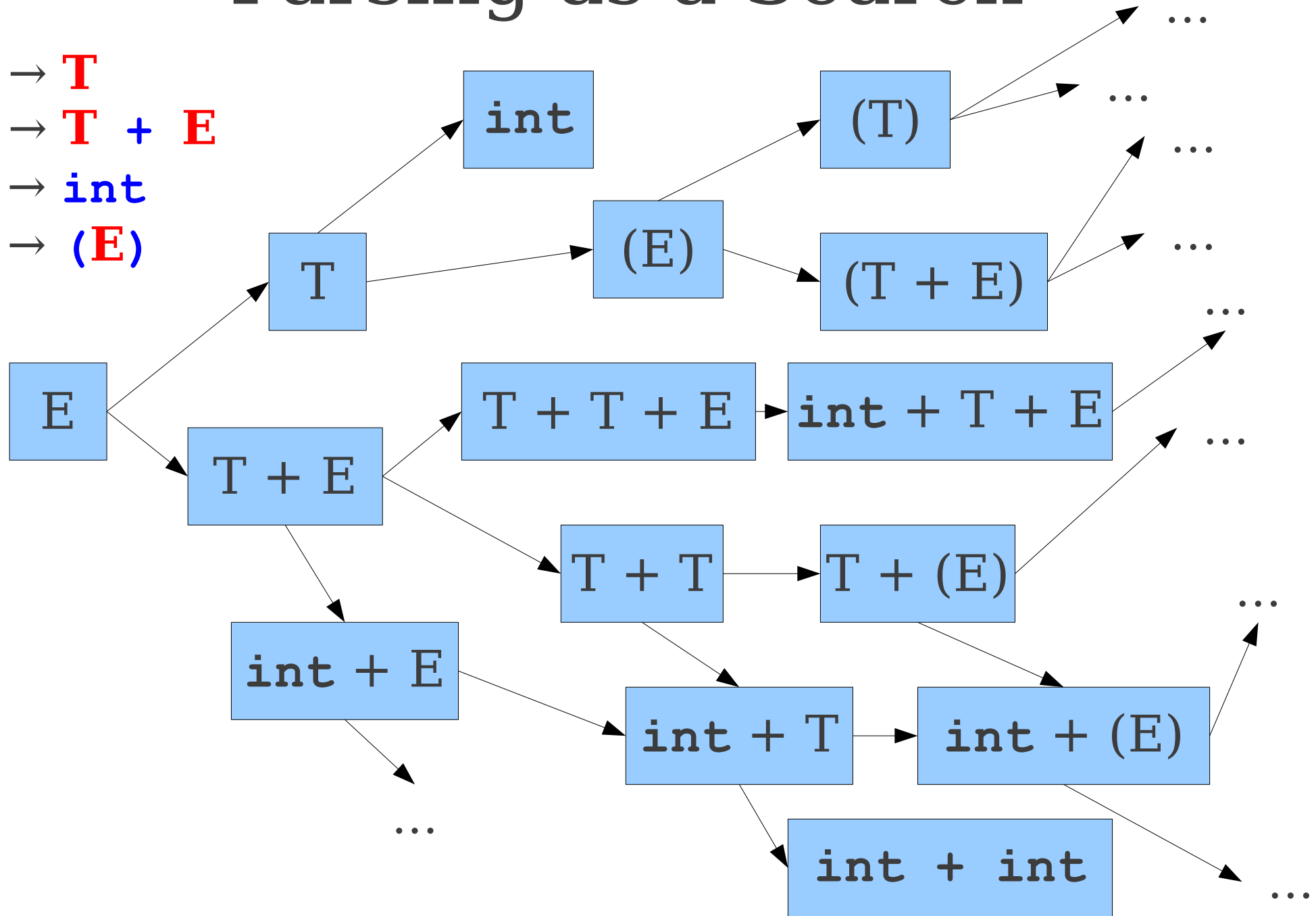
- Top-down parsing begins with virtually no information.
 - Begins with just the start symbol, which matches *every* program.
- How can we know which productions to apply?
- In general, we can't.
 - There are some grammars for which the best we can do is guess and backtrack if we're wrong.
 - If we have to guess, how do we do it?

Parsing as a Search

- An idea: **treat parsing as a graph search**.
- Each node is a **sentential form** (a string of terminals and nonterminals derivable from the start symbol).
- There is an edge from node α to node β iff $\alpha \Rightarrow \beta$.

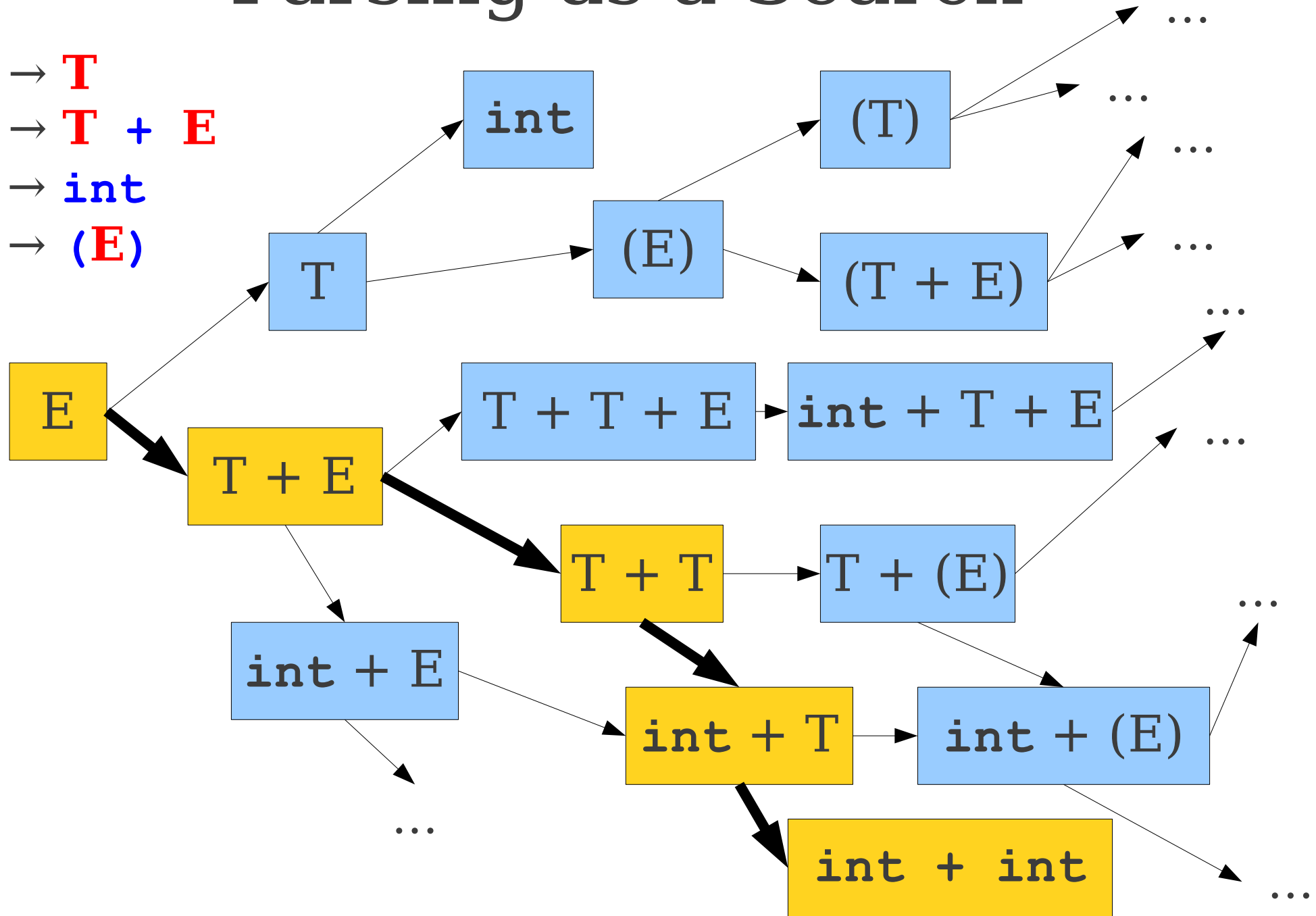
Parsing as a Search

$E \rightarrow T$
 $E \rightarrow T + E$
 $T \rightarrow \text{int}$
 $T \rightarrow (E)$



Parsing as a Search

$E \rightarrow T$
 $E \rightarrow T + E$
 $T \rightarrow \text{int}$
 $T \rightarrow (E)$



BFS is Slow

- Enormous time and memory usage:
 - Lots of **wasted effort**:
 - Generates a lot of sentential forms that couldn't possibly match.
 - But in general, extremely hard to tell whether a sentential form can match – that's the job of parsing!
 - High **branching factor**:
 - Each sentential form can expand in (potentially) many ways for each nonterminal it contains.

Leftmost Derivations

- Recall: A **leftmost derivation** is one where we always expand the leftmost symbol first.
- Updated algorithm:
 - Do a breadth-first search, **only considering leftmost derivations**.
 - Dramatically drops branching factor.
 - Increases likelihood that we get a prefix of nonterminals.
 - Prune sentential forms that can't possibly match.
 - Avoids wasted effort.

Leftmost BFS

- Substantial improvement over naïve algorithm.
- Will always find a valid parse of a program if one exists.
- Can easily be modified to find if a program can't be parsed.
- But, there are still problems.

Leftmost DFS

- Idea: Use **depth-first** search.
- Advantages:
 - Lower memory usage: Only considers one branch at a time.
 - High performance: On many grammars, runs very quickly.
 - Easy to implement: Can be written as a set of mutually recursive functions.

Left Recursion

- A nonterminal **A** is said to be **left-recursive** iff

$$\mathbf{A} \Rightarrow^* \mathbf{A}\omega$$

for some string ω .

- Leftmost DFS may fail on left-recursive grammars.
- Fortunately, in many cases it is possible to eliminate left recursion.

Summary of Leftmost BFS/DFS

- Leftmost BFS works on all grammars.
- Worst-case runtime is exponential.
- Worst-case memory usage is exponential.
- Rarely used in practice.
- Leftmost DFS works on grammars without left recursion.
- Worst-case runtime is exponential.
- Worst-case memory usage is linear.
- Often used in a limited form as **recursive descent**.

Predictive Parsing

Predictive Parsing

- The leftmost DFS/BFS algorithms are **backtracking** algorithms.
 - Guess which production to use, then back up if it doesn't work.
 - Try to match a prefix by sheer dumb luck.
- There is another class of parsing algorithms called **predictive** algorithms.
 - Based on remaining input, predict (*without backtracking*) which production to use.

Tradeoffs in Prediction

- Predictive parsers are *fast*.
 - Many predictive algorithms can be made to run in linear time.
 - Often can be table-driven for extra performance.
- Predictive parsers are *weak*.
 - Not all grammars can be accepted by predictive parsers.
- Trade *expressiveness* for *speed*.

Exploiting Lookahead

- Given just the start symbol, how do you know which productions to use to get to the input program?
- Idea: Use **lookahead tokens**.
- When trying to decide which production to use, look at some number of tokens of the input to help make the decision.

Implementing Predictive Parsing

- Predictive parsing is only possible if we can predict which production to use given some number of lookahead tokens.
- Increasing the number of lookahead tokens increases the number of grammars we can parse, but complicates the parser.
- Decreasing the number of lookahead tokens decreases the number of grammars we can parse, but simplifies the parser.

A Simple Predictive Parser: **LL(1)**

- Top-down, predictive parsing:
 - **L**: Left-to-right scan of the tokens
 - **L**: Leftmost derivation.
 - **(1)**: One token of lookahead
- Construct a leftmost derivation for the sequence of tokens.
- When expanding a nonterminal, we predict the production to use by looking at the next token of the input. **The decision is forced.**

LL(1) Parse Tables

LL(1) Parse Tables

E \rightarrow **int**

E \rightarrow **(E Op E)**

Op \rightarrow **+**

Op \rightarrow *****

LL(1) Parse Tables

E \rightarrow **int**

E \rightarrow (**E Op E**)

Op \rightarrow **+**

Op \rightarrow *****

| | int | (|) | + | * |
|----|-----|----------|---|---|---|
| E | int | (E Op E) | | | |
| Op | | | | + | * |

LL(1) Parsing

(int + (int * int))

(1) **E** → int

(2) **E** → (E **Op** E)

(3) **Op** → +

(4) **Op** → *

LL(1) Parsing

| | |
|---|---------------------|
| E | (int + (int * int)) |
|---|---------------------|

(1) **E** → int

(2) **E** → (E **Op** E)

(3) **Op** → +

(4) **Op** → *

LL(1) Parsing

| | |
|---|---------------------|
| E | (int + (int * int)) |
|---|---------------------|

(1) **E** \rightarrow int

(2) **E** \rightarrow (**E Op E**)

(3) **Op** \rightarrow +

(4) **Op** \rightarrow *

| | int | (|) | + | * |
|----|-----|---|---|---|---|
| E | 1 | 2 | | | |
| Op | | | | 3 | 4 |

LL(1) Parsing

E\$

(int + (int * int))\$

(1) **E** → int

(2) **E** → (E **Op** E)

(3) **Op** → +

(4) **Op** → *

| | int | (|) | + | * |
|----|-----|---|---|---|---|
| E | 1 | 2 | | | |
| Op | | | | 3 | 4 |

LL(1) Parsing

E\$

(int + (int * int))\$

(1) **E** → int

(2) **E** → (**E Op E**)

(3) **Op** → +

(4) **Op** → *

| | int | (|) | + | * |
|----|-----|---|---|---|---|
| E | 1 | 2 | | | |
| Op | | | | 3 | 4 |

The **\$** symbol is the end-of-input marker and is used by the parser to detect when we have reached the end of the input. It is not a part of the grammar.

LL(1) Parsing

E\$

(int + (int * int))\$

(1) **E** → int

(2) **E** → (E **Op** E)

(3) **Op** → +

(4) **Op** → *

| | int | (|) | + | * |
|----|-----|---|---|---|---|
| E | 1 | 2 | | | |
| Op | | | | 3 | 4 |

LL(1) Parsing

E\$

(int + (int * int))\$

(1) **E** \rightarrow int

(2) **E** \rightarrow (**E Op E**)

(3) **Op** \rightarrow +

(4) **Op** \rightarrow *

| | int | (|) | + | * |
|----|-----|---|---|---|---|
| E | 1 | 2 | | | |
| Op | | | | 3 | 4 |

The first symbol of our guess is a nonterminal. We then look at our parsing table to see what production to use.

This is called a **predict** step.

LL(1) Parsing

E\$

(int + (int * int))\$

(1) **E** → int

(2) **E** → (E **Op** E)

(3) **Op** → +

(4) **Op** → *

| | int | (|) | + | * |
|----|-----|---|---|---|---|
| E | 1 | 2 | | | |
| Op | | | | 3 | 4 |

LL(1) Parsing

(1) **E** \rightarrow **int**

(2) **E** \rightarrow (**E Op E**)

(3) **Op** \rightarrow **+**

(4) **Op** \rightarrow *****

| | |
|------------|-----------------------|
| E\$ | (int + (int * int))\$ |
| (E Op E)\$ | (int + (int * int))\$ |

| | | | | | |
|----|-----|---|---|---|---|
| | int | (|) | + | * |
| E | 1 | 2 | | | |
| Op | | | | 3 | 4 |

LL(1) Parsing

- (1) **E** \rightarrow **int**
- (2) **E** \rightarrow (**E Op E**)
- (3) **Op** \rightarrow **+**
- (4) **Op** \rightarrow *****

| | |
|------------|-----------------------|
| E\$ | (int + (int * int))\$ |
| (E Op E)\$ | (int + (int * int))\$ |

The first symbol of our guess is now a terminal symbol. We thus match it against the first symbol of the string to parse.

This is called a **match** step.

| | int | (|) | + | * |
|----|-----|---|---|---|---|
| E | 1 | 2 | | | |
| Op | | | | 3 | 4 |

LL(1) Parsing

(1) **E** → **int**

(2) **E** → (**E Op E**)

(3) **Op** → **+**

(4) **Op** → *****

| | |
|------------|-----------------------|
| E\$ | (int + (int * int))\$ |
| (E Op E)\$ | (int + (int * int))\$ |
| E Op E)\$ | int + (int * int))\$ |

| | | | | | |
|----|-----|---|---|---|---|
| | int | (|) | + | * |
| E | 1 | 2 | | | |
| Op | | | | 3 | 4 |

LL(1) Parsing

(1) **E** → **int**

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(3) **Op** → **+**

(4) **Op** → *****

| | |
|------------|-----------------------|
| E\$ | (int + (int * int))\$ |
| (E Op E)\$ | (int + (int * int))\$ |
| E Op E)\$ | int + (int * int))\$ |

| | | | | | |
|----|-----|---|---|---|---|
| | int | (|) | + | * |
| E | 1 | 2 | | | |
| Op | | | | 3 | 4 |

LL(1) Parsing

(1) **E** → **int**

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| | |
|-------------|-----------------------|
| E\$ | (int + (int * int))\$ |
| (E Op E)\$ | (int + (int * int))\$ |
| E Op E)\$ | int + (int * int))\$ |
| int Op E)\$ | int + (int * int))\$ |

| | | | | | |
|----|-----|---|---|---|---|
| | int | (|) | + | * |
| E | 1 | 2 | | | |
| Op | | | | 3 | 4 |

LL(1) Parsing

(1) **E** → **int**

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| | |
|-------------|-----------------------|
| E\$ | (int + (int * int))\$ |
| (E Op E)\$ | (int + (int * int))\$ |
| E Op E)\$ | int + (int * int))\$ |
| int Op E)\$ | int + (int * int))\$ |
| Op E)\$ | + (int * int))\$ |

| | | | | | |
|----|-----|---|---|---|---|
| | int | (|) | + | * |
| E | 1 | 2 | | | |
| Op | | | | 3 | 4 |

LL(1) Parsing

(1) **E** → **int**

(2) **E** → (**E Op E**)

(3) **Op** → **+**

(4) **Op** → *****

| | |
|-------------|-----------------------|
| E\$ | (int + (int * int))\$ |
| (E Op E)\$ | (int + (int * int))\$ |
| E Op E)\$ | int + (int * int))\$ |
| int Op E)\$ | int + (int * int))\$ |
| Op E)\$ | + (int * int))\$ |

| | | | | | |
|----|-----|---|---|---|---|
| | int | (|) | + | * |
| E | 1 | 2 | | | |
| Op | | | | 3 | 4 |

LL(1) Parsing

(1) **E** \rightarrow **int**

(2) **E** \rightarrow (**E Op E**)

(3) **Op** \rightarrow **+**

(4) **Op** \rightarrow *****

| | |
|-------------|-----------------------|
| E\$ | (int + (int * int))\$ |
| (E Op E)\$ | (int + (int * int))\$ |
| E Op E)\$ | int + (int * int))\$ |
| int Op E)\$ | int + (int * int))\$ |
| Op E)\$ | + (int * int))\$ |
| + E)\$ | + (int * int))\$ |

| | | | | | |
|----|-----|---|---|---|---|
| | int | (|) | + | * |
| E | 1 | 2 | | | |
| Op | | | | 3 | 4 |

LL(1) Parsing

(1) **E** → **int**

(2) **E** → (**E Op E**)

(3) **Op** → **+**

(4) **Op** → *****

| | |
|-------------|-----------------------|
| E\$ | (int + (int * int))\$ |
| (E Op E)\$ | (int + (int * int))\$ |
| E Op E)\$ | int + (int * int))\$ |
| int Op E)\$ | int + (int * int))\$ |
| Op E)\$ | + (int * int))\$ |
| + E)\$ | + (int * int))\$ |
| E)\$ | (int * int))\$ |

| | | | | | |
|----|-----|---|---|---|---|
| | int | (|) | + | * |
| E | 1 | 2 | | | |
| Op | | | | 3 | 4 |

LL(1) Parsing

(1) **E** → **int**

(2) **E** → (**E Op E**)

(3) **Op** → **+**

(4) **Op** → *****

| | |
|-------------|-----------------------|
| E\$ | (int + (int * int))\$ |
| (E Op E)\$ | (int + (int * int))\$ |
| E Op E)\$ | int + (int * int))\$ |
| int Op E)\$ | int + (int * int))\$ |
| Op E)\$ | + (int * int))\$ |
| + E)\$ | + (int * int))\$ |
| E)\$ | (int * int))\$ |

| | | | | | |
|----|-----|---|---|---|---|
| | int | (|) | + | * |
| E | 1 | 2 | | | |
| Op | | | | 3 | 4 |

LL(1) Parsing

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| | |
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| E\$ | (int + (int * int))\$ |
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| E Op E)\$ | int + (int * int))\$ |
| int Op E)\$ | int + (int * int))\$ |
| Op E)\$ | + (int * int))\$ |
| + E)\$ | + (int * int))\$ |
| E)\$ | (int * int))\$ |
| (E Op E))\$ | (int * int))\$ |

| | | | | | |
|----|-----|---|---|---|---|
| | int | (|) | + | * |
| E | 1 | 2 | | | |
| Op | | | | 3 | 4 |

LL(1) Parsing

(1) **E** → **int**

(2) **E** → (**E Op E**)

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(4) **Op** → *****

| | int | (|) | + | * |
|----|-----|---|---|---|---|
| E | 1 | 2 | | | |
| Op | | | | 3 | 4 |

| | |
|-------------|-----------------------|
| E\$ | (int + (int * int))\$ |
| (E Op E)\$ | (int + (int * int))\$ |
| E Op E)\$ | int + (int * int))\$ |
| int Op E)\$ | int + (int * int))\$ |
| Op E)\$ | + (int * int))\$ |
| + E)\$ | + (int * int))\$ |
| E)\$ | (int * int))\$ |
| (E Op E))\$ | (int * int))\$ |
| E Op E))\$ | int * int))\$ |

LL(1) Parsing

(1) **E** → **int**

(2) **E** → (**E Op E**)

(3) **Op** → **+**

(4) **Op** → *****

| | int | (|) | + | * |
|----|-----|---|---|---|---|
| E | 1 | 2 | | | |
| Op | | | | 3 | 4 |

| | |
|-------------|-----------------------|
| E\$ | (int + (int * int))\$ |
| (E Op E)\$ | (int + (int * int))\$ |
| E Op E)\$ | int + (int * int))\$ |
| int Op E)\$ | int + (int * int))\$ |
| Op E)\$ | + (int * int))\$ |
| + E)\$ | + (int * int))\$ |
| E)\$ | (int * int))\$ |
| (E Op E))\$ | (int * int))\$ |
| E Op E))\$ | int * int))\$ |

LL(1) Parsing

(1) **E** → **int**

(2) **E** → (**E Op E**)

(3) **Op** → **+**

(4) **Op** → *****

| | int | (|) | + | * |
|----|-----|---|---|---|---|
| E | 1 | 2 | | | |
| Op | | | | 3 | 4 |

| | |
|--------------|-----------------------|
| E\$ | (int + (int * int))\$ |
| (E Op E)\$ | (int + (int * int))\$ |
| E Op E)\$ | int + (int * int))\$ |
| int Op E)\$ | int + (int * int))\$ |
| Op E)\$ | + (int * int))\$ |
| + E)\$ | + (int * int))\$ |
| E)\$ | (int * int))\$ |
| (E Op E))\$ | (int * int))\$ |
| E Op E))\$ | int * int))\$ |
| int Op E))\$ | int * int))\$ |

LL(1) Parsing

(1) **E** → **int**

(2) **E** → (**E Op E**)

(3) **Op** → **+**

(4) **Op** → *****

| | int | (|) | + | * |
|----|-----|---|---|---|---|
| E | 1 | 2 | | | |
| Op | | | | 3 | 4 |

| | |
|--------------|-----------------------|
| E\$ | (int + (int * int))\$ |
| (E Op E)\$ | (int + (int * int))\$ |
| E Op E)\$ | int + (int * int))\$ |
| int Op E)\$ | int + (int * int))\$ |
| Op E)\$ | + (int * int))\$ |
| + E)\$ | + (int * int))\$ |
| E)\$ | (int * int))\$ |
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| E Op E))\$ | int * int))\$ |
| int Op E))\$ | int * int))\$ |
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LL(1) Parsing

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(4) **Op** → *****

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| Op | | | | 3 | 4 |

| | |
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LL(1) Parsing

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| | * int))\$ |
| | * int))\$ |
| | int))\$ |
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|)\$ |)\$ |
|)\$ |)\$ |
| \$ | \$ |

The Limits of LL(1)

A Grammar that is Not LL(1)

- Consider the following (left-recursive) grammar:

$$A \rightarrow Ab \mid c$$

- $\text{FIRST}(A) = \{c\}$
- However, we cannot build an LL(1) parse table.
- **Why?**

A Grammar that is Not LL(1)

- Consider the following (left-recursive) grammar:

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- $\text{FIRST}(A) = \{c\}$
- However, we cannot build an LL(1) parse table.
- Why?**

| | b | c |
|---|---|---|
| A | | $A \rightarrow Ab$ $A \rightarrow c$ |

A Grammar that is Not LL(1)

- Consider the following (left-recursive) grammar:

$$A \rightarrow Ab \mid c$$

- $\text{FIRST}(A) = \{c\}$
- However, we cannot build an LL(1) parse table.
- Why?**

| | b | c |
|---|---|---|
| A | | $A \rightarrow Ab$ $A \rightarrow c$ |

- Cannot uniquely predict production!**
- This is called a **FIRST/FIRST conflict**.

The Strengths of LL(1)

LL(1) is Straightforward

- Can be implemented quickly with a table-driven design.
- Can be implemented by **recursive descent**:
 - Define a function for each nonterminal.
 - Have these functions call each other based on the lookahead token.

LL(1) is Fast

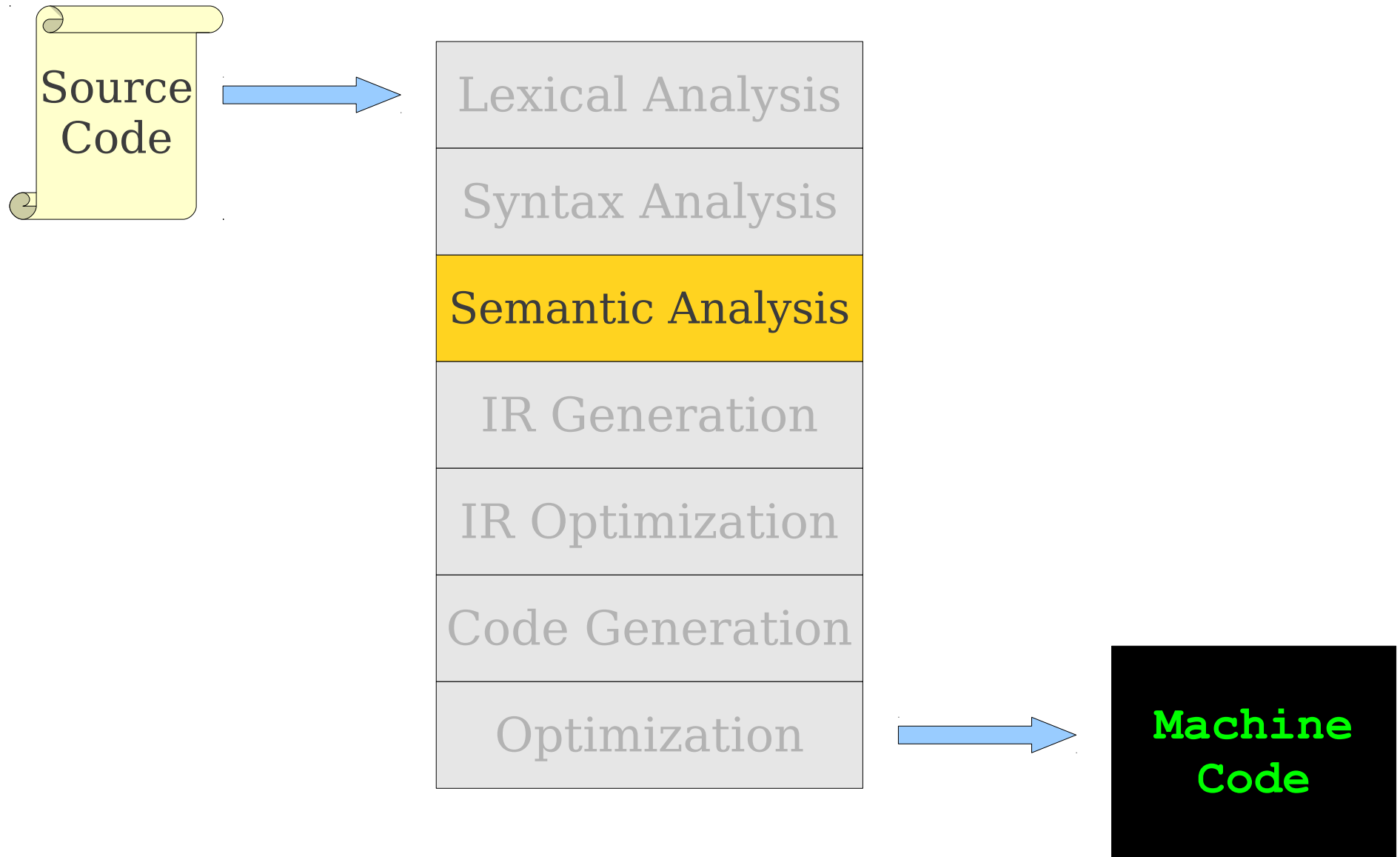
- Both table-driven LL(1) and recursive-descent-powered LL(1) are fast.
- Can parse in $O(n |G|)$ time, where n is the length of the string and $|G|$ is the size of the grammar.

Summary

- **Top-down parsing** tries to derive the user's program from the start symbol.
- **Leftmost BFS** is one approach to top-down parsing; it is mostly of theoretical interest.
- **Leftmost DFS** is another approach to top-down parsing that is uncommon in practice.
- **LL(1)** parsing scans from left-to-right, using one token of lookahead to find a leftmost derivation.
- **Left recursion** and **left factorability** cause LL(1) to fail and can be mechanically eliminated in some cases.

Semantic Analysis

Where We Are



Where We Are

- Program is *lexically* well-formed:
 - Identifiers have valid names.
 - Strings are properly terminated.
 - No stray characters.
- Program is *syntactically* well-formed:
 - Class declarations have the correct structure.
 - Expressions are syntactically valid.
- Does this mean that the program is legal?

Semantic Analysis

- Ensure that the program has a well-defined **meaning**.
- Verify properties of the program that aren't caught during the earlier phases:
 - Variables are declared before they're used.
 - Expressions have the right types.
 - Classes don't inherit from nonexistent base classes
 - ...
- Once we finish semantic analysis, we know that the user's input program is legal.

Challenges in Semantic Analysis

- Reject the largest number of incorrect programs.
- Accept the largest number of correct programs.


Validity versus Correctness

```
int Fibonacci(int n) {  
    if (n <= 1) return 0;  
  
    return Fibonacci(n - 1) + Fibonacci(n - 2);  
}  
  
int main() {  
    Print(Fibonacci(40));  
}
```

Validity versus Correctness

```
int Fibonacci(int n) {  
    if (n <= 1) return 0;  
  
    return Fibonacci(n - 1) + Fibonacci(n - 2);  
}  
  
int main() {  
    Print(Fibonacci(40));  
}
```

Incorrect,
should be
"return n;"

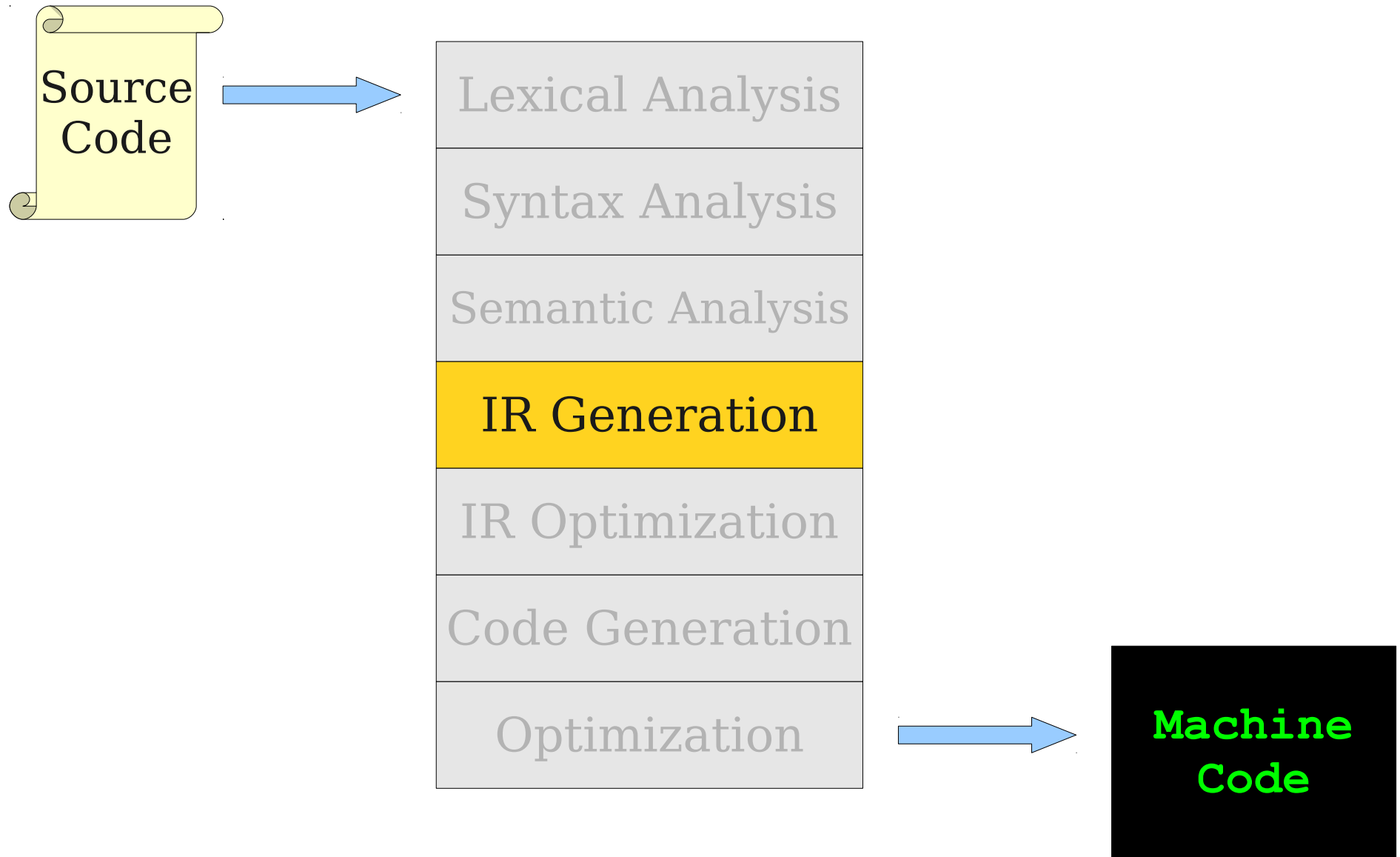


Challenges in Semantic Analysis

- Reject the largest number of incorrect programs.
- Accept the largest number of correct programs.
- Do so quickly.

Three-Address Code IR

Where We Are



An Important Detail

- When generating IR at this level, you do **not** need to worry about optimizing it.
- It's okay to generate IR that has lots of unnecessary assignments, redundant computations, etc.
- We'll see how to optimize IR code later.
 - It's tricky, but extremely cool!

Temporary Variables

- The “three” in “three-address code” refers to the number of operands in any instruction.
- Evaluating an expression with more than three subexpressions requires the introduction of temporary variables.
- This is actually a lot easier than you might think; we'll see how to do it later on.

Sample TAC Code

```
int a;
```

```
int b;
```

```
a = 5 + 2 * b;
```

Sample TAC Code

```
int a;  
int b;
```

```
a = 5 + 2 * b;
```

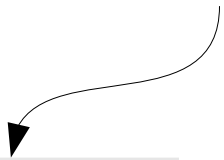
```
_t0 = 5;  
_t1 = 2 * b;  
a = _t0 + _t1;
```

Sample TAC Code

```
int a;  
int b;
```

```
a = 5 + 2 * b;
```

TAC allows for
instructions with two
operands.



```
_t0 = 5;  
_t1 = 2 * b;  
a = _t0 + _t1;
```

Simple TAC Instructions

- **Variable assignment** allows assignments of the form
 - $\text{var} = \text{constant};$
 - $\text{var}_1 = \text{var}_2;$
 - $\text{var}_1 = \text{var}_2 \text{ **op** } \text{var}_3;$
 - $\text{var}_1 = \text{constant} \text{ **op** } \text{var}_2;$
 - $\text{var}_1 = \text{var}_2 \text{ **op** } \text{constant};$
 - $\text{var} = \text{constant}_1 \text{ **op** } \text{constant}_2;$
- Permitted operators are $+$, $-$, $*$, $/$, $\%$.
- How would you compile $\mathbf{y} = -\mathbf{x};$?

Control Flow Statements

```
int x;
```

```
int y;
```

```
int z;
```

```
if (x < y)
```

```
    z = x;
```

```
else
```

```
    z = y;
```

```
z = z * z;
```

Control Flow Statements

```
int x;  
int y;  
int z;  
  
if (x < y)  
    z = x;  
else  
    z = y;  
  
z = z * z;
```

```
    _t0 = x < y;  
    IfZ _t0 Goto _L0;  
    z = x;  
    Goto _L1;  
_L0:  
    z = y;  
_L1:  
    z = z * z;
```

Control Flow Statements

```
int x;  
int y;  
int z;  
  
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    z = x;  
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```


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    Goto _L1;  
_L0:  
    z = y;  
_L1:  
    z = z * z;
```

Labels

- TAC allows for **named labels** indicating particular points in the code that can be jumped to.
- There are two control flow instructions:
 - `Goto label;`
 - `IfZ value Goto label;`
- Note that **IfZ** is always paired with **Goto**.

Control Flow Statements

```
int x;
```

```
int y;
```

```
while (x < y) {  
    x = x * 2;  
}
```

```
y = x;
```

Control Flow Statements

```
int x;
```

```
int y;
```

```
while (x < y) {  
    x = x * 2;  
}
```

```
y = x;
```

```
_L0:
```

```
    t0 = x < y;
```

```
    IfZ t0 Goto _L1;
```

```
    x = x * 2;
```

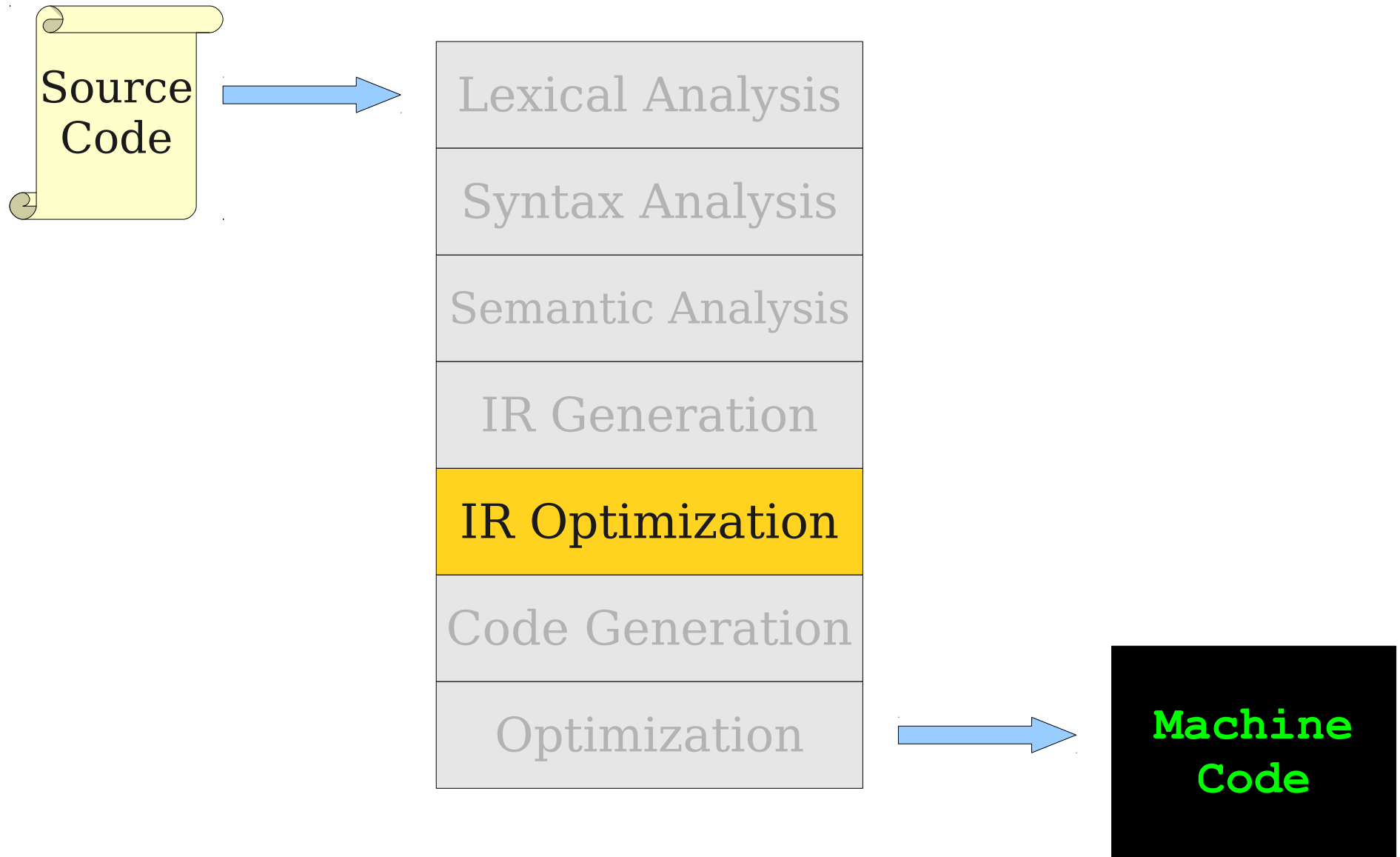
```
    Goto _L0;
```

```
_L1:
```

```
    y = x;
```

IR Optimization

Where We Are



IR Optimization

- **Goal:** Improve the IR generated by the previous step to take better advantage of resources.
- One of the most important and complex parts of any modern compiler.
- A very active area of research.
- There is a whole class (CS243) dedicated to this material.

Sources of Optimization

- In order to optimize our IR, we need to understand why it can be improved in the first place.
- **Reason one:** IR generation introduces redundancy.
 - A naïve translation of high-level language features into IR often introduces subcomputations.
 - Those subcomputations can often be sped up, shared, or eliminated.
- **Reason two:** Programmers are lazy.
 - Code executed inside of a loop can often be factored out of the loop.
 - Language features with side effects often used for purposes other than those side effects.

Optimizations from IR Generation

```
int x;  
int y;  
bool b1;  
bool b2;  
bool b3;
```

```
b1 = x + x < y  
b2 = x + x == y  
b3 = x + x > y
```

Optimizations from IR Generation

```
int x;  
int y;  
bool b1;  
bool b2;  
bool b3;
```

```
b1 = x + x < y  
b2 = x + x == y  
b3 = x + x > y
```

```
_t0 = x + x;  
_t1 = y;  
b1 = _t0 < _t1;
```

```
_t2 = x + x;  
_t3 = y;  
b2 = _t2 == _t3;
```

```
_t4 = x + x;  
_t5 = y;  
b3 = _t5 < _t4;
```

Optimizations from IR Generation

```
int x;  
int y;  
bool b1;  
bool b2;  
bool b3;
```

```
b1 = x + x < y  
b2 = x + x == y  
b3 = x + x > y
```

```
_t0 = x + x;  
_t1 = y;  
b1 = _t0 < _t1;
```

```
_t2 = x + x;  
_t3 = y;  
b2 = _t2 == _t3;
```

```
_t4 = x + x;  
_t5 = y;  
b3 = _t5 < _t4;
```

Optimizations from IR Generation

```
int x;  
int y;  
bool b1;  
bool b2;  
bool b3;  
  
b1 = x + x < y  
b2 = x + x == y  
b3 = x + x > y
```

```
_t0 = x + x;  
_t1 = y;  
b1 = _t0 < _t1;  
  
b2 = _t0 == _t1;  
  
b3 = _t0 > _t1;
```

```
while (x < y + z) {  
    x = x - y;  
}
```



Optimizations from Lazy Coders

```
while (x < y + z) {  
    x = x - y;  
}
```

```
_L0:  
    _t0 = y + z;  
    _t1 = x < _t0;  
    IfZ _t1 Goto _L1;  
    x = x - y;  
    Goto _L0;  
_L1:
```

Optimizations from Lazy Coders

```
while (x < y + z) {  
    x = x - y;  
}
```

```
_L0:  
    _t0 = y + z;  
    _t1 = x < _t0;  
    IfZ _t1 Goto _L1;  
    x = x - y;  
    Goto _L0;  
_L1:
```

Optimizations from Lazy Coders

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while (x < y + z) {  
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    _t0 = y + z;  
_L0:  
    _t1 = x < _t0;  
    IfZ _t1 Goto _L1;  
    x = x - y;  
    Goto _L0;  
_L1:
```


Optimizations from Lazy Coders

```
while (x < y + z) {  
    x = x - y;  
}
```

```
    _t0 = y + z;  
_L0:  
    _t1 = x < _t0;  
    IfZ _t1 Goto _L1;  
    x = x - y;  
    Goto _L0;  
_L1:
```

A Note on Terminology

- The term “optimization” implies looking for an “optimal” piece of code for a program.
- This is, in general, undecidable.
 - e.g. create a program that can be simplified iff some other program halts.
- Our goal will be IR *improvement* rather than IR *optimization*.

The Challenge of Optimization

- A good optimizer
 - Should never change the observable behavior of a program.
 - Should produce IR that is as efficient as possible.
 - Should not take too long to process inputs.
- Unfortunately:
 - Even good optimizers sometimes introduce bugs into code.
 - Optimizers often miss “easy” optimizations due to limitations of their algorithms.
 - Almost all interesting optimizations are **NP**-hard or undecidable.

What are we Optimizing?

- Optimizers can try to improve code usage with respect to many observable properties.
- What are some quantities we might want to optimize?

What are we Optimizing?

- Optimizers can try to improve code usage with respect to many observable properties.
- What are some quantities we might want to optimize?
- **Runtime** (make the program as fast as possible at the expense of time and power)
- **Memory usage** (generate the smallest possible executable at the expense of time and power)
- **Power consumption** (choose simple instructions at the expense of speed and memory usage)
- Plus a lot more (minimize function calls, reduce use of floating-point hardware, etc.)

Overview of IR Optimization

- **Formalisms and Terminology**
 - Control-flow graphs.
 - Basic blocks.
- **Local optimizations**
 - Speeding up small pieces of a function.
- **Global optimizations**
 - Speeding up functions as a whole.

Formalisms and Terminology

Analyzing a Program

- In order to optimize a program, the compiler has to be able to reason about the properties of that program.
- An analysis is called **sound** if it never asserts an incorrect fact about a program.
- All the analyses we will discuss in this class are sound.
 - *(Why?)*

Soundness


```
int x;  
int y;  
  
if (y < 5)  
    x = 137;  
else  
    x = 42;  
  
Print(x);
```

Soundness

```
int x;  
int y;  
  
if (y < 5)  
    x = 137;  
else  
    x = 42;  
  
Print (x) ;
```

Soundness

```
int x;  
int y;  
  
if (y < 5)  
    x = 137;  
else  
    x = 42;  
Print (x) ;
```



“At this point in the program, **x** holds some integer value.”

Soundness

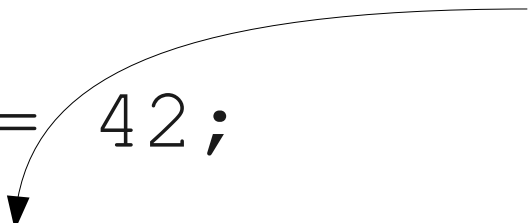
```
int x;  
int y;
```

```
if (y < 5)  
    x = 137;
```

```
else  
    x = 42;
```

```
Print (x) ;
```

“At this point in the
program, **x** is either 137
or 42”



Semantics-Preserving Optimizations

- An optimization is **semantics-preserving** if it does not alter the semantics of the original program.
- Examples:
 - Eliminating unnecessary temporary variables.
 - Computing values that are known statically at compile-time instead of runtime.
 - Evaluating constant expressions outside of a loop instead of inside.
- Non-examples:
 - Replacing bubble sort with quicksort.
- The optimizations we will consider in this class are all semantics-preserving.

A Formalism for IR Optimization

- Every phase of the compiler uses some new abstraction:
 - Scanning uses regular expressions.
 - Parsing uses CFGs.
 - Semantic analysis uses proof systems and symbol tables.
 - IR generation uses ASTs.
- In optimization, we need a formalism that captures the structure of a program in a way amenable to optimization.

Visualizing IR

```
main:
    BeginFunc 40;
    _tmp0 = LCall _ReadInteger;
    a = _tmp0;
    _tmp1 = LCall _ReadInteger;
    b = _tmp1;
_L0:
    _tmp2 = 0;
    _tmp3 = b == _tmp2;
    _tmp4 = 0;
    _tmp5 = _tmp3 == _tmp4;
    IfZ _tmp5 Goto _L1;
    c = a;
    a = b;
    _tmp6 = c % a;
    b = _tmp6;
    Goto _L0;
_L1:
    PushParam a;
    LCall _PrintInt;
    PopParams 4;
    EndFunc;
```

Visualizing IR

```
main:
  BeginFunc 40;
  _tmp0 = LCall _ReadInteger;
  a = _tmp0;
  _tmp1 = LCall _ReadInteger;
  b = _tmp1;
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  _tmp2 = 0;
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  IfZ _tmp5 Goto _L1;
  c = a;
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  _tmp6 = c % a;
  b = _tmp6;
  Goto _L0;
_L1:
  PushParam a;
  LCall _PrintInt;
  PopParams 4;
  EndFunc;
```


Visualizing IR

main:

BeginFunc 40;

_tmp0 = LCall _ReadInteger;

a = _tmp0;

_tmp1 = LCall _ReadInteger;

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_L0:

_tmp2 = 0;

_tmp3 = b == _tmp2;

_tmp4 = 0;

_tmp5 = _tmp3 == _tmp4;

IfZ _tmp5 Goto _L1;

c = a;

a = b;

_tmp6 = c % a;

b = _tmp6;

Goto _L0;

_L1:

PushParam a;

LCall _PrintInt;

PopParams 4;

EndFunc;

```
_tmp0 = LCall _ReadInteger;  
a = _tmp0 ;  
_tmp1 = LCall _ReadInteger;  
b = _tmp1 ;
```

```
_tmp2 = 0 ;  
_tmp3 = b == _tmp2 ;  
_tmp4 = 0 ;  
_tmp5 = _tmp3 == _tmp4 ;  
IfZ _tmp5 Goto _L1 ;
```

```
c = a ;  
a = b ;  
_tmp6 = c % a ;  
b = _tmp6 ;  
Goto _L0 ;
```

```
PushParam a ;  
LCall _PrintInt ;  
PopParams 4 ;
```

Visualizing IR

main:

BeginFunc 40;

_tmp0 = LCall _ReadInteger;

a = _tmp0;

_tmp1 = LCall _ReadInteger;

b = _tmp1;

_L0:

_tmp2 = 0;

_tmp3 = b == _tmp2;

_tmp4 = 0;

_tmp5 = _tmp3 == _tmp4;

IfZ _tmp5 Goto _L1;

c = a;

a = b;

_tmp6 = c % a;

b = _tmp6;

Goto _L0;

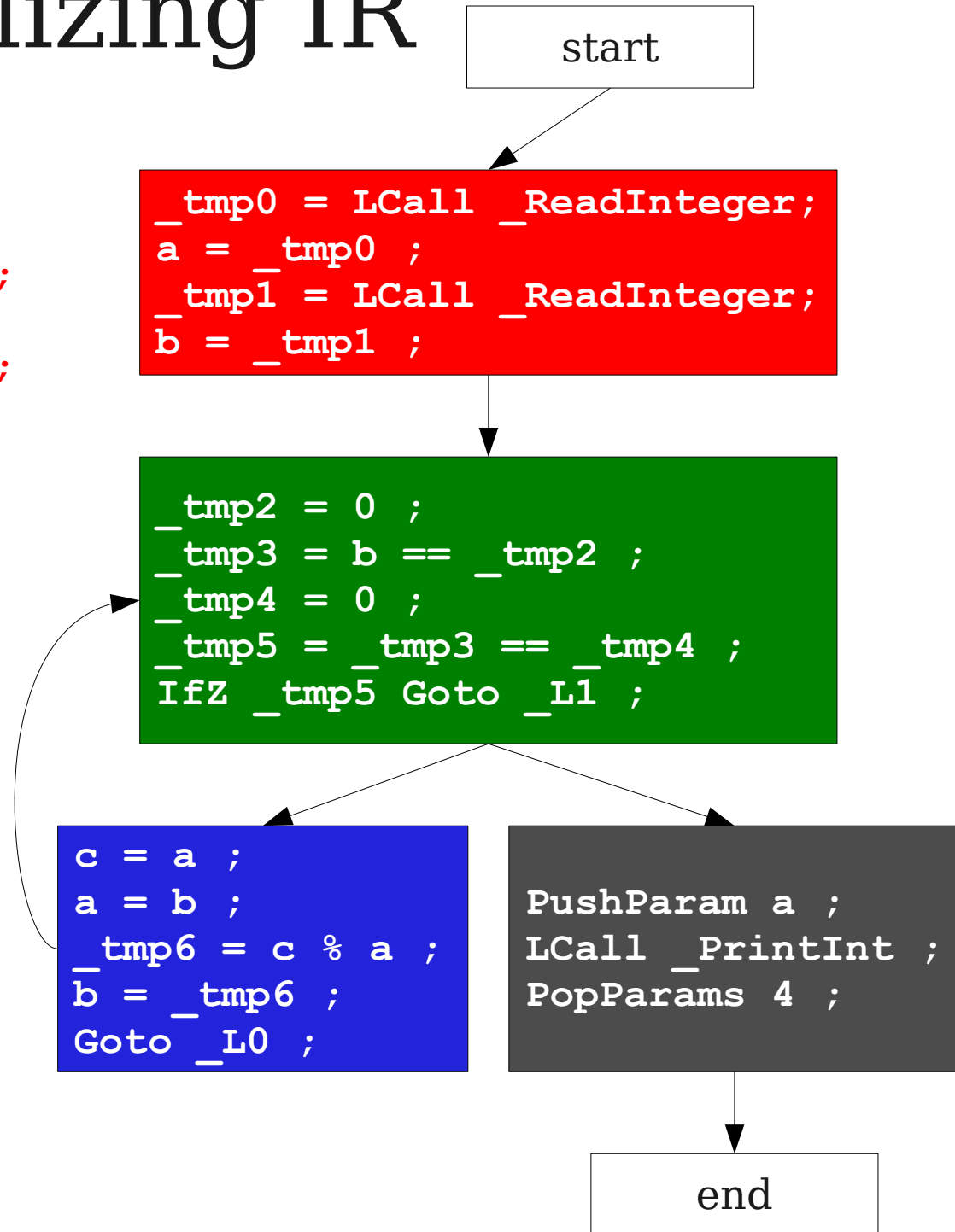
_L1:

PushParam a;

LCall _PrintInt;

PopParams 4;

EndFunc;



Basic Blocks

- A **basic block** is a sequence of IR instructions where
 - There is exactly one spot where control enters the sequence, which must be at the start of the sequence.
 - There is exactly one spot where control leaves the sequence, which must be at the end of the sequence.
- Informally, a sequence of instructions that always execute as a group.

Control-Flow Graphs

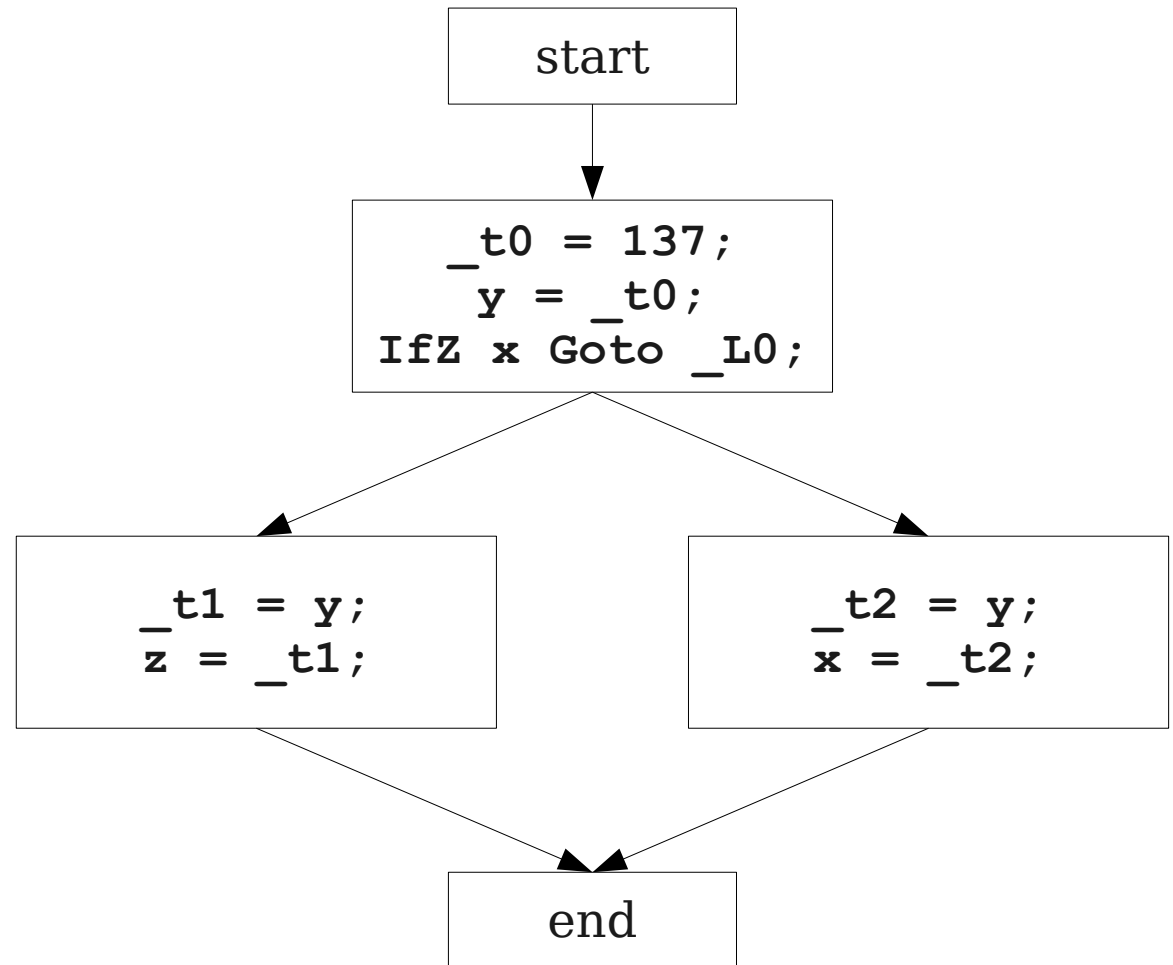
- A **control-flow graph** (CFG) is a graph of the basic blocks in a function.
 - The term CFG is overloaded – from here on out, we'll mean “control-flow graph” and not “context-free grammar.”
- Each edge from one basic block to another indicates that control can flow from the end of the first block to the start of the second block.
- There is a dedicated node for the start and end of a function.

Types of Optimizations

- An optimization is **local** if it works on just a single basic block.
- An optimization is **global** if it works on an entire control-flow graph.
- An optimization is **interprocedural** if it works across the control-flow graphs of multiple functions.
 - We won't talk about this in this course.

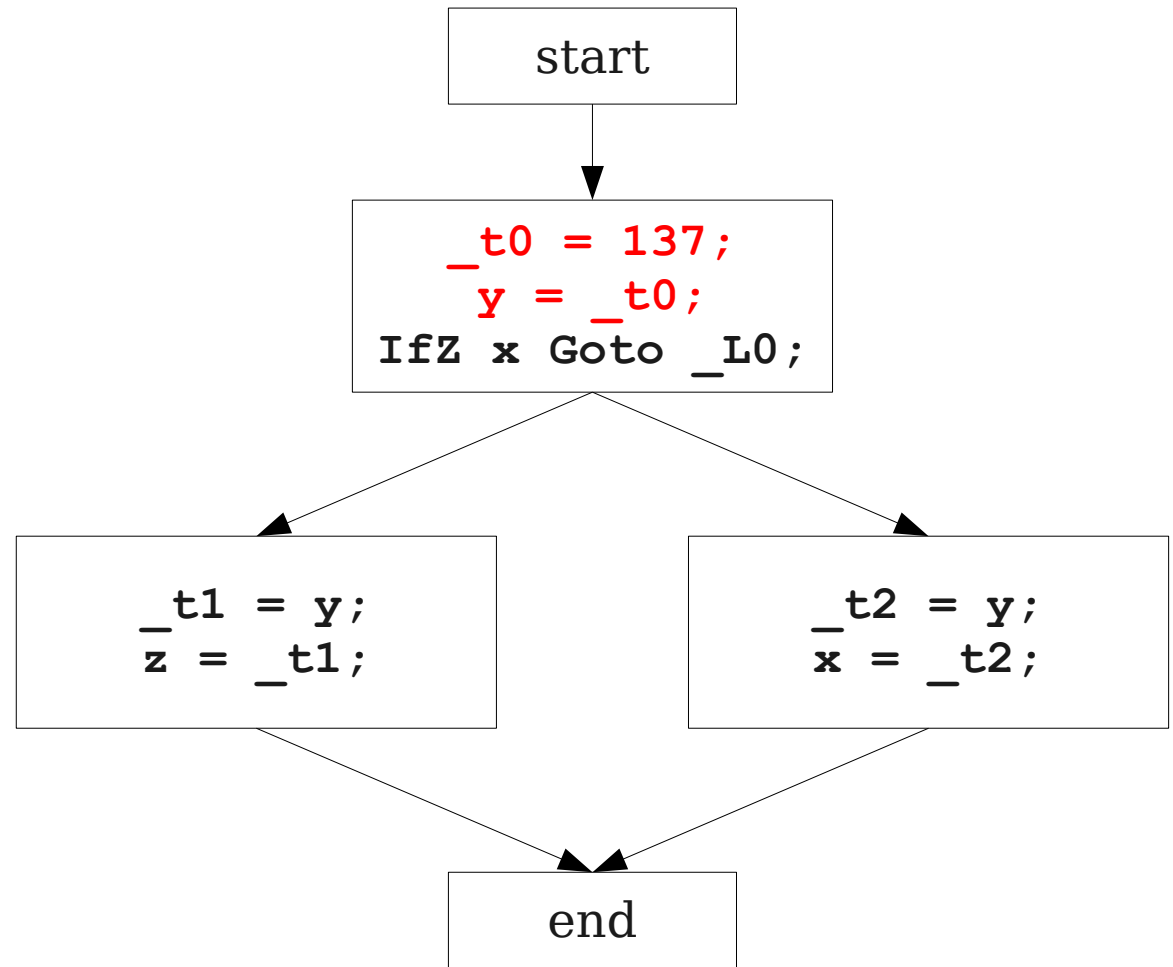
Local Optimizations

```
int main() {  
    int x;  
    int y;  
    int z;  
  
    y = 137;  
    if (x == 0)  
        z = y;  
    else  
        x = y;  
}
```



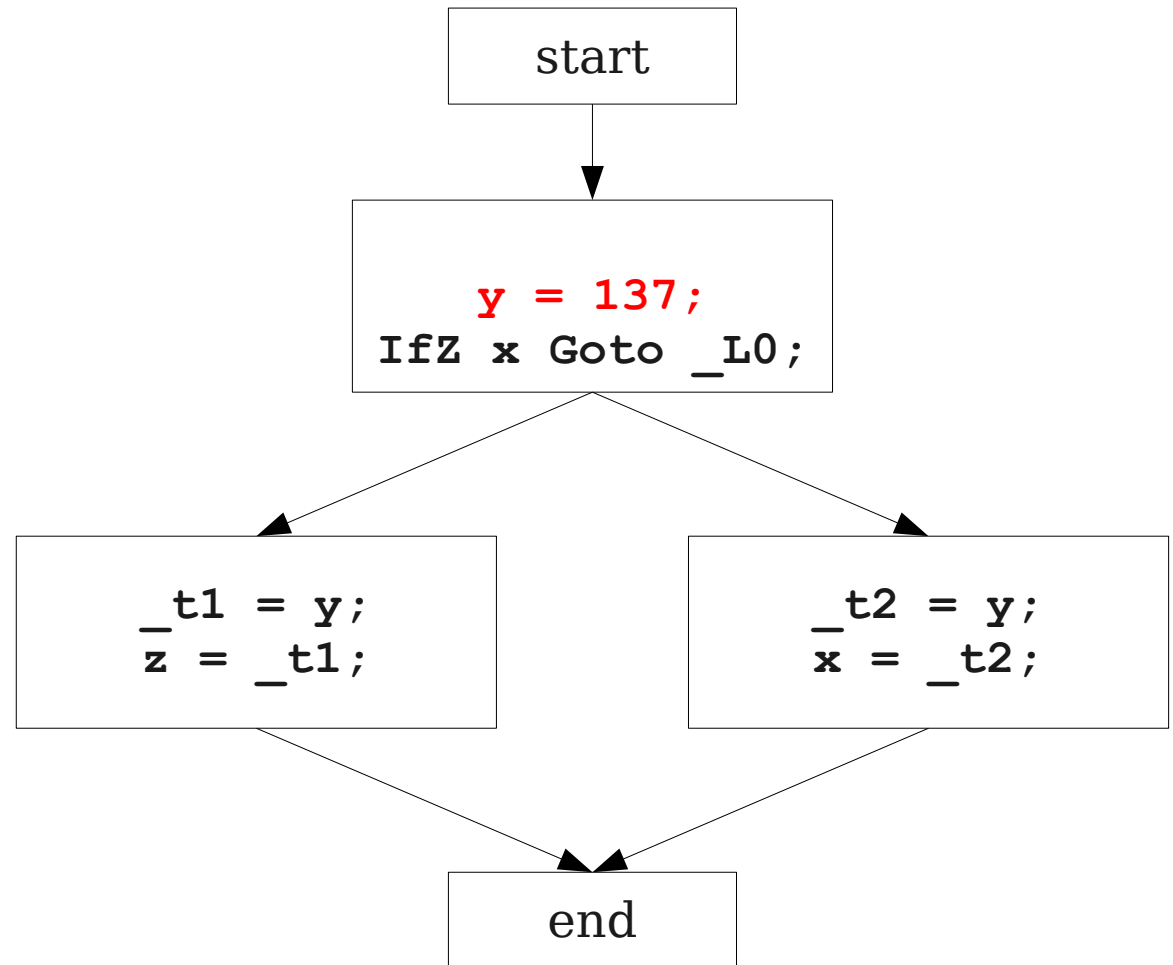
Local Optimizations

```
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    int x;  
    int y;  
    int z;  
  
    y = 137;  
    if (x == 0)  
        z = y;  
    else  
        x = y;  
}
```



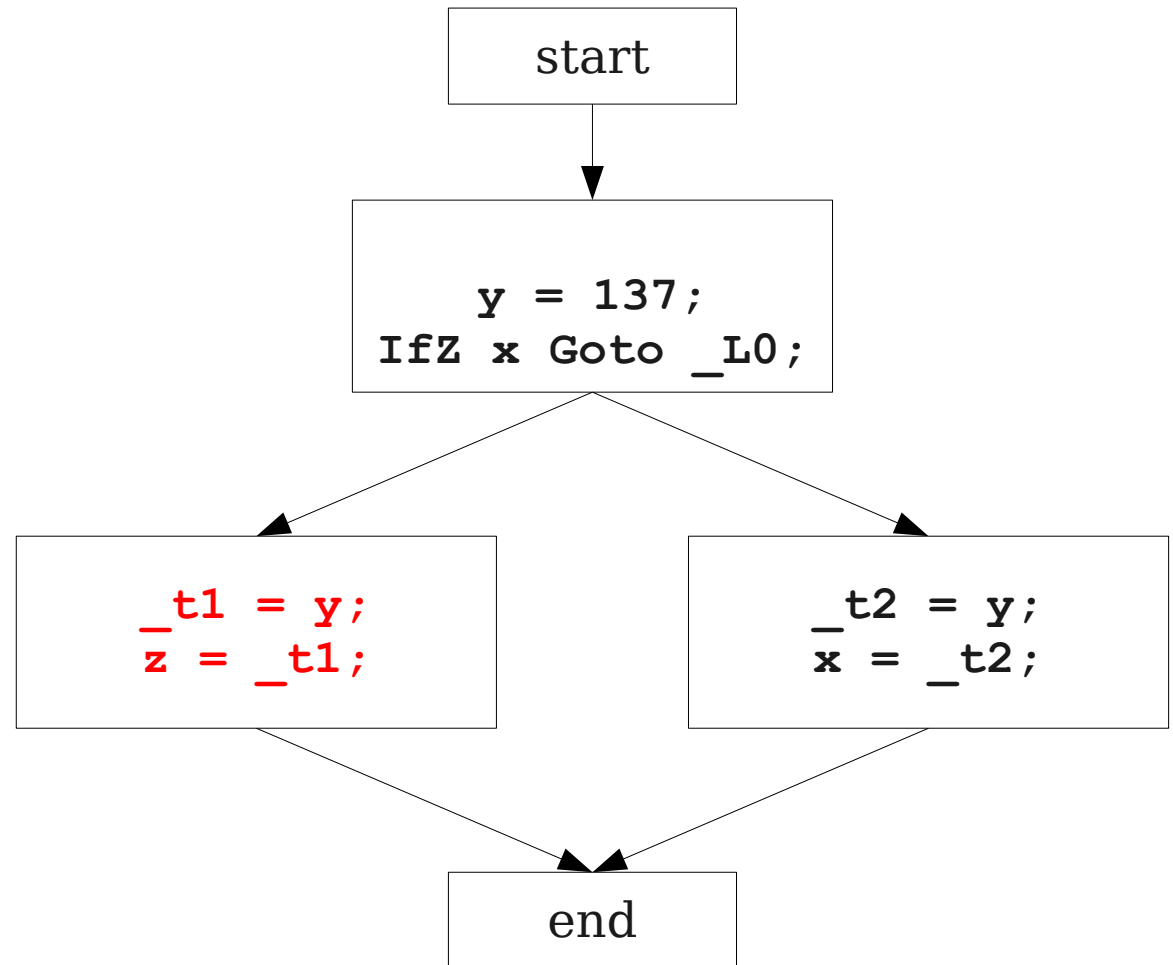
Local Optimizations

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    y = 137;  
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```



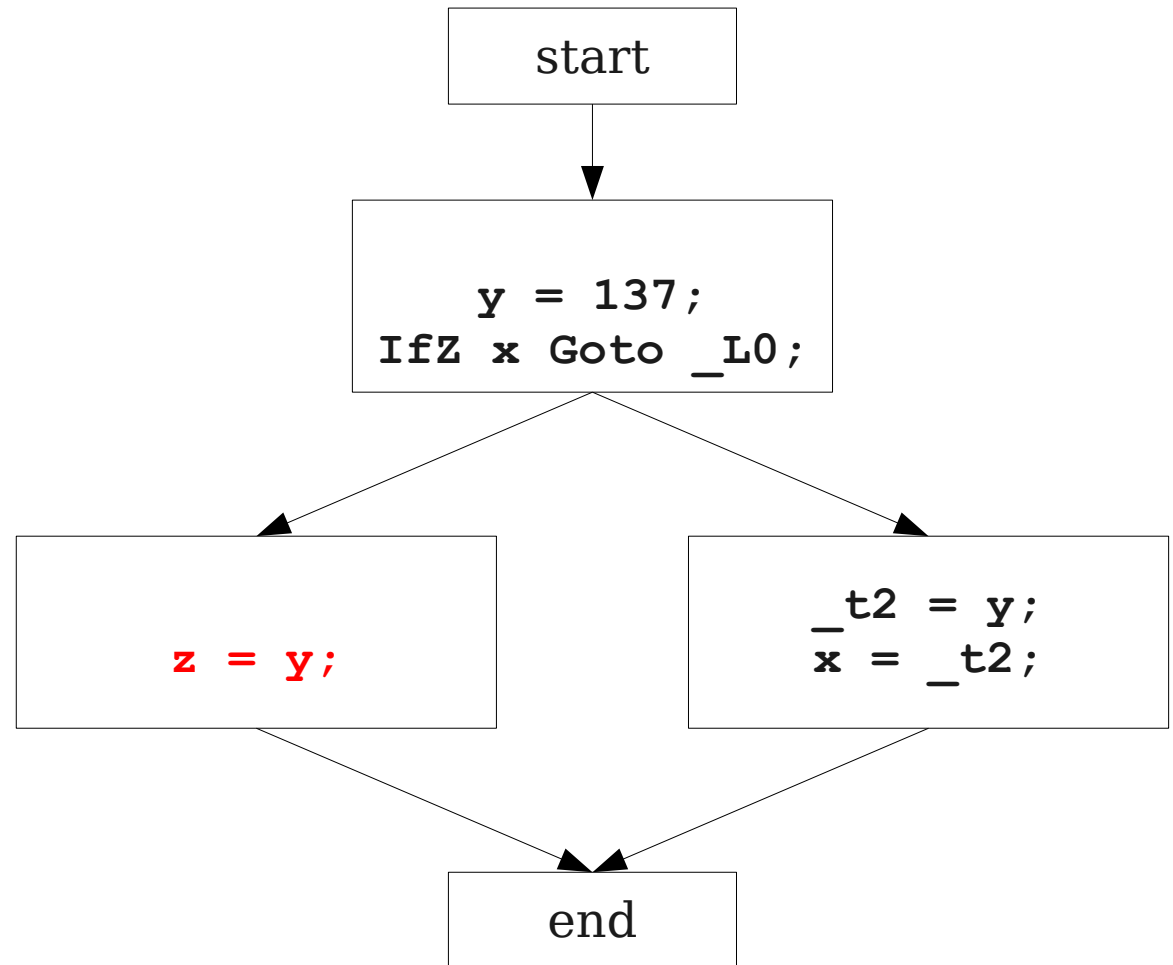
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}
```



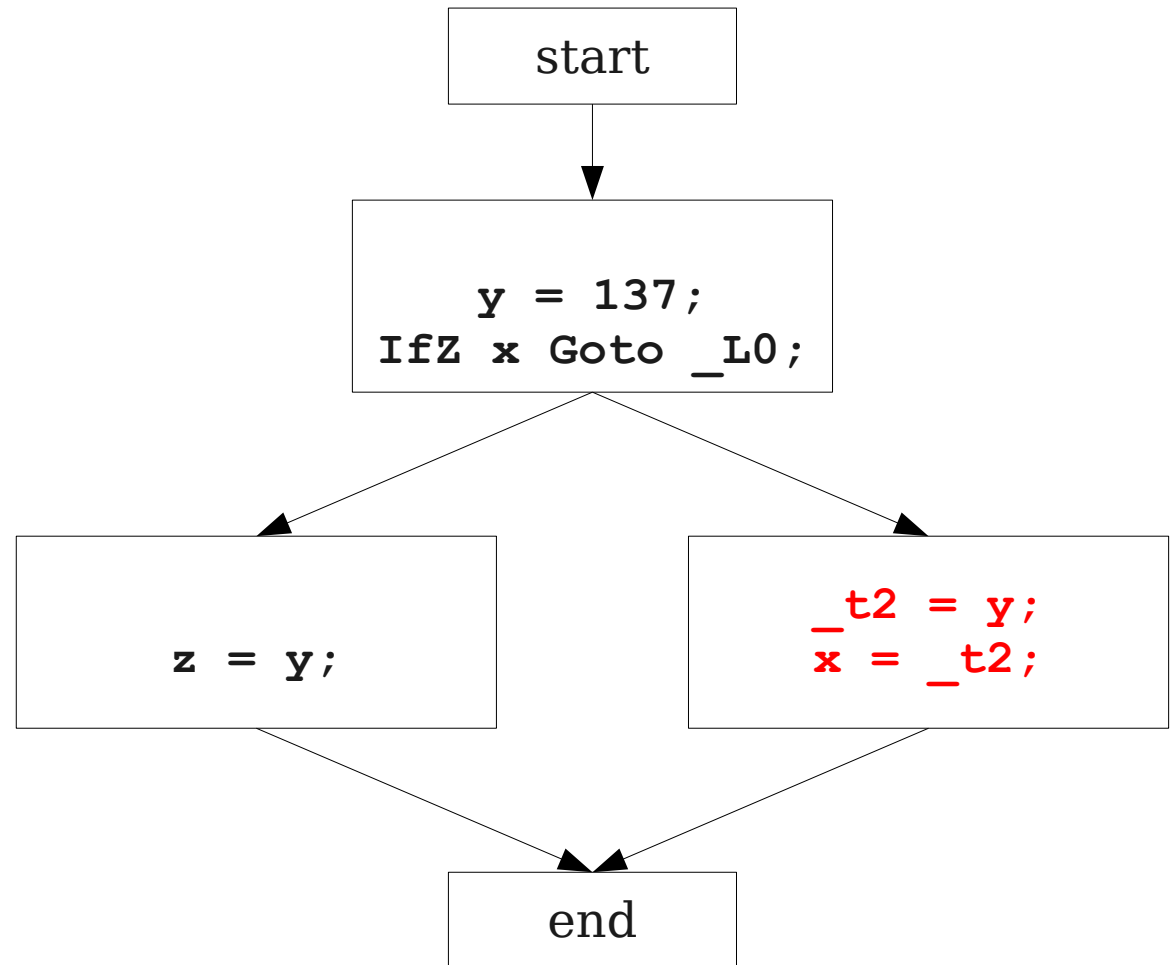
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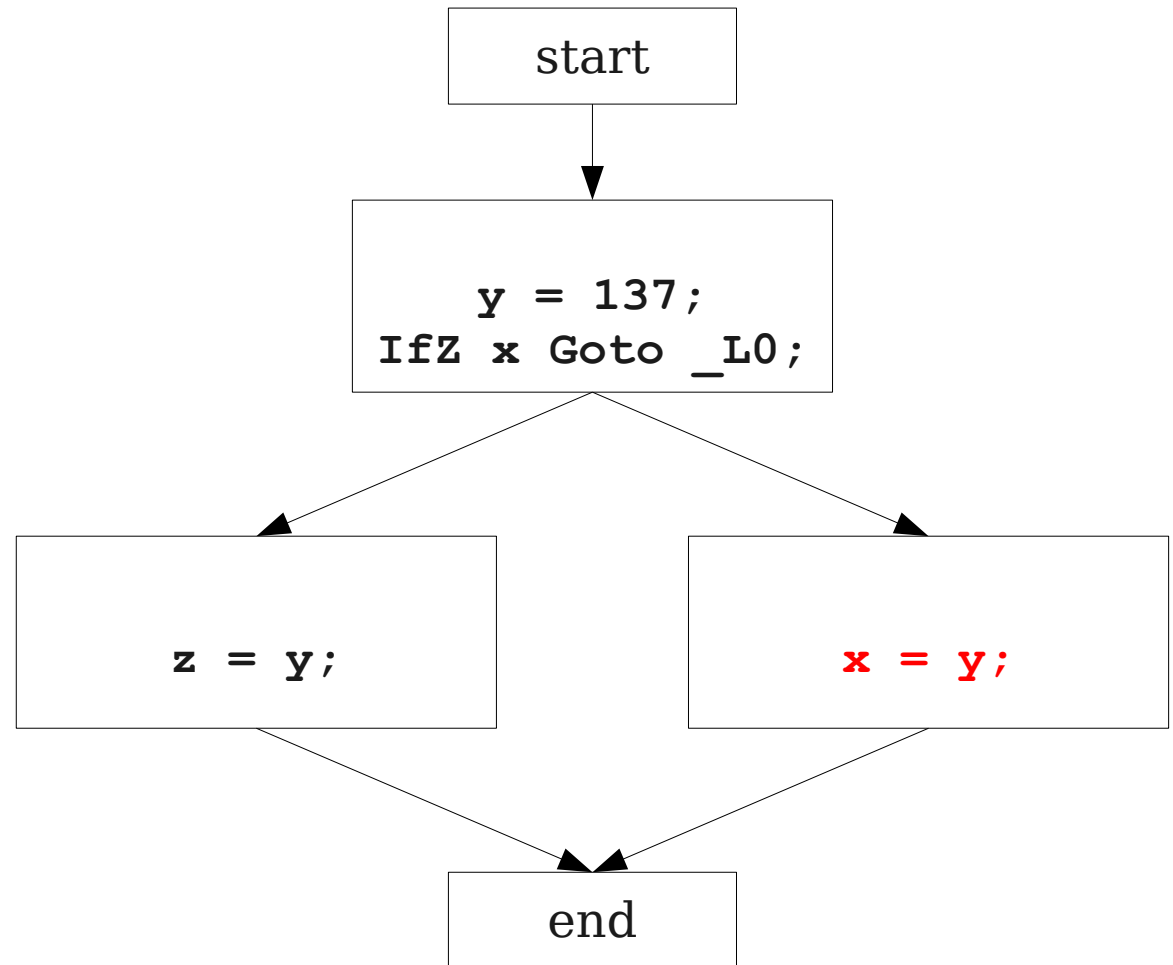
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    if (x == 0)  
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        x = y;  
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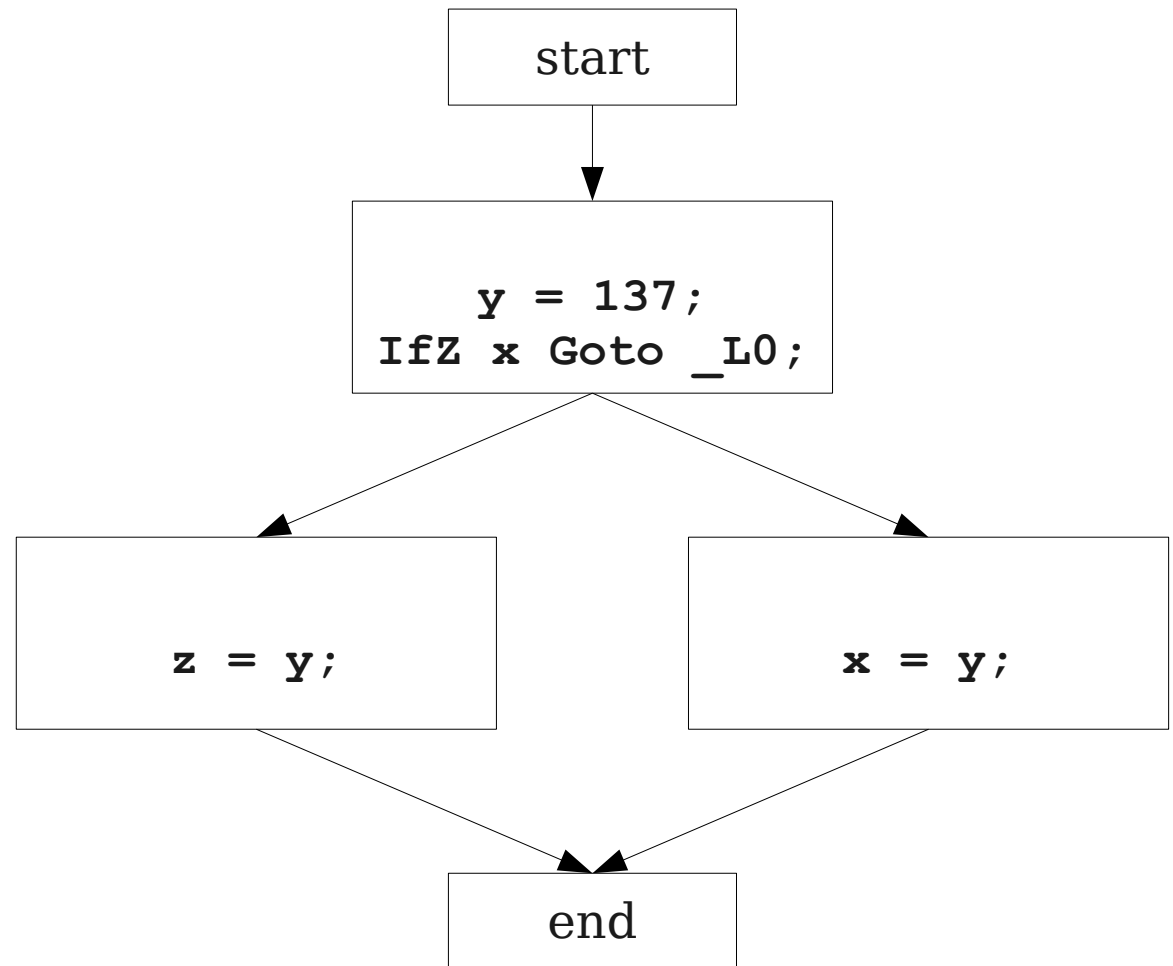
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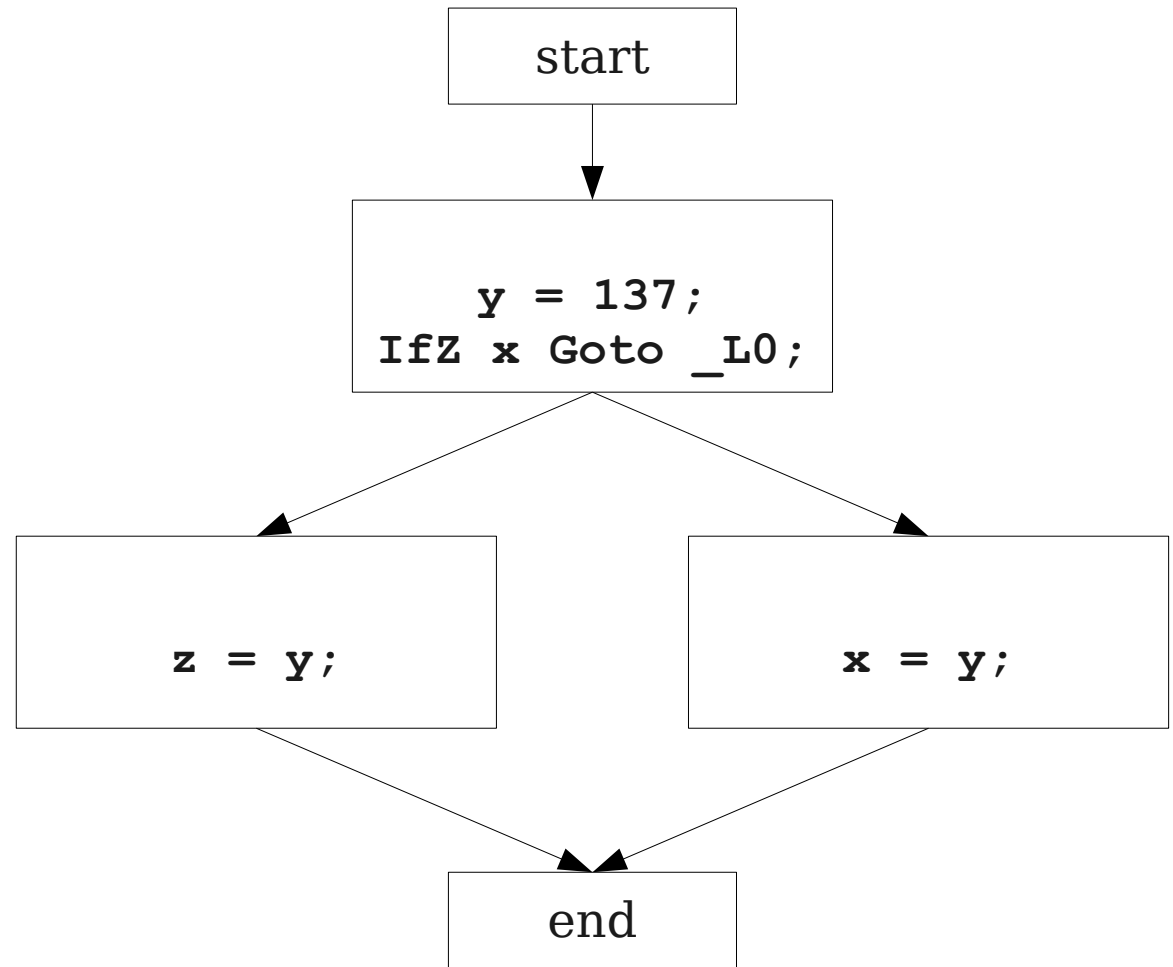
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```



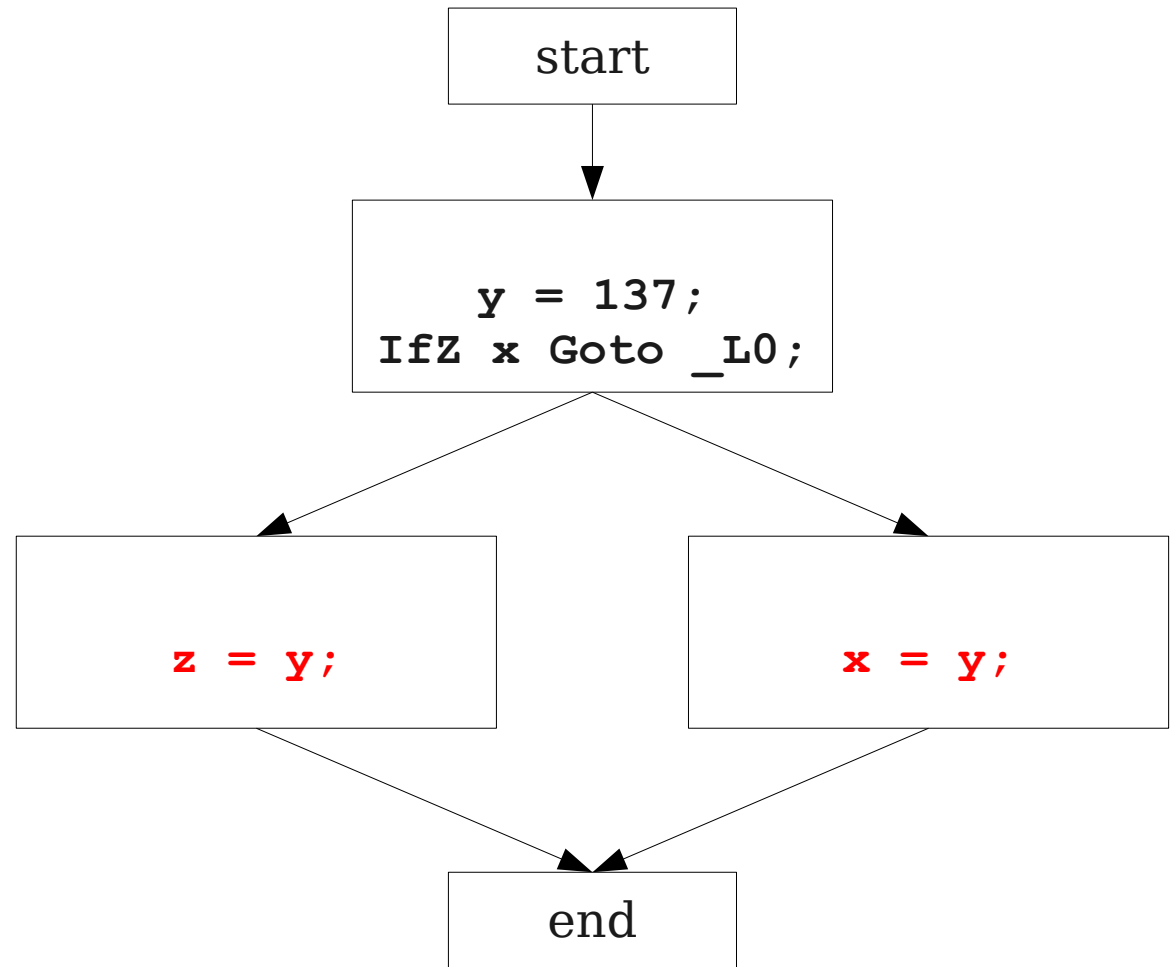
Global Optimizations

```
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    int x;  
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    int z;  
  
    y = 137;  
    if (x == 0)  
        z = y;  
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        x = y;  
}
```



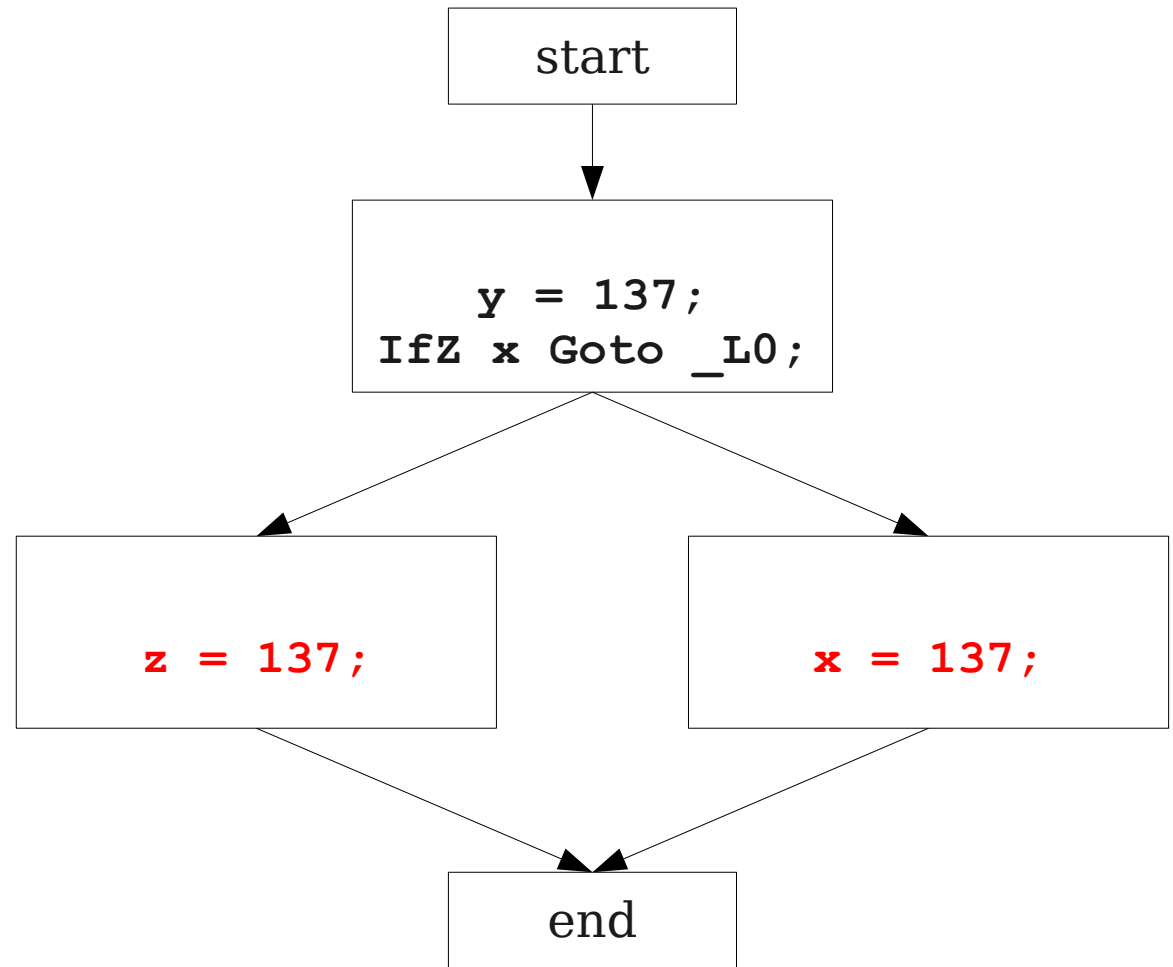
Global Optimizations

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    int x;  
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    int z;  
  
    y = 137;  
    if (x == 0)  
        z = y;  
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}
```



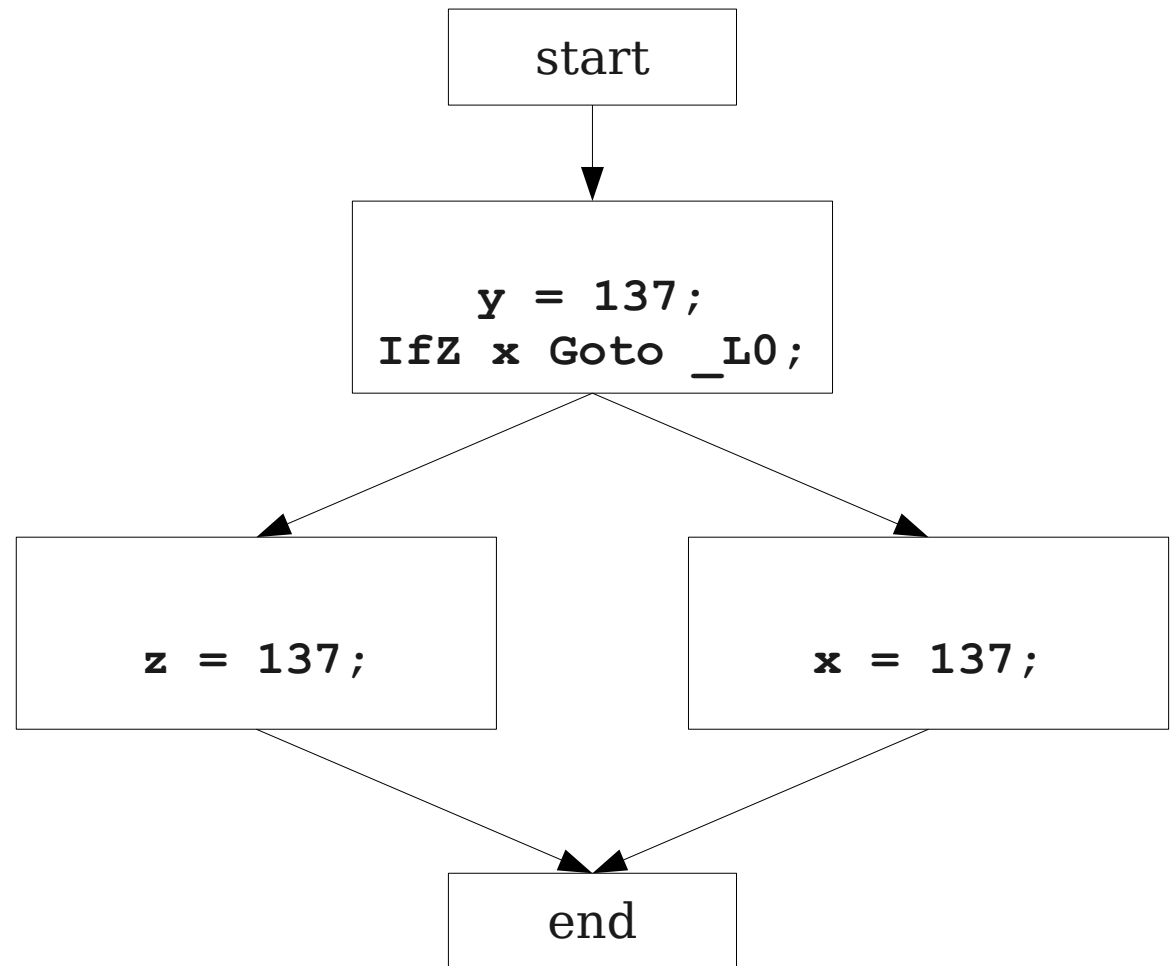
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```
int main() {  
    int x;  
    int y;  
    int z;  
  
    y = 137;  
    if (x == 0)  
        z = y;  
    else  
        x = y;  
}
```



Local Optimizations

Common Subexpression Elimination

```
Object x;
```

```
int a;
```

```
int b;
```

```
int c;
```

```
x = new Object;
```

```
a = 4;
```

```
c = a + b;
```

```
x.fn(a + b);
```

Common Subexpression Elimination

```
Object x;
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x = new Object;
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```
a = 4;
```

```
c = a + b;
```

```
x.fn(a + b);
```

```
_tmp0 = 4 ;
```

```
PushParam _tmp0 ;
```

```
_tmp1 = LCall _Alloc ;
```

```
PopParams 4 ;
```

```
_tmp2 = Object ;
```

```
*(_tmp1) = _tmp2 ;
```

```
x = _tmp1 ;
```

```
_tmp3 = 4 ;
```

```
a = _tmp3 ;
```

```
_tmp4 = a + b ;
```

```
c = _tmp4 ;
```

```
_tmp5 = a + b ;
```

```
_tmp6 = *(x) ;
```

```
_tmp7 = *(_tmp6) ;
```

```
PushParam _tmp5 ;
```

```
PushParam x ;
```

```
ACall _tmp7 ;
```

```
PopParams 8 ;
```

Common Subexpression Elimination

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Object x;  
int a;  
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_tmp3 = 4 ;  
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Common Subexpression Elimination

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Object x;  
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x.fn(a + b);
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_tmp6 = *(x) ;  
_tmp7 = *(_tmp6) ;  
PushParam _tmp5 ;  
PushParam x ;  
ACall _tmp7 ;  
PopParams 8 ;
```

Common Subexpression Elimination

- If we have two variable assignments

$\mathbf{v_1 = a \ op \ b}$

...

$\mathbf{v_2 = a \ op \ b}$

and the values of $\mathbf{v_1}$, \mathbf{a} , and \mathbf{b} have not changed between the assignments, rewrite the code as

$\mathbf{v_1 = a \ op \ b}$

...

$\mathbf{v_2 = v_1}$

- Eliminates useless recalculation.
- Paves the way for later optimizations.

Copy Propagation

```
Object x;  
int a;  
int b;  
int c;  
  
x = new Object;  
a = 4;  
c = a + b;  
x.fn(a + b);
```

```
_tmp0 = 4 ;  
PushParam _tmp0 ;  
_tmp1 = LCall _Alloc ;  
PopParams 4 ;  
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*(_tmp1) = _tmp2 ;  
x = _tmp1 ;  
_tmp3 = _tmp0 ;  
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_tmp4 = a + b ;  
c = _tmp4 ;  
_tmp5 = c ;  
_tmp6 = *(x) ;  
_tmp7 = *(_tmp6) ;  
PushParam _tmp5 ;  
PushParam x ;  
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PopParams 8 ;
```

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_tmp5 = c ;  
_tmp6 = *(_tmp1) ;  
_tmp7 = *(_tmp6) ;  
PushParam _tmp5 ;  
PushParam _tmp1 ;  
ACall _tmp7 ;  
PopParams 8 ;
```

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```
Object x;  
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_tmp4 = _tmp3 + b ;  
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_tmp3 = _tmp0 ;  
a = _tmp3 ;  
_tmp4 = _tmp3 + b ;  
c = _tmp4 ;  
_tmp5 = c ;  
_tmp6 = *(_tmp1) ;  
_tmp7 = *(_tmp6) ;  
PushParam c ;  
PushParam _tmp1 ;  
ACall _tmp7 ;  
PopParams 8 ;
```

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```
Object x;  
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c = _tmp4 ;  
_tmp5 = c ;  
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_tmp7 = *(_tmp6) ;  
PushParam c ;  
PushParam _tmp1 ;  
ACall _tmp7 ;  
PopParams 8 ;
```

Copy Propagation

```
Object x;  
int a;  
int b;  
int c;  
  
x = new Object;  
a = 4;  
c = a + b;  
x.fn(a + b);
```

```
_tmp0 = 4 ;  
PushParam _tmp0 ;  
_tmp1 = LCall _Alloc ;  
PopParams 4 ;  
_tmp2 = Object ;  
*(_tmp1) = _tmp2 ;  
x = _tmp1 ;  
_tmp3 = _tmp0 ;  
a = _tmp3 ;  
_tmp4 = _tmp3 + b ;  
c = _tmp4 ;  
_tmp5 = c ;  
_tmp6 = _tmp2 ;  
_tmp7 = *(_tmp6) ;  
PushParam c ;  
PushParam _tmp1 ;  
ACall _tmp7 ;  
PopParams 8 ;
```

Copy Propagation

```
Object x;  
int a;  
int b;  
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x = new Object;  
a = 4;  
c = a + b;  
x.fn(a + b);
```

```
_tmp0 = 4 ;  
PushParam _tmp0 ;  
_tmp1 = LCall _Alloc ;  
PopParams 4 ;  
_tmp2 = Object ;  
*(_tmp1) = _tmp2 ;  
x = _tmp1 ;  
_tmp3 = _tmp0 ;  
a = _tmp3 ;  
_tmp4 = _tmp3 + b ;  
c = _tmp4 ;  
_tmp5 = c ;  
_tmp6 = _tmp2 ;  
_tmp7 = *(_tmp2) ;  
PushParam c ;  
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Copy Propagation

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Object x;  
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x = new Object;  
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```
_tmp0 = 4 ;  
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PopParams 4 ;  
_tmp2 = Object ;  
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x = _tmp1 ;  
_tmp3 = _tmp0 ;  
a = _tmp3 ;  
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_tmp5 = c ;  
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ACall _tmp7 ;  
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Object x;  
int a;  
int b;  
int c;  
  
x = new Object;  
a = 4;  
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x.fn(a + b);
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PopParams 4 ;  
_tmp2 = Object ;  
*(_tmp1) = _tmp2 ;  
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_tmp3 = _tmp0 ;  
a = _tmp0 ;  
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c = _tmp4 ;  
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Copy Propagation

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int a;  
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int c;  
  
x = new Object;  
a = 4;  
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x.fn(a + b);
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_tmp0 = 4 ;  
PushParam _tmp0 ;  
_tmp1 = LCall _Alloc ;  
PopParams 4 ;  
_tmp2 = Object ;  
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x = _tmp1 ;  
_tmp3 = _tmp0 ;  
a = _tmp0 ;  
_tmp4 = _tmp0 + b ;  
c = _tmp4 ;  
_tmp5 = c ;  
_tmp6 = _tmp2 ;  
_tmp7 = *(_tmp2) ;  
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PushParam _tmp1 ;  
ACall _tmp7 ;  
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```

Copy Propagation

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Object x;  
int a;  
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x = new Object;  
a = 4;  
c = a + b;  
x.fn(a + b);
```

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_tmp0 = 4 ;  
PushParam _tmp0 ;  
_tmp1 = LCall _Alloc ;  
PopParams 4 ;  
_tmp2 = Object ;  
*(_tmp1) = _tmp2 ;  
x = _tmp1 ;  
_tmp3 = 4 ;  
a = 4 ;  
_tmp4 = _tmp0 + b ;  
c = _tmp4 ;  
_tmp5 = c ;  
_tmp6 = _tmp2 ;  
_tmp7 = *(_tmp2) ;  
PushParam c ;  
PushParam _tmp1 ;  
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PopParams 8 ;
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Copy Propagation

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Object x;  
int a;  
int b;  
int c;  
  
x = new Object;  
a = 4;  
c = a + b;  
x.fn(a + b);
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_tmp0 = 4 ;  
PushParam _tmp0 ;  
_tmp1 = LCall _Alloc ;  
PopParams 4 ;  
_tmp2 = Object ;  
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x = _tmp1 ;  
_tmp3 = 4 ;  
a = 4 ;  
_tmp4 = _tmp0 + b ;  
c = _tmp4 ;  
_tmp5 = c ;  
_tmp6 = _tmp2 ;  
_tmp7 = *(_tmp2) ;  
PushParam c ;  
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Copy Propagation

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_tmp0 = 4 ;  
PushParam _tmp0 ;  
_tmp1 = LCall _Alloc ;  
PopParams 4 ;  
_tmp2 = Object ;  
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x = _tmp1 ;  
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a = 4 ;  
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Copy Propagation

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Object x;  
int a;  
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x = new Object;  
a = 4;  
c = a + b;  
x.fn(a + b);
```

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_tmp0 = 4 ;  
PushParam _tmp0 ;  
_tmp1 = LCall _Alloc ;  
PopParams 4 ;  
_tmp2 = Object ;  
*(_tmp1) = _tmp2 ;  
x = _tmp1 ;  
_tmp3 = 4 ;  
a = 4 ;  
_tmp4 = _tmp0 + b ;  
c = _tmp4 ;  
_tmp5 = _tmp4 ;  
_tmp6 = _tmp2 ;  
_tmp7 = *(_tmp2) ;  
PushParam _tmp4 ;  
PushParam _tmp1 ;  
ACall _tmp7 ;  
PopParams 8 ;
```

Copy Propagation

- If we have a variable assignment

$$v_1 = v_2$$

then as long as v_1 and v_2 are not reassigned,
we can rewrite expressions of the form

$$a = \dots v_1 \dots$$

as

$$a = \dots v_2 \dots$$

provided that such a rewrite is legal.

- This will help immensely later on, as you'll see.

Dead Code Elimination

```
Object x;  
int a;  
int b;  
int c;  
  
x = new Object;  
a = 4;  
c = a + b;  
x.fn(a + b);
```

```
_tmp0 = 4 ;  
PushParam _tmp0 ;  
_tmp1 = LCall _Alloc ;  
PopParams 4 ;  
_tmp2 = Object ;  
*(_tmp1) = _tmp2 ;  
x = _tmp1 ;  
_tmp3 = 4 ;  
a = 4 ;  
_tmp4 = _tmp0 + b ;  
c = _tmp4 ;  
_tmp5 = _tmp4 ;  
_tmp6 = _tmp2 ;  
_tmp7 = *(_tmp2) ;  
PushParam _tmp4 ;  
PushParam _tmp1 ;  
ACall _tmp7 ;  
PopParams 8 ;
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Dead Code Elimination

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Object x;  
int a;  
int b;  
int c;  
  
x = new Object;  
a = 4;  
c = a + b;  
x.fn(a + b);
```

```
_tmp0 = 4 ;  
PushParam _tmp0 ;  
_tmp1 = LCall _Alloc ;  
PopParams 4 ;  
_tmp2 = Object ;  
*(_tmp1) = _tmp2 ;  
x = _tmp1 ;  
_tmp3 = 4 ;  
a = 4 ;  
_tmp4 = _tmp0 + b ;  
c = _tmp4 ;  
_tmp5 = _tmp4 ;  
_tmp6 = _tmp2 ;  
_tmp7 = *(_tmp2) ;  
PushParam _tmp4 ;  
PushParam _tmp1 ;  
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```

Dead Code Elimination

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Object x;  
int a;  
int b;  
int c;  
  
x = new Object;  
a = 4;  
c = a + b;  
x.fn(a + b);
```

```
_tmp0 = 4 ;  
PushParam _tmp0 ;  
_tmp1 = LCall _Alloc ;  
PopParams 4 ;  
_tmp2 = Object ;  
*(_tmp1) = _tmp2 ;  
  
_tmp3 = 4 ;  
a = 4 ;  
_tmp4 = _tmp0 + b ;  
c = _tmp4 ;  
_tmp5 = _tmp4 ;  
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_tmp7 = *(_tmp2) ;  
PushParam _tmp4 ;  
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Dead Code Elimination

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Object x;  
int a;  
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x = new Object;  
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Object x;  
int a;  
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int c;  
  
x = new Object;  
a = 4;  
c = a + b;  
x.fn(a + b);
```

```
_tmp0 = 4 ;  
PushParam _tmp0 ;  
_tmp1 = LCall _Alloc ;  
PopParams 4 ;  
_tmp2 = Object ;  
*(_tmp1) = _tmp2 ;
```

```
a = 4 ;  
_tmp4 = _tmp0 + b ;  
c = _tmp4 ;  
_tmp5 = _tmp4 ;  
_tmp6 = _tmp2 ;  
_tmp7 = *(_tmp2) ;  
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Object x;  
int a;  
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x = new Object;  
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x.fn(a + b);
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```
_tmp0 = 4 ;  
PushParam _tmp0 ;  
_tmp1 = LCall _Alloc ;  
PopParams 4 ;  
_tmp2 = Object ;  
*(_tmp1) = _tmp2 ;
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a = 4 ;  
_tmp4 = _tmp0 + b ;  
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_tmp5 = _tmp4 ;  
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_tmp4 = _tmp0 + b ;  
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Dead Code Elimination

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Object x;  
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PopParams 4 ;  
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*(_tmp1) = _tmp2 ;
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_tmp4 = _tmp0 + b ;  
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_tmp5 = _tmp4 ;  
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Dead Code Elimination

```
Object x;  
int a;  
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x = new Object;  
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x.fn(a + b);
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_tmp0 = 4 ;  
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_tmp1 = LCall _Alloc ;  
PopParams 4 ;  
_tmp2 = Object ;  
*(_tmp1) = _tmp2 ;
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_tmp4 = _tmp0 + b ;  
  
_tmp5 = _tmp4 ;  
_tmp6 = _tmp2 ;  
_tmp7 = *(_tmp2) ;  
PushParam _tmp4 ;  
PushParam _tmp1 ;  
ACall _tmp7 ;  
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```


Dead Code Elimination

```
Object x;  
int a;  
int b;  
int c;
```

```
x = new Object;  
a = 4;  
c = a + b;  
x.fn(a + b);
```

```
_tmp0 = 4 ;  
PushParam _tmp0 ;  
_tmp1 = LCall _Alloc ;  
PopParams 4 ;  
_tmp2 = Object ;  
*(_tmp1) = _tmp2 ;
```

```
_tmp4 = _tmp0 + b ;
```

```
_tmp5 = _tmp4 ;  
_tmp6 = _tmp2 ;  
_tmp7 = *(_tmp2) ;  
PushParam _tmp4 ;  
PushParam _tmp1 ;  
ACall _tmp7 ;  
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```

Dead Code Elimination

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Object x;  
int a;  
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int c;
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x = new Object;  
a = 4;  
c = a + b;  
x.fn(a + b);
```

```
_tmp0 = 4 ;  
PushParam _tmp0 ;  
_tmp1 = LCall _Alloc ;  
PopParams 4 ;  
_tmp2 = Object ;  
*(_tmp1) = _tmp2 ;
```

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_tmp4 = _tmp0 + b ;
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```
_tmp6 = _tmp2 ;  
_tmp7 = *(_tmp2) ;  
PushParam _tmp4 ;  
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ACall _tmp7 ;  
PopParams 8 ;
```

Dead Code Elimination

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int a;  
int b;  
int c;
```

```
x = new Object;  
a = 4;  
c = a + b;  
x.fn(a + b);
```

```
_tmp0 = 4 ;  
PushParam _tmp0 ;  
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PopParams 4 ;  
_tmp2 = Object ;  
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```
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```
_tmp6 = _tmp2 ;  
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PopParams 8 ;
```

Dead Code Elimination

```
Object x;  
int a;  
int b;  
int c;
```

```
x = new Object;  
a = 4;  
c = a + b;  
x.fn(a + b);
```

```
_tmp0 = 4 ;  
PushParam _tmp0 ;  
_tmp1 = LCall _Alloc ;  
PopParams 4 ;  
_tmp2 = Object ;  
*(_tmp1) = _tmp2 ;
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```
_tmp4 = _tmp0 + b ;
```

```
_tmp7 = *(_tmp2) ;  
PushParam _tmp4 ;  
PushParam _tmp1 ;  
ACall _tmp7 ;  
PopParams 8 ;
```

Dead Code Elimination

- An assignment to a variable v is called **dead** if the value of that assignment is never read anywhere.
- **Dead code elimination** removes dead assignments from IR.
- Determining whether an assignment is dead depends on what variable is being assigned to and when it's being assigned.

Applying Local Optimizations

- The different optimizations we've seen so far all take care of just a small piece of the optimization.
 - Common subexpression elimination eliminates unnecessary statements.
 - Copy propagation helps identify dead code.
 - Dead code elimination removes statements that are no longer needed.
- To get maximum effect, we may have to apply these optimizations numerous times.

Applying Local Optimizations

```
b = a * a;  
c = a * a;  
d = b + c;  
e = b + b;
```

Applying Local Optimizations

```
b = a * a;  
c = a * a;  
d = b + c;  
e = b + b;
```


Applying Local Optimizations

```
b = a * a;  
c = a * a;  
d = b + c;  
e = b + b;
```

Common Subexpression Elimination

Applying Local Optimizations

```
b = a * a;  
c = b;  
d = b + c;  
e = b + b;
```

Common Subexpression Elimination

Applying Local Optimizations

```
b = a * a;  
c = b;  
d = b + c;  
e = b + b;
```

Applying Local Optimizations

```
b = a * a;  
c = b;  
d = b + c;  
e = b + b;
```

Applying Local Optimizations

```
b = a * a;  
c = b;  
d = b + c;  
e = b + b;
```

Copy Propagation

Applying Local Optimizations

```
b = a * a;  
c = b;  
d = b + b;  
e = b + b;
```

Copy Propagation

Applying Local Optimizations

```
b = a * a;  
c = b;  
d = b + b;  
e = b + b;
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Applying Local Optimizations

```
b = a * a;
```

```
c = b;
```

```
d = b + b;
```

```
e = b + b;
```


Applying Local Optimizations

```
b = a * a;  
c = b;  
d = b + b;  
e = b + b;
```

Common Subexpression Elimination (Again)

Applying Local Optimizations

```
b = a * a;  
c = b;  
d = b + b;  
e = d;
```

Common Subexpression Elimination (Again)

Applying Local Optimizations

```
b = a * a;  
c = b;  
d = b + b;  
e = d;
```

Other Types of Local Optimization

- **Arithmetic Simplification**

- Replace “hard” operations with easier ones.
- e.g. rewrite `x = 4 * a;` as `x = a << 2;`

- **Constant Folding**

- Evaluate expressions at compile-time if they have a constant value.
- e.g. rewrite `x = 4 * 5;` as `x = 20;`.

Optimizations and Analyses

- Most optimizations are only possible given some analysis of the program's behavior.
- In order to implement an optimization, we will talk about the corresponding program analyses.

Available Expressions

- Both common subexpression elimination and copy propagation depend on an analysis of the **available expressions** in a program.
- An expression is called **available** if some variable in the program holds the value of that expression.
- In common subexpression elimination, we replace an available expression by the variable holding its value.
- In copy propagation, we replace the use of a variable by the available expression it holds.

Finding Available Expressions

- Initially, no expressions are available.
- Whenever we execute a statement **$a = b + c$** :
 - Any expression holding **a** is invalidated.
 - The expression **$a = b + c$** becomes available.
- **Idea**: Iterate across the basic block, beginning with the empty set of expressions and updating available expressions at each variable.

Live Variables

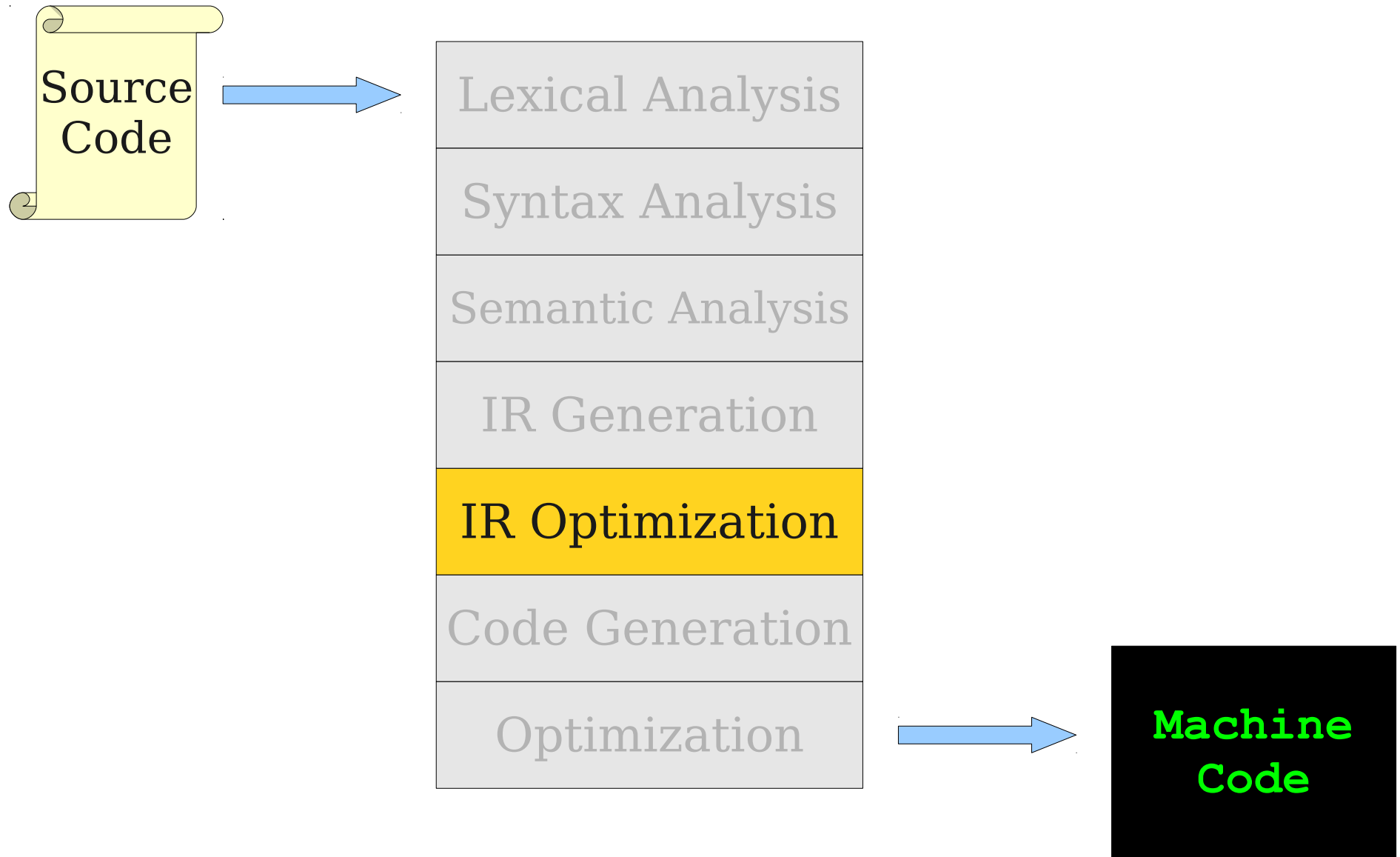
- The analysis corresponding to dead code elimination is called **liveness analysis**.
- A variable is **live** at a point in a program if later in the program its value will be read before it is written to again.
- Dead code elimination works by computing liveness for each variable, then eliminating assignments to dead variables.

Computing Live Variables

- To know if a variable will be used at some point, we iterate across the statements in a basic block in reverse order.
- Initially, some small set of values are known to be live (which ones depends on the particular program).
- When we see the statement **a = b + c**:
 - Just before the statement, **a** is not alive, since its value is about to be overwritten.
 - Just before the statement, both **b** and **c** are alive, since we're about to read their values.
 - *(what if we have **a = a + b**?)*

Global Optimization

Where We Are



Global Analysis

- A **global analysis** is an analysis that works on a control-flow graph as a whole.
- Substantially more powerful than a local analysis.
 - (Why?)
- Substantially more complicated than a local analysis.
 - (Why?)

Local vs. Global Analysis

- Many of the optimizations from local analysis can still be applied globally.
 - We'll see how to do this later today.
- Certain optimizations are possible in global analysis that aren't possible locally:
 - e.g. **code motion**: Moving code from one basic block into another to avoid computing values unnecessarily.

Major Changes, Part One

- In a local analysis, each statement has exactly one predecessor.
- In a global analysis, each statement may have **multiple** predecessors.
- A global analysis must have some means of combining information from all predecessors of a basic block.

Major Changes, Part II

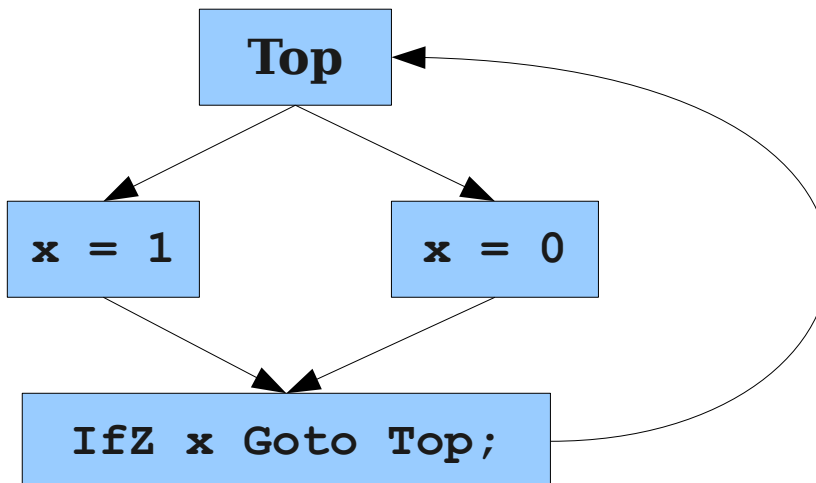
- In a local analysis, there is only one possible path through a basic block.
- In a global analysis, there may be **many** paths through a CFG.
- May need to recompute values multiple times as more information becomes available.
- Need to be careful when doing this not to loop infinitely!
 - (More on that later)

CFGs with Loops

- Up to this point, we've considered loop-free CFGs, which have only finitely many possible paths.
- When we add loops into the picture, this is no longer true.
- Not all possible loops in a CFG can be realized in the actual program.

CFGs with Loops

- Up to this point, we've considered loop-free CFGs, which have only finitely many possible paths.
- When we add loops into the picture, this is no longer true.
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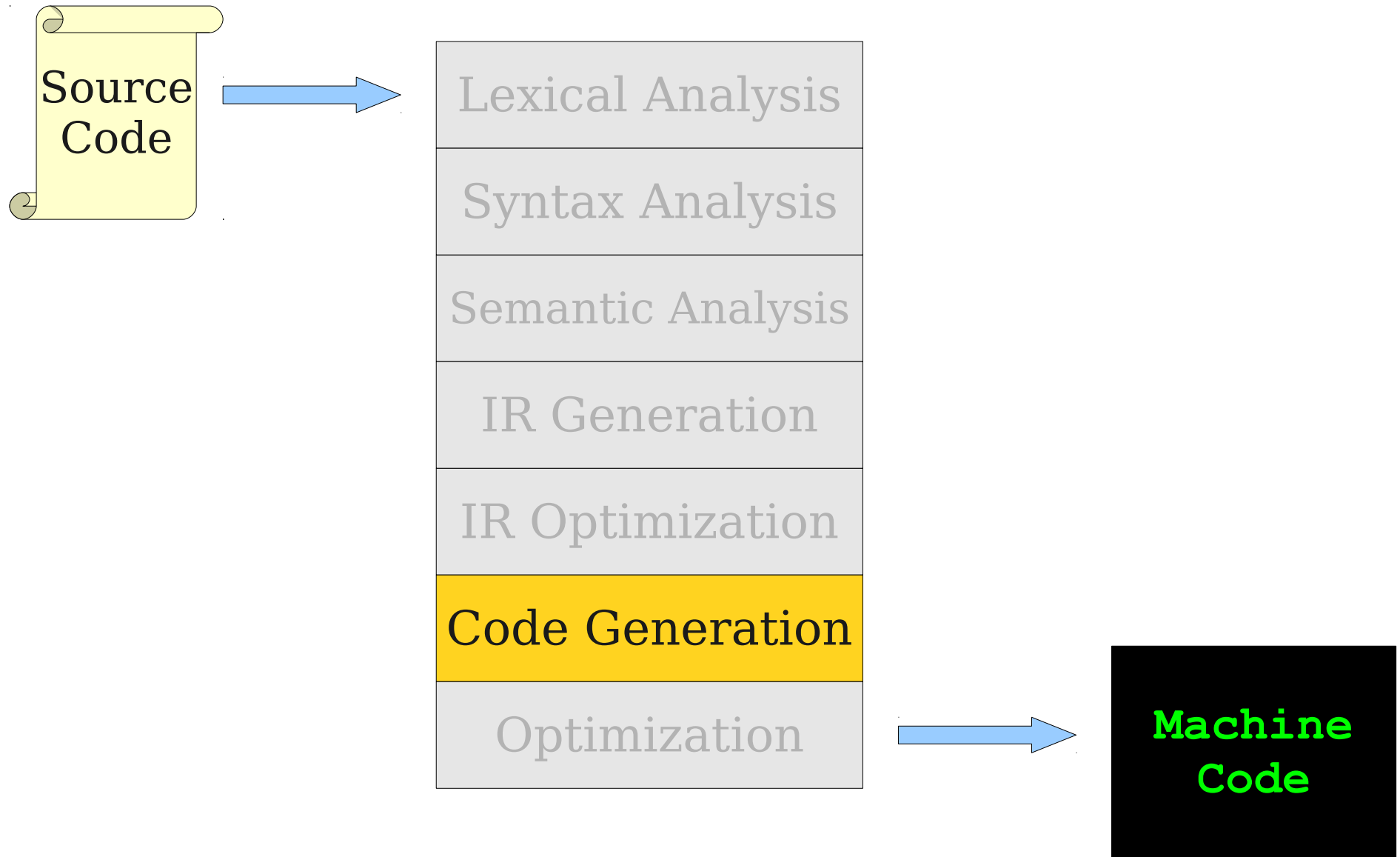


Major Changes, Part III

- In a local analysis, there is always a well-defined “first” statement to begin processing.
- In a global analysis with loops, every basic block might depend on every other basic block.
- To fix this, we need to assign initial values to all of the blocks in the CFG.

Register Allocation

Where We Are



Code Generation at a Glance

- At this point, we have optimized IR code that needs to be converted into the target language (e.g. assembly, machine code).
- Goal of this stage:
 - Choose the appropriate machine instructions for each IR instruction.
 - Divvy up finite machine resources (registers, caches, etc.)
 - Implement low-level details of the runtime environment.
- Machine-specific optimizations are often done here, though some are treated as part of a final optimization phase.

Memory Tradeoffs

- There is an enormous tradeoff between *speed* and *size* in memory.
- SRAM is fast but very expensive:
 - Can keep up with processor speeds in the GHz.
 - As of 2007, cost is \$10/MB
 - Good luck buying 1TB of the stuff!
- Hard disks are cheap but very slow:
 - As of 2012, you can buy a 2TB hard drive for about \$100
 - As of 2012, good disk seek times are measured in ms (about two to four million times slower than a processor cycle!)

Registers

- Most machines have a set of **registers**, dedicated memory locations that
 - can be accessed quickly,
 - can have computations performed on them, and
 - exist in small quantity.
- Using registers intelligently is a critical step in any compiler.
 - A good register allocator can generate code orders of magnitude better than a bad register allocator.

Register Allocation

- In TAC, there are an unlimited number of variables.
- On a physical machine there are a small number of registers:
 - x86 has four general-purpose registers and a number of specialized registers.
 - MIPS has twenty-four general-purpose registers and eight special-purpose registers.
- **Register allocation** is the process of assigning variables to registers and managing data transfer in and out of registers.

Challenges in Register Allocation

- **Registers are scarce.**
 - Often substantially more IR variables than registers.
 - Need to find a way to reuse registers whenever possible.
- **Registers are complicated.**
 - x86: Each register made of several smaller registers; can't use a register and its constituent registers at the same time.
 - x86: Certain instructions must store their results in specific registers; can't store values there if you want to use those instructions.
 - MIPS: Some registers reserved for the assembler or operating system.
 - Most architectures: Some registers must be preserved across function calls.

An Initial Register Allocator

- **Idea:** Store every value in main memory, loading values only when they're needed.
- To generate a code that performs a computation:
 - Generate **load** instructions to pull the values from main memory into registers.
 - Generate code to perform the computation on the registers.
 - Generate **store** instructions to store the result back into main memory.

Analysis of our Allocator

- Disadvantage: **Gross inefficiency**.
 - Issues unnecessary loads and stores by the dozen.
 - Wastes space on values that could be stored purely in registers.
 - Easily an order of magnitude or two slower than necessary.
 - Unacceptable in any production compiler.
- Advantage: **Simplicity**.
 - Can translate each piece of IR directly to assembly as we go.
 - Never need to worry about running out of registers.
 - Never need to worry about function calls or special-purpose registers.
 - Good if you just needed to get a prototype compiler up and running.

Building a Better Allocator

- **Goal:** Try to hold as many variables in registers as possible.
 - Reduces memory reads/writes.
 - Reduces total memory usage.
- We will need to address these questions:
 - Which registers do we put variables in?
 - What do we do when we run out of registers?

Live Ranges and Live Intervals

- Recall: A variable is **live** at a particular program point if its value may be read later before it is written.
 - Can find this using global liveness analysis.
- The **live range** for a variable is the set of program points at which that variable is live.
- The **live interval** for a variable is the smallest subrange of the IR code containing all a variable's live ranges.
 - A property of the IR code, **not** the CFG.
 - Less precise than live ranges, but simpler to work with.

Live Ranges and Live Intervals

```
e = d + a
```

```
f = b + c
```

```
f = f + b
```

```
IfZ e Goto _L0
```

```
d = e + f
```

```
Goto _L1;
```

```
_L0:
```

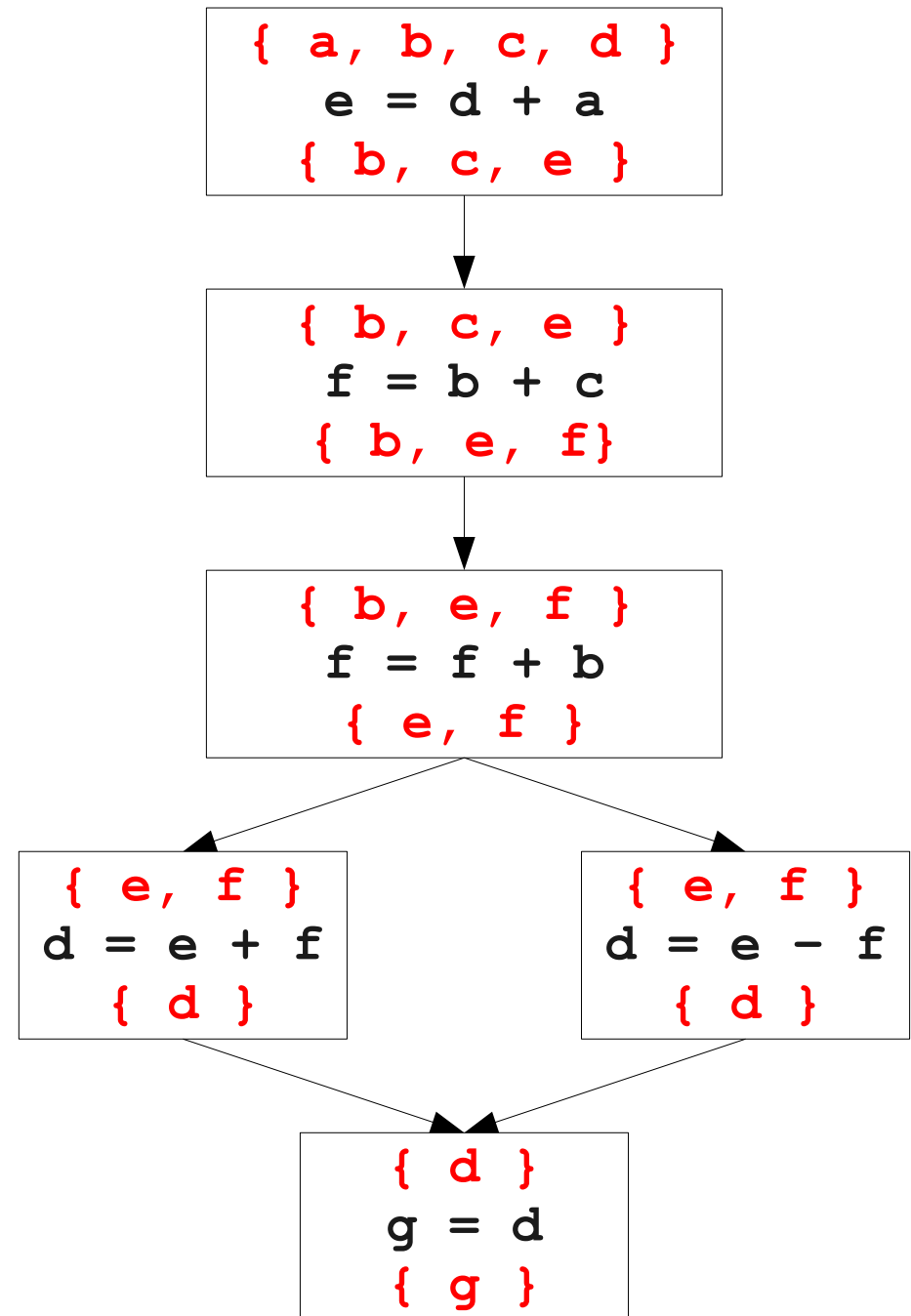
```
d = e - f
```

```
_L1:
```

```
g = d
```

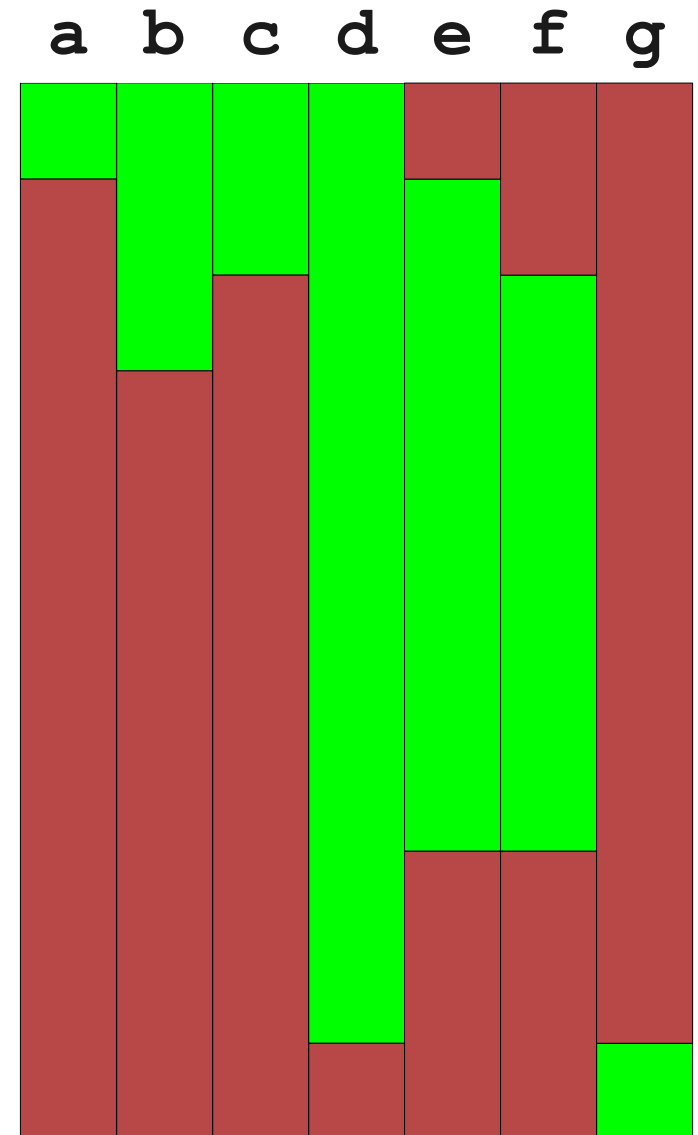
Live Ranges and Live Intervals

| | a | b | c | d | e | f | g |
|-----------------------------|---|---|---|---|---|---|---|
| | | | | | | | |
| <code>e = d + a</code> | | | | | | | |
| <code>f = b + c</code> | | | | | | | |
| <code>f = f + b</code> | | | | | | | |
| <code>IfZ e Goto _L0</code> | | | | | | | |
| <code>d = e + f</code> | | | | | | | |
| <code>Goto _L1;</code> | | | | | | | |
| <code>_L0:</code> | | | | | | | |
| <code>d = e - f</code> | | | | | | | |
| <code>_L1:</code> | | | | | | | |
| <code>g = d</code> | | | | | | | |

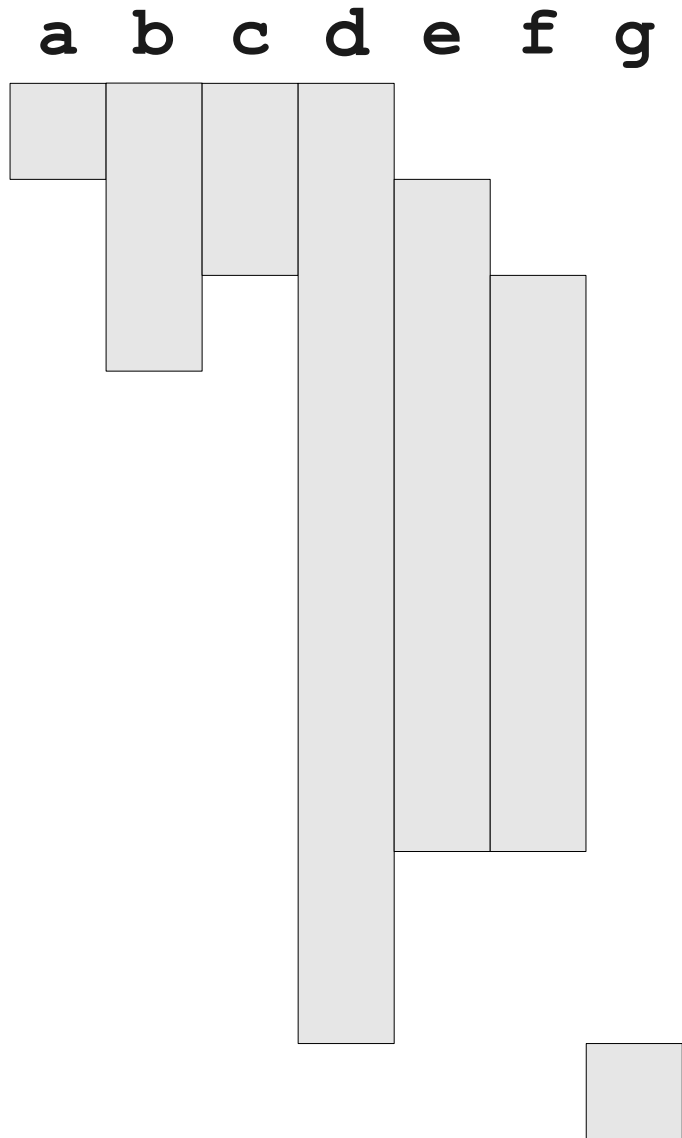


Register Allocation with Live Intervals

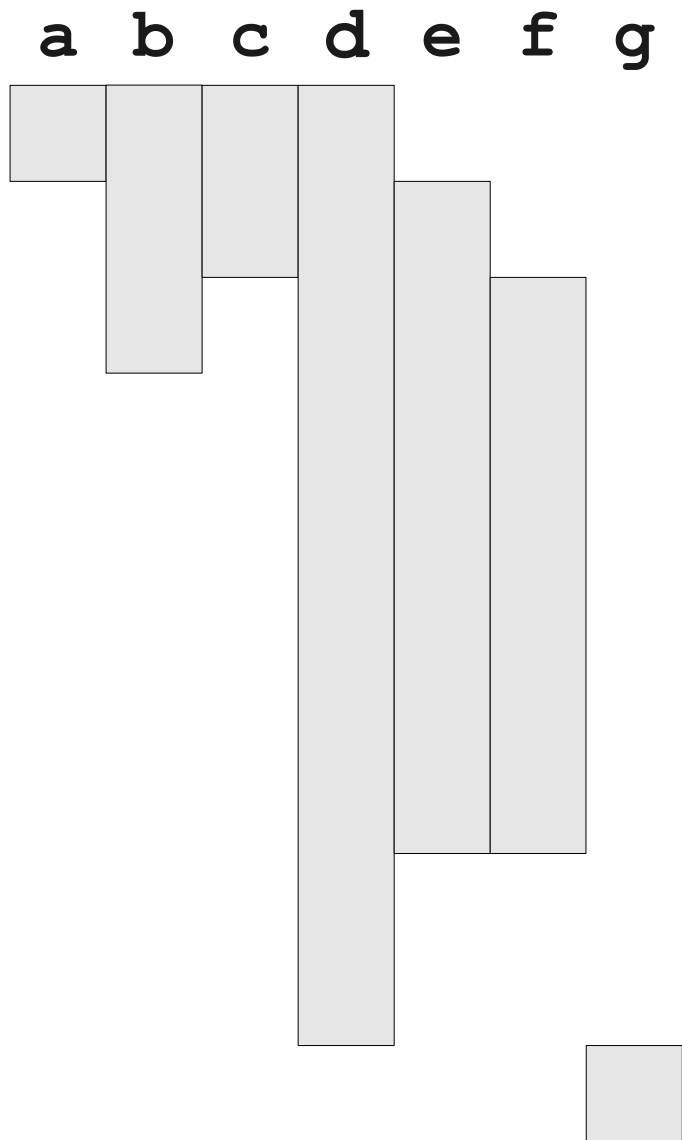
- Given the live intervals for all the variables in the program, we can allocate registers using a simple greedy algorithm.
- Idea: Track which registers are free at each point.
- When a live interval begins, give that variable a free register.
- When a live interval ends, the register is once again free.
- We can't always fit everything into a register; we'll see what to do in a minute.



Register Allocation with Live Intervals



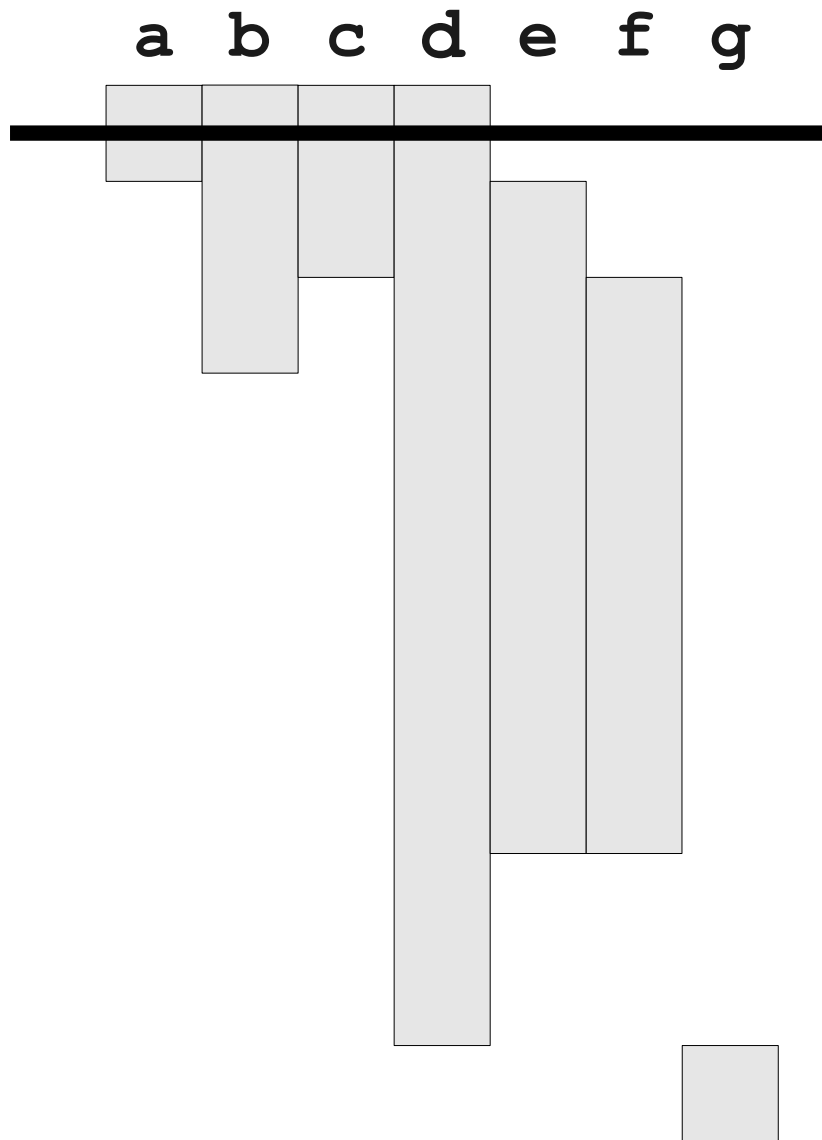
Register Allocation with Live Intervals



Free Registers



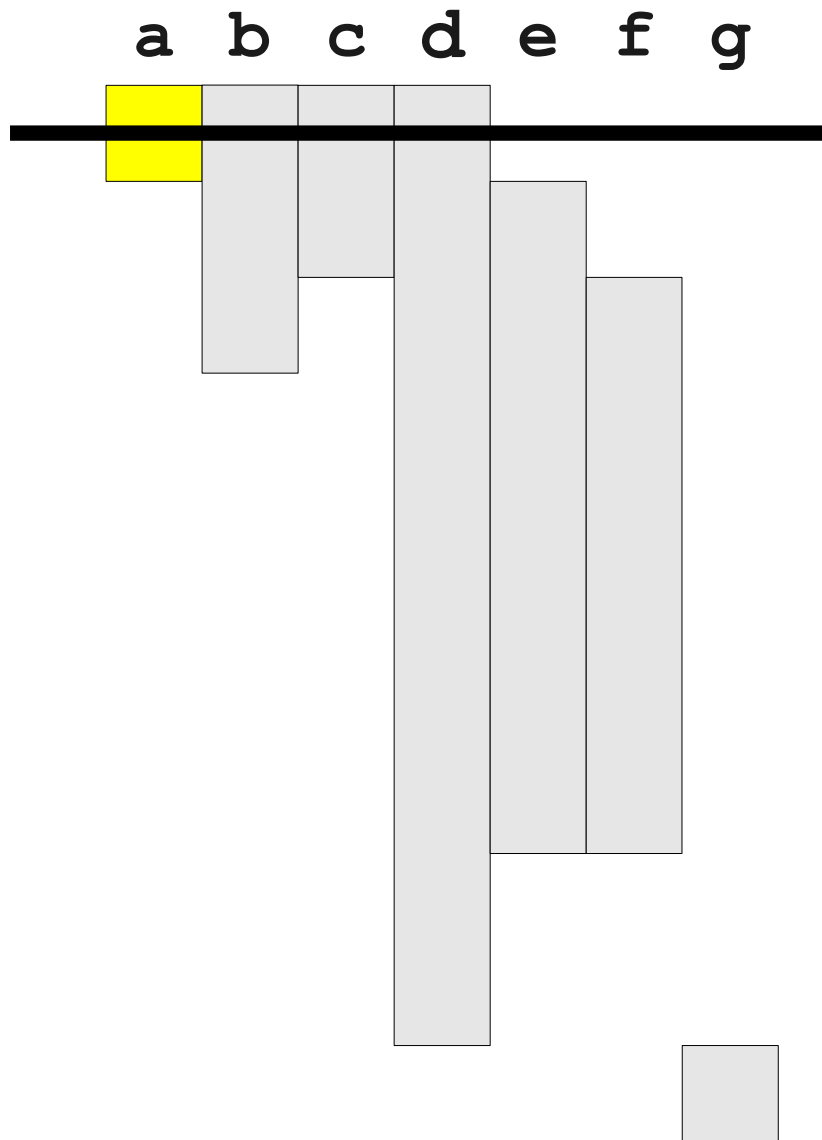
Register Allocation with Live Intervals



Free Registers



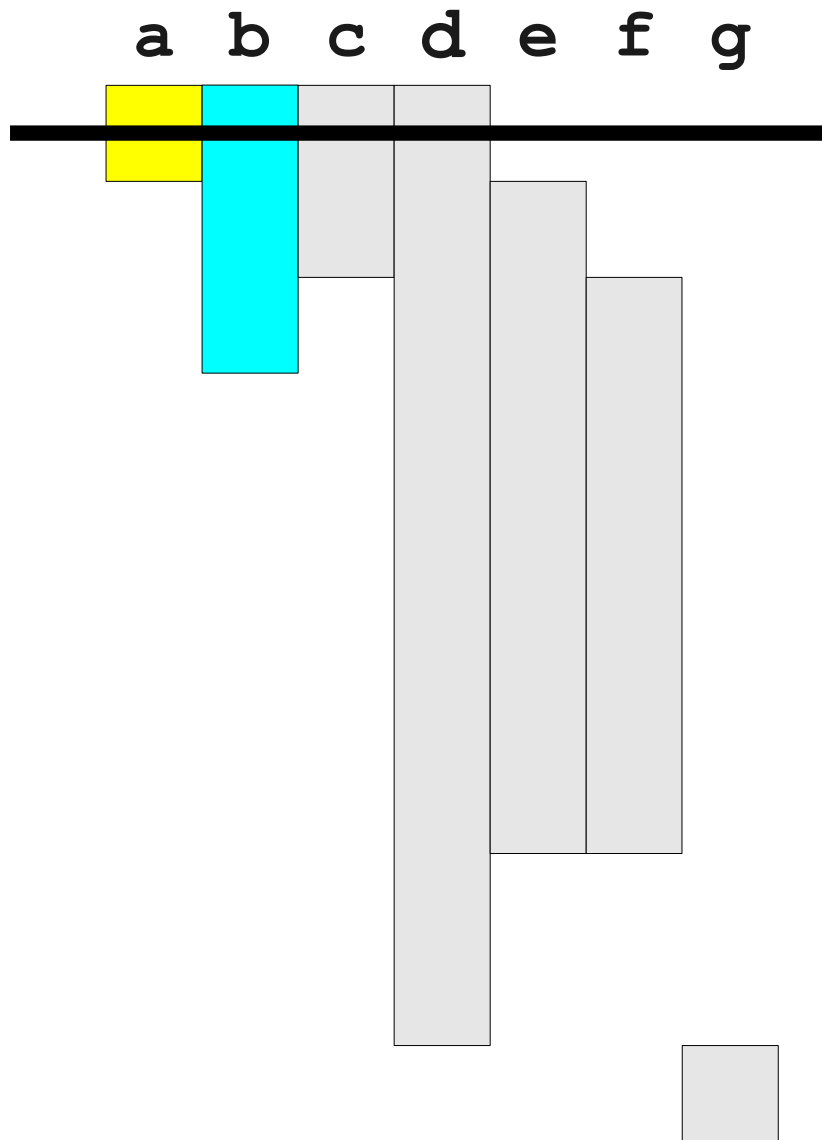
Register Allocation with Live Intervals



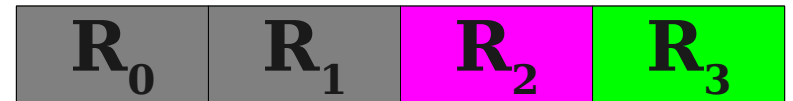
Free Registers



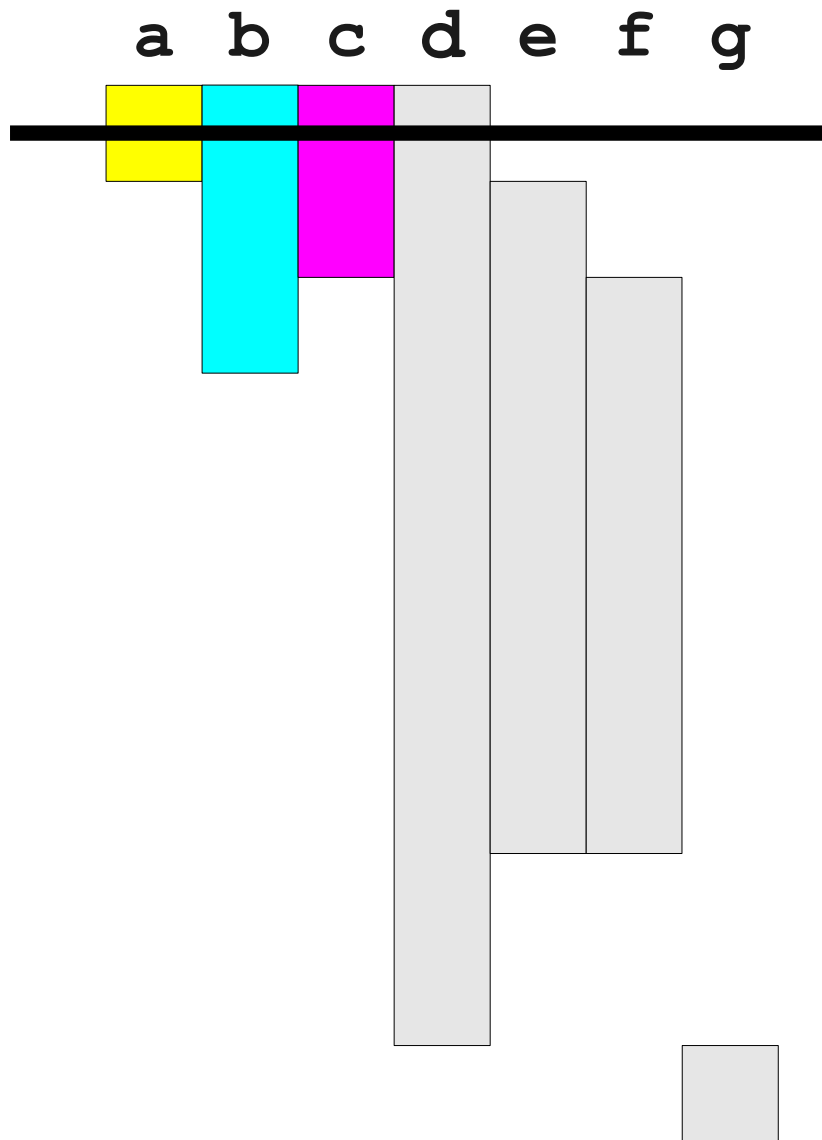
Register Allocation with Live Intervals



Free Registers



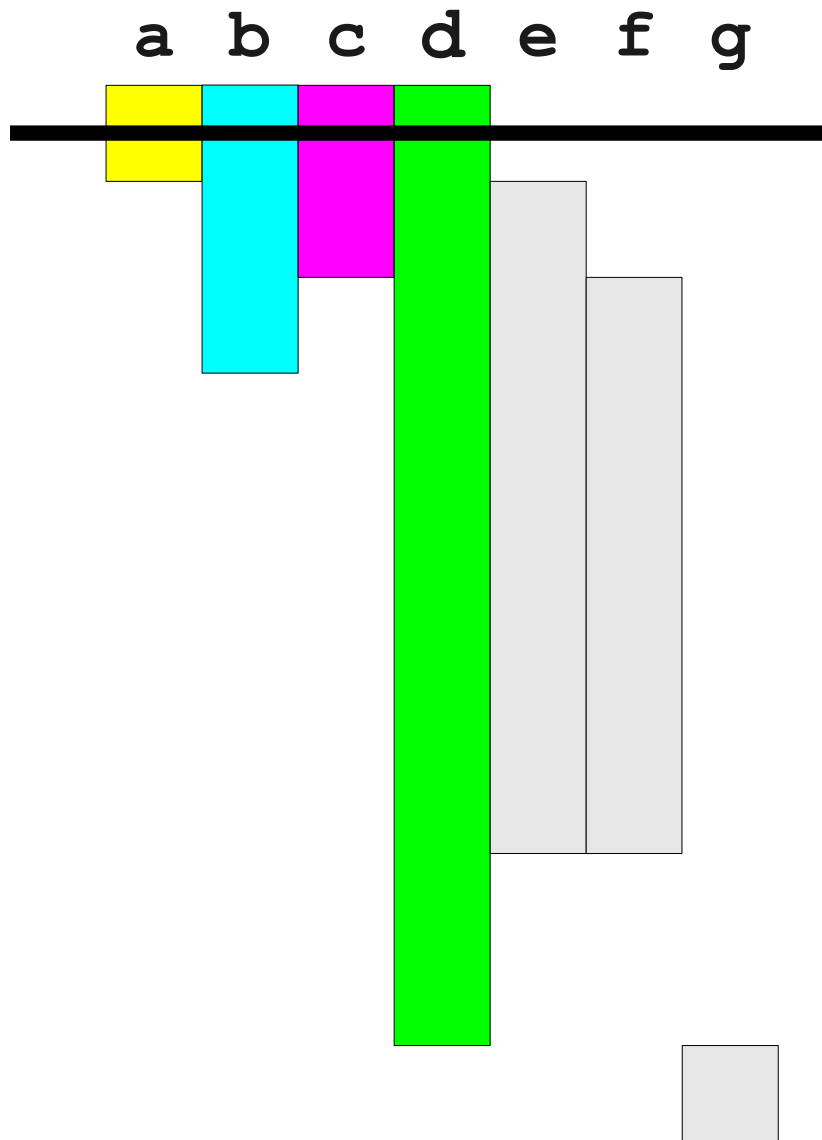
Register Allocation with Live Intervals



Free Registers



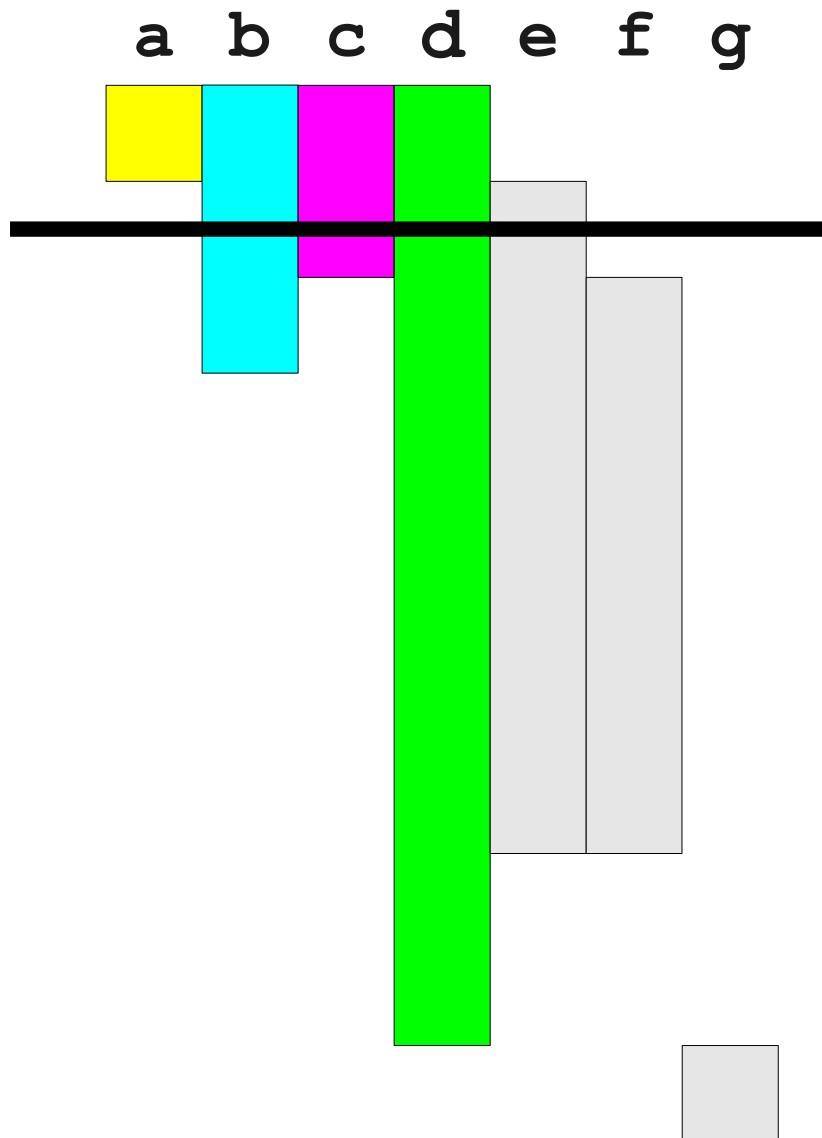
Register Allocation with Live Intervals



Free Registers

| | | | |
|-------|-------|-------|-------|
| R_0 | R_1 | R_2 | R_2 |
|-------|-------|-------|-------|

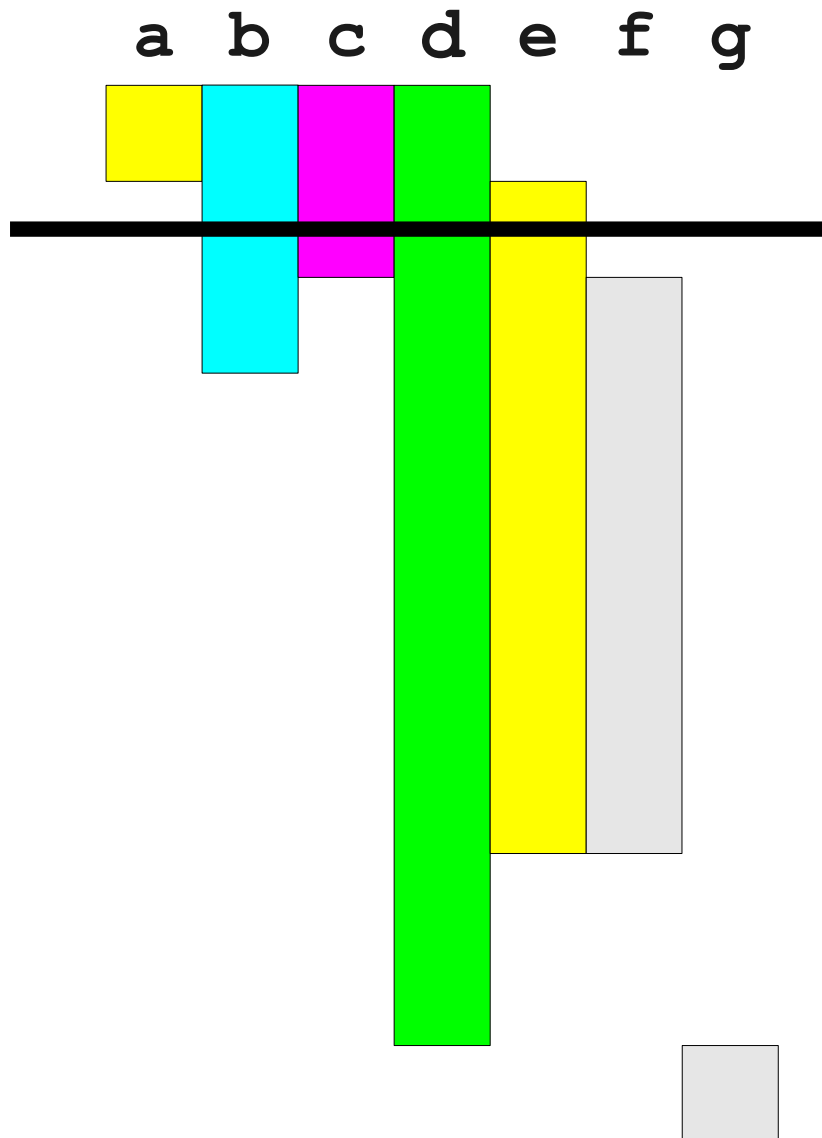
Register Allocation with Live Intervals



Free Registers



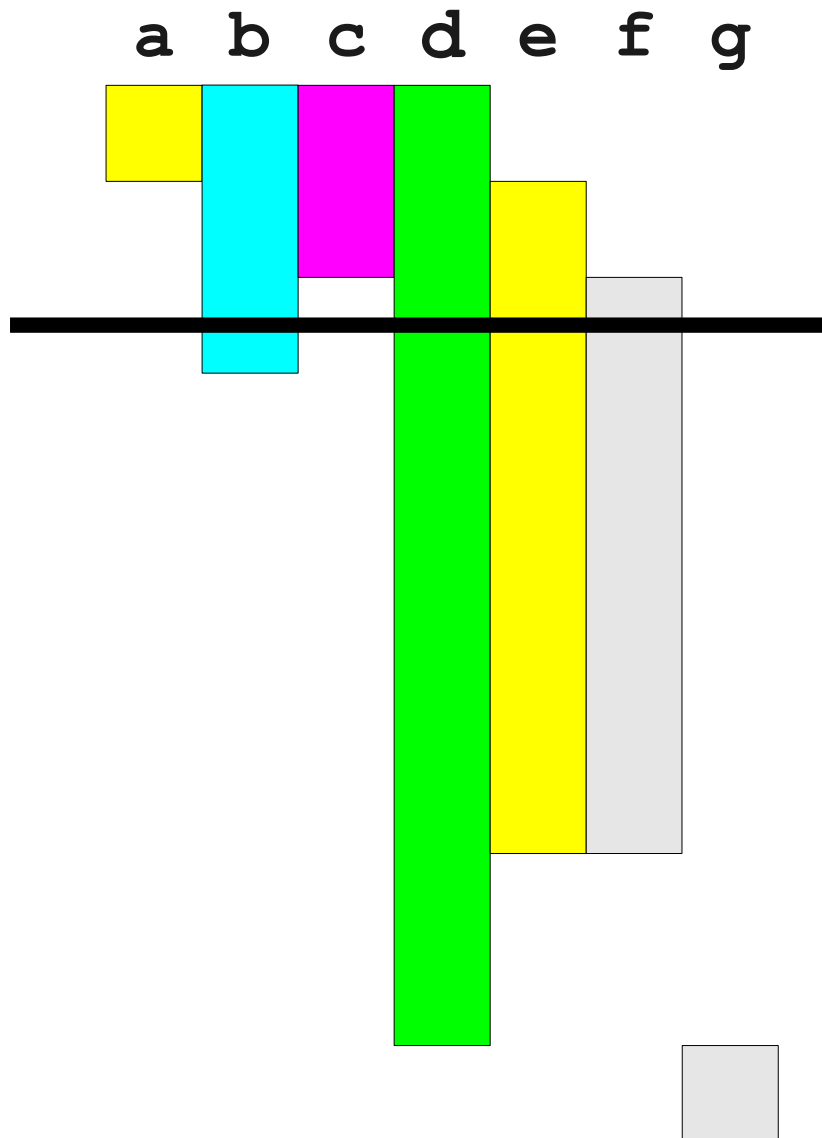
Register Allocation with Live Intervals



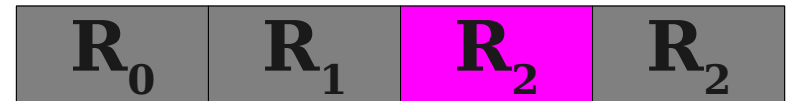
Free Registers

| | | | |
|-------|-------|-------|-------|
| R_0 | R_1 | R_2 | R_2 |
|-------|-------|-------|-------|

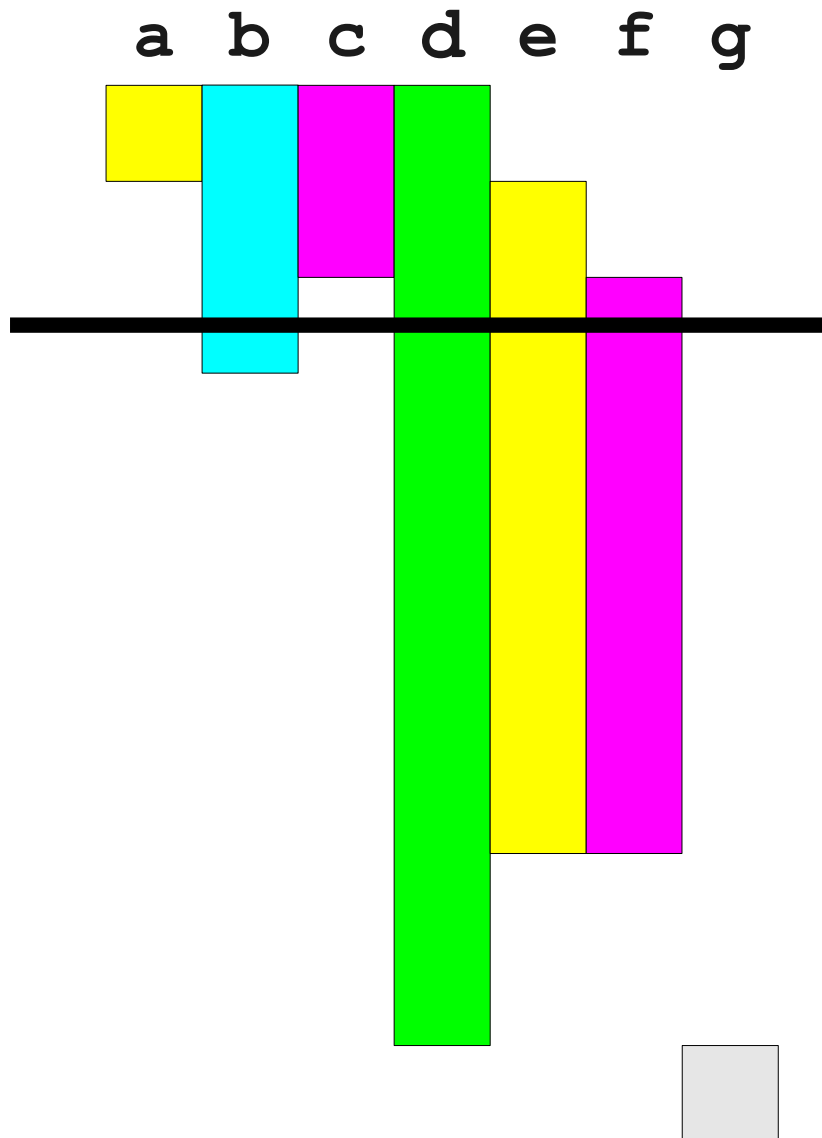
Register Allocation with Live Intervals



Free Registers



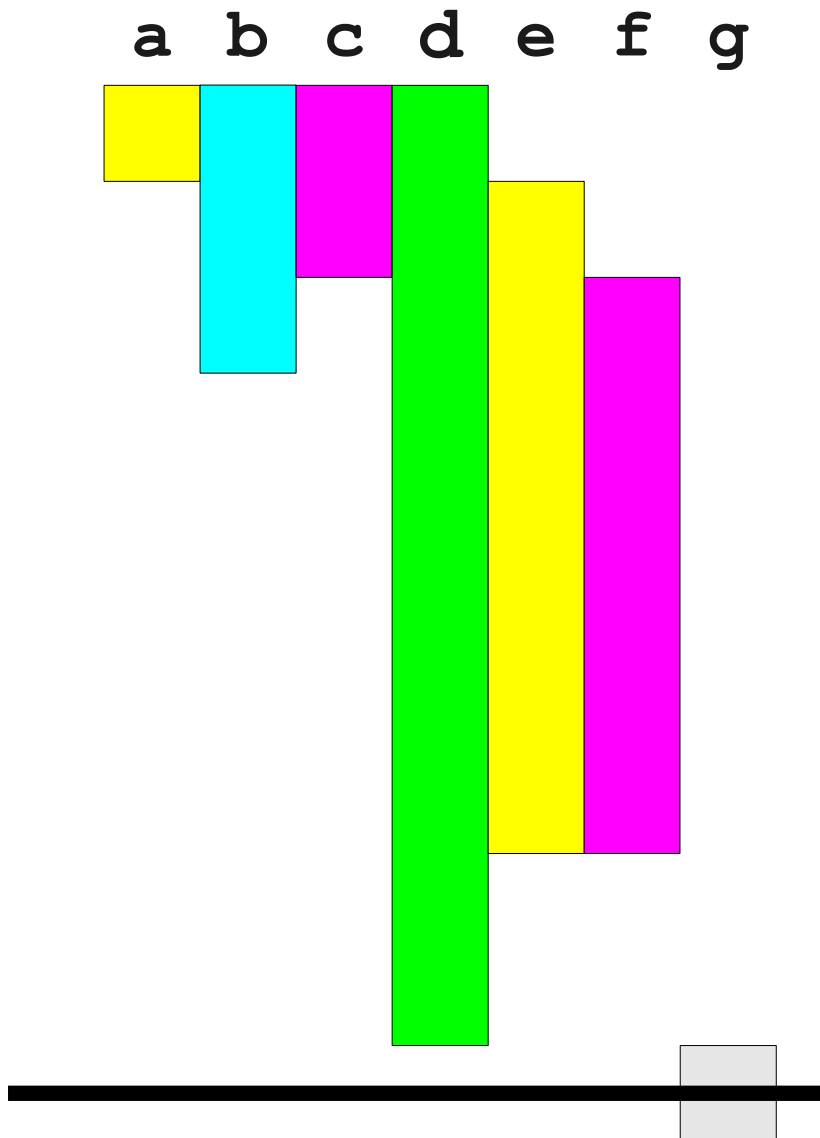
Register Allocation with Live Intervals



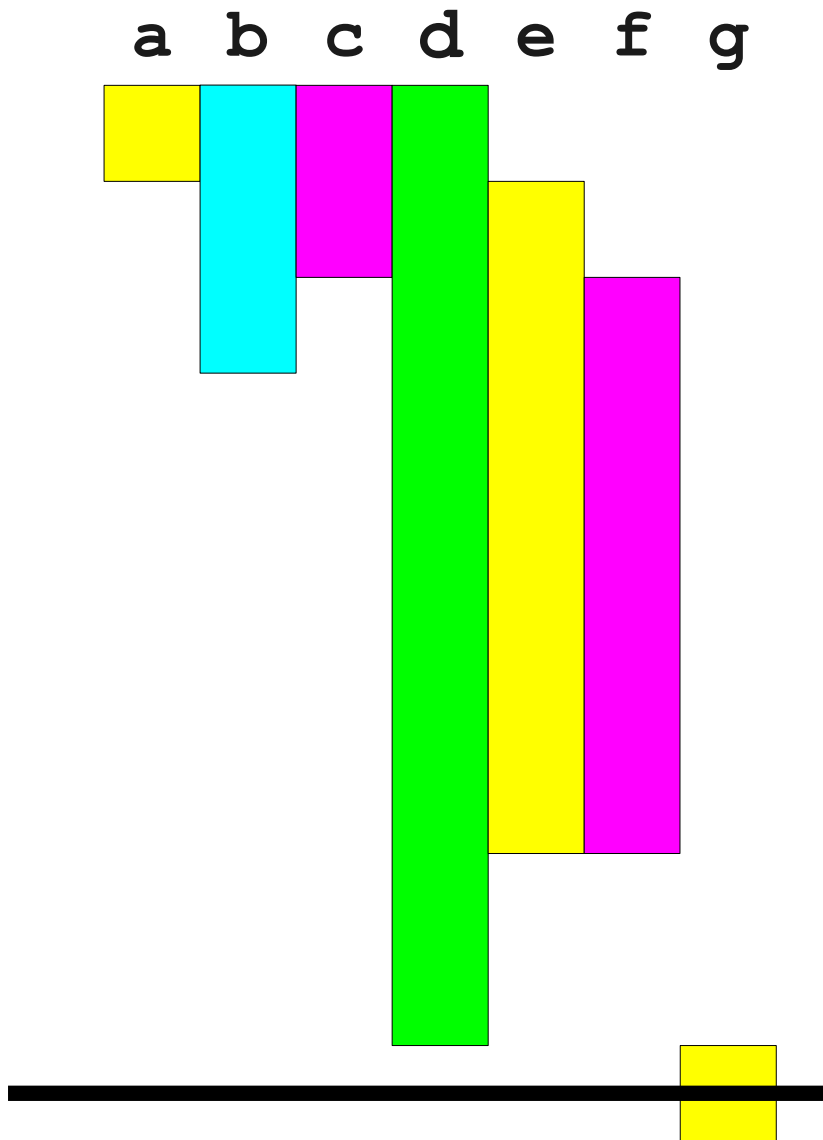
Free Registers

| | | | |
|-------|-------|-------|-------|
| R_0 | R_1 | R_2 | R_2 |
|-------|-------|-------|-------|

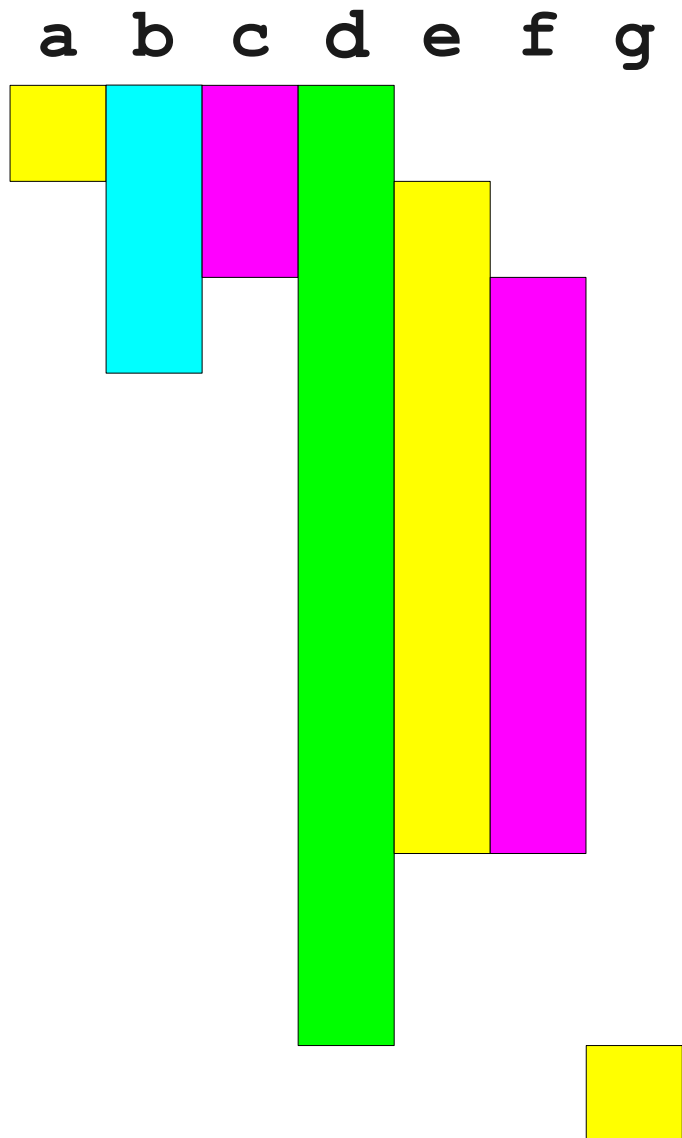
Register Allocation with Live Intervals



Register Allocation with Live Intervals

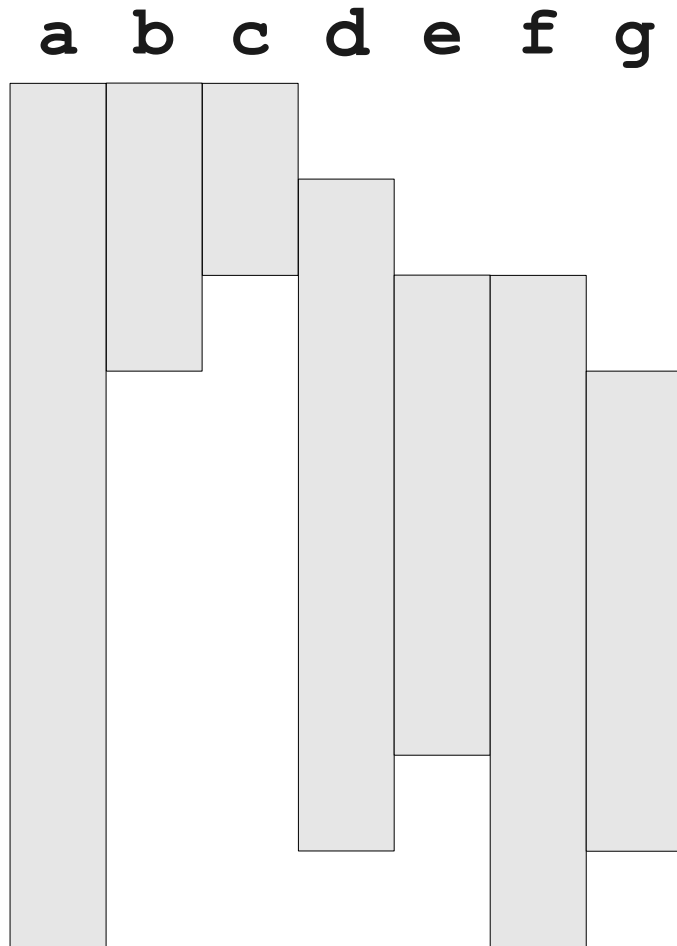


Register Allocation with Live Intervals

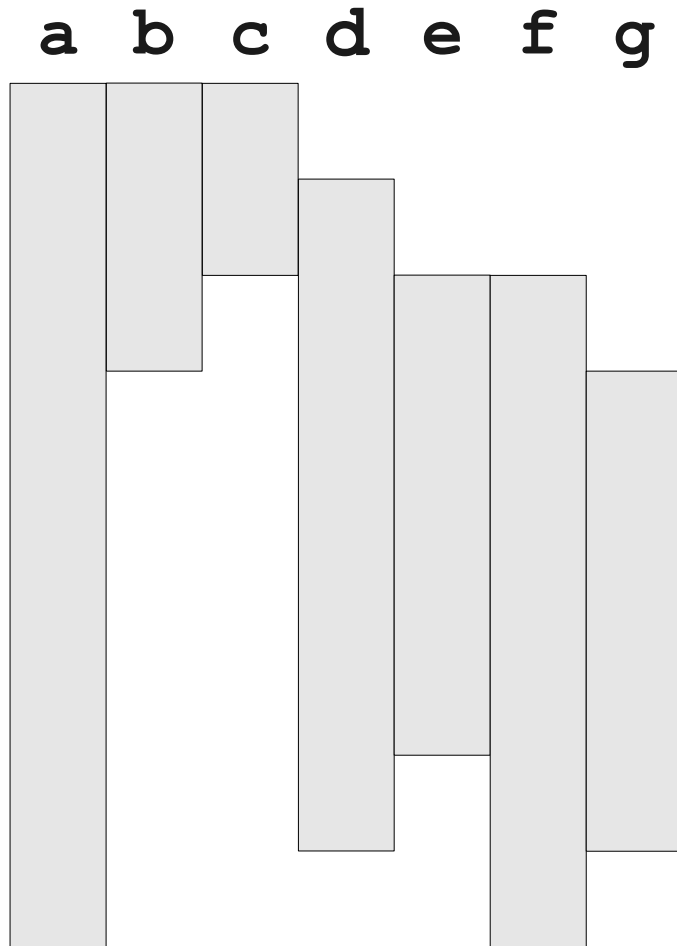


Another Example

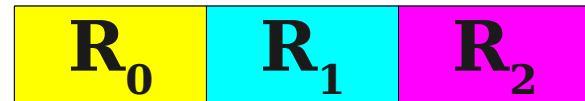
Another Example



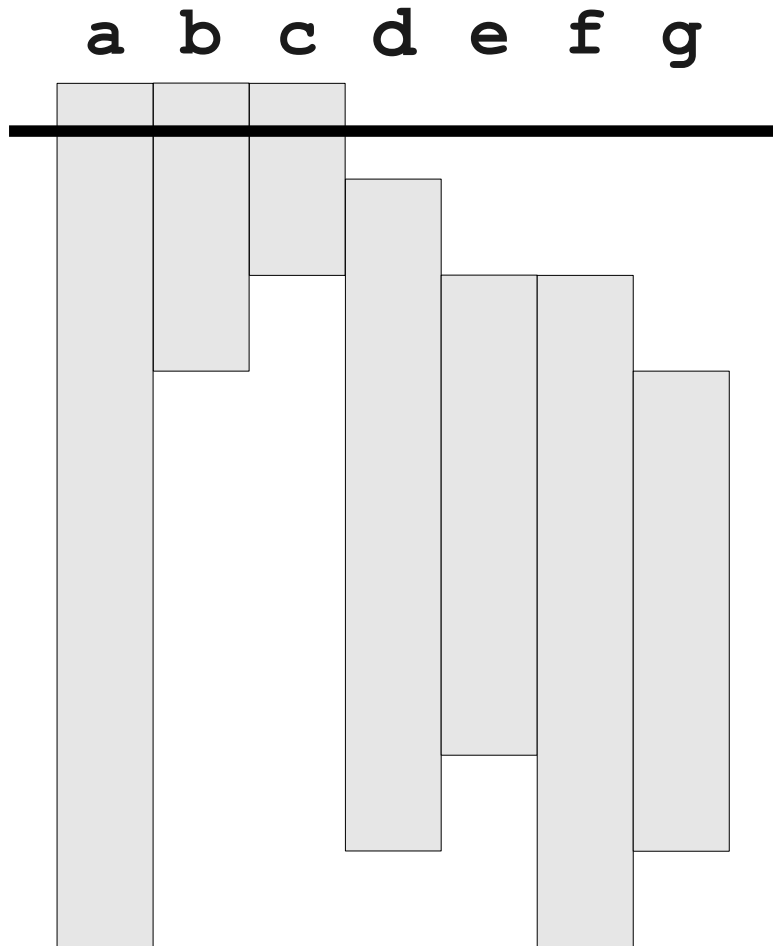
Another Example



Free Registers



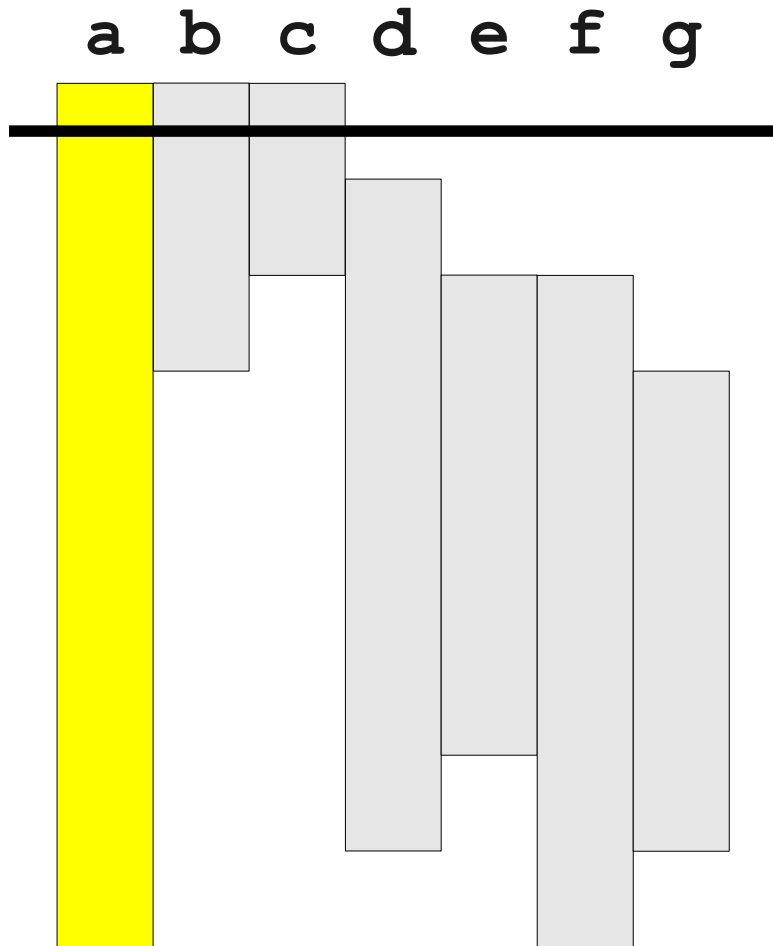
Another Example



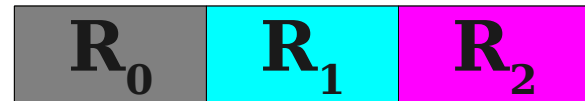
Free Registers



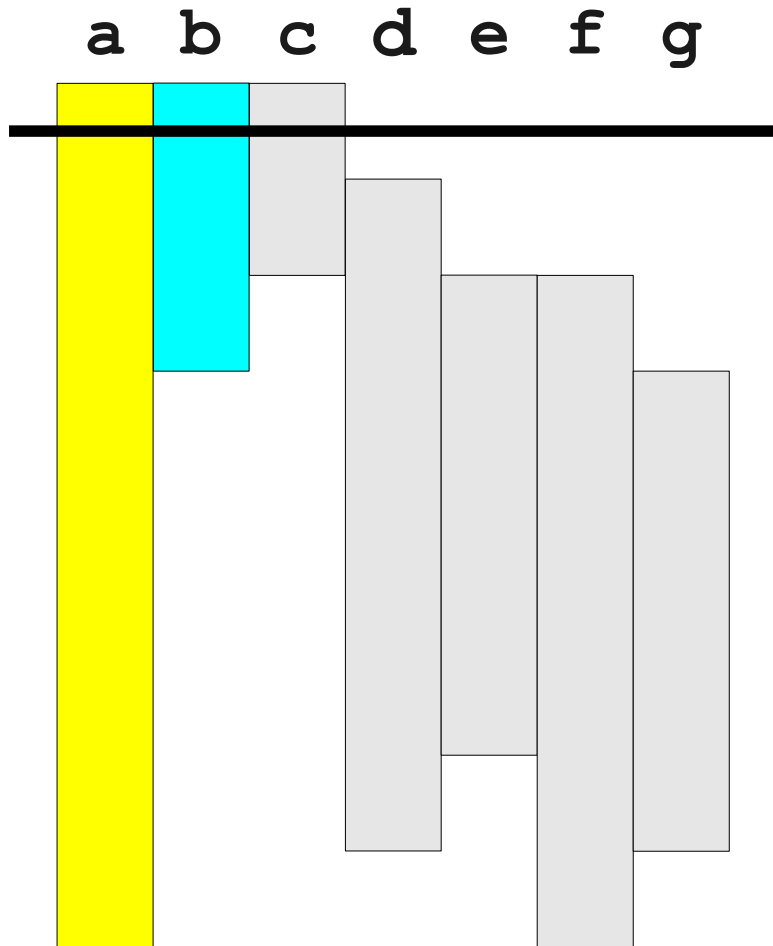
Another Example



Free Registers



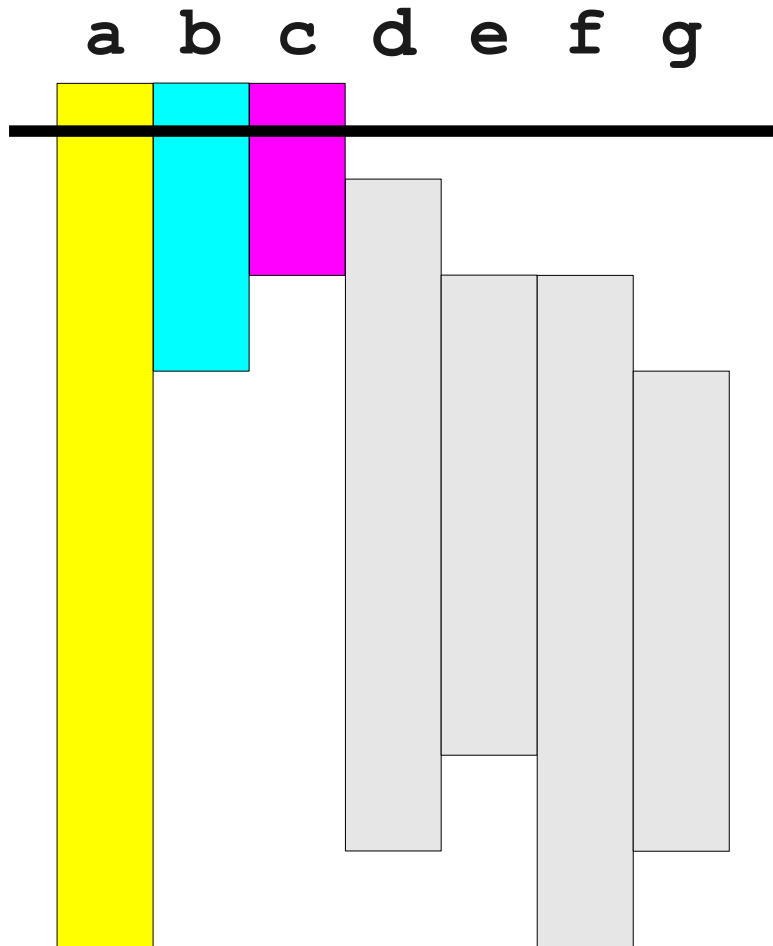
Another Example



Free Registers



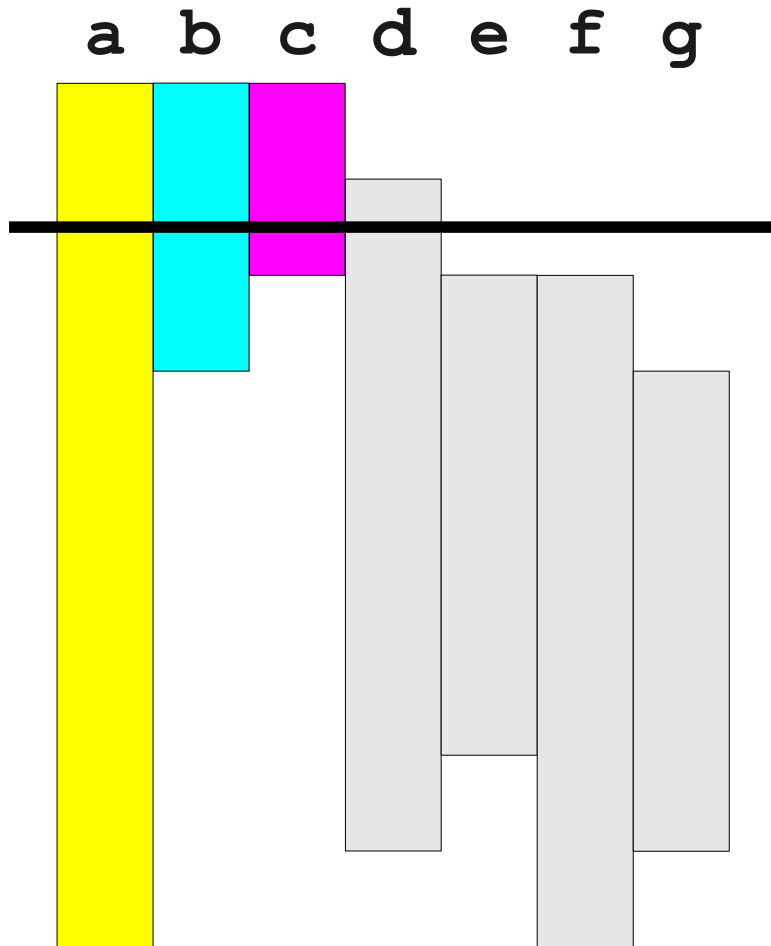
Another Example



Free Registers



Another Example



Free Registers

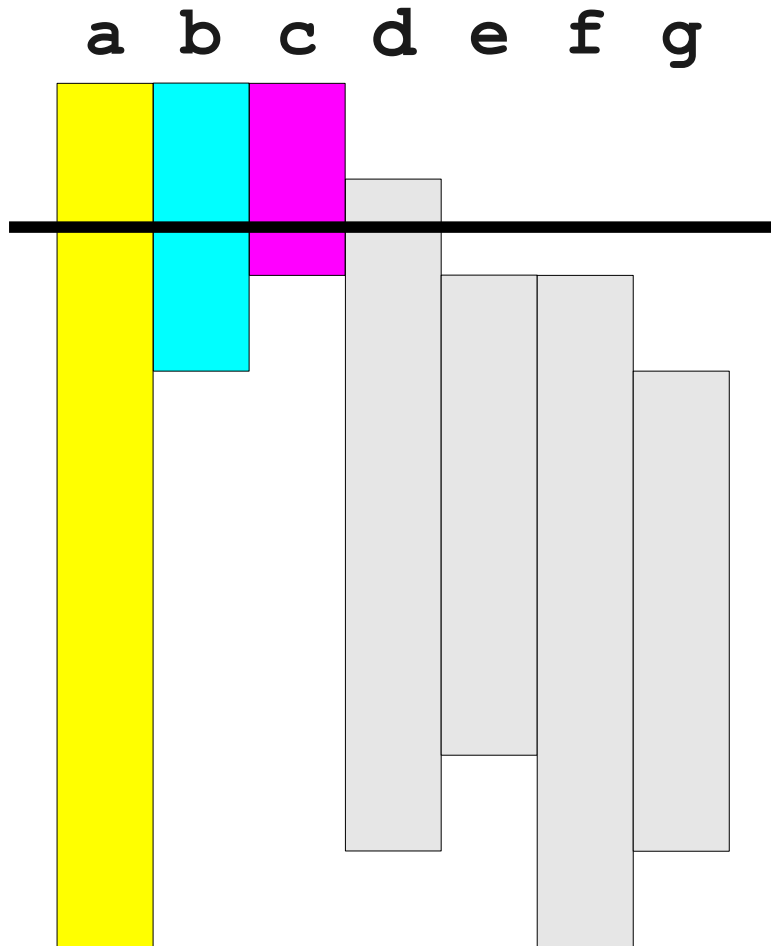


What do we do
now?

Register Spilling

- If a register cannot be found for a variable v , we may need to **spill** a variable.
- When a variable is spilled, it is stored in memory rather than a register.
- When we need a register for the spilled variable:
 - Evict some existing register to memory.
 - Load the variable into the register.
 - When done, write the register back to memory and reload the register with its original value.
- Spilling is slow, but sometimes necessary.

Another Example

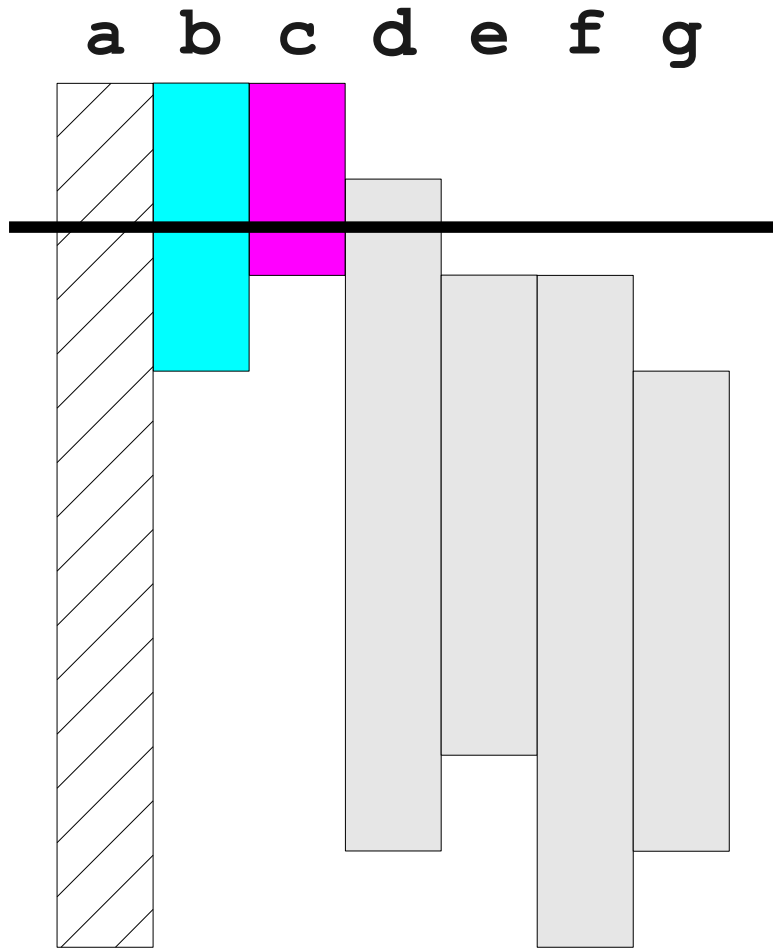


Free Registers



What do we do
now?

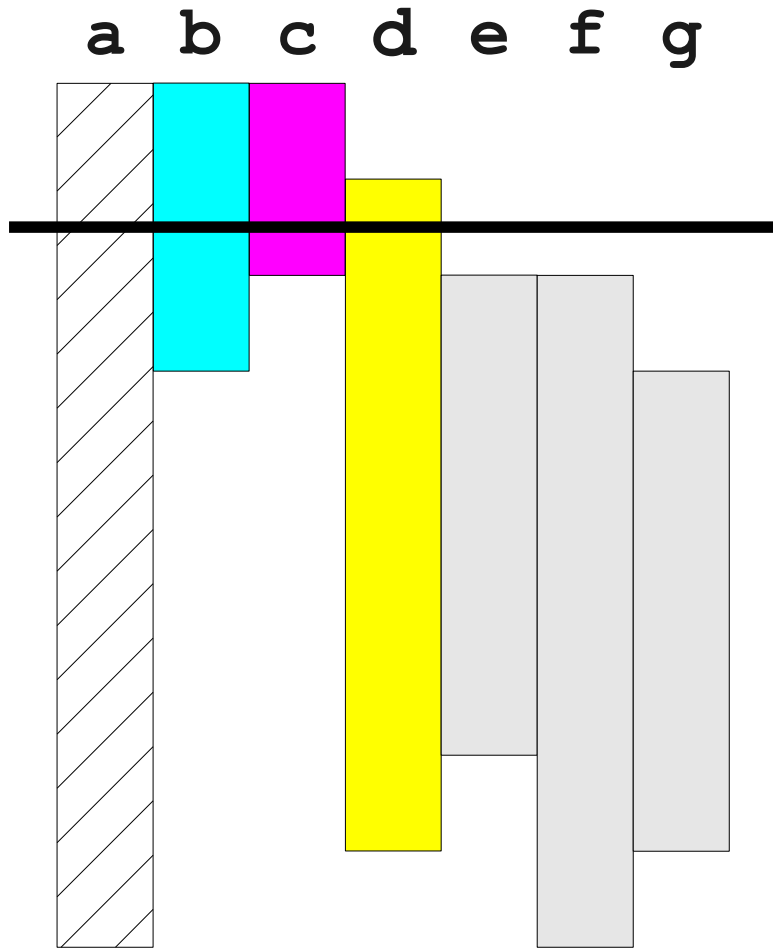
Another Example



Free Registers



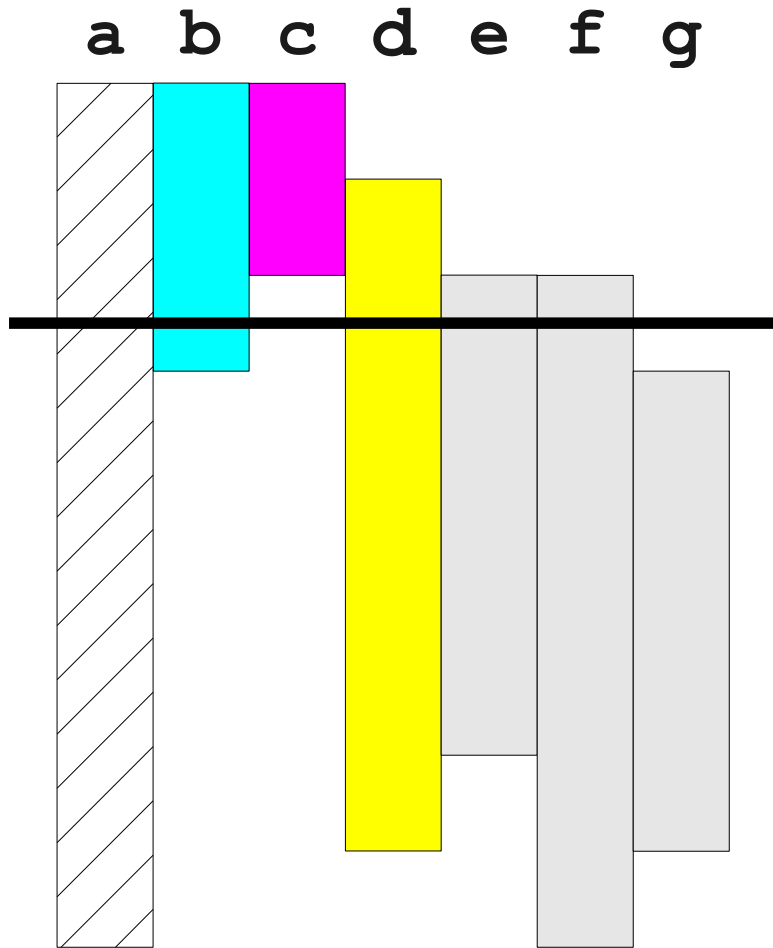
Another Example



Free Registers



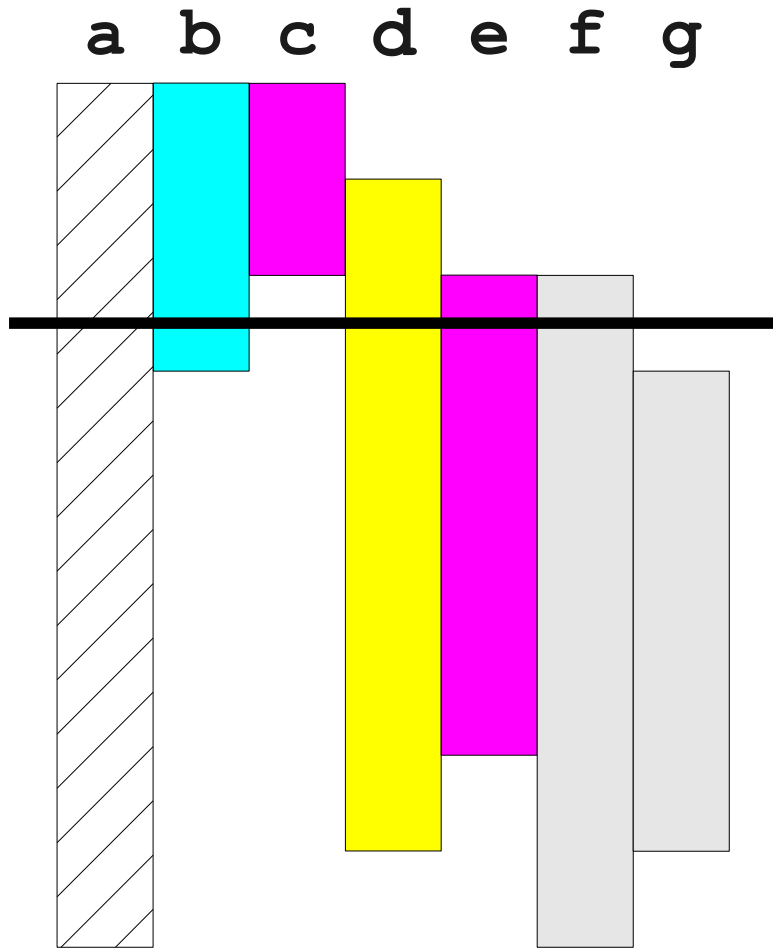
Another Example



Free Registers



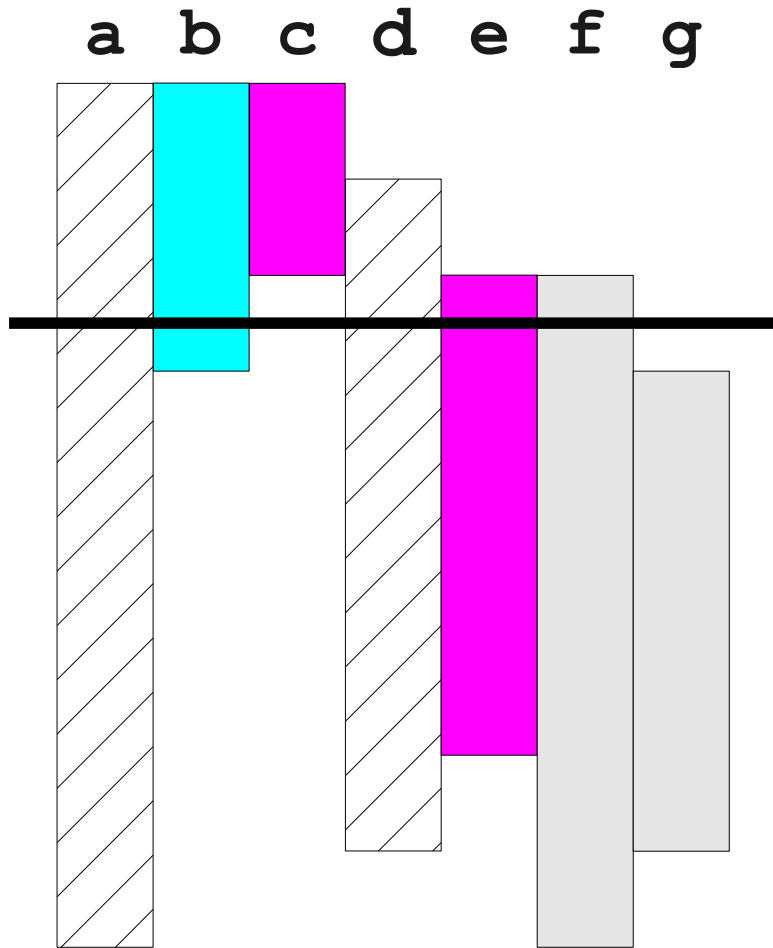
Another Example



Free Registers



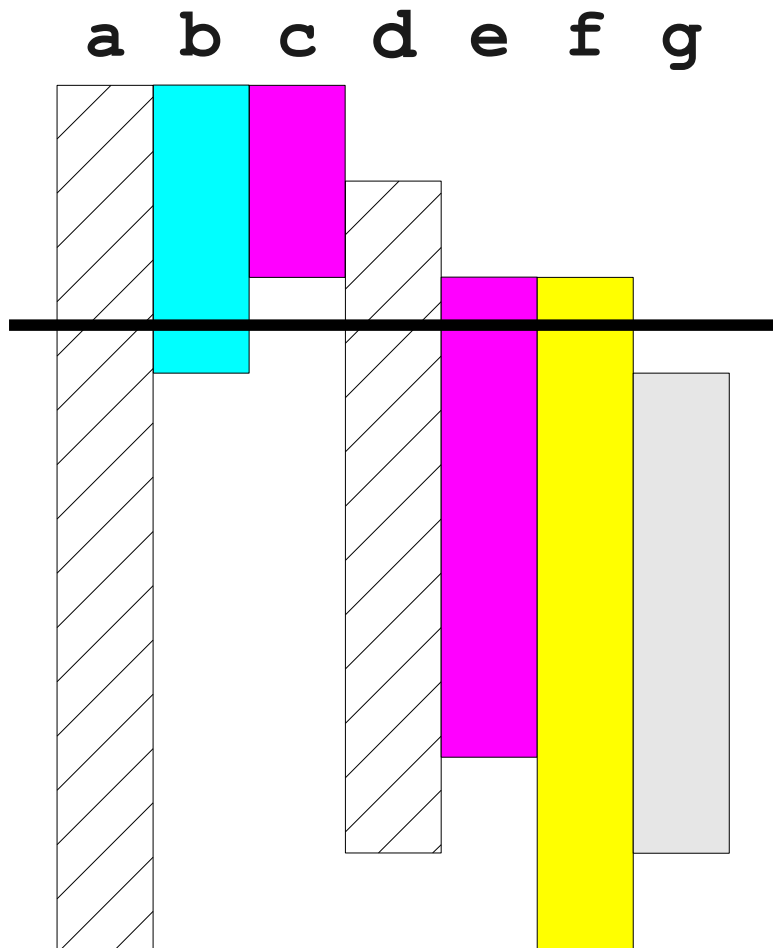
Another Example



Free Registers



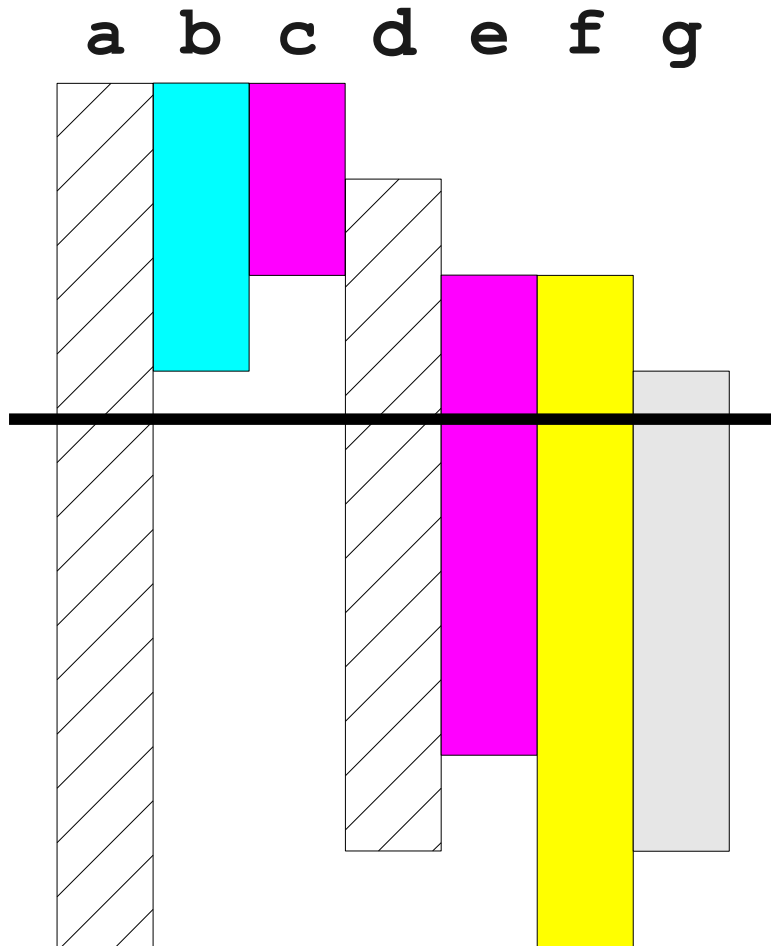
Another Example



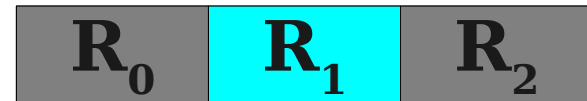
Free Registers



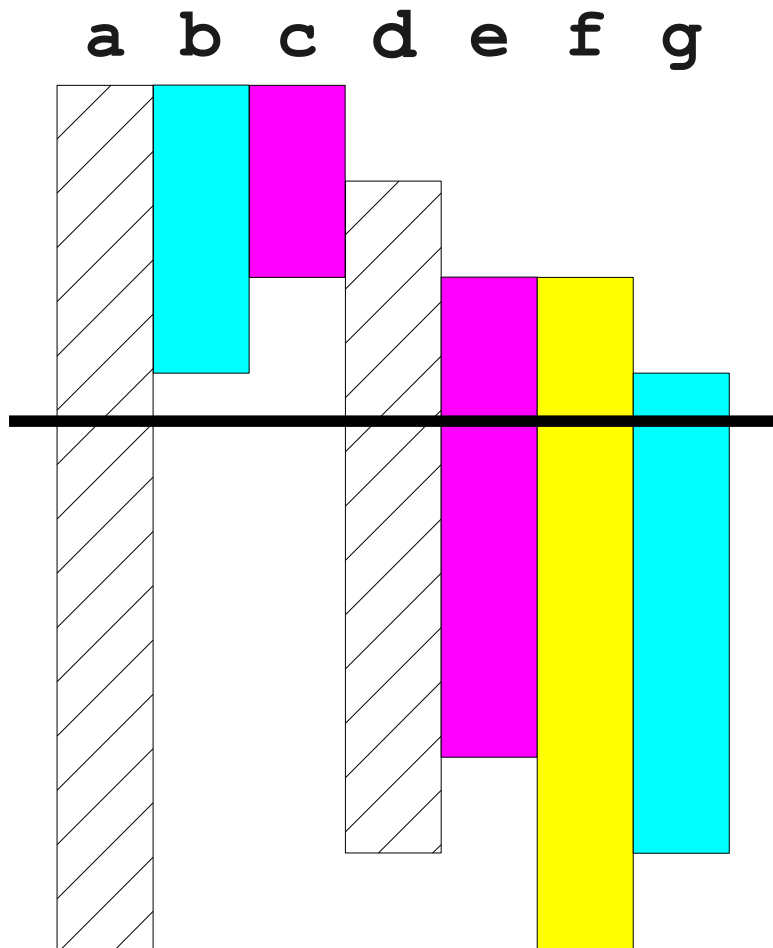
Another Example



Free Registers



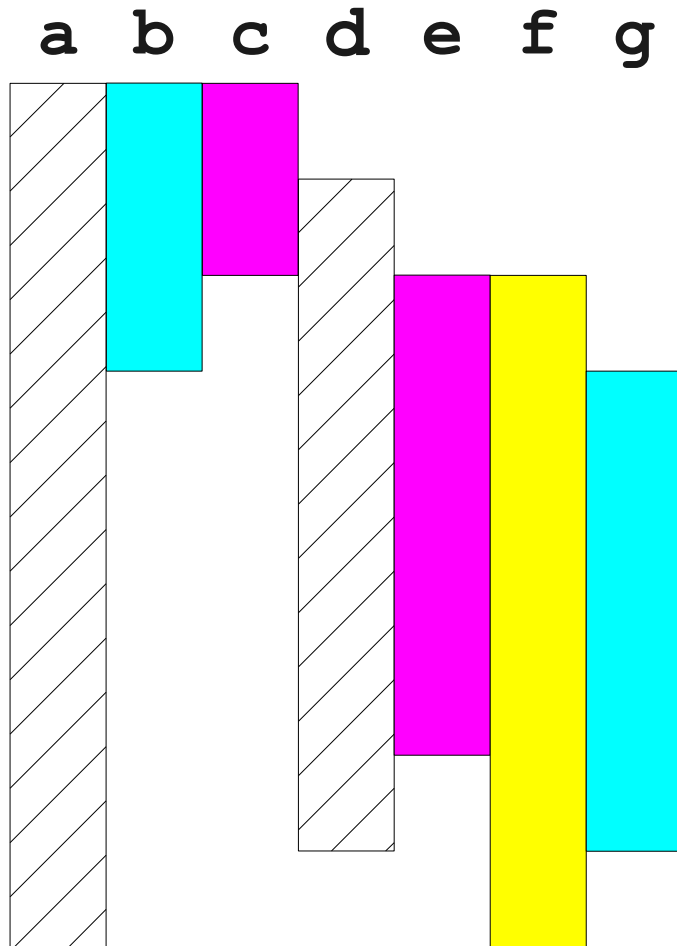
Another Example



Free Registers



Another Example



Free Registers

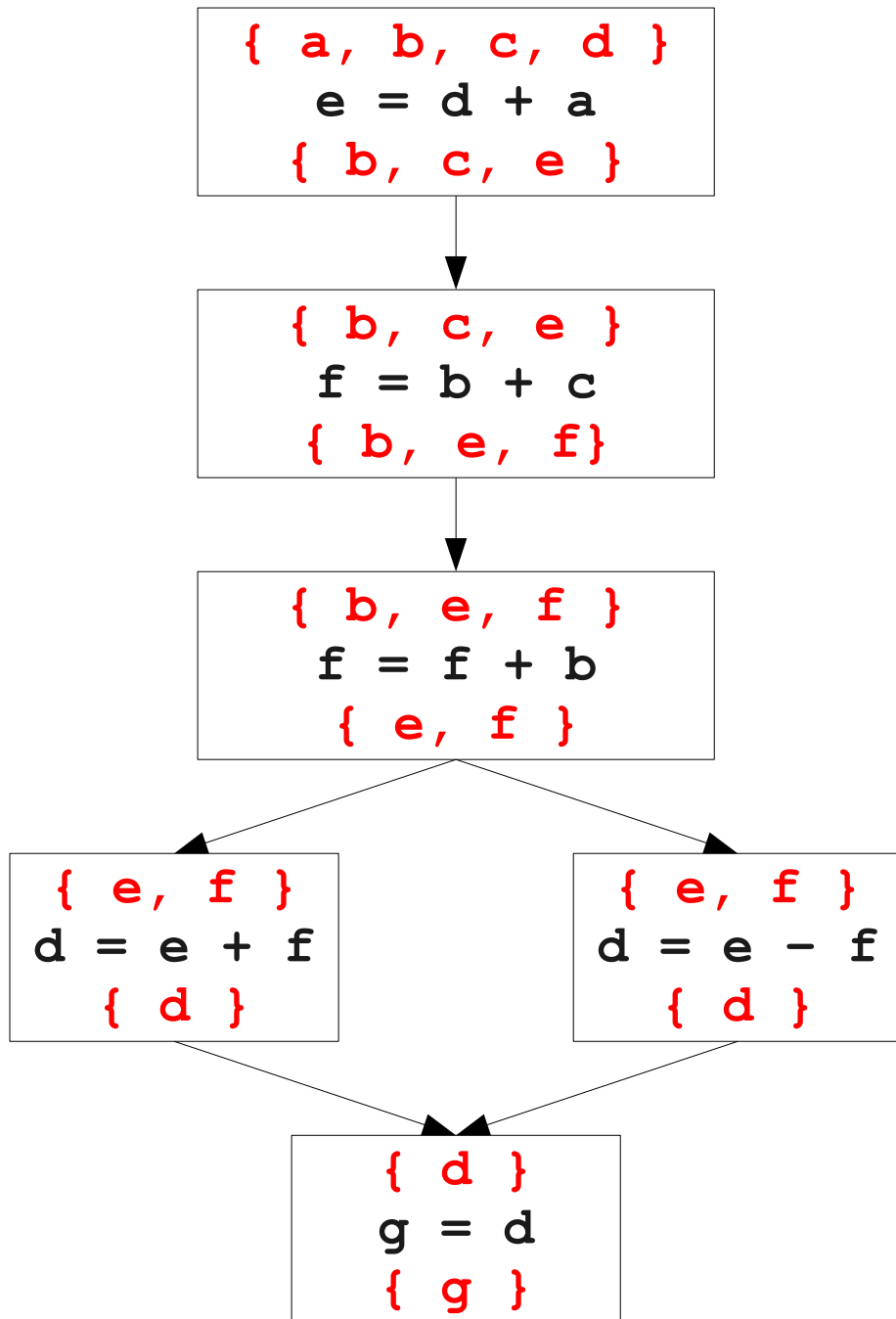


Linear Scan Register Allocation

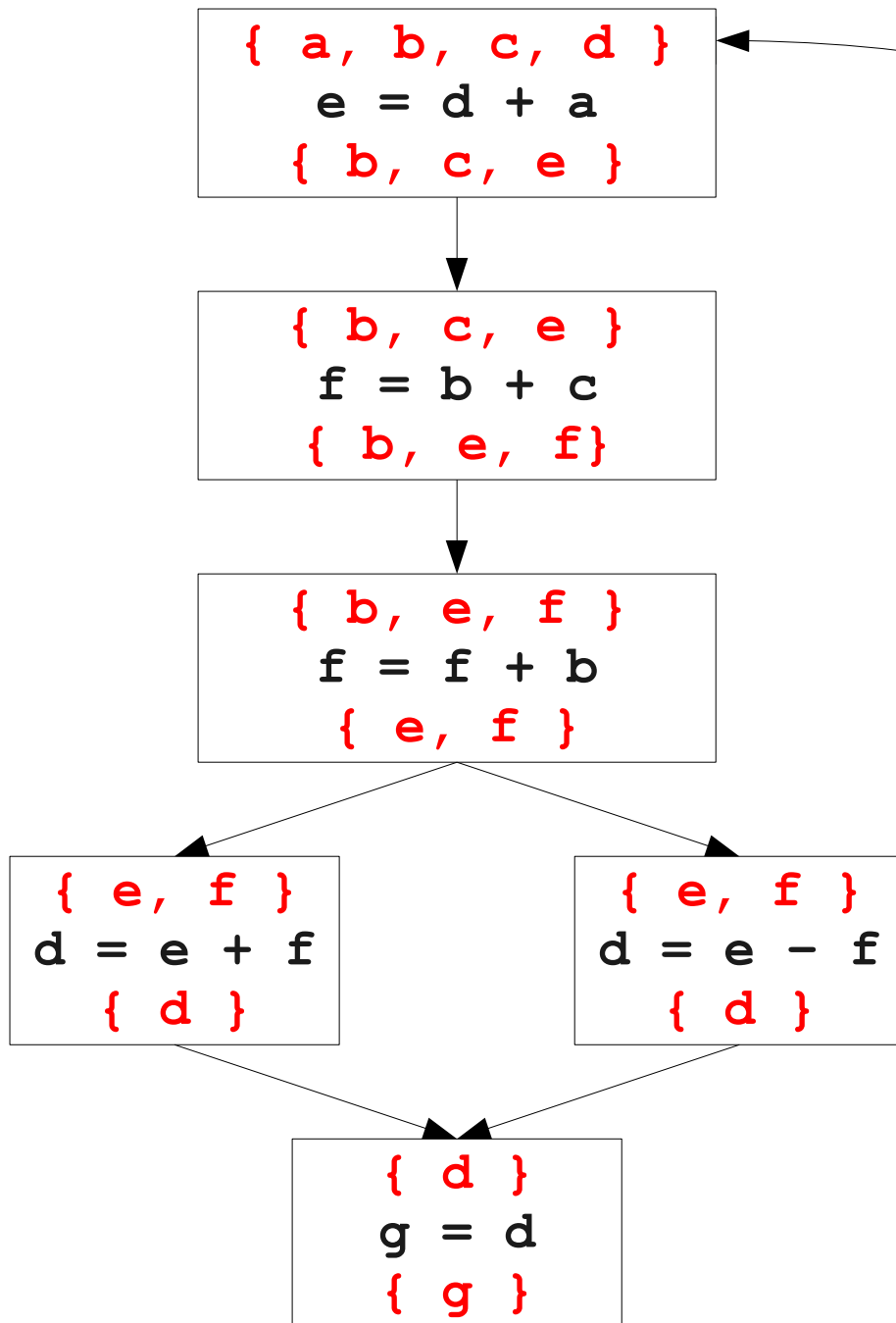
- This algorithm is called **linear scan register allocation** and is a comparatively new algorithm.
- Advantages:
 - Very efficient (after computing live intervals, runs in linear time)
 - Produces good code in many instances.
 - Allocation step works in one pass; can generate code during iteration.
 - Often used in JIT compilers like Java HotSpot.
- Disadvantages:
 - Imprecise due to use of live **intervals** rather than live **ranges**.
 - Other techniques known to be superior in many cases.

An Entirely Different Approach

An Entirely Different Approach

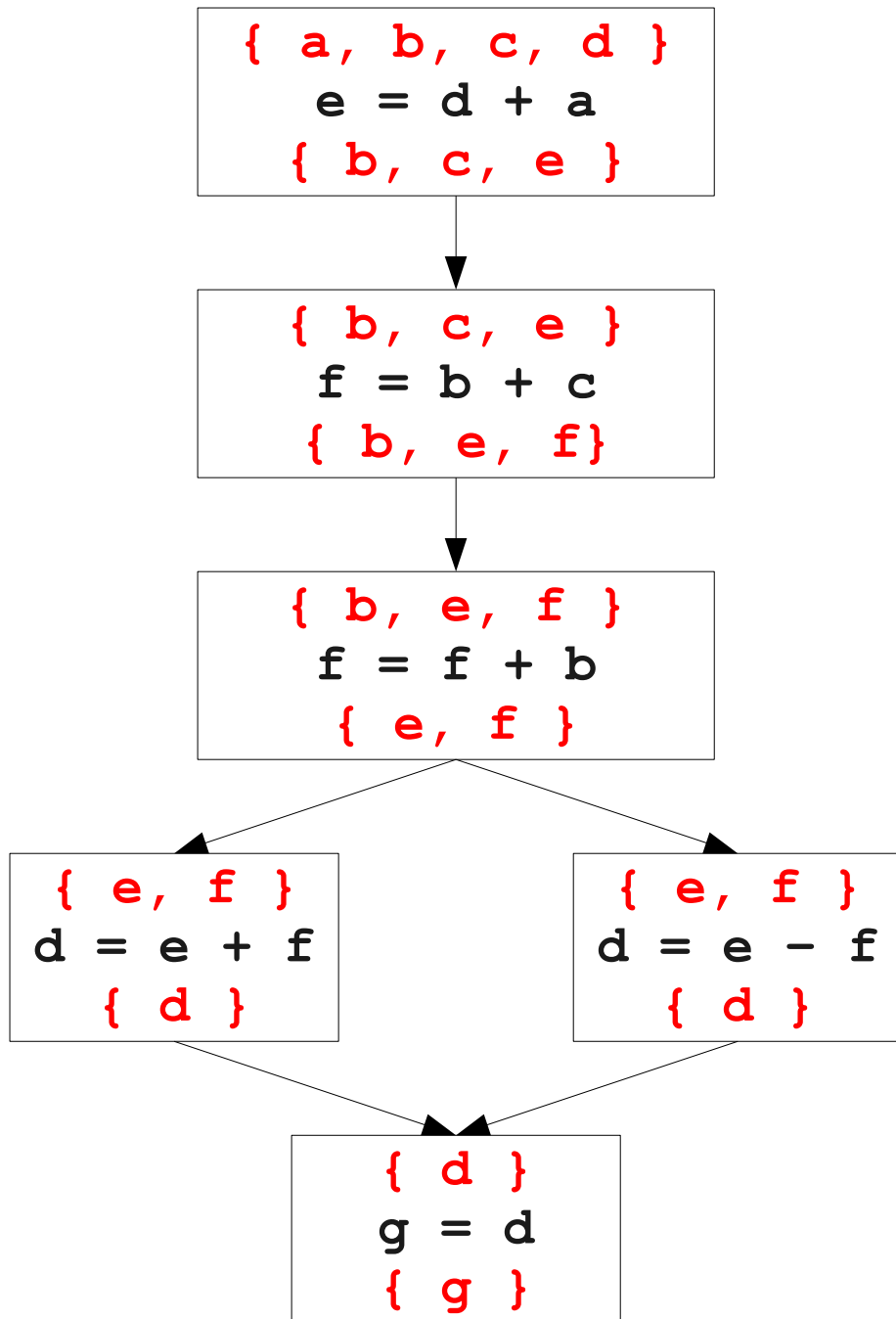


An Entirely Different Approach

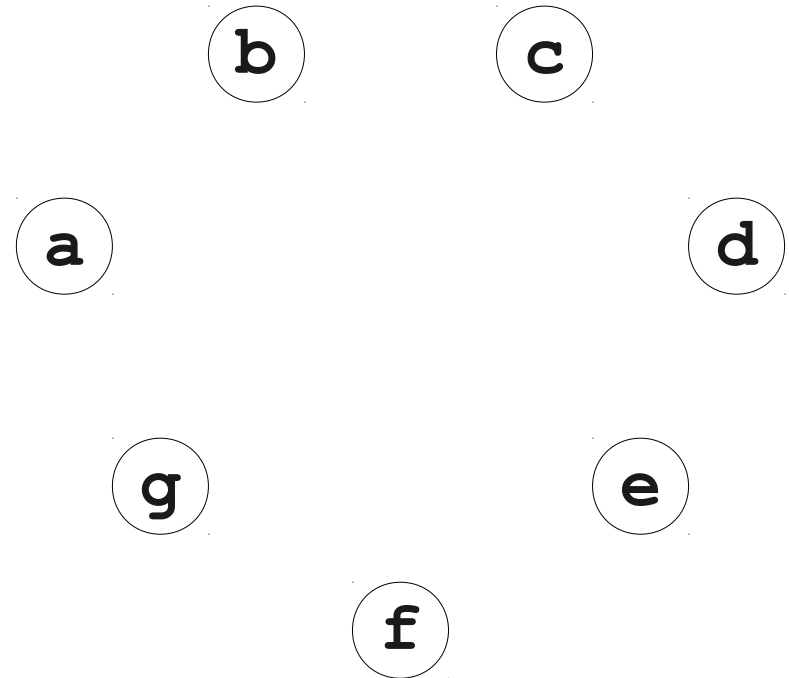
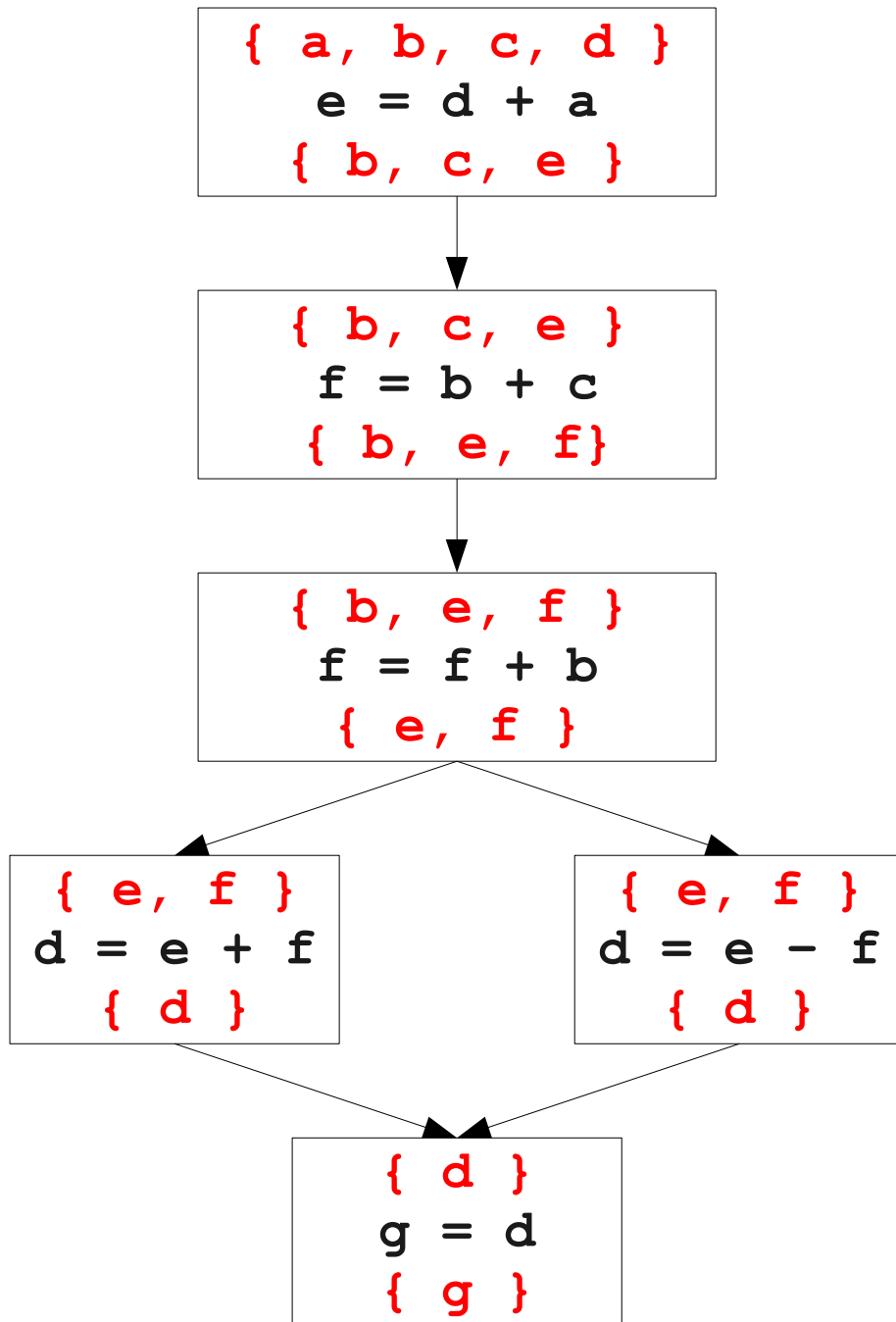


What can we infer from
all these variables being
live at this point?

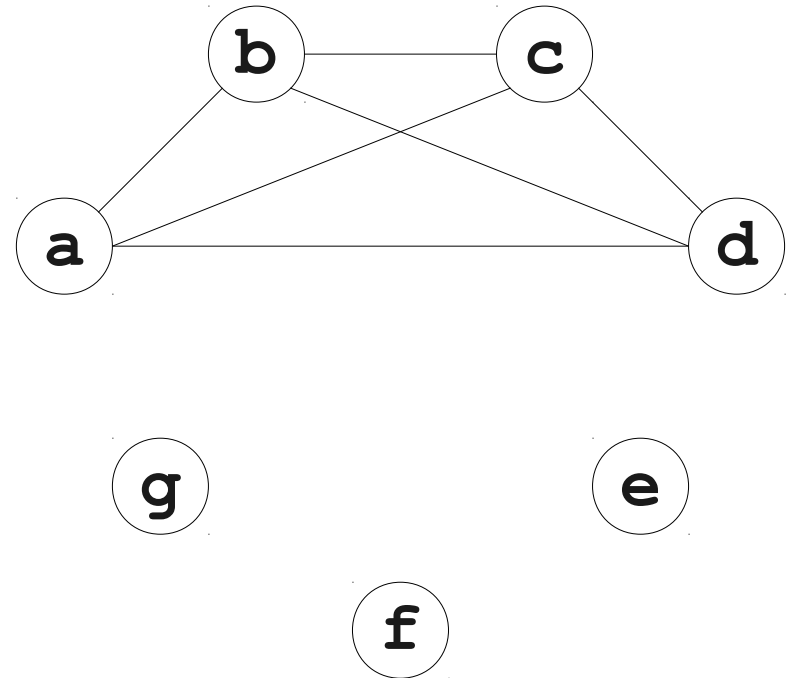
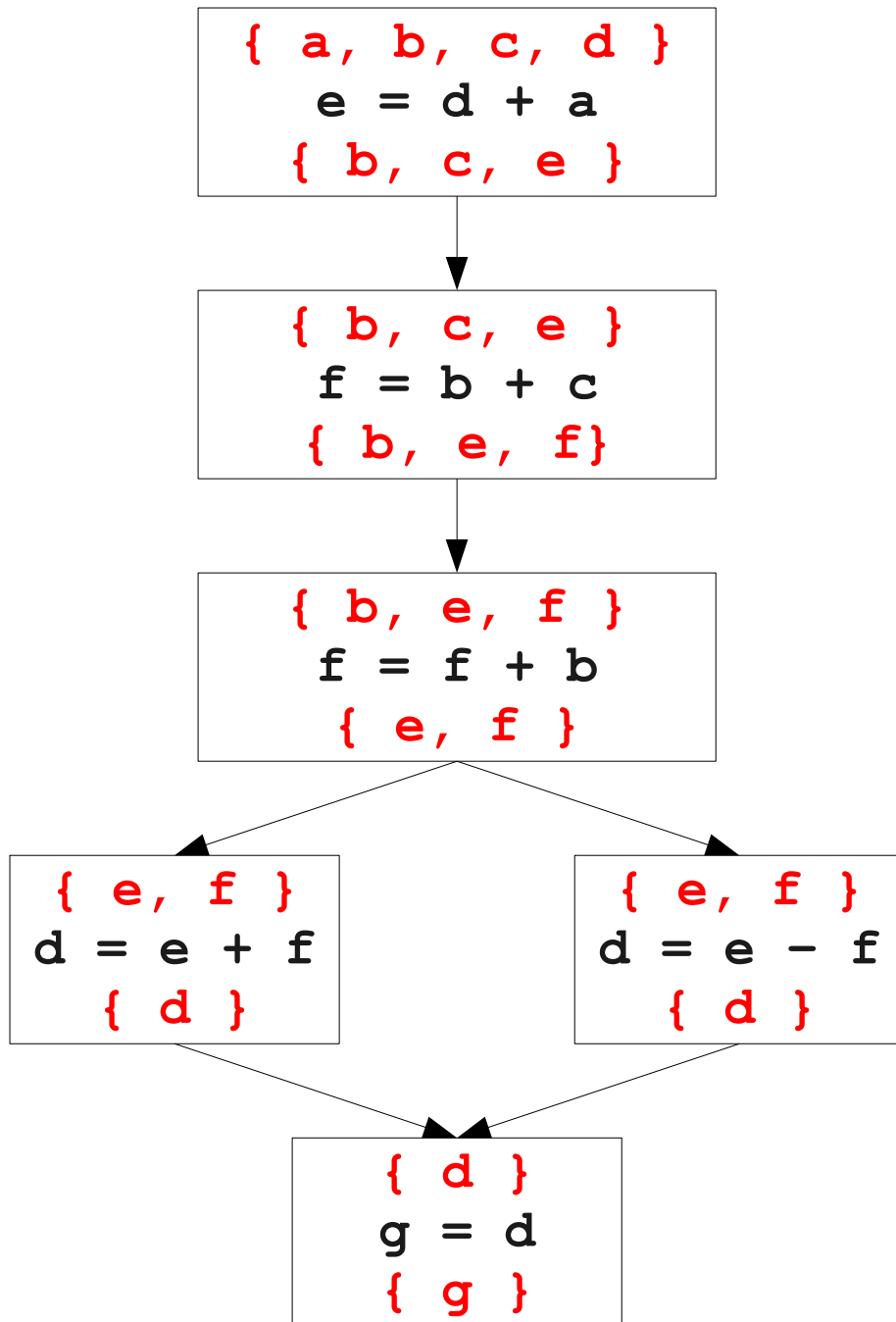
An Entirely Different Approach



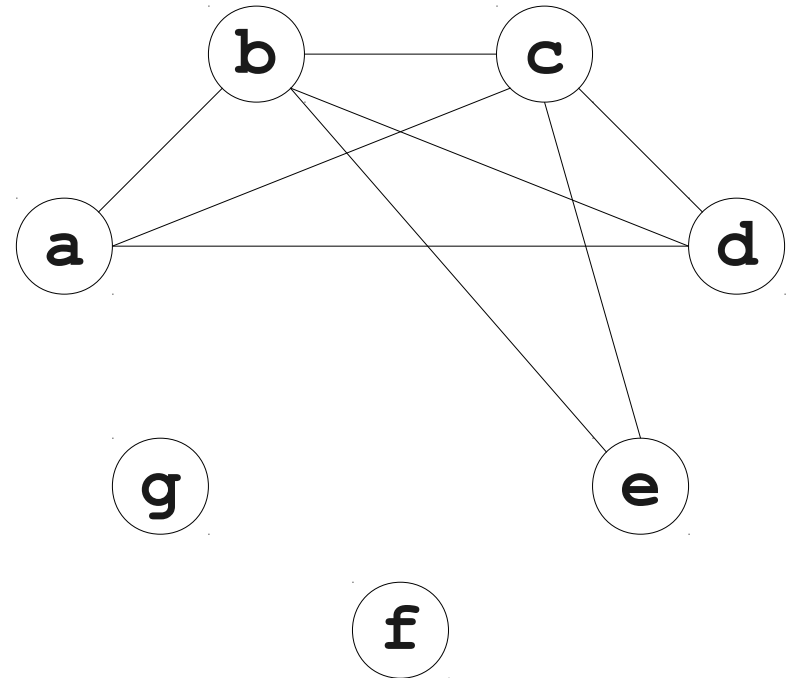
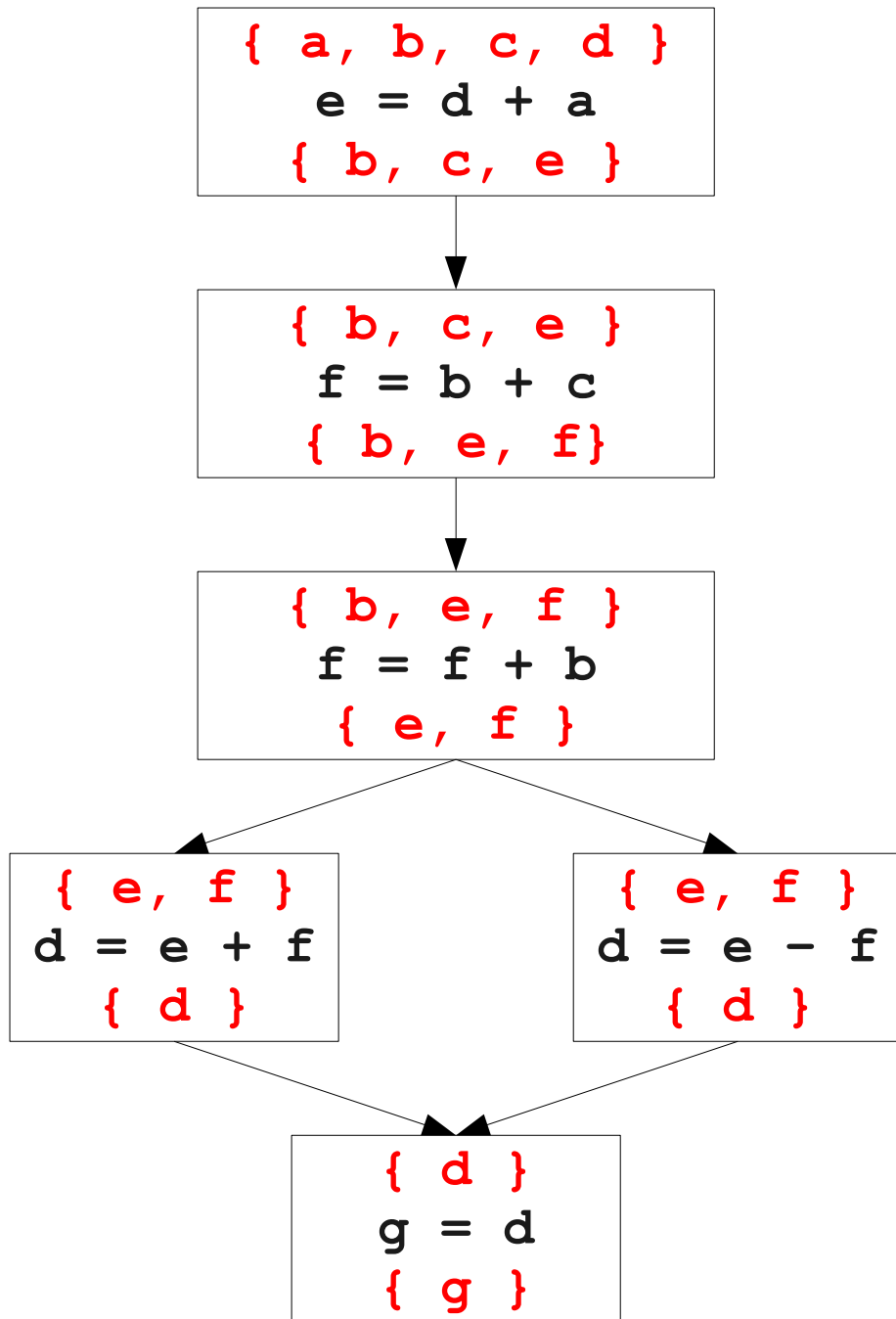
An Entirely Different Approach



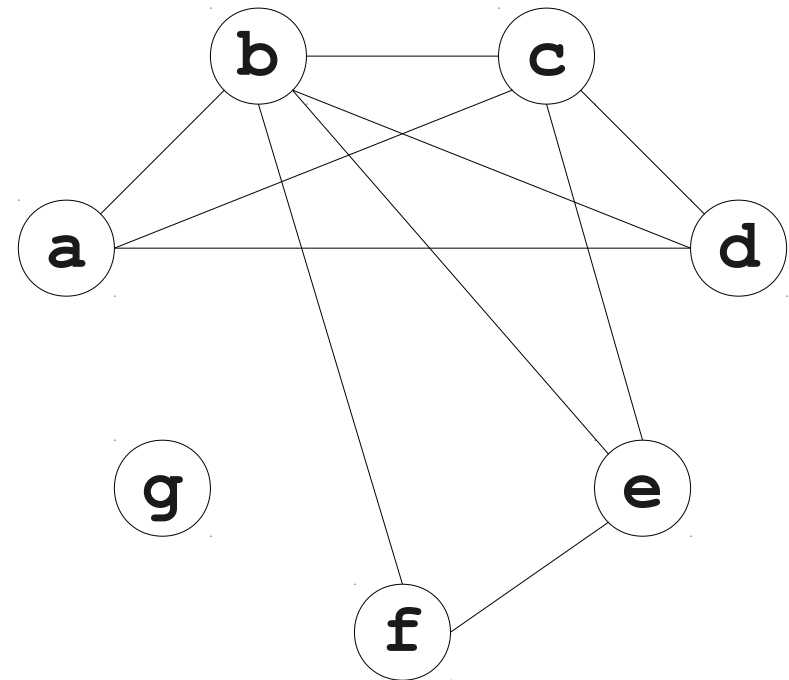
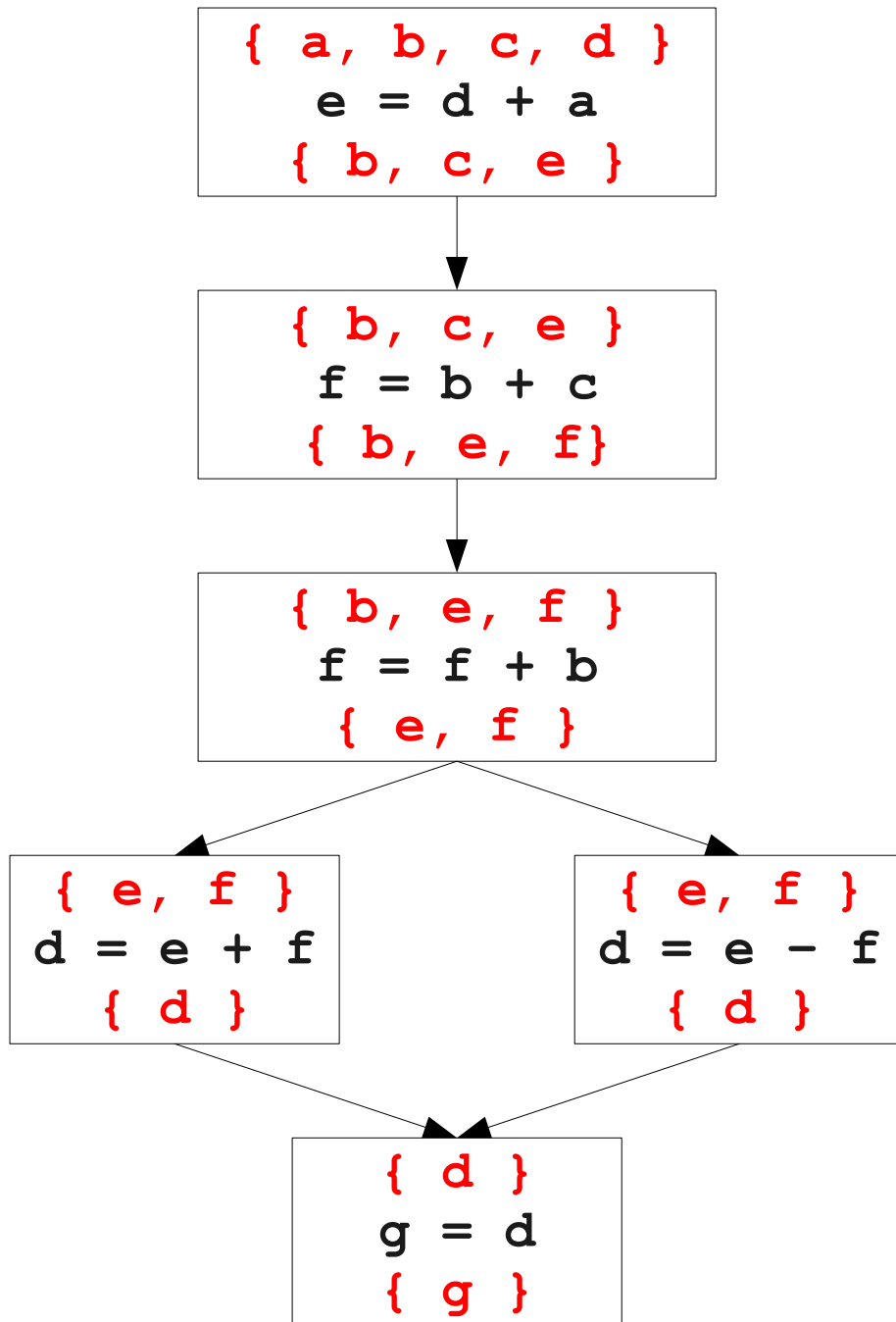
An Entirely Different Approach



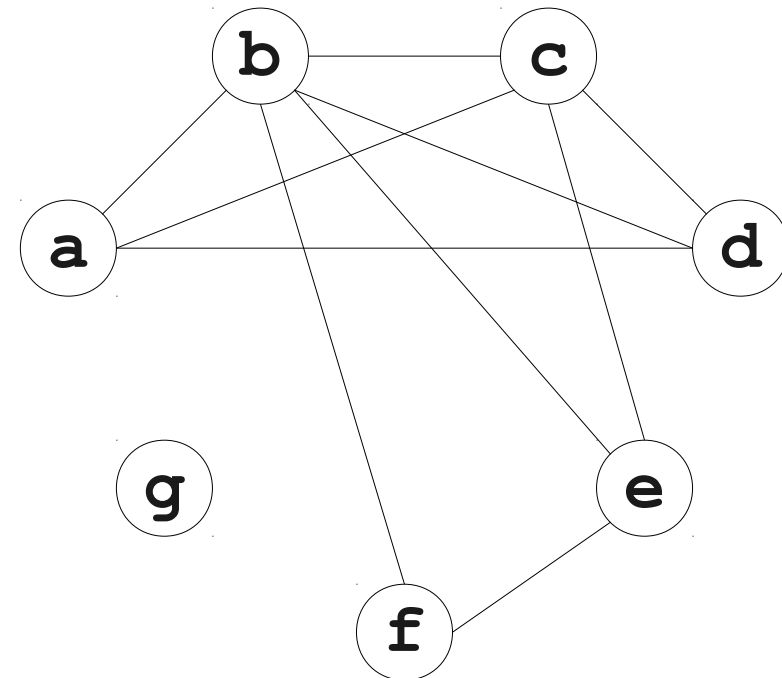
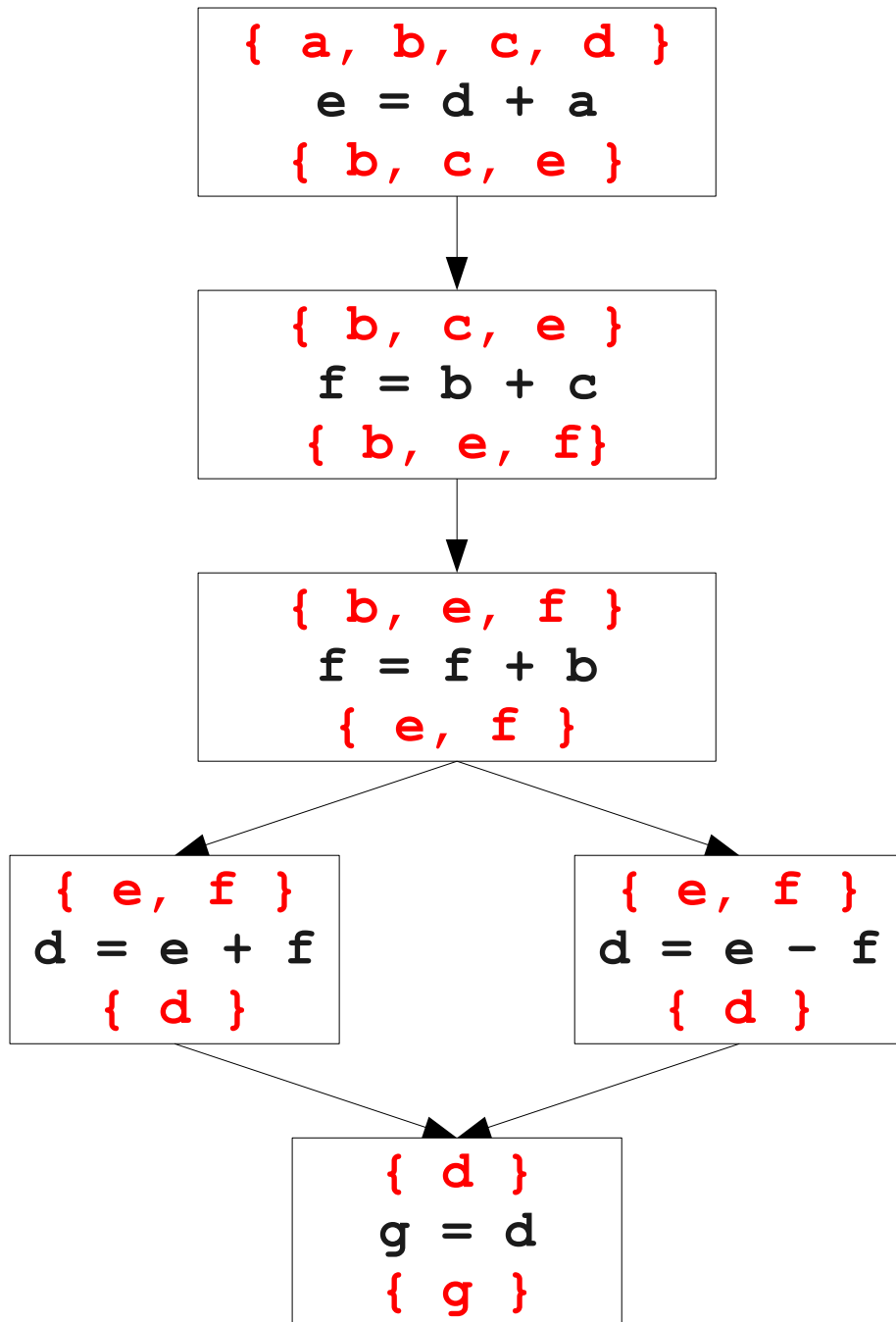
An Entirely Different Approach



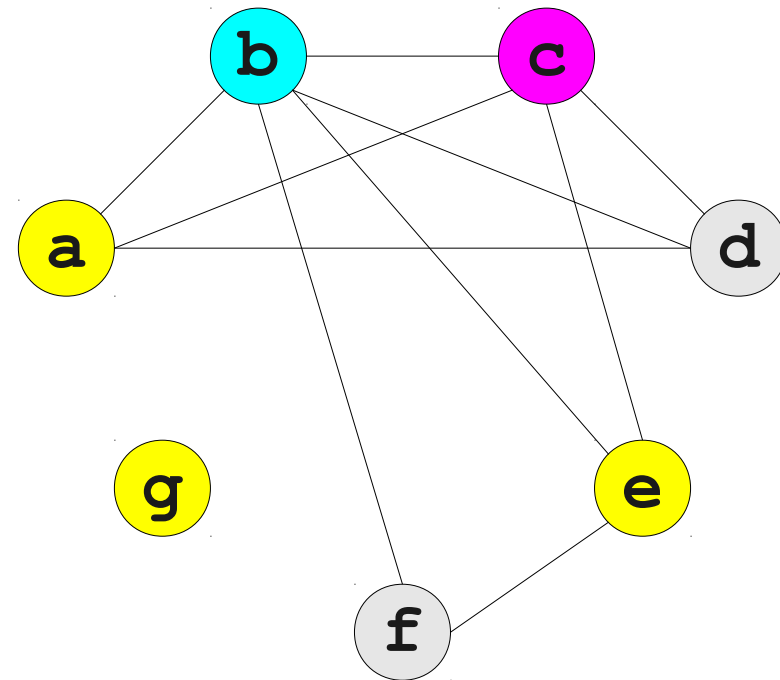
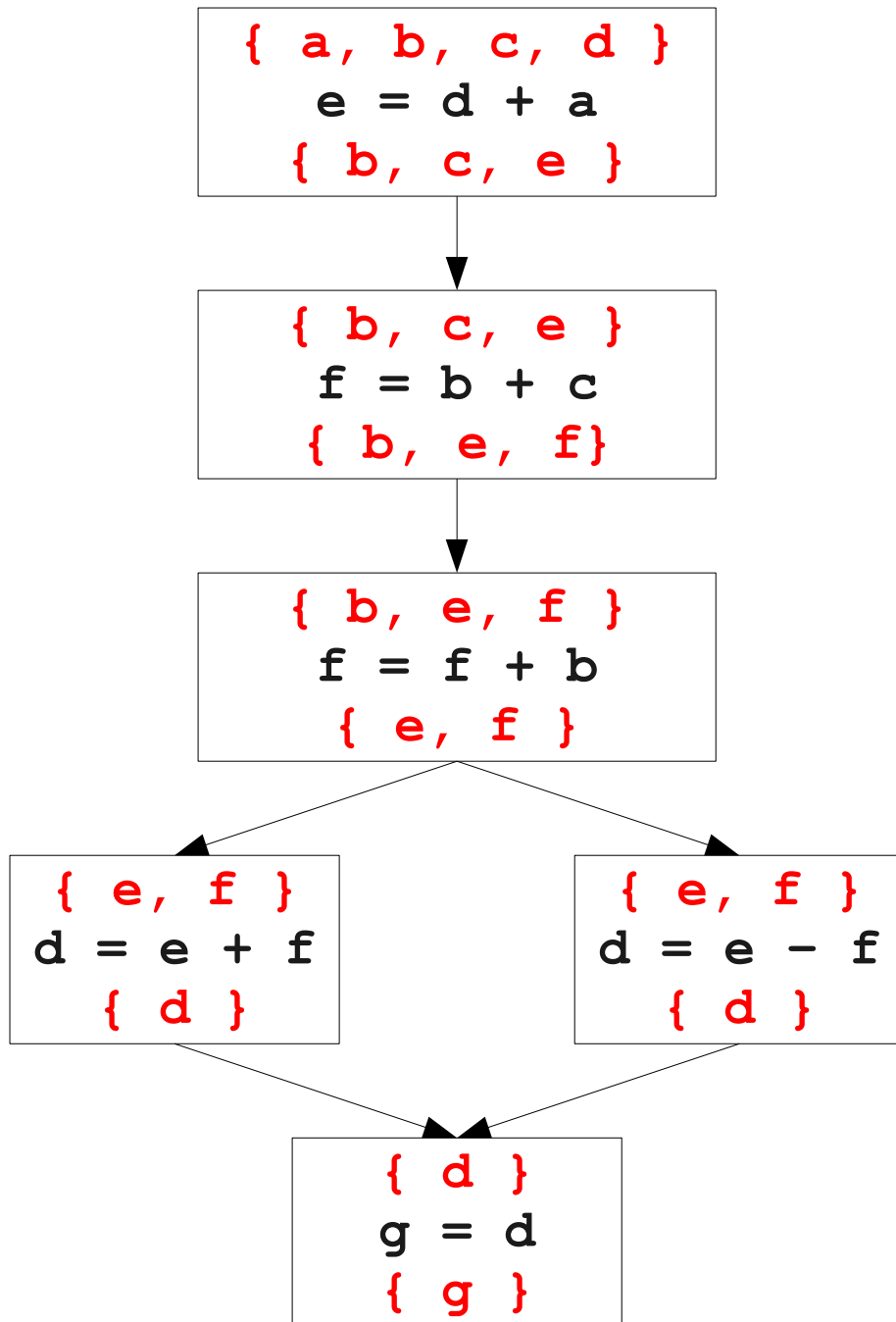
An Entirely Different Approach



An Entirely Different Approach



An Entirely Different Approach



The Register Interference Graph

- The **register interference graph** (RIG) of a control-flow graph is an undirected graph where
 - Each node is a variable.
 - There is an edge between two variables that are live at the same program point.
- Perform register allocation by assigning each variable a different register from all of its neighbors.
- There's just one catch...

The One Catch

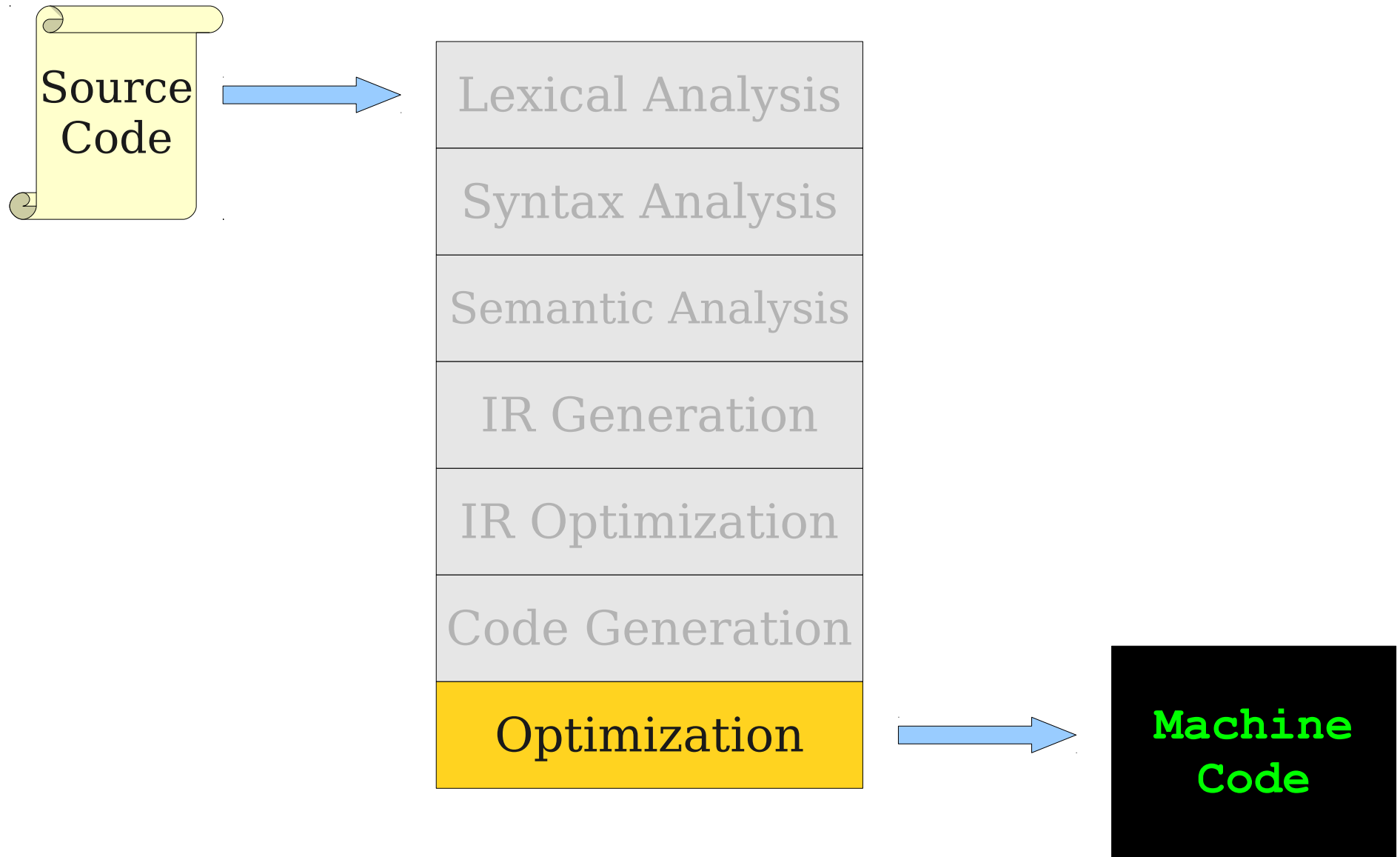
- This problem is equivalent to **graph-coloring**, which is **NP-hard** if there are at least three registers.
- No good polynomial-time algorithms (or even good approximations!) are known for this problem.
- We have to be content with a heuristic that is good enough for RIGs that arise in practice.

Summary of Register Allocation

- Critical step in all optimizing compilers.
- The **linear scan** algorithm uses **live intervals** to greedily assign variables to registers.
 - Often used in JIT compilers due to efficiency.
- **Chaitin's algorithm** uses the **register interference graph** (based on **live ranges**) and **graph coloring** to assign registers.
 - The basis for the technique used in GCC.

Code Optimization

Where We Are



Final Code Optimization

- **Goal:** Optimize generated code by exploiting machine-dependent properties not visible at the IR level.
- Critical step in most compilers, but often very messy:
 - Techniques developed for one machine may be completely useless on another.
 - Techniques developed for one language may be completely useless with another.

Optimizations for Pipelining

Processor Pipelines

Processor Pipelines

```
add $t2, $t0, $t1    # $t2 = $t0 + $t1
add $t5, $t3, $t4    # $t5 = $t3 + $t4
add $t8, $t6, $t7    # $t8 = $t6 + $t7
```

Processor Pipelines

**Instruction
Decoder**

```
add $t2, $t0, $t1    # $t2 = $t0 + $t1
add $t5, $t3, $t4    # $t5 = $t3 + $t4
add $t8, $t6, $t7    # $t8 = $t6 + $t7
```

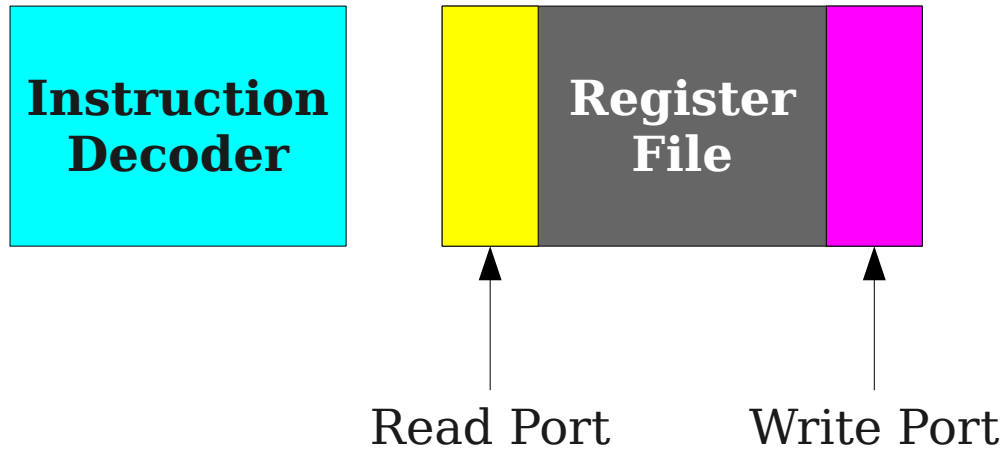
Processor Pipelines

**Instruction
Decoder**

**Register
File**

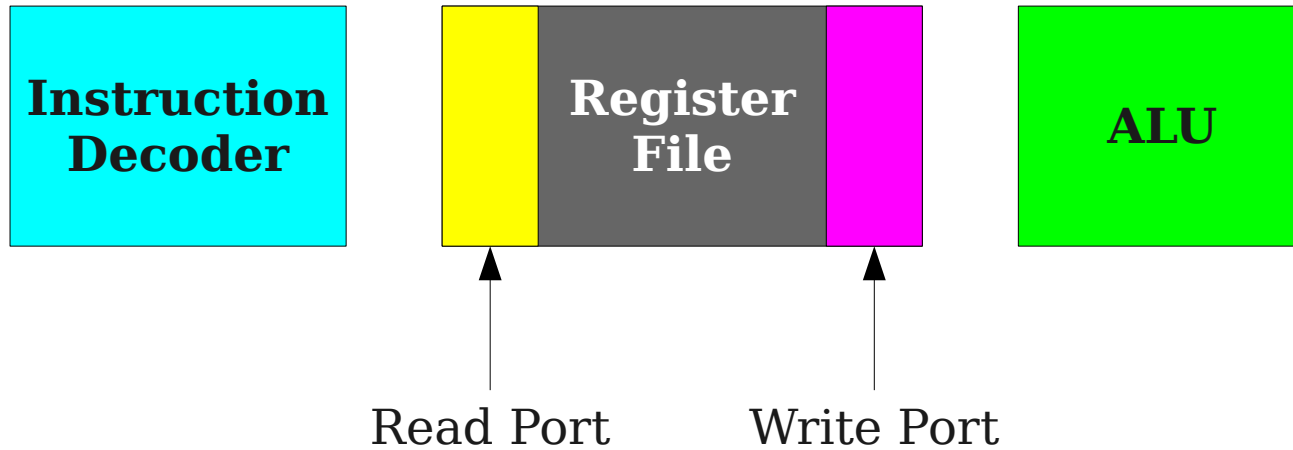
```
add $t2, $t0, $t1    # $t2 = $t0 + $t1
add $t5, $t3, $t4    # $t5 = $t3 + $t4
add $t8, $t6, $t7    # $t8 = $t6 + $t7
```

Processor Pipelines



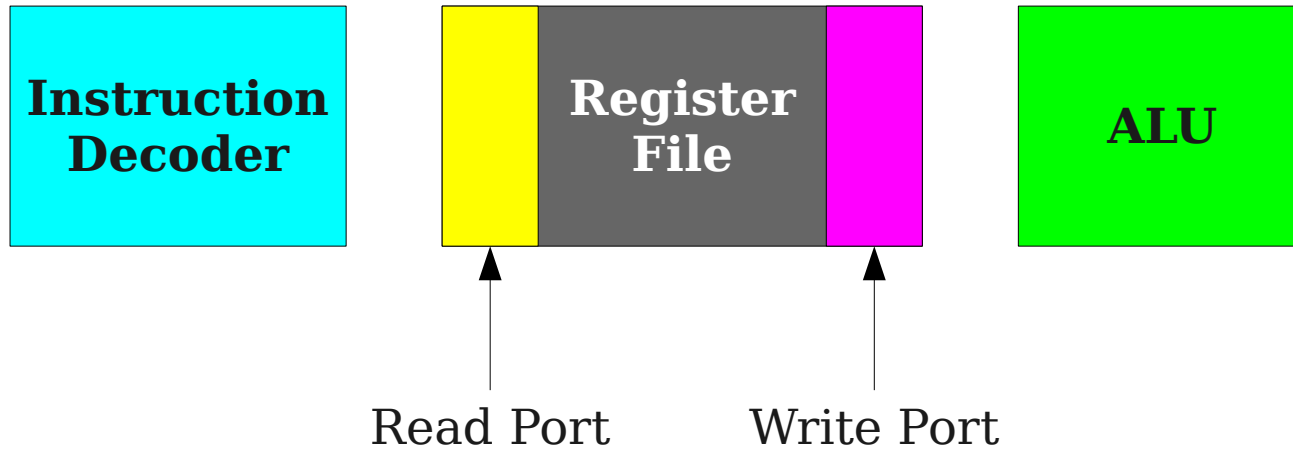
```
add $t2, $t0, $t1    # $t2 = $t0 + $t1
add $t5, $t3, $t4    # $t5 = $t3 + $t4
add $t8, $t6, $t7    # $t8 = $t6 + $t7
```


Processor Pipelines



```
add $t2, $t0, $t1    # $t2 = $t0 + $t1
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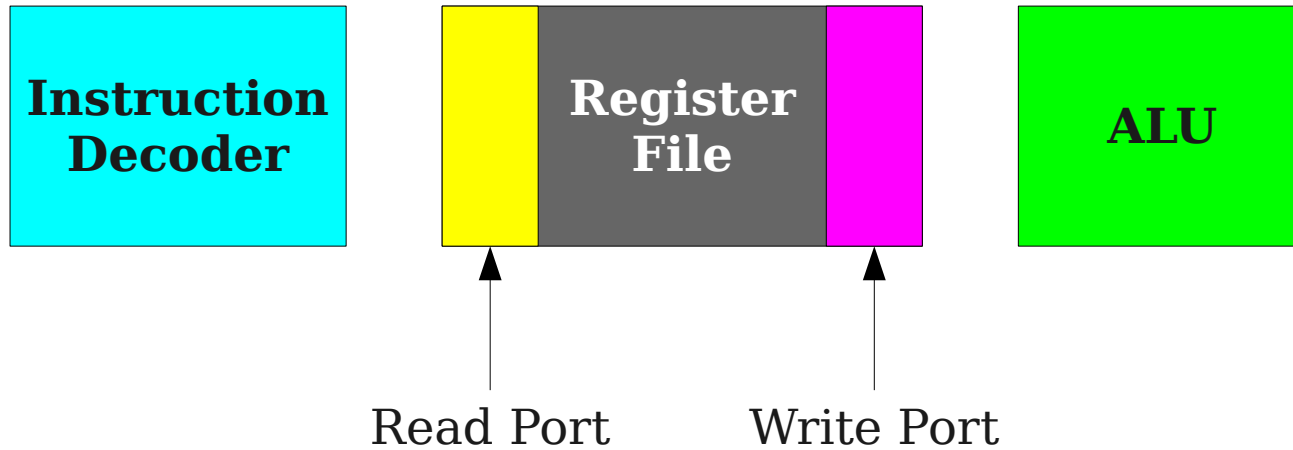
Processor Pipelines



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add $t8, $t6, $t7    # $t8 = $t6 + $t7
```

[illegible]

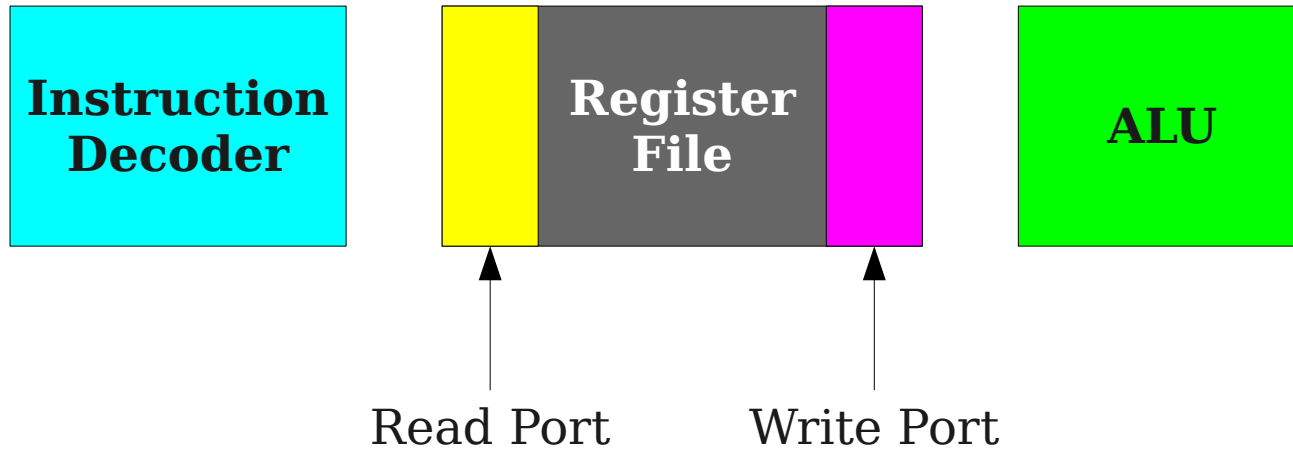
Processor Pipelines



```
add $t2, $t0, $t1    # $t2 = $t0 + $t1
add $t5, $t3, $t4    # $t5 = $t3 + $t4
add $t8, $t6, $t7    # $t8 = $t6 + $t7
```

[illegible]

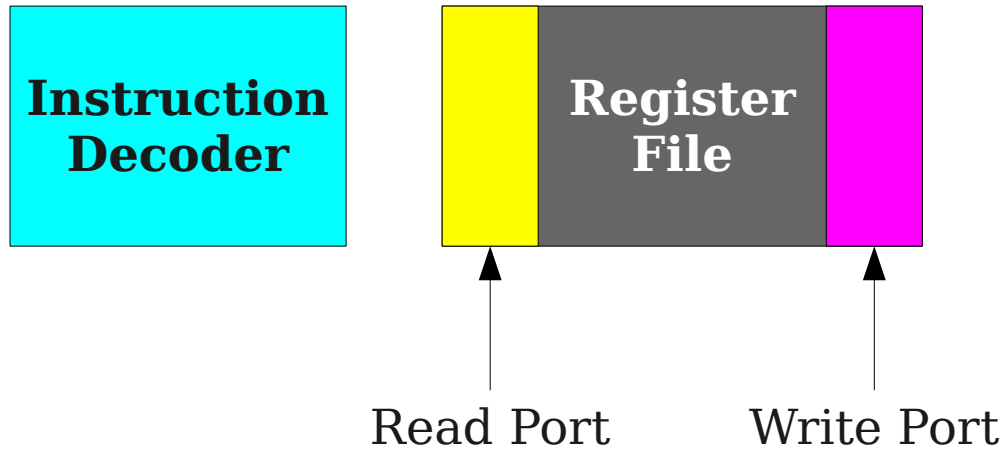
Processor Pipelines



```
add $t2, $t0, $t1    # $t2 = $t0 + $t1
add $t5, $t3, $t4    # $t5 = $t3 + $t4
add $t8, $t6, $t7    # $t8 = $t6 + $t7
```

[illegible]

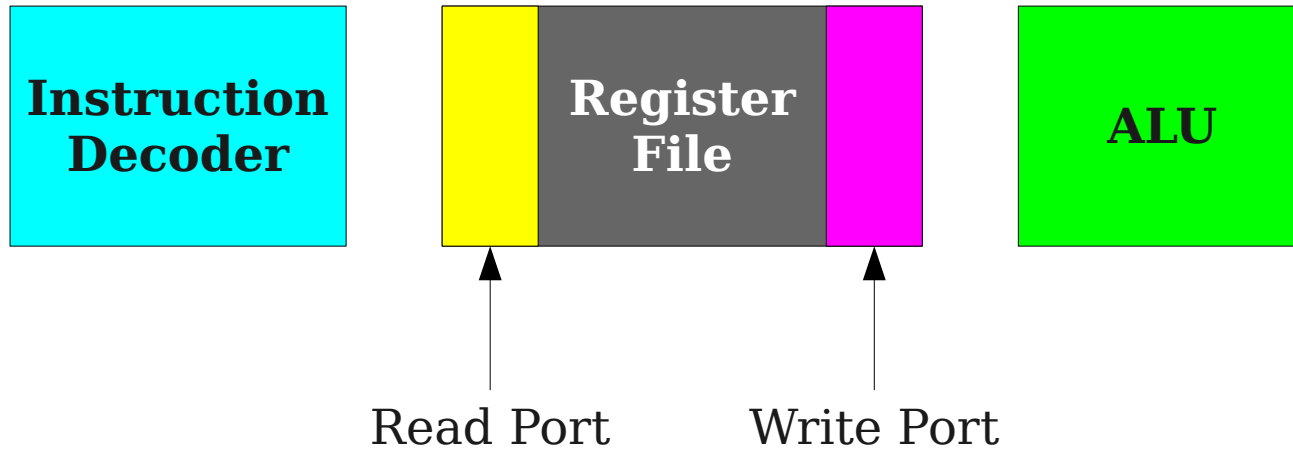
Processor Pipelines



```
add $t2, $t0, $t1    # $t2 = $t0 + $t1
add $t5, $t3, $t4    # $t5 = $t3 + $t4
add $t8, $t6, $t7    # $t8 = $t6 + $t7
```

| ID | RR | ALU | RW |
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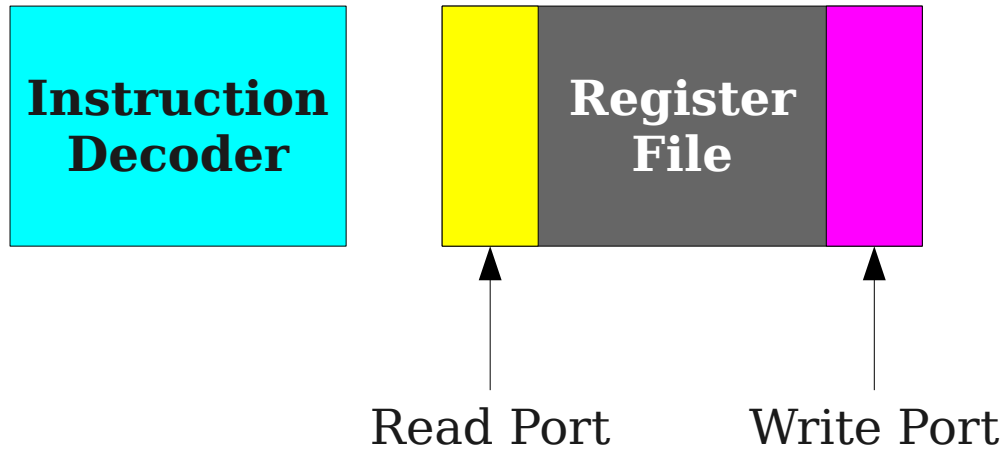
Processor Pipelines



```
add $t2, $t0, $t1    # $t2 = $t0 + $t1
add $t5, $t3, $t4    # $t5 = $t3 + $t4
add $t8, $t6, $t7    # $t8 = $t6 + $t7
```

| ID | RR | ALU | RW |
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Processor Pipelines

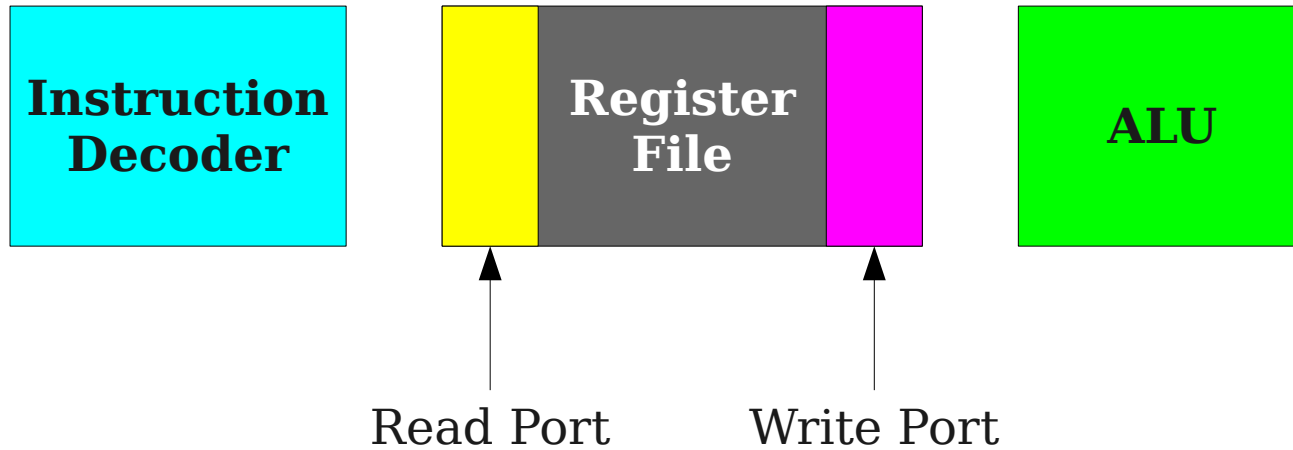


```
add $t2, $t0, $t1    # $t2 = $t0 + $t1
add $t5, $t3, $t4    # $t5 = $t3 + $t4
add $t8, $t6, $t7    # $t8 = $t6 + $t7
```

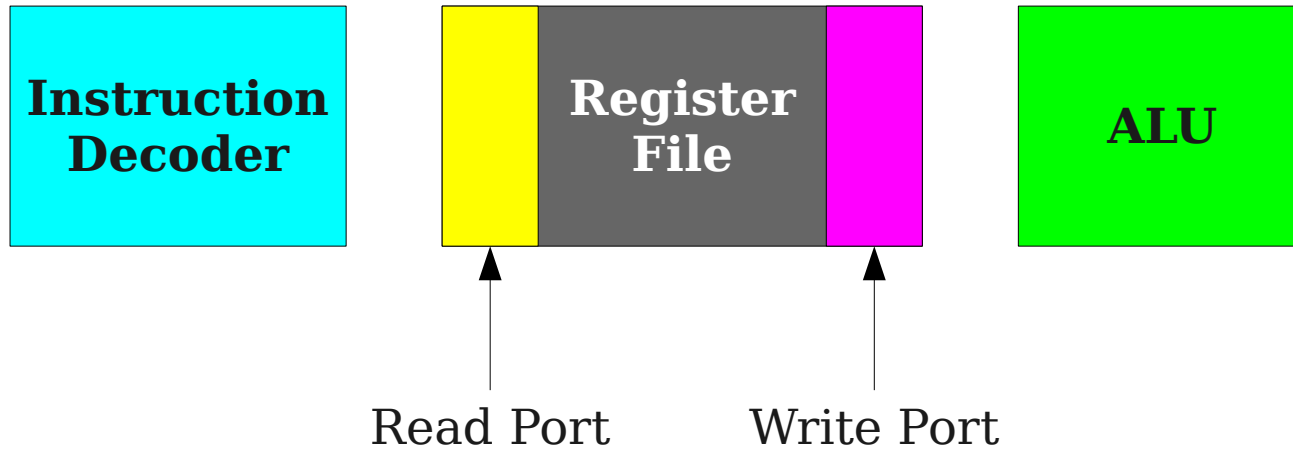
[illegible]

Pipeline Hazards

Pipeline Hazards

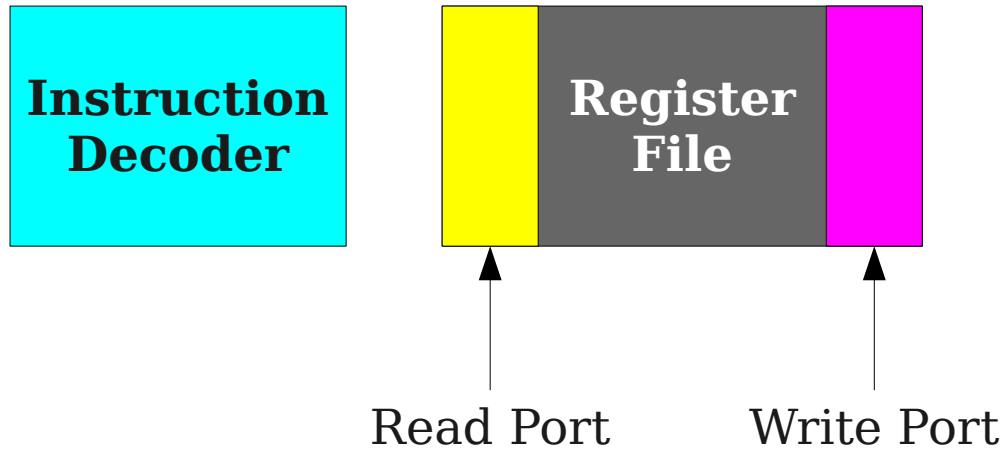


Pipeline Hazards



```
add $t2, $t0, $t1    # $t2 = $t0 + $t1
add $t4, $t3, $t2    # $t5 = $t3 + $t2
add $t7, $t5, $t6    # $t7 = $t5 + $t6
add $t0, $t0, $t7    # $t0 = $t0 + $t7
```

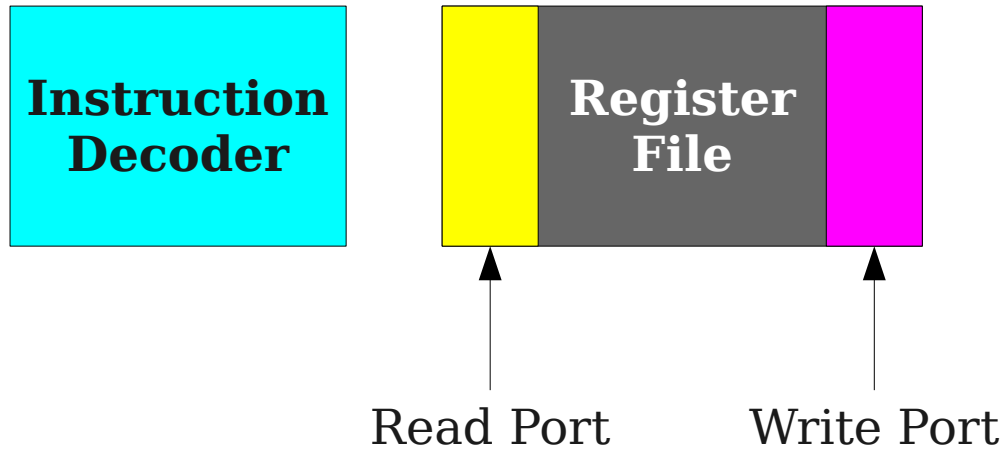
Pipeline Hazards



```
add $t2, $t0, $t1    # $t2 = $t0 + $t1
add $t4, $t3, $t2    # $t5 = $t3 + $t2
add $t7, $t5, $t6    # $t7 = $t5 + $t6
add $t0, $t0, $t7    # $t0 = $t0 + $t7
```

[illegible]

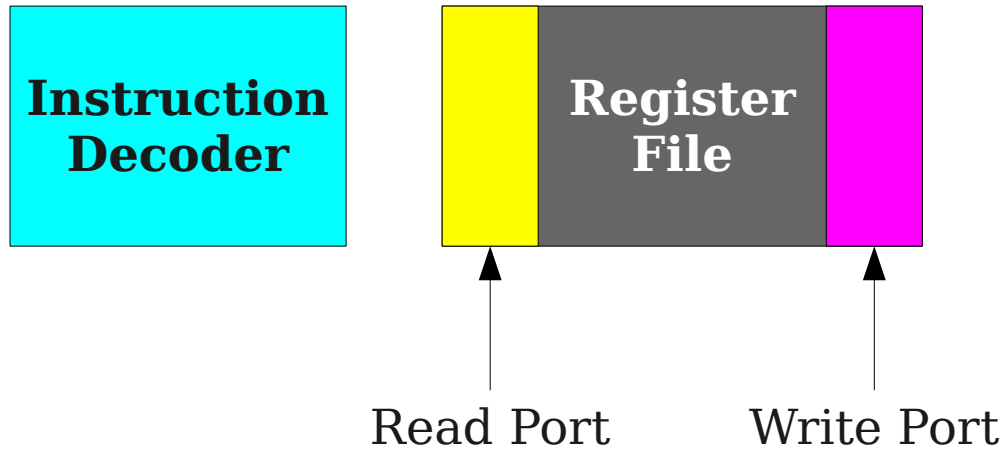
Pipeline Hazards



```
add $t2, $t0, $t1    # $t2 = $t0 + $t1
add $t4, $t3, $t2    # $t5 = $t3 + $t2
add $t7, $t5, $t6    # $t7 = $t5 + $t6
add $t0, $t0, $t7    # $t0 = $t0 + $t7
```

[illegible]

Pipeline Hazards

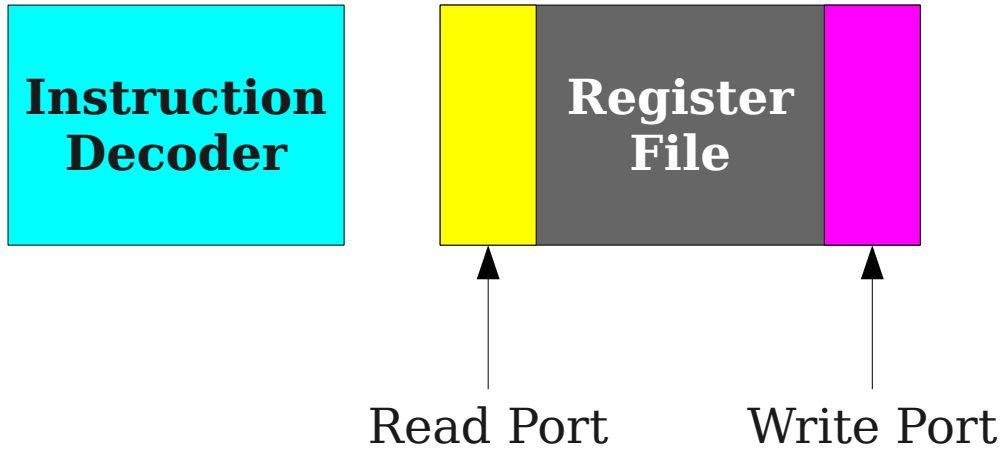


```
add $t2, $t0, $t1    # $t2 = $t0 + $t1
add $t4, $t3, $t2    # $t5 = $t3 + $t2
add $t7, $t5, $t6    # $t7 = $t5 + $t6
add $t0, $t0, $t7    # $t0 = $t0 + $t7
```

[illegible]

Pipeline Hazards

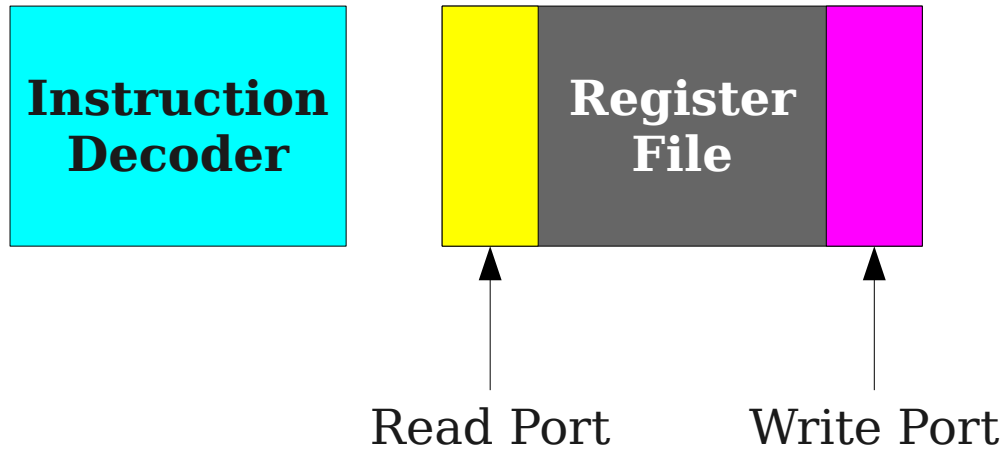
This value isn't ready yet!



```
add $t2, $t0, $t1    # $t2 = $t0 + $t1
add $t4, $t3, $t2    # $t5 = $t3 + $t2
add $t7, $t5, $t6    # $t7 = $t5 + $t6
add $t0, $t0, $t7    # $t0 = $t0 + $t7
```

[illegible]

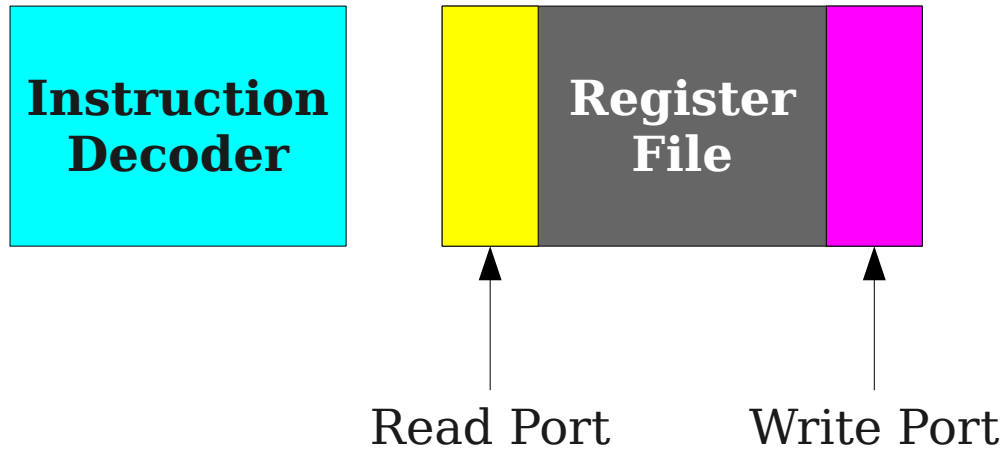
Pipeline Hazards



```
add $t2, $t0, $t1    # $t2 = $t0 + $t1
add $t4, $t3, $t2    # $t5 = $t3 + $t2
add $t7, $t5, $t6    # $t7 = $t5 + $t6
add $t0, $t0, $t7    # $t0 = $t0 + $t7
```

[illegible]

Pipeline Hazards

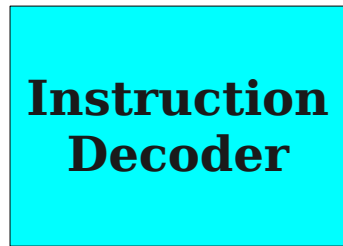


```
add $t2, $t0, $t1    # $t2 = $t0 + $t1
add $t4, $t3, $t2    # $t5 = $t3 + $t2
add $t7, $t5, $t6    # $t7 = $t5 + $t6
add $t0, $t0, $t7    # $t0 = $t0 + $t7
```

| ID | RR | ALU | RW |
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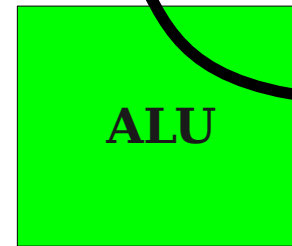
Pipeline Hazards

stall pipeline until
value is ready



Read Port

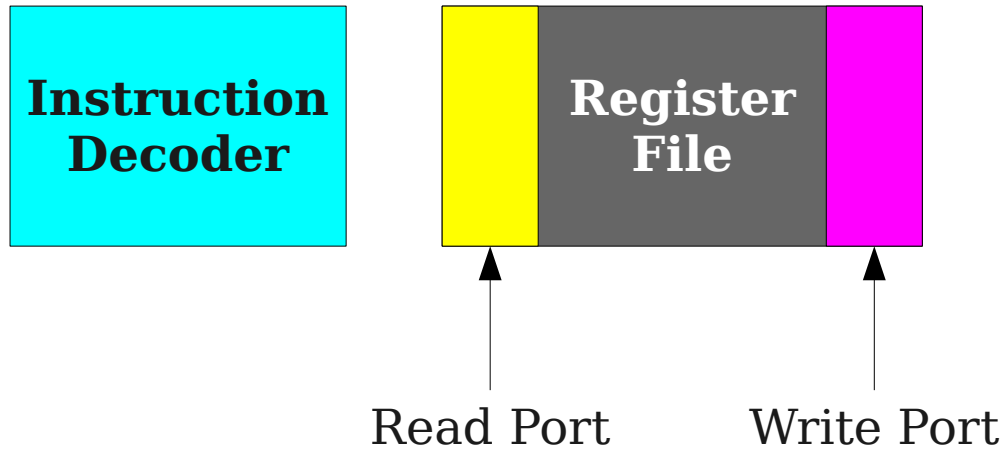
Write Port



| | ID | RR | ALU | RW |
|---|-------|-------|-------|-------|
| | Red | | | |
| | Green | Red | | |
| ▶ | Green | | Red | |
| | Green | | | Red |
| | | Green | | |
| | | | Green | |
| | | | | Green |
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```
add $t2, $t0, $t1    # $t2 = $t0 + $t1
add $t4, $t3, $t2    # $t5 = $t3 + $t2
add $t7, $t5, $t6    # $t7 = $t5 + $t6
add $t0, $t0, $t7    # $t0 = $t0 + $t7
```

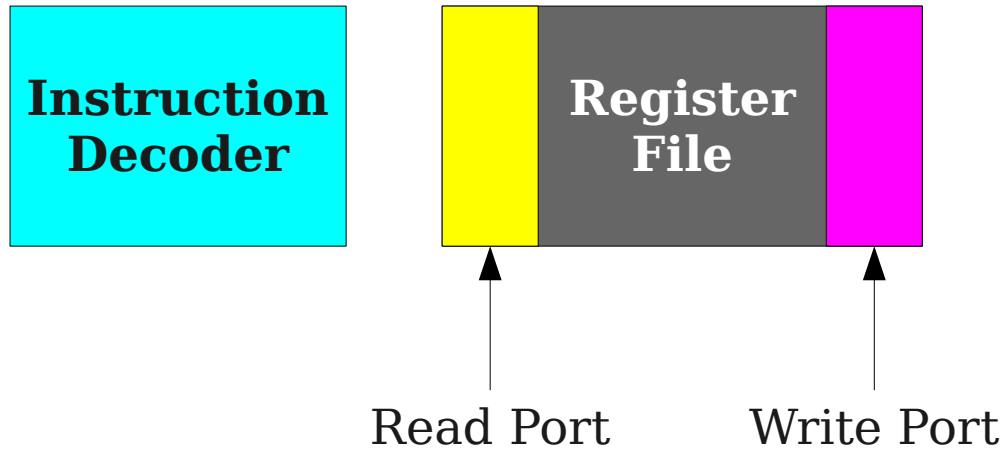
Pipeline Hazards



```
add $t2, $t0, $t1    # $t2 = $t0 + $t1
add $t4, $t3, $t2    # $t5 = $t3 + $t2
add $t7, $t5, $t6    # $t7 = $t5 + $t6
add $t0, $t0, $t7    # $t0 = $t0 + $t7
```

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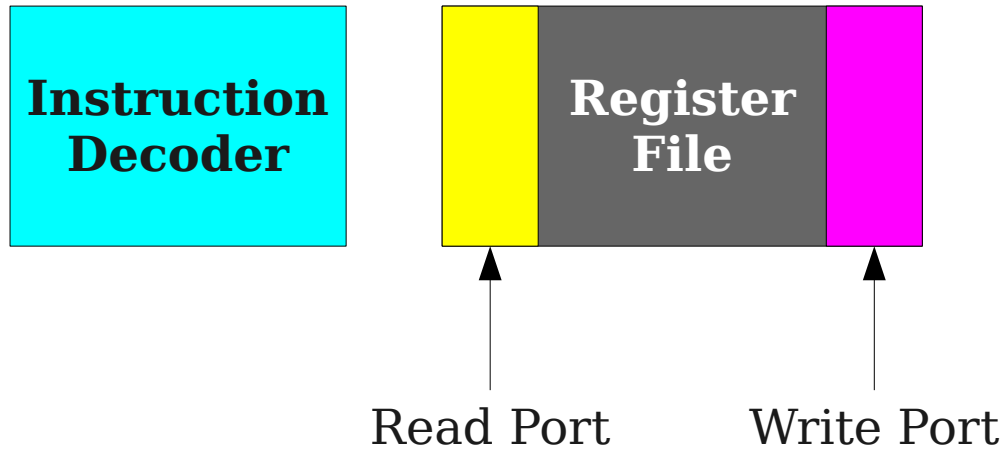
Pipeline Hazards



```
add $t2, $t0, $t1    # $t2 = $t0 + $t1
add $t4, $t3, $t2    # $t5 = $t3 + $t2
add $t7, $t5, $t6    # $t7 = $t5 + $t6
add $t0, $t0, $t7    # $t0 = $t0 + $t7
```

| ID | RR | ALU | RW |
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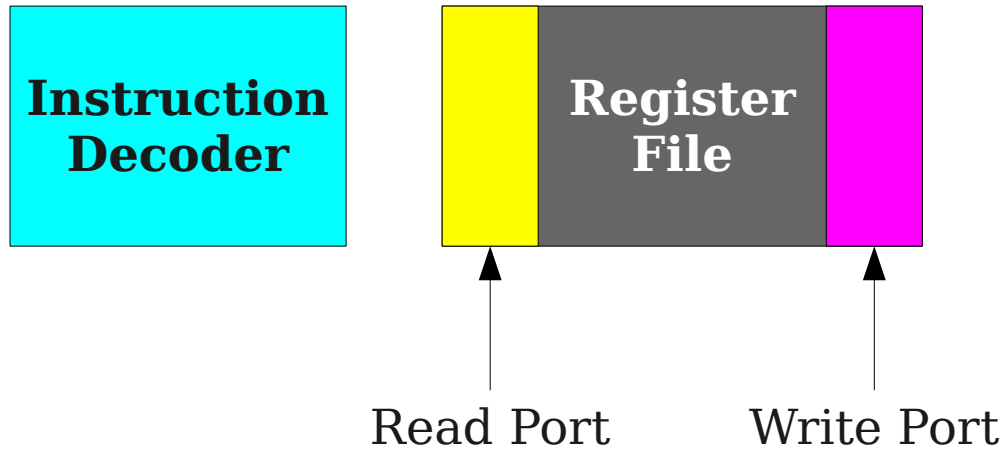
Pipeline Hazards



```
add $t2, $t0, $t1    # $t2 = $t0 + $t1
add $t4, $t3, $t2    # $t5 = $t3 + $t2
add $t7, $t5, $t6    # $t7 = $t5 + $t6
add $t0, $t0, $t7    # $t0 = $t0 + $t7
```

| ID | RR | ALU | RW |
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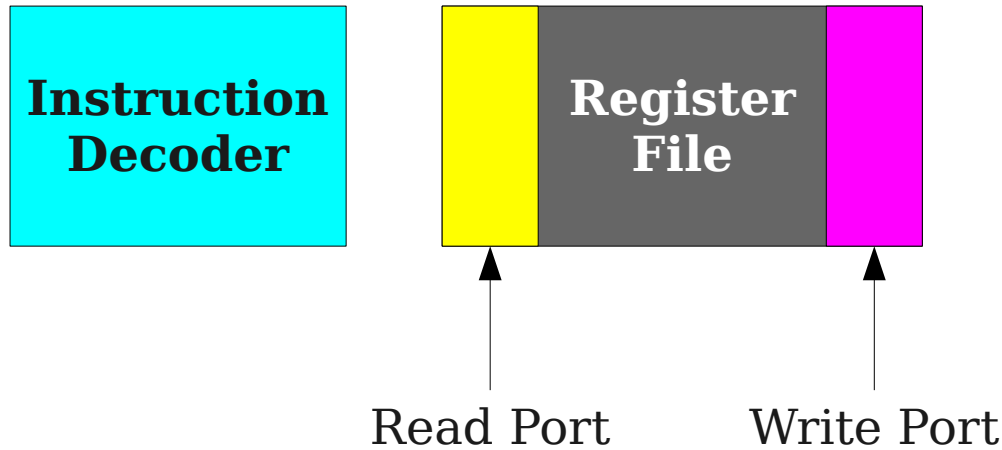
Pipeline Hazards



```
add $t2, $t0, $t1    # $t2 = $t0 + $t1
add $t4, $t3, $t2    # $t5 = $t3 + $t2
add $t7, $t5, $t6    # $t7 = $t5 + $t6
add $t0, $t0, $t7    # $t0 = $t0 + $t7
```

[illegible]

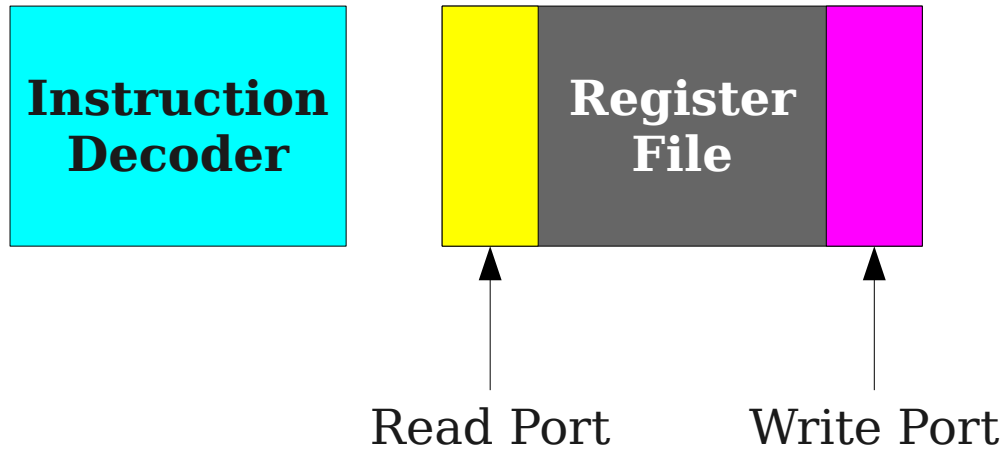
Pipeline Hazards



```
add $t2, $t0, $t1    # $t2 = $t0 + $t1
add $t7, $t5, $t6    # $t7 = $t5 + $t6
add $t4, $t3, $t2    # $t5 = $t3 + $t2
add $t0, $t0, $t7    # $t0 = $t0 + $t7
```

[illegible]

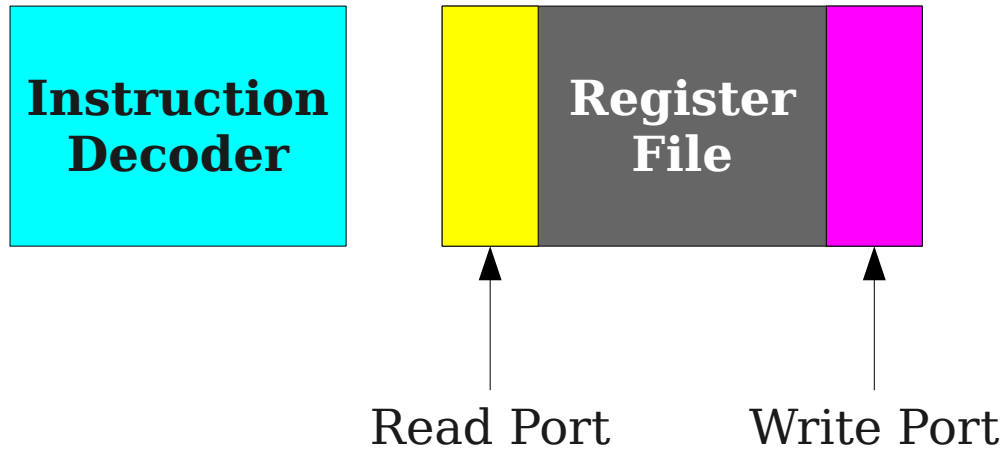
Pipeline Hazards



```
add $t2, $t0, $t1    # $t2 = $t0 + $t1
add $t7, $t5, $t6    # $t7 = $t5 + $t6
add $t4, $t3, $t2    # $t5 = $t3 + $t2
add $t0, $t0, $t7    # $t0 = $t0 + $t7
```

[illegible]

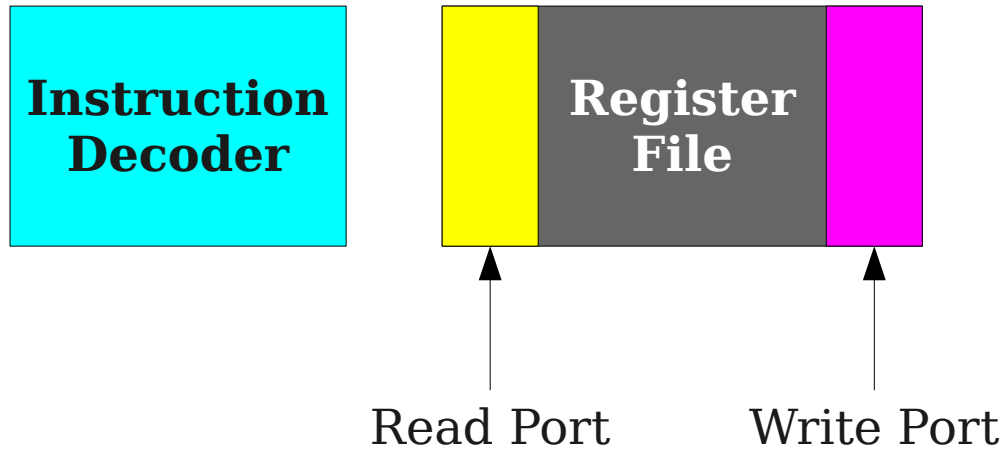
Pipeline Hazards



```
add $t2, $t0, $t1    # $t2 = $t0 + $t1
add $t7, $t5, $t6    # $t7 = $t5 + $t6
add $t4, $t3, $t2    # $t5 = $t3 + $t2
add $t0, $t0, $t7    # $t0 = $t0 + $t7
```

[illegible]

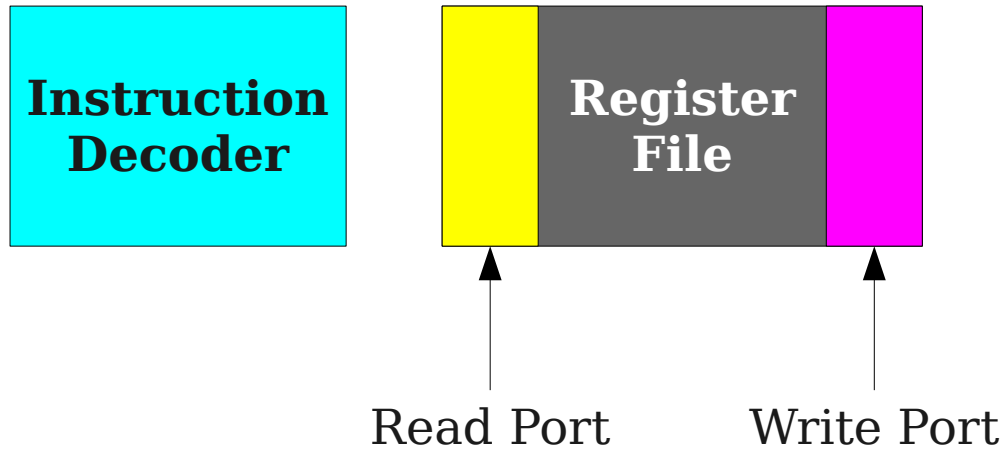
Pipeline Hazards



```
add $t2, $t0, $t1    # $t2 = $t0 + $t1
add $t7, $t5, $t6    # $t7 = $t5 + $t6
add $t4, $t3, $t2    # $t5 = $t3 + $t2
add $t0, $t0, $t7    # $t0 = $t0 + $t7
```

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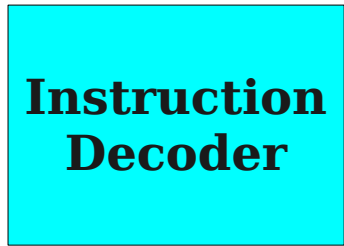
Pipeline Hazards



```
add $t2, $t0, $t1    # $t2 = $t0 + $t1
add $t7, $t5, $t6    # $t7 = $t5 + $t6
add $t4, $t3, $t2    # $t5 = $t3 + $t2
add $t0, $t0, $t7    # $t0 = $t0 + $t7
```

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Pipeline Hazards



Read Port

Write Port

```
add $t2, $t0, $t1    # $t2 = $t0 + $t1
add $t7, $t5, $t6    # $t7 = $t5 + $t6
add $t4, $t3, $t2    # $t5 = $t3 + $t2
add $t0, $t0, $t7    # $t0 = $t0 + $t7
```

Two clock cycles faster!

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Instruction Scheduling

- Because of processor pipelining, the order in which instructions are executed can impact performance.
- **Instruction scheduling** is the reordering or insertion of machine instructions to increase performance.
- All good optimizing compilers have some sort of instruction scheduling support.

Data Dependencies

- A **data dependency** in machine code is a set of instructions whose behavior depends on one another.
- Intuitively, a set of instructions that cannot be reordered around each other.
- Three types of data dependencies:

**Read-after-Write
(RAW)**

$x = \dots$
 $\dots = x$

**Write-after-Read
(WAR)**

$\dots = x$
 $x = \dots$

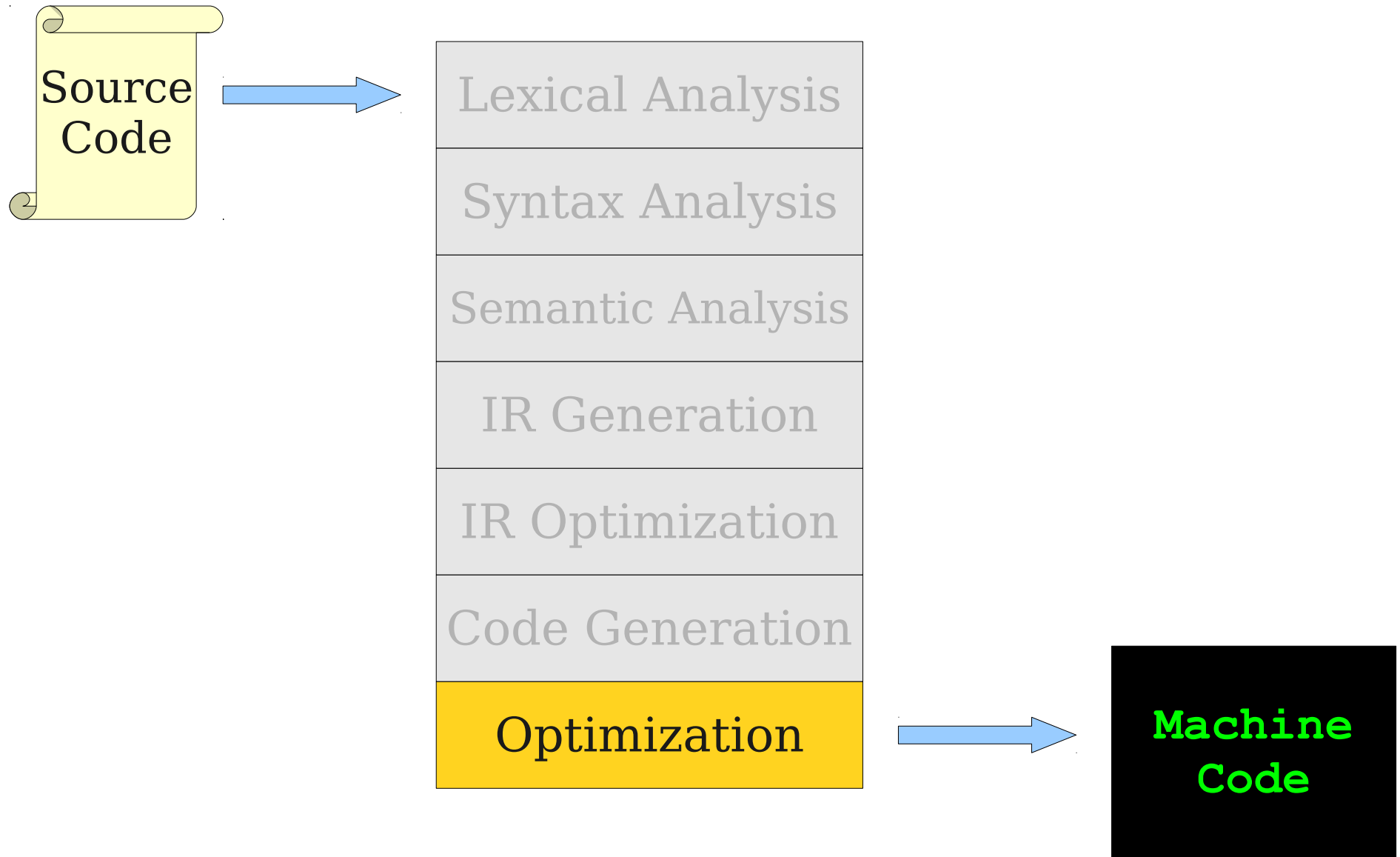
**Write-after-Write
(WAW)**

$x = \dots$
 $x = \dots$

Summary

- **Instruction scheduling** optimizations try to take advantage of the processor pipeline.
- **Locality** optimizations try to take advantage of cache behavior.
- **Parallelism** optimizations try to take advantage of multicore machines.
- There are *many more* optimizations out there!

Where We Are



Where We Are

