

$$T_n(Z) = -\frac{1}{n^2} \sum_{i,j=1}^n \ln(1 + (X_i - Y_j)^2) + \\ + \frac{1}{n(n-1)} \sum_{i < j}^n \ln(1 + (X_i - X_j)^2) + \frac{1}{n(n-1)} \sum_{i < j}^n \ln(1 + (Y_i - Y_j)^2).$$

Critical value of T_n simulation criterion calculated by 1'000'000 iterations. 1000 iterations was performed in each case, 800 permutations used for $T_{n, perm}$.

$$formulae = P(z > z_{1-\alpha/2} - \frac{h}{\sqrt{6 \ln 3}}) + P(z < -z_{1-\alpha/2} - \frac{h}{\sqrt{6 \ln 3}}),$$

where $z \sim N(0, 1)$, $1/\sqrt{6 \ln 3} \approx 0.389$.

Cauchy Mean

Table 1. Cauchy distribution, $X \sim C(0, 10)$, $Y \sim C(10 * h/\sqrt{n}, 10)$, $n = 100$

h	$T_{n, perm}$	$T_{n, sim}$	$formulae$	$wilcox.test$	$ks.test$
1	7	6.8	6.8	6.6	6.3
2	12.6	12.7	12.2	11.9	11.1
3	21.8	21.3	21.5	20.5	20.2
5	53.8	54	49.5	48.5	53.1
7	86.4	86.4	77.8	77.2	83.6
9	97.6	97.8	93.9	91.5	96.5

Table 2. Power of tests for the Cauchy distribution, $X \sim C(0, 0.1)$, $Y \sim C(h/(10\sqrt{n}), 0.1)$, $n = 100$

h	$T_{n, perm}$	$T_{n, sim}$	$formula$	$wilcox.test$	$ks.test$
1	5.8	6.8	6.8	6.6	6.3
2	6.8	35.2	12.2	11.9	11.1
3	9.6	40.2	21.5	20.5	20.2
5	20	56.5	49.5	48.5	53.1
7	35.4	74.4	77.8	77.2	83.6
9	54.9	87.7	93.9	91.5	96.5