Мощность тестов для распределения Коши, $X \sim C(0,1)$, $Y \sim C(h/\sqrt{n},1)$, размер выборок n

$$L_2(Z) = \sum_{i,j=1}^{n} \ln(1+|X_i - Y_j|^2), \tag{1}$$

$$n.pL_2 = -\frac{1}{n^2}\sum_{i,j=1}^{n} \ln(1+(X_i-Y_j)^2) + \frac{1}{n(n-1)}\sum_{i<j}^{n} \ln(1+(X_i-X_j)^2) + \frac{1}{n(n-1)}\sum_{i<j}^{n} \ln(1+(Y_i-Y_j)^2), \tag{2}$$

$$formula = P(z > z_{1-\alpha/2} - h/\sqrt{6} - h^2/6\sqrt{n}), \quad z \sim N(0,1), \quad 1/\sqrt{6} = 0.4082, \tag{3}$$

$$ah + bh^2 = P(z > z_{1-\alpha/2} - ah - bh^2), \tag{4}$$

$$ah = P(z > z_{1-\alpha/2} - ah), \tag{5}$$

$$n.pLL_{cauchy} = -2\sum_{i=1}^{n} \ln \frac{Y_i^2+1}{(Y_i-\mu_x)^2+1}, \tag{6}$$

$$cv(criterion) = \text{critical value of a non-permutation criterion calculated by 1'000'000 iterations} \tag{7}$$

Таблица 1: n=1000, 1000 итераций, 800 перестановок в L_2
$formula$: $1/\sqrt{6} = 0.4082$, $1/6\sqrt{n} = 0.0053$
$ah + bh^2$: $a = 0.3823$, $b = 0.0045$
ah: $a = 0.419$

h	L_2	$n.pL_2$	$formula$	$ah + bh^2$	ah	$n.pLL_{cauchy}$	$wilcox.test$	$ks.test$
1	6.3	6	6.1	5.79	6.17	16.9	6.8	8.1
2	11.4	11.9	13.08	11.96	13.09	42.6	12.9	13.4
3	21	20.9	24.58	22.01	24.1	71.6	22.8	26.2
4	34.9	34.6	40.41	36.02	38.82	90	36.1	43
7	84	84.5	87.62	82.62	83.47	99.9	79.5	87.6
10	99	98.9	99.6	98.98	98.71	100	96.8	99.2

Таблица 2: n=1000, 10'000 итераций
$formula$: $1/\sqrt{6} = 0.4082$, $1/6\sqrt{n} = 0.0053$
$ah + bh^2$: $a = 0.3658$, $b = 0.0062$
ah: $a = 0.416$

h	$n.pL_2$	$formula$	$ah + bh^2$	ah	$n.pLL_{cauchy}$	$cv(n.pL_2)$	$cv(n.pLL_{cauchy})$
2	11.46	13.08	11.44	12.97	40.8	0.0022	2.6553
3	20.65	24.58	20.99	23.83	67.97	0.0022	2.482
4	34.04	40.41	34.55	38.36	88.02	0.0022	1.3133
7	82.41	87.62	81.71	82.95	99.98	0.0022	-8.124
10	98.95	99.6	98.98	98.61	100	0.0022	-26.3019

Таблица 3: n=500, 1000 итераций, 800 перестановок в L_2
$formula$: $1/\sqrt{6} = 0.4082$, $1/6\sqrt{n} = 0.0075$
$ah + bh^2$: $a = 0.3784$, $b = 0.0046$
ah: $a = 0.4118$

h	L_2	$n.pL_2$	$formula$	$ah + bh^2$	ah	$n.pLL_{cauchy}$	$cv(n.pL_2)$	$cv(n.pLL_{cauchy})$	$wilcox.test$	$ks.test$
1	5.8	6.1	6.13	5.74	6.08	16.1	0.0044	1.8238	6.4	6.4
2	11.6	11.6	13.27	11.8	12.79	38.9	0.0044	2.6468	12.6	13.9
3	21	21.8	25.2	21.67	23.44	68.7	0.0044	2.4709	22.2	24.3
5	50.9	51	60.55	51.85	53.94	97.4	0.0044	-0.852	48	57.9
7	82.2	82.4	89.67	81.94	82.19	99.9	0.0044	-8.0712	75.6	85.9
9	96.2	96.5	98.98	96.53	95.96	100	0.0044	-19.0575	93.2	97.2

Таблица 4: n=250, 1000 итераций
$formula$: $1/\sqrt{6} = 0.4082$, $1/6\sqrt{n} = 0.0105$
$ah + bh^2$: $a = 0.377$, $b = 0.0046$
ah: $a = 0.4138$

h	$n.pL_2$	$formula$	$ah + bh^2$	ah	$n.pLL_{cauchy}$
1	6.5	0.0616	5.72	6.1	16.7
2	11.5	0.1354	11.75	12.87	40.3
3	21.2	0.261	21.54	23.62	69.4
4	34	0.4371	35.24	38.03	88.3
7	82.5	0.9214	81.66	82.55	99.9
10	98.8	0.9993	98.83	98.53	100

Таблица 5: n=50, 1000 итераций
$formula$: $1/\sqrt{6} = 0.4082$, $1/6\sqrt{n} = 0.0236$
$ah + bh^2$: $a = 0.3813$, $b = 0$
ah: $a = 0.3813$

h	$n.pL_2$	$formula$	$ah + bh^2$	ah	$n.pLL_{cauchy}$
2	12.5	14.7	11.56	11.56	40.4
4	33.2	52	33.19	33.19	87.3
10	97.9	100	96.81	96.81	100
12	99.3	100	99.55	99.55	100

Таблица 6: Мощность тестов для распределения Коши без рандомизации, размер выборок n, 1000 итераций, 800 перестановок в каждой итерации

n	F_2	L_2	LL_{cauchy}	wilcox.test	ks.test
50	C(1,1)	79.6	85.3	74.6	79.9
200	C(0.5,1)	80.2	87.8	74.7	83.5
1250	C(0.2,1)	84.1	89.7	77.5	88.2
5000	C(0.1,1)	82.7	90.7	78.9	87.3

Таблица 7: Мощность тестов для распределения Коши без рандомизации, размер выборок $n = 1000$, 1000 итераций, 800 перестановок в каждой итерации

F_2	L_2	LL_{cauchy}	wilcox.test	ks.test
C(0, 1)	4.7	5.4	4.8	6
C(0.05, 1)	9.2	10.6	10.2	10.7
C(0.1, 1)	23.1	27.8	24.8	28.7
C(0.15, 1)	48.7	56.8	47.5	55.7
C(0.2, 1)	76	81.9	70.1	81.2

Old results

$$L_1(Z) = \sum_{i,j=1}^{n} \ln(1+|X_i - Y_j|), \tag{8}$$

$$L_2^*(Z) = \sum_{i,j=1}^{n} \ln(1+|X_i - Y_j|^2)/n^2 - \sqrt{B_1 B_2}, \tag{9}$$

$$B_1 = \sum_{i,j=1}^{n} \ln(1+|X_i - X_j|^2)/n(n-1), \tag{10}$$

$$B_2 = \sum_{i,j=1}^{n} \ln(1+|Y_i - Y_j|^2)/n(n-1), \tag{11}$$

$$L_\infty(Z) = \sum_{i,j=1}^{n} \ln(|X_i - Y_j|), \tag{12}$$

$$LL_{distribution} = \text{maximum log likelyhood permutation criterion based on the distribution} \tag{13}$$

Таблица 8: Мощность тестов для распределения Коши без рандомизации, размер выборок $n = 200$, 1000 итераций, 800 перестановок в каждой итерации

F_2	L_1	L_2	L_2^*	L_∞	LL_{norm}	LL_{cauchy}	$LL_{laplace}$	LL_{levy}	wilcox.test	ks.test
C(0, 1)	5.9	5.5	5.2	5.6	4.4	5.9	4.9	3.7	4.8	3.6
C(0.1, 1)	8.9	7.9	5.7	8.9	4.2	9.7	4.9	3.7	8	6.7
C(0.2, 1)	18.3	17.9	9.2	18.1	4.2	21.8	5.1	3.6	17.6	17.4
C(0.3, 1)	37.6	36.2	18.4	38.1	4.2	43	5.8	3.6	35	38
C(0.4, 1)	61.5	60.1	33.8	61.4	4.2	69.3	6.1	3.5	57.1	63.2
C(0, 1.2)	16.6	16.4	15.3	16.2	5	19.3	6.6	5	5.1	7.4
C(0, 1.4)	50.4	48.7	46.6	49.9	7.6	54.5	12.4	7.3	5.4	16.7
C(0, 1.6)	78.3	77.1	74.8	78.6	10.5	83.6	20.5	9.2	5.4	34.8
C(0, 1.8)	93.6	93.4	91.6	93.7	13.6	96	27.6	11.5	5.5	56.8
C(0.1, 1.1)	11.9	11.2	7.8	11.5	4.4	12.6	5.4	4.3	7.7	7.5
C(0.2, 1.2)	28.4	28	21	28.7	5	33.4	7.2	5.2	15.1	21.5
C(0.3, 1.3)	56.8	54.5	43.7	55.8	6.6	60.6	9.9	6.1	27.1	40.9
C(0.4, 1.4)	78.3	77.7	67.3	79.1	7.6	83.7	13.7	7.8	41.4	64.4
C(0.1, 1.2)	19.4	19	16.9	19.7	5	23.2	6.9	5.1	7.8	10.3
C(0.2, 1.4)	58.3	56.9	50	57.3	7.6	63.1	12.5	7.7	13.8	31.3
C(0.3, 1.6)	87.4	85.5	80.3	86.9	10.5	90.4	20.7	9.7	22.5	58.3
C(0.4, 1.8)	98.5	98.2	96.3	97.9	13.6	98.8	28.7	11.7	31.6	80.4

Таблица 9: Мощность тестов для распределения Коши без рандомизации, размер выборок $n = 50$, 1000 итераций, 800 перестановок в каждой итерации

F_2	L_1	L_2	L_2^*	L_∞	LL_{norm}	LL_{cauchy}	$LL_{laplace}$	LL_{levy}	wilcox.test	ks.test
C(0, 1)	5	5	4.6	5.1	5.4	4.9	5.1	5	5	4.3
C(0.25, 1)	10.7	10.5	6.4	10.7	5.5	11.3	5.9	4.9	10.9	10.3
C(0.5, 1)	28.4	26.7	14.4	27.6	5.5	30	7.5	4.9	28.9	30
C(0.75, 1)	56	53.7	32.1	55.6	5.8	61.1	10.1	5	52.2	57.1
C(1, 1)	80.7	79.6	56.5	80.6	6.3	85.3	14.6	5.2	74.6	79.9
C(0, 1.5)	22.7	21.7	21.5	22.2	11.2	22.9	13.9	7.9	5.5	9.8
C(0, 2)	54.7	53	52.5	53.7	18.8	60.5	28.1	14	6	19.6
C(0, 2.5)	80.6	79.8	79	79.9	24.7	84.7	40.9	19.5	6.4	31.8
C(0, 3)	92.2	92.3	91.8	91.8	32.8	95.2	52.5	26.2	6.6	49.4
C(0.25, 1.25)	15	13.9	13.1	14	6.9	15.4	9	5.1	9.5	10.8
C(0.5, 1.5)	38.1	38.8	30	36.9	11.3	42.4	15.5	7.4	20.9	27.2
C(0.75, 1.75)	64.6	63.8	54.8	63.5	15.7	70.3	23.6	10.9	33.5	48.1
C(1, 2)	83.3	82	75.7	82.2	19.2	86.9	32.4	14	45.5	66.2
C(0.25, 1.5)	26.4	25.3	24.3	26.5	11.4	29	14.7	7.8	9	12.8
C(0.5, 2)	64.6	64.6	59.5	63.2	19.1	70.9	29.1	14	16.8	34.3
C(0.75, 2.5)	87.2	86.2	85.5	86.2	25.2	90	43.2	19.8	23.9	57.3
C(1, 3)	96.7	96.8	95.3	96.1	33.5	97.1	55.3	24.7	30	72.3

Таблица 10: Мощность тестов для Нормального распределения без рандомизации, размер выборок $n = 50$, 1000 итераций, 800 перестановок в каждой итерации

F_2	L_1	L_2	L_2^*	L_∞	LL_{norm}	$LL_{norm}^{var.eq}$	LL_{cauchy}	$LL_{laplace}$	LL_{levy}	wilcox.test	ks.test
N(0, 1)	5.2	5.4	5.2	4.5	5.2	5.1	5.6	5.8	5	5.4	4.3
N(0.25, 1)	19.4	21.8	20.9	15	19.5	23.5	15.6	17.5	7.9	23.4	16.5
N(0.5, 1)	62.7	66.5	63.6	50.6	62.3	70.6	49.1	53.5	15.2	70	53.2
N(0.75, 1)	93.9	95.2	93.7	86.7	93.7	96.6	83.9	89.6	26.1	96.5	90.1
N(1, 1)	98.9	99.1	99	98.4	98.9	99.8	97.5	98.5	43.6	99.4	98.7
N(0, 1.5)	35.7	33.2	54.2	33.8	69.9	4.7	20.8	39.3	34.9	5.5	11.2
N(0, 2)	89.5	89.9	96.7	84.3	99.1	4.8	68.8	91.4	75.2	5.9	37
N(0, 2.5)	99.3	99.4	100	97.7	100	4.7	93.6	99.8	93.7	6.3	69
N(0, 3)	100	100	100	100	100	4.9	99.3	100	98.6	6.8	89.2
N(0.25, 1.25)	24.5	25.3	31.7	20.2	38.6	20.5	19.2	26.1	8.3	18.9	16.5
N(0.5, 1.5)	74.2	75.4	83.6	63.7	89.2	50.8	51.8	71.8	16.7	48.2	54.2
N(0.75, 1.75)	95.9	96.3	97.9	91.8	98.9	75.2	86	95.4	26.3	73	84.1
N(1, 2)	99.5	99.6	99.9	98.2	100	88.3	95.9	99.3	36.7	86.7	96
N(0.25, 1.5)	47	46.5	63.3	41.2	76.1	16.5	32	49.5	24.1	17.1	23.2
N(0.5, 2)	95	95.8	98.5	91.1	99.6	36	82.3	95.8	56	34.7	68.1
N(0.75, 2.5)	99.8	99.9	100	98.7	100	51.4	96.8	99.8	81.2	49.5	91.1
N(1, 3)	100	100	100	100	100	60.4	99.7	100	91.6	58.1	98.2

Таблица 11: Мощность тестов для распределения Леви без рандомизации, размер выборок $n = 50$, 1000 итераций, 800 перестановок в каждой итерации

F_2	L_1	L_2	L_2^*	L_∞	LL_{norm}	LL_{cauchy}	$LL_{laplace}$	LL_{levy}	wilcox.test	ks.test
Le(0, 1)	5.5	5.7	4.8	5.8	5.6	5.8	5.6	6.2	4.6	4.7
Le(0.25, 1)	7.1	6.9	5.4	13.8	5.6	6.1	5.5	65.3	10.1	8.5
Le(0.5, 1)	16.2	10.8	7.5	45.9	5.6	10.7	5.5	98.9	22.7	36.8
Le(0.75, 1)	41	27.8	9.8	81.8	5.6	23.3	5.7	100	38.7	74.5
Le(1, 1)	69.3	53.2	17.6	94.9	5.6	43.8	5.7	100	51.9	92.3
Le(0, 1.5)	13.4	12.7	9.8	15.6	6.9	14.3	6.7	22.9	20.5	13.4
Le(0, 2)	32.9	31.5	23.5	40.7	7.3	35	8.9	57.5	48.2	37.9
Le(0, 2.5)	54.3	52.3	39.7	64.3	8.5	53.4	10.4	81.9	69.5	61.1

Le(0, 3)	71.8	69.7	54.8	81.1	9.5	70.8	12.2	93	84.3	77.9
Le(0.25, 1.25)	12.8	11.2	6.9	22.3	6.7	11.2	5.7	75.5	22.4	17.7
Le(0.5, 1.5)	37.6	32.1	15.9	67.6	6.8	30.6	6.8	99.3	54.6	64
Le(0.75, 1.75)	72.3	64.8	35.3	92.6	6.9	52.2	7.6	100	75.1	90.2
Le(1, 2)	90	85.4	57.8	97.9	7.3	73.4	8.9	100	89.2	97.3
Le(0.25, 1.5)	21.3	19.5	11.9	35.4	6.9	20.7	6.8	82.7	37.4	30.8
Le(0.5, 2)	63.7	57.4	35.5	84.7	7.3	50.2	8.9	99.6	73.8	82
Le(0.75, 2.5)	89.2	86.6	63.1	97.2	8.5	77.4	10.6	100	92	96.3
Le(1, 3)	97.4	96.8	84	99.5	9.5	88.9	12.6	100	96.7	99.4

Таблица 12: Мощность тестов для распределения Лапласа без рандомизации, размер выборок $n = 50$, 1000 итераций, 800 перестановок в каждой итерации

F_2	L_1	L_2	L_2^*	L_∞	LL_{norm}	LL_{cauchy}	$LL_{laplace}$	LL_{levy}	wilcox.test	ks.test
La(0, 1)	5.9	5.4	6.5	5.3	5.4	5.1	6.3	5.6	4.6	4.7
La(0.25, 1)	17.6	17.1	15.5	15.4	9.5	19.2	16.9	5.5	19.5	16.3
La(0.5, 1)	54.6	53.9	47.8	49.5	25.9	56.6	53.4	6.8	56.7	54.2
La(0.75, 1)	86.9	87	83.5	83.3	52.9	87.7	86.6	9.6	88.8	86.1
La(1, 1)	97.7	98.1	96.9	96.6	79.6	97.8	97.6	13.4	97.8	97.5
La(0, 1.5)	24.3	24.6	38.2	22.6	42.4	22.3	38.5	17.4	5.7	9.2
La(0, 2)	70.6	71.3	84.3	63.8	86.5	61.5	86.8	42.5	5.8	21.9
La(0, 2.5)	93.1	93.3	97.6	89.2	98.1	87.7	98.2	64	6.4	40.3
La(0, 3)	98.6	98.7	99.5	97.5	99.7	97.5	99.7	79.3	7.1	60.4
La(0.25, 1.25)	21.8	21.5	23.8	18.8	20.5	21.7	25.4	7.6	15.9	16.1
La(0.5, 1.5)	59.7	58.4	63.8	54.3	55.2	57.1	67.5	13.5	39.8	45.9
La(0.75, 1.75)	87.7	88.1	90.3	83.7	82.5	86.3	92.3	21.8	62.9	72.6
La(1, 2)	97.3	97.6	98.4	95.5	95.6	96.5	98.8	29.4	78.4	91
La(0.25, 1.5)	37.4	37	45.8	32.6	46.3	33	47.5	16	13.9	19.3
La(0.5, 2)	83	82.8	91.2	78.1	89.9	78.7	92.5	35.3	30	50.2
La(0.75, 2.5)	98.1	98.1	99.2	96.3	99.2	96.4	99.7	55	45.3	76
La(1, 3)	99.9	99.9	100	99.6	100	99.8	100	69.8	54.8	91.8

Таблица 13: Мощность тестов для распределения Лог-Коши без рандомизации, размер выборок $n = 50$, 1000 итераций, 800 перестановок в каждой итерации

F_2	L_1	L_2	L_2^*	L_∞	LL_{norm}	LL_{cauchy}	$LL_{laplace}$	LL_{levy}	$LL_{logcauchy}$	wilcox.test	ks.test
LC(0, 1)	6.7	6.4	6.1	6.9	4.4	6	4.6	4.5	5.8	4.8	4.5
LC(0.25, 1)	9.4	9.2	6.9	8.9	4.7	12.3	4.9	5.7	11.7	8.6	9.8
LC(0.5, 1)	21.4	20.6	9.6	24	4.9	38.3	5.8	11.5	32.7	27.6	30
LC(0.75, 1)	42.4	41.6	13.9	48.2	5.8	68	6.5	22.4	63.1	53	58.5
LC(1, 1)	66.6	65.6	20.5	72.4	6.1	88	7.2	38.5	85.7	75	80.7
LC(0, 1.5)	15.2	14.2	12.1	17.3	8.7	11.9	9.2	7.8	24.9	5.1	9.5
LC(0, 2)	30.4	28.3	22.3	39.8	15.1	21.2	16.1	15.9	58.6	5.1	20.6
LC(0, 2.5)	45.7	42.3	30.5	61	19.9	33.6	21	24.6	84.2	5.7	32.4
LC(0, 3)	57.8	54.7	38	77.6	24.1	43.5	26	35.1	92.6	5.7	47.1
LC(0.25, 1.25)	12.9	13.3	10.7	11.3	6.8	12.9	7.1	4.9	16.1	7.9	10.6
LC(0.5, 1.5)	28.1	28.2	18.8	29.4	10.2	34	10.9	6.1	44.1	19.2	26.2
LC(0.75, 1.75)	47.6	47.7	27.7	48.7	13.8	56.6	15.4	9.3	68.9	32.4	47.9
LC(1, 2)	64	64.2	37.6	66.3	18.3	72.9	19.9	12	86.4	44.4	67
LC(0.25, 1.5)	19.7	18.9	15.6	18.9	9.2	16.1	10.5	6	28.9	7.7	12.9
LC(0.5, 2)	41.8	41.3	27.9	47	16.4	39.9	17.7	9.6	69.9	14.6	33.4
LC(0.75, 2.5)	63.8	62.9	40.9	70.6	21.9	57.8	23.1	16.1	90.5	22.1	56.1
LC(1, 3)	78.4	77.4	52.5	84.8	27.2	70.8	28.1	21.7	97.1	27.4	71.9