

$$L_1(Z) = \sum_{i,j=1}^n \ln(1 + |X_i - Y_j|), \quad (1)$$

$$L_2(Z) = \sum_{i,j=1}^n \ln(1 + |X_i - Y_j|^2), \quad (2)$$

$$L_2^*(Z) = \sum_{i,j=1}^n \ln(1 + |X_i - Y_j|^2/n^2 - \sqrt{B_1 B_2}), \quad (3)$$

$$B_1 = \sum_{i,j=1}^n \ln(1 + |X_i - X_j|^2)/n(n-1), \quad (4)$$

$$B_2 = \sum_{i,j=1}^n \ln(1 + |Y_i - Y_j|^2)/n(n-1), \quad (5)$$

$$L_\infty(Z) = \sum_{i,j=1}^n \ln(|X_i - Y_j|), \quad (6)$$

$$LL_{distribution} = \text{maximum log likelihood permutation criterion based on the distribution} \quad (7)$$

Таблица 1: Мощность тестов для распределения Коши без рандомизации, размер выборок  $n$ , 1000 итераций, 800 перестановок в каждой итерации

n	$F_2$	$L_2$	$LL_{cauchy}$	wilcox.test	ks.test
50	C(1,1)	79.6	85.3	74.6	79.9
200	C(0.5,1)	80.2	87.8	74.7	83.5
1250	C(0.2,1)	84.1	89.7	77.5	88.2
5000	C(0.1,1)	82.7	90.7	78.9	87.3

Таблица 2: Мощность тестов для распределения Коши без рандомизации, размер выборок  $n = 1000$ , 1000 итераций, 800 перестановок в каждой итерации

$F_2$	$L_2$	$LL_{cauchy}$	wilcox.test	ks.test
C(0, 1)	4.7	5.4	4.8	6
C(0.05, 1)	9.2	10.6	10.2	10.7
C(0.1, 1)	23.1	27.8	24.8	28.7
C(0.15, 1)	48.7	56.8	47.5	55.7
C(0.2, 1)	76	81.9	70.1	81.2

Таблица 3: Мощность тестов для распределения Коши без рандомизации, размер выборок  $n = 200$ , 1000 итераций, 800 перестановок в каждой итерации

$F_2$	$L_1$	$L_2$	$L_2^*$	$L_\infty$	$LL_{norm}$	$LL_{cauchy}$	$LL_{laplace}$	$LL_{levy}$	wilcox.test	ks.test
C(0, 1)	5.9	5.5	5.2	5.6	4.4	5.9	4.9	3.7	4.8	3.6
C(0.1, 1)	8.9	7.9	5.7	8.9	4.2	9.7	4.9	3.7	8	6.7
C(0.2, 1)	18.3	17.9	9.2	18.1	4.2	21.8	5.1	3.6	17.6	17.4
C(0.3, 1)	37.6	36.2	18.4	38.1	4.2	43	5.8	3.6	35	38
C(0.4, 1)	61.5	60.1	33.8	61.4	4.2	69.3	6.1	3.5	57.1	63.2
C(0, 1.2)	16.6	16.4	15.3	16.2	5	19.3	6.6	5	5.1	7.4
C(0, 1.4)	50.4	48.7	46.6	49.9	7.6	54.5	12.4	7.3	5.4	16.7
C(0, 1.6)	78.3	77.1	74.8	78.6	10.5	83.6	20.5	9.2	5.4	34.8
C(0, 1.8)	93.6	93.4	91.6	93.7	13.6	96	27.6	11.5	5.5	56.8

C(0.1, 1.1)	11.9	11.2	7.8	11.5	4.4	12.6	5.4	4.3	7.7	7.5
C(0.2, 1.2)	28.4	28	21	28.7	5	33.4	7.2	5.2	15.1	21.5
C(0.3, 1.3)	56.8	54.5	43.7	55.8	6.6	60.6	9.9	6.1	27.1	40.9
C(0.4, 1.4)	78.3	77.7	67.3	79.1	7.6	83.7	13.7	7.8	41.4	64.4
C(0.1, 1.2)	19.4	19	16.9	19.7	5	23.2	6.9	5.1	7.8	10.3
C(0.2, 1.4)	58.3	56.9	50	57.3	7.6	63.1	12.5	7.7	13.8	31.3
C(0.3, 1.6)	87.4	85.5	80.3	86.9	10.5	90.4	20.7	9.7	22.5	58.3
C(0.4, 1.8)	98.5	98.2	96.3	97.9	13.6	98.8	28.7	11.7	31.6	80.4

Таблица 4: Мощность тестов для распределения Коши без рандомизации, размер выборок  $n = 50, 1000$  итераций, 800 перестановок в каждой итерации

$F_2$	$L_1$	$L_2$	$L_2^*$	$L_\infty$	$LL_{norm}$	$LL_{cauchy}$	$LL_{laplace}$	$LL_{levy}$	wilcox.test	ks.test
C(0, 1)	5	5	4.6	5.1	5.4	4.9	5.1	5	5	4.3
C(0.25, 1)	10.7	10.5	6.4	10.7	5.5	11.3	5.9	4.9	10.9	10.3
C(0.5, 1)	28.4	26.7	14.4	27.6	5.5	30	7.5	4.9	28.9	30
C(0.75, 1)	56	53.7	32.1	55.6	5.8	61.1	10.1	5	52.2	57.1
C(1, 1)	80.7	79.6	56.5	80.6	6.3	85.3	14.6	5.2	74.6	79.9
C(0, 1.5)	22.7	21.7	21.5	22.2	11.2	22.9	13.9	7.9	5.5	9.8
C(0, 2)	54.7	53	52.5	53.7	18.8	60.5	28.1	14	6	19.6
C(0, 2.5)	80.6	79.8	79	79.9	24.7	84.7	40.9	19.5	6.4	31.8
C(0, 3)	92.2	92.3	91.8	91.8	32.8	95.2	52.5	26.2	6.6	49.4
C(0.25, 1.25)	15	13.9	13.1	14	6.9	15.4	9	5.1	9.5	10.8
C(0.5, 1.5)	38.1	38.8	30	36.9	11.3	42.4	15.5	7.4	20.9	27.2
C(0.75, 1.75)	64.6	63.8	54.8	63.5	15.7	70.3	23.6	10.9	33.5	48.1
C(1, 2)	83.3	82	75.7	82.2	19.2	86.9	32.4	14	45.5	66.2
C(0.25, 1.5)	26.4	25.3	24.3	26.5	11.4	29	14.7	7.8	9	12.8
C(0.5, 2)	64.6	64.6	59.5	63.2	19.1	70.9	29.1	14	16.8	34.3
C(0.75, 2.5)	87.2	86.2	85.5	86.2	25.2	90	43.2	19.8	23.9	57.3
C(1, 3)	96.7	96.8	95.3	96.1	33.5	97.1	55.3	24.7	30	72.3

Таблица 5: Мощность тестов для Нормального распределения без рандомизации, размер выборок  $n = 50, 1000$  итераций, 800 перестановок в каждой итерации

$F_2$	$L_1$	$L_2$	$L_2^*$	$L_\infty$	$LL_{norm}$	$LL_{var.eq}^{norm}$	$LL_{cauchy}$	$LL_{laplace}$	$LL_{levy}$	wilcox.test	ks.test
N(0, 1)	5.2	5.4	5.2	4.5	5.2	5.1	5.6	5.8	5	5.4	4.3
N(0.25, 1)	19.4	21.8	20.9	15	19.5	23.5	15.6	17.5	7.9	23.4	16.5
N(0.5, 1)	62.7	66.5	63.6	50.6	62.3	70.6	49.1	53.5	15.2	70	53.2
N(0.75, 1)	93.9	95.2	93.7	86.7	93.7	96.6	83.9	89.6	26.1	96.5	90.1
N(1, 1)	98.9	99.1	99	98.4	98.9	99.8	97.5	98.5	43.6	99.4	98.7
N(0, 1.5)	35.7	33.2	54.2	33.8	69.9	4.7	20.8	39.3	34.9	5.5	11.2
N(0, 2)	89.5	89.9	96.7	84.3	99.1	4.8	68.8	91.4	75.2	5.9	37
N(0, 2.5)	99.3	99.4	100	97.7	100	4.7	93.6	99.8	93.7	6.3	69
N(0, 3)	100	100	100	100	100	4.9	99.3	100	98.6	6.8	89.2
N(0.25, 1.25)	24.5	25.3	31.7	20.2	38.6	20.5	19.2	26.1	8.3	18.9	16.5
N(0.5, 1.5)	74.2	75.4	83.6	63.7	89.2	50.8	51.8	71.8	16.7	48.2	54.2
N(0.75, 1.75)	95.9	96.3	97.9	91.8	98.9	75.2	86	95.4	26.3	73	84.1
N(1, 2)	99.5	99.6	99.9	98.2	100	88.3	95.9	99.3	36.7	86.7	96
N(0.25, 1.5)	47	46.5	63.3	41.2	76.1	16.5	32	49.5	24.1	17.1	23.2
N(0.5, 2)	95	95.8	98.5	91.1	99.6	36	82.3	95.8	56	34.7	68.1
N(0.75, 2.5)	99.8	99.9	100	98.7	100	51.4	96.8	99.8	81.2	49.5	91.1
N(1, 3)	100	100	100	100	100	60.4	99.7	100	91.6	58.1	98.2

Таблица 6: Мощность тестов для распределения Леви без рандомизации, размер выборок  $n = 50, 1000$  итераций, 800 перестановок в каждой итерации

$F_2$	$L_1$	$L_2$	$L_2^*$	$L_\infty$	$LL_{norm}$	$LL_{cauchy}$	$LL_{laplace}$	$LL_{levy}$	wilcox.test	ks.test
Le(0, 1)	5.5	5.7	4.8	5.8	5.6	5.8	5.6	6.2	4.6	4.7
Le(0.25, 1)	7.1	6.9	5.4	13.8	5.6	6.1	5.5	65.3	10.1	8.5
Le(0.5, 1)	16.2	10.8	7.5	45.9	5.6	10.7	5.5	98.9	22.7	36.8
Le(0.75, 1)	41	27.8	9.8	81.8	5.6	23.3	5.7	100	38.7	74.5
Le(1, 1)	69.3	53.2	17.6	94.9	5.6	43.8	5.7	100	51.9	92.3
Le(0, 1.5)	13.4	12.7	9.8	15.6	6.9	14.3	6.7	22.9	20.5	13.4
Le(0, 2)	32.9	31.5	23.5	40.7	7.3	35	8.9	57.5	48.2	37.9
Le(0, 2.5)	54.3	52.3	39.7	64.3	8.5	53.4	10.4	81.9	69.5	61.1
Le(0, 3)	71.8	69.7	54.8	81.1	9.5	70.8	12.2	93	84.3	77.9
Le(0.25, 1.25)	12.8	11.2	6.9	22.3	6.7	11.2	5.7	75.5	22.4	17.7
Le(0.5, 1.5)	37.6	32.1	15.9	67.6	6.8	30.6	6.8	99.3	54.6	64
Le(0.75, 1.75)	72.3	64.8	35.3	92.6	6.9	52.2	7.6	100	75.1	90.2
Le(1, 2)	90	85.4	57.8	97.9	7.3	73.4	8.9	100	89.2	97.3
Le(0.25, 1.5)	21.3	19.5	11.9	35.4	6.9	20.7	6.8	82.7	37.4	30.8
Le(0.5, 2)	63.7	57.4	35.5	84.7	7.3	50.2	8.9	99.6	73.8	82
Le(0.75, 2.5)	89.2	86.6	63.1	97.2	8.5	77.4	10.6	100	92	96.3
Le(1, 3)	97.4	96.8	84	99.5	9.5	88.9	12.6	100	96.7	99.4

Таблица 7: Мощность тестов для распределения Лапласа без рандомизации, размер выборок  $n = 50, 1000$  итераций, 800 перестановок в каждой итерации

$F_2$	$L_1$	$L_2$	$L_2^*$	$L_\infty$	$LL_{norm}$	$LL_{cauchy}$	$LL_{laplace}$	$LL_{levy}$	wilcox.test	ks.test
La(0, 1)	5.9	5.4	6.5	5.3	5.4	5.1	6.3	5.6	4.6	4.7
La(0.25, 1)	17.6	17.1	15.5	15.4	9.5	19.2	16.9	5.5	19.5	16.3
La(0.5, 1)	54.6	53.9	47.8	49.5	25.9	56.6	53.4	6.8	56.7	54.2
La(0.75, 1)	86.9	87	83.5	83.3	52.9	87.7	86.6	9.6	88.8	86.1
La(1, 1)	97.7	98.1	96.9	96.6	79.6	97.8	97.6	13.4	97.8	97.5
La(0, 1.5)	24.3	24.6	38.2	22.6	42.4	22.3	38.5	17.4	5.7	9.2
La(0, 2)	70.6	71.3	84.3	63.8	86.5	61.5	86.8	42.5	5.8	21.9
La(0, 2.5)	93.1	93.3	97.6	89.2	98.1	87.7	98.2	64	6.4	40.3
La(0, 3)	98.6	98.7	99.5	97.5	99.7	97.5	99.7	79.3	7.1	60.4
La(0.25, 1.25)	21.8	21.5	23.8	18.8	20.5	21.7	25.4	7.6	15.9	16.1
La(0.5, 1.5)	59.7	58.4	63.8	54.3	55.2	57.1	67.5	13.5	39.8	45.9
La(0.75, 1.75)	87.7	88.1	90.3	83.7	82.5	86.3	92.3	21.8	62.9	72.6
La(1, 2)	97.3	97.6	98.4	95.5	95.6	96.5	98.8	29.4	78.4	91
La(0.25, 1.5)	37.4	37	45.8	32.6	46.3	33	47.5	16	13.9	19.3
La(0.5, 2)	83	82.8	91.2	78.1	89.9	78.7	92.5	35.3	30	50.2
La(0.75, 2.5)	98.1	98.1	99.2	96.3	99.2	96.4	99.7	55	45.3	76
La(1, 3)	99.9	99.9	100	99.6	100	99.8	100	69.8	54.8	91.8

Таблица 8: Мощность тестов для распределения Лог-Коши без рандомизации, размер выборок  $n = 50, 1000$  итераций, 800 перестановок в каждой итерации

$F_2$	$L_1$	$L_2$	$L_2^*$	$L_\infty$	$LL_{norm}$	$LL_{cauchy}$	$LL_{laplace}$	$LL_{levy}$	$LL_{logcauchy}$	wilcox.test	ks.test
LC(0, 1)	6.7	6.4	6.1	6.9	4.4	6	4.6	4.5	5.8	4.8	4.5
LC(0.25, 1)	9.4	9.2	6.9	8.9	4.7	12.3	4.9	5.7	11.7	8.6	9.8
LC(0.5, 1)	21.4	20.6	9.6	24	4.9	38.3	5.8	11.5	32.7	27.6	30
LC(0.75, 1)	42.4	41.6	13.9	48.2	5.8	68	6.5	22.4	63.1	53	58.5
LC(1, 1)	66.6	65.6	20.5	72.4	6.1	88	7.2	38.5	85.7	75	80.7
LC(0, 1.5)	15.2	14.2	12.1	17.3	8.7	11.9	9.2	7.8	24.9	5.1	9.5
LC(0, 2)	30.4	28.3	22.3	39.8	15.1	21.2	16.1	15.9	58.6	5.1	20.6
LC(0, 2.5)	45.7	42.3	30.5	61	19.9	33.6	21	24.6	84.2	5.7	32.4
LC(0, 3)	57.8	54.7	38	77.6	24.1	43.5	26	35.1	92.6	5.7	47.1
LC(0.25, 1.25)	12.9	13.3	10.7	11.3	6.8	12.9	7.1	4.9	16.1	7.9	10.6
LC(0.5, 1.5)	28.1	28.2	18.8	29.4	10.2	34	10.9	6.1	44.1	19.2	26.2
LC(0.75, 1.75)	47.6	47.7	27.7	48.7	13.8	56.6	15.4	9.3	68.9	32.4	47.9
LC(1, 2)	64	64.2	37.6	66.3	18.3	72.9	19.9	12	86.4	44.4	67
LC(0.25, 1.5)	19.7	18.9	15.6	18.9	9.2	16.1	10.5	6	28.9	7.7	12.9
LC(0.5, 2)	41.8	41.3	27.9	47	16.4	39.9	17.7	9.6	69.9	14.6	33.4
LC(0.75, 2.5)	63.8	62.9	40.9	70.6	21.9	57.8	23.1	16.1	90.5	22.1	56.1
LC(1, 3)	78.4	77.4	52.5	84.8	27.2	70.8	28.1	21.7	97.1	27.4	71.9