

$$L_1(Z) = \sum_{i,j=1}^n \ln(1 + |X_i - Y_j|), \quad (1)$$

$$L_2(Z) = \sum_{i,j=1}^n \ln(1 + |X_i - Y_j|^2), \quad (2)$$

$$L_\infty(Z) = \sum_{i,j=1}^n \ln(|X_i - Y_j|), \quad (3)$$

$$LL_{distribution} = \text{maximum log likelihood permutation criterion based on the distribution} \quad (4)$$

Таблица 1: Мощность тестов для Нормального распределения с рандомизацией, размер выборок $n = 50, 1000$ итераций, 800 перестановок в каждой итерации

F_2	L_1	L_2	L_∞	LL_{norm}	$LL_{norm}^{var.eq}$	LL_{cauchy}	$LL_{laplace}$	LL_{levy}	wilcox.test	ks.test
N(0, 1)	5.4	5.6	4.8	5.2	5.1	5.9	5.8	5.2	5.4	4.3
N(0.25, 1)	19.8	22.1	15.1	19.6	23.7	15.7	17.8	8.1	23.4	16.5
N(0.5, 1)	63	67	50.7	62.8	70.9	49.6	54.4	15.6	70	53.2
N(0.75, 1)	94	95.2	86.9	93.7	96.8	84.1	89.6	26.6	96.5	90.1
N(1, 1)	98.9	99.1	98.4	98.9	99.8	97.6	98.6	44.2	99.4	98.7
N(0, 1.5)	36.1	34	34.1	70.3	4.7	21.2	40	35.2	5.5	11.2
N(0, 2)	89.7	90.2	84.6	99.1	4.8	69.7	91.6	75.5	5.9	37
N(0, 2.5)	99.3	99.4	97.7	100	4.8	93.6	99.9	94	6.3	69
N(0, 3)	100	100	100	100	4.9	99.5	100	98.6	6.8	89.2
N(0.25, 1.25)	24.9	25.5	20.8	39	20.9	19.4	26.7	8.6	18.9	16.5
N(0.5, 1.5)	74.9	75.6	64.2	89.3	51	52.1	72.5	16.9	48.2	54.2
N(0.75, 1.75)	96	96.3	92	98.9	75.5	86.2	95.4	26.5	73	84.1
N(1, 2)	99.5	99.6	98.4	100	88.7	96	99.3	37.3	86.7	96
N(0.25, 1.5)	47.5	46.7	41.8	76.2	16.7	32.1	49.9	24.3	17.1	23.2
N(0.5, 2)	95	95.8	91.3	99.6	36.2	82.9	95.9	56.4	34.7	68.1
N(0.75, 2.5)	99.8	99.9	98.7	100	51.6	96.9	99.8	81.5	49.5	91.1
N(1, 3)	100	100	100	100	61.1	99.7	100	91.6	58.1	98.2

Таблица 2: Мощность тестов для Нормального распределения без рандомизации, размер выборок $n = 50, 1000$ итераций, 800 перестановок в каждой итерации

F_2	L_1	L_2	L_∞	LL_{norm}	$LL_{norm}^{var.eq}$	LL_{cauchy}	$LL_{laplace}$	LL_{levy}	wilcox.test	ks.test
N(0, 1)	5.2	5.4	4.5	5.2	5.1	5.6	5.8	5	5.4	4.3
N(0.25, 1)	19.4	21.8	15	19.5	23.5	15.6	17.5	7.9	23.4	16.5
N(0.5, 1)	62.7	66.5	50.6	62.3	70.6	49.1	53.5	15.2	70	53.2
N(0.75, 1)	93.9	95.2	86.7	93.7	96.6	83.9	89.6	26.1	96.5	90.1
N(1, 1)	98.9	99.1	98.4	98.9	99.8	97.5	98.5	43.6	99.4	98.7
N(0, 1.5)	35.7	33.2	33.8	69.9	4.7	20.8	39.3	34.9	5.5	11.2
N(0, 2)	89.5	89.9	84.3	99.1	4.8	68.8	91.4	75.2	5.9	37
N(0, 2.5)	99.3	99.4	97.7	100	4.7	93.6	99.8	93.7	6.3	69
N(0, 3)	100	100	100	100	4.9	99.3	100	98.6	6.8	89.2
N(0.25, 1.25)	24.5	25.3	20.2	38.6	20.5	19.2	26.1	8.3	18.9	16.5
N(0.5, 1.5)	74.2	75.4	63.7	89.2	50.8	51.8	71.8	16.7	48.2	54.2
N(0.75, 1.75)	95.9	96.3	91.8	98.9	75.2	86	95.4	26.3	73	84.1
N(1, 2)	99.5	99.6	98.2	100	88.3	95.9	99.3	36.7	86.7	96
N(0.25, 1.5)	47	46.5	41.2	76.1	16.5	32	49.5	24.1	17.1	23.2
N(0.5, 2)	95	95.8	91.1	99.6	36	82.3	95.8	56	34.7	68.1
N(0.75, 2.5)	99.8	99.9	98.7	100	51.4	96.8	99.8	81.2	49.5	91.1
N(1, 3)	100	100	100	100	60.4	99.7	100	91.6	58.1	98.2

Таблица 3: Мощность тестов для распределения Коши с рандомизацией, размер выборок $n = 50, 1000$ итераций, 800 перестановок в каждой итерации

F_2	L_1	L_2	L_∞	LL_{norm}	LL_{cauchy}	$LL_{laplace}$	LL_{levy}	wilcox.test	ks.test
C(0, 1)	5	5.1	5.1	5.7	5	5.2	5.2	5	4.3
C(0.25, 1)	10.8	10.8	11.1	5.6	11.4	6.2	4.9	10.9	10.3
C(0.5, 1)	28.7	27.1	27.8	5.7	30.5	7.6	5	28.9	30
C(0.75, 1)	56.2	54.2	56	5.8	61.4	10.3	5.1	52.2	57.1
C(1, 1)	81	79.7	81	6.4	86	14.7	5.2	74.6	79.9
C(0, 1.5)	23.2	22	22.5	11.6	23.1	14.1	8.1	5.5	9.8
C(0, 2)	55	53.4	54.1	18.9	61	28.5	14.1	6	19.6
C(0, 2.5)	80.6	80	80	24.8	84.8	41.1	19.7	6.4	31.8
C(0, 3)	92.3	92.4	91.8	33.1	95.3	52.8	26.3	6.6	49.4
C(0.25, 1.25)	15.2	14.1	14.3	7	15.8	9.1	5.3	9.5	10.8
C(0.5, 1.5)	38.2	39.4	37.2	11.4	43.3	15.7	7.6	20.9	27.2
C(0.75, 1.75)	65.5	64.2	63.9	16	70.8	24	11	33.5	48.1
C(1, 2)	83.3	82.3	82.5	19.4	87.1	32.5	14	45.5	66.2
C(0.25, 1.5)	26.7	25.7	26.8	11.6	29.7	14.7	7.8	9	12.8
C(0.5, 2)	65.1	65	63.7	19.2	71.3	29.4	14	16.8	34.3
C(0.75, 2.5)	87.3	86.5	86.3	25.3	90	43.4	20.1	23.9	57.3
C(1, 3)	96.7	96.8	96.2	33.6	97.1	55.8	24.9	30	72.3

Таблица 4: Мощность тестов для распределения Коши без рандомизации, размер выборок $n = 50, 1000$ итераций, 800 перестановок в каждой итерации

F_2	L_1	L_2	L_∞	LL_{norm}	LL_{cauchy}	$LL_{laplace}$	LL_{levy}	wilcox.test	ks.test
C(0, 1)	5	5	5.1	5.4	4.9	5.1	5	5	4.3
C(0.25, 1)	10.7	10.5	10.7	5.5	11.3	5.9	4.9	10.9	10.3
C(0.5, 1)	28.4	26.7	27.6	5.5	30	7.5	4.9	28.9	30
C(0.75, 1)	56	53.7	55.6	5.8	61.1	10.1	5	52.2	57.1
C(1, 1)	80.7	79.6	80.6	6.3	85.3	14.6	5.2	74.6	79.9
C(0, 1.5)	22.7	21.7	22.2	11.2	22.9	13.9	7.9	5.5	9.8
C(0, 2)	54.7	53	53.7	18.8	60.5	28.1	14	6	19.6
C(0, 2.5)	80.6	79.8	79.9	24.7	84.7	40.9	19.5	6.4	31.8
C(0, 3)	92.2	92.3	91.8	32.8	95.2	52.5	26.2	6.6	49.4
C(0.25, 1.25)	15	13.9	14	6.9	15.4	9	5.1	9.5	10.8
C(0.5, 1.5)	38.1	38.8	36.9	11.3	42.4	15.5	7.4	20.9	27.2
C(0.75, 1.75)	64.6	63.8	63.5	15.7	70.3	23.6	10.9	33.5	48.1
C(1, 2)	83.3	82	82.2	19.2	86.9	32.4	14	45.5	66.2
C(0.25, 1.5)	26.4	25.3	26.5	11.4	29	14.7	7.8	9	12.8
C(0.5, 2)	64.6	64.6	63.2	19.1	70.9	29.1	14	16.8	34.3
C(0.75, 2.5)	87.2	86.2	86.2	25.2	90	43.2	19.8	23.9	57.3
C(1, 3)	96.7	96.8	96.1	33.5	97.1	55.3	24.7	30	72.3

Таблица 5: Мощность тестов для распределения Лапласа с рандомизацией, размер выборок $n = 50, 1000$ итераций, 800 перестановок в каждой итерации

F_2	L_1	L_2	L_∞	LL_{norm}	LL_{cauchy}	$LL_{laplace}$	LL_{levy}	wilcox.test	ks.test
La(0, 1)	5.9	5.5	5.5	5.4	5.1	6.6	5.8	4.6	4.7
La(0.25, 1)	17.9	17.8	15.6	9.8	19.5	17.4	5.7	19.5	16.3
La(0.5, 1)	54.8	54.9	50	26.3	57.1	53.8	7	56.7	54.2
La(0.75, 1)	87	87.3	83.8	53.3	88.1	86.6	9.9	88.8	86.1
La(1, 1)	97.8	98.1	96.8	79.9	97.9	97.6	13.5	97.8	97.5
La(0, 1.5)	24.9	25.1	23	42.5	23	39.1	17.7	5.7	9.2
La(0, 2)	70.9	71.6	64.4	86.7	62	87	42.6	5.8	21.9
La(0, 2.5)	93.2	93.6	89.4	98.2	88.2	98.2	64.3	6.4	40.3
La(0, 3)	98.6	98.7	97.8	99.7	97.5	99.7	79.6	7.1	60.4
La(0.25, 1.25)	22.2	21.6	18.9	20.9	22	25.8	7.9	15.9	16.1
La(0.5, 1.5)	60	59.1	54.9	55.5	57.5	67.8	14	39.8	45.9
La(0.75, 1.75)	88.3	88.4	83.9	82.8	86.6	92.5	22	62.9	72.6
La(1, 2)	97.3	97.6	95.8	95.8	96.5	98.8	29.9	78.4	91
La(0.25, 1.5)	37.9	37.2	32.8	46.7	33.3	47.8	16.6	13.9	19.3
La(0.5, 2)	83.3	83	78.5	90	79.1	92.7	35.8	30	50.2
La(0.75, 2.5)	98.1	98.2	96.3	99.2	96.4	99.7	55.6	45.3	76
La(1, 3)	99.9	99.9	99.6	100	99.8	100	70	54.8	91.8

Таблица 6: Мощность тестов для распределения Лапласа без рандомизации, размер выборок $n = 50, 10000$ итераций, 1600 перестановок в каждой итерации

F_2	L_1	L_2	L_∞	LL_{norm}	LL_{cauchy}	$LL_{laplace}$	LL_{levy}	wilcox.test	ks.test
La(0, 1)	5.5	5.6	5.1	5	5	5.2	4.8	5.5	4.1
La(0.25, 1)	18	17.8	16.3	9.3	18.8	17.2	5.4	19.1	17.2
La(0.5, 1)	54.6	53.6	49.9	23.7	56.8	53.6	6.4	55.3	54.1
La(0.75, 1)	87	86.7	83	51.3	87.8	86.6	8.7	87.3	86.3
La(1, 1)	98.3	98.2	97.1	78.8	98.3	98.2	12	98	98.1
La(0, 1.5)	28.1	28	25.3	44.3	24.8	41.8	19.4	5.7	9.4
La(0, 2)	72.1	72.8	65.6	87.5	64.2	87.2	44.6	5.9	22.8
La(0, 2.5)	94	94.5	90.4	98.4	89.4	98.6	66.1	6.3	41.9
La(0, 3)	99.1	99.2	97.8	99.8	97.3	99.9	80.7	6.5	61.4
La(0.25, 1.25)	22.2	21.8	20.3	21.1	22.1	26.1	7.6	16.3	16.5
La(0.5, 1.5)	61	60.6	56.2	56	59.6	69.6	14	39.6	45.7
La(0.75, 1.75)	88.2	88.3	84.4	83.9	86.6	92.9	22.2	61.9	73.4
La(1, 2)	97.7	97.8	96.3	95.6	96.6	98.9	30.6	78.2	89.8
La(0.25, 1.5)	37.8	37.7	34	47.3	34.4	49.9	16.6	14.2	19.2
La(0.5, 2)	84.4	84.8	80.2	90.3	80	93	37.7	30	51.5
La(0.75, 2.5)	98.3	98.5	96.9	99	96.6	99.7	56.8	43.9	77.3
La(1, 3)	99.9	99.9	99.7	99.9	99.6	100	70.6	54.5	91.6

Таблица 7: Мощность тестов для распределения Леви с рандомизацией, размер выборок $n = 50, 1000$ итераций, 800 перестановок в каждой итерации

F_2	L_1	L_2	L_∞	LL_{norm}	LL_{cauchy}	$LL_{laplace}$	LL_{levy}	wilcox.test	ks.test
Le(0, 1)	5.9	5.7	5.9	5.7	5.9	5.9	6.2	4.6	4.7
Le(0.25, 1)	7.2	7.1	14	5.7	6.2	5.9	66	10.1	8.5
Le(0.5, 1)	17.1	11.3	46.6	5.7	11.1	6.1	99	22.7	36.8
Le(0.75, 1)	41.3	28.1	82.4	5.7	23.9	6.1	100	38.7	74.5
Le(1, 1)	69.7	54	95.1	5.7	44.6	6.1	100	51.9	92.3
Le(0, 1.5)	13.6	13.2	16.2	6.9	14.7	6.8	23	20.5	13.4
Le(0, 2)	33.1	32.1	40.9	7.3	35.4	8.9	58.2	48.2	37.9
Le(0, 2.5)	54.9	52.7	64.7	8.7	54	10.6	82.4	69.5	61.1
Le(0, 3)	72.3	70	81.7	9.7	71	12.3	93.1	84.3	77.9
Le(0.25, 1.25)	13.4	11.2	23	6.8	11.9	5.9	75.9	22.4	17.7
Le(0.5, 1.5)	37.8	32.5	68	6.9	30.9	6.9	99.3	54.6	64
Le(0.75, 1.75)	72.8	64.9	93.1	7	52.4	7.9	100	75.1	90.2
Le(1, 2)	90.4	85.9	97.9	7.4	73.5	8.9	100	89.2	97.3
Le(0.25, 1.5)	21.6	19.5	35.8	6.9	21	6.9	83	37.4	30.8
Le(0.5, 2)	63.9	58.1	85	7.4	50.7	8.9	99.6	73.8	82
Le(0.75, 2.5)	89.4	86.8	97.2	8.7	77.5	10.8	100	92	96.3
Le(1, 3)	97.5	96.9	99.5	9.7	89	12.8	100	96.7	99.4

Таблица 8: Мощность тестов для распределения Леви без рандомизации, размер выборок $n = 50, 1000$ итераций, 800 перестановок в каждой итерации

F_2	L_1	L_2	L_∞	LL_{norm}	LL_{cauchy}	$LL_{laplace}$	LL_{levy}	wilcox.test	ks.test
Le(0, 1)	5.5	5.7	5.8	5.6	5.8	5.6	6.2	4.6	4.7
Le(0.25, 1)	7.1	6.9	13.8	5.6	6.1	5.5	65.3	10.1	8.5
Le(0.5, 1)	16.2	10.8	45.9	5.6	10.7	5.5	98.9	22.7	36.8
Le(0.75, 1)	41	27.8	81.8	5.6	23.3	5.7	100	38.7	74.5
Le(1, 1)	69.3	53.2	94.9	5.6	43.8	5.7	100	51.9	92.3
Le(0, 1.5)	13.4	12.7	15.6	6.9	14.3	6.7	22.9	20.5	13.4
Le(0, 2)	32.9	31.5	40.7	7.3	35	8.9	57.5	48.2	37.9
Le(0, 2.5)	54.3	52.3	64.3	8.5	53.4	10.4	81.9	69.5	61.1
Le(0, 3)	71.8	69.7	81.1	9.5	70.8	12.2	93	84.3	77.9
Le(0.25, 1.25)	12.8	11.2	22.3	6.7	11.2	5.7	75.5	22.4	17.7
Le(0.5, 1.5)	37.6	32.1	67.6	6.8	30.6	6.8	99.3	54.6	64
Le(0.75, 1.75)	72.3	64.8	92.6	6.9	52.2	7.6	100	75.1	90.2
Le(1, 2)	90	85.4	97.9	7.3	73.4	8.9	100	89.2	97.3
Le(0.25, 1.5)	21.3	19.5	35.4	6.9	20.7	6.8	82.7	37.4	30.8
Le(0.5, 2)	63.7	57.4	84.7	7.3	50.2	8.9	99.6	73.8	82
Le(0.75, 2.5)	89.2	86.6	97.2	8.5	77.4	10.6	100	92	96.3
Le(1, 3)	97.4	96.8	99.5	9.5	88.9	12.6	100	96.7	99.4

Таблица 9: Мощность тестов для распределения Лог-Коши с
рандомизацией, размер выборок $n = 50, 1000$ итераций, 800
перестановок в каждой итерации

F_2	L_1	L_2	L_∞	LL_{norm}	LL_{cauchy}	$LL_{laplace}$	LL_{levy}	$LL_{logcauchy}$	wilcox.test	ks.test
LC(0, 1)	6.8	6.7	6.9	4.5	6.1	4.7	4.9	6	4.8	4.5
LC(0.25, 1)	9.6	9.5	9	4.9	12.5	4.9	5.8	11.8	8.6	9.8
LC(0.5, 1)	22.1	20.9	24.4	4.9	38.6	5.9	11.6	33.1	27.6	30
LC(0.75, 1)	42.9	41.8	48.7	5.9	68.5	6.6	22.7	63.4	53	58.5
LC(1, 1)	66.9	65.9	72.9	6.5	88.1	7.4	38.7	85.9	75	80.7
LC(0, 1.5)	15.2	14.3	17.5	9	12.4	9.5	8	25.2	5.1	9.5
LC(0, 2)	30.9	28.8	40.2	15.4	21.5	15.9	16	58.8	5.1	20.6
LC(0, 2.5)	46.8	43	61.6	20.2	34.1	21.3	24.8	84.4	5.7	32.4
LC(0, 3)	58.4	55.1	77.9	24.3	44	26.1	35.5	92.8	5.7	47.1
LC(0.25, 1.25)	13.1	13.6	11.7	6.8	13.4	7.5	5	16.1	7.9	10.6
LC(0.5, 1.5)	28.4	28.6	29.5	10.4	34.4	11	6.4	44.3	19.2	26.2
LC(0.75, 1.75)	47.9	47.9	49.5	13.9	56.9	15.5	9.6	69.2	32.4	47.9
LC(1, 2)	64.3	64.3	66.7	18.6	73	19.9	12.2	86.7	44.4	67
LC(0.25, 1.5)	19.9	19	19.5	9.7	16.2	10.6	6.2	29.1	7.7	12.9
LC(0.5, 2)	42.2	41.7	47.5	16.8	40.1	17.6	9.7	70.1	14.6	33.4
LC(0.75, 2.5)	64.4	63	70.9	22	58.4	23.4	16.2	90.9	22.1	56.1
LC(1, 3)	78.5	77.8	85.2	27.3	71.5	28	21.8	97.4	27.4	71.9

Таблица 10: Мощность тестов для распределения Лог-Коши без
рандомизации, размер выборок $n = 50, 1000$ итераций, 800 пе-
рестановок в каждой итерации

F_2	L_1	L_2	L_∞	LL_{norm}	LL_{cauchy}	$LL_{laplace}$	LL_{levy}	$LL_{logcauchy}$	wilcox.test	ks.test
LC(0, 1)	6.7	6.4	6.9	4.4	6	4.6	4.5	5.8	4.8	4.5
LC(0.25, 1)	9.4	9.2	8.9	4.7	12.3	4.9	5.7	11.7	8.6	9.8
LC(0.5, 1)	21.4	20.6	24	4.9	38.3	5.8	11.5	32.7	27.6	30
LC(0.75, 1)	42.4	41.6	48.2	5.8	68	6.5	22.4	63.1	53	58.5
LC(1, 1)	66.6	65.6	72.4	6.1	88	7.2	38.5	85.7	75	80.7
LC(0, 1.5)	15.2	14.2	17.3	8.7	11.9	9.2	7.8	24.9	5.1	9.5
LC(0, 2)	30.4	28.3	39.8	15.1	21.2	16.1	15.9	58.6	5.1	20.6
LC(0, 2.5)	45.7	42.3	61	19.9	33.6	21	24.6	84.2	5.7	32.4
LC(0, 3)	57.8	54.7	77.6	24.1	43.5	26	35.1	92.6	5.7	47.1
LC(0.25, 1.25)	12.9	13.3	11.3	6.8	12.9	7.1	4.9	16.1	7.9	10.6
LC(0.5, 1.5)	28.1	28.2	29.4	10.2	34	10.9	6.1	44.1	19.2	26.2
LC(0.75, 1.75)	47.6	47.7	48.7	13.8	56.6	15.4	9.3	68.9	32.4	47.9
LC(1, 2)	64	64.2	66.3	18.3	72.9	19.9	12	86.4	44.4	67
LC(0.25, 1.5)	19.7	18.9	18.9	9.2	16.1	10.5	6	28.9	7.7	12.9
LC(0.5, 2)	41.8	41.3	47	16.4	39.9	17.7	9.6	69.9	14.6	33.4
LC(0.75, 2.5)	63.8	62.9	70.6	21.9	57.8	23.1	16.1	90.5	22.1	56.1
LC(1, 3)	78.4	77.4	84.8	27.2	70.8	28.1	21.7	97.1	27.4	71.9