$$T_n(Z) = -\frac{1}{n^2}\sum_{i,j=1}^{n} \ln(1+(X_i-Y_j)^2) +$$
$$+\frac{1}{n(n-1)}\sum_{i<j}^{n} \ln(1+(X_i-X_j)^2) + \frac{1}{n(n-1)}\sum_{i<j}^{n} \ln(1+(Y_i-Y_j)^2).$$

Critical value of Tn simulation criterion calculated by 1'000'000 iterations. 1000 iterations was performed in each case, 800 permutations used for $T_n, perm$.

$$formulae = P(z > z_{1-\alpha/2} - \frac{h}{\sqrt{6\ln 3}}) + P(z < -z_{1-\alpha/2} - \frac{h}{\sqrt{6\ln 3}}),$$

where $z \sim N(0,1), \quad 1/\sqrt{6\ln 3} \approx 0.389.$

Cauchy Mean

Table 1. Power of tests for the Cauchy distribution, $X \sim C(0,1)$, $Y \sim C(h/\sqrt{n},1)$, $n = 100$

h	$T_n, perm$	T_n, sim	$formulae$	$wilcox.test$	$ks.test$
1	6.4	6.3	6.8	6.6	6.3
3	19.6	20.3	21.5	20.5	20.2
5	50.9	50.5	49.5	48.5	53.1
7	82	82.3	77.8	77.2	83.6
9	96.7	96.8	93.9	91.5	96.5

Table 2. Power of tests for the Cauchy distribution, $X \sim C(0,1)$, $Y \sim C(h/\sqrt{n},1)$, $n = 500$

h	$T_n, perm$	T_n, sim	$formula$	$wilcox.test$	$ks.test$
1	5.8	6.1	6.8	6.4	6.4
3	21	21.8	21.5	22.2	24.3
5	50.9	51	49.5	48	57.9
7	82.2	82.4	77.8	75.6	85.9
9	96.2	96.5	93.9	93.2	97.2

Table 3. Power of tests for the Cauchy distribution, $X \sim C(0,1)$, $Y \sim C(h/\sqrt{n},1)$, $n = 1000$

h	$T_n, perm$	T_n, sim	$formula$	$wilcox.test$	$ks.test$
1	6.3	6	6.8	6.8	8.1
3	21	20.9	21.5	22.8	26.2
5	53.6	53.6	49.5	50.8	59.6
7	84	84.5	77.8	79.5	87.6
9	96.6	96.6	93.9	93.2	98.3

Cauchy Var

Table 4. Power of tests for the Cauchy distribution, $X \sim C(0,1)$, $Y \sim C(0, 1+h/\sqrt{n})$, $n = 100$

h	$T_n, perm$	T_n, sim	$wilcox.test$	$ks.test$
2	10.6	11.9	5.4	5.4
4	27.6	29.8	5.5	8.7
6	49.4	53.6	5.5	15.9
8	68.8	73.5	5.5	25
10	84.2	87.1	5.2	36.4

Table 5. Power of tests for the Cauchy distribution, $X \sim C(0,1)$, $Y \sim C(0, 1+h/\sqrt{n})$, $n = 500$

h	$T_n, perm$	T_n, sim	$wilcox.test$	$ks.test$
2	9.4	10	4.5	6.3
4	28.5	30.6	4.8	14
6	54.5	56.5	5	26.1
8	79.5	80.5	5.2	43.3
10	93	94	5.2	62.2

Table 6. Power of tests for the Cauchy distribution, $X \sim C(0,1)$, $Y \sim C(0, 1+h/\sqrt{n})$, $n = 1000$

h	$T_n, perm$	T_n, sim	$wilcox.test$	$ks.test$
2	10.2	10.5	5	7.6
4	32.4	33.8	5.2	13.8
6	61.1	62.8	5.2	27.9
8	84.8	85.6	5.2	47.4
10	96.1	97.1	5.4	67.9

Norm Mean

Table 7. Power of tests for the Normal distribution, $X \sim N(0,1)$, $Y \sim N(h/\sqrt{n}, 1)$, $n = 100$

h	$T_n, perm$	T_n, sim	$wilcox.test$	$ks.test$
1	11.1	11.3	12.5	9.5
2	29.3	29	31.1	20.5
3	52.4	53.4	55.8	42
4	77.5	77.5	80.6	64.9
5	91.9	92.5	93.1	84.7

Table 8. Power of tests for the Normal distribution, $X \sim N(0,1)$, $Y \sim N(h/\sqrt{n}, 1)$, $n = 500$

h	$T_n, perm$	T_n, sim	$wilcox.test$	$ks.test$
1	9.2	8.9	9.6	8.3
2	23.9	23.9	26.3	20.6
3	47.3	48.9	51.7	41.4
4	75.3	75.1	77.8	66.9
5	91.1	91	92.8	86.1

Table 9. Power of tests for the Normal distribution, $X \sim N(0,1)$, $Y \sim N(h/\sqrt{n}, 1)$, $n = 1000$

h	$T_n, perm$	T_n, sim	$wilcox.test$	$ks.test$
1	11	11.3	11.5	10
2	26.4	27.4	28.5	22
3	51.3	51.6	54.2	44.6
4	76.7	77	79.3	68.9
5	91.6	91.2	92.7	86.6

Norm Var

Table 10. Power of tests for the Normal distribution, $X \sim N(0,1)$, $Y \sim N(0, 1 + h/\sqrt{n})$, $n = 100$

h	$T_n, perm$	T_n, sim	$wilcox.test$	$ks.test$
1	8.1	8.7	6.4	5.3
2	15	17.4	6.3	7.2
3	30.5	34.2	6.6	10.7
4	50.6	57.1	6.7	16.7
5	70.8	76.7	6.5	24.8

Table 11. Power of tests for the Normal distribution, $X \sim N(0,1)$, $Y \sim N(0, 1 + h/\sqrt{n})$, $n = 500$

h	$T_n, perm$	T_n, sim	$wilcox.test$	$ks.test$
1	8.3	8.4	5	7.4
2	15.4	16.7	5.1	10.3
3	33.2	34.7	5.4	16.4
4	60	63.3	5.6	25.3
5	83.1	86.3	5.5	40.4

Table 12. Power of tests for the Normal distribution, $X \sim N(0,1)$, $Y \sim N(0, 1 + h/\sqrt{n})$, $n = 1000$

h	$T_n, perm$	T_n, sim	$wilcox.test$	$ks.test$
1	6.7	6.9	5.4	6
2	15.1	16.4	5.5	9.9
3	33.2	36	5.4	16.1
4	62.2	64	5.6	27.5
5	84.6	86.6	5.4	43.6