$$L_1(Z) = \sum_{i,j=1}^n \ln(1+|X_i-Y_j|), \tag{1}$$

$$L_2(Z) = \sum_{i,j=1}^n \ln(1+|X_i-Y_j|^2), \tag{2}$$

$$L_{\infty}(Z) = \sum_{i,j=1}^{n} \ln(|X_i - Y_j|),$$
 (3)

(4)

 $LL_{distribution}$ = maximum log likelyhood permutation criterion based on the distribution

Таблица 1: Мощность тестов для Нормального распределения, размер выборок $n=50,\,1000$ итераций, 800 перестановок в каждой итерации

| F_2 | L_1 | L_2 | L_{∞} | LL_{norm} | LL_{cauchy} | $LL_{laplace}$ | LL_{levy} | wilcox.test | ks.test |
|---------------|-------|-------|--------------|-------------|---------------|----------------|-------------|-------------|---------|
| N(0, 1) | 5.2 | 5.4 | 4.5 | 5.2 | 5.6 | 5.8 | 4.9 | 5.4 | 4.3 |
| N(0.25, 1) | 19.4 | 21.8 | 15 | 19.5 | 15.6 | 17.5 | 5 | 23.4 | 16.5 |
| N(0.5, 1) | 62.7 | 66.5 | 50.6 | 62.3 | 49.1 | 53.5 | 6.7 | 70 | 53.2 |
| N(0.75, 1) | 93.9 | 95.2 | 86.7 | 93.7 | 83.9 | 89.6 | 7.5 | 96.5 | 90.1 |
| N(1, 1) | 98.9 | 99.1 | 98.4 | 98.9 | 97.5 | 98.5 | 9 | 99.4 | 98.7 |
| N(0, 1.5) | 35.7 | 33.2 | 33.8 | 69.9 | 20.8 | 39.3 | 14.6 | 5.5 | 11.2 |
| N(0, 2) | 89.5 | 89.9 | 84.3 | 99.1 | 68.8 | 91.4 | 26.2 | 5.9 | 37 |
| N(0, 2.5) | 99.3 | 99.4 | 97.7 | 100 | 93.6 | 99.8 | 34.2 | 6.3 | 69 |
| N(0, 3) | 100 | 100 | 100 | 100 | 99.3 | 100 | 41.4 | 6.8 | 89.2 |
| N(0.25, 1.25) | 24.5 | 25.3 | 20.2 | 38.6 | 19.2 | 26.1 | 7.6 | 18.9 | 16.5 |
| N(0.5, 1.5) | 74.2 | 75.4 | 63.7 | 89.2 | 51.8 | 71.8 | 10.7 | 48.2 | 54.2 |
| N(0.75, 1.75) | 95.9 | 96.3 | 91.8 | 98.9 | 86 | 95.4 | 13.2 | 73 | 84.1 |
| N(1, 2) | 99.5 | 99.6 | 98.2 | 100 | 95.9 | 99.3 | 16.1 | 86.7 | 96 |
| N(0.25, 1.5) | 47 | 46.5 | 41.2 | 76.1 | 32 | 49.5 | 12.3 | 17.1 | 23.2 |
| N(0.5, 2) | 95 | 95.8 | 91.1 | 99.6 | 82.3 | 95.8 | 21.8 | 34.7 | 68.1 |
| N(0.75, 2.5) | 99.8 | 99.9 | 98.7 | 100 | 96.8 | 99.8 | 29.9 | 49.5 | 91.1 |
| N(1, 3) | 100 | 100 | 100 | 100 | 99.7 | 100 | 35.8 | 58.1 | 98.2 |

Таблица 2: Мощность тестов для распределения Коши, размер выборок $n=50,\,1000$ итераций, 800 перестановок в каждой итерации

| F_2 | L_1 | L_2 | L_{∞} | LL_{norm} | LL_{cauchy} | $LL_{laplace}$ | LL_{levy} | wilcox.test | ks.test |
|---------------|-------|-------|--------------|-------------|---------------|----------------|-------------|-------------|---------|
| C(0, 1) | 5 | 5 | 5.1 | 5.4 | 4.9 | 5.1 | 4.5 | 5 | 4.3 |
| C(0.25, 1) | 10.7 | 10.5 | 10.7 | 5.5 | 11.3 | 5.9 | 4.4 | 10.9 | 10.3 |
| C(0.5, 1) | 28.4 | 26.7 | 27.6 | 5.5 | 30 | 7.5 | 4.4 | 28.9 | 30 |
| C(0.75, 1) | 56 | 53.7 | 55.6 | 5.8 | 61.1 | 10.1 | 4 | 52.2 | 57.1 |
| C(1, 1) | 80.7 | 79.6 | 80.6 | 6.3 | 85.3 | 14.6 | 4.5 | 74.6 | 79.9 |
| C(0, 1.5) | 22.7 | 21.7 | 22.2 | 11.2 | 22.9 | 13.9 | 6 | 5.5 | 9.8 |
| C(0, 2) | 54.7 | 53 | 53.7 | 18.8 | 60.5 | 28.1 | 7.9 | 6 | 19.6 |
| C(0, 2.5) | 80.6 | 79.8 | 79.9 | 24.7 | 84.7 | 40.9 | 10.1 | 6.4 | 31.8 |
| C(0, 3) | 92.2 | 92.3 | 91.8 | 32.8 | 95.2 | 52.5 | 11.4 | 6.6 | 49.4 |
| C(0.25, 1.25) | 15 | 13.9 | 14 | 6.9 | 15.4 | 9 | 4.8 | 9.5 | 10.8 |
| C(0.5, 1.5) | 38.1 | 38.8 | 36.9 | 11.3 | 42.4 | 15.5 | 5.6 | 20.9 | 27.2 |
| C(0.75, 1.75) | 64.6 | 63.8 | 63.5 | 15.7 | 70.3 | 23.6 | 6.8 | 33.5 | 48.1 |
| C(1, 2) | 83.3 | 82 | 82.2 | 19.2 | 86.9 | 32.4 | 7.7 | 45.5 | 66.2 |
| C(0.25, 1.5) | 26.4 | 25.3 | 26.5 | 11.4 | 29 | 14.7 | 5.8 | 9 | 12.8 |
| C(0.5, 2) | 64.6 | 64.6 | 63.2 | 19.1 | 70.9 | 29.1 | 7.3 | 16.8 | 34.3 |
| C(0.75, 2.5) | 87.2 | 86.2 | 86.2 | 25.2 | 90 | 43.2 | 10.1 | 23.9 | 57.3 |
| C(1, 3) | 96.7 | 96.8 | 96.1 | 33.5 | 97.1 | 55.3 | 11.6 | 30 | 72.3 |

Таблица 3: Мощность тестов для распределения Лапласа, размер выборок $n=50,\,10000$ итераций, 1600 перестановок в каждой итерации

| F_2 | L_1 | L_2 | L_{∞} | LL_{norm} | LL_{cauchy} | $LL_{laplace}$ | LL_{levy} | wilcox.test | ks.test |
|----------------|-------|-------|--------------|-------------|---------------|----------------|-------------|-------------|---------|
| La(0, 1) | 5.5 | 5.6 | 5.1 | 5 | 5 | 5.2 | 5 | 5.5 | 4.1 |
| La(0.25, 1) | 18 | 17.8 | 16.3 | 9.3 | 18.8 | 17.2 | 5.1 | 19.1 | 17.2 |
| La(0.5, 1) | 54.6 | 53.6 | 49.9 | 23.7 | 56.8 | 53.6 | 5.4 | 55.3 | 54.1 |
| La(0.75, 1) | 87 | 86.7 | 83 | 51.3 | 87.8 | 86.6 | 5.8 | 87.3 | 86.3 |
| La(1, 1) | 98.3 | 98.2 | 97.1 | 78.8 | 98.3 | 98.2 | 5.8 | 98 | 98.1 |
| La(0, 1.5) | 28.1 | 28 | 25.3 | 44.3 | 24.8 | 41.8 | 10 | 5.7 | 9.4 |
| La(0, 2) | 72.1 | 72.8 | 65.6 | 87.5 | 64.2 | 87.2 | 17 | 5.9 | 22.8 |
| La(0, 2.5) | 94 | 94.5 | 90.4 | 98.4 | 89.4 | 98.6 | 23.6 | 6.3 | 41.9 |
| La(0, 3) | 99.1 | 99.2 | 97.8 | 99.8 | 97.3 | 99.9 | 28.6 | 6.5 | 61.4 |
| La(0.25, 1.25) | 22.2 | 21.8 | 20.3 | 21.1 | 22.1 | 26.1 | 6.4 | 16.3 | 16.5 |
| La(0.5, 1.5) | 61 | 60.6 | 56.2 | 56 | 59.6 | 69.6 | 8.8 | 39.6 | 45.7 |
| La(0.75, 1.75) | 88.2 | 88.3 | 84.4 | 83.9 | 86.6 | 92.9 | 11.1 | 61.9 | 73.4 |
| La(1, 2) | 97.7 | 97.8 | 96.3 | 95.6 | 96.6 | 98.9 | 13.5 | 78.2 | 89.8 |
| La(0.25, 1.5) | 37.8 | 37.7 | 34 | 47.3 | 34.4 | 49.9 | 9.5 | 14.2 | 19.2 |
| La(0.5, 2) | 84.4 | 84.8 | 80.2 | 90.3 | 80 | 93 | 15.3 | 30 | 51.5 |
| La(0.75, 2.5) | 98.3 | 98.5 | 96.9 | 99 | 96.6 | 99.7 | 20.8 | 43.9 | 77.3 |
| La(1, 3) | 99.9 | 99.9 | 99.7 | 99.9 | 99.6 | 100 | 25.7 | 54.5 | 91.6 |

Таблица 4: Мощность тестов для распределения Леви, размер выборок $n=50,\,1000$ итераций, 800 перестановок в каждой итерации

| F_2 | L_1 | L_2 | L_{∞} | LL_{norm} | LL_{cauchy} | $LL_{laplace}$ | LL_{levy} | wilcox.test | ks.test |
|----------------|-------|-------|--------------|-------------|---------------|----------------|-------------|-------------|---------|
| Le(0, 1) | 5.5 | 5.7 | 5.8 | 5.6 | 5.8 | 5.6 | 5.8 | 4.6 | 4.7 |
| Le(0.25, 1) | 7.1 | 6.9 | 13.8 | 5.6 | 6.1 | 5.5 | 31.6 | 10.1 | 8.5 |
| Le(0.5, 1) | 16.2 | 10.8 | 45.9 | 5.6 | 10.7 | 5.5 | 62.6 | 22.7 | 36.8 |
| Le(0.75, 1) | 41 | 27.8 | 81.8 | 5.6 | 23.3 | 5.7 | 79.7 | 38.7 | 74.5 |
| Le(1, 1) | 69.3 | 53.2 | 94.9 | 5.6 | 43.8 | 5.7 | 89.5 | 51.9 | 92.3 |
| Le(0, 1.5) | 13.4 | 12.7 | 15.6 | 6.9 | 14.3 | 6.7 | 7.9 | 20.5 | 13.4 |
| Le(0, 2) | 32.9 | 31.5 | 40.7 | 7.3 | 35 | 8.9 | 11.9 | 48.2 | 37.9 |
| Le(0, 2.5) | 54.3 | 52.3 | 64.3 | 8.5 | 53.4 | 10.4 | 17.3 | 69.5 | 61.1 |
| Le(0, 3) | 71.8 | 69.7 | 81.1 | 9.5 | 70.8 | 12.2 | 22.8 | 84.3 | 77.9 |
| Le(0.25, 1.25) | 12.8 | 11.2 | 22.3 | 6.7 | 11.2 | 5.7 | 28.8 | 22.4 | 17.7 |
| Le(0.5, 1.5) | 37.6 | 32.1 | 67.6 | 6.8 | 30.6 | 6.8 | 52.1 | 54.6 | 64 |
| Le(0.75, 1.75) | 72.3 | 64.8 | 92.6 | 6.9 | 52.2 | 7.6 | 63.6 | 75.1 | 90.2 |
| Le(1, 2) | 90 | 85.4 | 97.9 | 7.3 | 73.4 | 8.9 | 72.6 | 89.2 | 97.3 |
| Le(0.25, 1.5) | 21.3 | 19.5 | 35.4 | 6.9 | 20.7 | 6.8 | 27.8 | 37.4 | 30.8 |
| Le(0.5, 2) | 63.7 | 57.4 | 84.7 | 7.3 | 50.2 | 8.9 | 48.5 | 73.8 | 82 |
| Le(0.75, 2.5) | 89.2 | 86.6 | 97.2 | 8.5 | 77.4 | 10.6 | 59.1 | 92 | 96.3 |
| Le(1, 3) | 97.4 | 96.8 | 99.5 | 9.5 | 88.9 | 12.6 | 67.8 | 96.7 | 99.4 |

Таблица 5: Мощность тестов для распределения Лог-Коши, размер выборок $n=50,\,1000$ итераций, 800 перестановок в каждой итерации

| F_2 | L_1 | L_2 | L_{∞} | LL_{norm} | LL_{cauchy} | $LL_{laplace}$ | LL_{levy} | $LL_{log cauchy}$ | wilcox.test | ks.test |
|----------------|-------|-------|--------------|-------------|---------------|----------------|-------------|-------------------|-------------|---------|
| LC(0, 1) | 6.7 | 6.4 | 6.9 | 4.4 | 6 | 4.6 | 4.2 | 5.8 | 4.8 | 4.5 |
| LC(0.25, 1) | 9.4 | 9.2 | 8.9 | 4.7 | 12.3 | 4.9 | 6.6 | 11.7 | 8.6 | 9.8 |
| LC(0.5, 1) | 21.4 | 20.6 | 24 | 4.9 | 38.3 | 5.8 | 12 | 32.7 | 27.6 | 30 |
| LC(0.75, 1) | 42.4 | 41.6 | 48.2 | 5.8 | 68 | 6.5 | 23.2 | 63.1 | 53 | 58.5 |
| LC(1, 1) | 66.6 | 65.6 | 72.4 | 6.1 | 88 | 7.2 | 34 | 85.7 | 75 | 80.7 |
| LC(0, 1.5) | 15.2 | 14.2 | 17.3 | 8.7 | 11.9 | 9.2 | 6.6 | 24.9 | 5.1 | 9.5 |
| LC(0, 2) | 30.4 | 28.3 | 39.8 | 15.1 | 21.2 | 16.1 | 13.6 | 58.6 | 5.1 | 20.6 |
| LC(0, 2.5) | 45.7 | 42.3 | 61 | 19.9 | 33.6 | 21 | 22 | 84.2 | 5.7 | 32.4 |
| LC(0, 3) | 57.8 | 54.7 | 77.6 | 24.1 | 43.5 | 26 | 34 | 92.6 | 5.7 | 47.1 |
| LC(0.25, 1.25) | 12.9 | 13.3 | 11.3 | 6.8 | 12.9 | 7.1 | 4.4 | 16.1 | 7.9 | 10.6 |
| LC(0.5, 1.5) | 28.1 | 28.2 | 29.4 | 10.2 | 34 | 10.9 | 6.5 | 44.1 | 19.2 | 26.2 |
| LC(0.75, 1.75) | 47.6 | 47.7 | 48.7 | 13.8 | 56.6 | 15.4 | 8.3 | 68.9 | 32.4 | 47.9 |
| LC(1, 2) | 64 | 64.2 | 66.3 | 18.3 | 72.9 | 19.9 | 10.6 | 86.4 | 44.4 | 67 |
| LC(0.25, 1.5) | 19.7 | 18.9 | 18.9 | 9.2 | 16.1 | 10.5 | 5.1 | 28.9 | 7.7 | 12.9 |
| LC(0.5, 2) | 41.8 | 41.3 | 47 | 16.4 | 39.9 | 17.7 | 8.3 | 69.9 | 14.6 | 33.4 |
| LC(0.75, 2.5) | 63.8 | 62.9 | 70.6 | 21.9 | 57.8 | 23.1 | 13.7 | 90.5 | 22.1 | 56.1 |
| LC(1, 3) | 78.4 | 77.4 | 84.8 | 27.2 | 70.8 | 28.1 | 19.2 | 97.1 | 27.4 | 71.9 |