$$T_n(Z) = -\frac{1}{n^2}\sum_{i,j=1}^{n}\ln(1+(X_i-Y_j)^2) +$$
$$+\frac{1}{n(n-1)}\sum_{i<j}^{n}\ln(1+(X_i-X_j)^2)+\frac{1}{n(n-1)}\sum_{i<j}^{n}\ln(1+(Y_i-Y_j)^2).$$

Critical value of Tn simulation criterion calculated by 1'000'000 iterations. 1000 iterations was performed in each case, 800 permutations used for $T_n, perm$.

ad.test comes from kSamples package with using only an asymptotic P-value approximation (see Pettitt, A.N. (1976), A two-sample Anderson_Darling rank statistic, Biometrika, 63, 161-168.).

Cauchy Mean

$$formulae = P(z > z_{1-\alpha/2} - \frac{h}{\sqrt{6 \ln 3}}) + P(z < -z_{1-\alpha/2} - \frac{h}{\sqrt{6 \ln 3}}),$$

where $z \sim N(0,1)$, $\quad 1/\sqrt{6 \ln 3} \approx 0.389$.

Table 1. Power of tests for the Cauchy distribution, $X \sim C(0,1)$, $Y \sim C(h/\sqrt{n},1)$, $n = 100$

h	$T_n, perm$	T_n, sim	$formulae$	$wilcox.test$	$ks.test$	$ad.test$
1	6.4	6.3	6.8	6.6	6.3	7.1
2	10.1	10.6	12.2	11.9	11.1	11.6
3	19.6	20.3	21.5	20.5	20.2	20.7
5	50.9	50.5	49.5	48.5	53.1	52.2
7	82	82.3	77.8	77.2	83.6	80.7
9	96.7	96.8	93.9	91.5	96.5	95.2

Table 2. Power of tests for the Cauchy distribution, $X \sim C(0,1)$, $Y \sim C(h/\sqrt{n},1)$, $n = 500$

h	$T_n, perm$	T_n, sim	$formula$	$wilcox.test$	$ks.test$	$ad.test$
1	5.8	6.1	6.8	6.4	6.4	7.1
2	11.6	11.6	12.2	12.6	13.9	12.2
3	21	21.8	21.5	22.2	24.3	22.8
5	50.9	51	49.5	48	57.9	50.3
7	82.2	82.4	77.8	75.6	85.9	81.1
9	96.2	96.5	93.9	93.2	97.2	96.0

Table 3. Power of tests for the Cauchy distribution, $X \sim C(0,1)$, $Y \sim C(h/\sqrt{n},1)$, $n = 1000$

h	$T_n, perm$	T_n, sim	$formula$	$wilcox.test$	$ks.test$	$ad.test$
1	6.3	6	6.8	6.8	8.1	6.8
2	11.4	11.9	12.2	12.9	13.4	12.9
3	21	20.9	21.5	22.8	26.2	22.2
5	53.6	53.6	49.5	50.8	59.6	54.2
7	84	84.5	77.8	79.5	87.6	84.4
9	96.6	96.6	93.9	93.2	98.3	96.3

Cauchy Var

Table 4. Power of tests for the Cauchy distribution, $X \sim C(0,1)$, $Y \sim C(0,1+h/\sqrt{n})$, $n = 100$

h	$T_n, perm$	T_n, sim	$wilcox.test$	$ks.test$	$ad.test$
2	10.6	11.9	5.4	5.4	6.9
4	27.6	29.8	5.5	8.7	11.3
6	49.4	53.6	5.5	15.9	22.2
8	68.8	73.5	5.5	25	37.7
10	84.2	87.1	5.2	36.4	55.4

Table 5. Power of tests for the Cauchy distribution, $X \sim C(0,1)$, $Y \sim C(0,1+h/\sqrt{n})$, $n = 500$

h	$T_n, perm$	T_n, sim	$wilcox.test$	$ks.test$	$ad.test$
2	9.4	10	4.5	6.3	6.2
4	28.5	30.6	4.8	14	12.3
6	54.5	56.5	5	26.1	29.7
8	79.5	80.5	5.2	43.3	51.0
10	93	94	5.2	62.2	74.2

Table 6. Power of tests for the Cauchy distribution, $X \sim C(0,1)$, $Y \sim C(0, 1+h/\sqrt{n})$, $n = 1000$

h	$T_n, perm$	T_n, sim	$wilcox.test$	$ks.test$	$ad.test$
2	10.2	10.5	5	7.6	7.3
4	32.4	33.8	5.2	13.8	14.9
6	61.1	62.8	5.2	27.9	32.8
8	84.8	85.6	5.2	47.4	59.7
10	96.1	97.1	5.4	67.9	82.8

Norm Mean

$$formulae = P(z > z_{1-\alpha/2} - \sqrt{\frac{3J^*(h)}{J_0}}) + P(z < -z_{1-\alpha/2} - \sqrt{\frac{3J^*(h)}{J_0}}),$$

where $z \sim N(0,1)$, $J^*(h) \approx h^2/4.4$, $J_0 \approx 0.81$.

Table 7. Power of tests for the Normal distribution, $X \sim N(0,1)$, $Y \sim N(h/\sqrt{n}, 1)$, $n = 100$

h	$T_n, perm$	T_n, sim	$formulae$	$wilcox.test$	$ks.test$	$ad.test$
1	11.1	11.3	15.1	12.5	9.5	12.2
2	29.3	29	45	31.1	20.5	29.6
3	52.4	53.4	78.6	55.8	42	55
4	77.5	77.5	95.6	80.6	64.9	78.9
5	91.9	92.5	99.6	93.1	84.7	93.1

Table 8. Power of tests for the Normal distribution, $X \sim N(0,1)$, $Y \sim N(h/\sqrt{n}, 1)$, $n = 500$

h	$T_n, perm$	T_n, sim	$formulae$	$wilcox.test$	$ks.test$	$ad.test$
1	9.2	8.9	15.1	9.6	8.3	9.0
2	23.9	23.9	45	26.3	20.6	25.4
3	47.3	48.9	78.6	51.7	41.4	49.7
4	75.3	75.1	95.6	77.8	66.9	76.9
5	91.1	91	99.6	92.8	86.1	92.6

Table 9. Power of tests for the Normal distribution, $X \sim N(0,1)$, $Y \sim N(h/\sqrt{n},1)$, $n = 1000$

h	$T_n, perm$	T_n, sim	$formulae$	$wilcox.test$	$ks.test$	$ad.test$
1	11	11.3	15.1	11.5	10	11.6
2	26.4	27.4	45	28.5	22	27.7
3	51.3	51.6	78.6	54.2	44.6	52.9
4	76.7	77	95.6	79.3	68.9	77.9
5	91.6	91.2	99.6	92.7	86.6	92.1

Norm Var

Table 10. Power of tests for the Normal distribution, $X \sim N(0,1)$, $Y \sim N(0, 1 + h/\sqrt{n})$, $n = 100$

h	$T_n, perm$	T_n, sim	$wilcox.test$	$ks.test$	$ad.test$
1	8.1	8.7	6.4	5.3	7.3
2	15	17.4	6.3	7.2	12.7
3	30.5	34.2	6.6	10.7	24.0
4	50.6	57.1	6.7	16.7	39.9
5	70.8	76.7	6.5	24.8	59.9

Table 11. Power of tests for the Normal distribution, $X \sim N(0,1)$, $Y \sim N(0, 1 + h/\sqrt{n})$, $n = 500$

h	$T_n, perm$	T_n, sim	$wilcox.test$	$ks.test$	$ad.test$
1	8.3	8.4	5	7.4	7.7
2	15.4	16.7	5.1	10.3	12.8
3	33.2	34.7	5.4	16.4	28.3
4	60	63.3	5.6	25.3	52.6
5	83.1	86.3	5.5	40.4	78.1

Table 12. Power of tests for the Normal distribution, $X \sim N(0,1)$, $Y \sim N(0, 1 + h/\sqrt{n})$, $n = 1000$

h	$T_n, perm$	T_n, sim	$wilcox.test$	$ks.test$	$ad.test$
1	6.7	6.9	5.4	6	6.7
2	15.1	16.4	5.5	9.9	13.1
3	33.2	36	5.4	16.1	30.6
4	62.2	64	5.6	27.5	56.8
5	84.6	86.6	5.4	43.6	81.1