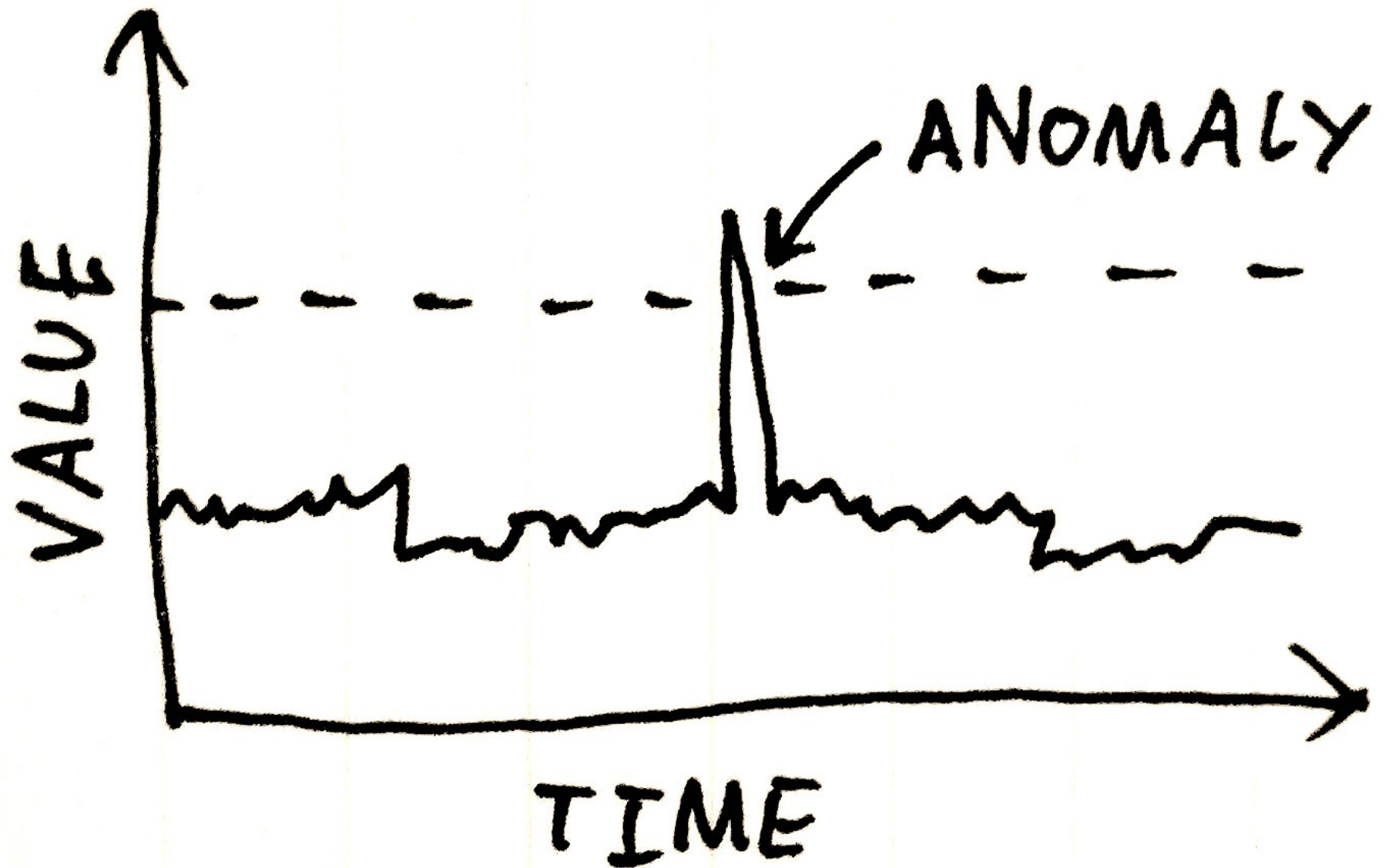


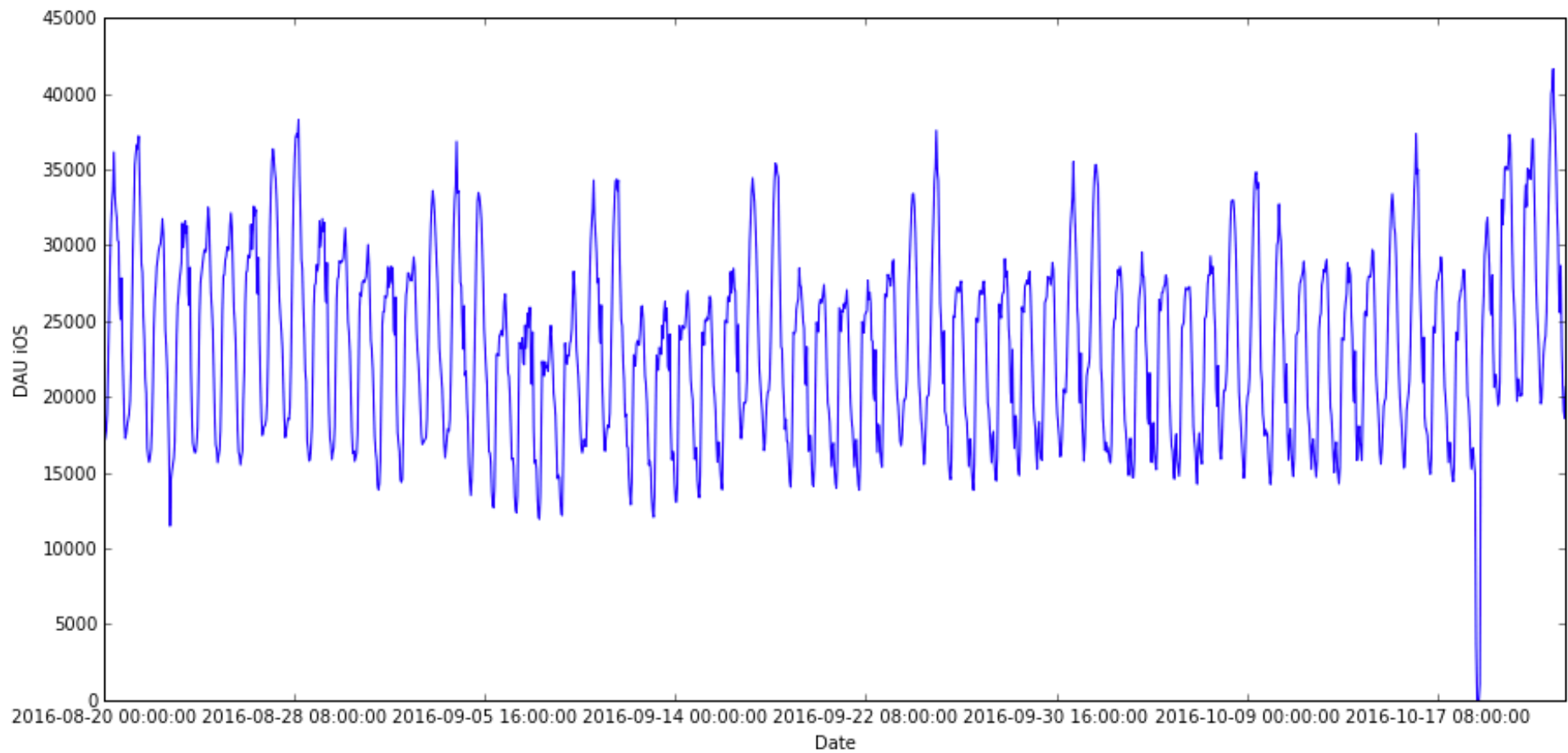
# Anomaly Detection in Time Series

Dmitry Sergeyev

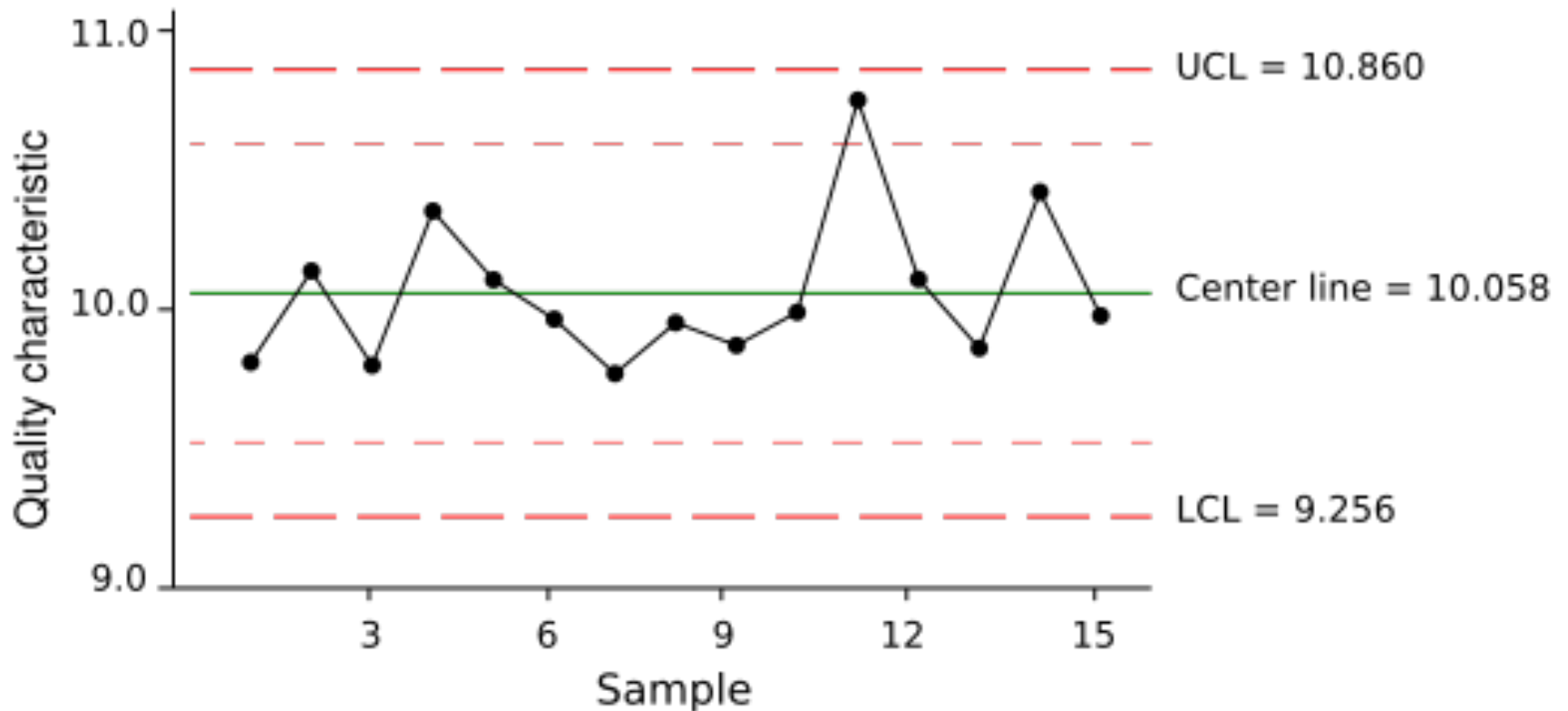
Why?



# Why?



# Statistical Process Control



# Statistical Process Control

The application of SPC involves three main phases of activity:

- Understanding the process and the specification limits
- Eliminating assignable (special) sources of variation, so that the process is stable
- Monitoring the ongoing production process, assisted by the use of control charts, to detect significant changes of mean or variation.

# Window estimations

# Window estimations

- Simple moving average
- Weighted moving average
- Exponentially weighted moving average
- etc.

# Window estimations

Good:

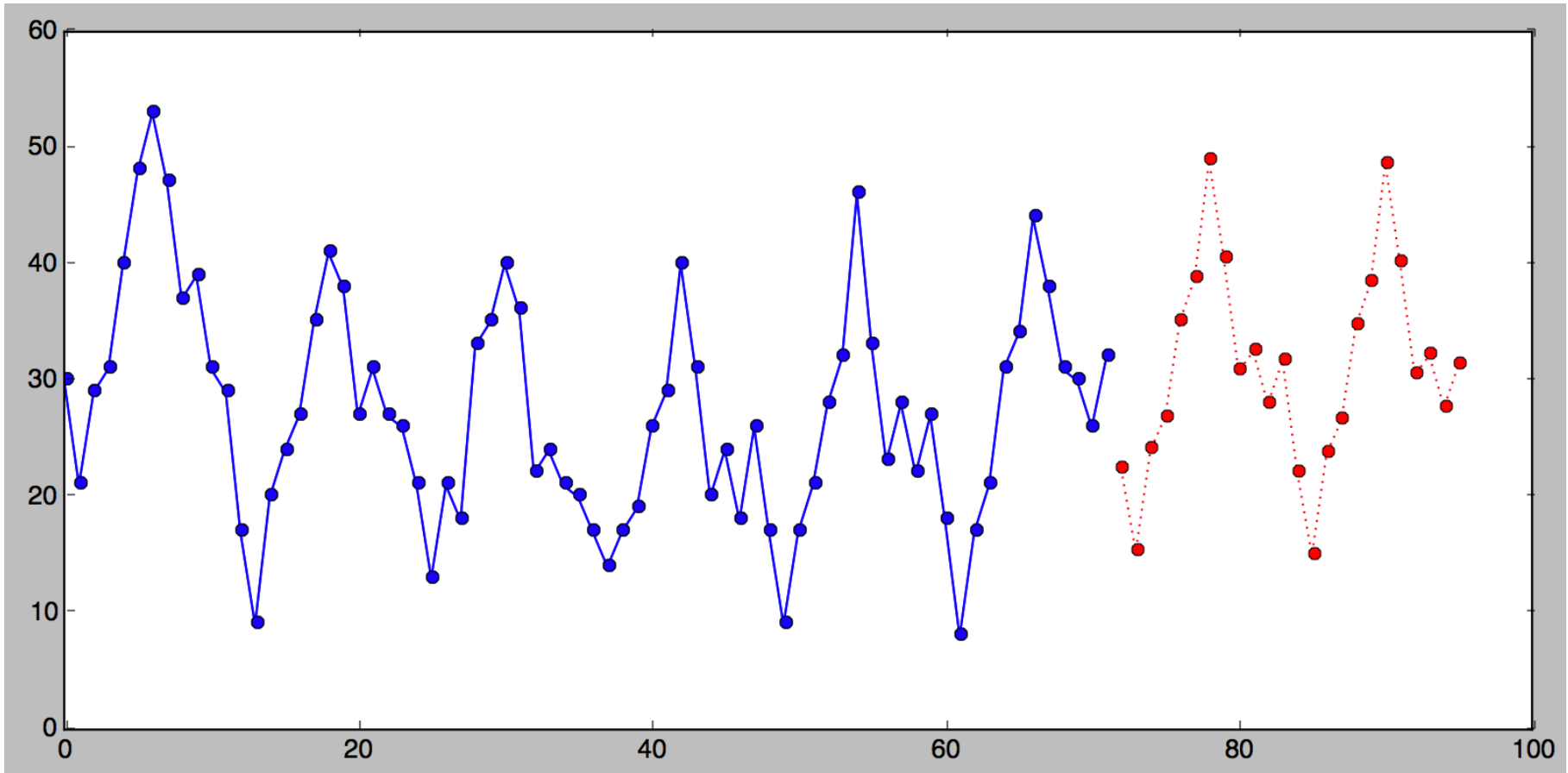
- Easy to use
- Fast calculations

Bad:

- One-step ahead predictions
- False positives

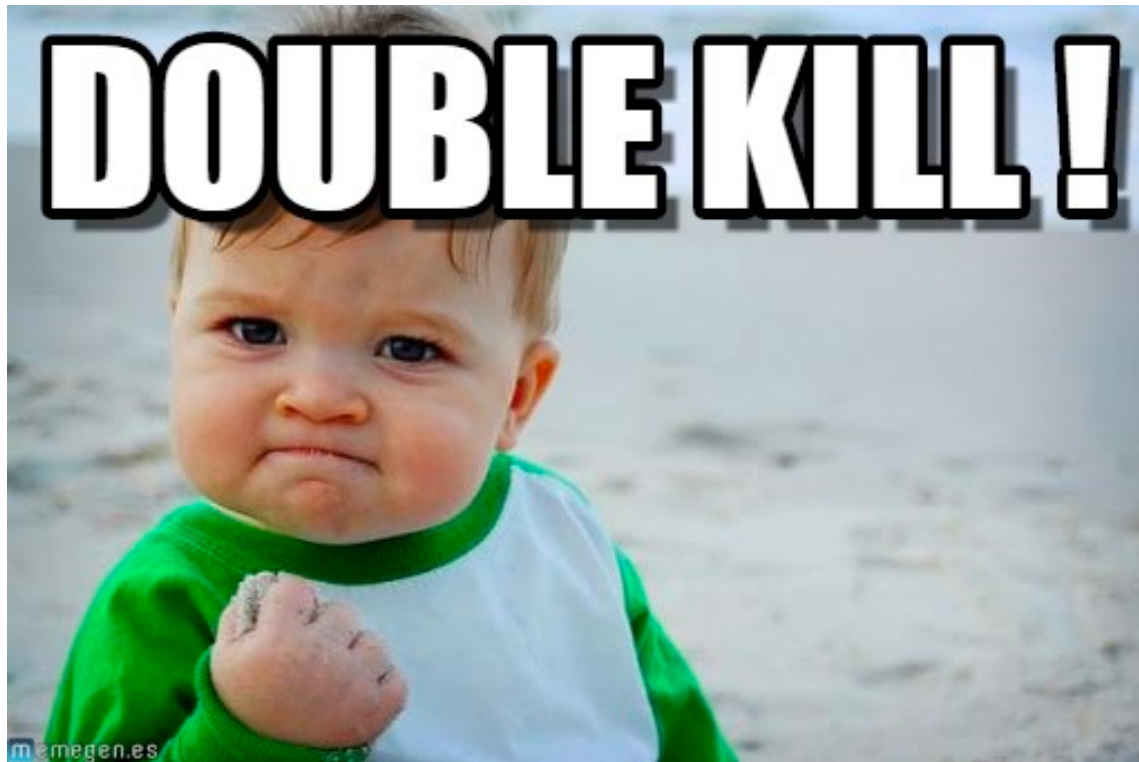


# Exponential Smoothing



# Exponential Smoothing

## Double Exponential Smoothing



# Exponential Smoothing

## Double Exponential Smoothing

Double exponential smoothing is nothing more than exponential smoothing applied to both level and trend.

- Level – expected value/slope/intercept
- Trend or Slope –  $y_t - y_{t-1}$

# Exponential Smoothing

## Double Exponential Smoothing

$$\begin{array}{ll}\ell_x = \alpha y_x + (1 - \alpha)(\ell_{x-1} + b_{x-1}) & \text{level} \\ b_x = \beta(\ell_x - \ell_{x-1}) + (1 - \beta)b_{x-1} & \text{trend} \\ \hat{y}_{x+1} = \ell_x + b_x & \text{forecast}\end{array}$$

- In the first equation we're using  $\ell$  instead of  $\hat{y}$  and on the right side the expected value becomes the sum of level and trend.
- The second equation introduces  $\beta$ , the trend factor (or coefficient). As with  $\alpha$ , some values of  $\beta$  work better than others depending on the series.

# Holt-Winters

## Triple Exponential Smoothing



# Holt-Winters

## Triple Exponential Smoothing

$\ell_x = \alpha(y_x - s_{x-L}) + (1 - \alpha)(\ell_{x-1} + b_{x-1})$	level
$b_x = \beta(\ell_x - \ell_{x-1}) + (1 - \beta)b_{x-1}$	trend
$s_x = \gamma(y_x - \ell_x) + (1 - \gamma)s_{x-L}$	seasonal
$\hat{y}_{x+m} = \ell_x + mb_x + s_{x-L+1+(m-1)modL}$	forecast

- $\gamma$  - the smoothing factor for the seasonal component
- We can forecast any number of points into the future
- The forecast equation now consists of level, trend and the seasonal component

# Brutlag

$$\hat{y}_{max_t} = L_{t-1} + P_{t-1} + S_{t-T} + m \cdot d_{t-T}$$

$$\hat{y}_{min_t} = L_{t-1} + P_{t-1} + S_{t-T} - m \cdot d_{t-T}$$

$d$  is predicted deviation given by:

$$d_t = \gamma |y_t - \hat{y}_t| + (1 - \gamma) d_{t-T}$$

$\gamma$  is the seasonal change smoothing factor

$m$  is the scaling factor for Brutlags confidence bands

# Kolmogorov-Smirnov

Good:

- works on periodic data when its period is smaller than the window size
- outlier resistant (less sensitive to small-scale changes with larger windows)
- good for data exploration (allows top-down exploration of large-scale changes first)

Bad:

- produces false positives on trends and seasonal changes
- needs many (hundreds) unique values (it won't work on metrics gathered with 5 minutes interval)
- bad for alerting because of false positives

<http://mabrek.github.io/blog/statistics-for-monitoring-anomaly-p2/>



# SARIMA

$$SARIMA(p, d, q)(P, D, Q)_s$$

- $p$  – AR( $p$ )
- $d$  – порядок интегрирования
- $q$  – MA( $q$ )
- $P$  – порядок сезонной составляющей SAR( $P$ )
- $D$  – порядок интегрирования сезонной составляющей
- $Q$  – порядок сезонной составляющей SMA( $Q$ )
- $s$  – размерность сезонности (месяц, квартал и т.д.)



# Prophet



[https://facebookincubator.github.io/prophet/static/prophet\\_paper\\_20170113.pdf](https://facebookincubator.github.io/prophet/static/prophet_paper_20170113.pdf)

<https://facebookincubator.github.io/prophet/>



# Prophet

Good:

- Stuff is done for you
- Custom parameters
- Easy to use

Bad:

- Less accurate
- Doesn't work with non-daily/monthly data