Lecture 4. Into to Neural Networks

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Who am I?

MIPT
MOSCOW INSTITUTE
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Yandex

ШКОЛА АНАЛИЗА ДАННЫХ

Dbrain

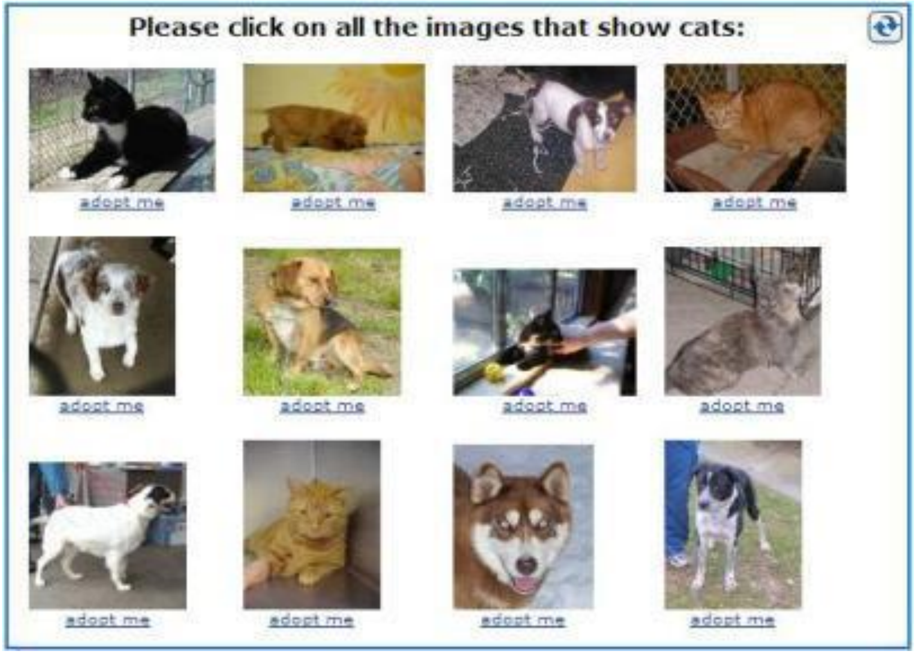
Skoltech
Skolkovo Institute of Science and Technology

Lecture plan

- Examples of NN usage
- Biological motivation
- Mathematical modelling
- Fully connected NNs
- Activation functions
- Loss functions and backprop
- Gradient descent
- Optimization
- Weight initialization

Once upon a time...

2006



Computer vision = 60%

$0.6^{12} = 0.00217$

2014

Completed • Swag • 215 teams

Dogs vs. Cats

Wed 25 Sep 2013 – Sat 1 Feb 2014 (8 months ago)

Dashboard

Private Leaderboard - Dogs vs. Cats

This competition has completed. This leaderboard reflects the final standings.

See someone

#	Δ1w	Team Name * in the money	Score	Entries	Last Submission UTC (Best – Las
1	—	Pierre Sermanet *	0.98914	5	Sat, 01 Feb 2014 21:43:19 (-
2	↑26	orchid *	0.98309	17	Sat, 01 Feb 2014 23:52:30
3	—	Owen	0.98171	15	Sat, 01 Feb 2014 17:04:40 (-
4	new	Paul Covington	0.98171	3	Sat, 01 Feb 2014 23:05:20
5	↓3	Maxim Milakov	0.98137	24	Sat, 01 Feb 2014 18:20:58

$$0.989^{12} = 0.875$$

2014

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ASIRRA

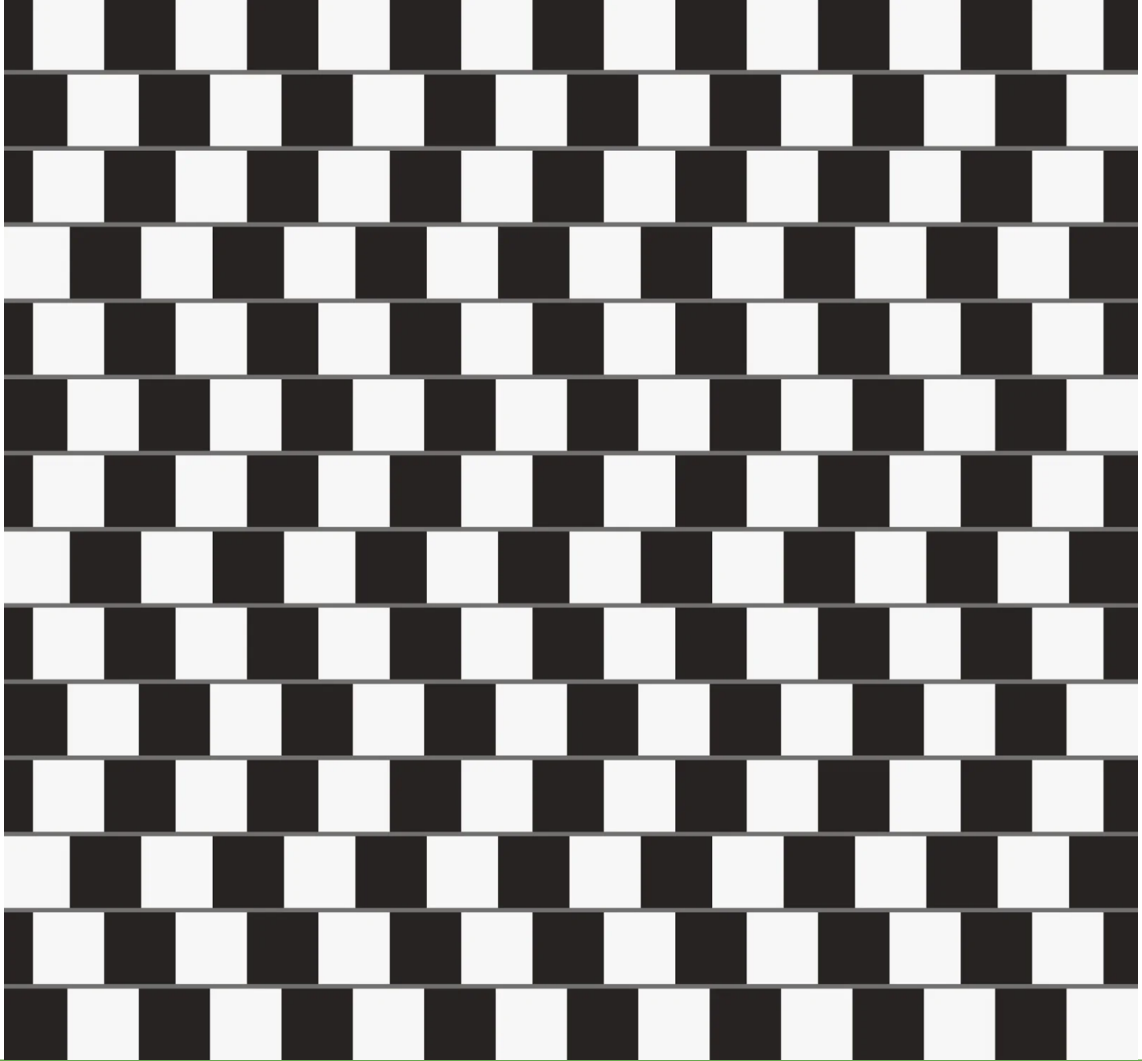
After 8 years of operation, Asirra is shutting down effective October 1, 2014. Thank you to all of our users!

Problem statement

What we see

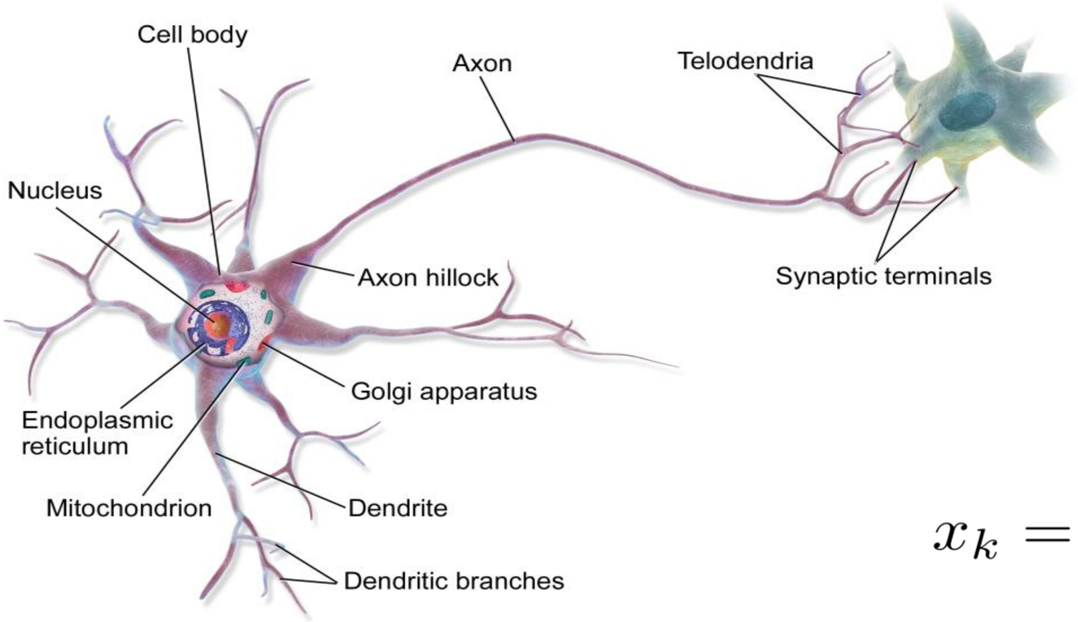
0	3	2	5	4	7	6	9	8
3	0	1	2	3	4	5	6	7
2	1	0	3	2	5	4	7	6
5	2	3	0	1	2	3	4	5
4	3	2	1	0	3	2	5	4
7	4	5	2	3	0	1	2	3
6	5	4	3	2	1	0	3	2
9	6	7	4	5	2	3	0	1
8	7	6	5	4	3	2	1	0

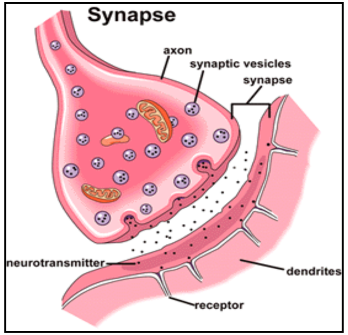
What a computer sees



Enough motivation!

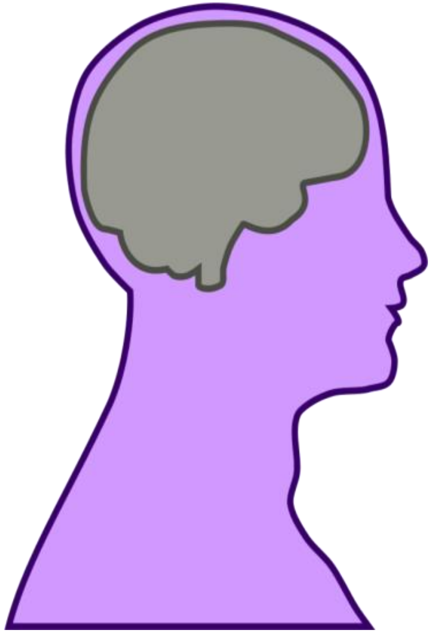
Neuron model





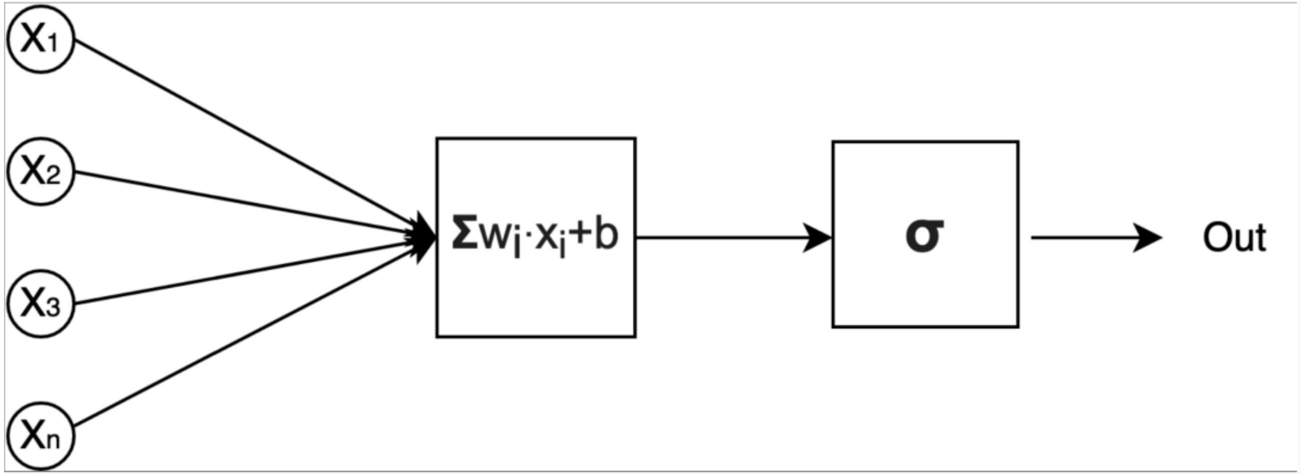
$$x_k = H\left(\sum_i w_{ik} x_i - \tau\right)$$

Our brain



- 100 billion neurons
- average neuron is connected to 1000-10000 other neurons
- 100 trillion synapses
- 10-25% is in visual cortex

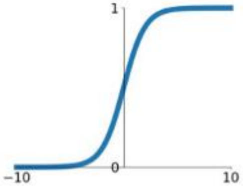
Mathematical model



Activation functions

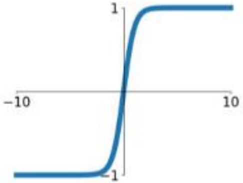
Sigmoid

$\sigma(x) = \frac{1}{1+e^{-x}}$



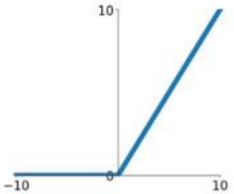
tanh

$\tanh(x)$



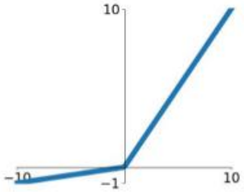
ReLU

$\max(0, x)$



Leaky ReLU

$\max(0.1x, x)$

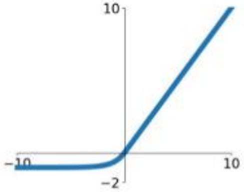


Maxout

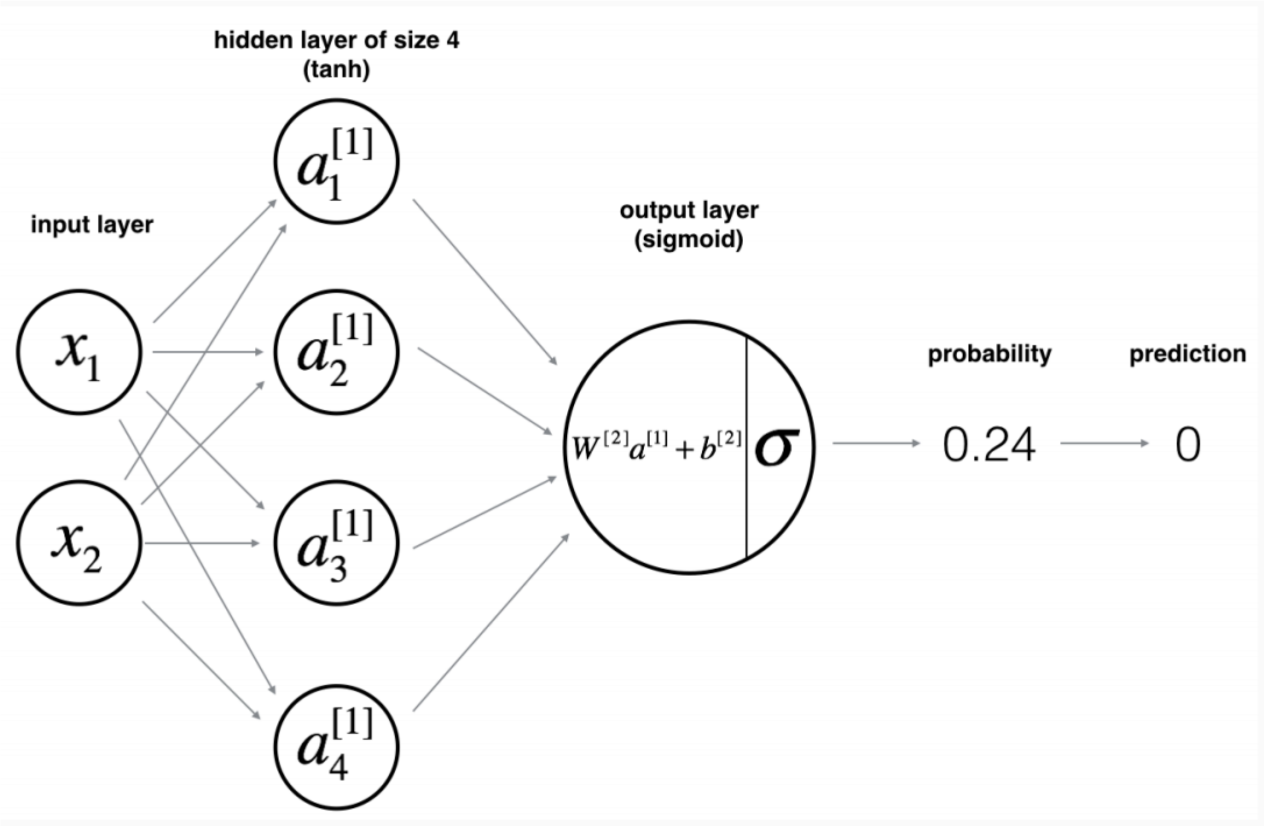
$\max(w_1^T x + b_1, w_2^T x + b_2)$

ELU

$$\begin{cases} x & x \geq 0 \\ \alpha(e^x - 1) & x < 0 \end{cases}$$



Multilayer perceptron



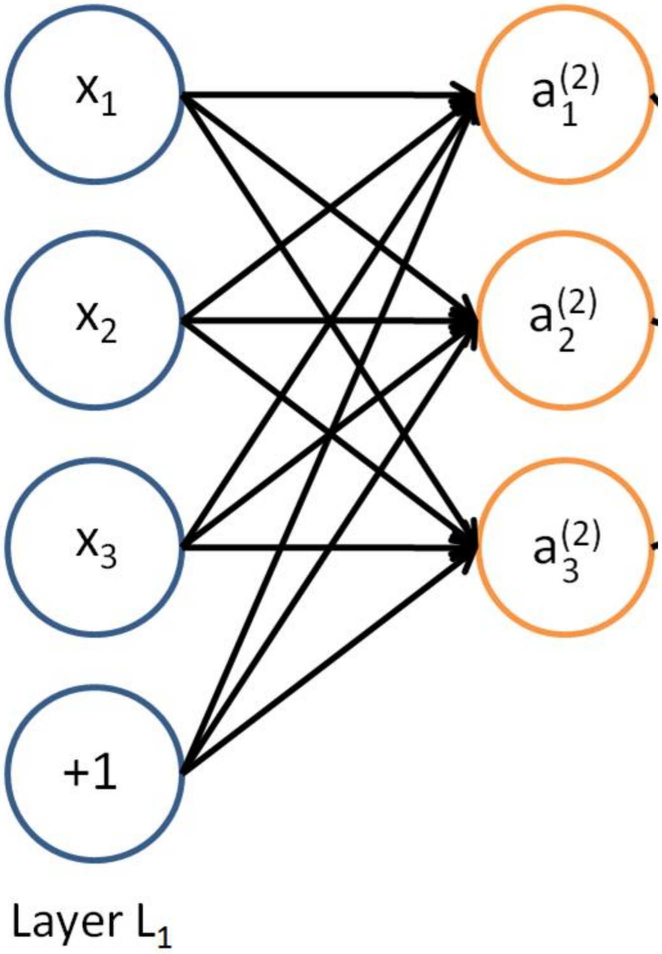
$$a = L(x), \quad x \in \mathcal{R}^m, a \in \mathcal{R}^n$$

$$a_i = \sigma\Big(\sum_j w_{ij} x_j + b_i\Big)$$

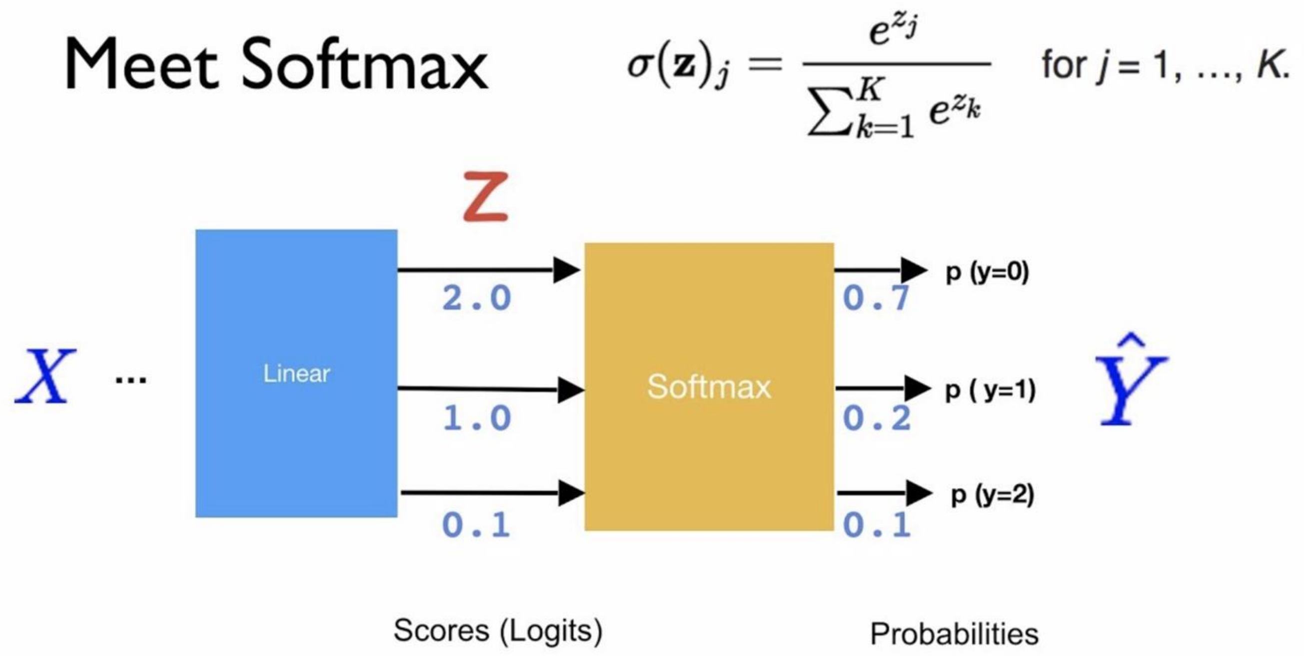
$$a = \sigma\Big(Wx + b\Big), \quad b \in \mathcal{R}^n$$

Hint, don’t look there!

$$\begin{bmatrix} 3 & 2 & 0 \\ 0 & 4 & 1 \\ 2 & 0 & 1 \end{bmatrix} \underset{x}{\begin{bmatrix} 4 \\ 3 \\ 1 \end{bmatrix}} = \underset{y}{\begin{bmatrix} 18 \\ 13 \\ 9 \end{bmatrix}}$$



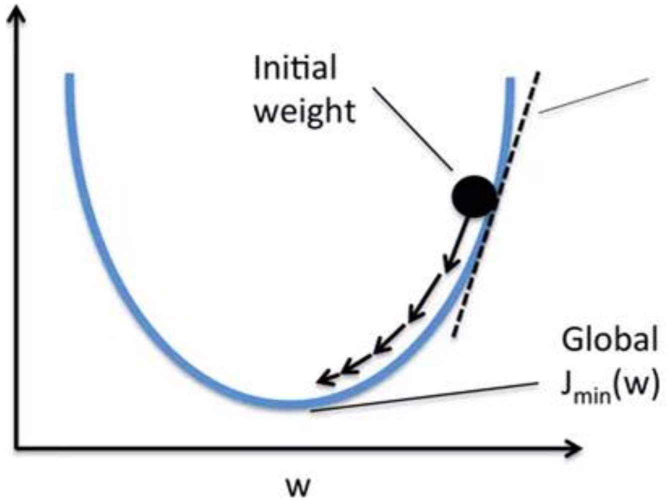
Multiclass case



Recap: Gradient decent

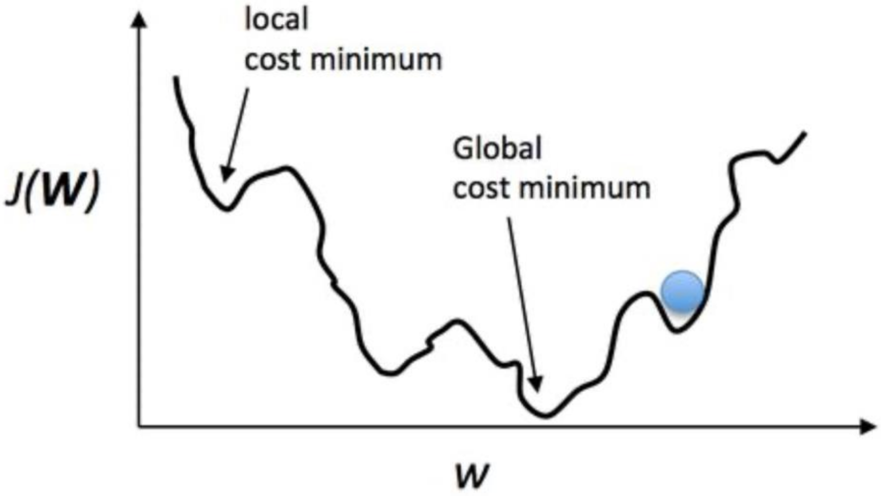
- We want to minimize $f(x)$
- Initialize x_0
- For each x_n do...
- Compute gradient (derivative) $f'(x_n)$
- Make a step:

$$x_{n+1} = x_n - \alpha f'(x_n)$$



- Just change the notation and minimize the loss:

$$w_n = w_{n-1} - \alpha \frac{\partial \mathcal{L}}{\partial w}(X, y, w_{n-1})$$


local
cost minimum
Global
cost minimum
J(W)
W

Backprop

$$x \to f \to y, \; y = f(x, w)$$

$$\frac{\partial L}{\partial y} \textbf{ is known}$$

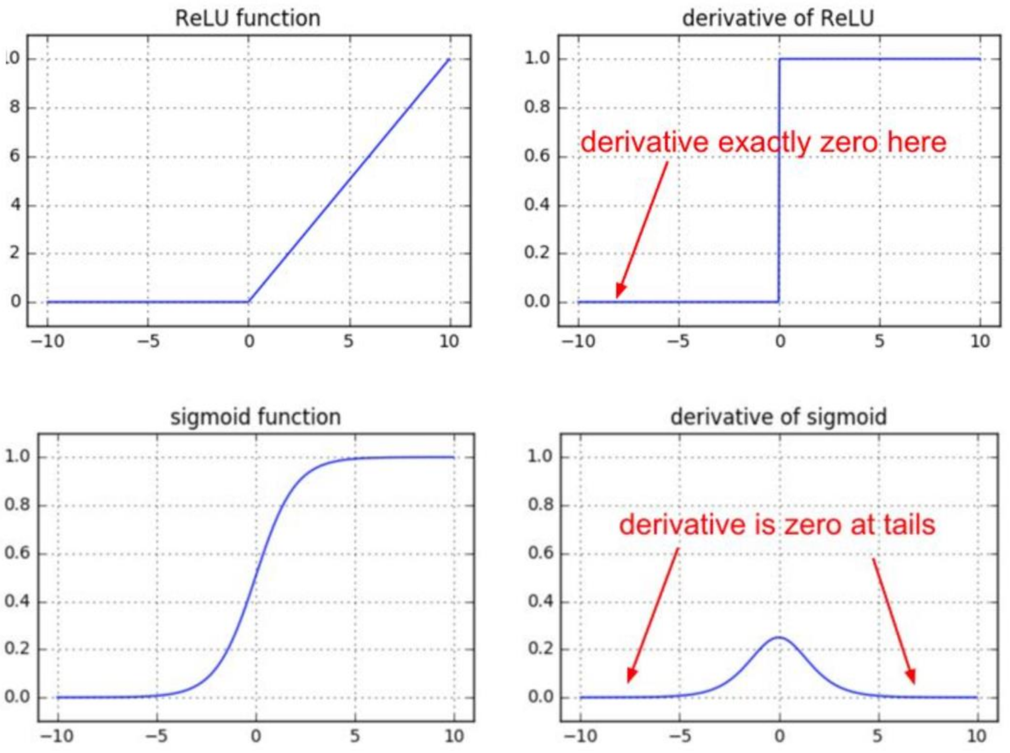
$$\frac{\partial L}{\partial x} = ?, \quad \frac{\partial L}{\partial w} = ?$$

$$\frac{\partial L}{\partial x} = \frac{\partial L}{\partial y}\frac{\partial y}{\partial x} = \frac{\partial L}{\partial y}\frac{\partial f}{\partial x}$$

$$\frac{\partial L}{\partial w} = \frac{\partial L}{\partial y}\frac{\partial y}{\partial w} = \frac{\partial L}{\partial y}\frac{\partial f}{\partial w}$$

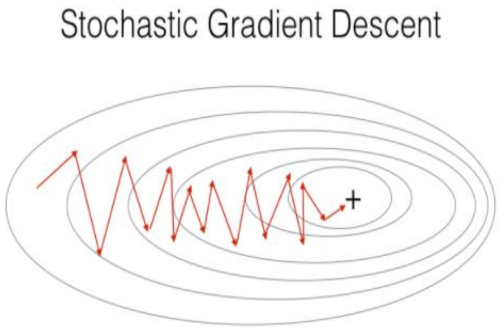
$$y = Wx \Rightarrow \frac{\partial L}{\partial x} = W^T \frac{\partial L}{\partial y}$$

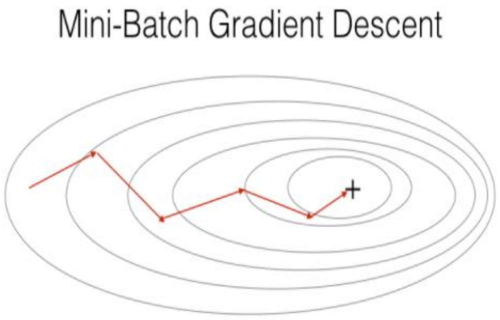
Vanishing gradients



Variations of stochastic gradient descent (SGD)

- Compute gradient using only a subsample of a smaller size (called batch size)
- It accelerates convergence because the algorithm usually converges in approximately the same number of steps
- Use always this method, never compute gradient by the entire sample

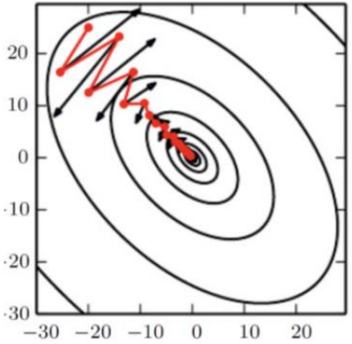




Optimizers

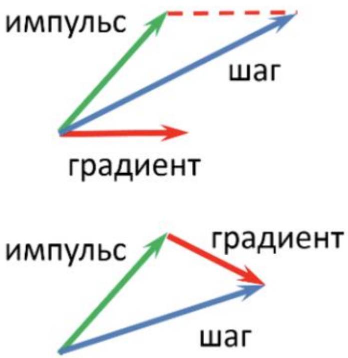
Momentum — экспоненциальное скользящее среднее градиента по $\approx \frac{1}{1-\gamma}$ последним итерациям [Б.Т.Поляк, 1964]:

$$v := \gamma v + \eta \mathscr{L}_i'(w)$$
$$w := w - v$$



NAG (Nesterov's accelerated gradient) — стохастический градиент с импульсом Нестерова [1983]:

$$v := \gamma v + \eta \mathscr{L}_i'(w - \gamma v)$$
$$w := w - v$$



Optimizers

RMSProp (running mean square) — адаптация скорости изменения весов, скользящим средним по $\approx \frac{1}{1-\alpha}$ итерациям:

$$G := \alpha G + (1-\alpha)\mathscr{L}_i'(w) \odot \mathscr{L}_i'(w)$$

$$w := w - \eta \mathscr{L}_i'(w) \oslash \left(\sqrt{G} + \varepsilon\right)$$

где $\odot$ и $\oslash$ — покоординатное умножение и деление векторов.

AdaDelta (adaptive learning rate) — двойная нормировка приращений весов, после которой можно брать $\eta = 1$:

$$G := \alpha G + (1-\alpha)\mathscr{L}_i'(w) \odot \mathscr{L}_i'(w)$$

$$\delta := \mathscr{L}_i'(w) \odot \frac{\sqrt{\Delta}+\varepsilon}{\sqrt{G}+\varepsilon}$$

$$\Delta := \alpha \Delta + (1-\alpha)\delta \odot \delta$$

$$w := w - \eta \delta$$

Optimizers

Adam (adaptive momentum) = импульс + RMSProp:

$$v := \gamma v + (1-\gamma)\mathscr{L}_i'(w) \qquad \hat{v} := v(1-\gamma^k)^{-1}$$

$$G := \alpha G + (1-\alpha)\mathscr{L}_i'(w) \odot \mathscr{L}_i'(w) \qquad \hat{G} := G(1-\alpha^k)^{-1}$$

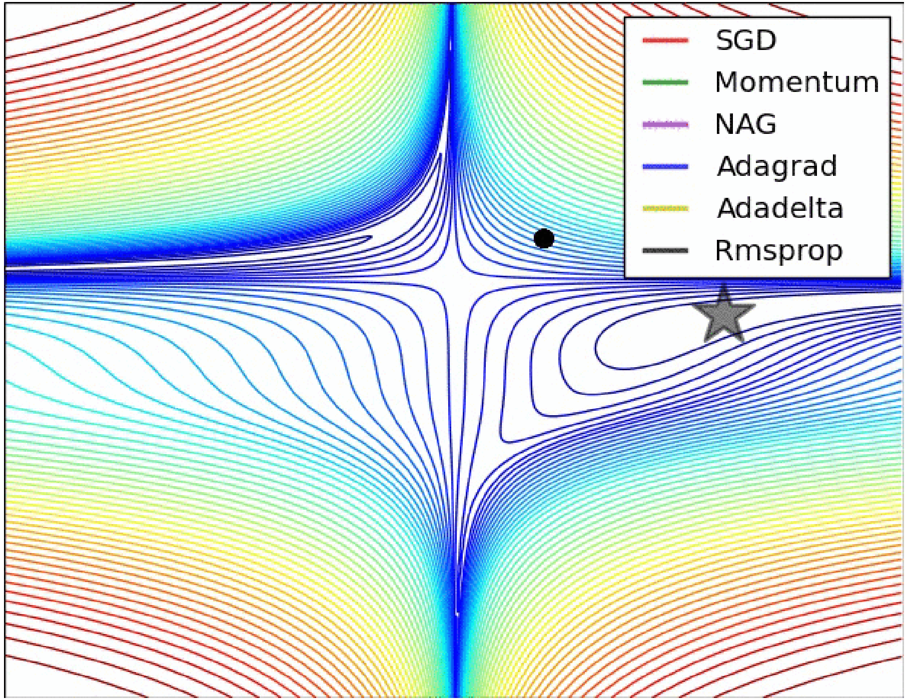
$$w := w - \eta \hat{v} \oslash \left(\sqrt{\hat{G}} + \varepsilon\right)$$

Калибровка $\hat{v}$, $\hat{G}$ увеличивает v, G на первых итерациях,
где k — номер итерации; $\gamma = 0.9$, $\alpha = 0.999$, $\varepsilon = 10^{-8}$

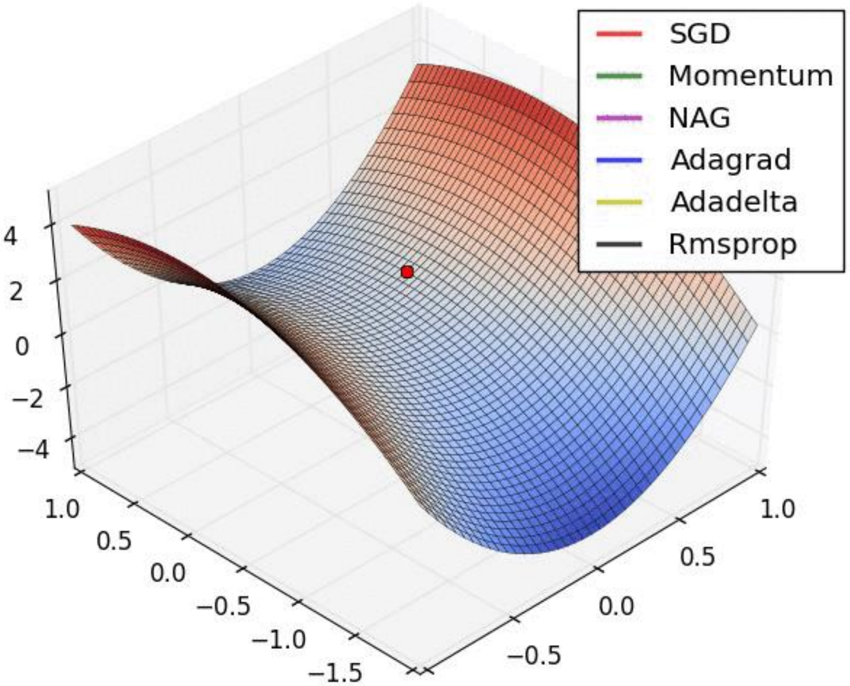
Nadam (Nesterov-accelerated adaptive momentum):
те же формулы для v, $\hat{v}$, G, $\hat{G}$,

$$w := w - \eta\left(\gamma\hat{v} + \tfrac{1-\gamma}{1-\gamma^k}\mathscr{L}_i'(w)\right) \oslash \left(\sqrt{\hat{G}} + \varepsilon\right)$$

Optimizers



SGD
Momentum
NAG
Adagrad
Adadelta
Rmsprop



SGD
Momentum
NAG
Adagrad
Adadelta
Rmsprop
4
2
0
−2
−4
1.0
0.5
0.0
−0.5
−1.0
−1.5
1.0
0.5
0.0
−0.5