1 Abstract

Wireless communication is increasingly being employed to transfer highly sensitive information. Systems such as ambient living assistance systems, emergency response systems employing wireless networks, contactless smart cards and military sensor networks all employ wireless communication to transmit potentially sensitive information, such as patient health information, banking or financial data or military information. Unlike wired networks, in which the link topology is fixed at the time the network is deployed, wireless networks have no fixed underlying topology. In addition, the relational disposition of wireless nodes is constantly changing. The temporary physical topology of the network is determined by the distribution of the wireless nodes, as well as the transmission range of each node. The ranges determine a communication graph, in which the nodes correspond to the transceivers and the edges correspond to the communication links. Unlike nodes in wired networks, wireless devices are typically equipped with limited energy supplies making energy efficiency one of the primary objectives in network design. Energy efficiency is especially important for networks where battery replacement is infeasible.

In NETWORK DESIGN problems the goal is to select a "cheap" network that satisfies some prescribed property. Many fundamental properties can be characterized by degree demands (number of edges incident to a node), pairwise connectivity demands (number of disjoint paths between node pairs), network lifetime (the time it takes the first node to run out of its battery charge), nodes proximity (dense networks decrease the number of transmissions that can happen simultaneously, while increasing the number of retransmissions), and others. The goal of this proposal is to consider a family of ACTIVATION NETWORK DESIGN problems, where we seek an assignment $\mathbf{a} = \{a_v \geq 0 : v \in V\}$ to the nodes, such that the activated graph $G_{\mathbf{a}} = (V, E_{\mathbf{a}})$ satisfies a given property, and the value $\mathbf{a}(V) = \sum_{v \in V} a_v$ of the assignment is minimized. Such problems include node-weighted problems, minpower problems, installation problems, and more. We *suggest to study approximability of activation problems in terms of the slope parameter* θ, that is the maximum ratio between activating thresholds of the endnodes of an edge in the network. Besides finding a unifying algorithmic idea that generalizes and improves previous results, we expect to find tractable special cases in this new direction. Moreover, we intend to focus on practical approximation algorithms that can be implemented for real-life scenarios that require maintenance of the network for a long period of time.

2 Brief Subject Description and Scientific and Technological Background

Wireless communication is increasingly being employed to transfer highly sensitive information. Systems such as ambient living assistance systems, emergency response systems employing wireless networks, contactless smart cards and military networks all employ wireless communication to transmit potentially sensitive information, such as patient health information, banking or financial data or military information. Unlike wired networks, in which the link topology is fixed at the time the network is deployed, wireless networks have no fixed underlying topology. In addition, the relational disposition of wireless nodes is constantly changing. The temporary physical topology of the network is determined by the distribution of the wireless nodes, as well as the transmission range of each node. The ranges determine a directed communication graph, in which the nodes correspond to the transceivers and the links correspond to the communication links.

In NETWORK DESIGN problems the goal is to select a "cheap" graph that satisfies some prescribed property. Many fundamental properties can be characterized by **coverage (degree) demands** (number of links incident to a node) or pairwise **connectivity demands** (number of disjoint paths between node pairs). Traditionally, "cheap" means that the links of the input graph $G = (V, E)$ have costs $\mathbf{c} = \{c_e : e \in E\}$, and the cost of a subgraph of G is the sum of the costs of its links. For example, in the context of wireless ad-hoc networks, wireless devices are typically equipped with limited energy supplies making *energy efficiency* one of the primary objectives in network design [8]. Energy efficiency is especially important for networks where battery replacement is infeasible. Accordingly, we can define the cost function on the link set E as $c_e = {d_e}^{\gamma}$, where d_e is the Euclidean distance between the nodes connected by e, and γ is the *path-loss exponent*, having value of at least 2, (see [47]). Note that in this case, the cost of the link e matches the amount of energy which is required to transmit between the endpoints of e.

Some classic examples of "low demands" problems are LINK-COVER (EDGE-COVER), st-PATH (SHORTEST PATH), SPANNING TREE, STEINER TREE, STEINER FOREST, and others. Examples of "high demands" problems are LINK-MULTI-COVER, k DISJOINT PATHS (MIN-COST k-FLOW), k-OUT-CONNECTED SUBGRAPH, k-CONNECTED SUBGRAPH, and others. See, for example, [53, 15] for polynomial time solvable problems of this type and a survey [44] for approximation algorithms for NP-hard connectivity problems. Motivated by various applications, among others in wireless networks, we consider ACTIVATION NETWORK DESIGN problems [48], where we seek an assignment $\mathbf{a} = \{a_v \geq 0 : v \in V\}$ to the nodes, such that the activated graph $G_{\mathbf{a}} = (V, E_{\mathbf{a}})$ satisfies a given property, and the value $\mathbf{a}(V) = \sum_{v \in V} a_v$ of the assignment is minimized. Let us give some examples of such problems.

Node-Weighted Network Design. Here we have node-weights $\mathbf{w} = \{w_v : v \in V\}$ instead of link-costs. The goal is to find a node subset $S \subseteq V$ of minimum total weight $\mathbf{w}(S) = \sum_{v \in S} w_v$, such that the graph $G[S]$ induced by S in G satisfies the given property. This can be formulated as an activation problem, where the graph $G_{\mathbf{a}} = (V, E_{\mathbf{a}})$ activated by an assignment $\mathbf{a}$ has link set $E_{\mathbf{a}} = \{uv : a_u \geq w_u, a_v \geq w_v\}$.

Min-Power Network Design. Consider the following scenario with motivation in wireless ad-hoc networks. We are given a set V of nodes (transmitters) and power thresholds $\mathbf{p} = \{p_{uv} : uv \in V\}$, where p_{uv} is the minimum power (energy level) needed at u to reach v. If u can reach v then we can include the directed link uv in the activated communication graph. The goal is to find an assignment $\mathbf{a} = \{a_v : v \in V\}$ of power levels to the nodes such that the activated directed graph $G_{\mathbf{a}} = (V, E_{\mathbf{a}})$, where $E_{\mathbf{a}} = \{uv : a_u \geq p_{uv},\ u, v \in V\}$, satisfies the given property.

Installation Network Design. Suppose that the installation cost of a wireless network is dominated by the cost of building towers at the nodes for mounting antennas, which in turn is proportional to the height of the towers. A link uv is activated if the towers at its endpoints u and v are tall enough to overcome obstructions in the middle and establish line-of-sight between the antennas mounted on the towers. This is modeled as each directed link uv has a height-threshold requirements.

Virtual Backbone Network Design. Consider a practical scenario when we need to choose a set R of terminals to install various expensive devices in some nodes of a network, in order to provide services to all users of the network. In addition to installation costs of the terminals, we also have service costs between the terminals and the users determined by activating functions. In order to provide services to users, R should be a dominating set in the activated network $G_{\mathbf{a}}$. This is a generalization of the classic FACILITY LOCATION problem, but note that in the FACILITY LOCATION problem the service costs are fixed, while we consider a more general realistic scenario when the service costs are dependent on our "investment" in the nodes of the network. If we require that the terminals will induce a connected graph to form a **virtual communication backbone**, then we get a generalization of the NODE WEIGHTED CONNECTED DOMINATING SET and CONNECTED FACILITY LOCATION problems. We might further require that R induces a k-connected graph in $G_{\mathbf{a}}$ and that every user has at least m neighbors in $G_{\mathbf{a}}$, generalizing the k-CONNECTED m-DOMINATING SET problem.

In many ACTIVATION NETWORK DESIGN problems the activated link set $E_{\mathbf{a}}$ can be defined as follows [29]. Each link $e = uv \in E$ has **activation thresholds** t^e_u, t^e_v and an assignment $\mathbf{a}$ activates e if $a_u \geq t^e_u$ and $a_v \geq t^e_v$; namely, $E_{\mathbf{a}} = \{e = uv : a_u \geq t^e_u, a_v \geq t^e_v\}$. E.g., in node weighted problems $t^e_u = w_u$ and $t^e_v = w_v$, while in min-power problems $t^e_u = t^e_v = p_{uv}$.

3 Objectives and Expected Significance of the Research

Most activation problems are NP-hard, so we intend to focus on **practical approximation algorithms**. We say that a polynomial time algorithm for a minimization problem has **approximation ratio** ρ or that it is a ρ**-approximation algorithm** if it produces a feasible solution of value at most ρ times the value of an optimal solution. By "**practical**" we mean a simple combinatorial algorithm, that does no need to solve linear programs; such an algorithm has a good chance to be time efficient, and moreover, might be implementable in a distributed environment.

As we will explain later, already a very specific type of activation problems is among the most fundamental problems in network design, that include NP-hard problems that arise in wireless networks (node weighted/min-power/installation problems), and many other problems. To bridge between the constant ratio for min-power and tight logarithmic ratios for more general activation thresholds with lifetime and interference constraints, it is suggested in [41] to study approximability of activation problems in terms of the **dilation** θ. For simplicity of exposition, we state the definition of this parameter in terms of activation thresholds, while more general and/or slightly modified definitions can be found in [41, 29].

Definition 1 ([41]) *The* **dilation** $\theta(e)$ *of a uv-link e is defined by* $\theta(e) = \frac{\max\{t^e_u, t^e_v\}}{\min\{t^e_u, t^e_v\}}$*. The dilation of an activation problem instance is defined by* $\max_{e \in E} \theta(e)$*. We say that an activation problem instance is* θ**-bounded** *if the instance dilation is at most* θ*; moreover, we assume by default that* $\theta = \max_{e \in E} \theta(e)$ *is the instance dilation.*

Note that min-power instances have dilation 1, while node-weighted problems may have large dilation. In [41] are derived some simple dilation based approximation ratios for various problems. The ***goal of the suggested research*** is to establish much better ratios, as was already done for some problems in [29].

Note that the focus of our proposal is not only technical, but also conceptual. We expect our main contribution to be giving unified algorithms for a large classes of problems – θ-BOUNDED ACTIVATION LINK-COVER/CONNECTIVITY problems, either substantially improving known ratios, or showing that many seemingly hard problems may be tractable in practice. The generalization to θ-bounded ACTIVATION LINK-COVER/CONNECTIVITY problems is different from earlier results; besides finding a unifying algorithmic idea that generalizes and improves previous results, we expect to find tractable special cases in a new direction. In order to make our solutions feasible, i.e. to allow them to work in real life node deployments, we also outline how it is possible to implement them in a decentralized (distributed – without the need for coordination by a central unit) and local environments. In particular, our techniques can be used for optimizing resources of airborne sensors (e.g. UAVs) or increasing the robustness of network by combining wireless and wired technologies.

We stress again that the implications of finding a unified approach for many seemingly unrelated problems are immense, from both practical and theoretical point of view.

4 Comprehensive description of the methodology and plan of operation

In what follows we will present the detailed approach for each one of the problems considered in this proposal.

4.1 Network design model

In general, the input graph G may have parallel links with distinct activation functions. We write $e = uv$ meaning that e is a uv-link (namely, that u, v are the endnodes of e), and $e = uv \in E$ means that e is a uv-link that belongs to E. Panigrahi [48] suggested the following common generalization of these and several other problems.

Definition 2 (Panigrahi [48]) *Let $G = (V, E)$ be a graph such that each link $e \in E$ has an* **activating function** *f^e from some range $L^e \subseteq \mathbb{R}^2_+$ to $\{0, 1\}$. Given a non-negative* **assignment a** $= \{a_v : v \in V\}$ *to the nodes, we say that a uv-link $e \in E$ is* **activated** *by* **a** *if $f^e(a_u, a_v) = 1$. Let $E_{\mathbf{a}} = \{e \in E : f^e(a_u, a_v) = 1\}$ denote the set of links activated by* **a**. *The* **value** *of an assignment* **a** *is* $\mathbf{a}(V) = \sum_{v \in V} a_v$.

ACTIVATION NETWORK DESIGN
Input: A graph $G = (V, E)$, a family $\mathbf{f} = \{f^e(x_u, x_v) : e = uv \in E\}$ of activating functions from a range $L^e \subseteq \mathbb{R}^2_+$ to $\{0, 1\}$ each, and a prescribed property (namely, a family of subgraphs of G).
Output: An assignment $\mathbf{a} = \{a_v \geq 0 : v \in V\}$ of minimum value $\mathbf{a}(V) = \sum_{v \in V} a_v$ such that the graph $G_{\mathbf{a}} = (V, E_{\mathbf{a}})$ activated by **a** satisfies the given property.

We make the following two assumptions about activating functions, which currently are standard in the literature; see the paper of Panigrahi [48] that introduced the problem, and a recent survey [41] on various activation problems.

Monotonicity Assumption. *For every $e \in E$, the activating function f^e of e is monotone non-decreasing, namely, $f^e(x_u, x_v) = 1$ implies $f^e(y_u, y_v) = 1$ if $y_u \geq x_u$ and $y_v \geq x_v$.*

Polynomial Domain Assumption. *Every $v \in V$ has a polynomial size in $n = |V|$ set L_v of "levels" and $L^e = L_u \times L_v$ for every uv link $e \in E$.*

In SUBMODULAR COST NETWORK DESIGN problems the goal is to satisfy some prescribed property while minimizing an increasing submodular function on the edges, given by a value oracle. Such problems are studied, for example, in [56, 30]. Given an ACTIVATION NETWORK DESIGN instance and $S \subseteq E$ let $\tau(S)$ be the minimum assignment value that activates S. It can be seen that under the above two assumptions, the set function τ is submodular. Furthermore, if $\tau(S)$ can be computed in polynomial time, which is the case for all practical problems we consider, then we get a SUBMODULAR COST NETWORK DESIGN instance. Thus ACTIVATION NETWORK DESIGN is a particular case of SUBMODULAR COST NETWORK DESIGN. However ACTIVATION NETWORK DESIGN is already general enough to capture a very large class of practical problems, in particular those that stem from wireless network design. Moreover, we do not know how the dilation parameter θ translates into SUBMODULAR COST NETWORK DESIGN problems.

Our target is to consider coverage and connectivity variants of the ACTIVATION NETWORK DESIGN problem. In both types of problems we are given a graph $G = (V, E)$ and certain non-negative integral demands (a.k.a. requirements). In coverage problems we have **degree demands** $\mathbf{r} = \{r_v : v \in V\}$ and in connectivity problems we have **connectivity demands** $\mathbf{r} = \{r_{st} : s, t \in V\}$. In both cases we use k to denote the maximum demand. In the case of degree demands we say that a graph (V, J) (or J) **satisfies r** if $\deg_J(v) \geq r_v$ for all $v \in V$, where $\deg_J(v)$ denotes the degree of v in the graph (V, J). In the case of connectivity demands we say that (V, J) (or J) **satisfies r** if the graph (V, J) contains r_{st} pairwise disjoint st-paths for all $s, t \in V$. In link-connectivity problems the path should be link disjoint, while in node-connectivity problems the paths should be internally disjoint.

The ACTIVATION LINK-COVER and the ACTIVATION STEINER FOREST problems can be restated as follows.

ACTIVATION LINK COVER
Input: A graph $G = (V, E)$, a set of terminals $R \subseteq V$, and thresholds $\{t_u^e, t_v^e\}$ for each uv-link $e \in E$.
Output: An assignment $\mathbf{a}$ of minimum value such that the link set $E_{\mathbf{a}} = \{e = uv \in E : a_u \geq t_u^e, a_v \geq t_v^e\}$ activated by $\mathbf{a}$ covers R.

ACTIVATION STEINER FOREST
Input: A graph $G = (V, E)$, a set of pairs $R \subseteq V \times V$, and thresholds $\{t_u^e, t_v^e\}$ for each uv-link $e \in E$.
Output: An assignment $\mathbf{a}$ of minimum value such that the link set $E_{\mathbf{a}} = \{e = uv \in E : a_u \geq t_u^e, a_v \geq t_v^e\}$ activated by $\mathbf{a}$ contains an st-path for every $(s, t) \in R$.

In order to make a connection with the wireless networks, we introduce additional requirements to the problems domain, such as the **lifetime** and **interference** demands. In general, we can assume that each node $v \in V$ has some initial battery charge b_v, which is sufficient for a limited amount of time, proportional to the assignment cost (in other words, the power) a_v. It is common to take the lifetime of a wireless node v to be $l_v = b_v / a_v$. The network lifetime is defined as the time it takes the first node to run out of its battery charge. For a cost assignment $\mathbf{a}$ and initial battery charges $\mathbf{b} = \{b_v : v \in V\}$, the network lifetime is defined as $l(\mathbf{a}) = \min_{v \in V} l(v)$.

Interference is a direct consequence of any assignment $\mathbf{a}$. The level of interference determines the length of the transmission schedule. In general, the level of interference depends on the transmitting nodes proximity and the transmission ranges. High levels of interference decrease the number of transmissions that can happen simultaneously, while increasing the number of retransmissions. All common models of interference define the notion of *coverage*, which is the number of nodes or links that are affected (interfered) by a transmission over a specific link in the activated link set $E_{\mathbf{a}}$. The authors in [4] defined the coverage of an undirected link $e = uv$ as the number of nodes covered by two transmission disks centered at u and v with radius d_e, (implying that u and v communicate with each other), $COV_I(E_{\mathbf{a}}, e) = |\{w \in V, \min\{d_{uw}, d_{vw}\} \leq d_{uv}\}|$. Another coverage model proposed by [33] puts emphasis on the transmission range of a single node. The measure is the number of nodes covered by a transmission from node u alone, $COV_{II}(E_{\mathbf{a}}, u) = |\{w \in V,\ d_{uw} \leq a_u\}|$. A third coverage model was introduced by [24]. They define the coverage as the number of links affected by a link $uv \in E_a$, $COV_{III}(E_{\mathbf{a}}, uv) = |\{u'v' \in E_a, d_{uv} \leq d_{u'v'}, \min\{d_{uu'}, d_{uv'}, d_{vu'}, d_{vv'}\} \leq d_{u'v'}\}|$. Let, $COV_I^*(E_{\mathbf{a}}) = \max_{uv \in E_{\mathbf{a}}} COV_I(E_{\mathbf{a}}, (u, v))$, $COV_{II}^*(E_{\mathbf{a}}) = \max_{u \in V} COV_{II}(E_{\mathbf{a}}, u)$, $COV_{III}^*(E_{\mathbf{a}}) = \max_{uv \in E_{\mathbf{a}}} COV_{III}(E_{\mathbf{a}}, uv)$. The interference of $E_{\mathbf{a}}$ is defined as $I(E_{\mathbf{a}}) = \max\{COV_I^*(E_{\mathbf{a}}), COV_{II}^*(E_{\mathbf{a}}), COV_{III}^*(E_{\mathbf{a}})\}$. In order to generalize the above mentioned activation problems having lifetime and interference demands, we add input thresholds B and I altogether with initial battery charges values $\mathbf{b} = \{b_v : v \in V\}$ and require to obtain a minimum cost assignment satisfying previous demands in addition to have lifetime at least B and interference at most I.

Under Rayleigh dissipation model, for a receiver located at location r and a transmitter located at a point t, the receiving power $P(r,t)$ at r is given by $P(r,t) = \tilde{P}\beta(\|r-t\|)^{-\gamma}$, where $\tilde{P}$ is the transmission power, β is a distance-independent constant representing factors such as transmitter/receiver gain and γ is the path-loss exponent. The additional communication model we intend to use in our interference analysis is the *Signal-to-Interference-plus-Noise-Ratio* (***SINR***) model. Under the ***SINR*** model, successful reception of a message at a receiver r depends on the received signal power $P(r)$, the ambient noise power N_a and the interference power caused by simultaneous transmissions IP. Formally, the receiver is able to successfully receive the message if ***SINR***$(r) > \sigma$ where ***SINR***$(r) = P(r,t)/(N_a + IP)$ denotes the ***SINR*** at r and σ is the minimum ***SINR*** at which the message is successfully received.

4.2 Previous best-known results

ACTIVATION LINK COVER problems are among the most fundamental problems in network design, that include NP-hard problems such as SET-COVER, FACILITY LOCATION, covering problems that arise in wireless networks (node weighted/min-power/installation problems), and many other problems. The ACTIVATION STEINER FOREST problem includes the node-weighted and the min-power variants of the st-PATH and the STEINER TREE problems.

We now describe some high demands node-connectivity problems.

- k DISJOINT PATHS: Here $r_{st} = k$ for a given pair of nodes $s,t \in V$ and $r_{uv} = 0$ otherwise, namely, the solution graph should contain k internally disjoint st-paths. For $k = 1$ we get the st-PATH problem.
- k-OUT-CONNECTIVITY: A graph is **k-out-connected from** s if it contains k internally disjoint paths from s to any other node. In k-OUT-CONNECTIVITY problems the activated graph should be k-out-connected from s, namely, it should satisfy the node-connectivity demands $r_{st} = k$ for all $t \in V \setminus \{s\}$.
- k-CONNECTIVITY: Here the graph should be k-connected, namely it should satisfy the node-connectivity demands $r_{st} = k$ for all $s,t \in V$. For $k = 1$ we get the SPANNING TREE problem.

The corresponding link-connectivity problems are k LINK-DISJOINT PATHS, k-LINK-OUT-CONNECTIVITY and k-LINK-CONNECTIVITY; the later two problems are equivalent. We summarize the currently best known ratios for high demands activation problems in the following table; for the best known approximation ratios for **min-cost** high demands connectivity problems c.f. [18, 23, 16, 10, 37, 9, 17, 38, 40, 45, 7, 2] and surveys [32, 44, 43].

problem/activation fn.	**general**	**node-weighted**	**power**
k DISJOINT PATHS	2 [39]	2 [39]	2 [21]
k-OUT-CONNECTIVITY	$O(k \ln n)$ [39]	in P	$\min\{k+1, O(\ln k)\}$ [35, 31, 12]
k-CONNECTIVITY	$O(k \ln n)$ [39]	in P	$O\left(\ln k \ln \frac{n}{n-k}\right)$ [12, 40]
k LINK-DISJOINT PATHS	k [31]	k [31, 36]	k [31]
k-LINK-CONNECTIVITY	$O(k \ln n)$ [39]	in P	$\min\{2k-1/2, O(\sqrt{n})\}$ [28, 21]
LINK-MULTI-COVER	$O(k \ln n)$	$O(\ln n)$	$\min\{k+1/2, O(\ln k)\}$ [12, 27]

Table 1: Best known ratios for high demands activation problems.

In the DIRECTED ACTIVATION NETWORK DESIGN problems each activating function f^{uv} depends only on the assignment at the tail u of the link uv, so it is a function $f^{uv}(x_u) = f^{uv}(x)$ of one variable. Then by the Monotonicity Assumption each link uv has a threshold p_{uv} such that $f^{uv}(x) = 1$ iff $x \geq p_{uv}$. This gives the directed min-power variant discussed earlier, where p_{uv} is the minimum power needed at u to reach to v. Consequently, the directed variant can be stated as follows.

DIRECTED ACTIVATION NETWORK DESIGN (DIRECTED MIN-POWER NETWORK DESIGN)
Input: A directed graph $G = (V, E)$ with power thresholds $\mathbf{p} = \{p_e : e \in E\}$ and a prescribed property.
Output: An assignment $\mathbf{a} = \{a_v \geq 0 : v \in V\}$ of minimum value $\mathbf{a}(V) = \sum_{v \in V} a_v$ such that the graph $G_{\mathbf{a}} = (V, E_{\mathbf{a}})$ activated by $\mathbf{a}$ satisfies the given property, where $E_{\mathbf{a}} = \{uv \in E : a_u \geq p_{uv},\ u, v \in V\}$.

Note that in directed LINK-MULTI-COVER problems we may have both outdegree and indegree demands $r_v, r^{in}(v)$, and $J \subseteq E$ is a feasible solution if $\deg_J(v) \geq r_v$ and $\deg_J^{in}(v) \geq r^{in}(v)$ for all $v \in V$, where $\deg_J(v)$ and $\deg_J^{in}(v)$ denote the outdegree and the indegree of v in the graph (V, J), respectively. Similarly, in the k-OUT-CONNECTIVITY problem we require k internally disjoint paths from s to any other node, while in the k-OUT-CONNECTIVITY problem the paths should be from any node to s. See [5, 21, 31, 34, 39, 41] for these problems. Let us summarize the best known ratios for these problems in the following table.

problem		**problem**	**node-connectivity**	**link-connectivity**
st-PATH	in P	k DISJOINT PATHS	in P [21]	k [31]
IN-ARBORESCENCE	in P	k-IN-CONNECTIVITY	in P [31]	k [34]
OUT-ARBORESCENCE	$O(\ln n)$ [5, 7]	k-OUT-CONNECTIVITY	$O(k \ln n)$ [39]	$O(k \ln n)$ [34]
STRONG CONNECTIVITY	$O(\ln n)$ [5, 7]	k-CONNECTIVITY	$O(k \ln n)$ [39]	$O(k \ln n)$ [34]

Table 2: Best known ratios for directed activation problems.

4.3 Coverage problems

Recall that in the ACTIVATION LINK COVER problem we are given a graph $G = (V, E)$, a set of terminals $R \subseteq V$, and thresholds $\{t_u^e, t_v^e\}$ for each uv-link $e \in E$. The goal is to find an assignment $\mathbf{a}$ of minimum value such that the link set $E_{\mathbf{a}} = \{e = uv \in E : a_u \geq t_u^e, a_v \geq t_v^e\}$ activated by $\mathbf{a}$ covers R. As said above, in the context of wireless networks, additional demands can be taken into account such as lifetime, interference, ***SINR*** levels and other.

Let us say that $v \in V$ is a **steady node** if the thresholds t_v^e of the links e incident to v are all equal to the same number w_v, which we call the **weight of** v. We may assume that all non-terminals are steady, by replacing each $v \in V \setminus R$ by L_v new nodes. Clearly, we may assume that $V \setminus R$ is an independent set in G. Let BIPARTITE ACTIVATION LINK-COVER be the restriction of ACTIVATION LINK-COVER to instances when also R is an independent set, namely, when G is bipartite with sides $R, V \setminus R$. Note that in this case G is a simple graph and all non-terminals are steady.

Just to illustrate the importance of the dilation definition, let us consider the following particular cases.

WEIGHTED SET-COVER

This is a particular case of BIPARTITE ACTIVATION LINK-COVER when all nodes are steady and nodes in R have weight 0. Note that in this case θ is infinite. Consider a modification of the problem, which we call θ-BOUNDED WEIGHTED SET-COVER: when we pick a set $v \in V \setminus R$, we pay w_v for v, and also pay w_v/θ for each element in R covered by v that was not yet covered. Then the corresponding ACTIVATION LINK-COVER instance is θ-bounded.

FACILITY LOCATION

Here we have a bipartite graph with sides R (clients) and $V \setminus R$ (facilities), weights (opening costs) $\mathbf{w} = \{w_v : v \in V \setminus R\}$, and distances (service costs) $\mathbf{d} = \{d_{uv} : u \in R, v \in V \setminus R\}$. We need to choose $S \subseteq V \setminus R$

with $\mathbf{w}(S) + \sum_{u \in R} d(u,S)$ minimal, where $d(u,S) = \min_{v \in S} d_{uv}$ is the minimal distance from u to S. This is equivalent to BIPARTITE ACTIVATION LINK-COVER. Note however that if for some constant θ we have $w_v \leq \theta d_{uv}$ for all $uv \in E$ with $u \in R$ and $v \in V \setminus R$, then the corresponding BIPARTITE ACTIVATION LINK-COVER instance is θ-bounded.

INSTALLATION LINK-COVER. Suppose that the installation cost of a wireless network is proportional to the total height of the towers for mounting antennas. An link uv is activated if the towers at u and v are tall enough to overcome obstructions and establish line of sight between the antennas. This is modeled as each pair $u,v \in V$ has a height demand h^{uv} and constants γ_{uv}, γ_{vu}, such that a uv-link is activated by **a** if the scaled heights $\gamma_{uv}a_u, \gamma_{vu}a_v$ sum to at least h^{uv}. In the INSTALLATION LINK-COVER problem, we need to assign heights to the antennas such that each terminal can communicate with some other node, while minimizing the total sum of the heights. The problem is SET-COVER hard even for $0,1$ thresholds and bipartite G [48]. But in a practical scenario, the quotient of the maximum tower height over the minimum tower height is a small constant; say, if possible tower heights are $5,15,20$, then $\theta = 4$.

MIN-POWER LINK-COVER. This problem is a particular case of ACTIVATION LINK-COVER when $t_u^e = t_v^e$ for every link $e = uv \in E$; note that $\theta = 1$ in this case. The motivation is to assign energy levels to the nodes of a wireless network while minimizing the total energy consumption, and enabling communication for every terminal. The MIN-POWER LINK-COVER problem is NP-hard even if $R = V$, or if R is an independent set in the input graph G and unit thresholds [21]. The problem admits ratio 2 by a trivial reduction to the min-cost case. This was improved to 1.5 in [28], and then to 1.406 in [6], which also establishes the ratio $73/60$ for the bipartite case and for unit thresholds.

Let H_k denote the k-th harmonic number. In [29], the approximation ratios for the problem are expressed in the terms of the following two functions $\omega(\theta)$ and $\bar{\omega}(\theta)$ defined for $\theta > 0$, where $\bar{\omega}$ is a "discrete version" of ω:

$$\omega(\theta) = \max_{x \geq 1} \frac{\ln x}{1 + x/\theta} \qquad \bar{\omega}(\theta) = \max_{k \in \mathbb{N}} \frac{H_k - 1}{1 + k/\theta}$$

In [29] is introduced a generic simplified version of the Relative Greedy Heuristic of Zelikovsky [58], and using it the following is proved.

Theorem 3 ([29]) (i) ACTIVATION LINK-COVER *admits ratio* $1 + \omega(\theta)$ *for* θ*-bounded instances.*

(ii) BIPARTITE ACTIVATION LINK-COVER *with locally uniform thresholds admits ratio* $1 + \bar{\omega}(\theta)$.

(iii) ACTIVATION LINK-COVER *with unit thresholds admits ratio* $\frac{1555}{1347} < 1.155$.

Corollary 4 ([29]) MIN-POWER LINK-COVER *admits ratio* $1 + \omega(1) < 1.2785$*, and the* θ*-bounded versions of each one of the problems* WEIGHTED SET-COVER, FACILITY LOCATION, *and* INSTALLATION LINK-COVER, *admits ratio* $1 + \omega(\theta)$. FACILITY LOCATION *with locally uniform thresholds admits ratio* $1 + \bar{\omega}(\theta)$. MIN-POWER LINK-COVER *with unit powers admits ratio* 1.155.

Let us illustrate this result on the FACILITY LOCATION problem. Note that the ratio $1 + \omega(\theta)$ is surprisingly low. Even if $\theta = 100$ (service costs are at least 1% of opening costs) then we get a small ratio $1 + \omega(100) < 3.636$. Even for $\theta = 10^4$ we still get a reasonable ratio $1 + \omega(10^4) < 7.3603$. All previous results for the problem are usually summarized by just two observations: the problem is SET-COVER hard (so has a logarithmic approximation threshold by [51, 14]), and that it admits a matching logarithmic ratio $1 + \ln |R|$ [11, 57]; see surveys on FACILITY LOCATION problems by Vygen [55] and Shmoys [54]. Due to this, almost all work focused on the more tractable

METRIC FACILITY LOCATION problem. Theorem 3 implies that many practical non-metric FACILITY LOCATION instances admit a reasonable small constant ratio.

While the *main focus of this proposal is obtaining unifying algorithms for many problems raised from wireless networks design context, by giving dilation based approximation ratios*, the PI also intends to study some old long standing problems in the field. Moreover, while the activation network design problems correspond to the omnidirectional communications between nodes, we plan also to study the unidirectional model where the nodes can choose specific links to communicate with the (subset of) neighbors. We note that in this case, in order to improve lifetime and interference values, we may end with the multiple topology version where the activation of links is performed several times, each time resulting in a possibly different (subset of) activated links, see [46].

In general, we are interested to study the following open questions by *providing dilation-based guaranteed performance bounds and efficient distributed implementation* having in mind an assignment satisfying lifetime, interference and *SINR* criteria:

- PARTIAL ACTIVATION LINK COVER: Here we are given an integer Q and require that at least Q terminals in R are covered. We expect that the ratios in [29] for the ordinary ACTIVATION LINK COVER problem (the case $Q = |R|$) can be extended to the more general partial case.

- BUDGETED ACTIVATION LINK COVER: Here we are given a budget BD and seek an assignment of value at most BD that maximizes the number of terminals covered. Note that the node weighted version includes the BUDGETED MAXIMUM COVERAGE problem that has approximation threshold $1 - 1/e$ (otherwise we could achieve ratio $o(\ln n)$ for SET COVER) [25]. However, this threshold does not apply for the min-power version, so it is interesting to study what ratio can be achieved in this case – when $\theta = 1$. And more generally, what ratio can be achieved for an arbitrary dilation θ?

- LINK MULTICOVER: For the min-power case (when $\theta = 1$) the best ratio known is $O(\ln k)$ [12]. Can we achieve a constant ratio, or is the problem $\Omega(\ln k)$ hard to approximate? What approximation ratio can we achieve when the maximum demand is 2? For the latter case, the best known ratio is 2.5 [12]. We believe that combining our simplification of the method of [29] with the ideas of [12] will give better ratios for small (practical) values of k in the min-power case, as well as good dilation based approximation ratios for the more general activation version.

4.4 Connectivity problems

Cornerstone problem in connectivity network design are the SPANNING TREE and the STEINER TREE problems. As was mentioned, MIN-POWER SPANNING TREE admits ratio 1.5 and MIN-POWER STEINER TREE admits ratio $3\ln 4 - \frac{9}{4} + \epsilon$ [19]. These algorithms are based on the Iterative Randomized Rounding method and are not practical. Earlier, [1] gave relatively efficient algorithms with ratios $5/3$ and $11/6$ for the MIN-POWER SPANNING TREE problem. On the other hand, the NODE-WEIGHTED STEINER TREE problem admits a tight ratio $\Theta(\log n)$, and the same ratio applies also for the more general NODE WEIGHTED STEINER FOREST problem [26]. The best ratio known for the MIN-POWER STEINER FOREST is 4, which is achieved by a folklore reduction to the min-cost case, c.f [41]. To bridge between the constant ratio for min-power and tight logarithmic ratio for more general activation connectivity problems, we need a slightly different definition of a dilation. Specifically, [41] suggested the following definition (for simplicity of exposition, we state the definition for the STEINER TREE problem, and in terms of thresholds rather than in terms of activating functions).

Definition 5 ([41]) *The dilation $\theta(e)$ (of an activating function) of an undirected link $e = uv$ is defined by $\theta(e) = \frac{t_u^e + t_v^e}{\min\{t_u^e, t_v^e\}} - 1$. The dilation of an* Activation Steiner Tree *instance is defined by $\theta = \max_{e \in E} \theta(e)$.*

The fault tolerant version of the Spanning Tree problem is the k-Connectivity problem. The min-cost version of this problem admits ratio $O\left(\ln k \ln \frac{n}{n-k}\right)$ [40] by a combinatorial algorithm; the problem also admits ratio $4+\epsilon$ for any constant k [45] and ratio 6 when $k \geq n^3$ [9] (see also [17]), but these algorithms are based on the Iterative Rounding method. For the min-power variant the best ratio known is $O\left(\ln k \ln \frac{n}{n-k}\right)$ [12, 40]. For "practical" small values of k the currently best known ratios are 3 for $k = 2$, 4 for $k = 3$, $k+3$ for $k \in \{4, 5\}$, $k+5$ for $k \in \{6, 7\}$, and $3(k-1)$ for any constant k [35]. All these algorithms are combinatorial and relatively simple. In [41] are derived some simple dilation based approximation ratios for various connectivity problems. However, we believe that much better ratios are possible. To bridge between the constant ratio for min-power and tight logarithmic ratio for more general activation connectivity problems, the PI intends to *study θ-bounded versions of these problems, with focus on practical approximation algorithms*. In particular, we intend to concentrate our efforts on the following.

- No practical approximation algorithm is known for the Min-Power Steiner Tree problem. The PI intends to design such an algorithm. Our goal is to design combinatorial algorithm, either using the method of [50], or by using the simplified variation [29] of the Relative Greedy Heuristic [58, 52, 20]. This may allow us to implement the algorithms in a distributed fashion. We intend to use similar methods to obtain good dilation based approximation ratios for the Activation Spanning Tree and the Activation Steiner Tree problems.
- Activation k-Out-Connectivity and Activation k-Connectivity problems. Our first goal here is to obtain good dilation based approximation ratios, extending [35, 31, 12]. In addition, the PI may try to improve the approximation ratios for particular activation functions, e.g., for the min-power case. Recently, the best known ratios for Min-Cost k-Connectivity problems were substantially improved [40, 9, 42, 45], while generating novel methods and new insights for these fundamental problems. Using these new methods and insights, or even just using the improved ratios in these papers, may lead to improved ratios for the activation version.

5 Distributiveness

In order to make our solutions feasible, i.e. to allow them to work in real life node deployments, we outline how it is possible to implement them in a decentralized (distributed) (without the need for coordination by a central unit) and local, based on neighbor knowledge manner. Some above-mentioned algorithms can be done easily distributed. The paper [3] shows how to find a leader in a distributed fashion (and also minimum spanning tree) in a network with n nodes in $O(n)$ time using $O(n \log n)$ messages. To establish connectivity, can we follow two different approaches. The first, described in Dolev et al. [13] forms a temporary underlying topology in $O(n)$ time using $O(n^3)$ messages. The second (better) approach is given by Halldórsson and Mitra [22] that shows how to do this in $O(poly(\log \gamma, \log n))$, where γ is the ratio between the longest and shortest distances among nodes. Given each node knows the total number of nodes in the network, sometimes we will require the knowledge of the local GPS coordinates of each wireless node. To retrieve this information, we can apply Peleg et al. [49] distributed algorithm for finding the graph's diameter and propagate it to all nodes. Further optimization of the algorithms can recalculate the information gain only for a specific areas or a subset of nodes and take the old measurement from the rest of the network.

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