本证明由我自己编写的定理证明器 https://github.com/DnailZ/WangProver 生成
所有 <8>、<20>、<5b> ..... 表示任意公式
(对考试作用不大～)

---------- 基本结论（不含否定）----------

prove List() ⊢ p → p

```
1: p -> (<8> -> p)                                                  [L1]
2: p -> ((<8> -> p) -> p)                                           [L1]
3: (p -> ((<8> -> p) -> p)) -> ((p -> (<8> -> p)) -> (p -> p))      [L2]
4: (p -> (<8> -> p)) -> (p -> p)                                    MP 3, 2
5: p -> p                                                           MP 4, 1
```

prove List(p) ⊢ q → p

```
1: p                                                                [AS]
2: p -> (q -> p)                                                    [L1]
3: q -> p                                                           MP 2, 1
```

prove List(q → r) ⊢ (p → q) → (p → r)

```
1: q -> r                                                           [AS]
2: (q -> r) -> (p -> (q -> r))                                      [L1]
3: p -> (q -> r)                                                    MP 2, 1
4: (p -> (q -> r)) -> ((p -> q) -> (p -> r))                        [L2]
5: (p -> q) -> (p -> r)                                             MP 4, 3
```

prove List(p → r) ⊢ p → (q → r)

```
1: p -> r                                                           [AS]
2: r -> (q -> r)                                                    [L1]
3: (r -> (q -> r)) -> (p -> (r -> (q -> r)))                        [L1]
4: p -> (r -> (q -> r))                                             MP 3, 2
5: (p -> (r -> (q -> r))) -> ((p -> r) -> (p -> (q -> r)))          [L2]
6: (p -> r) -> (p -> (q -> r))                                      MP 5, 4
7: p -> (q -> r)                                                    MP 6, 1
```

prove List(p → q, p → (q → r)) ⊢ p → r

```
1: p -> q                                                           [AS]
2: p -> (q -> r)                                                    [AS]
3: (p -> (q -> r)) -> ((p -> q) -> (p -> r))                        [L2]
4: (p -> q) -> (p -> r)                                             MP 3, 2
5: p -> r                                                           MP 4, 1
```

prove List(p → (q → r)) ⊢ q → (p → r)

```
1: q -> (p -> q)                                                    [L1]
2: p -> (q -> r)                                                    [AS]
3: (p -> (q -> r)) -> ((p -> q) -> (p -> r))                        [L2]
4: (p -> q) -> (p -> r)                                             MP 3, 2
5: ((p -> q) -> (p -> r)) -> (q -> ((p -> q) -> (p -> r)))          [L1]
6: q -> ((p -> q) -> (p -> r))                                      MP 5, 4
7: (q -> ((p -> q) -> (p -> r))) -> ((q -> (p -> q)) -> (q -> (p -> r)))   [L2]
8: (q -> (p -> q)) -> (q -> (p -> r))                               MP 7, 6
9: q -> (p -> r)                                                    MP 8, 1
```

prove List(p → q, q → r) ⊢ p → r

```
1: p -> q                                                           [AS]
2: q -> r                                                           [AS]
3: (q -> r) -> (p -> (q -> r))                                      [L1]
4: p -> (q -> r)                                                    MP 3, 2
5: (p -> (q -> r)) -> ((p -> q) -> (p -> r))                        [L2]
6: (p -> q) -> (p -> r)                                             MP 5, 4
7: p -> r                                                           MP 6, 1
```

prove List(p → (q → r), p → (q → (r → s))) ⊢ p → (q → s)

```
 1: p -> (q -> r)                                                                   [AS]
 2: p -> (q -> (r -> s))                                                            [AS]
 3: (q -> (r -> s)) -> ((q -> r) -> (q -> s))                                       [L2]
 4: ((q -> (r -> s)) -> ((q -> r) -> (q -> s))) -> (p -> ((q -> (r -> s)) -> ((q -> r) -> (q -> s))))   [L1]
 5: p -> ((q -> (r -> s)) -> ((q -> r) -> (q -> s)))                                MP 4, 3
 6: (p -> ((q -> (r -> s)) -> ((q -> r) -> (q -> s)))) -> ((p -> (q -> (r -> s))) -> (p -> ((q -> r) -> (q -> s))))   [L2]
 7: (p -> (q -> (r -> s))) -> (p -> ((q -> r) -> (q -> s)))                         MP 6, 5
 8: p -> ((q -> r) -> (q -> s))                                                     MP 7, 2
 9: (p -> ((q -> r) -> (q -> s))) -> ((p -> (q -> r)) -> (p -> (q -> s)))           [L2]
10: (p -> (q -> r)) -> (p -> (q -> s))                                              MP 9, 8
11: p -> (q -> s)                                                                   MP 10, 1
```

---------- 作业题（不含否定）----------

prove List() ⊢ (p → q) → (p → p)

```
1: p -> (q -> p)                                                                [L1]
2: (p -> (q -> p)) -> ((p -> q) -> (p -> p))                                     [L2]
3: (p -> q) -> (p -> p)                                                         MP 2, 1
```

prove List(p, q → (p → r)) ⊢ q → r

```
1: p                                                                            [AS]
2: p -> (q -> p)                                                                [L1]
3: q -> p                                                                       MP 2, 1
4: q -> (p -> r)                                                                [AS]
5: (q -> (p -> r)) -> ((q -> p) -> (q -> r))                                    [L2]
6: (q -> p) -> (q -> r)                                                         MP 5, 4
7: q -> r                                                                       MP 6, 3
```

prove List() ⊢ (p → q) → ((¬p → ¬q) → (q → p))

```
1: (¬p -> ¬q) -> (q -> p)                                                       [L3]
2: ((¬p -> ¬q) -> (q -> p)) -> ((p -> q) -> ((¬p -> ¬q) -> (q -> p)))           [L1]
3: (p -> q) -> ((¬p -> ¬q) -> (q -> p))                                         MP 2, 1
```

prove List() ⊢ p → (q → (p → q))

```
1: q -> (p -> q)                                                                [L1]
2: (q -> (p -> q)) -> (p -> (q -> (p -> q)))                                    [L1]
3: p -> (q -> (p -> q))                                                         MP 2, 1
```

prove List() ⊢ ((p → (q → r)) → (p → q)) → ((p → (q → r)) → (p → r))

```
1: (p -> (q -> r)) -> ((p -> q) -> (p -> r))                                    [L2]
2: ((p -> (q -> r)) -> ((p -> q) -> (p -> r))) -> (((p -> (q -> r)) -> (p -> q)) -> ((p -> (q -> r)) -> (p -> r)))  [L2]
3: ((p -> (q -> r)) -> (p -> q)) -> ((p -> (q -> r)) -> (p -> r))               MP 2, 1
```

--------------- 基本结论（含否定）---------------

prove List(¬q, q) ⊢ p

```
1: q                                    [AS]
2: ¬q                                   [AS]
3: ¬q -> (¬p -> ¬q)                     [L1]
4: ¬p -> ¬q                             MP 3, 2
5: (¬p -> ¬q) -> (q -> p)               [L3]
6: q -> p                               MP 5, 4
7: p                                    MP 6, 1
```

prove List(¬q) ⊢ q → p

```
1: ¬q                                   [AS]
2: ¬q -> (¬p -> ¬q)                     [L1]
3: ¬p -> ¬q                             MP 2, 1
4: (¬p -> ¬q) -> (q -> p)               [L3]
5: q -> p                               MP 4, 3
```

prove List() ⊢ ¬q → (q → p)

```
1: ¬q -> (¬p -> ¬q)                     [L1]
2: (¬p -> ¬q) -> (q -> p)               [L3]
3: ((¬p -> ¬q) -> (q -> p)) -> (¬q -> ((¬p -> ¬q) -> (q -> p)))     [L1]
4: ¬q -> ((¬p -> ¬q) -> (q -> p))       MP 3, 2
5: (¬q -> ((¬p -> ¬q) -> (q -> p))) -> ((¬q -> (¬p -> ¬q)) -> (¬q -> (q -> p)))     [L2]
6: (¬q -> (¬p -> ¬q)) -> (¬q -> (q -> p))     MP 5, 4
7: ¬q -> (q -> p)                       MP 6, 1
```

prove List(¬p → p) ⊢ p

```
 1: ¬p -> p                             [AS]
 2: ¬p -> p                             [AS]
 3: ¬p -> (¬¬(¬p -> p) -> ¬p)           [L1]
 4: (¬¬(¬p -> p) -> ¬p) -> (p -> ¬(¬p -> p))     [L3]
 5: ((¬¬(¬p -> p) -> ¬p) -> (p -> ¬(¬p -> p))) -> (¬p -> ((¬¬(¬p -> p) -> ¬p) -> (p -> ¬(¬p -> p))))     [L1]
 6: ¬p -> ((¬¬(¬p -> p) -> ¬p) -> (p -> ¬(¬p -> p)))     MP 5, 4
 7: (¬p -> ((¬¬(¬p -> p) -> ¬p) -> (p -> ¬(¬p -> p)))) -> ((¬p -> (¬¬(¬p -> p) -> ¬p)) -> (¬p -> (p -> ¬(¬p -> p))))     [L2]
 8: (¬p -> (¬¬(¬p -> p) -> ¬p)) -> (¬p -> (p -> ¬(¬p -> p)))     MP 7, 6
 9: ¬p -> (p -> ¬(¬p -> p))             MP 8, 3
10: (¬p -> (p -> ¬(¬p -> p))) -> ((¬p -> p) -> (¬p -> ¬(¬p -> p)))     [L2]
11: (¬p -> p) -> (¬p -> ¬(¬p -> p))     MP 10, 9
12: ¬p -> ¬(¬p -> p)                    MP 11, 2
13: (¬p -> ¬(¬p -> p)) -> ((¬p -> p) -> p)     [L3]
14: (¬p -> p) -> p                      MP 13, 12
15: p                                   MP 14, 2
```

prove List() ⊢ (¬p → p) → p

```
 1: (¬p -> p) -> (<20> -> (¬p -> p))    [L1]
 2: ¬p -> (¬¬(<20> -> (¬p -> p)) -> ¬p)     [L1]
 3: (¬¬(<20> -> (¬p -> p)) -> ¬p) -> (p -> ¬(<20> -> (¬p -> p)))     [L3]
 4: ((¬¬(<20> -> (¬p -> p)) -> ¬p) -> (p -> ¬(<20> -> (¬p -> p)))) -> (¬p -> ((¬¬(<20> -> (¬p -> p)) -> ¬p) -> (p -> ¬(<20> -> (¬p -> p)))))     [L1]
 5: ¬p -> ((¬¬(<20> -> (¬p -> p)) -> ¬p) -> (p -> ¬(<20> -> (¬p -> p))))     MP 4, 3
 6: (¬p -> ((¬¬(<20> -> (¬p -> p)) -> ¬p) -> (p -> ¬(<20> -> (¬p -> p))))) -> ((¬p -> (¬¬(<20> -> (¬p -> p)) -> ¬p)) -> (¬p -> (p -> ¬(<20> -> (¬p -> p)))
 6: ))                                  [L2]
 7: (¬p -> (¬¬(<20> -> (¬p -> p)) -> ¬p)) -> (¬p -> (p -> ¬(<20> -> (¬p -> p))))     MP 6, 5
 8: ¬p -> (p -> ¬(<20> -> (¬p -> p)))   MP 7, 2
 9: (¬p -> (p -> ¬(<20> -> (¬p -> p)))) -> ((¬p -> p) -> (¬p -> ¬(<20> -> (¬p -> p))))     [L2]
10: (¬p -> p) -> (¬p -> ¬(<20> -> (¬p -> p)))     MP 9, 8
11: (¬p -> ¬(<20> -> (¬p -> p))) -> ((<20> -> (¬p -> p)) -> p)     [L3]
12: ((¬p -> ¬(<20> -> (¬p -> p))) -> ((<20> -> (¬p -> p)) -> p)) -> ((¬p -> p) -> ((¬p -> ¬(<20> -> (¬p -> p))) -> ((<20> -> (¬p -> p)) -> p)))     [L1]
13: (¬p -> p) -> ((¬p -> ¬(<20> -> (¬p -> p))) -> ((<20> -> (¬p -> p)) -> p))     MP 12, 11
14: ((¬p -> p) -> ((¬p -> ¬(<20> -> (¬p -> p))) -> ((<20> -> (¬p -> p)) -> p))) -> (((¬p -> p) -> (¬p -> ¬(<20> -> (¬p -> p)))) -> ((¬p -> p) -> ((<20> ->
14:  (¬p -> p)) -> p)))                 [L2]
15: ((¬p -> p) -> (¬p -> ¬(<20> -> (¬p -> p)))) -> ((¬p -> p) -> ((<20> -> (¬p -> p)) -> p))     MP 14, 13
16: (¬p -> p) -> ((<20> -> (¬p -> p)) -> p)     MP 15, 10
17: ((¬p -> p) -> ((<20> -> (¬p -> p)) -> p)) -> (((¬p -> p) -> (<20> -> (¬p -> p))) -> ((¬p -> p) -> p))     [L2]
18: ((¬p -> p) -> (<20> -> (¬p -> p))) -> ((¬p -> p) -> p)     MP 17, 16
19: (¬p -> p) -> p                      MP 18, 1
```

prove List(¬¬p) ⊢ p

```
1: ¬¬p                                  [AS]
2: ¬¬p                                  [AS]
3: ¬¬p -> (¬¬¬¬p -> ¬¬p)                [L1]
4: ¬¬¬¬p -> ¬¬p                         MP 3, 2
5: (¬¬¬¬p -> ¬¬p) -> (¬p -> ¬¬¬p)       [L3]
6: ¬p -> ¬¬¬p                           MP 5, 4
7: (¬p -> ¬¬¬p) -> (¬¬p -> p)           [L3]
8: ¬¬p -> p                             MP 7, 6
9: p                                    MP 8, 2
```

prove List() ⊢ ¬¬p → p

```
 1: ¬¬p -> (<1c> -> ¬¬p)                [L1]
 2: ¬¬p -> (¬¬(<1c> -> ¬¬p) -> ¬¬p)     [L1]
 3: (¬¬(<1c> -> ¬¬p) -> ¬¬p) -> (¬p -> ¬(<1c> -> ¬¬p))     [L3]
 4: ((¬¬(<1c> -> ¬¬p) -> ¬¬p) -> (¬p -> ¬(<1c> -> ¬¬p))) -> (¬¬p -> ((¬¬(<1c> -> ¬¬p) -> ¬¬p) -> (¬p -> ¬(<1c> -> ¬¬p))))     [L1]
 5: ¬¬p -> ((¬¬(<1c> -> ¬¬p) -> ¬¬p) -> (¬p -> ¬(<1c> -> ¬¬p)))     MP 4, 3
 6: (¬¬p -> ((¬¬(<1c> -> ¬¬p) -> ¬¬p) -> (¬p -> ¬(<1c> -> ¬¬p)))) -> ((¬¬p -> (¬¬(<1c> -> ¬¬p) -> ¬¬p)) -> (¬¬p -> (¬p -> ¬(<1c> -> ¬¬p))))     [L2]
 7: (¬¬p -> (¬¬(<1c> -> ¬¬p) -> ¬¬p)) -> (¬¬p -> (¬p -> ¬(<1c> -> ¬¬p)))     MP 6, 5
 8: ¬¬p -> (¬p -> ¬(<1c> -> ¬¬p))       MP 7, 2
 9: (¬p -> ¬(<1c> -> ¬¬p)) -> ((<1c> -> ¬¬p) -> p)     [L3]
10: ((¬p -> ¬(<1c> -> ¬¬p)) -> ((<1c> -> ¬¬p) -> p)) -> (¬¬p -> ((¬p -> ¬(<1c> -> ¬¬p)) -> ((<1c> -> ¬¬p) -> p)))     [L1]
11: ¬¬p -> ((¬p -> ¬(<1c> -> ¬¬p)) -> ((<1c> -> ¬¬p) -> p))     MP 10, 9
12: (¬¬p -> ((¬p -> ¬(<1c> -> ¬¬p)) -> ((<1c> -> ¬¬p) -> p))) -> ((¬¬p -> (¬p -> ¬(<1c> -> ¬¬p))) -> (¬¬p -> ((<1c> -> ¬¬p) -> p)))     [L2]
13: (¬¬p -> (¬p -> ¬(<1c> -> ¬¬p))) -> (¬¬p -> ((<1c> -> ¬¬p) -> p))     MP 12, 11
14: ¬¬p -> ((<1c> -> ¬¬p) -> p)         MP 13, 8
15: (¬¬p -> ((<1c> -> ¬¬p) -> p)) -> ((¬¬p -> (<1c> -> ¬¬p)) -> (¬¬p -> p))     [L2]
16: (¬¬p -> (<1c> -> ¬¬p)) -> (¬¬p -> p)     MP 15, 14
17: ¬¬p -> p                            MP 16, 1
```

prove List(p) ⊢ ¬¬p

```
1: p    [AS]
2: p    [AS]
3: p -> (¬¬¬p -> p)    [L1]
4: ¬¬¬p -> p    MP 3, 2
5: ¬¬¬p -> (¬¬¬¬¬p -> ¬¬¬p)    [L1]
6: (¬¬¬¬¬p -> ¬¬¬p) -> (¬¬p -> ¬¬¬¬p)    [L3]
7: ((¬¬¬¬¬p -> ¬¬¬p) -> (¬¬p -> ¬¬¬¬p)) -> (¬¬¬p -> ((¬¬¬¬¬p -> ¬¬¬p) -> (¬¬p -> ¬¬¬¬p)))    [L1]
8: ¬¬¬p -> ((¬¬¬¬¬p -> ¬¬¬p) -> (¬¬p -> ¬¬¬¬p))    MP 7, 6
9: (¬¬¬p -> ((¬¬¬¬¬p -> ¬¬¬p) -> (¬¬p -> ¬¬¬¬p))) -> ((¬¬¬p -> (¬¬¬¬¬p -> ¬¬¬p)) -> (¬¬¬p -> (¬¬p -> ¬¬¬¬p)))    [L2]
10: (¬¬¬p -> (¬¬¬¬¬p -> ¬¬¬p)) -> (¬¬¬p -> (¬¬p -> ¬¬¬¬p))    MP 9, 8
11: ¬¬¬p -> (¬¬p -> ¬¬¬¬p)    MP 10, 5
12: (¬¬p -> ¬¬¬¬p) -> (¬¬¬p -> ¬p)    [L3]
13: ((¬¬p -> ¬¬¬¬p) -> (¬¬¬p -> ¬p)) -> (¬¬¬p -> ((¬¬p -> ¬¬¬¬p) -> (¬¬¬p -> ¬p)))    [L1]
14: ¬¬¬p -> ((¬¬p -> ¬¬¬¬p) -> (¬¬¬p -> ¬p))    MP 13, 12
15: (¬¬¬p -> ((¬¬p -> ¬¬¬¬p) -> (¬¬¬p -> ¬p))) -> ((¬¬¬p -> (¬¬p -> ¬¬¬¬p)) -> (¬¬¬p -> (¬¬¬p -> ¬p)))    [L2]
16: (¬¬¬p -> (¬¬p -> ¬¬¬¬p)) -> (¬¬¬p -> (¬¬¬p -> ¬p))    MP 15, 14
17: ¬¬¬p -> (¬¬¬p -> ¬p)    MP 16, 11
18: (¬¬¬p -> ¬p) -> (p -> ¬¬p)    [L3]
19: ((¬¬¬p -> ¬p) -> (p -> ¬¬p)) -> (¬¬¬p -> ((¬¬¬p -> ¬p) -> (p -> ¬¬p)))    [L1]
20: ¬¬¬p -> ((¬¬¬p -> ¬p) -> (p -> ¬¬p))    MP 19, 18
21: (¬¬¬p -> ((¬¬¬p -> ¬p) -> (p -> ¬¬p))) -> ((¬¬¬p -> (¬¬¬p -> ¬p)) -> (¬¬¬p -> (p -> ¬¬p)))    [L2]
22: (¬¬¬p -> (¬¬¬p -> ¬p)) -> (¬¬¬p -> (p -> ¬¬p))    MP 21, 20
23: ¬¬¬p -> (p -> ¬¬p)    MP 22, 17
24: (¬¬¬p -> (p -> ¬¬p)) -> ((¬¬¬p -> p) -> (¬¬¬p -> ¬¬p))    [L2]
25: (¬¬¬p -> p) -> (¬¬¬p -> ¬¬p)    MP 24, 23
26: ¬¬¬p -> ¬¬p    MP 25, 4
27: ¬¬¬p -> (¬¬p -> ¬¬¬p)    [L1]
28: (¬¬p -> ¬¬¬p) -> (¬¬p -> ¬p)    [L3]
29: ((¬¬p -> ¬¬¬p) -> (¬¬p -> ¬p)) -> (¬¬¬p -> ((¬¬p -> ¬¬¬p) -> (¬¬p -> ¬p)))    [L1]
30: ¬¬¬p -> ((¬¬p -> ¬¬¬p) -> (¬¬p -> ¬p))    MP 29, 28
31: (¬¬¬p -> ((¬¬p -> ¬¬¬p) -> (¬¬p -> ¬p))) -> ((¬¬¬p -> (¬¬p -> ¬¬¬p)) -> (¬¬¬p -> (¬¬p -> ¬p)))    [L2]
32: (¬¬¬p -> (¬¬p -> ¬¬¬p)) -> (¬¬¬p -> (¬¬p -> ¬p))    MP 31, 30
33: ¬¬¬p -> (¬¬p -> ¬p)    MP 32, 27
34: (¬¬¬p -> (¬¬p -> ¬p)) -> ((¬¬¬p -> ¬¬p) -> (¬¬¬p -> ¬p))    [L2]
35: (¬¬¬p -> ¬¬p) -> (¬¬¬p -> ¬p)    MP 34, 33
36: ¬¬¬p -> ¬p    MP 35, 26
37: (¬¬¬p -> ¬p) -> (p -> ¬¬p)    [L3]
38: p -> ¬¬p    MP 37, 36
39: ¬¬p    MP 38, 2
```

prove List() ⊢ p → ¬¬p

```
1: ¬¬¬p -> (<1f> -> ¬¬¬p)    [L1]
2: ¬¬¬p -> (¬¬(<1f> -> ¬¬¬p) -> ¬¬¬p)    [L1]
3: (¬¬(<1f> -> ¬¬¬p) -> ¬¬¬p) -> (¬¬p -> ¬(<1f> -> ¬¬¬p))    [L3]
4: ((¬¬(<1f> -> ¬¬¬p) -> ¬¬¬p) -> (¬¬p -> ¬(<1f> -> ¬¬¬p))) -> (¬¬¬p -> ((¬¬(<1f> -> ¬¬¬p) -> ¬¬¬p) -> (¬¬p -> ¬(<1f> -> ¬¬¬p))))    [L1]
5: ¬¬¬p -> ((¬¬(<1f> -> ¬¬¬p) -> ¬¬¬p) -> (¬¬p -> ¬(<1f> -> ¬¬¬p)))    MP 4, 3
6: (¬¬¬p -> ((¬¬(<1f> -> ¬¬¬p) -> ¬¬¬p) -> (¬¬p -> ¬(<1f> -> ¬¬¬p)))) -> ((¬¬¬p -> (¬¬(<1f> -> ¬¬¬p) -> ¬¬¬p)) -> (¬¬¬p -> (¬¬p -> ¬(<1f> -> ¬¬¬p))))    [L2]
7: (¬¬¬p -> (¬¬(<1f> -> ¬¬¬p) -> ¬¬¬p)) -> (¬¬¬p -> (¬¬p -> ¬(<1f> -> ¬¬¬p)))    MP 6, 5
8: ¬¬¬p -> (¬¬p -> ¬(<1f> -> ¬¬¬p))    MP 7, 2
9: (¬¬p -> ¬(<1f> -> ¬¬¬p)) -> ((<1f> -> ¬¬¬p) -> ¬p)    [L3]
10: ((¬¬p -> ¬(<1f> -> ¬¬¬p)) -> ((<1f> -> ¬¬¬p) -> ¬p)) -> (¬¬¬p -> ((¬¬p -> ¬(<1f> -> ¬¬¬p)) -> ((<1f> -> ¬¬¬p) -> ¬p)))    [L1]
11: ¬¬¬p -> ((¬¬p -> ¬(<1f> -> ¬¬¬p)) -> ((<1f> -> ¬¬¬p) -> ¬p))    MP 10, 9
12: (¬¬¬p -> ((¬¬p -> ¬(<1f> -> ¬¬¬p)) -> ((<1f> -> ¬¬¬p) -> ¬p))) -> ((¬¬¬p -> (¬¬p -> ¬(<1f> -> ¬¬¬p))) -> (¬¬¬p -> ((<1f> -> ¬¬¬p) -> ¬p)))    [L2]
13: (¬¬¬p -> (¬¬p -> ¬(<1f> -> ¬¬¬p))) -> (¬¬¬p -> ((<1f> -> ¬¬¬p) -> ¬p))    MP 12, 11
14: ¬¬¬p -> ((<1f> -> ¬¬¬p) -> ¬p)    MP 13, 8
15: (¬¬¬p -> ((<1f> -> ¬¬¬p) -> ¬p)) -> ((¬¬¬p -> (<1f> -> ¬¬¬p)) -> (¬¬¬p -> ¬p))    [L2]
16: (¬¬¬p -> (<1f> -> ¬¬¬p)) -> (¬¬¬p -> ¬p)    MP 15, 14
17: ¬¬¬p -> ¬p    MP 16, 1
18: (¬¬¬p -> ¬p) -> (p -> ¬¬p)    [L3]
19: p -> ¬¬p    MP 18, 17
```

--------------- 作业题（含否定）---------------

prove List() ⊢ ¬(p → q) → p

```
 1: ¬p -> (¬q -> ¬p)                                                    [L1]
 2: (¬q -> ¬p) -> (p -> q)                                              [L3]
 3: ((¬q -> ¬p) -> (p -> q)) -> (¬p -> ((¬q -> ¬p) -> (p -> q)))        [L1]
 4: ¬p -> ((¬q -> ¬p) -> (p -> q))                                      MP 3, 2
 5: (¬p -> ((¬q -> ¬p) -> (p -> q))) -> ((¬p -> (¬q -> ¬p)) -> (¬p -> (p -> q)))   [L2]
 6: (¬p -> (¬q -> ¬p)) -> (¬p -> (p -> q))                              MP 5, 4
 7: ¬p -> (p -> q)                                                      MP 6, 1
 8: (¬p -> (p -> q)) -> (¬(p -> q) -> (¬p -> (p -> q)))                 [L1]
 9: ¬(p -> q) -> (¬p -> (p -> q))                                       MP 8, 7
10: ¬(p -> q) -> (¬¬¬(p -> q) -> ¬(p -> q))                             [L1]
11: (¬¬¬(p -> q) -> ¬(p -> q)) -> ((p -> q) -> ¬¬(p -> q))              [L3]
12: ((¬¬¬(p -> q) -> ¬(p -> q)) -> ((p -> q) -> ¬¬(p -> q))) -> (¬(p -> q) -> ((¬¬¬(p -> q) -> ¬(p -> q)) -> ((p -> q) -> ¬¬(p -> q))))   [L1]
13: ¬(p -> q) -> ((¬¬¬(p -> q) -> ¬(p -> q)) -> ((p -> q) -> ¬¬(p -> q)))   MP 12, 11
14: (¬(p -> q) -> ((¬¬¬(p -> q) -> ¬(p -> q)) -> ((p -> q) -> ¬¬(p -> q)))) -> ((¬(p -> q) -> (¬¬¬(p -> q) -> ¬(p -> q))) -> (¬(p -> q) -> ((p -> q) -> ¬¬
14: (p -> q))))                                                         [L2]
15: (¬(p -> q) -> (¬¬¬(p -> q) -> ¬(p -> q))) -> (¬(p -> q) -> ((p -> q) -> ¬¬(p -> q)))   MP 14, 13
16: ¬(p -> q) -> ((p -> q) -> ¬¬(p -> q))                               MP 15, 10
17: ((p -> q) -> ¬¬(p -> q)) -> (¬p -> ((p -> q) -> ¬¬(p -> q)))        [L1]
18: (((p -> q) -> ¬¬(p -> q)) -> (¬p -> ((p -> q) -> ¬¬(p -> q)))) -> (¬(p -> q) -> (((p -> q) -> ¬¬(p -> q)) -> (¬p -> ((p -> q) -> ¬¬(p -> q)))))   [L1]
19: ¬(p -> q) -> (((p -> q) -> ¬¬(p -> q)) -> (¬p -> ((p -> q) -> ¬¬(p -> q))))   MP 18, 17
20: (¬(p -> q) -> (((p -> q) -> ¬¬(p -> q)) -> (¬p -> ((p -> q) -> ¬¬(p -> q))))) -> ((¬(p -> q) -> ((p -> q) -> ¬¬(p -> q))) -> (¬(p -> q) -> (¬p -> ((p
20: -> q) -> ¬¬(p -> q)))))                                             [L2]
21: (¬(p -> q) -> ((p -> q) -> ¬¬(p -> q))) -> (¬(p -> q) -> (¬p -> ((p -> q) -> ¬¬(p -> q))))   MP 20, 19
22: ¬(p -> q) -> (¬p -> ((p -> q) -> ¬¬(p -> q)))                       MP 21, 16
23: (¬p -> ((p -> q) -> ¬¬(p -> q))) -> ((¬p -> (p -> q)) -> (¬p -> ¬¬(p -> q)))   [L2]
24: ((¬p -> ((p -> q) -> ¬¬(p -> q))) -> ((¬p -> (p -> q)) -> (¬p -> ¬¬(p -> q)))) -> (¬(p -> q) -> ((¬p -> ((p -> q) -> ¬¬(p -> q))) -> ((¬p -> (p -> q))
24:  -> (¬p -> ¬¬(p -> q)))))                                           [L1]
25: ¬(p -> q) -> ((¬p -> ((p -> q) -> ¬¬(p -> q))) -> ((¬p -> (p -> q)) -> (¬p -> ¬¬(p -> q))))   MP 24, 23
26: (¬(p -> q) -> ((¬p -> ((p -> q) -> ¬¬(p -> q))) -> ((¬p -> (p -> q)) -> (¬p -> ¬¬(p -> q))))) -> ((¬(p -> q) -> (¬p -> ((p -> q) -> ¬¬(p -> q)))) -> (
26: ¬(p -> q) -> ((¬p -> (p -> q)) -> (¬p -> ¬¬(p -> q)))))             [L2]
27: (¬(p -> q) -> (¬p -> ((p -> q) -> ¬¬(p -> q)))) -> (¬(p -> q) -> ((¬p -> (p -> q)) -> (¬p -> ¬¬(p -> q))))   MP 26, 25
28: ¬(p -> q) -> ((¬p -> (p -> q)) -> (¬p -> ¬¬(p -> q)))               MP 27, 22
29: (¬(p -> q) -> ((¬p -> (p -> q)) -> (¬p -> ¬¬(p -> q)))) -> ((¬(p -> q) -> (¬p -> (p -> q))) -> (¬(p -> q) -> (¬p -> ¬¬(p -> q))))   [L2]
30: (¬(p -> q) -> (¬p -> (p -> q))) -> (¬(p -> q) -> (¬p -> ¬¬(p -> q)))   MP 29, 28
31: ¬(p -> q) -> (¬p -> ¬¬(p -> q))                                     MP 30, 9
32: ¬(p -> q) -> ((¬p -> ¬¬(p -> q)) -> ¬(p -> q))                      [L1]
33: (¬p -> ¬¬(p -> q)) -> (¬(p -> q) -> p)                              [L3]
34: ((¬p -> ¬¬(p -> q)) -> (¬(p -> q) -> p)) -> (((¬p -> ¬¬(p -> q)) -> ¬(p -> q)) -> ((¬p -> ¬¬(p -> q)) -> p))   [L2]
35: ((¬p -> ¬¬(p -> q)) -> ¬(p -> q)) -> ((¬p -> ¬¬(p -> q)) -> p)     MP 34, 33
36: (((¬p -> ¬¬(p -> q)) -> ¬(p -> q)) -> ((¬p -> ¬¬(p -> q)) -> p)) -> (¬(p -> q) -> (((¬p -> ¬¬(p -> q)) -> ¬(p -> q)) -> ((¬p -> ¬¬(p -> q)) -> p)))   [L1]
37: ¬(p -> q) -> (((¬p -> ¬¬(p -> q)) -> ¬(p -> q)) -> ((¬p -> ¬¬(p -> q)) -> p))   MP 36, 35
38: (¬(p -> q) -> (((¬p -> ¬¬(p -> q)) -> ¬(p -> q)) -> ((¬p -> ¬¬(p -> q)) -> p))) -> ((¬(p -> q) -> ((¬p -> ¬¬(p -> q)) -> ¬(p -> q))) -> (¬(p -> q) ->
38: ((¬p -> ¬¬(p -> q)) -> p)))                                         [L2]
39: (¬(p -> q) -> ((¬p -> ¬¬(p -> q)) -> ¬(p -> q))) -> (¬(p -> q) -> ((¬p -> ¬¬(p -> q)) -> p))   MP 38, 37
40: ¬(p -> q) -> ((¬p -> ¬¬(p -> q)) -> p)                              MP 39, 32
41: (¬(p -> q) -> ((¬p -> ¬¬(p -> q)) -> p)) -> ((¬(p -> q) -> (¬p -> ¬¬(p -> q))) -> (¬(p -> q) -> p))   [L2]
42: (¬(p -> q) -> (¬p -> ¬¬(p -> q))) -> (¬(p -> q) -> p)               MP 41, 40
43: ¬(p -> q) -> p                                                      MP 42, 31
```

prove List() ⊢ ¬(p → q) → ¬q

```
 1: ¬¬q -> (¬¬p -> ¬¬q)                                                 [L1]
 2: (¬¬p -> ¬¬q) -> (¬q -> ¬p)                                          [L3]
 3: ((¬¬p -> ¬¬q) -> (¬q -> ¬p)) -> (¬¬q -> ((¬¬p -> ¬¬q) -> (¬q -> ¬p)))   [L1]
 4: ¬¬q -> ((¬¬p -> ¬¬q) -> (¬q -> ¬p))                                 MP 3, 2
 5: (¬¬q -> ((¬¬p -> ¬¬q) -> (¬q -> ¬p))) -> ((¬¬q -> (¬¬p -> ¬¬q)) -> (¬¬q -> (¬q -> ¬p)))   [L2]
 6: (¬¬q -> (¬¬p -> ¬¬q)) -> (¬¬q -> (¬q -> ¬p))                        MP 5, 4
 7: ¬¬q -> (¬q -> ¬p)                                                   MP 6, 1
 8: (¬q -> ¬p) -> (p -> q)                                              [L3]
 9: ((¬q -> ¬p) -> (p -> q)) -> (¬¬q -> ((¬q -> ¬p) -> (p -> q)))       [L1]
10: ¬¬q -> ((¬q -> ¬p) -> (p -> q))                                     MP 9, 8
11: (¬¬q -> ((¬q -> ¬p) -> (p -> q))) -> ((¬¬q -> (¬q -> ¬p)) -> (¬¬q -> (p -> q)))   [L2]
12: (¬¬q -> (¬q -> ¬p)) -> (¬¬q -> (p -> q))                            MP 11, 10
13: ¬¬q -> (p -> q)                                                     MP 12, 7
14: (¬¬q -> (p -> q)) -> (¬(p -> q) -> (¬¬q -> (p -> q)))               [L1]
15: ¬(p -> q) -> (¬¬q -> (p -> q))                                      MP 14, 13
16: ¬(p -> q) -> (¬¬¬(p -> q) -> ¬(p -> q))                             [L1]
17: (¬¬¬(p -> q) -> ¬(p -> q)) -> ((p -> q) -> ¬¬(p -> q))              [L3]
18: ((¬¬¬(p -> q) -> ¬(p -> q)) -> ((p -> q) -> ¬¬(p -> q))) -> (¬(p -> q) -> ((¬¬¬(p -> q) -> ¬(p -> q)) -> ((p -> q) -> ¬¬(p -> q))))   [L1]
19: ¬(p -> q) -> ((¬¬¬(p -> q) -> ¬(p -> q)) -> ((p -> q) -> ¬¬(p -> q)))   MP 18, 17
20: (¬(p -> q) -> ((¬¬¬(p -> q) -> ¬(p -> q)) -> ((p -> q) -> ¬¬(p -> q)))) -> ((¬(p -> q) -> (¬¬¬(p -> q) -> ¬(p -> q))) -> (¬(p -> q) -> ((p -> q) -> ¬¬
20: (p -> q))))                                                         [L2]
21: (¬(p -> q) -> (¬¬¬(p -> q) -> ¬(p -> q))) -> (¬(p -> q) -> ((p -> q) -> ¬¬(p -> q)))   MP 20, 19
22: ¬(p -> q) -> ((p -> q) -> ¬¬(p -> q))                               MP 21, 16
23: ((p -> q) -> ¬¬(p -> q)) -> (¬¬q -> ((p -> q) -> ¬¬(p -> q)))       [L1]
24: (((p -> q) -> ¬¬(p -> q)) -> (¬¬q -> ((p -> q) -> ¬¬(p -> q)))) -> (¬(p -> q) -> (((p -> q) -> ¬¬(p -> q)) -> (¬¬q -> ((p -> q) -> ¬¬(p -> q)))))   [L1]
25: ¬(p -> q) -> (((p -> q) -> ¬¬(p -> q)) -> (¬¬q -> ((p -> q) -> ¬¬(p -> q))))   MP 24, 23
26: (¬(p -> q) -> (((p -> q) -> ¬¬(p -> q)) -> (¬¬q -> ((p -> q) -> ¬¬(p -> q))))) -> ((¬(p -> q) -> ((p -> q) -> ¬¬(p -> q))) -> (¬(p -> q) -> (¬¬q -> ((
26: p -> q) -> ¬¬(p -> q)))))                                           [L2]
27: (¬(p -> q) -> ((p -> q) -> ¬¬(p -> q))) -> (¬(p -> q) -> (¬¬q -> ((p -> q) -> ¬¬(p -> q))))   MP 26, 25
28: ¬(p -> q) -> (¬¬q -> ((p -> q) -> ¬¬(p -> q)))                      MP 27, 22
29: (¬¬q -> ((p -> q) -> ¬¬(p -> q))) -> ((¬¬q -> (p -> q)) -> (¬¬q -> ¬¬(p -> q)))   [L2]
30: ((¬¬q -> ((p -> q) -> ¬¬(p -> q))) -> ((¬¬q -> (p -> q)) -> (¬¬q -> ¬¬(p -> q)))) -> (¬(p -> q) -> ((¬¬q -> ((p -> q) -> ¬¬(p -> q))) -> ((¬¬q -> (p -
30: > q)) -> (¬¬q -> ¬¬(p -> q)))))                                     [L1]
31: ¬(p -> q) -> ((¬¬q -> ((p -> q) -> ¬¬(p -> q))) -> ((¬¬q -> (p -> q)) -> (¬¬q -> ¬¬(p -> q))))   MP 30, 29
32: (¬(p -> q) -> ((¬¬q -> ((p -> q) -> ¬¬(p -> q))) -> ((¬¬q -> (p -> q)) -> (¬¬q -> ¬¬(p -> q))))) -> ((¬(p -> q) -> (¬¬q -> ((p -> q) -> ¬¬(p -> q))))
32: -> (¬(p -> q) -> ((¬¬q -> (p -> q)) -> (¬¬q -> ¬¬(p -> q)))))       [L2]
33: (¬(p -> q) -> (¬¬q -> ((p -> q) -> ¬¬(p -> q)))) -> (¬(p -> q) -> ((¬¬q -> (p -> q)) -> (¬¬q -> ¬¬(p -> q))))   MP 32, 31
34: ¬(p -> q) -> ((¬¬q -> (p -> q)) -> (¬¬q -> ¬¬(p -> q)))             MP 33, 28
35: (¬(p -> q) -> ((¬¬q -> (p -> q)) -> (¬¬q -> ¬¬(p -> q)))) -> ((¬(p -> q) -> (¬¬q -> (p -> q))) -> (¬(p -> q) -> (¬¬q -> ¬¬(p -> q))))   [L2]
36: (¬(p -> q) -> (¬¬q -> (p -> q))) -> (¬(p -> q) -> (¬¬q -> ¬¬(p -> q)))   MP 35, 34
37: ¬(p -> q) -> (¬¬q -> ¬¬(p -> q))                                    MP 36, 15
38: ¬(p -> q) -> ((¬¬q -> ¬¬(p -> q)) -> ¬(p -> q))                     [L1]
39: (¬¬q -> ¬¬(p -> q)) -> (¬(p -> q) -> ¬q)                            [L3]
40: ((¬¬q -> ¬¬(p -> q)) -> (¬(p -> q) -> ¬q)) -> (((¬¬q -> ¬¬(p -> q)) -> ¬(p -> q)) -> ((¬¬q -> ¬¬(p -> q)) -> ¬q))   [L2]
41: ((¬¬q -> ¬¬(p -> q)) -> ¬(p -> q)) -> ((¬¬q -> ¬¬(p -> q)) -> ¬q)   MP 40, 39
42: (((¬¬q -> ¬¬(p -> q)) -> ¬(p -> q)) -> ((¬¬q -> ¬¬(p -> q)) -> ¬q)) -> (¬(p -> q) -> (((¬¬q -> ¬¬(p -> q)) -> ¬(p -> q)) -> ((¬¬q -> ¬¬(p -> q)) -> ¬q
42: )))                                                                 [L1]
43: ¬(p -> q) -> (((¬¬q -> ¬¬(p -> q)) -> ¬(p -> q)) -> ((¬¬q -> ¬¬(p -> q)) -> ¬q))   MP 42, 41
44: (¬(p -> q) -> (((¬¬q -> ¬¬(p -> q)) -> ¬(p -> q)) -> ((¬¬q -> ¬¬(p -> q)) -> ¬q))) -> ((¬(p -> q) -> ((¬¬q -> ¬¬(p -> q)) -> ¬(p -> q))) -> (¬(p -> q)
44:  -> ((¬¬q -> ¬¬(p -> q)) -> ¬q)))                                   [L2]
45: (¬(p -> q) -> ((¬¬q -> ¬¬(p -> q)) -> ¬(p -> q))) -> (¬(p -> q) -> ((¬¬q -> ¬¬(p -> q)) -> ¬q))   MP 44, 43
46: ¬(p -> q) -> ((¬¬q -> ¬¬(p -> q)) -> ¬q)                            MP 45, 38
47: (¬(p -> q) -> ((¬¬q -> ¬¬(p -> q)) -> ¬q)) -> ((¬(p -> q) -> (¬¬q -> ¬¬(p -> q))) -> (¬(p -> q) -> ¬q))   [L2]
48: (¬(p -> q) -> (¬¬q -> ¬¬(p -> q))) -> (¬(p -> q) -> ¬q)             MP 47, 46
49: ¬(p -> q) -> ¬q                                                     MP 48, 37
```

prove List() ⊢ (¬p → ¬q) → ((¬p → q) → p)

```
 1: (¬p -> ¬q) -> ((¬p -> q) -> (¬p -> ¬q))                                                  [L1]
 2: ¬q -> (¬¬(¬p -> ¬q) -> ¬q)                                                               [L1]
 3: (¬q -> (¬¬(¬p -> ¬q) -> ¬q)) -> (¬p -> (¬q -> (¬¬(¬p -> ¬q) -> ¬q)))                      [L1]
 4: ¬p -> (¬q -> (¬¬(¬p -> ¬q) -> ¬q))                                                       MP 3, 2
 5: (¬p -> (¬q -> (¬¬(¬p -> ¬q) -> ¬q))) -> ((¬p -> ¬q) -> (¬p -> (¬¬(¬p -> ¬q) -> ¬q)))      [L2]
 6: (¬p -> ¬q) -> (¬p -> (¬¬(¬p -> ¬q) -> ¬q))                                               MP 5, 4
 7: (¬¬(¬p -> ¬q) -> ¬q) -> (q -> ¬(¬p -> ¬q))                                               [L3]
 8: ((¬¬(¬p -> ¬q) -> ¬q) -> (q -> ¬(¬p -> ¬q))) -> (¬p -> ((¬¬(¬p -> ¬q) -> ¬q) -> (q -> ¬(¬p -> ¬q))))      [L1]
 9: ¬p -> ((¬¬(¬p -> ¬q) -> ¬q) -> (q -> ¬(¬p -> ¬q)))                                       MP 8, 7
10: (¬p -> ((¬¬(¬p -> ¬q) -> ¬q) -> (q -> ¬(¬p -> ¬q)))) -> ((¬p -> (¬¬(¬p -> ¬q) -> ¬q)) -> (¬p -> (q -> ¬(¬p -> ¬q))))      [L2]
11: (¬p -> (¬¬(¬p -> ¬q) -> ¬q)) -> (¬p -> (q -> ¬(¬p -> ¬q)))                               MP 10, 9
12: ((¬p -> (¬¬(¬p -> ¬q) -> ¬q)) -> (¬p -> (q -> ¬(¬p -> ¬q)))) -> ((¬p -> ¬q) -> ((¬p -> (¬¬(¬p -> ¬q) -> ¬q)) -> (¬p -> (q -> ¬(¬p -> ¬q)))))      [L1]
13: (¬p -> ¬q) -> ((¬p -> (¬¬(¬p -> ¬q) -> ¬q)) -> (¬p -> (q -> ¬(¬p -> ¬q))))               MP 12, 11
14: ((¬p -> ¬q) -> ((¬p -> (¬¬(¬p -> ¬q) -> ¬q)) -> (¬p -> (q -> ¬(¬p -> ¬q))))) -> (((¬p -> ¬q) -> (¬p -> (¬¬(¬p -> ¬q) -> ¬q))) -> ((¬p -> ¬q) -> (¬p ->
14:  (q -> ¬(¬p -> ¬q)))))                                                                   [L2]
15: ((¬p -> ¬q) -> (¬p -> (¬¬(¬p -> ¬q) -> ¬q))) -> ((¬p -> ¬q) -> (¬p -> (q -> ¬(¬p -> ¬q))))      MP 14, 13
16: (¬p -> ¬q) -> (¬p -> (q -> ¬(¬p -> ¬q)))                                                 MP 15, 6
17: (¬p -> (q -> ¬(¬p -> ¬q))) -> ((¬p -> q) -> (¬p -> ¬(¬p -> ¬q)))                         [L2]
18: ((¬p -> (q -> ¬(¬p -> ¬q))) -> ((¬p -> q) -> (¬p -> ¬(¬p -> ¬q)))) -> ((¬p -> ¬q) -> ((¬p -> (q -> ¬(¬p -> ¬q))) -> ((¬p -> q) -> (¬p -> ¬(¬p -> ¬q)))
18: ))                                                                                       [L1]
19: (¬p -> ¬q) -> ((¬p -> (q -> ¬(¬p -> ¬q))) -> ((¬p -> q) -> (¬p -> ¬(¬p -> ¬q))))         MP 18, 17
20: ((¬p -> ¬q) -> ((¬p -> (q -> ¬(¬p -> ¬q))) -> ((¬p -> q) -> (¬p -> ¬(¬p -> ¬q))))) -> (((¬p -> ¬q) -> (¬p -> (q -> ¬(¬p -> ¬q)))) -> ((¬p -> ¬q) -> ((
20: ¬p -> q) -> (¬p -> ¬(¬p -> ¬q)))))                                                       [L2]
21: ((¬p -> ¬q) -> (¬p -> (q -> ¬(¬p -> ¬q)))) -> ((¬p -> ¬q) -> ((¬p -> q) -> (¬p -> ¬(¬p -> ¬q))))      MP 20, 19
22: (¬p -> ¬q) -> ((¬p -> q) -> (¬p -> ¬(¬p -> ¬q)))                                         MP 21, 16
23: (¬p -> ¬(¬p -> ¬q)) -> ((¬p -> ¬q) -> p)                                                 [L3]
24: ((¬p -> ¬(¬p -> ¬q)) -> ((¬p -> ¬q) -> p)) -> ((¬p -> q) -> ((¬p -> ¬(¬p -> ¬q)) -> ((¬p -> ¬q) -> p)))      [L1]
25: (¬p -> q) -> ((¬p -> ¬(¬p -> ¬q)) -> ((¬p -> ¬q) -> p))                                  MP 24, 23
26: ((¬p -> q) -> ((¬p -> ¬(¬p -> ¬q)) -> ((¬p -> ¬q) -> p))) -> (((¬p -> q) -> (¬p -> ¬(¬p -> ¬q))) -> ((¬p -> q) -> ((¬p -> ¬q) -> p)))      [L2]
27: ((¬p -> q) -> (¬p -> ¬(¬p -> ¬q))) -> ((¬p -> q) -> ((¬p -> ¬q) -> p))                   MP 26, 25
28: (((¬p -> q) -> (¬p -> ¬(¬p -> ¬q))) -> ((¬p -> q) -> ((¬p -> ¬q) -> p))) -> ((¬p -> ¬q) -> (((¬p -> q) -> (¬p -> ¬(¬p -> ¬q))) -> ((¬p -> q) -> ((¬p -
28: > ¬q) -> p))))                                                                           [L1]
29: (¬p -> ¬q) -> (((¬p -> q) -> (¬p -> ¬(¬p -> ¬q))) -> ((¬p -> q) -> ((¬p -> ¬q) -> p)))   MP 28, 27
30: ((¬p -> ¬q) -> (((¬p -> q) -> (¬p -> ¬(¬p -> ¬q))) -> ((¬p -> q) -> ((¬p -> ¬q) -> p)))) -> (((¬p -> ¬q) -> ((¬p -> q) -> (¬p -> ¬(¬p -> ¬q)))) -> ((¬
30: p -> ¬q) -> ((¬p -> q) -> ((¬p -> ¬q) -> p))))                                           [L2]
31: ((¬p -> ¬q) -> ((¬p -> q) -> (¬p -> ¬(¬p -> ¬q)))) -> ((¬p -> ¬q) -> ((¬p -> q) -> ((¬p -> ¬q) -> p)))      MP 30, 29
32: (¬p -> ¬q) -> ((¬p -> q) -> ((¬p -> ¬q) -> p))                                           MP 31, 22
33: ((¬p -> q) -> ((¬p -> ¬q) -> p)) -> (((¬p -> q) -> (¬p -> ¬q)) -> ((¬p -> q) -> p))      [L2]
34: (((¬p -> q) -> ((¬p -> ¬q) -> p)) -> (((¬p -> q) -> (¬p -> ¬q)) -> ((¬p -> q) -> p))) -> ((¬p -> ¬q) -> (((¬p -> q) -> ((¬p -> ¬q) -> p)) -> (((¬p ->
34: q) -> (¬p -> ¬q)) -> ((¬p -> q) -> p))))                                                 [L1]
35: (¬p -> ¬q) -> (((¬p -> q) -> ((¬p -> ¬q) -> p)) -> (((¬p -> q) -> (¬p -> ¬q)) -> ((¬p -> q) -> p)))      MP 34, 33
36: ((¬p -> ¬q) -> (((¬p -> q) -> ((¬p -> ¬q) -> p)) -> (((¬p -> q) -> (¬p -> ¬q)) -> ((¬p -> q) -> p)))) -> (((¬p -> ¬q) -> ((¬p -> q) -> ((¬p -> ¬q) ->
36: p))) -> ((¬p -> ¬q) -> (((¬p -> q) -> (¬p -> ¬q)) -> ((¬p -> q) -> p))))                 [L2]
37: ((¬p -> ¬q) -> ((¬p -> q) -> ((¬p -> ¬q) -> p))) -> ((¬p -> ¬q) -> (((¬p -> q) -> (¬p -> ¬q)) -> ((¬p -> q) -> p)))      MP 36, 35
38: (¬p -> ¬q) -> (((¬p -> q) -> (¬p -> ¬q)) -> ((¬p -> q) -> p))                            MP 37, 32
39: ((¬p -> ¬q) -> (((¬p -> q) -> (¬p -> ¬q)) -> ((¬p -> q) -> p))) -> (((¬p -> ¬q) -> ((¬p -> q) -> (¬p -> ¬q))) -> ((¬p -> ¬q) -> ((¬p -> q) -> p)))      [L2]
40: ((¬p -> ¬q) -> ((¬p -> q) -> (¬p -> ¬q))) -> ((¬p -> ¬q) -> ((¬p -> q) -> p))            MP 39, 38
41: (¬p -> ¬q) -> ((¬p -> q) -> p)                                                           MP 40, 1
```

prove List() ⊢ (p → ¬q) → (q → ¬p)

```
 1: ¬¬p -> (<3b> -> ¬¬p)                                                                     [L1]
 2: ¬¬p -> (¬¬(<3b> -> ¬¬p) -> ¬¬p)                                                          [L1]
 3: (¬¬(<3b> -> ¬¬p) -> ¬¬p) -> (¬p -> ¬(<3b> -> ¬¬p))                                       [L3]
 4: ((¬¬(<3b> -> ¬¬p) -> ¬¬p) -> (¬p -> ¬(<3b> -> ¬¬p))) -> (¬¬p -> ((¬¬(<3b> -> ¬¬p) -> ¬¬p) -> (¬p -> ¬(<3b> -> ¬¬p))))      [L1]
 5: ¬¬p -> ((¬¬(<3b> -> ¬¬p) -> ¬¬p) -> (¬p -> ¬(<3b> -> ¬¬p)))                              MP 4, 3
 6: (¬¬p -> ((¬¬(<3b> -> ¬¬p) -> ¬¬p) -> (¬p -> ¬(<3b> -> ¬¬p)))) -> ((¬¬p -> (¬¬(<3b> -> ¬¬p) -> ¬¬p)) -> (¬¬p -> (¬p -> ¬(<3b> -> ¬¬p))))      [L2]
 7: (¬¬p -> (¬¬(<3b> -> ¬¬p) -> ¬¬p)) -> (¬¬p -> (¬p -> ¬(<3b> -> ¬¬p)))                     MP 6, 5
 8: ¬¬p -> (¬p -> ¬(<3b> -> ¬¬p))                                                            MP 7, 2
 9: (¬p -> ¬(<3b> -> ¬¬p)) -> ((<3b> -> ¬¬p) -> p)                                           [L3]
10: ((¬p -> ¬(<3b> -> ¬¬p)) -> ((<3b> -> ¬¬p) -> p)) -> (¬¬p -> ((¬p -> ¬(<3b> -> ¬¬p)) -> ((<3b> -> ¬¬p) -> p)))      [L1]
11: ¬¬p -> ((¬p -> ¬(<3b> -> ¬¬p)) -> ((<3b> -> ¬¬p) -> p))                                  MP 10, 9
12: (¬¬p -> ((¬p -> ¬(<3b> -> ¬¬p)) -> ((<3b> -> ¬¬p) -> p))) -> ((¬¬p -> (¬p -> ¬(<3b> -> ¬¬p))) -> (¬¬p -> ((<3b> -> ¬¬p) -> p)))      [L2]
13: (¬¬p -> (¬p -> ¬(<3b> -> ¬¬p))) -> (¬¬p -> ((<3b> -> ¬¬p) -> p))                         MP 12, 11
14: ¬¬p -> ((<3b> -> ¬¬p) -> p)                                                              MP 13, 8
15: (¬¬p -> ((<3b> -> ¬¬p) -> p)) -> ((¬¬p -> (<3b> -> ¬¬p)) -> (¬¬p -> p))                  [L2]
16: (¬¬p -> (<3b> -> ¬¬p)) -> (¬¬p -> p)                                                     MP 15, 14
17: ¬¬p -> p                                                                                 MP 16, 1
18: (¬¬p -> p) -> ((p -> ¬q) -> (¬¬p -> p))                                                  [L1]
19: (p -> ¬q) -> (¬¬p -> p)                                                                  MP 18, 17
20: (p -> ¬q) -> (¬¬p -> (p -> ¬q))                                                          [L1]
21: (¬¬p -> (p -> ¬q)) -> ((¬¬p -> p) -> (¬¬p -> ¬q))                                        [L2]
22: ((¬¬p -> (p -> ¬q)) -> ((¬¬p -> p) -> (¬¬p -> ¬q))) -> ((p -> ¬q) -> ((¬¬p -> (p -> ¬q)) -> ((¬¬p -> p) -> (¬¬p -> ¬q))))      [L1]
23: (p -> ¬q) -> ((¬¬p -> (p -> ¬q)) -> ((¬¬p -> p) -> (¬¬p -> ¬q)))                         MP 22, 21
24: ((p -> ¬q) -> ((¬¬p -> (p -> ¬q)) -> ((¬¬p -> p) -> (¬¬p -> ¬q)))) -> (((p -> ¬q) -> (¬¬p -> (p -> ¬q))) -> ((p -> ¬q) -> ((¬¬p -> p) -> (¬¬p -> ¬q)))
24: )                                                                                        [L2]
25: ((p -> ¬q) -> (¬¬p -> (p -> ¬q))) -> ((p -> ¬q) -> ((¬¬p -> p) -> (¬¬p -> ¬q)))          MP 24, 23
26: (p -> ¬q) -> ((¬¬p -> p) -> (¬¬p -> ¬q))                                                 MP 25, 20
27: ((p -> ¬q) -> ((¬¬p -> p) -> (¬¬p -> ¬q))) -> (((p -> ¬q) -> (¬¬p -> p)) -> ((p -> ¬q) -> (¬¬p -> ¬q)))      [L2]
28: ((p -> ¬q) -> (¬¬p -> p)) -> ((p -> ¬q) -> (¬¬p -> ¬q))                                  MP 27, 26
29: (p -> ¬q) -> (¬¬p -> ¬q)                                                                 MP 28, 19
30: (¬¬p -> ¬q) -> (q -> ¬p)                                                                 [L3]
31: ((¬¬p -> ¬q) -> (q -> ¬p)) -> ((p -> ¬q) -> ((¬¬p -> ¬q) -> (q -> ¬p)))                  [L1]
32: (p -> ¬q) -> ((¬¬p -> ¬q) -> (q -> ¬p))                                                  MP 31, 30
33: ((p -> ¬q) -> ((¬¬p -> ¬q) -> (q -> ¬p))) -> (((p -> ¬q) -> (¬¬p -> ¬q)) -> ((p -> ¬q) -> (q -> ¬p)))      [L2]
34: ((p -> ¬q) -> (¬¬p -> ¬q)) -> ((p -> ¬q) -> (q -> ¬p))                                   MP 33, 32
35: (p -> ¬q) -> (q -> ¬p)                                                                   MP 34, 29
```

prove List() ⊢ (¬p → q) → (¬q → p)

```
 1: ¬¬¬q -> (<30> -> ¬¬¬q)                                                                   [L1]
 2: ¬¬¬q -> (¬¬(<30> -> ¬¬¬q) -> ¬¬¬q)                                                       [L1]
 3: (¬¬(<30> -> ¬¬¬q) -> ¬¬¬q) -> (¬¬q -> ¬(<30> -> ¬¬¬q))                                   [L3]
 4: ((¬¬(<30> -> ¬¬¬q) -> ¬¬¬q) -> (¬¬q -> ¬(<30> -> ¬¬¬q))) -> (¬¬¬q -> ((¬¬(<30> -> ¬¬¬q) -> ¬¬¬q) -> (¬¬q -> ¬(<30> -> ¬¬¬q))))      [L1]
 5: ¬¬¬q -> ((¬¬(<30> -> ¬¬¬q) -> ¬¬¬q) -> (¬¬q -> ¬(<30> -> ¬¬¬q)))                         MP 4, 3
 6: (¬¬¬q -> ((¬¬(<30> -> ¬¬¬q) -> ¬¬¬q) -> (¬¬q -> ¬(<30> -> ¬¬¬q)))) -> ((¬¬¬q -> (¬¬(<30> -> ¬¬¬q) -> ¬¬¬q)) -> (¬¬¬q -> (¬¬q -> ¬(<30> -> ¬¬¬q))))      [L2]
 7: (¬¬¬q -> (¬¬(<30> -> ¬¬¬q) -> ¬¬¬q)) -> (¬¬¬q -> (¬¬q -> ¬(<30> -> ¬¬¬q)))               MP 6, 5
 8: ¬¬¬q -> (¬¬q -> ¬(<30> -> ¬¬¬q))                                                         MP 7, 2
 9: (¬¬q -> ¬(<30> -> ¬¬¬q)) -> ((<30> -> ¬¬¬q) -> ¬q)                                       [L3]
10: ((¬¬q -> ¬(<30> -> ¬¬¬q)) -> ((<30> -> ¬¬¬q) -> ¬q)) -> (¬¬¬q -> ((¬¬q -> ¬(<30> -> ¬¬¬q)) -> ((<30> -> ¬¬¬q) -> ¬q)))      [L1]
11: ¬¬¬q -> ((¬¬q -> ¬(<30> -> ¬¬¬q)) -> ((<30> -> ¬¬¬q) -> ¬q))                            MP 10, 9
12: (¬¬¬q -> ((¬¬q -> ¬(<30> -> ¬¬¬q)) -> ((<30> -> ¬¬¬q) -> ¬q))) -> ((¬¬¬q -> (¬¬q -> ¬(<30> -> ¬¬¬q))) -> (¬¬¬q -> ((<30> -> ¬¬¬q) -> ¬q)))      [L2]
13: (¬¬¬q -> (¬¬q -> ¬(<30> -> ¬¬¬q))) -> (¬¬¬q -> ((<30> -> ¬¬¬q) -> ¬q))                  MP 12, 11
14: ¬¬¬q -> ((<30> -> ¬¬¬q) -> ¬q)                                                           MP 13, 8
15: (¬¬¬q -> ((<30> -> ¬¬¬q) -> ¬q)) -> ((¬¬¬q -> (<30> -> ¬¬¬q)) -> (¬¬¬q -> ¬q))          [L2]
16: (¬¬¬q -> (<30> -> ¬¬¬q)) -> (¬¬¬q -> ¬q)                                                 MP 15, 14
17: ¬¬¬q -> ¬q                                                                               MP 16, 1
18: (¬¬¬q -> ¬q) -> (q -> ¬¬q)                                                               [L3]
19: q -> ¬¬q                                                                                 MP 18, 17
20: (q -> ¬¬q) -> (¬p -> (q -> ¬¬q))                                                         [L1]
21: ¬p -> (q -> ¬¬q)                                                                         MP 20, 19
22: (¬p -> (q -> ¬¬q)) -> ((¬p -> q) -> (¬p -> ¬¬q))                                         [L2]
23: (¬p -> q) -> (¬p -> ¬¬q)                                                                 MP 22, 21
24: (¬p -> ¬¬q) -> (¬q -> p)                                                                 [L3]
25: ((¬p -> ¬¬q) -> (¬q -> p)) -> ((¬p -> q) -> ((¬p -> ¬¬q) -> (¬q -> p)))                  [L1]
26: (¬p -> q) -> ((¬p -> ¬¬q) -> (¬q -> p))                                                  MP 25, 24
27: ((¬p -> q) -> ((¬p -> ¬¬q) -> (¬q -> p))) -> (((¬p -> q) -> (¬p -> ¬¬q)) -> ((¬p -> q) -> (¬q -> p)))      [L2]
28: ((¬p -> q) -> (¬p -> ¬¬q)) -> ((¬p -> q) -> (¬q -> p))                                   MP 27, 26
29: (¬p -> q) -> (¬q -> p)                                                                   MP 28, 23
```

--------------- 析取运算 ---------------

prove List() ⊢ p → (¬p → q)

```
 1: p -> (¬p -> p)                                                        [L1]
 2: ¬p -> (¬q -> ¬p)                                                      [L1]
 3: (¬q -> ¬p) -> (p -> q)                                                [L3]
 4: ((¬q -> ¬p) -> (p -> q)) -> (¬p -> ((¬q -> ¬p) -> (p -> q)))          [L1]
 5: ¬p -> ((¬q -> ¬p) -> (p -> q))                                        MP 4, 3
 6: (¬p -> ((¬q -> ¬p) -> (p -> q))) -> ((¬p -> (¬q -> ¬p)) -> (¬p -> (p -> q)))   [L2]
 7: (¬p -> (¬q -> ¬p)) -> (¬p -> (p -> q))                                MP 6, 5
 8: ¬p -> (p -> q)                                                        MP 7, 2
 9: (¬p -> (p -> q)) -> ((¬p -> p) -> (¬p -> q))                          [L2]
10: (¬p -> p) -> (¬p -> q)                                                MP 9, 8
11: ((¬p -> p) -> (¬p -> q)) -> (p -> ((¬p -> p) -> (¬p -> q)))           [L1]
12: p -> ((¬p -> p) -> (¬p -> q))                                         MP 11, 10
13: (p -> ((¬p -> p) -> (¬p -> q))) -> ((p -> (¬p -> p)) -> (p -> (¬p -> q)))   [L2]
14: (p -> (¬p -> p)) -> (p -> (¬p -> q))                                  MP 13, 12
15: p -> (¬p -> q)                                                        MP 14, 1
```

prove List() ⊢ q → (¬p → q)

```
1: q -> (¬p -> q)                                                         [L1]
```

prove List() ⊢ (¬p → q) → (¬q → p)

```
 1: ¬¬¬q -> (<30> -> ¬¬¬q)                                                [L1]
 2: ¬¬¬q -> (¬¬(<30> -> ¬¬¬q) -> ¬¬¬q)                                    [L1]
 3: (¬¬(<30> -> ¬¬¬q) -> ¬¬¬q) -> (¬¬q -> ¬(<30> -> ¬¬¬q))                [L3]
 4: ((¬¬(<30> -> ¬¬¬q) -> ¬¬¬q) -> (¬¬q -> ¬(<30> -> ¬¬¬q))) -> (¬¬¬q -> ((¬¬(<30> -> ¬¬¬q) -> ¬¬¬q) -> (¬¬q -> ¬(<30> -> ¬¬¬q))))   [L1]
 5: ¬¬¬q -> ((¬¬(<30> -> ¬¬¬q) -> ¬¬¬q) -> (¬¬q -> ¬(<30> -> ¬¬¬q)))     MP 4, 3
 6: (¬¬¬q -> ((¬¬(<30> -> ¬¬¬q) -> ¬¬¬q) -> (¬¬q -> ¬(<30> -> ¬¬¬q)))) -> ((¬¬¬q -> (¬¬(<30> -> ¬¬¬q) -> ¬¬¬q)) -> (¬¬¬q -> (¬¬q -> ¬(<30> -> ¬¬¬q))))   [L2]
 7: (¬¬¬q -> (¬¬(<30> -> ¬¬¬q) -> ¬¬¬q)) -> (¬¬¬q -> (¬¬q -> ¬(<30> -> ¬¬¬q)))   MP 6, 5
 8: ¬¬¬q -> (¬¬q -> ¬(<30> -> ¬¬¬q))                                      MP 7, 2
 9: (¬¬q -> ¬(<30> -> ¬¬¬q)) -> ((<30> -> ¬¬¬q) -> ¬q)                    [L3]
10: ((¬¬q -> ¬(<30> -> ¬¬¬q)) -> ((<30> -> ¬¬¬q) -> ¬q)) -> (¬¬¬q -> ((¬¬q -> ¬(<30> -> ¬¬¬q)) -> ((<30> -> ¬¬¬q) -> ¬q)))   [L1]
11: ¬¬¬q -> ((¬¬q -> ¬(<30> -> ¬¬¬q)) -> ((<30> -> ¬¬¬q) -> ¬q))         MP 10, 9
12: (¬¬¬q -> ((¬¬q -> ¬(<30> -> ¬¬¬q)) -> ((<30> -> ¬¬¬q) -> ¬q))) -> ((¬¬¬q -> (¬¬q -> ¬(<30> -> ¬¬¬q))) -> (¬¬¬q -> ((<30> -> ¬¬¬q) -> ¬q)))   [L2]
13: (¬¬¬q -> (¬¬q -> ¬(<30> -> ¬¬¬q))) -> (¬¬¬q -> ((<30> -> ¬¬¬q) -> ¬q))   MP 12, 11
14: ¬¬¬q -> ((<30> -> ¬¬¬q) -> ¬q)                                        MP 13, 8
15: (¬¬¬q -> ((<30> -> ¬¬¬q) -> ¬q)) -> ((¬¬¬q -> (<30> -> ¬¬¬q)) -> (¬¬¬q -> ¬q))   [L2]
16: (¬¬¬q -> (<30> -> ¬¬¬q)) -> (¬¬¬q -> ¬q)                              MP 15, 14
17: ¬¬¬q -> ¬q                                                            MP 16, 1
18: (¬¬¬q -> ¬q) -> (q -> ¬¬q)                                            [L3]
19: q -> ¬¬q                                                              MP 18, 17
20: (q -> ¬¬q) -> (¬p -> (q -> ¬¬q))                                      [L1]
21: ¬p -> (q -> ¬¬q)                                                      MP 20, 19
22: (¬p -> (q -> ¬¬q)) -> ((¬p -> q) -> (¬p -> ¬¬q))                      [L2]
23: (¬p -> q) -> (¬p -> ¬¬q)                                              MP 22, 21
24: (¬p -> ¬¬q) -> (¬q -> p)                                              [L3]
25: ((¬p -> ¬¬q) -> (¬q -> p)) -> ((¬p -> q) -> ((¬p -> ¬¬q) -> (¬q -> p)))   [L1]
26: (¬p -> q) -> ((¬p -> ¬¬q) -> (¬q -> p))                               MP 25, 24
27: ((¬p -> q) -> ((¬p -> ¬¬q) -> (¬q -> p))) -> (((¬p -> q) -> (¬p -> ¬¬q)) -> ((¬p -> q) -> (¬q -> p)))   [L2]
28: ((¬p -> q) -> (¬p -> ¬¬q)) -> ((¬p -> q) -> (¬q -> p))                MP 27, 26
29: (¬p -> q) -> (¬q -> p)                                                MP 28, 23
```

prove List() ⊢ (¬p → p) → p

```
 1: (¬p -> p) -> (<20> -> (¬p -> p))                                      [L1]
 2: ¬p -> (¬¬(<20> -> (¬p -> p)) -> ¬p)                                   [L1]
 3: (¬¬(<20> -> (¬p -> p)) -> ¬p) -> (p -> ¬(<20> -> (¬p -> p)))          [L3]
 4: ((¬¬(<20> -> (¬p -> p)) -> ¬p) -> (p -> ¬(<20> -> (¬p -> p)))) -> (¬p -> ((¬¬(<20> -> (¬p -> p)) -> ¬p) -> (p -> ¬(<20> -> (¬p -> p)))))   [L1]
 5: ¬p -> ((¬¬(<20> -> (¬p -> p)) -> ¬p) -> (p -> ¬(<20> -> (¬p -> p))))  MP 4, 3
 6: (¬p -> ((¬¬(<20> -> (¬p -> p)) -> ¬p) -> (p -> ¬(<20> -> (¬p -> p))))) -> ((¬p -> (¬¬(<20> -> (¬p -> p)) -> ¬p)) -> (¬p -> (p -> ¬(<20> -> (¬p -> p)))
 6: ))                                                                    [L2]
 7: (¬p -> (¬¬(<20> -> (¬p -> p)) -> ¬p)) -> (¬p -> (p -> ¬(<20> -> (¬p -> p))))   MP 6, 5
 8: ¬p -> (p -> ¬(<20> -> (¬p -> p)))                                     MP 7, 2
 9: (¬p -> (p -> ¬(<20> -> (¬p -> p)))) -> ((¬p -> p) -> (¬p -> ¬(<20> -> (¬p -> p))))   [L2]
10: (¬p -> p) -> (¬p -> ¬(<20> -> (¬p -> p)))                             MP 9, 8
11: (¬p -> ¬(<20> -> (¬p -> p))) -> ((<20> -> (¬p -> p)) -> p)            [L3]
12: ((¬p -> ¬(<20> -> (¬p -> p))) -> ((<20> -> (¬p -> p)) -> p)) -> ((¬p -> p) -> ((¬p -> ¬(<20> -> (¬p -> p))) -> ((<20> -> (¬p -> p)) -> p)))   [L1]
13: (¬p -> p) -> ((¬p -> ¬(<20> -> (¬p -> p))) -> ((<20> -> (¬p -> p)) -> p))   MP 12, 11
14: ((¬p -> p) -> ((¬p -> ¬(<20> -> (¬p -> p))) -> ((<20> -> (¬p -> p)) -> p))) -> (((¬p -> p) -> (¬p -> ¬(<20> -> (¬p -> p)))) -> ((¬p -> p) -> ((<20> ->
14:  (¬p -> p)) -> p)))                                                   [L2]
15: ((¬p -> p) -> (¬p -> ¬(<20> -> (¬p -> p)))) -> ((¬p -> p) -> ((<20> -> (¬p -> p)) -> p))   MP 14, 13
16: (¬p -> p) -> ((<20> -> (¬p -> p)) -> p)                               MP 15, 10
17: ((¬p -> p) -> ((<20> -> (¬p -> p)) -> p)) -> (((¬p -> p) -> (<20> -> (¬p -> p))) -> ((¬p -> p) -> p))   [L2]
18: ((¬p -> p) -> (<20> -> (¬p -> p))) -> ((¬p -> p) -> p)                MP 17, 16
19: (¬p -> p) -> p                                                        MP 18, 1
```

prove List() ⊢ ¬¬p → p

```
 1: ¬¬p -> (<1c> -> ¬¬p)                                                  [L1]
 2: ¬¬p -> (¬¬(<1c> -> ¬¬p) -> ¬¬p)                                       [L1]
 3: (¬¬(<1c> -> ¬¬p) -> ¬¬p) -> (¬p -> ¬(<1c> -> ¬¬p))                    [L3]
 4: ((¬¬(<1c> -> ¬¬p) -> ¬¬p) -> (¬p -> ¬(<1c> -> ¬¬p))) -> (¬¬p -> ((¬¬(<1c> -> ¬¬p) -> ¬¬p) -> (¬p -> ¬(<1c> -> ¬¬p))))   [L1]
 5: ¬¬p -> ((¬¬(<1c> -> ¬¬p) -> ¬¬p) -> (¬p -> ¬(<1c> -> ¬¬p)))          MP 4, 3
 6: (¬¬p -> ((¬¬(<1c> -> ¬¬p) -> ¬¬p) -> (¬p -> ¬(<1c> -> ¬¬p)))) -> ((¬¬p -> (¬¬(<1c> -> ¬¬p) -> ¬¬p)) -> (¬¬p -> (¬p -> ¬(<1c> -> ¬¬p))))   [L2]
 7: (¬¬p -> (¬¬(<1c> -> ¬¬p) -> ¬¬p)) -> (¬¬p -> (¬p -> ¬(<1c> -> ¬¬p)))  MP 6, 5
 8: ¬¬p -> (¬p -> ¬(<1c> -> ¬¬p))                                         MP 7, 2
 9: (¬p -> ¬(<1c> -> ¬¬p)) -> ((<1c> -> ¬¬p) -> p)                        [L3]
10: ((¬p -> ¬(<1c> -> ¬¬p)) -> ((<1c> -> ¬¬p) -> p)) -> (¬¬p -> ((¬p -> ¬(<1c> -> ¬¬p)) -> ((<1c> -> ¬¬p) -> p)))   [L1]
11: ¬¬p -> ((¬p -> ¬(<1c> -> ¬¬p)) -> ((<1c> -> ¬¬p) -> p))              MP 10, 9
12: (¬¬p -> ((¬p -> ¬(<1c> -> ¬¬p)) -> ((<1c> -> ¬¬p) -> p))) -> ((¬¬p -> (¬p -> ¬(<1c> -> ¬¬p))) -> (¬¬p -> ((<1c> -> ¬¬p) -> p)))   [L2]
13: (¬¬p -> (¬p -> ¬(<1c> -> ¬¬p))) -> (¬¬p -> ((<1c> -> ¬¬p) -> p))     MP 12, 11
14: ¬¬p -> ((<1c> -> ¬¬p) -> p)                                           MP 13, 8
15: (¬¬p -> ((<1c> -> ¬¬p) -> p)) -> ((¬¬p -> (<1c> -> ¬¬p)) -> (¬¬p -> p))   [L2]
16: (¬¬p -> (<1c> -> ¬¬p)) -> (¬¬p -> p)                                  MP 15, 14
17: ¬¬p -> p                                                              MP 16, 1
```

--------------- 合取运算 ---------------

prove List() ⊢ ¬(p → ¬q) → p

```
1: ¬p -> (¬¬q -> ¬p)    [L1]
2: (¬¬q -> ¬p) -> (p -> ¬q)    [L3]
3: ((¬¬q -> ¬p) -> (p -> ¬q)) -> (¬p -> ((¬¬q -> ¬p) -> (p -> ¬q)))    [L1]
4: ¬p -> ((¬¬q -> ¬p) -> (p -> ¬q))    MP 3, 2
5: (¬p -> ((¬¬q -> ¬p) -> (p -> ¬q))) -> ((¬p -> (¬¬q -> ¬p)) -> (¬p -> (p -> ¬q)))    [L2]
6: (¬p -> (¬¬q -> ¬p)) -> (¬p -> (p -> ¬q))    MP 5, 4
7: ¬p -> (p -> ¬q)    MP 6, 1
8: (¬p -> (p -> ¬q)) -> (¬(p -> ¬q) -> (¬p -> (p -> ¬q)))    [L1]
9: ¬(p -> ¬q) -> (¬p -> (p -> ¬q))    MP 8, 7
10: ¬(p -> ¬q) -> (¬¬(¬p -> ¬p) -> ¬(p -> ¬q))    [L1]
11: (¬¬(¬p -> ¬p) -> ¬(p -> ¬q)) -> ((p -> ¬q) -> ¬(¬p -> ¬p))    [L3]
12: ((¬¬(¬p -> ¬p) -> ¬(p -> ¬q)) -> ((p -> ¬q) -> ¬(¬p -> ¬p))) -> (¬(p -> ¬q) -> ((¬¬(¬p -> ¬p) -> ¬(p -> ¬q)) -> ((p -> ¬q) -> ¬(¬p -> ¬p))))    [L1]
13: ¬(p -> ¬q) -> ((¬¬(¬p -> ¬p) -> ¬(p -> ¬q)) -> ((p -> ¬q) -> ¬(¬p -> ¬p)))    MP 12, 11
14: (¬(p -> ¬q) -> ((¬¬(¬p -> ¬p) -> ¬(p -> ¬q)) -> ((p -> ¬q) -> ¬(¬p -> ¬p)))) -> ((¬(p -> ¬q) -> (¬¬(¬p -> ¬p) -> ¬(p -> ¬q))) -> (¬(p -> ¬q) -> ((p ->
14:  ¬q) -> ¬(¬p -> ¬p))))    [L2]
15: (¬(p -> ¬q) -> (¬¬(¬p -> ¬p) -> ¬(p -> ¬q))) -> (¬(p -> ¬q) -> ((p -> ¬q) -> ¬(¬p -> ¬p)))    MP 14, 13
16: ¬(p -> ¬q) -> ((p -> ¬q) -> ¬(¬p -> ¬p))    MP 15, 10
17: ((p -> ¬q) -> ¬(¬p -> ¬p)) -> (¬p -> ((p -> ¬q) -> ¬(¬p -> ¬p)))    [L1]
18: (((p -> ¬q) -> ¬(¬p -> ¬p)) -> (¬p -> ((p -> ¬q) -> ¬(¬p -> ¬p)))) -> (¬(p -> ¬q) -> (((p -> ¬q) -> ¬(¬p -> ¬p)) -> (¬p -> ((p -> ¬q) -> ¬(¬p -> ¬p)))
18: ))    [L1]
19: ¬(p -> ¬q) -> (((p -> ¬q) -> ¬(¬p -> ¬p)) -> (¬p -> ((p -> ¬q) -> ¬(¬p -> ¬p))))    MP 18, 17
20: (¬(p -> ¬q) -> (((p -> ¬q) -> ¬(¬p -> ¬p)) -> (¬p -> ((p -> ¬q) -> ¬(¬p -> ¬p))))) -> ((¬(p -> ¬q) -> ((p -> ¬q) -> ¬(¬p -> ¬p))) -> (¬(p -> ¬q) -> (¬
20: p -> ((p -> ¬q) -> ¬(¬p -> ¬p)))))    [L2]
21: (¬(p -> ¬q) -> ((p -> ¬q) -> ¬(¬p -> ¬p))) -> (¬(p -> ¬q) -> (¬p -> ((p -> ¬q) -> ¬(¬p -> ¬p))))    MP 20, 19
22: ¬(p -> ¬q) -> (¬p -> ((p -> ¬q) -> ¬(¬p -> ¬p)))    MP 21, 16
23: (¬p -> ((p -> ¬q) -> ¬(¬p -> ¬p))) -> ((¬p -> (p -> ¬q)) -> (¬p -> ¬(¬p -> ¬p)))    [L2]
24: ((¬p -> ((p -> ¬q) -> ¬(¬p -> ¬p))) -> ((¬p -> (p -> ¬q)) -> (¬p -> ¬(¬p -> ¬p)))) -> (¬(p -> ¬q) -> ((¬p -> ((p -> ¬q) -> ¬(¬p -> ¬p))) -> ((¬p -> (p
24:  -> ¬q)) -> (¬p -> ¬(¬p -> ¬p)))))    [L1]
25: ¬(p -> ¬q) -> ((¬p -> ((p -> ¬q) -> ¬(¬p -> ¬p))) -> ((¬p -> (p -> ¬q)) -> (¬p -> ¬(¬p -> ¬p))))    MP 24, 23
26: (¬(p -> ¬q) -> ((¬p -> ((p -> ¬q) -> ¬(¬p -> ¬p))) -> ((¬p -> (p -> ¬q)) -> (¬p -> ¬(¬p -> ¬p))))) -> ((¬(p -> ¬q) -> (¬p -> ((p -> ¬q) -> ¬(¬p -> ¬p)
26: ))) -> (¬(p -> ¬q) -> ((¬p -> (p -> ¬q)) -> (¬p -> ¬(¬p -> ¬p)))))    [L2]
27: (¬(p -> ¬q) -> (¬p -> ((p -> ¬q) -> ¬(¬p -> ¬p)))) -> (¬(p -> ¬q) -> ((¬p -> (p -> ¬q)) -> (¬p -> ¬(¬p -> ¬p))))    MP 26, 25
28: ¬(p -> ¬q) -> ((¬p -> (p -> ¬q)) -> (¬p -> ¬(¬p -> ¬p)))    MP 27, 22
29: (¬(p -> ¬q) -> ((¬p -> (p -> ¬q)) -> (¬p -> ¬(¬p -> ¬p)))) -> ((¬(p -> ¬q) -> (¬p -> (p -> ¬q))) -> (¬(p -> ¬q) -> (¬p -> ¬(¬p -> ¬p))))    [L2]
30: (¬(p -> ¬q) -> (¬p -> (p -> ¬q))) -> (¬(p -> ¬q) -> (¬p -> ¬(¬p -> ¬p)))    MP 29, 28
31: ¬(p -> ¬q) -> (¬p -> ¬(¬p -> ¬p))    MP 30, 9
32: ¬p -> (¬(¬p -> ¬p) -> ¬p)    [L1]
33: (¬p -> (¬(¬p -> ¬p) -> ¬p)) -> ((¬p -> ¬(¬p -> ¬p)) -> (¬p -> ¬p))    [L2]
34: (¬p -> ¬(¬p -> ¬p)) -> (¬p -> ¬p)    MP 33, 32
35: (¬p -> ¬(¬p -> ¬p)) -> ((¬p -> ¬p) -> p)    [L3]
36: ((¬p -> ¬(¬p -> ¬p)) -> ((¬p -> ¬p) -> p)) -> (((¬p -> ¬(¬p -> ¬p)) -> (¬p -> ¬p)) -> ((¬p -> ¬(¬p -> ¬p)) -> p))    [L2]
37: ((¬p -> ¬(¬p -> ¬p)) -> (¬p -> ¬p)) -> ((¬p -> ¬(¬p -> ¬p)) -> p)    MP 36, 35
38: (¬p -> ¬(¬p -> ¬p)) -> p    MP 37, 34
39: ((¬p -> ¬(¬p -> ¬p)) -> p) -> (¬(p -> ¬q) -> ((¬p -> ¬(¬p -> ¬p)) -> p))    [L1]
40: ¬(p -> ¬q) -> ((¬p -> ¬(¬p -> ¬p)) -> p)    MP 39, 38
41: (¬(p -> ¬q) -> ((¬p -> ¬(¬p -> ¬p)) -> p)) -> ((¬(p -> ¬q) -> (¬p -> ¬(¬p -> ¬p))) -> (¬(p -> ¬q) -> p))    [L2]
42: (¬(p -> ¬q) -> (¬p -> ¬(¬p -> ¬p))) -> (¬(p -> ¬q) -> p)    MP 41, 40
43: ¬(p -> ¬q) -> p    MP 42, 31
```

prove List() ⊢ ¬(p → ¬q) → q

```
1: ¬q -> (p -> ¬q)    [L1]
2: (¬q -> (p -> ¬q)) -> (¬(p -> ¬q) -> (¬q -> (p -> ¬q)))    [L1]
3: ¬(p -> ¬q) -> (¬q -> (p -> ¬q))    MP 2, 1
4: ¬(p -> ¬q) -> (¬¬(¬q -> ¬q) -> ¬(p -> ¬q))    [L1]
5: (¬¬(¬q -> ¬q) -> ¬(p -> ¬q)) -> ((p -> ¬q) -> ¬(¬q -> ¬q))    [L3]
6: ((¬¬(¬q -> ¬q) -> ¬(p -> ¬q)) -> ((p -> ¬q) -> ¬(¬q -> ¬q))) -> (¬(p -> ¬q) -> ((¬¬(¬q -> ¬q) -> ¬(p -> ¬q)) -> ((p -> ¬q) -> ¬(¬q -> ¬q))))    [L1]
7: ¬(p -> ¬q) -> ((¬¬(¬q -> ¬q) -> ¬(p -> ¬q)) -> ((p -> ¬q) -> ¬(¬q -> ¬q)))    MP 6, 5
8: (¬(p -> ¬q) -> ((¬¬(¬q -> ¬q) -> ¬(p -> ¬q)) -> ((p -> ¬q) -> ¬(¬q -> ¬q)))) -> ((¬(p -> ¬q) -> (¬¬(¬q -> ¬q) -> ¬(p -> ¬q))) -> (¬(p -> ¬q) -> ((p ->
8:  ¬q) -> ¬(¬q -> ¬q))))    [L2]
9: (¬(p -> ¬q) -> (¬¬(¬q -> ¬q) -> ¬(p -> ¬q))) -> (¬(p -> ¬q) -> ((p -> ¬q) -> ¬(¬q -> ¬q)))    MP 8, 7
10: ¬(p -> ¬q) -> ((p -> ¬q) -> ¬(¬q -> ¬q))    MP 9, 4
11: ((p -> ¬q) -> ¬(¬q -> ¬q)) -> (¬q -> ((p -> ¬q) -> ¬(¬q -> ¬q)))    [L1]
12: (((p -> ¬q) -> ¬(¬q -> ¬q)) -> (¬q -> ((p -> ¬q) -> ¬(¬q -> ¬q)))) -> (¬(p -> ¬q) -> (((p -> ¬q) -> ¬(¬q -> ¬q)) -> (¬q -> ((p -> ¬q) -> ¬(¬q -> ¬q)))
12: ))    [L1]
13: ¬(p -> ¬q) -> (((p -> ¬q) -> ¬(¬q -> ¬q)) -> (¬q -> ((p -> ¬q) -> ¬(¬q -> ¬q))))    MP 12, 11
14: (¬(p -> ¬q) -> (((p -> ¬q) -> ¬(¬q -> ¬q)) -> (¬q -> ((p -> ¬q) -> ¬(¬q -> ¬q))))) -> ((¬(p -> ¬q) -> ((p -> ¬q) -> ¬(¬q -> ¬q))) -> (¬(p -> ¬q) -> (¬
14: q -> ((p -> ¬q) -> ¬(¬q -> ¬q)))))    [L2]
15: (¬(p -> ¬q) -> ((p -> ¬q) -> ¬(¬q -> ¬q))) -> (¬(p -> ¬q) -> (¬q -> ((p -> ¬q) -> ¬(¬q -> ¬q))))    MP 14, 13
16: ¬(p -> ¬q) -> (¬q -> ((p -> ¬q) -> ¬(¬q -> ¬q)))    MP 15, 10
17: (¬q -> ((p -> ¬q) -> ¬(¬q -> ¬q))) -> ((¬q -> (p -> ¬q)) -> (¬q -> ¬(¬q -> ¬q)))    [L2]
18: ((¬q -> ((p -> ¬q) -> ¬(¬q -> ¬q))) -> ((¬q -> (p -> ¬q)) -> (¬q -> ¬(¬q -> ¬q)))) -> (¬(p -> ¬q) -> ((¬q -> ((p -> ¬q) -> ¬(¬q -> ¬q))) -> ((¬q -> (p
18:  -> ¬q)) -> (¬q -> ¬(¬q -> ¬q)))))    [L1]
19: ¬(p -> ¬q) -> ((¬q -> ((p -> ¬q) -> ¬(¬q -> ¬q))) -> ((¬q -> (p -> ¬q)) -> (¬q -> ¬(¬q -> ¬q))))    MP 18, 17
20: (¬(p -> ¬q) -> ((¬q -> ((p -> ¬q) -> ¬(¬q -> ¬q))) -> ((¬q -> (p -> ¬q)) -> (¬q -> ¬(¬q -> ¬q))))) -> ((¬(p -> ¬q) -> (¬q -> ((p -> ¬q) -> ¬(¬q -> ¬q)
20: ))) -> (¬(p -> ¬q) -> ((¬q -> (p -> ¬q)) -> (¬q -> ¬(¬q -> ¬q)))))    [L2]
21: (¬(p -> ¬q) -> (¬q -> ((p -> ¬q) -> ¬(¬q -> ¬q)))) -> (¬(p -> ¬q) -> ((¬q -> (p -> ¬q)) -> (¬q -> ¬(¬q -> ¬q))))    MP 20, 19
22: ¬(p -> ¬q) -> ((¬q -> (p -> ¬q)) -> (¬q -> ¬(¬q -> ¬q)))    MP 21, 16
23: (¬(p -> ¬q) -> ((¬q -> (p -> ¬q)) -> (¬q -> ¬(¬q -> ¬q)))) -> ((¬(p -> ¬q) -> (¬q -> (p -> ¬q))) -> (¬(p -> ¬q) -> (¬q -> ¬(¬q -> ¬q))))    [L2]
24: (¬(p -> ¬q) -> (¬q -> (p -> ¬q))) -> (¬(p -> ¬q) -> (¬q -> ¬(¬q -> ¬q)))    MP 23, 22
25: ¬(p -> ¬q) -> (¬q -> ¬(¬q -> ¬q))    MP 24, 3
26: ¬q -> (¬(¬q -> ¬q) -> ¬q)    [L1]
27: (¬q -> (¬(¬q -> ¬q) -> ¬q)) -> ((¬q -> ¬(¬q -> ¬q)) -> (¬q -> ¬q))    [L2]
28: (¬q -> ¬(¬q -> ¬q)) -> (¬q -> ¬q)    MP 27, 26
29: (¬q -> ¬(¬q -> ¬q)) -> ((¬q -> ¬q) -> q)    [L3]
30: ((¬q -> ¬(¬q -> ¬q)) -> ((¬q -> ¬q) -> q)) -> (((¬q -> ¬(¬q -> ¬q)) -> (¬q -> ¬q)) -> ((¬q -> ¬(¬q -> ¬q)) -> q))    [L2]
31: ((¬q -> ¬(¬q -> ¬q)) -> (¬q -> ¬q)) -> ((¬q -> ¬(¬q -> ¬q)) -> q)    MP 30, 29
32: (¬q -> ¬(¬q -> ¬q)) -> q    MP 31, 28
33: ((¬q -> ¬(¬q -> ¬q)) -> q) -> (¬(p -> ¬q) -> ((¬q -> ¬(¬q -> ¬q)) -> q))    [L1]
34: ¬(p -> ¬q) -> ((¬q -> ¬(¬q -> ¬q)) -> q)    MP 33, 32
35: (¬(p -> ¬q) -> ((¬q -> ¬(¬q -> ¬q)) -> q)) -> ((¬(p -> ¬q) -> (¬q -> ¬(¬q -> ¬q))) -> (¬(p -> ¬q) -> q))    [L2]
36: (¬(p -> ¬q) -> (¬q -> ¬(¬q -> ¬q))) -> (¬(p -> ¬q) -> q)    MP 35, 34
37: ¬(p -> ¬q) -> q    MP 36, 25
```

prove List() ⊢ ¬¬(p → ¬¬p)

```
   1: (¬<5e> -> ¬<5d>) -> (<5d> -> <5e>)                                                                                                                  [L3]
   2: ¬¬¬p -> (<5c> -> ¬¬¬p)                                                                                                                             [L1]
   3: ¬¬¬p -> (¬¬(<5c> -> ¬¬¬p) -> ¬¬¬p)                                                                                                                 [L1]
   4: (¬¬(<5c> -> ¬¬¬p) -> ¬¬¬p) -> (¬¬p -> ¬(<5c> -> ¬¬¬p))                                                                                             [L3]
   5: ((¬¬(<5c> -> ¬¬¬p) -> ¬¬¬p) -> (¬¬p -> ¬(<5c> -> ¬¬¬p))) -> (¬¬¬p -> ((¬¬(<5c> -> ¬¬¬p) -> ¬¬¬p) -> (¬¬p -> ¬(<5c> -> ¬¬¬p))))                   [L1]
   6: ¬¬¬p -> ((¬¬(<5c> -> ¬¬¬p) -> ¬¬¬p) -> (¬¬p -> ¬(<5c> -> ¬¬¬p)))                                                                                  MP 5, 4
   7: (¬¬¬p -> ((¬¬(<5c> -> ¬¬¬p) -> ¬¬¬p) -> (¬¬p -> ¬(<5c> -> ¬¬¬p)))) -> ((¬¬¬p -> (¬¬(<5c> -> ¬¬¬p) -> ¬¬¬p)) -> (¬¬¬p -> (¬¬p -> ¬(<5c> -> ¬¬¬p))))  [L2]
   8: (¬¬¬p -> (¬¬(<5c> -> ¬¬¬p) -> ¬¬¬p)) -> (¬¬¬p -> (¬¬p -> ¬(<5c> -> ¬¬¬p)))                                                                        MP 7, 6
   9: ¬¬¬p -> (¬¬p -> ¬(<5c> -> ¬¬¬p))                                                                                                                  MP 8, 3
  10: (¬¬p -> ¬(<5c> -> ¬¬¬p)) -> ((<5c> -> ¬¬¬p) -> ¬p)                                                                                                [L3]
  11: ((¬¬p -> ¬(<5c> -> ¬¬¬p)) -> ((<5c> -> ¬¬¬p) -> ¬p)) -> (¬¬¬p -> ((¬¬p -> ¬(<5c> -> ¬¬¬p)) -> ((<5c> -> ¬¬¬p) -> ¬p)))                           [L1]
  12: ¬¬¬p -> ((¬¬p -> ¬(<5c> -> ¬¬¬p)) -> ((<5c> -> ¬¬¬p) -> ¬p))                                                                                      MP 11, 10
  13: (¬¬¬p -> ((¬¬p -> ¬(<5c> -> ¬¬¬p)) -> ((<5c> -> ¬¬¬p) -> ¬p))) -> ((¬¬¬p -> (¬¬p -> ¬(<5c> -> ¬¬¬p))) -> (¬¬¬p -> ((<5c> -> ¬¬¬p) -> ¬p)))     [L2]
  14: (¬¬¬p -> (¬¬p -> ¬(<5c> -> ¬¬¬p))) -> (¬¬¬p -> ((<5c> -> ¬¬¬p) -> ¬p))                                                                           MP 13, 12
  15: ¬¬¬p -> ((<5c> -> ¬¬¬p) -> ¬p)                                                                                                                   MP 14, 9
  16: (¬¬¬p -> ((<5c> -> ¬¬¬p) -> ¬p)) -> ((¬¬¬p -> (<5c> -> ¬¬¬p)) -> (¬¬¬p -> ¬p))                                                                    [L2]
  17: (¬¬¬p -> (<5c> -> ¬¬¬p)) -> (¬¬¬p -> ¬p)                                                                                                         MP 16, 15
  18: ¬¬¬p -> ¬p                                                                                                                                      MP 17, 2
  19: (¬¬¬p -> ¬p) -> (p -> ¬¬p)                                                                                                                      [L3]
  20: p -> ¬¬p                                                                                                                                         MP 19, 18
  21: (p -> ¬¬p) -> (¬¬¬(p -> ¬¬p) -> (p -> ¬¬p))                                                                                                       [L1]
  22: ¬¬¬(p -> ¬¬p) -> (p -> ¬¬p)                                                                                                                      MP 21, 20
  23: ¬¬¬(p -> ¬¬p) -> (¬¬¬¬¬(p -> ¬¬p) -> ¬¬¬(p -> ¬¬p))                                                                                                [L1]
  24: (¬¬¬¬¬(p -> ¬¬p) -> ¬¬¬(p -> ¬¬p)) -> (¬¬(p -> ¬¬p) -> ¬¬¬¬(p -> ¬¬p))                                                                            [L3]
  25: ((¬¬¬¬¬(p -> ¬¬p) -> ¬¬¬(p -> ¬¬p)) -> (¬¬(p -> ¬¬p) -> ¬¬¬¬(p -> ¬¬p))) -> (¬¬¬(p -> ¬¬p) -> ((¬¬¬¬¬(p -> ¬¬p) -> ¬¬¬(p -> ¬¬p)) -> (¬¬(p -> ¬¬p) ->
  25: ¬¬¬¬(p -> ¬¬p))))                                                                                                                                [L1]
  26: ¬¬¬(p -> ¬¬p) -> ((¬¬¬¬¬(p -> ¬¬p) -> ¬¬¬(p -> ¬¬p)) -> (¬¬(p -> ¬¬p) -> ¬¬¬¬(p -> ¬¬p)))                                                          MP 25, 24
  27: (¬¬¬(p -> ¬¬p) -> ((¬¬¬¬¬(p -> ¬¬p) -> ¬¬¬(p -> ¬¬p)) -> (¬¬(p -> ¬¬p) -> ¬¬¬¬(p -> ¬¬p)))) -> ((¬¬¬(p -> ¬¬p) -> (¬¬¬¬¬(p -> ¬¬p) -> ¬¬¬(p -> ¬¬p)))
  27: -> (¬¬¬(p -> ¬¬p) -> (¬¬(p -> ¬¬p) -> ¬¬¬¬(p -> ¬¬p))))                                                                                          [L2]
  28: (¬¬¬(p -> ¬¬p) -> (¬¬¬¬¬(p -> ¬¬p) -> ¬¬¬(p -> ¬¬p))) -> (¬¬¬(p -> ¬¬p) -> (¬¬(p -> ¬¬p) -> ¬¬¬¬(p -> ¬¬p)))                                      MP 27, 26
  29: ¬¬¬(p -> ¬¬p) -> (¬¬(p -> ¬¬p) -> ¬¬¬¬(p -> ¬¬p))                                                                                                 MP 28, 23
  30: (¬¬(p -> ¬¬p) -> ¬¬¬¬(p -> ¬¬p)) -> (¬¬¬(p -> ¬¬p) -> ¬(p -> ¬¬p))                                                                              [L3]
  31: ((¬¬(p -> ¬¬p) -> ¬¬¬¬(p -> ¬¬p)) -> (¬¬¬(p -> ¬¬p) -> ¬(p -> ¬¬p))) -> (¬¬¬(p -> ¬¬p) -> ((¬¬(p -> ¬¬p) -> ¬¬¬¬(p -> ¬¬p)) -> (¬¬¬(p -> ¬¬p) -> ¬(p -
  31: > ¬¬p))))                                                                                                                                        [L1]
  32: ¬¬¬(p -> ¬¬p) -> ((¬¬(p -> ¬¬p) -> ¬¬¬¬(p -> ¬¬p)) -> (¬¬¬(p -> ¬¬p) -> ¬(p -> ¬¬p)))                                                              MP 31, 30
  33: (¬¬¬(p -> ¬¬p) -> ((¬¬(p -> ¬¬p) -> ¬¬¬¬(p -> ¬¬p)) -> (¬¬¬(p -> ¬¬p) -> ¬(p -> ¬¬p)))) -> ((¬¬¬(p -> ¬¬p) -> (¬¬(p -> ¬¬p) -> ¬¬¬¬(p -> ¬¬p))) -> (¬¬
  33: ¬(p -> ¬¬p) -> (¬¬¬(p -> ¬¬p) -> ¬(p -> ¬¬p))))                                                                                                   [L2]
  34: (¬¬¬(p -> ¬¬p) -> (¬¬(p -> ¬¬p) -> ¬¬¬¬(p -> ¬¬p))) -> (¬¬¬(p -> ¬¬p) -> (¬¬¬(p -> ¬¬p) -> ¬(p -> ¬¬p)))                                          MP 33, 32
  35: ¬¬¬(p -> ¬¬p) -> (¬¬¬(p -> ¬¬p) -> ¬(p -> ¬¬p))                                                                                                   MP 34, 29
  36: (¬¬¬(p -> ¬¬p) -> ¬(p -> ¬¬p)) -> ((p -> ¬¬p) -> ¬¬(p -> ¬¬p))                                                                                   [L3]
  37: ((¬¬¬(p -> ¬¬p) -> ¬(p -> ¬¬p)) -> ((p -> ¬¬p) -> ¬¬(p -> ¬¬p))) -> (¬¬¬(p -> ¬¬p) -> ((¬¬¬(p -> ¬¬p) -> ¬(p -> ¬¬p)) -> ((p -> ¬¬p) -> ¬¬(p -> ¬¬p)))
  37: )                                                                                                                                                [L1]
  38: ¬¬¬(p -> ¬¬p) -> ((¬¬¬(p -> ¬¬p) -> ¬(p -> ¬¬p)) -> ((p -> ¬¬p) -> ¬¬(p -> ¬¬p)))                                                                MP 37, 36
  39: (¬¬¬(p -> ¬¬p) -> ((¬¬¬(p -> ¬¬p) -> ¬(p -> ¬¬p)) -> ((p -> ¬¬p) -> ¬¬(p -> ¬¬p)))) -> ((¬¬¬(p -> ¬¬p) -> (¬¬¬(p -> ¬¬p) -> ¬(p -> ¬¬p))) -> (¬¬¬(p ->
  39:  ¬¬p) -> ((p -> ¬¬p) -> ¬¬(p -> ¬¬p))))                                                                                                            [L2]
  40: (¬¬¬(p -> ¬¬p) -> (¬¬¬(p -> ¬¬p) -> ¬(p -> ¬¬p))) -> (¬¬¬(p -> ¬¬p) -> ((p -> ¬¬p) -> ¬¬(p -> ¬¬p)))                                              MP 39, 38
  41: ¬¬¬(p -> ¬¬p) -> ((p -> ¬¬p) -> ¬¬(p -> ¬¬p))                                                                                                     MP 40, 35
  42: (¬¬¬(p -> ¬¬p) -> ((p -> ¬¬p) -> ¬¬(p -> ¬¬p))) -> ((¬¬¬(p -> ¬¬p) -> (p -> ¬¬p)) -> (¬¬¬(p -> ¬¬p) -> ¬¬(p -> ¬¬p)))                             [L2]
  43: (¬¬¬(p -> ¬¬p) -> (p -> ¬¬p)) -> (¬¬¬(p -> ¬¬p) -> ¬¬(p -> ¬¬p))                                                                                  MP 42, 41
  44: ¬¬¬(p -> ¬¬p) -> ¬¬(p -> ¬¬p)                                                                                                                     MP 43, 22
  45: ¬¬¬(p -> ¬¬p) -> (¬¬((¬<5e> -> ¬<5d>) -> (<5d> -> <5e>)) -> ¬¬¬(p -> ¬¬p))                                                                        [L1]
  46: (¬¬((¬<5e> -> ¬<5d>) -> (<5d> -> <5e>)) -> ¬¬¬(p -> ¬¬p)) -> (¬¬(p -> ¬¬p) -> ¬((¬<5e> -> ¬<5d>) -> (<5d> -> <5e>)))                              [L3]
  47: ((¬¬((¬<5e> -> ¬<5d>) -> (<5d> -> <5e>)) -> ¬¬¬(p -> ¬¬p)) -> (¬¬(p -> ¬¬p) -> ¬((¬<5e> -> ¬<5d>) -> (<5d> -> <5e>)))) -> (¬¬¬(p -> ¬¬p) -> ((¬¬((¬<5e
  47: > -> ¬<5d>) -> (<5d> -> <5e>)) -> ¬¬¬(p -> ¬¬p)) -> (¬¬(p -> ¬¬p) -> ¬((¬<5e> -> ¬<5d>) -> (<5d> -> <5e>)))))                                    [L1]
  48: ¬¬¬(p -> ¬¬p) -> ((¬¬((¬<5e> -> ¬<5d>) -> (<5d> -> <5e>)) -> ¬¬¬(p -> ¬¬p)) -> (¬¬(p -> ¬¬p) -> ¬((¬<5e> -> ¬<5d>) -> (<5d> -> <5e>))))           MP 47, 46
  49: (¬¬¬(p -> ¬¬p) -> ((¬¬((¬<5e> -> ¬<5d>) -> (<5d> -> <5e>)) -> ¬¬¬(p -> ¬¬p)) -> (¬¬(p -> ¬¬p) -> ¬((¬<5e> -> ¬<5d>) -> (<5d> -> <5e>))))) -> ((¬¬¬(p -
  49: > ¬¬p) -> (¬¬((¬<5e> -> ¬<5d>) -> (<5d> -> <5e>)) -> ¬¬¬(p -> ¬¬p))) -> (¬¬¬(p -> ¬¬p) -> (¬¬(p -> ¬¬p) -> ¬((¬<5e> -> ¬<5d>) -> (<5d> -> <5e>)))))  [L2]
  50: (¬¬¬(p -> ¬¬p) -> (¬¬((¬<5e> -> ¬<5d>) -> (<5d> -> <5e>)) -> ¬¬¬(p -> ¬¬p))) -> (¬¬¬(p -> ¬¬p) -> (¬¬(p -> ¬¬p) -> ¬((¬<5e> -> ¬<5d>) -> (<5d> -> <5e>
  50: ))))                                                                                                                                             MP 49, 48
  51: ¬¬¬(p -> ¬¬p) -> (¬¬(p -> ¬¬p) -> ¬((¬<5e> -> ¬<5d>) -> (<5d> -> <5e>)))                                                                          MP 50, 45
  52: (¬¬¬(p -> ¬¬p) -> (¬¬(p -> ¬¬p) -> ¬((¬<5e> -> ¬<5d>) -> (<5d> -> <5e>)))) -> ((¬¬¬(p -> ¬¬p) -> ¬¬(p -> ¬¬p)) -> (¬¬¬(p -> ¬¬p) -> ¬((¬<5e> -> ¬<5d>)
  52:  -> (<5d> -> <5e>))))                                                                                                                            [L2]
  53: (¬¬¬(p -> ¬¬p) -> ¬¬(p -> ¬¬p)) -> (¬¬¬(p -> ¬¬p) -> ¬((¬<5e> -> ¬<5d>) -> (<5d> -> <5e>)))                                                        MP 52, 51
  54: ¬¬¬(p -> ¬¬p) -> ¬((¬<5e> -> ¬<5d>) -> (<5d> -> <5e>))                                                                                            MP 53, 44
  55: (¬¬¬(p -> ¬¬p) -> ¬((¬<5e> -> ¬<5d>) -> (<5d> -> <5e>))) -> (((¬<5e> -> ¬<5d>) -> (<5d> -> <5e>)) -> ¬¬(p -> ¬¬p))                                 [L3]
  56: ((¬<5e> -> ¬<5d>) -> (<5d> -> <5e>)) -> ¬¬(p -> ¬¬p)                                                                                             MP 55, 54
  57: ¬¬(p -> ¬¬p)                                                                                                                                     MP 56, 1
```