

----- 基本结论 (不含否定) -----

prove List() $\vdash p \rightarrow p$

1	1: $p \rightarrow (<8> \rightarrow p)$	[L1]
2	2: $p \rightarrow ((<8> \rightarrow p) \rightarrow p)$	[L1]
3	3: $(p \rightarrow ((<8> \rightarrow p) \rightarrow p)) \rightarrow ((p \rightarrow (<8> \rightarrow p)) \rightarrow (p \rightarrow p))$	[L2]
4	4: $(p \rightarrow (<8> \rightarrow p)) \rightarrow (p \rightarrow p)$	MP 3, 2
5	5: $p \rightarrow p$	MP 4, 1

prove List(p) $\vdash q \rightarrow p$

1	1: p	[AS]
2	2: $p \rightarrow (q \rightarrow p)$	[L1]
3	3: $q \rightarrow p$	MP 2, 1

prove List($q \rightarrow r$) $\vdash (p \rightarrow q) \rightarrow (p \rightarrow r)$

1	1: $q \rightarrow r$	[AS]
2	2: $(q \rightarrow r) \rightarrow (p \rightarrow (q \rightarrow r))$	[L1]
3	3: $p \rightarrow (q \rightarrow r)$	MP 2, 1
4	4: $(p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r))$	[L2]
5	5: $(p \rightarrow q) \rightarrow (p \rightarrow r)$	MP 4, 3

prove List($p \rightarrow r$) $\vdash p \rightarrow (q \rightarrow r)$

1	1: $p \rightarrow r$	[AS]
2	2: $r \rightarrow (q \rightarrow r)$	[L1]
3	3: $(r \rightarrow (q \rightarrow r)) \rightarrow (p \rightarrow (r \rightarrow (q \rightarrow r)))$	[L1]
4	4: $p \rightarrow (r \rightarrow (q \rightarrow r))$	MP 3, 2
5	5: $(p \rightarrow (r \rightarrow (q \rightarrow r))) \rightarrow ((p \rightarrow r) \rightarrow (p \rightarrow (q \rightarrow r)))$	[L2]
6	6: $(p \rightarrow r) \rightarrow (p \rightarrow (q \rightarrow r))$	MP 5, 4
7	7: $p \rightarrow (q \rightarrow r)$	MP 6, 1

prove List($p \rightarrow q, p \rightarrow (q \rightarrow r)$) $\vdash p \rightarrow r$

1	1: $p \rightarrow q$	[AS]
2	2: $p \rightarrow (q \rightarrow r)$	[AS]
3	3: $(p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r))$	[L2]
4	4: $(p \rightarrow q) \rightarrow (p \rightarrow r)$	MP 3, 2
5	5: $p \rightarrow r$	MP 4, 1

prove List($p \rightarrow (q \rightarrow r)$) $\vdash q \rightarrow (p \rightarrow r)$

1	1: $q \rightarrow (p \rightarrow q)$	[L1]
2	2: $p \rightarrow (q \rightarrow r)$	[AS]
3	3: $(p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r))$	[L2]
4	4: $(p \rightarrow q) \rightarrow (p \rightarrow r)$	MP 3, 2
5	5: $((p \rightarrow q) \rightarrow (p \rightarrow r)) \rightarrow (q \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r)))$	[L1]
6	6: $q \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r))$	MP 5, 4
7	7: $(q \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r))) \rightarrow ((q \rightarrow (p \rightarrow q)) \rightarrow (q \rightarrow (p \rightarrow r)))$	[L2]
8	8: $(q \rightarrow (p \rightarrow q)) \rightarrow (q \rightarrow (p \rightarrow r))$	MP 7, 6
9	9: $q \rightarrow (p \rightarrow r)$	MP 8, 1

prove List($p \rightarrow q, q \rightarrow r$) $\vdash p \rightarrow r$

1	1: $p \rightarrow q$	[AS]
2	2: $q \rightarrow r$	[AS]
3	3: $(q \rightarrow r) \rightarrow (p \rightarrow (q \rightarrow r))$	[L1]
4	4: $p \rightarrow (q \rightarrow r)$	MP 3, 2
5	5: $(p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r))$	[L2]
6	6: $(p \rightarrow q) \rightarrow (p \rightarrow r)$	MP 5, 4
7	7: $p \rightarrow r$	MP 6, 1

prove List($p \rightarrow (q \rightarrow r), p \rightarrow (q \rightarrow (r \rightarrow s))$) $\vdash p \rightarrow (q \rightarrow s)$

1	1: $p \rightarrow (q \rightarrow r)$	[AS]
2	2: $p \rightarrow (q \rightarrow (r \rightarrow s))$	[AS]
3	3: $(q \rightarrow (r \rightarrow s)) \rightarrow ((q \rightarrow r) \rightarrow (q \rightarrow s))$	[L1]
4	4: $((q \rightarrow (r \rightarrow s)) \rightarrow ((q \rightarrow r) \rightarrow (q \rightarrow s))) \rightarrow (p \rightarrow ((q \rightarrow (r \rightarrow s)) \rightarrow ((q \rightarrow r) \rightarrow (q \rightarrow s))))$	[L2]
5	5: $p \rightarrow ((q \rightarrow (r \rightarrow s)) \rightarrow ((q \rightarrow r) \rightarrow (q \rightarrow s)))$	MP 4, 3
6	6: $(p \rightarrow ((q \rightarrow (r \rightarrow s)) \rightarrow ((q \rightarrow r) \rightarrow (q \rightarrow s)))) \rightarrow ((p \rightarrow (q \rightarrow (r \rightarrow s))) \rightarrow (p \rightarrow ((q \rightarrow r) \rightarrow (q \rightarrow s))))$	[L2]
7	7: $(p \rightarrow (q \rightarrow (r \rightarrow s))) \rightarrow (p \rightarrow ((q \rightarrow r) \rightarrow (q \rightarrow s)))$	MP 6, 5
8	8: $p \rightarrow ((q \rightarrow r) \rightarrow (q \rightarrow s))$	MP 7, 2
9	9: $p \rightarrow ((q \rightarrow r) \rightarrow (q \rightarrow s)) \rightarrow ((p \rightarrow (q \rightarrow r)) \rightarrow (p \rightarrow (q \rightarrow s)))$	[L2]
10	10: $(p \rightarrow (q \rightarrow r)) \rightarrow (p \rightarrow (q \rightarrow s))$	MP 9, 8
11	11: $p \rightarrow (q \rightarrow s)$	MP 10, 1

----- 作业题（不含否定） -----

prove List() $\vdash (p \rightarrow q) \rightarrow (p \rightarrow p)$

1		1: $p \rightarrow (q \rightarrow p)$	[L1]
2		2: $(p \rightarrow (q \rightarrow p)) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow p))$	[L2]
3		3: $(p \rightarrow q) \rightarrow (p \rightarrow p)$	MP 2, 1

prove List(p, $q \rightarrow (p \rightarrow r)$) $\vdash q \rightarrow r$

1		1: p	[AS]
2		2: $p \rightarrow (q \rightarrow p)$	[L1]
3		3: $q \rightarrow p$	MP 2, 1
4		4: $q \rightarrow (p \rightarrow r)$	[AS]
5		5: $(q \rightarrow (p \rightarrow r)) \rightarrow ((q \rightarrow p) \rightarrow (q \rightarrow r))$	[L2]
6		6: $(q \rightarrow p) \rightarrow (q \rightarrow r)$	MP 5, 4
7		7: $q \rightarrow r$	MP 6, 3

prove List() $\vdash (p \rightarrow q) \rightarrow ((\neg p \rightarrow \neg q) \rightarrow (q \rightarrow p))$

1		1: $(\neg p \rightarrow \neg q) \rightarrow (q \rightarrow p)$	[L3]
2		2: $((\neg p \rightarrow \neg q) \rightarrow (q \rightarrow p)) \rightarrow ((p \rightarrow q) \rightarrow ((\neg p \rightarrow \neg q) \rightarrow (q \rightarrow p)))$	[L1]
3		3: $(p \rightarrow q) \rightarrow ((\neg p \rightarrow \neg q) \rightarrow (q \rightarrow p))$	MP 2, 1

prove List() $\vdash p \rightarrow (q \rightarrow (p \rightarrow q))$

1		1: $q \rightarrow (p \rightarrow q)$	[L1]
2		2: $(q \rightarrow (p \rightarrow q)) \rightarrow (p \rightarrow (q \rightarrow (p \rightarrow q)))$	[L1]
3		3: $p \rightarrow (q \rightarrow (p \rightarrow q))$	MP 2, 1

prove List() $\vdash ((p \rightarrow (q \rightarrow r)) \rightarrow (p \rightarrow q)) \rightarrow ((p \rightarrow (q \rightarrow r)) \rightarrow (p \rightarrow r))$

1		1: $(p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r))$	[L2]
2		2: $((p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r))) \rightarrow (((p \rightarrow (q \rightarrow r)) \rightarrow (p \rightarrow q)) \rightarrow ((p \rightarrow (q \rightarrow r)) \rightarrow (p \rightarrow r)))$	[L2]
3		3: $((p \rightarrow (q \rightarrow r)) \rightarrow (p \rightarrow q)) \rightarrow ((p \rightarrow (q \rightarrow r)) \rightarrow (p \rightarrow r))$	MP 2, 1

----- 基本结论 (含否定) -----

prove List($\neg q, q$) $\vdash p$

1	1: q	[AS]
2	2: $\neg q$	[AS]
3	3: $\neg q \rightarrow (p \rightarrow q)$	[L1]
4	4: $p \rightarrow q$	MP 3, 2
5	5: $(p \rightarrow \neg q) \rightarrow (q \rightarrow p)$	[L3]
6	6: $q \rightarrow p$	MP 5, 4
7	7: p	MP 6, 1

prove List($\neg q$) $\vdash q \rightarrow p$

1	1: $\neg q$	[AS]
2	2: $\neg q \rightarrow (p \rightarrow q)$	[L1]
3	3: $p \rightarrow q$	MP 2, 1
4	4: $(p \rightarrow \neg q) \rightarrow (q \rightarrow p)$	[L3]
5	5: $q \rightarrow p$	MP 4, 3

prove List() $\vdash \neg q \rightarrow (q \rightarrow p)$

1	1: $\neg q \rightarrow (p \rightarrow q)$	[L1]
2	2: $(\neg q \rightarrow q) \rightarrow (q \rightarrow p)$	[L3]
3	3: $((p \rightarrow \neg q) \rightarrow (q \rightarrow p)) \rightarrow (\neg q \rightarrow ((p \rightarrow \neg q) \rightarrow (q \rightarrow p)))$	[L1]
4	4: $\neg q \rightarrow ((p \rightarrow \neg q) \rightarrow (q \rightarrow p))$	MP 3, 2
5	5: $(\neg q \rightarrow ((p \rightarrow \neg q) \rightarrow (q \rightarrow p))) \rightarrow ((\neg q \rightarrow (p \rightarrow \neg q)) \rightarrow (\neg q \rightarrow (q \rightarrow p)))$	[L2]
6	6: $(\neg q \rightarrow (p \rightarrow \neg q)) \rightarrow (\neg q \rightarrow (q \rightarrow p))$	MP 5, 4
7	7: $\neg q \rightarrow (q \rightarrow p)$	MP 6, 1

prove List($\neg p \rightarrow p$) $\vdash p$

1	1: $p \rightarrow p$	[AS]
2	2: $\neg p$	[AS]
3	3: p	[L1]
4	4: $((p \rightarrow p) \rightarrow p) \rightarrow (p \rightarrow (p \rightarrow p))$	[L3]
5	5: $((p \rightarrow (p \rightarrow p)) \rightarrow p) \rightarrow (p \rightarrow ((p \rightarrow (p \rightarrow p)) \rightarrow p))$	[L1]
6	6: $p \rightarrow ((p \rightarrow (p \rightarrow p)) \rightarrow p)$	MP 5, 4
7	7: $p \rightarrow ((p \rightarrow (p \rightarrow p)) \rightarrow p)$	[L2]
8	8: $p \rightarrow ((p \rightarrow (p \rightarrow p)) \rightarrow p)$	MP 7, 6
9	9: $p \rightarrow (p \rightarrow (p \rightarrow p))$	MP 8, 3
10	10: $p \rightarrow (p \rightarrow (p \rightarrow p))$	[L2]
11	11: $p \rightarrow (p \rightarrow (p \rightarrow p))$	MP 10, 9
12	12: $p \rightarrow (p \rightarrow (p \rightarrow p))$	MP 11, 2
13	13: $p \rightarrow (p \rightarrow (p \rightarrow p))$	[L3]
14	14: $p \rightarrow (p \rightarrow p)$	MP 13, 12
15	15: p	MP 14, 2

prove List() $\vdash (\neg p \rightarrow p) \rightarrow p$

1	1: $(p \rightarrow p) \rightarrow (p \rightarrow p)$	[L1]
2	2: $p \rightarrow ((p \rightarrow p) \rightarrow p)$	[L1]
3	3: $((p \rightarrow p) \rightarrow p) \rightarrow (p \rightarrow ((p \rightarrow p) \rightarrow p))$	[L3]
4	4: $((p \rightarrow p) \rightarrow p) \rightarrow (p \rightarrow ((p \rightarrow p) \rightarrow p))$	[L1]
5	5: $p \rightarrow ((p \rightarrow p) \rightarrow p)$	MP 4, 3
6	6: $p \rightarrow ((p \rightarrow p) \rightarrow p)$	[L2]
7	7: $p \rightarrow ((p \rightarrow p) \rightarrow p)$	MP 6, 5
8	8: $p \rightarrow (p \rightarrow (p \rightarrow p))$	MP 7, 2
9	9: $p \rightarrow (p \rightarrow (p \rightarrow p))$	[L2]
10	10: $p \rightarrow (p \rightarrow (p \rightarrow p))$	MP 9, 8
11	11: $p \rightarrow (p \rightarrow (p \rightarrow p))$	[L3]
12	12: $((p \rightarrow p) \rightarrow p) \rightarrow ((p \rightarrow p) \rightarrow p)$	[L1]
13	13: $((p \rightarrow p) \rightarrow p) \rightarrow ((p \rightarrow p) \rightarrow p)$	MP 12, 11
14	14: $((p \rightarrow p) \rightarrow p) \rightarrow ((p \rightarrow p) \rightarrow p)$	[L2]
15	15: $((p \rightarrow p) \rightarrow p) \rightarrow ((p \rightarrow p) \rightarrow p)$	MP 14, 13
16	16: $((p \rightarrow p) \rightarrow p) \rightarrow ((p \rightarrow p) \rightarrow p)$	MP 15, 10
17	17: $((p \rightarrow p) \rightarrow p) \rightarrow ((p \rightarrow p) \rightarrow p)$	[L2]
18	18: $((p \rightarrow p) \rightarrow p) \rightarrow ((p \rightarrow p) \rightarrow p)$	MP 17, 16
19	19: $(p \rightarrow p) \rightarrow p$	MP 18, 1

prove List($\neg\neg p$) $\vdash p$

1	1: $\neg\neg p$	[AS]
2	2: $\neg\neg p$	[AS]
3	3: $\neg\neg p \rightarrow (\neg\neg p \rightarrow p)$	[L1]
4	4: $\neg\neg p \rightarrow (\neg\neg p \rightarrow p)$	MP 3, 2
5	5: $\neg\neg p \rightarrow (\neg\neg p \rightarrow p)$	[L3]
6	6: $\neg\neg p \rightarrow (\neg\neg p \rightarrow p)$	MP 5, 4
7	7: $\neg\neg p \rightarrow (\neg\neg p \rightarrow p)$	[L3]
8	8: $\neg\neg p \rightarrow p$	MP 7, 6
9	9: p	MP 8, 2

prove List() $\vdash \neg\neg p \rightarrow p$

1	1: $\neg\neg p \rightarrow p$	[L1]
2	2: $\neg\neg p \rightarrow p$	[L1]
3	3: $((\neg\neg p \rightarrow p) \rightarrow p) \rightarrow (\neg\neg p \rightarrow p)$	[L3]
4	4: $((\neg\neg p \rightarrow p) \rightarrow p) \rightarrow (\neg\neg p \rightarrow p)$	[L1]
5	5: $((\neg\neg p \rightarrow p) \rightarrow p) \rightarrow (\neg\neg p \rightarrow p)$	MP 4, 3
6	6: $((\neg\neg p \rightarrow p) \rightarrow p) \rightarrow (\neg\neg p \rightarrow p)$	[L2]
7	7: $((\neg\neg p \rightarrow p) \rightarrow p) \rightarrow (\neg\neg p \rightarrow p)$	MP 6, 5
8	8: $((\neg\neg p \rightarrow p) \rightarrow p) \rightarrow (\neg\neg p \rightarrow p)$	MP 7, 2
9	9: $((\neg\neg p \rightarrow p) \rightarrow p) \rightarrow (\neg\neg p \rightarrow p)$	[L3]
10	10: $((\neg\neg p \rightarrow p) \rightarrow p) \rightarrow (\neg\neg p \rightarrow p)$	[L1]
11	11: $((\neg\neg p \rightarrow p) \rightarrow p) \rightarrow (\neg\neg p \rightarrow p)$	MP 10, 9
12	12: $((\neg\neg p \rightarrow p) \rightarrow p) \rightarrow (\neg\neg p \rightarrow p)$	[L2]
13	13: $((\neg\neg p \rightarrow p) \rightarrow p) \rightarrow (\neg\neg p \rightarrow p)$	MP 12, 11
14	14: $((\neg\neg p \rightarrow p) \rightarrow p) \rightarrow (\neg\neg p \rightarrow p)$	MP 13, 8
15	15: $((\neg\neg p \rightarrow p) \rightarrow p) \rightarrow (\neg\neg p \rightarrow p)$	[L2]
16	16: $((\neg\neg p \rightarrow p) \rightarrow p) \rightarrow (\neg\neg p \rightarrow p)$	MP 15, 14
17	17: $\neg\neg p \rightarrow p$	MP 16, 1

prove List(p) ⊢ קרר

1:	p	[AS]
2:	p	[AS]
3:	3: p → (קרר → p)	[L1]
4:	4: p → קרר	MP 3, 2
5:	5: (קרר → קרררר) → קרר	[L1]
6:	6: (קרר → קרר) → (קרר → קרר) → (קרר → קרררר)	[L3]
7:	7: (קרר → קרר) → (קרר → קרר) → (קרר → קרררר)	[L1]
8:	8: (קרר → קרר) → (קרר → קרר) → (קרר → קרררר)	MP 7, 6
9:	9: (קרר → קרר) → (קרר → קרר) → (קרר → קרר) → (קרר → קרררר)	[L2]
10:	10: (קרר → קרר) → (קרר → קרר) → (קרר → קרר) → (קרר → קרררר)	MP 9, 8
11:	11: (קרר → קרר) → (קרר → קרר) → (קרר → קרר) → (קרר → קרררר)	MP 10, 5
12:	12: (קרר → קרר) → (קרר → קרר) → (קרר → קרר) → (קרר → קרררר)	[L3]
13:	13: (קרר → קרר) → (קרר → קרר) → (קרר → קרר) → (קרר → קרררר)	[L1]
14:	14: (קרר → קרר) → (קרר → קרר) → (קרר → קרר) → (קרר → קרררר)	MP 13, 12
15:	15: (קרר → קרר) → (קרר → קרר) → (קרר → קרר) → (קרר → קרררר)	[L2]
16:	16: (קרר → קרר) → (קרר → קרר) → (קרר → קרר) → (קרר → קרררר)	MP 15, 14
17:	17: (קרר → קרר) → (קרר → קרר) → (קרר → קרר) → (קרר → קרררר)	MP 16, 11
18:	18: (קרר → קרר) → (קרר → קרר) → (קרר → קרר) → (קרר → קרררר)	[L3]
19:	19: (קרר → קרר) → (קרר → קרר) → (קרר → קרר) → (קרר → קרררר)	[L1]
20:	20: (קרר → קרר) → (קרר → קרר) → (קרר → קרר) → (קרר → קרררר)	MP 19, 18
21:	21: (קרר → קרר) → (קרר → קרר) → (קרר → קרר) → (קרר → קרררר)	[L2]
22:	22: (קרר → קרר) → (קרר → קרר) → (קרר → קרר) → (קרר → קרררר)	MP 21, 20
23:	23: (קרר → קרר) → (קרר → קרר) → (קרר → קרר) → (קרר → קרררר)	MP 22, 17
24:	24: (קרר → קרר) → (קרר → קרר) → (קרר → קרר) → (קרר → קרררר)	[L2]
25:	25: (קרר → קרר) → (קרר → קרר) → (קרר → קרר) → (קרר → קרררר)	MP 24, 23
26:	26: קרר → קרר	MP 25, 4
27:	27: קרר → קרר	[L1]
28:	28: קרר → קרר	[L3]
29:	29: (קרר → קרר) → (קרר → קרר) → (קרר → קרר) → (קרר → קרררר)	[L1]
30:	30: (קרר → קרר) → (קרר → קרר) → (קרר → קרר) → (קרר → קרררר)	MP 29, 28
31:	31: (קרר → קרר) → (קרר → קרר) → (קרר → קרר) → (קרר → קרררר)	[L2]
32:	32: (קרר → קרר) → (קרר → קרר) → (קרר → קרר) → (קרר → קרררר)	MP 31, 30
33:	33: קרר → קרר	MP 32, 27
34:	34: (קרר → קרר) → (קרר → קרר) → (קרר → קרר) → (קרר → קרררר)	[L2]
35:	35: (קרר → קרר) → (קרר → קרר) → (קרר → קרר) → (קרר → קרררר)	MP 34, 33
36:	36: קרר → קרר	MP 35, 26
37:	37: קרר → קרר	[L3]
38:	38: קרר → p	MP 37, 36
39:	39: קרר	MP 38, 2

prove List() ⊢ p → קרר

1:	1: (קרר → <lf>) → קרר	[L1]
2:	2: (קרר → <lf>) → קרר	[L1]
3:	3: (קרר → <lf>) → קרר	[L3]
4:	4: (קרר → <lf>) → קרר	[L1]
5:	5: (קרר → <lf>) → קרר	MP 4, 3
6:	6: (קרר → <lf>) → קרר	[L2]
7:	7: (קרר → <lf>) → קרר	MP 6, 5
8:	8: (קרר → <lf>) → קרר	MP 7, 2
9:	9: (קרר → <lf>) → קרר	[L3]
10:	10: (קרר → <lf>) → קרר	[L1]
11:	11: (קרר → <lf>) → קרר	MP 10, 9
12:	12: (קרר → <lf>) → קרר	[L2]
13:	13: (קרר → <lf>) → קרר	MP 12, 11
14:	14: קרר → קרר	MP 13, 8
15:	15: (קרר → <lf>) → קרר	[L2]
16:	16: (קרר → <lf>) → קרר	MP 15, 14
17:	17: קרר → קרר	MP 16, 1
18:	18: קרר → קרר	[L3]
19:	19: p → קרר	MP 18, 17

----- 作业题 (含否定) -----

prove List() $\vdash \neg(p \rightarrow q) \rightarrow p$

1	1: $\neg(p \rightarrow q) \rightarrow p$	[L1]
2	2: $\neg(p \rightarrow q) \rightarrow p$	[L3]
3	3: $\neg(p \rightarrow q) \rightarrow p$	[L1]
4	4: $\neg(p \rightarrow q) \rightarrow p$	MP 3, 2
5	5: $\neg(p \rightarrow q) \rightarrow p$	[L2]
6	6: $\neg(p \rightarrow q) \rightarrow p$	MP 5, 4
7	7: $\neg(p \rightarrow q) \rightarrow p$	MP 6, 1
8	8: $\neg(p \rightarrow q) \rightarrow p$	[L1]
9	9: $\neg(p \rightarrow q) \rightarrow p$	MP 8, 7
10	10: $\neg(p \rightarrow q) \rightarrow p$	[L1]
11	11: $\neg(p \rightarrow q) \rightarrow p$	[L3]
12	12: $\neg(p \rightarrow q) \rightarrow p$	MP 12, 11
13	13: $\neg(p \rightarrow q) \rightarrow p$	[L1]
14	14: $\neg(p \rightarrow q) \rightarrow p$	[L2]
15	15: $\neg(p \rightarrow q) \rightarrow p$	[L1]
16	16: $\neg(p \rightarrow q) \rightarrow p$	MP 14, 13
17	17: $\neg(p \rightarrow q) \rightarrow p$	MP 15, 10
18	18: $\neg(p \rightarrow q) \rightarrow p$	[L1]
19	19: $\neg(p \rightarrow q) \rightarrow p$	MP 18, 17
20	20: $\neg(p \rightarrow q) \rightarrow p$	[L1]
21	21: $\neg(p \rightarrow q) \rightarrow p$	[L2]
22	22: $\neg(p \rightarrow q) \rightarrow p$	MP 20, 19
23	23: $\neg(p \rightarrow q) \rightarrow p$	MP 21, 16
24	24: $\neg(p \rightarrow q) \rightarrow p$	[L2]
25	25: $\neg(p \rightarrow q) \rightarrow p$	[L1]
26	26: $\neg(p \rightarrow q) \rightarrow p$	[L1]
27	27: $\neg(p \rightarrow q) \rightarrow p$	MP 24, 23
28	28: $\neg(p \rightarrow q) \rightarrow p$	[L2]
29	29: $\neg(p \rightarrow q) \rightarrow p$	MP 26, 25
30	30: $\neg(p \rightarrow q) \rightarrow p$	MP 27, 22
31	31: $\neg(p \rightarrow q) \rightarrow p$	[L2]
32	32: $\neg(p \rightarrow q) \rightarrow p$	MP 29, 28
33	33: $\neg(p \rightarrow q) \rightarrow p$	MP 30, 9
34	34: $\neg(p \rightarrow q) \rightarrow p$	[L1]
35	35: $\neg(p \rightarrow q) \rightarrow p$	[L3]
36	36: $\neg(p \rightarrow q) \rightarrow p$	[L2]
37	37: $\neg(p \rightarrow q) \rightarrow p$	MP 34, 33
38	38: $\neg(p \rightarrow q) \rightarrow p$	[L1]
39	39: $\neg(p \rightarrow q) \rightarrow p$	MP 36, 35
40	40: $\neg(p \rightarrow q) \rightarrow p$	[L2]
41	41: $\neg(p \rightarrow q) \rightarrow p$	MP 38, 37
42	42: $\neg(p \rightarrow q) \rightarrow p$	MP 39, 32
43	43: $\neg(p \rightarrow q) \rightarrow p$	[L2]
44	44: $\neg(p \rightarrow q) \rightarrow p$	MP 41, 40
45	45: $\neg(p \rightarrow q) \rightarrow p$	MP 42, 31

prove List() $\vdash \neg(p \rightarrow q) \rightarrow \neg p$

1	1: $\neg(p \rightarrow q) \rightarrow \neg p$	[L1]
2	2: $\neg(p \rightarrow q) \rightarrow \neg p$	[L3]
3	3: $\neg(p \rightarrow q) \rightarrow \neg p$	[L1]
4	4: $\neg(p \rightarrow q) \rightarrow \neg p$	MP 3, 2
5	5: $\neg(p \rightarrow q) \rightarrow \neg p$	[L2]
6	6: $\neg(p \rightarrow q) \rightarrow \neg p$	MP 5, 4
7	7: $\neg(p \rightarrow q) \rightarrow \neg p$	MP 6, 1
8	8: $\neg(p \rightarrow q) \rightarrow \neg p$	[L3]
9	9: $\neg(p \rightarrow q) \rightarrow \neg p$	[L1]
10	10: $\neg(p \rightarrow q) \rightarrow \neg p$	MP 9, 8
11	11: $\neg(p \rightarrow q) \rightarrow \neg p$	[L2]
12	12: $\neg(p \rightarrow q) \rightarrow \neg p$	MP 11, 10
13	13: $\neg(p \rightarrow q) \rightarrow \neg p$	MP 12, 7
14	14: $\neg(p \rightarrow q) \rightarrow \neg p$	[L1]
15	15: $\neg(p \rightarrow q) \rightarrow \neg p$	MP 14, 13
16	16: $\neg(p \rightarrow q) \rightarrow \neg p$	[L1]
17	17: $\neg(p \rightarrow q) \rightarrow \neg p$	[L3]
18	18: $\neg(p \rightarrow q) \rightarrow \neg p$	[L1]
19	19: $\neg(p \rightarrow q) \rightarrow \neg p$	MP 18, 17
20	20: $\neg(p \rightarrow q) \rightarrow \neg p$	[L2]
21	21: $\neg(p \rightarrow q) \rightarrow \neg p$	MP 20, 19
22	22: $\neg(p \rightarrow q) \rightarrow \neg p$	MP 21, 16
23	23: $\neg(p \rightarrow q) \rightarrow \neg p$	[L1]
24	24: $\neg(p \rightarrow q) \rightarrow \neg p$	[L1]
25	25: $\neg(p \rightarrow q) \rightarrow \neg p$	[L1]
26	26: $\neg(p \rightarrow q) \rightarrow \neg p$	MP 24, 23
27	27: $\neg(p \rightarrow q) \rightarrow \neg p$	[L2]
28	28: $\neg(p \rightarrow q) \rightarrow \neg p$	MP 26, 25
29	29: $\neg(p \rightarrow q) \rightarrow \neg p$	MP 27, 22
30	30: $\neg(p \rightarrow q) \rightarrow \neg p$	[L2]
31	31: $\neg(p \rightarrow q) \rightarrow \neg p$	[L1]
32	32: $\neg(p \rightarrow q) \rightarrow \neg p$	MP 30, 29
33	33: $\neg(p \rightarrow q) \rightarrow \neg p$	[L1]
34	34: $\neg(p \rightarrow q) \rightarrow \neg p$	MP 32, 31
35	35: $\neg(p \rightarrow q) \rightarrow \neg p$	MP 33, 28
36	36: $\neg(p \rightarrow q) \rightarrow \neg p$	[L2]
37	37: $\neg(p \rightarrow q) \rightarrow \neg p$	MP 35, 34
38	38: $\neg(p \rightarrow q) \rightarrow \neg p$	MP 36, 15
39	39: $\neg(p \rightarrow q) \rightarrow \neg p$	[L1]
40	40: $\neg(p \rightarrow q) \rightarrow \neg p$	[L3]
41	41: $\neg(p \rightarrow q) \rightarrow \neg p$	[L2]
42	42: $\neg(p \rightarrow q) \rightarrow \neg p$	MP 40, 39
43	43: $\neg(p \rightarrow q) \rightarrow \neg p$	[L1]
44	44: $\neg(p \rightarrow q) \rightarrow \neg p$	MP 42, 41
45	45: $\neg(p \rightarrow q) \rightarrow \neg p$	[L2]
46	46: $\neg(p \rightarrow q) \rightarrow \neg p$	MP 44, 43
47	47: $\neg(p \rightarrow q) \rightarrow \neg p$	MP 45, 38
48	48: $\neg(p \rightarrow q) \rightarrow \neg p$	[L2]
49	49: $\neg(p \rightarrow q) \rightarrow \neg p$	MP 47, 46

prove $\text{List}() \vdash (\neg p \rightarrow \neg q) \rightarrow ((\neg p \rightarrow q) \rightarrow p)$

1	11: (q → p) → ((p → q) → (q → p))	[L1]
2	21: (q → p) → ((p → q) → (q → p))	[L1]
3	31: (q → p) → ((p → q) → (q → p))	[L1]
4	41: (q → p) → ((p → q) → (q → p))	MP 3, 2
5	51: (q → p) → ((p → q) → (q → p))	[L2]
6	61: (q → p) → ((p → q) → (q → p))	MP 5, 4
7	71: (q → p) → ((p → q) → (q → p))	[L3]
8	81: (q → p) → ((p → q) → (q → p))	[L3]
9	91: (q → p) → ((p → q) → (q → p))	MP 8, 7
10	101: (q → p) → ((p → q) → (q → p))	[L2]
11	111: (q → p) → ((p → q) → (q → p))	MP 10, 9
12	121: (q → p) → ((p → q) → (q → p))	[L1]
13	131: (q → p) → ((p → q) → (q → p))	MP 12, 11
14	141: (q → p) → ((p → q) → (q → p))	[L1]
15	142: (q → p) → ((p → q) → (q → p))	[L2]
16	151: (q → p) → ((p → q) → (q → p))	MP 14, 13
17	161: (q → p) → ((p → q) → (q → p))	MP 15, 6
18	171: (q → p) → ((p → q) → (q → p))	[L2]
19	181: (q → p) → ((p → q) → (q → p))	[L1]
20	182: (q → p) → ((p → q) → (q → p))	[L1]
21	191: (q → p) → ((p → q) → (q → p))	MP 18, 17
22	201: (q → p) → ((p → q) → (q → p))	[L2]
23	202: (q → p) → ((p → q) → (q → p))	MP 20, 19
24	211: (q → p) → ((p → q) → (q → p))	MP 21, 16
25	221: (q → p) → ((p → q) → (q → p))	[L3]
26	231: (q → p) → ((p → q) → (q → p))	[L1]
27	241: (q → p) → ((p → q) → (q → p))	MP 24, 23
28	251: (q → p) → ((p → q) → (q → p))	MP 26, 25
29	261: (q → p) → ((p → q) → (q → p))	[L1]
30	271: (q → p) → ((p → q) → (q → p))	[L2]
31	281: (q → p) → ((p → q) → (q → p))	[L1]
32	282: (q → p) → ((p → q) → (q → p))	MP 28, 27
33	291: (q → p) → ((p → q) → (q → p))	[L2]
34	301: (q → p) → ((p → q) → (q → p))	MP 30, 29
35	302: (q → p) → ((p → q) → (q → p))	MP 31, 22
36	311: (q → p) → ((p → q) → (q → p))	[L1]
37	321: (q → p) → ((p → q) → (q → p))	MP 34, 33
38	331: (q → p) → ((p → q) → (q → p))	[L2]
39	341: (q → p) → ((p → q) → (q → p))	MP 36, 35
40	351: (q → p) → ((p → q) → (q → p))	MP 37, 32
41	361: (q → p) → ((p → q) → (q → p))	[L2]
42	371: (q → p) → ((p → q) → (q → p))	MP 39, 38
43	381: (q → p) → ((p → q) → (q → p))	MP 40, 1
44	391: (q → p) → ((p → q) → (q → p))	[L1]
45	401: (q → p) → ((p → q) → (q → p))	[L2]
46	411: (q → p) → ((p → q) → (q → p))	[L1]
47		
48		

prove $\text{List}() \vdash (p \rightarrow \neg q) \rightarrow (q \rightarrow \neg p)$

1	11: (p → ¬q) → (q → ¬p)	[L1]
2	21: (p → ¬q) → (q → ¬p)	[L1]
3	31: (p → ¬q) → (q → ¬p)	[L1]
4	41: (p → ¬q) → (q → ¬p)	MP 4, 3
5	51: (p → ¬q) → (q → ¬p)	[L2]
6	61: (p → ¬q) → (q → ¬p)	MP 6, 5
7	71: (p → ¬q) → (q → ¬p)	MP 7, 2
8	81: (p → ¬q) → (q → ¬p)	[L3]
9	91: (p → ¬q) → (q → ¬p)	[L3]
10	101: (p → ¬q) → (q → ¬p)	MP 10, 9
11	111: (p → ¬q) → (q → ¬p)	[L2]
12	121: (p → ¬q) → (q → ¬p)	MP 12, 11
13	131: (p → ¬q) → (q → ¬p)	MP 13, 8
14	141: (p → ¬q) → (q → ¬p)	[L2]
15	151: (p → ¬q) → (q → ¬p)	MP 15, 14
16	161: (p → ¬q) → (q → ¬p)	MP 16, 1
17	171: (p → ¬q) → (q → ¬p)	[L1]
18	181: (p → ¬q) → (q → ¬p)	MP 18, 17
19	191: (p → ¬q) → (q → ¬p)	[L1]
20	201: (p → ¬q) → (q → ¬p)	MP 24, 23
21	211: (p → ¬q) → (q → ¬p)	MP 25, 20
22	221: (p → ¬q) → (q → ¬p)	[L2]
23	231: (p → ¬q) → (q → ¬p)	MP 27, 26
24	241: (p → ¬q) → (q → ¬p)	MP 28, 19
25	242: (p → ¬q) → (q → ¬p)	[L3]
26	251: (p → ¬q) → (q → ¬p)	[L1]
27	261: (p → ¬q) → (q → ¬p)	MP 31, 30
28	271: (p → ¬q) → (q → ¬p)	[L2]
29	281: (p → ¬q) → (q → ¬p)	MP 33, 32
30	291: (p → ¬q) → (q → ¬p)	MP 34, 29
31	301: (p → ¬q) → (q → ¬p)	[L1]
32	311: (p → ¬q) → (q → ¬p)	[L2]
33	321: (p → ¬q) → (q → ¬p)	[L1]
34	331: (p → ¬q) → (q → ¬p)	[L2]
35	341: (p → ¬q) → (q → ¬p)	MP 33, 32
36	351: (p → ¬q) → (q → ¬p)	MP 34, 29

prove $\text{List}() \vdash (\neg p \rightarrow q) \rightarrow (\neg q \rightarrow p)$

1	11: (¬p → q) → (¬q → p)	[L1]
2	21: (¬p → q) → (¬q → p)	[L1]
3	31: (¬p → q) → (¬q → p)	[L1]
4	41: (¬p → q) → (¬q → p)	MP 4, 3
5	51: (¬p → q) → (¬q → p)	[L2]
6	61: (¬p → q) → (¬q → p)	MP 6, 5
7	71: (¬p → q) → (¬q → p)	MP 7, 2
8	81: (¬p → q) → (¬q → p)	[L3]
9	91: (¬p → q) → (¬q → p)	[L3]
10	101: (¬p → q) → (¬q → p)	MP 10, 9
11	111: (¬p → q) → (¬q → p)	[L2]
12	121: (¬p → q) → (¬q → p)	MP 12, 11
13	131: (¬p → q) → (¬q → p)	MP 13, 8
14	141: (¬p → q) → (¬q → p)	[L2]
15	151: (¬p → q) → (¬q → p)	MP 15, 14
16	161: (¬p → q) → (¬q → p)	MP 16, 1
17	171: (¬p → q) → (¬q → p)	[L1]
18	181: (¬p → q) → (¬q → p)	MP 18, 17
19	191: (¬p → q) → (¬q → p)	[L1]
20	201: (¬p → q) → (¬q → p)	MP 20, 19
21	211: (¬p → q) → (¬q → p)	[L2]
22	221: (¬p → q) → (¬q → p)	MP 22, 21
23	231: (¬p → q) → (¬q → p)	[L3]
24	241: (¬p → q) → (¬q → p)	[L1]
25	251: (¬p → q) → (¬q → p)	MP 25, 24
26	261: (¬p → q) → (¬q → p)	[L2]
27	271: (¬p → q) → (¬q → p)	MP 27, 26
28	281: (¬p → q) → (¬q → p)	MP 28, 23
29	291: (¬p → q) → (¬q → p)	[L1]

-----析取运算-----

```
prove List() ⊢ p → (¬p → q)
```

1:	$p \rightarrow (\neg p \rightarrow p)$	[L1]
2:	$\neg p \rightarrow (\neg q \rightarrow \neg p)$	[L1]
3:	$(\neg q \rightarrow \neg p) \rightarrow (p \rightarrow q)$	[L3]
4:	$((\neg q \rightarrow \neg p) \rightarrow (p \rightarrow q)) \rightarrow (\neg p \rightarrow ((\neg q \rightarrow \neg p) \rightarrow (p \rightarrow q)))$	[L1]
5:	$\neg p \rightarrow ((\neg q \rightarrow \neg p) \rightarrow (p \rightarrow q))$	MP 4, 3
6:	$(\neg p \rightarrow ((\neg q \rightarrow \neg p) \rightarrow (p \rightarrow q))) \rightarrow ((\neg p \rightarrow (\neg q \rightarrow \neg p)) \rightarrow (\neg p \rightarrow (p \rightarrow q)))$	[L2]
7:	$\neg p \rightarrow ((\neg q \rightarrow \neg p) \rightarrow (\neg p \rightarrow (p \rightarrow q)))$	MP 6, 5
8:	$\neg p \rightarrow (p \rightarrow q)$	MP 7, 2
9:	$(\neg p \rightarrow (p \rightarrow q)) \rightarrow ((\neg p \rightarrow p) \rightarrow (\neg p \rightarrow q))$	[L2]
10:	$(\neg p \rightarrow p) \rightarrow (\neg p \rightarrow q)$	MP 9, 8
11:	$((\neg p \rightarrow p) \rightarrow (\neg p \rightarrow q)) \rightarrow (p \rightarrow ((\neg p \rightarrow p) \rightarrow (\neg p \rightarrow q)))$	[L1]
12:	$p \rightarrow ((\neg p \rightarrow p) \rightarrow (\neg p \rightarrow q))$	MP 11, 10
13:	$(p \rightarrow ((\neg p \rightarrow p) \rightarrow (\neg p \rightarrow q))) \rightarrow ((p \rightarrow (\neg p \rightarrow p)) \rightarrow (p \rightarrow (\neg p \rightarrow q)))$	[L2]
14:	$(p \rightarrow (\neg p \rightarrow p)) \rightarrow (p \rightarrow (\neg p \rightarrow q))$	MP 13, 12
15:	$p \rightarrow (\neg p \rightarrow q)$	MP 14, 1

```
prove List() ⊢ q → (¬p → q)
```

1		1: $q \rightarrow (\neg p \rightarrow q)$	[L1]
---	--	---	------

```
prove List() ⊢ (¬p → q) → (¬q → p)
```

1	q → (p → q)	[L1]
2	q → (p → q)	[L1]
3	q → (p → q)	[L3]
4	q → (p → q)	[L1]
5	q → (p → q)	MP 4, 3
6	q → (p → q)	[L2]
7	q → (p → q)	MP 6, 5
8	q → (p → q)	MP 7, 2
9	q → (p → q)	[L3]
10	q → (p → q)	[L1]
11	q → (p → q)	MP 10, 9
12	q → (p → q)	[L2]
13	q → (p → q)	MP 12, 11
14	q → (p → q)	MP 13, 8
15	q → (p → q)	[L2]
16	q → (p → q)	MP 15, 14
17	q → (p → q)	MP 16, 1
18	q → (p → q)	[L3]
19	q → (p → q)	MP 18, 17
20	q → (p → q)	[L1]
21	q → (p → q)	MP 20, 19
22	q → (p → q)	[L2]
23	q → (p → q)	MP 22, 21
24	q → (p → q)	[L3]
25	q → (p → q)	[L1]
26	q → (p → q)	MP 25, 24
27	q → (p → q)	[L2]
28	q → (p → q)	MP 27, 26
29	q → (p → q)	MP 28, 2

```
prove List() ⊢ (¬p → p) → p
```

[illegible]

```
prove List() ⊢ ¬¬p → p
```

1:	(קרי → (←סל → קרי))	[L1]
2:	קרי → (קרי → (←סל → קרי))	[L1]
3:	((קרי → (←סל → קרי)) → קרי) → (קרי → (←סל → קרי))	[L1]
4:	((קרי → (←סל → קרי)) → קרי) → (קרי → (←סל → קרי))	[L1]
5:	((קרי → (←סל → קרי)) → קרי) → (קרי → (←סל → קרי))	MP 4, 3
6:	((קרי → (←סל → קרי)) → קרי) → (קרי → (←סל → קרי))	[L2]
7:	((קרי → (←סל → קרי)) → קרי) → (קרי → (←סל → קרי))	MP 6, 5
8:	((קרי → (←סל → קרי)) → קרי) → (קרי → (←סל → קרי))	MP 7, 2
9:	((קרי → (←סל → קרי)) → קרי) → (קרי → (←סל → קרי))	[L3]
10:	((קרי → (←סל → קרי)) → קרי) → (קרי → (←סל → קרי))	[L1]
11:	((קרי → (←סל → קרי)) → קרי) → (קרי → (←סל → קרי))	MP 10, 9
12:	((קרי → (←סל → קרי)) → קרי) → (קרי → (←סל → קרי))	[L2]
13:	((קרי → (←סל → קרי)) → קרי) → (קרי → (←סל → קרי))	MP 12, 11
14:	((קרי → (←סל → קרי)) → קרי) → (קרי → (←סל → קרי))	MP 13, 8
15:	((קרי → (←סל → קרי)) → קרי) → (קרי → (←סל → קרי))	[L2]
16:	((קרי → (←סל → קרי)) → קרי) → (קרי → (←סל → קרי))	MP 15, 14
17:	קרי → p	MP 16, 1

----- 合取运算 -----

```
prove List() ⊢ ¬(p → ¬q) → p
```

[illegible]

```
prove List() ⊢ ¬(p → ¬q) → q
```

[illegible]

