# Special Relativity - Summary 3 - Space-Time Diagrams

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This summary is based on the book Robert Resnick: Introduction to Special Relativity, chapters A-1 - A-3 and B-1 - B-5

# 1 Space-Time Diagrams

In special relativity space and time are closely related, which is also reflected in the Lorentz transformations. One way to represent this close relation is space time diagrams, which were developed by the mathematician Herman Minkowski and are also called **Minkowski diagrams.** These diagrams are a geometrical representation of space-time. Typically the diagrams are simplified to one spatial axis (e.g. x-axis) and one time axis (e.g. w=ct axis).

The Minkowski diagram can represent several inertial frames at a time. An inertial frame at rest has an orthogonal frame, an inertial frame moving with a constant v speed is a non-orthogonal coordinate system (Fig. 1) and we can transform between them, using the Lorentz transformations. To determine the scale of the axes of the non-othogonal frames we can use the calibration hyperboles (Fig. 2).

The motion of objects or particles is represented by world lines in the diagram. The motion os light is the world line of light, which is at a 45° angle compared to the orthogonal x-axis. The Minkowski diagram can be divided into **four regions** by the world lines of light: The **absolute future**, the **absolute past** (time like regions) and two regions of **present** (space like regions).

# 2 Paradoxes

## 2.1 Twin Paradox

**The setup:** There is a pair of twins, where one of the twins remains stationary (e.g. on Earth) and the other twin travels away in a very fast spaceship to a certain distance and then returns to Earth. When the twins are together again, they find that the one that stayed on Earth is older compared to the one that has travelled.

**The paradox:** Shouldn't the position of the twins be interchangeable if motion is relative? Could we not just assume that Earth was moving compared to the "travelling" twin?

**Explanation:** The situation is not symmetric. The stationary twin remained in one inertial frame the whole time, while the travelling twin has been in two different inertial frames. One frame while going away and a different frame while returning. The travelling twin has moved along a certain path with a certain velocity and time has been dilated for this twin → twin is younger.

**Experimental test:** Two identical amounts of **radioactive material** on a rotor, where one sample of the radioactive material is stationary and the other one is moving. If the material is identical (e.g. uranium), then they have the same half life and the same amount of material should decay in a unit length of time. The experimental results show that the moving sample has decayed less compared to the stationary sample.

## 2.2 Bell's Spaceship paradox

We have two spaceships in S inertial frame at rest. The spaceships are separated by an L distance and connected by a thin thread. The two spaceships start to accelerate at the same time and at the same rate. Will the thread between them break?

**Solution:** The thread connecting the spaceships is going to break. There are several effects working here: length contraction of the thread and the relativity of simultaneity.

More information: `https://en.wikipedia.org/wiki/Bell%27s_spaceship_paradox`

## 2.3 Ehrenfest paradox

**Paradox:** If we have a rotating disk, the edge/circumference of the disk should get length contracted, however the radius of the disk should not get length contracted. Since the circumference depends on the radius ($2\pi R$) the disk should experience relativistic tension and shatter.

**Solution:** The disk is not an inertial frame, this is actually not a paradox.

More information: `https://en.wikipedia.org/wiki/Ehrenfest_paradox`

## 2.4 Ladder paradox

We have a ladder that is moving horizontally at a constant relativistic velocity. The ladder is passing trough a garage that is shorter than the rest length of the ladder. The garage has two doors, one in the front and one in the back so that the ladder can go trough. Because the ladder is moving relativisticly, it experiences length contraction and is going to fully fit inside the garage.

**The paradox:** This situation is symmetric. We could view the event from the point of view of the rest frame of the ladder, so that the garage is moving compared to the ladder. In this case the garage is getting length contracted and the ladder should not fit into the garage.

To analyse if the ladder is inside the garage we can close both doors of the garage at the same time and see if the ladder fits. In the rest frame of the garage the ladder fits because of length contraction. In the rest frame of the ladder we have a different situation. **Solution:** Simultaneity is relative. The garage doors are not closing at the same time in the frame of the ladder and the ladder can pass trough the garage with no problems.

**Similar paradoxes:** rod falling into hole, bar passing trough a ring.

More information: `https://en.wikipedia.org/wiki/Ladder_paradox`

## 2.5 Right-angle lever paradox

Also known as "Trouton-Noble paradox" or "Lewis-Tolman paradox"

If we have a balanced lever (two equal length arms at right angles) that is moving in the direction of one of its arms, the arm parallel to the motion is going to get length contracted compared to the arm perpendicular to the motion.

**Paradox:** The lever should get off balance and turn because the length change in the contracted arm. However, experiments confirm that the lever does not get off balance.

**Solution:** The forces, the momentum and the acceleration involved need to be Lorentz transformed as well, which gives the result that the lever should not get off balance.

More information: `https://en.wikipedia.org/wiki/Trouton%E2%80%93Noble_experiment#Right-angle_lever_paradox`

[2] Source of figures: `https://tikz.net/relativity_minkowski_diagram/#Coordinate_transformations`

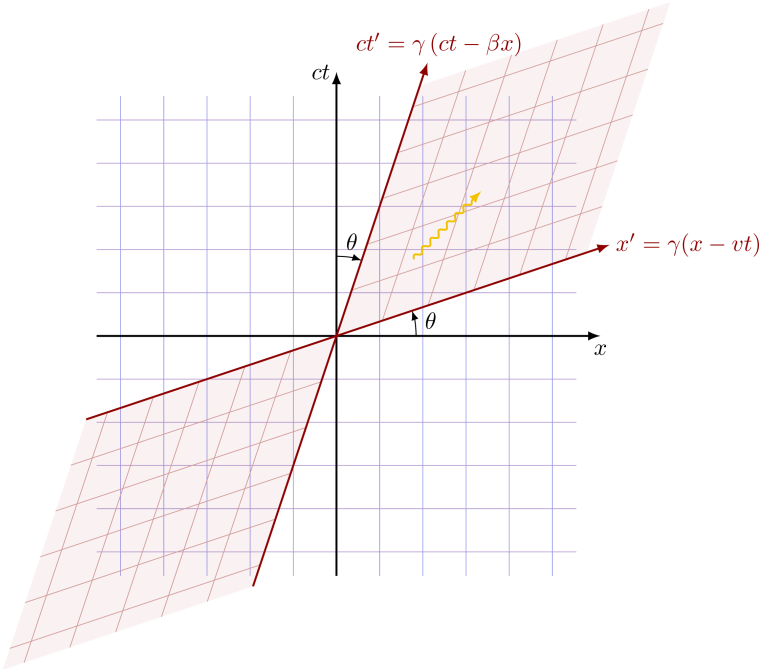


Figure 1: Illustration of the Lorentz transformation.

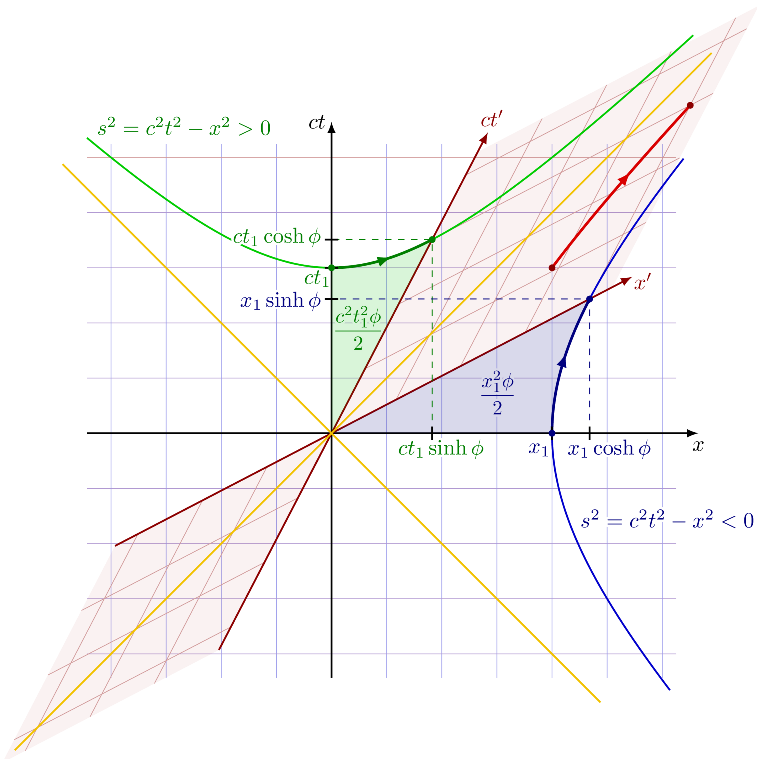


Figure 2: Hyperbolae in Minkowski spacetime to derive the Lorentz transformation in terms of hyperbolic functions sinh and cosh in analogy to rotation. The red point is boosted along the red hyperbola, such that its spacetime seperation from the origin remains constant.

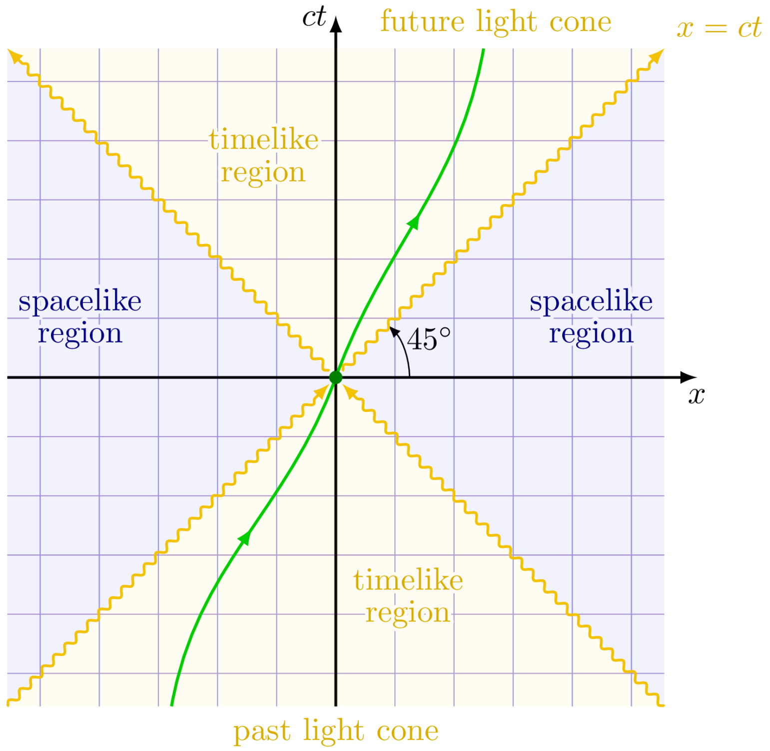


Figure 3: Future and past lightcone at a 45 degrees angle to illustrate the causal structure of Minkowski space.[2]