

Quiz 2

NAME: _____ SCORE: _____

Subject: Relativity

Date: Wednesday 1 March 2023

Duration: 60 minutes

Credits: 16 points, each question is one point.

This quiz consists of closed-book concept questions and short problems. Provide answers to the following items.

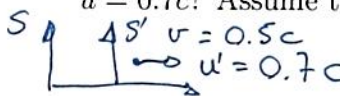
1. What is the factor for time dilation?

$$\sqrt{1 - \frac{v^2}{c^2}}$$

2. Name an example for when it is important to use special relativity for calculations.

electromagnetism
GPS

3. What velocity would an observer in the S frame observe for an object that moves in the S' frame. If the S' frame moves with $v = 0.5c$ with respect to S and the object moves with $u = 0.7c$? Assume that both v and u point in the same direction.

S 

$$u = \frac{u' + v}{1 + \frac{u'v}{c^2}} = \frac{0.7c + 0.5c}{1 + \frac{0.7 \cdot 0.5 c^2}{c^2}} = \frac{1.2c}{1 + 0.35} = 0.88c$$

4. Is acceleration invariant under the Lorentz transformation? Shortly explain.

No, acceleration is not invariant. The acceleration depends on the inertial frame.

5. What does the relativistic aberration describe?

Relates the propagation angle θ, θ' as seen from two inertial frames.

6. What is the transverse Doppler effect?

If the propagation of waves is $\theta = 90^\circ$ compared to the observer. The observer receives a lower frequency if $v \sim c$ compared to the proper frequency. This is a purely relativistic effect.

7. What is the difference between the classical momentum and the relativistic momentum?

for the relativistic momentum we need to use the relativistic mass instead of the classical mass:

$$\vec{p} = m \vec{u} = \frac{m_0 \vec{u}}{\sqrt{1 - \frac{u^2}{c^2}}}$$

8. What is the total energy of an object?

The total energy is the sum of the rest energy and the kinetic energy. $E = m_0 c^2 + K$

9. What is the proper mass of an object?

The mass measured when the object is at rest.

10. Is $\vec{F} = m\vec{a}$ in relativity?

No. $\vec{F} = m \frac{d\vec{u}}{dt} + \frac{\vec{u} (\vec{F} \cdot \vec{u})}{c^2}$

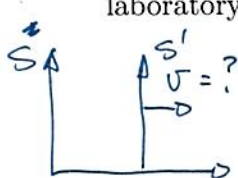
11. Can we simply substitute m for m_0 in classical equations to get the correct relativistic expression? Name an example.

No. Example: kinetic energy: $K = m_0 c^2 \left[\frac{1}{\sqrt{1 - \frac{u^2}{c^2}}} - 1 \right]$

12. Can photons have a speed other than c ? Shortly explain.

They can have a lower speed, when propagating in nonvacuum, e.g. in water.

13. The average lifetime of a μ meson at rest is 2.3×10^{-6} s. A laboratory measurement on μ mesons yields an average lifetime of 6.9×10^{-6} s. What is the speed of the meson in the laboratory frame?



$$t' = 2.3 \cdot 10^{-6} \text{ s}$$

$$t = 6.9 \cdot 10^{-6} \text{ s}$$

$$t' = t \sqrt{1 - \frac{v^2}{c^2}}$$

$$v = \sqrt{c^2 \left(1 - \frac{t'^2}{t^2} \right)}$$

$$= \sqrt{\left(3 \cdot 10^8 \frac{\text{m}}{\text{s}} \right)^2 \left(1 - \frac{2.3^2}{6.9^2} \right)}$$

$$= \underline{\underline{0.94 c}} = 2.8 \cdot 10^8 \frac{\text{m}}{\text{s}}$$

14. The rest mass of the μ meson is $207 m_e$. What is the effective mass of this meson if it is moving with the speed calculated in the previous question?

$$\begin{aligned}
 m_0 &= 207 m_e \\
 m &= ? \\
 v &= 0.94c \\
 m &= \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{207 m_e}{\sqrt{1 - \frac{0.94^2 c^2}{c^2}}} = \\
 &= \underline{\underline{621 m_e}}
 \end{aligned}$$

15. What is the kinetic energy and the momentum of the μ meson in the previous questions?

$$\begin{aligned}
 K &= ? \quad K = m_0 c^2 \left[\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 \right] = 207 m_e \cdot (3 \cdot 10^8)^2 \frac{\text{kg}}{\text{s}^2} \left[\frac{1}{\sqrt{1 - 0.94^2}} - 1 \right] = \\
 p &= ? \quad = \underline{\underline{3.726 \cdot 10^{19} \text{ J } m_e}} = 3.39 \cdot 10^{11} \text{ J} = 212 \text{ MeV}
 \end{aligned}$$

$$p = \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}} = \underline{\underline{1.756 \cdot 10^{19} \text{ kg } \frac{\text{m}}{\text{s}} m_e}} = 1.6 \cdot 10^{-19} \text{ kg } \frac{\text{m}}{\text{s}}$$

16. A radiative nucleus moves in the laboratory frame with a uniform velocity of $0.05c$. The nucleus decays by emitting an electron with speed $0.8c$ along the direction of motion (x - x' axis). What is the velocity of the electron in the lab frame?

$$\begin{aligned}
 v &= 0.05c \\
 u' &= 0.8c \\
 u &= ? \\
 u &= \frac{u' + v}{1 + \frac{u'v}{c^2}} = \frac{0.8c + 0.05c}{1 + \frac{0.8c \cdot 0.05c}{c^2}} = \underline{\underline{0.817c}}
 \end{aligned}$$