Special Relativity - Summary 1

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This summary is based on the book Robert Resnick: Introduction to Special Relativity, chapters 1.1- 1.9 and 2.1-2.7

1 Galilean transformations

Events in inertial systems can be represented by 4 coordinates: x, y, z, t. In classical physics ($v << c$) we can transform coordinates between two inertial systems using the **Galilean transformations**.
If we have two inertial frames, where S' is moving with v velocity relative to S along the joint x, x' axis:
S: x, y, z, t
S': x', y', z', t'

- $x' = x - vt$
- $y' = y$
- $z' = z$
- $t' = t$ (implicit)

Invariant quantities are quantities that are unchanged by transformations between inertial systems.

Under the Galilean transformations we have the following invariant quantities:
- time
- mass
- acceleration

Newton's laws of motion are exactly the same in all inertial systems. → Laws of mechanics are the same.
There is no way of determining an absolute velocity frame → The principle that we can only speak of the relative velocity of inertial frames is called **Newtonian relativity**.

Issue with classical mechanics: electromagnetism is not invariant under Galilean transformation. The speed of light should change in different inertial frames based on the motion of the frame. In addition, classical mechanics breaks down at velocities $v \sim c$.

2 Michelson-Morley experiment

To figure out the problem with the Galilean transformation → experiments.

The Michelson-Morley experiment was designed to test the change in the speed of light in a moving inertial frame.

Assumptions:
- Light propagates in a medium called **ether**.
- The ether frame is an absolute reference frame.
- The ether frame is attached to something in space, e.g. the Sun. → the Earth moves trough the ether with a certain speed, since the Earth is orbiting the Sun. Which also means that the relative speed of Earth trough the ether should change throughout the year due to Earth orbiting the Sun.

Method: Interferometry experiment with a beam splitter and two mirrors. The light travels two paths: l_1 and l_2, which are almost the same length. $l_1 \sim l_2$. This results in an interference pattern (also called fringe pattern).

Expectation: Due to Earths motion in the ether there should be a velocity difference for the speed of light in the direction of motion → the interference pattern should change if we rotate the instrument.

Result: No change in the interference pattern.

Implication: The ether theory may be wrong. → **There is no preferred inertial system and the speed of light is the same in all inertial systems.**

3 Alternative ideas to explain the Michelson-Morley experiment

3.1 The Lorentz-Fitzgerald contraction hypothesis

Idea: All bodies are contracted in the direction of motion by a factor of $\sqrt{1 - v^2/c^2}$

Issue: Doesn't explain the Michelson-Morley experiment if l_1 is significantly different from l_2.

3.2 The Ether drag hypothesis

Idea: All bodies with mass have ether attached to them and they drag the ether along with themself.

Issue: Does not explain stellar aberration and the Fizeau coefficient.

3.3 Modifying electrodynamics

Idea: The speed of light does not depend on the propagation medium, but the relative velocity of the light source.

Issue: Does not explain Michelson-Morley experiment with a moving light sources (e.g. the Sun), or the light received from binary stars (de Sitter experiment).

4 Special Relativity

Two postulates:
- The laws of physics are the same in all inertial systems. There is no preferred inertial system.
- The speed of light in vacuum has the same value (c) in all inertial systems.

In addition to the two postulates we also assume that space and time are homogeneous, e.g. there is no preferred direction, location or time. This also implies that the Lorentz transformation equations need to be linear.

5 Relativistic kinematics

5.1 The relativity of simultaneity

Two events that appear simultaneous in one frame are not necessarily simultaneous in another frame. → Event times will depend on the motion of the observers.

5.2 The Lorentz transformation

In special relativity the Galilean transformations get replaced by the Lorentz transformations:

If we have two inertial frames, where S' is moving with v velocity relative to S along the joint x, x' axis:
S: x, y, z, t
S': x', y', z', t'

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}} \qquad x = \frac{x' + vt'}{\sqrt{1 - v^2/c^2}}$$
$$y' = y$$
$$z' = z$$
$$t' = \frac{t - (v/c^2)x}{\sqrt{1 - v^2/c^2}} \qquad t = \frac{t' + (v/c^2)x'}{\sqrt{1 - v^2/c^2}}$$

In the classical case, where v << c, we get the Galilean transformation.

5.3 Consequences of the Lorentz transformation

- **Time dilation** for a moving object (clock) by a factor of $\sqrt{1 - v^2/c^2}$. The time intervals indicated by a clock are longer if the clock is moving with respect to the observer, e.g. moving clocks run slower.
- Two clocks which are synchronised and separated in one inertial frame are observed to be out of sync in another inertial frame. The difference is a **phase constant**, which depends on their location. This comes from the $(v/c^2)x$ term in the time transformation.
- **Length contraction** of a moving body parallel to the relative motion by a factor of $\sqrt{1 - v^2/c^2}$. Length contraction is a necessary consequence of time dilation.
- Lengths perpendicular to the relative motion do not change.

In relativity the frame attached to the observer is the **"proper" frame**. In this frame the length of a body is the **"proper" length**. The mass is the **"proper" mass**.

The time interval measured by a stationary clock is the **"proper" time**. A time interval measured by two different clocks (e.g. at two different locations) is the **"nonproper"** time or **"inproper"** time.

Similarly, **"rest"** frame, "rest" mass etc. is also used for the same things.

Examples for special relativity:
- Electromagnetism
- GPS - the calculations need to take relativity into account for accurate location determination
- High energy (particle) physics, where particles move with relativistic speeds. Particle physics is the combination of quantum mechanics and special relativity.

An **observer** in special relativity is effectively a measurement at a certain point in space and time.

6 Relativistic kinematics

6.1 Relativistic addition of velocities

Relativistic or Einstein velocity addition:

$$u = \frac{u' + v}{1 + \frac{u'v}{c^2}}$$

Here v is the velocity of the frame and u is the velocity of an object.
- If u and v are smaller than c $\rightarrow$ classical velocity addition $u = u' + v$
- If $u' = c \rightarrow u = c$ regardless of v (Light is moving)
- The velocity of light is always the same, no mater if the light source is moving. This is in agreement with experiments.
- Gives back one of the postulates $\rightarrow$ speed of light is always c in all inertial systems.
- The addition of 2 velocities $< c$ can not exceed the speed of light.
- It is possible to observe a faster speed than c, however this is always a geometric effect. E.g. superluminar motion in astrophysics, where a jet ejected from an AGN (active galactic nucleus, which is an accreting supermassive black hole in the centre of a galaxy) seems to be moving faster than the speed of light. This is a geometric effect of the projection. In 3D space the jet moves slower than c.

General velocity addition:

$$u'_x = \frac{u_x - v}{1 - u_x v/c^2} \qquad u_x = \frac{u'_x + v}{1 + u'_x v/c^2}$$

$$u'_y = \frac{u_y\sqrt{1 - v^2/c^2}}{1 - u_x v/c^2} \qquad u_y = \frac{u'_y\sqrt{1 - v^2/c^2}}{1 + u'_x v/c^2}$$

$$u'_z = \frac{u_z\sqrt{1 - v^2/c^2}}{1 - u_x v/c^2} \qquad u_z = \frac{u'_z\sqrt{1 - v^2/c^2}}{1 + u'_x v/c^2}$$

Transverse velocity components (u_y, u_z) of an object in the S frame are related to both the transverse component (u'_y, u'_z) and to the parallel component (u'_x) of the velocity of the object in the S' frame. $\rightarrow$ The result is not simple because neither observer is proper.

If we choose a frame where $u'_x = 0 \rightarrow u_y = u'_y\sqrt{1 - v^2/c^2}$ and $u_z = u'_z\sqrt{1 - v^2/c^2}$
- No length contraction is involved in the transverse direction
- Time dilation is responsible for the $\sqrt{1 - v^2/c^2}$ factor

6.2 Relativistic acceleration transformation

$$a'_x = a_x \frac{(1 - v^2/c^2)^{3/2}}{(1 - u_x v/c^2)^3}$$

Similar formula for a_y and a_z.
- Acceleration depends on the inertial frame in which it is measured
- When u and v $<< c$ we get back the classical result
- In SR frames do not accelerate, but objects in the frame can accelerate

6.3 Relativistic aberration of light

Relativistic equation for aberration of light:

$$tan\theta = \frac{sin\theta'\sqrt{1 - \beta^2}}{cos\theta' + \beta} \qquad tan\theta' = \frac{sin\theta\sqrt{1 - \beta^2}}{cos\theta - \beta}$$

Relates the propagation angle θ, θ' as seen from two inertial frames. $\beta = v/c$

6.4 Relativistic equation for the Doppler effect

$$\nu = \frac{\nu'(1 + \beta cos\theta')}{\sqrt{1 - \beta^2}} \qquad \nu' = \frac{\nu(1 - \beta cos\theta')}{\sqrt{1 - \beta^2}}$$

Classical case:
If $v << c$ we get the classical formula: $\nu = \nu'(1 + \beta cos\theta)$
- If $\theta = 0°$ $\rightarrow$ source is moving towards the observer: $\nu = \nu'(1 + \beta)$ $\rightarrow$ frequency is grater than the proper frequency (ν') (**"blueshift"**)
- If $\theta = 180°$ $\rightarrow$ source is moving away from the observer: $\nu = \nu'(1 - \beta)$ $\rightarrow$ frequency is less than the proper frequency (**"redshift"**)
- If $\theta = 90°$ $\rightarrow$ source is moving perpendicular compared to the observer $\rightarrow$ no Doppler effect.

SR:
If v is not small, we get relativistic or second order effects.
- **Longitudinal Doppler effect:**
- If $\theta = 0°$ $\rightarrow$ source is moving towards the observer: $\nu = \nu'\sqrt{\frac{c+v}{c-v}}$ $\rightarrow$ frequency is grater than the proper frequency (ν') (**"blueshift"**)
- If $\theta = 180°$ $\rightarrow$ source is moving away from the observer: $\nu = \nu'\sqrt{\frac{c-v}{c+v}}$ $\rightarrow$ frequency is less than the proper frequency (**"redshift"**)
- **Transverse Doppler effect:**
- If $\theta = 90°$ $\rightarrow \nu = \nu'\sqrt{1 - \beta^2}$ $\rightarrow$ Purely relativistic effect. $\rightarrow$ We observe a lower frequency compared to the proper frequency. (**"redshift"**)
- The transverse Doppler effect can be interpreted as a time dilation effect. $\rightarrow$ Experiments confirm relativistic time dilation.