Final

NAME: _____ SCORE: _____

Subject: Relativity
Date: Wednesday 8 March 2023
Duration: 120 minutes
Credits: 22 points, each question is one point.

This quiz consists of closed-book concept questions and short problems. Provide answers to the following items.

1. What is the longitudinal Doppler effect?
The frequency of a wave is changing if a source is moving towards or away from an observer. The longitudinal case is $\theta = 0°$ and $\theta = 180°$

2. What is the "proper" time?
A time that can be measured with one clock that is fixed in a frame or time measured at ~~a~~ one location in a frame

3. Which experiment proved that light propagates in all inertial frames with the same velocity?
Michelson - Morley experiment

4. Can we apply special relativity to a non inertial frame?
No, special relativity is only relevant for inertial frames

5. Briefly explain what does time dilation mean?
Moving clocks run slower. An "outside" observer perceives that time runs slower for a moving inertial frame.

6. If event A happens before event B in S frame, is there an S' frame where B happens first? Briefly explain.
Yes, if A and B are not in the same place, then such a frame can exist.

7. The Lorentz-Fitzgerald contraction hypothesis has the same assumption as the result of the length contraction from special relativity. What was the problem with the Lorentz-Fitzgerald contraction or what is the difference with special relativity?

The Lorentz-Fitzgerald contraction doesn't accept that the speed of light is the same in all inertial frames.

8. If an object moves with velocity v=0.2c, is its rest mass going to be smaller or larger than its relativistic mass? Briefly explain.

smaller. The rest mass is always smaller than the relativistic mass.

9. What form does the Lorentz transformation take if v << c?

The Galilean transformation

10. Briefly explain what does the $x'v/c^2$ term mean in the Lorentz transformation?

phase difference from the time transformation

11. Briefly explain why objects with mass can not travel faster than the speed of light?

As objects with mass get accelerated their relativistic mass increases. As $v \to c$ the mass $\to \infty$

12. Briefly explain what is the mass-energy?

Mass and energy are equivalent and form a single invariant in SR the mass-energy

13. Briefly explain what is the twin paradox and how does special relativity explain it?

If we have two twins, where one stays stationary and the other travels away with $v \sim c$ and then returns to the stationary twin, then the traveling twin is younger. Paradox resolution: the situation is not symmetric, time gets dilated for the traveling twin.

14. How does the frequency of light change if it is emitted by a star that is moving away from Earth?

The frequency becomes smaller due to the Doppler effect.
The light gets redshifted.

15. Does the speed of light depend on the motion of the light source? Briefly explain.

No. The speed of light is constant in vacuum in all inertial frames, regardless of the motion of the source

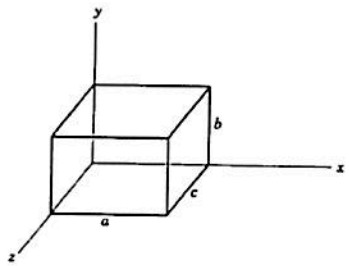
16. If the energy is conserved in an interaction in one inertial frame (e.g. S frame), is the energy also conserved in another inertial frame (e.g. S' frame)?

yes

17. Briefly explain what are space-time diagrams?

Are diagrams of wher space and time coordinates are the axes: the simple 2D version has x and ct as axes. We can use them to geometrically represent SR and the Lorentz transformations.

18. We have a box at rest with sides a, b, c, a rest mass m_0. What is the volume of the box as viewed by an observer moving relative to the box with speed u in the x-direction? What is the mass measured by this observer?

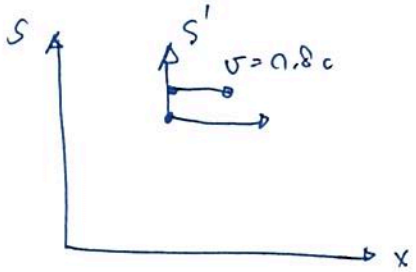


original volume: abc

volume for moving observer: $abc\sqrt{1-\frac{u^2}{c^2}}$

mass: $\frac{m_0}{\sqrt{1-\frac{u^2}{c^2}}}$

19. An e^- moves in the positive x-direction in frame S at a speed v=0.8c. What are it's momentum and energy in frame S? What is the momentum and energy of the e^- in it's rest frame? The rest mass of the electron is $9.1093837015 \times 10^{31}$ kg.



$p' = 0$

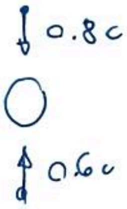
$E' = m_0 c^2 = 8.19 \cdot 10^{-14}$ J

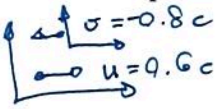
$p = \frac{m_0 v}{\sqrt{1-\frac{v^2}{c^2}}} = 3.64 \cdot 10^{-22}\ kg\frac{m}{s} = 4 \cdot 10^{8}\frac{m}{s} m_e$

$= 1.33 \cdot c \cdot m_e$

$E = c\sqrt{p^2 + m_0^2 c^2} = 1.36 \cdot 10^{-13}$ J

20. One cosmic-ray particle approaches the Earth along its axis with a velocity of 0.8c toward the North Pole and another with a velocity of 0.6c toward the South Pole. What is the relative speed of approach of one particle with respect to the other particle? Consider the Earth and one of the particles as two inertial frames.





$$u' = \frac{u - v}{1 - \frac{uv}{c^2}} = \frac{0.6c + 0.8c}{1 - \frac{(0.6 \cdot -0.8)c^2}{c^2}} =$$

$$= 0.946c$$

21. A ^{12}C nucleus consists of 6 p^+ and 6 n^0. What is the binding energy of the ^{12}C nucleus? The binding energy is equivalent to the energy that would be released if the nucleus broke spontaneously into its individual components. The mass of the ^{12}C nucleus is 12u the mass of the p^+ is 1.007825u and the mass of the n^0 is 1.008665u. (1u $\times c^2 = 931.5$ MeV)

$12u - 6 \cdot p - 6 \cdot n = -0.0989\,u \Rightarrow -92.11\,\text{MeV}$

22. If the radius of our Galaxy is 3 $\times 10^{20}$ m. What constant speed would a spaceship need to travel to go from the centre of the Galaxy to its edge in 30 years?

$x = r = 3 \cdot 10^{20}$ m $\quad v = ? \quad t = 30$ years proper time

→ distance contracts : $x\sqrt{1 - \frac{v^2}{c^2}} = vt$

$$\sqrt{1 - \frac{v^2}{c^2}} = \frac{vt}{x}$$

$$v = \sqrt{\frac{1}{\frac{t^2}{x^2} + \frac{1}{c^2}}} = 0.99c$$

non proper time : $v = \frac{x}{t} = 1056c$ → not possible