Special Relativity - Summary 1

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This summary is based on the book chapters 1.1- 1.9 and 2.1-2.3 of Robert Resnick: Introduction to Special Relativity

1 Galilean transformations

Events in inertial systems can be represented by 4 coordinates: x, y, z, t. In classical physics ($v << c$) we can transform coordinates between two inertial systems using the Galilean transformations.
If we have two inertial frames, where S' is moving with v velocity relative to S along the joint x, x' axis :
S: x, y, z, t
S': x',y',z',t'

- $x' = x - vt$
- $y' = y$
- $z' = z$
- $t' = t$ (implicit)

Invariant quantities are quantities that are unchanged by transformations between inertial systems.

Under the Galilean transformations we have the following invariant quantities:

- time
- mass
- acceleration

Newton's laws of motion are exactly the same in all inertial systems. → Laws of mechanics are the same. There is no way of determining an absolute velocity frame → The principle that we can only speak of the relative velocity of inertial frames is called **Newtonian relativity**.

Issue with classical mechanics: electromagnetism is not invariant under Galilean transformation. The speed of light should change in different inertial frames based on the motion of the frame. In addition, classical mechanism breaks down at velocities $v \sim c$.

2 Michelson-Morley experiment

To figure out the problem → experiments.

The Michelson-Morley experiment is a test that was designed to test the change in the speed of light in a moving inertial frame.

Assumptions:

- Light propagates in a medium called ether.
- The ether frame is an absolute reference frame.
- The ether frame is attached to something in Space, e.g. the Sun. → the Earth moves trough the ether with a certain speed, since the Earth is orbiting the Sun. Which also means that the relative speed of Earth trough the ether should change throughout one year due to Earth orbiting the Sun.

Method: Interferometry experiment with a beam splitter and two mirrors. The light travels two paths: l_1 and l_2, which are almost the same length. $l_1 \sim l_2$, which results in an interference pattern.

Expectation: Due to Earths motion in the Ether there should be a velocity difference for the speed of light in the direction of motion → the interference pattern should change if we rotate the instrument.

Result: No change in the interference pattern.

Implication: The ether theory may be wrong. → There is no preferred inertial system and the speed of light may be the same in all inertial systems.

3 Alternative ideas to explain the Michelson-Morley experiment

3.1 The Lorentz-Fitzgerald contraction hypothesis

Idea: All bodies are contracted in the direction of motion by a factor of $\sqrt{1 - v^2/c^2}$

Issue:doesn't explain the Michelson-Morley experiment if l_1 is significantly different from l_2.

3.2 The Ether drag hypothesis

Idea: All bodies with mass have ether attached to them and they drag the ether along with themself.

Issue: Does not explain stellar aberration and the Fizeau coefficient.

3.3 Modifying electrodynamics

Idea: The speed of light does not depend on the propagation medium, but the relative velocity of the light source.

Issue: Does not explain Michelson-Morley experiment with a moving light sources (e.g. the Sun), or the light received from binary stars (de Sitter experiment).

4 Special Relativity

Two postulates:
- The laws of physics are the same in all inertial systems. There is no preferred inertial system.
- The speed of light in vacuum has the same value (c) in all inertial systems.

In addition to the two postulates we also assume that space and time are homogenous, e.g. there is no preferred direction, location or time. This also implies that the Lorentz transformation equations need to be linear.

5 Relativistic kinematics

5.1 The relativity of simultaneity

Two events that appear simultaneous in one frame are not necessarily simultaneous in another frame. → Event times will depend on the motion of the observers.

5.2 The Lorentz transformation

In special relativity the Galilean transformations get replaced by the Lorentz transformations:

If we have two inertial frames, where S' is moving with v velocity relative to S along the joint x, x' axis
:
S: x, y, z, t
S': x',y',z',t'

- $x' = \frac{x-vt}{\sqrt{1-v^2/c^2}}$
- $y' = y$
- $z' = z$
- $t' = \frac{t-(v/c^2)x}{\sqrt{1-v^2/c^2}}$
- the inverse of the transformation is also valid.
- in the classical case, where v << c, we get the Galilean transformation.

5.3 Consequences of the Lorentz transformation

- Length contraction of a moving body in the direction of the motion by a factor of $\sqrt{1 - v^2/c^2}$.
- Time dilation for a moving object (clock) by a factor of $\sqrt{1 - v^2/c^2}$. (Moving clock will appear to go at a slower rate.)
- Moving clocks appear to differ from one another in their readings by a phase constant, which depends on their location. This comes from the $(v/c^2)x$ term in the time transformation.
- Lengths perpendicular to the relative motion do not change.

In relativity the frame attached to the observer is the ”proper” frame. In this frame the length of a body is the ”proper” length. The mass is the ”proper” mass.

The time interval measured by a stationary clock is the ”proper” time. A time interval measured by two different clocks (e.g. at two different locations) is the ”nonproper” time or ”inproper” time.

Similarly, sometimes ”rest” frame, ”rest” mass etc. is also used for the same things.