Special Relativity - Summary 3

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This summary is based on the book chapters 3.6 - 3.7 of Robert Resnick: Introduction to Special Relativity

1 Relativistic dynamics

1.1 The equivalence of mass and energy

The rest mass of a body is equivalent to energy → **rest-mass energy**.

- This follows from the Lorentz transformation and the momentum conservation.
- The total energy is conserved in inelastic collisions
- The conservation of total energy is equivalent to conservation of rest mass
- Mass and energy are equivalent → they form **a single invariant: mass-energy**
- relation: $E = mc^2$ or $m = E/c^2$ → we can convert one into the other
- The conversion of mass into energy is often used in nuclear physics: e.g. eV or MeV units for the mass of particles
- Or a mass less particle (e.g. photon) can get a "mass" assigned to it based on its energy
- We can think of the kinetic energy as an **external energy** and of the rest-mass energy as an internal energy.
- **Internal energy** is to the largest part mad up of the total rest-mass energy of the constituent fundamental particles, but it can also include:
  - molecular motion,
  - intermolecular potential energy,
  - atomic potential energy,
  - nuclear potential energy
- The rest mass or (proper mass) of a body is generally not constant.
- We can assign a rest mass to any collection of particles in motion in a frame at which the centre of mass is at rest.
- In classical physics we have 2 conservation laws: energy and mass conservation → in SR we have 1: mass-energy conservation.

1.2 Lorentz transformation of momentum, energy, mass and force

1.2.1 Momentum and energy

$$p_x = \frac{1}{\sqrt{1 - v^2/c^2}}\left(p'_x + \frac{E'v}{c^2}\right) \qquad p'_x = \frac{1}{\sqrt{1 - v^2/c^2}}\left(p_x - \frac{Ev}{c^2}\right)$$

$$p_y = p'_y$$

$$p_z = p'_z$$

$$E = \frac{1}{\sqrt{1 - v^2/c^2}}\left(E' + vp_x\right) \qquad E' = \frac{1}{\sqrt{1 - v^2/c^2}}\left(E - vp_x\right)$$

Very similar to the Lorentz transformation of x, y, z, t:

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}} \qquad p'_x = \frac{p_x - v(E/c^2)}{\sqrt{1 - v^2/c^2}}$$

$$t' = \frac{t - (v/c^2)x}{\sqrt{1 - v^2/c^2}} \qquad \frac{E'}{c^2} = \frac{(E/c^2) - (v/c^2)p_x}{\sqrt{1 - v^2/c^2}}$$

- When relativity is put into 4 dimensional space-time form a four-vector can be created out of the space and time coordinates
- Similarly a four-vector can be created out of the three momentum components and the energy
- If energy and momentum are conserved in one interaction in an inertial frame → then they are also conserved for the interaction in another inertial frame
- If momentum is conserved → energy must be conserved as well.

1.2.2 Mass

The transformation of mass follows directly from the transformation of energy:

$$m = \frac{m'(1 + u'_x v/c^2)}{\sqrt{1 - v^2/c^2}} \qquad m' = \frac{m(1 - u_x v/c^2)}{\sqrt{1 - v^2/c^2}}$$

1.2.3 Force

$$F_x = F'_x + \frac{u'_y v}{c^2 + u'_x v} F'_y + \frac{u'_z v}{c^2 + u'_x v} F'_z \qquad F'_x = F_x - \frac{u_y v}{c^2 - u_x v} F_y - \frac{u_z v}{c^2 - u_x v} F_z$$

$$F_y = \frac{\sqrt{1 - v^2/c^2}}{1 + u'_x v/c^2} F'_y \qquad F'_y = \frac{\sqrt{1 - v^2/c^2}}{1 - u_x v/c^2} F_y$$

$$F_z = \frac{\sqrt{1 - v^2/c^2}}{1 + u'_x v/c^2} F'_z \qquad F'_z = \frac{\sqrt{1 - v^2/c^2}}{1 - u_x v/c^2} F_z$$

A more **compact version**:

$$\frac{1}{\sqrt{1 - v^2/c^2}} = \gamma$$

$$F_x = \frac{F'_x + (v/c^2)\bar{u}' \cdot \bar{F}'}{1 + u'_x v/c^2} \qquad F'_x = \frac{F_x - (v/c^2)\bar{u} \cdot \bar{F}}{1 - u_x v/c^2}$$

$$F_y = \frac{F'_y}{1 + u'_x v/c^2} \qquad F'_y = \frac{F_y}{1 - u_x v/c^2}$$

$$F_z = \frac{F'_z}{1 + u'_x v/c^2} \qquad F'_z = \frac{F_z}{1 - u_x v/c^2}$$

F_x depends on $\bar{u} \cdot \bar{F}$ the power developed by the force in another frame $\rightarrow$ power and force are related in 4D space, similar to momentum and energy.

The 4th relation that goes together with the force is:

$$\bar{u} \cdot \bar{F} = \frac{\bar{u}' \cdot \bar{F}' + vF_x}{1 + u'_x v/c^2} \qquad \bar{u}' \cdot \bar{F}' = \frac{\bar{u} \cdot \bar{F} - vF_x}{1 - u'_x v/c^2}$$

Special case: if a body is at rest ($u' = 0$) in the S' frame and it is subjected to a force with components F'_x, F'_y, F'_z

$$F_x = F'_x$$

$$F_y = \frac{F'_y}{\gamma} = \frac{F'_y}{\sqrt{1 - v^2/c^2}}$$

$$F_z = \frac{F'_z}{\gamma} = \frac{F'_z}{\sqrt{1 - v^2/c^2}}$$

The force is the gratest in the rest frame of the object and smaller in all other inertial frames.