Nomes:

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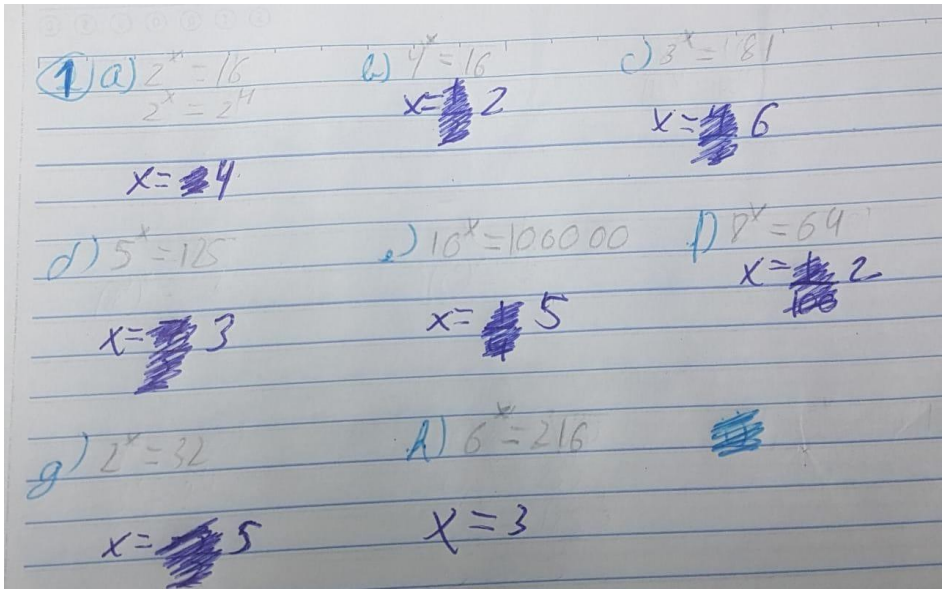


2. Use a definição para calcular:

a) $\log_2 \frac{1}{4}$

$2^x = \frac{1}{4}$

$2^x = \frac{1}{2^2}$

$2^x = 2^{-2}$

$x = -2$ //

b) $\log_3 \sqrt{3}$

$3^x = \sqrt{3}$

$3^x = 3^{\frac{1}{2}}$

$x = \frac{1}{2}$ //

c) $\log_8 16$

$8^x = 16$ ()

$(2^3)^x = 2^4$

$3x = 4$

$x = \frac{4}{3}$ //

d) $\log_4 128$

$4^x = 128$

$(2^2)^x = 2^7$

$2x = 7$

$x = \frac{7}{2}$ //

e) $\log_{36} \sqrt{6}$

$36^x = \sqrt{36}$

$(6^2)^x = 6^{\frac{1}{2}}$

$2x = \frac{1}{2}$

$x = \frac{\frac{1}{2}}{2} \rightarrow x = \frac{1}{4}$ //

f) $\log 0,01$

$10^x = 0,01$

$10^x = 10^{-2}$

$x = -2$ //

3. $A = \log_{25} 0,2$ | $B = \log_7 \frac{1}{49}$ | $C = \log_{0,25} \sqrt{8}$ | $D = \log 0,1$

$25^x = 0,2$	$7^x = \frac{1}{49}$	$0,25^x = \sqrt{8}$	$10^x = 0,1$
$5^{2x} = 5^{-1}$	$7^x = 7^{-2}$	$2^{-2x} = 2^{\frac{3}{2}}$	$x = -1$
$2x = -1 \rightarrow -\frac{1}{2}$	$x = -7$	$-2x = \frac{3}{2} \rightarrow x = -\frac{3}{4}$	

Resposta: $-7, -1, -\frac{3}{4}, -\frac{1}{2}$

4. a) $\log_5 5 + \log_3 1 - \log 10 \to 1 + 0 - 1 = 0$

b) $\log_{\frac{1}{4}} 4 + \log_4 \frac{1}{4} \to \log_{2^{-2}} 2^2 + \log_{2^2} 2^{-2} \to -1 - 1 = -2$

c) $\log 1000 + \log 100 + \log 10 + \log 1 \to 3 + 2 + 1 + 0 = 0$

d) $3^{\log_3 2} + 2^{\log_2 3} \to 2 + 3 = 5$

e) $\log_8 (\log_3 9) \to \log_8 2 \to \log_{2^3} 2 = \frac{1}{3}$

f) $\log_9 (\log_4 64) + \log_4 (\log_3 81) \to \log_9 3 + \log_4 4 \to$

$\to \log_{3^2} 3 + 1 \to \frac{1}{2} + 1 = \frac{3}{2}$

5. Sabendo que $\log a = 2$ e $\log b = -1$, calcule o valor de:

$\log_b(a) = \frac{\log(a)}{\log(b)}$

a) $\log_b a$

$\log_b(a) = \frac{2}{-1} = -2$ //

b) $\log_a b$

$\log_a b = \frac{-1}{2}$ //

c) $\log_a b^2$

$\log_a(b^2) = 2 \cdot \log_a(b)$

$\log_a(b^2) = 2 \cdot \frac{-1}{2} = -1$ //

d) $\log(a \cdot b)$

$\log(a \cdot b) = 2 - 1$

$\log(a \cdot b) = 1$ //

e) $\log\left(\frac{a}{b}\right)$

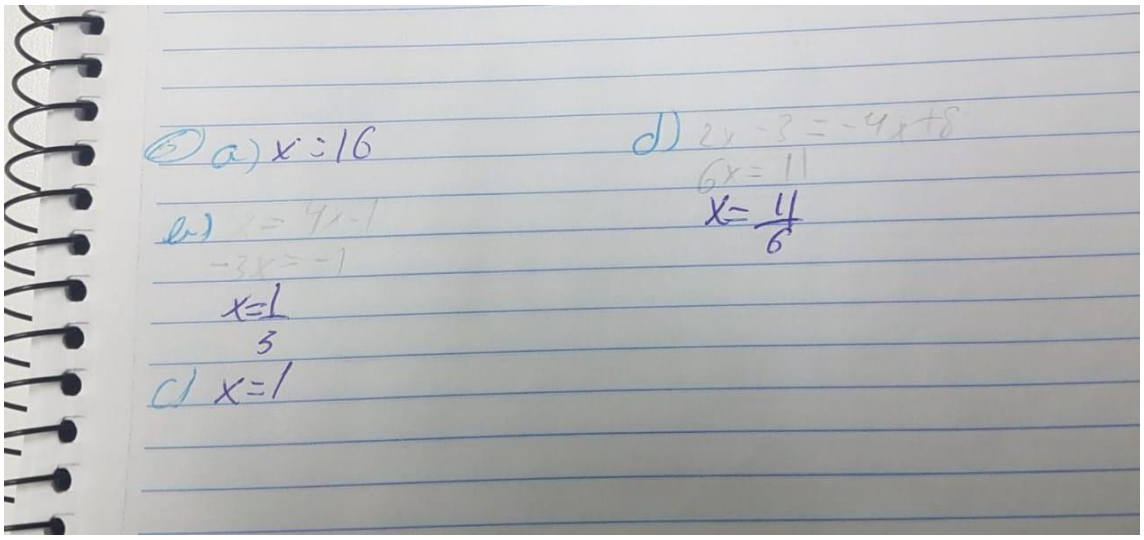
$\log \frac{a}{b} = 2 - (-1)$

$\log \frac{a}{b} = 3$ //

f) $\log_{\sqrt{b}} a$

$b > 0$

$x \in \mathbb{R}$



6) a) $x = 16$

b) $x = 4x - 1$

$-3x = -1$

$x = \frac{1}{3}$

c) $x = 1$

d) $2x - 3 = -4x + 8$

$6x = 11$

$x = \frac{11}{6}$

1. Determine o número real x tal que:

$\log_a b = x \Leftrightarrow b^x = b$

a) $\log_3 x = 4$

$3^4 = x$

$x = 3^4$

$x = 81$ //

b) $\log_{\frac{1}{2}} x = -2$

$\left(\frac{1}{2}\right)^{-2} = x$

$x = \left(\frac{1}{2}\right)^{-2}$

$x = 4$ //

c) $\log_x 2 = 1$

$x^1 = 2$

$x = 2$ //

d) $\log_x 0{,}25 = -1$

$x^{-1} = 0{,}25$

$\frac{1}{x} = \frac{25}{100} \Rightarrow 25x = 100$

$x = \frac{100}{25} = 4$ //

e) $\log_x 1 = 0$

$x^0 = 1$

$x \in \mathbb{R}$ //

f) $\log_3 (2x-1) = 2$

$3^2 = 2x - 1$

$2x - 1 = 9$

$2x = 9 + 1$

$2x = 10$

$x = \frac{10}{5} = 2$ //

8. a) $x = 1/25$

$\log_5 \frac{1}{25} = x$

$5^x = 5^{-2}$

$x = -2$

b) $x = \sqrt[7]{5}$

$\log_5 \sqrt[7]{5} = x$

$5^x = 5^{1/7}$

$x = 1/7$

c) $x = 5^{12}$

$\log_5 5^{12} = x$

$5^x = 5^{12}$

$x = 12$

d) $x = 1/\sqrt[9]{625}$

$\log_5 \frac{1}{\sqrt[9]{625}} = x$

$\log_5 \sqrt[9]{625}^{-1} = x$

$\log_5 (625^{1/9})^{-1} = x$

$\log_5 625^{-1/9} = x$

$5^x = (5^4)^{-1/9}$

$5^x = 5^{-4/9}$

$x = -4/9$

e) $x = 0,2$

$\log_5 0,2 = x$

$5^x = 0,2$

$5^x = 2/10$

$5^x = 1/5$

$5^x = 5^{-1}$

$x = -1$

9. $a = 1 \quad b = 4 \quad c = \log_2 m + 4$

Para ter raiz dupla, $\Delta = 0$

$\Delta = b^2 - 4 \cdot a \cdot c$

$\Delta = 4^2 - 4 \cdot 1 \cdot \log_2 m$

$\Delta = 16 - 4 \cdot \log_2 m$

como $\Delta = 0$

$16 - 4\log_2 m = 0$

$-4\log_2 m = -16$

$4\log_2 m = 16$

$\log_2 m = 16/4$

$\log_2 m = 4$

$\rightarrow m = 2^4 = 16$

$x = \frac{-b \pm \sqrt{\Delta}}{2a}$

$x = \frac{-4 \pm 0}{2} \rightarrow x_1 \text{ e } x_2 = -2$

10. a) $4^{3+\log_4 2}$

$(2^2)^{3+\log_2 2(2)} \rightarrow (2^2)^{3+\frac{1}{2}} \rightarrow 2^7$

b) $5^{1-\log_5 4}$

$5 \cdot 5^{-\log_5 4} \rightarrow 5 \cdot 5^{\log_5 4^{-1}} \rightarrow 5 \cdot \frac{1}{4} = \frac{5}{4}$

c) $8^{\log_2 7} \rightarrow (2^3)^{\log_2 7} \rightarrow 2^{\log_2 7^3} \rightarrow 7^3$

d) $81^{\log_3 2} \rightarrow 3^{4 \cdot \log_3 2} \rightarrow 3^{\log_3 2^4} = 2^4$

e) $5^{\log_{25} 7} \rightarrow 25^{\frac{1}{2} \cdot \log_{25} 7} \rightarrow 7^{\frac{1}{2}}$

12. a) $\log_b (x \cdot y)$

$\log_b x + \log_b y = -2 + 3$

1_z

b) $\log_b (x/y)$

$\log_b x - \log_b y$

$-2 - 3$

-1_z

c) $\log_b (x^3 \cdot y^2)$

$3 \log_b x + 2 \log_b y$

$3 \cdot (-2) + 2 \cdot 3$

0_z

d) $\log_b \left(\frac{y^2}{\sqrt{x}}\right)$

$2 \log_b y - \frac{1}{2} \log_b x$

$2 \cdot 3 - \frac{1}{2} \cdot -2$

7_z

e) $\log_b \left(\frac{x \cdot \sqrt{y}}{b}\right)$

$(\log_b x + \log_b y^{1/2}) - \log_b b$

$(-2 + \frac{1}{2} \log_b y) - \log_b b$

$-2 + \frac{1}{2} \cdot 3 - 1$

$-2 + 1{,}5 - 1$

$1{,}5_z$

f) $\log_b \sqrt{\sqrt{x} \cdot y^3}$

$\log_b (\sqrt{x} \cdot y^3)^{1/2}$

$\log_b (x^{1/2} \cdot y^3)^{1/2}$

$\frac{1}{4} \log_b x \cdot 1{,}5 \log y$

$\frac{1}{4} \cdot (-2) \cdot 1{,}5 \cdot 3$

$-0{,}5 \cdot 4{,}5$

$-2{,}25_z$

3) a) $\log_5(5a) - \log_5(bc)$

$(\log_5(5) + (\log_5(a)) - (\log_5(b)) + (\log_5(c))$

b) $(\log b + \log(b)) - (\log 10 + \log(a))$

c) $\log_3 a + \log_3 b - \log_3 c$

d) $\log_2 8 + \log_2 a - 3\log_2 b + 2\log_2 c$

$3 + \log_2 a - 3\log_2 b + 2\log_2 c$

19. a) $\log 6$

$\log 2.3$

$\log 2 + \log 3$

$a + b$ //

b) $\log 1,5$

$\log \frac{3}{2}$

$\log 3 - \log 2$

$b - a$ //

c) $\log 5$

$\log \frac{10}{2}$

$\log 10 - \log 2$

$1 - a$ //

d) $\log 30$

$\log 2.3.5$

$\log 2 + \log 3 + \log 5$

$a + b + (1 - a)$

$a + b + 1 - a$

$b + 1$ //

e) $\log \frac{1}{4}$

$\log 1 + \log 2^2$

$0 + 2.\log 2$

$2a$ //

f) $\log 72$

$\log 3^2 . 2^2 . 2$

$\log 3^2 + \log 2^2 + \log 2$

$2\log 3 + 2\log 2 + \log 2$

$2b + 2a + a$

$2b + 3a$ //

g) $\log 0,3$

$\log \frac{3}{10}$

$\log 3 - \log 10$

$b - 1$ //

h) $\log \sqrt[3]{1,8}$

$\log 1,8^{\frac{1}{3}}$

$\frac{1}{3}\log\left(\frac{18}{10}\right)$

$\frac{1}{3} \cdot (\log 18 - \log 10)$

$\frac{1}{3}(\log 3^2 . 2 - 1)$

$\frac{1}{3} \cdot (2\log 3 . a - 1)$

$\frac{2b}{3} + \frac{a}{3} - \frac{1}{3}$

i) $\log 0{,}024$

$\log \frac{3 \cdot 4}{1000} \cdot 10$

$\log \frac{3}{125}$

$\log 3 - \log 125$

$b - \log 5^3$

$b - 3 \cdot \log 5$

$b - 3 \cdot (1 - a)$

$b - 3 + 3a$

j) $\log 0{,}75$

$\log \frac{75}{100}$

$\log \frac{3}{4}$

$\log 3 - \log 4$

$b - \log 2^2$

$b - 2 \log 2$

$b - 2a$

K) $\log 20\,000$

$\log 2 \cdot 10^4$

$\log 2 + \log 10^4$

$\log 2 + 4 \log 10$

$a + 4 \cdot 1$

$a + \frac{4}{2}$

15. a) $\log a + \log b + \log c$

$\log a \cdot b \cdot c$

15. b) $3\log_2 a + 2\log_2 c - \log_2 b$

$(\log_2 a^3 \cdot c^2) - \log_2 b$

$\log_2 \left(\frac{a^3 c^2}{b}\right)$

c) $\log_3 a - \log_3 b - 2$

$\log_3 \frac{a}{b} - 2$

$\log_3 \frac{a}{b} / 2$

d) $\frac{1}{2} \cdot \log a - \log b$

$\log a^{\frac{1}{2}} - \log b$

$\log \sqrt{a} - \log b$

$\log \frac{\sqrt{a}}{b}$

16. a) $\log_{15} 3 + \log_{15} 5 \rightarrow \log_{15} 15 = 1$

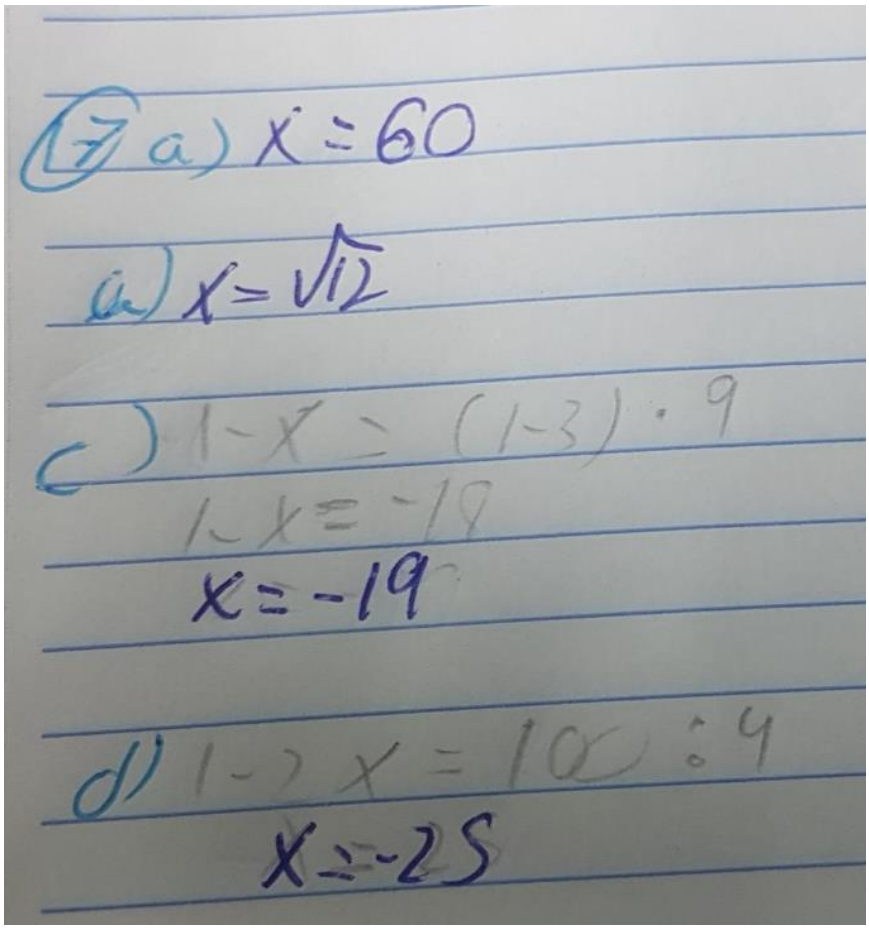
b) $\log_3 72 - \log_3 12 - \log_3 2 \rightarrow \log_3 \left(\frac{72}{12 \cdot 2}\right) \rightarrow \log_3 \left(\frac{6}{2}\right)$

$\log_3 3 \rightarrow 1$

c) $\frac{1}{3} \cdot \log_{15} 8 + 2 \cdot \log_{15} 2 + \log_{15} 5 - \log_{15} 9000$

$\log_{15} \left(\frac{8^{\frac{1}{3}} \cdot 2^2 \cdot 5}{9000}\right) \rightarrow \log_{15} \left(\frac{2^{\frac{3}{3}} \cdot 2^2 \cdot 5}{9000}\right) \rightarrow \log_{15} \left(\frac{2 \cdot 2^2 \cdot 5}{9000}\right) \rightarrow$

$\rightarrow \log_{15} \left(\frac{2^2 \cdot 5}{4500}\right) \rightarrow \log_{15} \left(\frac{2^2}{900}\right) \rightarrow \log_{15} (15^{-2}) = -2$


7) a) x = 60
b) x = √12
c) 1 - x = (1-3) · 9
1 - x = -18
x = -19
d) 1 - x = 10 : 4
x = -2,5

1º. Considerando os valores $\log 2 = 0{,}3$ e $\log 3 \approx 0{,}48$, calcule:

a) $\log 3000$

$\log 3 + \log 1000 = 0{,}48 + 3 =$
$3{,}48$ //

b) $\log 0{,}002$

$\log 2 - \log 1000 =$
$0{,}3 - 3 = -2{,}70$ //

c) $\log \sqrt{3}$

$\log 3^{\frac{1}{2}} \div 2 = 0{,}48 \div 2 = 0{,}24$ //

d) $\log 20$

$\log 2 + \log 10 =$
$0{,}30 + 1 = 1{,}30$ //

e) $\log 0{,}06$

$\log 6 \div 10 = \log 2 + \log 3 - \log 10$
$0{,}30 + 0{,}48 - 1 = -0{,}22$ //

f) $\log \sqrt[4]{2}$

$\frac{1}{4} \log 2 + \log 3$
$\frac{1}{4} \cdot 0{,}30 + 0{,}48 = 1{,}08$ //

g) $\log 125$

$3 \log 5 = 3 \cdot \log 10 \div 2 =$
$3 \log 10 - 3 \log 2 = 3 - 0{,}30 = 2{,}10$ //

19 a) $\log_2 10 = x$

$\log_2 (2 \cdot 5) = x$

$\log_2 2 + \log_2 5 = x$

$1 + 2{,}32 = x$

$3{,}32$

b) $\log_2 500 = x$

$\log_2 (2^2 \cdot 5^3) = x$

$\log_2 2^2 \cdot \log_2 5^3 = x$

$2\log_2 2 \cdot 3\log_2 5 = x$

$2 \cdot 1 + 3 \cdot 2{,}32 = x$

$x = 8{,}96$

c) $\log_2 1600 = x$

$\log_2 (2^6 \cdot 5^2) = x$

$\log_2 2^6 + \log_2 5^2 = x$

$6\log_2 2 + 2\log_2 5 = x$

$6 \cdot 1 + 2 \cdot 2{,}32 = x$

$x = 6 + 4{,}64$

$x = 10{,}64$

d) $\log_2 \sqrt[3]{0{,}2}$

$\log_2 \sqrt[3]{1/5}$

$\log_2 [(1/5)^{1/3}]$

$1/3 \log_2 1/5$

$1/3 (\log_2 1 - \log_2 5)$

$\frac{0 - 2{,}32}{3} \Rightarrow \log_2 \sqrt[3]{0{,}2} = -0{,}77$

e) $\log_2 (64/125) = x$

$\log_2 (2^6/5^3) = x$

$\log_2 2^6 - \log_2 5^3 = x$

$6\log_2 2 - 3\log_2 5 = x$

$6 \cdot 1 - 3 \cdot 2{,}32 = x$

$x = 6 - 6{,}96$

$x = 0{,}96$

20. a) $\log 26 = \log 20 + \log 6 \rightarrow \log 26 \neq \log(20 \cdot 6)$ (F)

b) $\log 5 + \log 8 + \log 2,5 = 2 \rightarrow \log(5 \cdot 8 \cdot 2,5) \rightarrow \frac{8 \cdot 8 \cdot 2,5}{10} \rightarrow$

$\frac{8 \cdot 2,5}{2} \rightarrow 4 \cdot 2,5 = 10 \neq 2$ (F)

c) $\log_2 4^{18} = 36 \rightarrow 18 \log_2 4 \rightarrow 18 \cdot 2 = 36$ (V)

d) $\log_3 \sqrt{\sqrt{3}} > 0,25 \rightarrow \frac{1}{27} \log_3 3 > 0,25 \rightarrow \frac{1}{27} > \frac{1}{4}$? (F)

e) $\log_5 35 - \log_5 7 = 1 \rightarrow \frac{35}{5} - \frac{7}{5} = 1$? (F)

f) $\log_3(\sqrt{2} + 1) + \log_3(\sqrt{2} - 1) = 0 \mid \log_3(2^{\frac{1}{2}} + 1) + \log_3(2^{\frac{1}{2}} - 1) \rightarrow$

$\frac{1}{2} \log_3(3) + \frac{1}{2} \log_3(1) \rightarrow \frac{1}{2} + 0 = 0$? (F)

21. (UFPR) Para determinar a rapidez com que se esquece de uma informação, foi efetuado um teste em que vinte palavras foram apresentadas a um grupo de pessoas e, uma manhã depois, verificou-se quantas (Pessoas) palavras eram lembradas. Uma análise mostrou que, de maneira aproximada, o percentual S de palavras, em função do tempo t, em minutos, após o teste foi efetuado, era dado pela expressão:

$$S = -19 \cdot \log(t+1) + 86$$

a) Após 9 minutos, que percentual da informação era lembrada?

$S = -19 \cdot \log(9+1) + 86$
$S = -19 \log 10 + 86$
$S = -19 \cdot 1 + 86$
$S = -19 + 86$
$S = 69\%$ R: 69%

b) Depois de quanto tempo o percentual S alcançou 50%?

$50 = -19 \cdot \log(t+1) + 86$
$-19 \cdot \log(t+1) = 86 - 50$
$-19 \cdot \log(t+1) = 36$
$\log(t+1) = \frac{36}{-19}$

$\log(t+1) = 2$
$10^2 = (t+1)$
$100 = t+1$

$t = 100 - 1$
$t = 99$ R: 99 minutos

22. a) $2 \cdot \log_3 (x+3) = \log_3 (x^2+45)$

$\log_3 (x+3)^2 = \log_3 (x^2+45)$

$x^2+6x+9 = x^2+45$

$x^2 - x^2 + 6x + 45 - 9$

$6x = 45 - 9$

$6x = 36$

$x = \frac{36}{6}$

$x = 6$

b) $\log (4x-1) - \log (x+2) = \log x$

$\log \frac{(4x-1)}{(x+2)} = \log x$

$\frac{4x-1}{x+2} = x$

$x \cdot (x+2) = 4x - 1$

$x^2 + 2x = 4x - 1$

$x^2 - 2x + 1 = 0$

$\Delta = b^2 - 4 \cdot a \cdot c$

$(-2)^2 - 4 \cdot 1 \cdot 1$

$4 - 4$

$\Delta = 0$

$x = \frac{-b \pm \sqrt{\Delta}}{2a}$

$x = \frac{-(-2) \pm 0}{2}$

$x' = \frac{2+0}{2} = \frac{2}{2} = 1$

$x'' = \frac{2-0}{2} = \frac{2}{2} = 1$

13. $\log_5 2 + \log_5 (x-1) = 0$

$\log_5 2^3 + \log_5 (x-1) = 0$

$\log_5 (8 \cdot (x-1)) = 0$

$5^0 = 8x - 8$

$1 = 8x - 8$

$8x - 8 - 1 = 0$

$8x - 9 = 0$

$8x = 9$

$x = \frac{9}{8}$

d) $2\log x = \log(2x-3) + \log(x+2)$

$\log x^2 = \log(2x-3)\cdot(x+2)$

$x^2 = 2x^2 + 4x - 3x - 6$

$x^2 = 2x^2 + x - 6$

$2x^2 + x - 6 - x^2 = 0$

$x^2 + x - 6 = 0$

$\Delta = b^2 - 4.a.c$

$\Delta = 1^2 - 4.1.(-6)$

$\Delta = 1 + 24$

$\Delta = 25$

$x = \frac{-b \pm \sqrt{\Delta}}{2a}$

$x = \frac{-1 \pm \sqrt{25}}{2}$

$x = \frac{-1 \pm 5}{2}$

$x_1 = \frac{-1+5}{2} = \frac{4}{2} = 2$

$x_2 = \frac{-1-5}{2} = \frac{-6}{2} = -3$

$S = \{2, -3\}$

e) $\log x + \log x^2 + \log x^3 = -6$

$\log(x \cdot x^2 \cdot x^3) = -6$

$\log_{10} x^6 = -6$

$10^{-6} = x^6$

$\frac{1}{10^6} = x^6$

$x = \frac{1}{10}$

23. a) $\begin{cases} x+y=10 \\ \log_4 x + \log_4 y = 2 \end{cases}$

$\log_4 (x \cdot y) = 2$

$4^2 = xy$

$xy = 16$

$(10-y)y = 16$

$10y - y^2 = 16$

$-y^2 + 10y - 16 = 0$

$y^2 - 10y + 16 = 0$

$x + y = 10$

$x = 10 - y$

$\Delta = b^2 - 4 \cdot a \cdot c$

$(-10)^2 - 4 \cdot 1 \cdot 16$

$100 - 64$

36

$y = \frac{-b \pm \sqrt{\Delta}}{2a}$

$\frac{-(-10) \pm \sqrt{36}}{2} \rightarrow \frac{10 \pm 6}{2}$

$\frac{16}{2} \rightarrow 8$

$\frac{4}{2} \rightarrow 2$

b) $\begin{cases} 4^{x-y} = 8 \\ \log_2 x - \log_2 y = 2 \end{cases}$

$\log_2 \left(\frac{x}{y}\right) = 2$

$2^2 = \frac{x}{y}$

$\frac{x}{y} = 4$

$x = 4y$

$3/2 + y = 4y$

$3/2 = 3y$

$y = (3/2)/3$

$y = (3/2) \cdot (1/3)$

$y = 1/2$

$4^{x-y} = 8$

$2^{2(x-y)} = 2^3$

$x - y = 3/2$

$x = 3/2 + y$

$x = 3/2 + y$

$x = 3/2 + 1/2$

$x = 4/2 \rightarrow 2$

$S = \{2, 1/2\}$