Stephen Dang
Chase Tolputt
Lucas Flynn

MXB242 EXPERIMENTAL STUDY

Middle Age Engineering

TREBUCHET RANGE OPTIMISATION:

IS INCREASING WEIGHT OR SWING-ARM LENGTH MORE EFFECTIVE?

AGENDA

Project Background

Exploratory Analysis

Formal Analysis

Results

PROJECT BACKGROUND

CONTEXT OF INVESTIGATION

- RockStellar: The weapons research and development company
- Improve the performance of RockStellar's defense missile systems
- Optimizing the design of the trebuchet
- The key variables influencing the projectile distance of trebuchet

QUESTIONS TO BE ANSWERED:

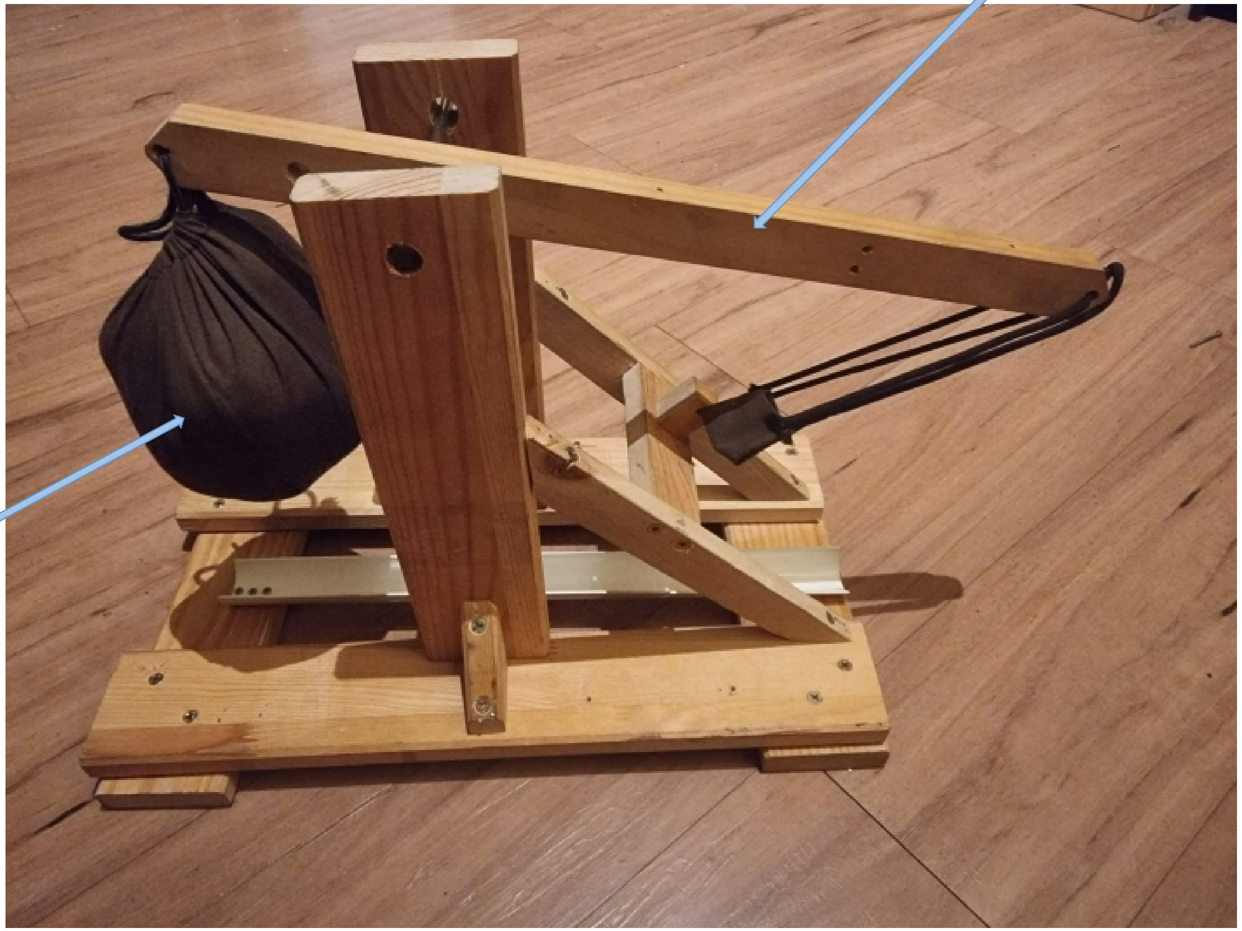
1. What is the best way to maximize the horizontal launching distance of a trebuchet?
2. Is there a significant difference in the trebuchet’s effectiveness by changing the length of the swing arm?
3. How much does the variance in counterweight affect the horizontal launching distance?
4. What effect does the projectile's weight have in consideration of its design?
5. Are there any interaction between counterweight and other predictors?

VARIABLES

PREDICTOR VARIABLES	RESPONSE VARIABLE
Swing-Arm Length	Distance
Counterweight	
Projectile weight	

VARIABLES

Swing Arm



Counterweight

Projectile (Marble)

LIMITATIONS AND ASSUMPTIONS

Assumptions

- The angle of release is constant across replicate(violated)
- The structure of the trebuchet is rigid and has no varying effects on the projectile launch

Limitations

- Many other factors are not included in the experiment, such as release angle and sling length because of the complexity of the experiment
- Only sand is the counterweight
- Using only one type of perfectly spherical object for payload does not accurately represent variation of real-world payloads

EXPERIMENTAL DESIGN

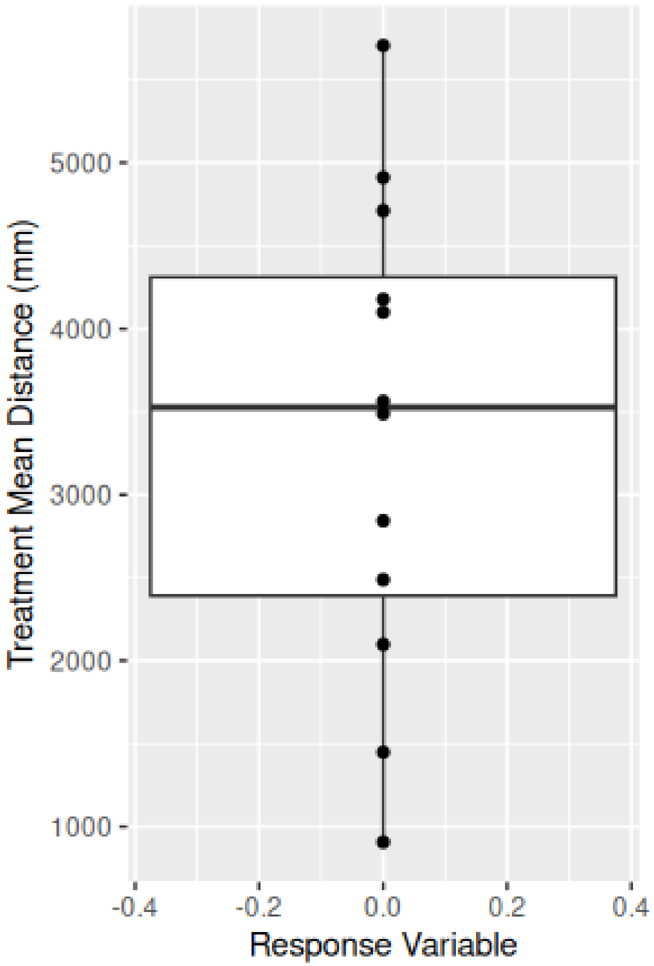
Factorial Design 3x2x2

Factor	Variable Type	Unit	Level	Factor Type
Swing Arm Length	Predictor	mm	320, 400	Continuous
Counterweight	Predictor	g	450, 600, 750	Continuous
Projectile weight	Predictor	g	6.5, 19.5	Continuous
Distance	Response	mm		Continuous

EXPLORATORY ANALYSIS

EXPLORATORY ANALYSIS

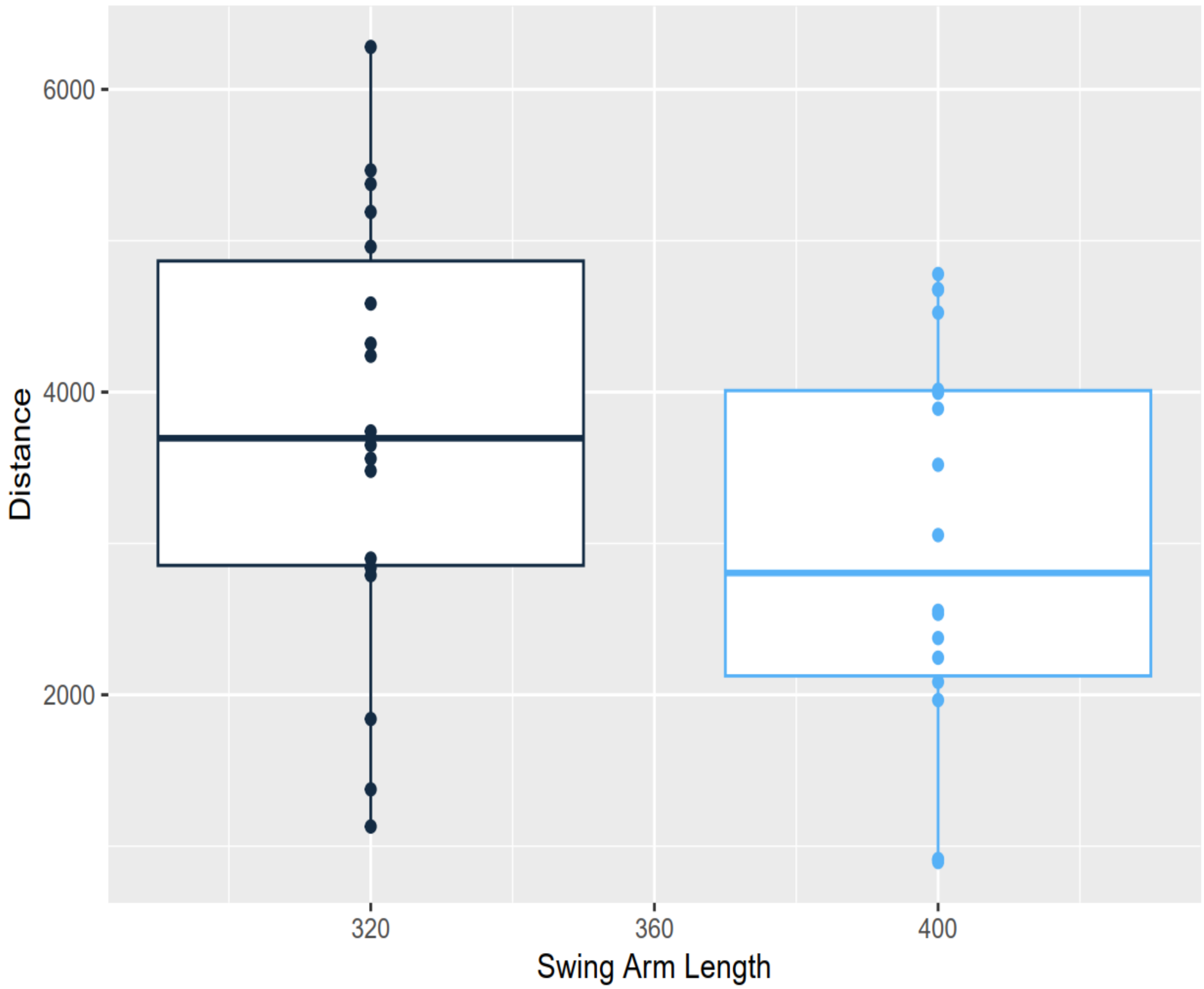
Treatment Mean Distance



Summary	Distance mm
Min	895
1st Quartile	2342
Median	3540
Mean	3370
3rd Quartile	4540
Max	6280

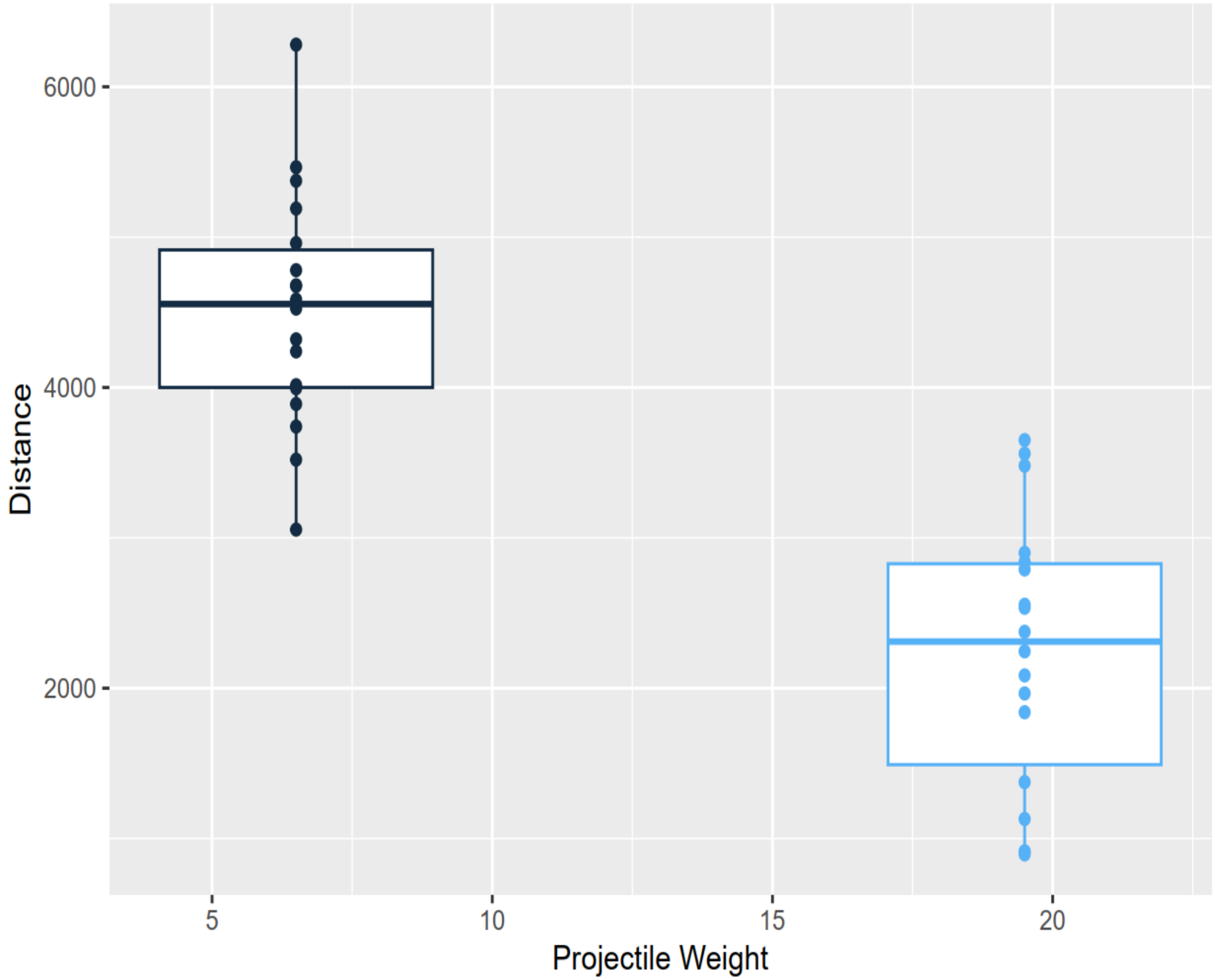
EXPLORATORY ANALYSIS

Swing-Arm Length vs Distance



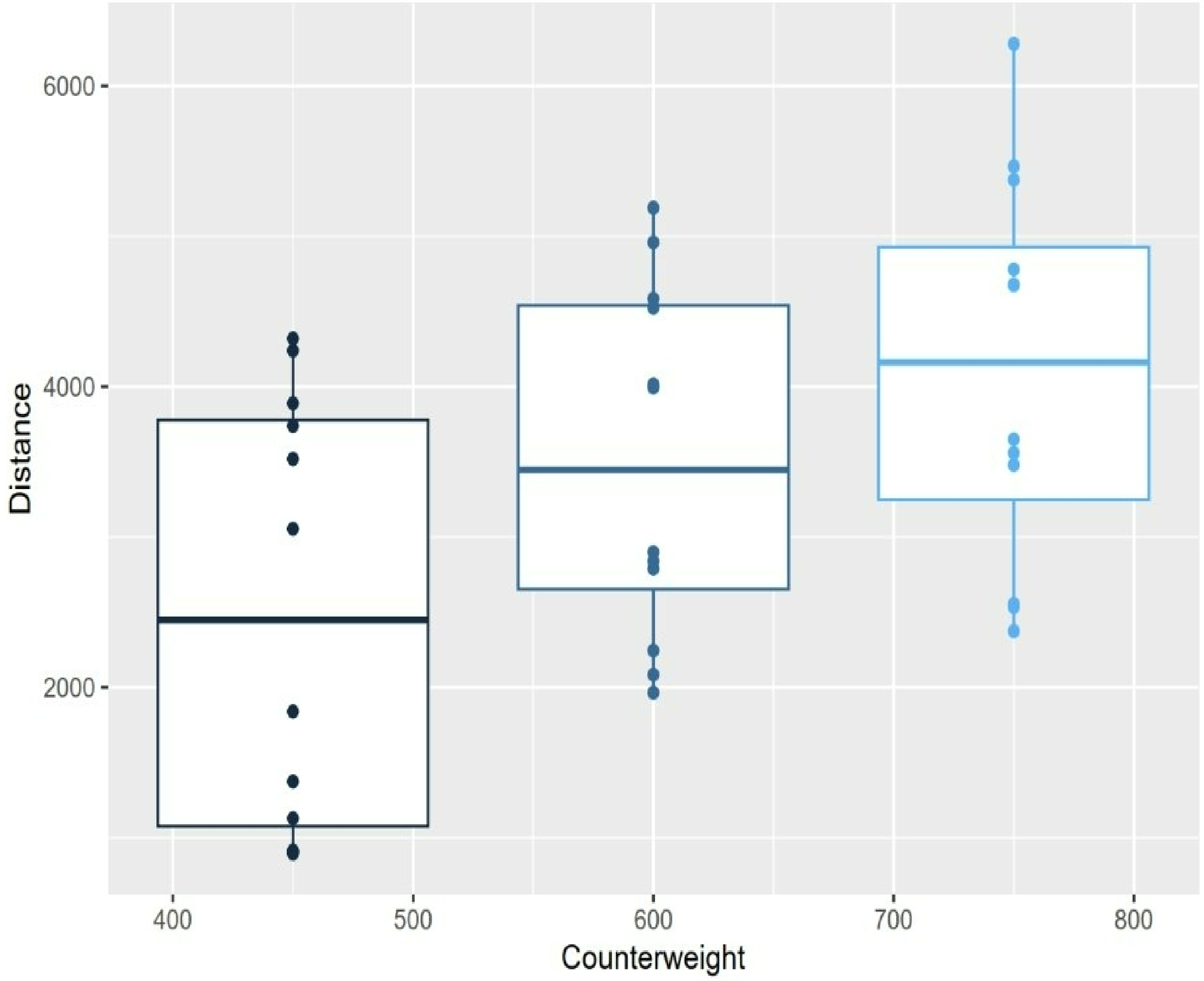
EXPLORATORY ANALYSIS

Projectile Weight vs Distance



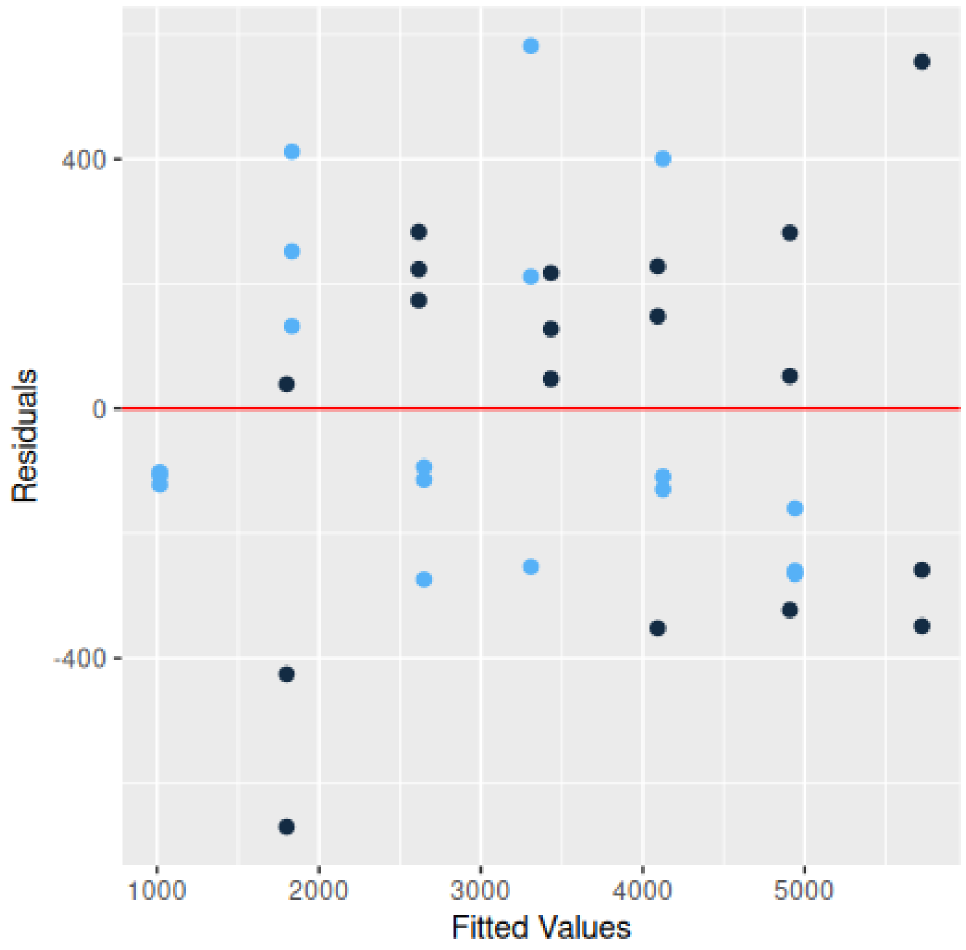
EXPLORATORY ANALYSIS

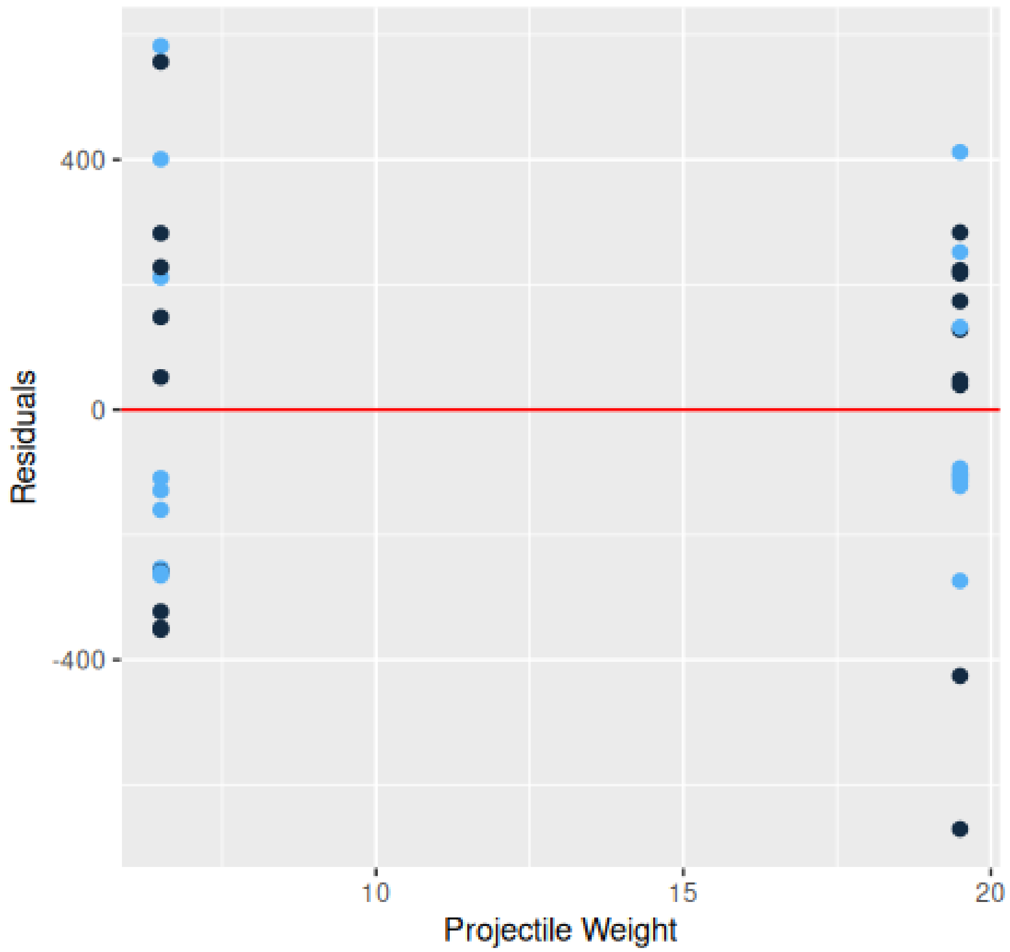
Counterweight vs Distance



EXPLORATORY ANALYSIS

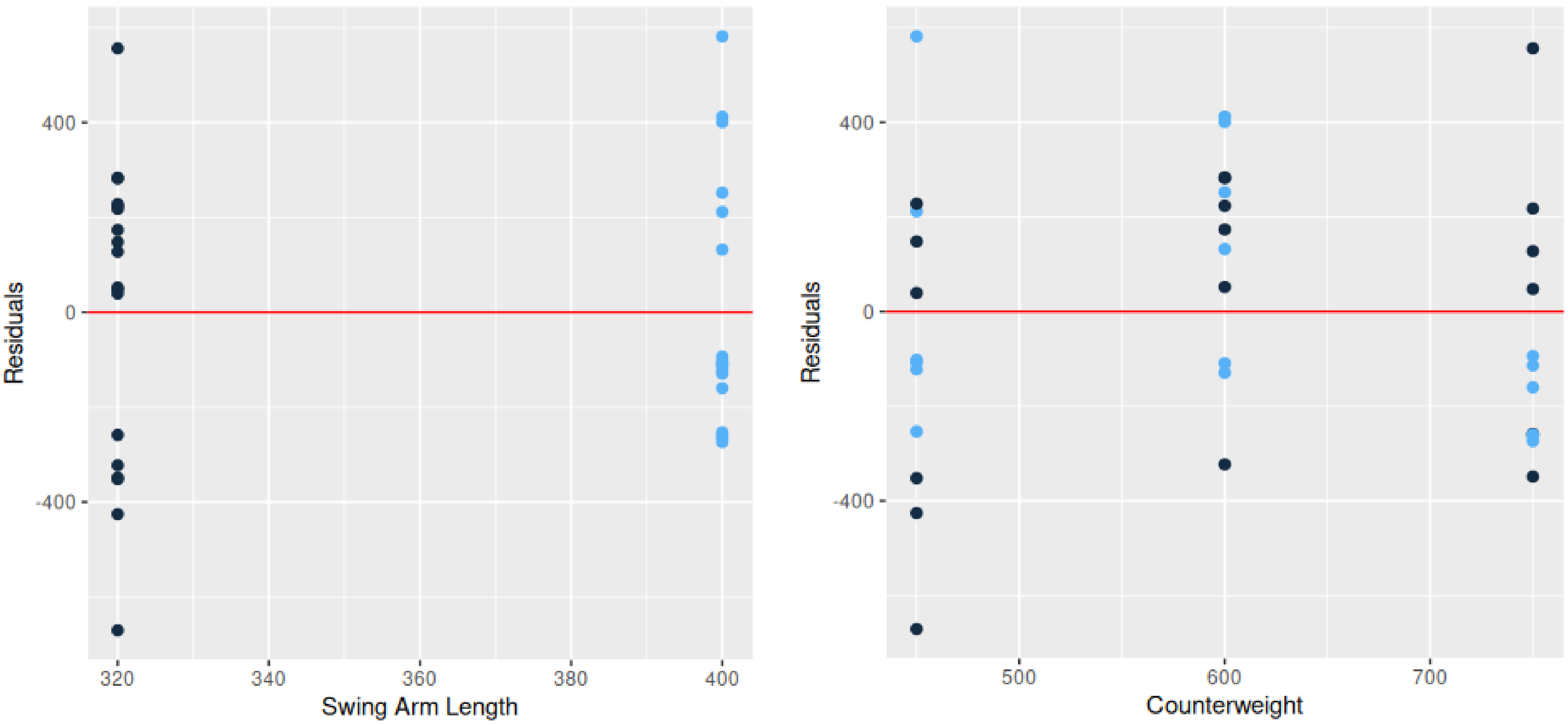
Residuals vs fitted





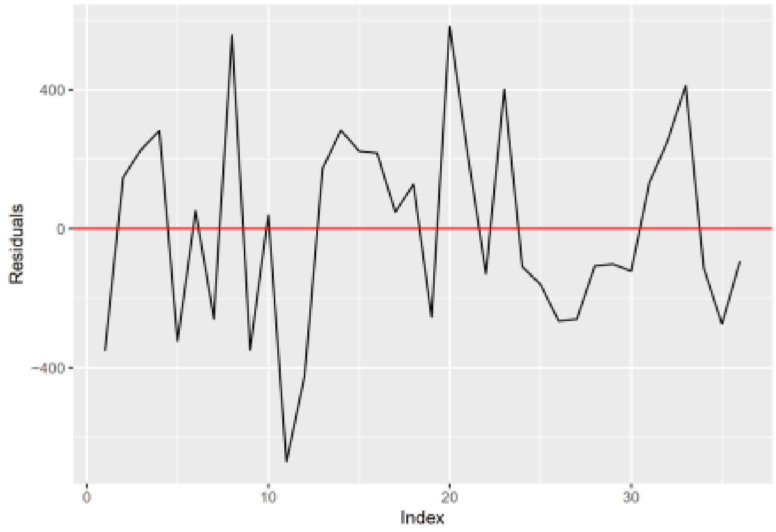
EXPLORATORY ANALYSIS

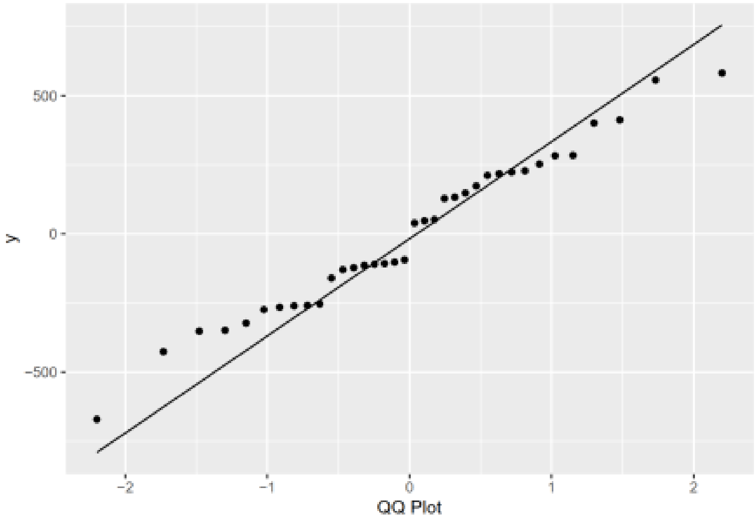
Residuals vs Predictors

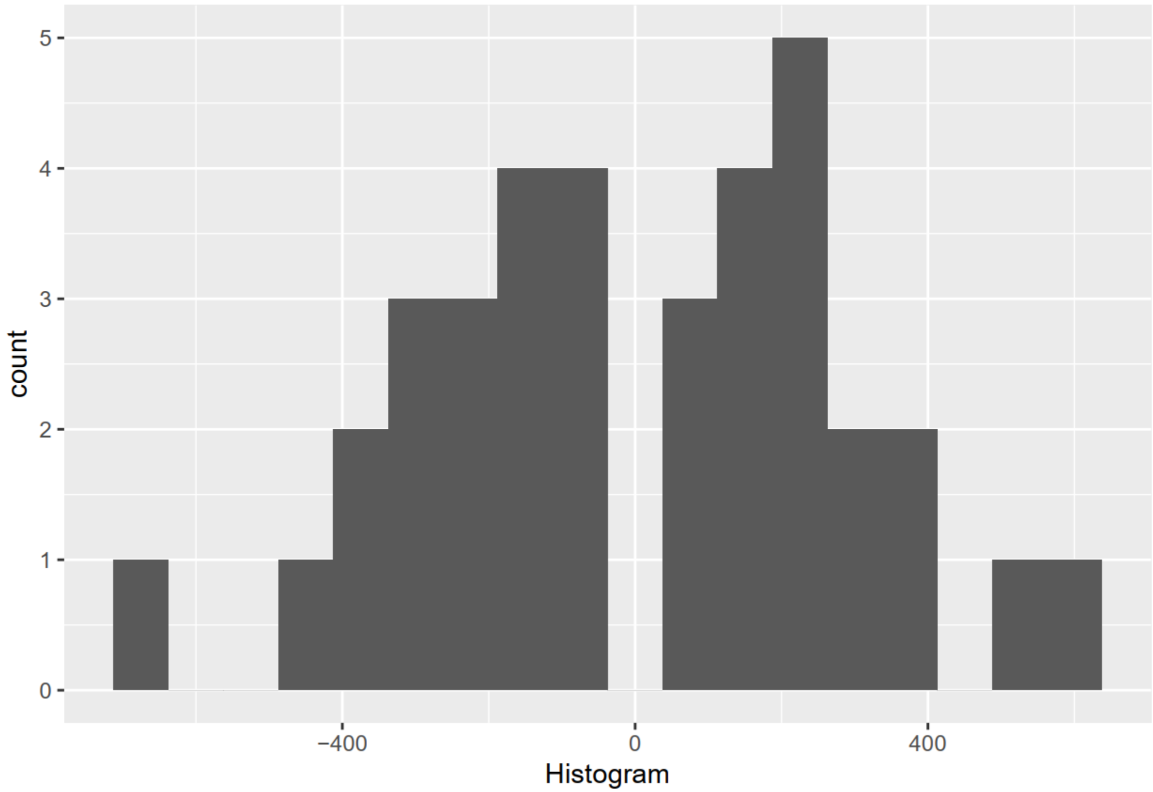


EXPLORATORY ANALYSIS

Plot to check assumptions







EXPLORATORY ANALYSIS

ASSUMPTIONS RESULTS

Linearity: Linearity looks good, there doesn't seem to be any obvious curvature or patterns in the residual plots. It does look like there may be a couple of outliers which will be checked in the formal analysis.

Independence: Independence plot looks good; Independence is satisfied by the context of the experiment.

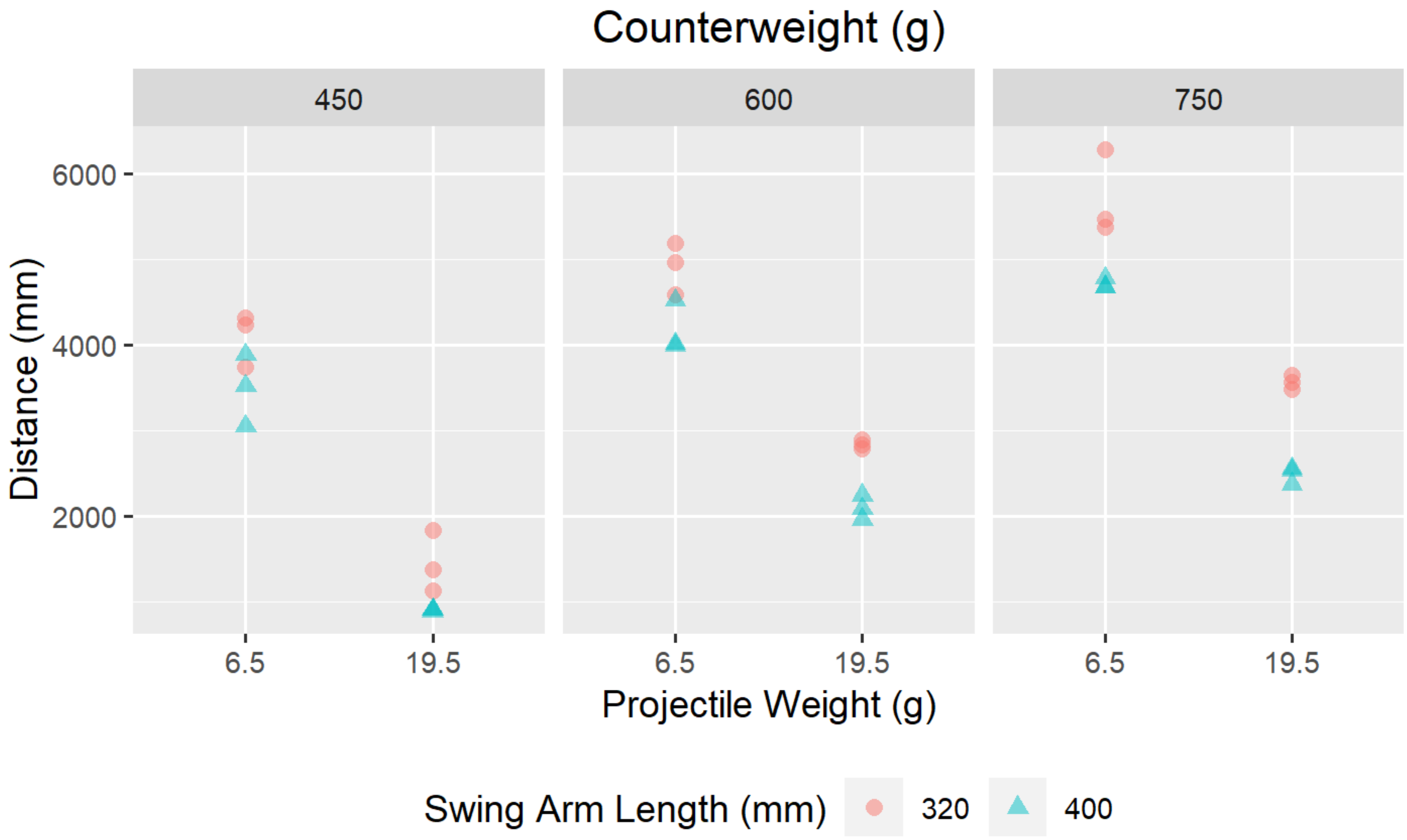
Homogeneity: Looks ok here, there may be some fanning/tapering in a couple of the residual plots.

Normality: The QQ plot shows that there looks to be some minor violations in normality. The histogram has no major skew but does have some gaps.

FORMAL ANALYSIS

TREATMENTS SNAPSHOT

Each of the 12 treatments has distinct launch distance results, supporting the literature.



SIMPLE LINEAR MODEL

```
Call:
lm(formula = Dist ~ CW + SAL + PW, data = df)

Residuals:
    Min      1Q  Median      3Q     Max 
-670.69 -254.79  -27.22  219.10  581.53 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept) 5924.7222   528.5322  11.210 1.29e-12 ***
CW             5.4389     0.4115  13.218 1.65e-14 ***
SAL           -9.7951     1.2598  -7.775 7.24e-09 ***
PW          -176.2607     7.7529 -22.735  < 2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 302.4 on 32 degrees of freedom
Multiple R-squared:  0.9592,	Adjusted R-squared:  0.9554 
F-statistic: 250.7 on 3 and 32 DF,  p-value: < 2.2e-16
```

BACKWARDS STEP-WISE REGRESSION

```
Start:  AIC=414.65
Dist ~ CW + SAL + PW + CW/PW + PW * SAL + CW * SAL + PW * CW *
    SAL

            Df Sum of Sq     RSS    AIC
- CW:SAL:PW  1    8437.5 2328629 412.78
<none>                   2320192 414.65

Step:  AIC=412.78
Dist ~ CW + SAL + PW + CW:PW + SAL:PW + CW:SAL

         Df Sum of Sq     RSS    AIC
- SAL:PW  1       117 2328747 410.78
<none>                2328629 412.78
- CW:PW   1    281667 2610296 414.89
- CW:SAL  1    315104 2643733 415.35

Step:  AIC=410.78
Dist ~ CW + SAL + PW + CW:PW + CW:SAL

         Df Sum of Sq     RSS    AIC
<none>                2328747 410.78
- CW:PW   1    281667 2610413 412.89
- CW:SAL  1    315104 2643851 413.35
```

STEP-WISE MODEL SUMMARY

```
Call:
lm(formula = Dist ~ CW + SAL + PW + CW:PW + CW:SAL, data = dataset)

Residuals:
    Min      1Q  Median      3Q     Max 
-476.39 -140.80  -39.72  226.15  550.00 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)    
(Intercept)  2.666e+03  2.153e+03   1.238  0.22518    
CW           1.087e+01  3.516e+00   3.091  0.00428 ** 
SAL          1.663e+00  5.804e+00   0.287  0.77643    
PW          -2.429e+02  3.572e+01  -6.801 1.52e-07 ***
CW:PW        1.111e-01  5.833e-02   1.905  0.06642 .  
CW:SAL      -1.910e-02  9.479e-03  -2.015  0.05297 .  
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 278.6 on 30 degrees of freedom
Multiple R-squared:  0.9675,    Adjusted R-squared:  0.9621 
F-statistic: 178.7 on 5 and 30 DF,  p-value: < 2.2e-16
```

PARTIAL F-TEST

```
{r}
# Take out CW:SA and see if it's statistically significant.
reduced.back.step <- lm(Dist ~ CW + PW + SAL + CW / PW, data = dataset)
anova(reduced.back.step, back.step)
```

```
Analysis of Variance Table

Model 1: Dist ~ CW + PW + SAL + CW/PW
Model 2: Dist ~ CW + SAL + PW + CW:PW + CW:SAL
  Res.Df     RSS Df Sum of Sq      F  Pr(>F)
1     31 2643851
2     30 2328747  1    315104 4.0593 0.05297 .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Statistically insignificant

REDUCED STEP-WISE MODEL

```
Call:
lm(formula = Dist ~ CW + PW + SAL + CW/PW, data = dataset)

Residuals:
    Min      1Q  Median      3Q     Max 
-562.36 -168.12    3.75  154.10  664.58 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept) 6791.38889  698.58616   9.722 6.26e-11 ***
CW             3.99444    0.88864   4.495 9.08e-05 ***
PW          -242.92735   37.44062  -6.488 3.08e-07 ***
SAL           -9.79514    1.21682  -8.050 4.33e-09 ***
CW:PW          0.11111    0.06114   1.817   0.0788 .  
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 292 on 31 degrees of freedom
Multiple R-squared:  0.9631,    Adjusted R-squared:  0.9584 
F-statistic: 202.4 on 4 and 31 DF,  p-value: < 2.2e-16
```

COMPARISON OF MODELS

Model	AIC	Adj R^2	RSE	DF
Simple Linear Model	519.16	0.9554	302.4	32
Backwards Step-wise Model	514.95	0.9621	278.6	30
Reduced Backwards Step-wise Model	517.52	0.9584	292	31

Based on the partial F-test,

FINAL MODEL: REDUCED STEP-WISE MODEL

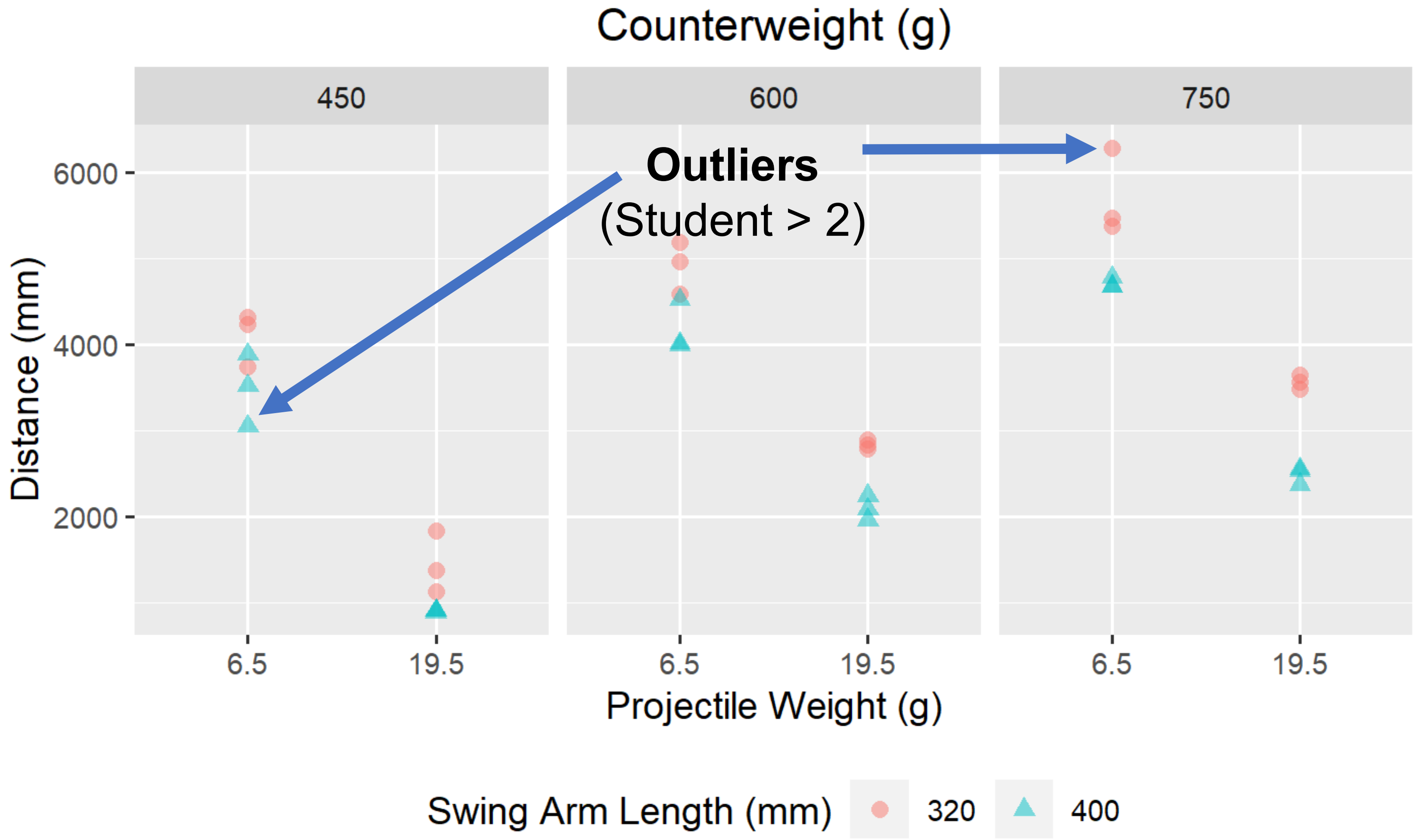
```
Call:
lm(formula = Dist ~ CW + PW + SAL + CW/PW, data = dataset)

Residuals:
    Min      1Q  Median      3Q     Max 
-562.36 -168.12    3.75  154.10  664.58 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept) 6791.38889  698.58616   9.722 6.26e-11 ***
CW             3.99444    0.88864   4.495 9.08e-05 ***
PW          -242.92735   37.44062  -6.488 3.08e-07 ***
SAL           -9.79514    1.21682  -8.050 4.33e-09 ***
CW:PW          0.11111    0.06114   1.817   0.0788 .  
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 292 on 31 degrees of freedom
Multiple R-squared:  0.9631,    Adjusted R-squared:  0.9584 
F-statistic: 202.4 on 4 and 31 DF,  p-value: < 2.2e-16
```

UNUSUAL OBSERVATIONS



RESULTS

ANSWER THE QUESTIONS

Shorter swing arm length

Heaviest counterweight

Smaller projectile weight

The best design of the trebuchet

The interaction between counterweight and projectile weight

The critical role of counterweight in the design of trebuchet

CONCLUSION

- Helping RockStellar with its new defence system
- There are some limitations from our experiment to consider
- Overall, RockStellar company is happy with The Middle Age Engineer experiment’s results

THANK YOU

The Middle Age Engineering

Chase

Stephen

Lucas