Ch3 (2018 02 19 22 29 25 UTC)

COMPUTER COMMUNICATION NETWORKS (University of Ottawa)

Solutions to Chapter 3

1. Suppose an uncompressed text file is 1 megabyte in size.

Solutions follow questions:

a. How long does it take to download the file over a 32 kilobit/second modem?

T_{32k} = 8(1024)(1024) / 32000 = 262.144 seconds

b. How long does it take to take to download the file over a 1 megabit/second modem?

T_{1M} = 8 (1024) (1024) bits / 1x10^6 bits/sec = 8.38 seconds

c. Suppose data compression is applied to the text file. How much do the transmission times in parts (a) and (b) change?

If we assume a maximum compression ratio of 1:6, then we have the following times for the 32 kilobit and 1 megabit lines respectively:

T_{32k} = 8 (1024) (1024) / (32000 x 6) = 43.69 sec
T_{1M} = 8 (1024) (1024) / ($1\text{x}10^{6\text{ x}}$6) = 1.4 sec

2. A scanner has a resolution of 600 x 600 pixels/square inch. How many bits are produced by an 8-inch x 10-inch image if scanning uses 8 bits/pixel? 24 bits/pixel?

Solution:

The number of pixels is 600x600x8x10 = $28.8\text{x}10^6$ pixels per picture.
With 8 bits/pixel representation, we have: $28.8\text{x}10^6$ x 8 = 230.4 bits per picture.
With 24 bits/pixel representation, we have: $28.8\text{x}10^6$ x 8 = 691.2 bits per picture.

3. Suppose a computer monitor has a screen resolution of 1200 x 800 pixels. How many bits are required if each pixel uses 256 colors? 65,536 colors?

Solution:

The number of pixels is 1200 x 800 = 960000 pixels per screen.
With 256 colors, we have: 9.6x105 x $\log_2 256$ = 7.68 x 106 bits per screen.
With 65536 colors, we have: 9.6x105 x $\log_2 65536$ = 15.36 x 106 bits per screen.

4. Explain the difference between facsimile, GIF, and JPEG coding. Give an example of an image that is appropriate to each of these three methods.

Solution:

Facsimile scans a black-and-white image into rows of black and white dots, which are then compressed. The scanning process introduces distortion in the sense that the scanned image is not the same as the original image.

GIF takes image data, in binary form, and applies a noiseless data compression scheme.

JPEG involves scanning an image and applying a noisy compression scheme that is designed so that the distortion is not visible.

5. A digital transmission system has a bit rate of 45 Megabits/second. How many PCM voice calls can be carried by the system?

Solution:

PCM channels = (45×10^6 bits/sec) / (64×10^3 bits/sec channel) = 703 channels.

6. Suppose a storage device has a capacity of 1 gigabyte. How many 1-minute songs can the device hold using conventional CD format? using MP3 coding?

Solution:

A stereo CD signal has a bit rate of 1.4 megabits per second, or 84 megabits per minute, which is approximately 10 megabytes per minute. Therefore a 1 gigabyte storage will hold 1 gigabyte/10 megabyte = 100 songs.

An MP3 signal has a lower bit rate than a CD signal by about a factor of 14, so 1 gigabyte storage will hold about 1400 songs.

7. How many high-quality audio channels could be transmitted using an HDTV channel?

Solution:

An audio channel is about 380 kbps and an HDTV channel is about 38 Mbps, so an HDTV channel can carry about 100 such audio channels. The number of audio channels increases by a factor of 10 if more compression is applied to the audio signals.

8. How many HDTV channels can be transmitted simultaneously over the optical fiber transmission systems in Table 3.3?

Solution:

Suppose that an optical fiber carries 1600×10^9 bps, and an HDTV channel is about 38 Mbps, then the fiber can carry about 1600000/38 = 40,000 HDTV channels.

9. Comment on the properties of the sequence of frame images and the associated bit rates in the following examples.

Solutions follow questions:

a. A children's cartoon program.

The frames consist of relatively simple objects and static backgrounds with little fast motion, so the differences between consecutive frames are small. Therefore these sequences are highly compressible.

b. A music video.

The frames consist of relatively complex images that may consist of the overlay/combination of multiple images, and there can be fast motion and sudden changes in scenes, so the differences between consecutive frames can be high. Therefore these sequences are not highly compressible.

c. A tennis game; a basketball game.

Both tennis and basketball have fast motion. In tennis there are fewer moving objects that in basketball, and in basketball the background can be more complex. Both sequences are not easy to compress.

d. A documentary on famous paintings.

The sequences are relatively slowly changing, with much panning and zooming. The degree of detail/complexity of the images can be high. The bit rate depends on the detail in the images.

10. Suppose that at a given time of the day, in a city with a population of 1 million, 1% of the people are on the phone.

Solutions follow questions:

a. What is the total bit rate generated by all these people if each voice call is encoded using PCM?

The number of people on the phone at a given time is 106x10 – 2 =104, so the total bit rate is 104x64x103 = 640 megabits per second.

b. What is the total bit rate if all of the telephones are replaced by H.261 videoconferencing terminals?

If the videoconferencing bit rate is 64 kbps, then the total bit rate generated by the city is unchanged. However is high-quality videoconferencing is used and the bit rate is increased to 1.5 Mbps, then the total bit rate increases to 15 Gbps.

11. Consider an analog repeater system in which the signal has power σ_x^2 and each stage adds noise with power σ_n^2. For simplicity assume that each repeater recovers the original signal without distortion but that the noise accumulates. Find the SNR after n repeater links. Write the expression in decibels: SNR dB = 10 $\log_{10}$SNR.

Solution:

After n stages, the signal power is πx2 and the noise power is $n\pi n^2$, so the SNR is:

SNR dB = 10 $\log_{10}\sigma_x^2/n\sigma_n^2$ = 10 $\log_{10}\sigma_x^2/\sigma_n^2$ +10 $\log_{10}1/n$ = 10 $\log_{10}\sigma_x^2/\sigma_n^2$ – 10 $\log_{10}n$

12. Suppose that a link between two telephone offices has 50 repeaters. Suppose that the probability that a repeater fails during a year is 0.01, and that repeaters fail independently of each other.

Solutions follow questions:

a. What is the probability that the link does not fail at all during one year?

Let p be the probability that a repeater fails during a year, then 1 – p is the probability that it does not fail, and the probability that all 50 repeaters do not fail is (1 – .01) 50 ≈ e^{-50}(.01) = 0.605 where we have used the approximation (1 – p)n ≈ e^{-np} which is valid for large n and small p.

b. Repeat (a) with 10 repeaters; with 1 repeater.

The probability that all 10 repeaters do not fail is (1 – .01) 10 ≈ e^{-10}(.01) = 0.905, and the probability that a single repeater does not fail is 0.99. The moral of the calculations is that a system that requires the functioning of a large number of relatively reliable components may be fairly unreliable. In terms of repeaters, this implies that minimizing the number of repeaters needed in a link is important from the point of view of reliability. Of course this also reduces the cost expended to install and maintain the repeaters.

13. Suppose that a signal has twice the power as a noise signal that is added to it. Find the SNR in decibels. Repeat if the signal has 10 times the noise power? 2^n times the noise power? 10^k times the noise power?

Solution:

SNR dB = 10 $\log_{10}\sigma_x^2/\sigma_n^2$ = 10 $\log_{10}2$ = 3.01 dB

SNR dB = 10 $\log_{10}10$ = 10 dB

SNR dB = 10 $\log_{10}2n$ = = 10n $\log_{10}2$ = 3.01n dB

SNR dB = 10 $\log_{10}10k$ = 3.01k dB

14. A square periodic signal is represented as the following sum of sinusoids:

$$g(t) = \frac{2}{\pi}\sum_{k=0}^{\infty}\frac{(-1)^k}{2k+1}\cos(2k+1)\pi t$$

Solutions follow questions:

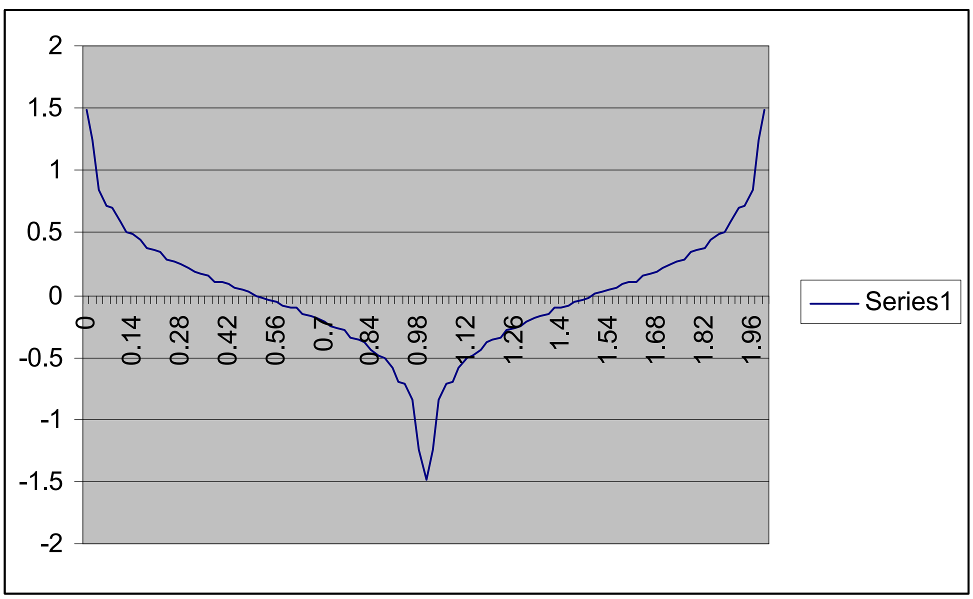
a. Suppose that the signal is applied to an ideal low-pass filter with bandwidth 15 Hz. Plot the output from the low-pass filter and compare to the original signal. Repeat for 5 Hz; for 3 Hz. What happens as W increases?

If we expand the above series to obtain the first few terms of $g(t)$, we get:

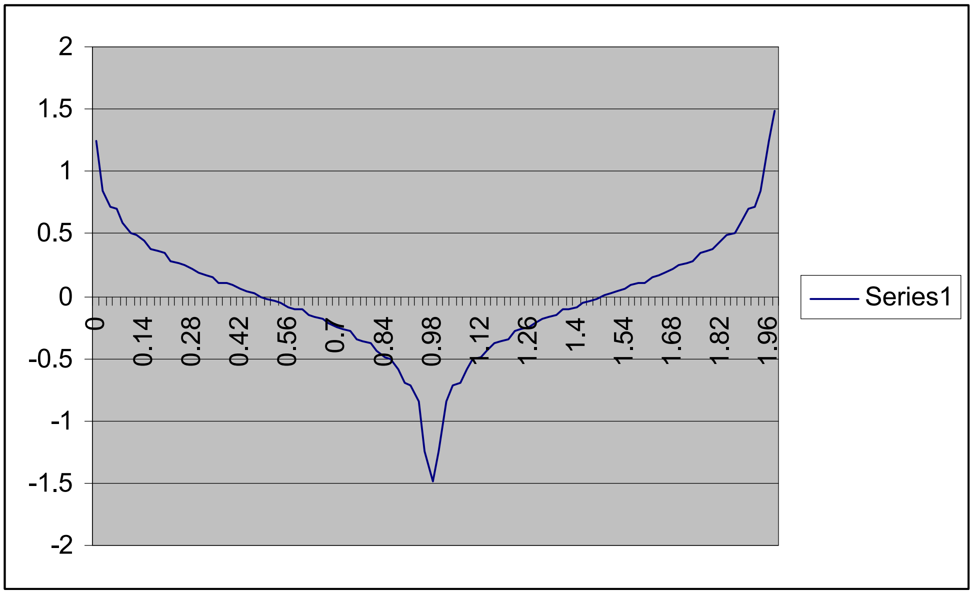
$g(t) = 2/\pi \{ \cos\pi t - (1/3)\cos 3\pi t + (1/5)\cos 5\pi t - (1/7)\cos 7\pi t + (1/9)\cos 9\pi t - \ldots\}$

so frequencies of the components in Hz are 1/2, 3/2, 5/2, 7/2, 9/2, and so on.

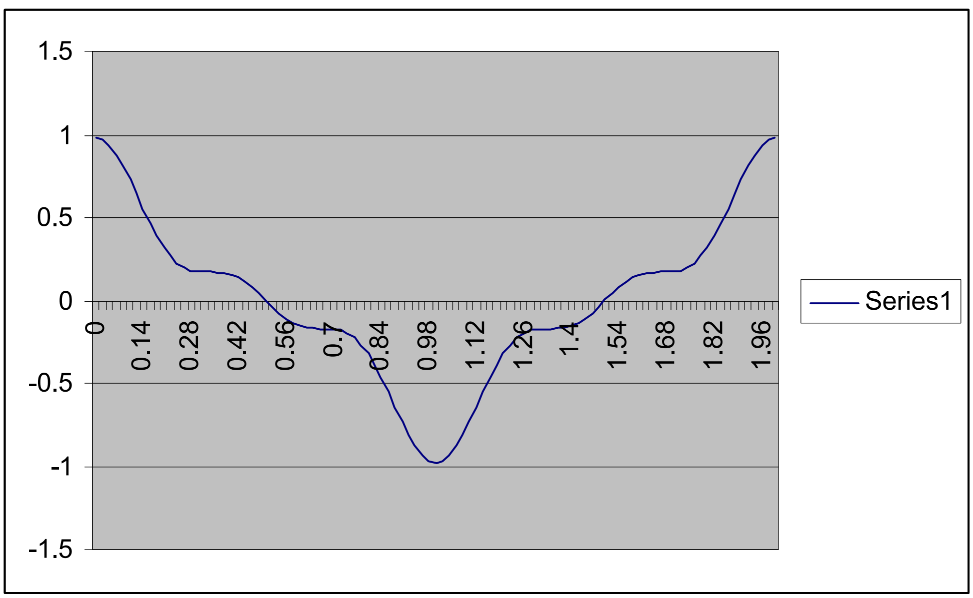
If the signal is applied to a low pass filter with bandwidth 15 Hz, then the output will consist of the sum of all components up to frequency 14.5 Hz, that is, up to component $(2k + 1)/2$ = 14.5, which gives k = 14. The resulting signal is shown below.



If the lowpass filter has bandwidth 5 Hz, then the first four components are passed. The resulting signal is shown below:

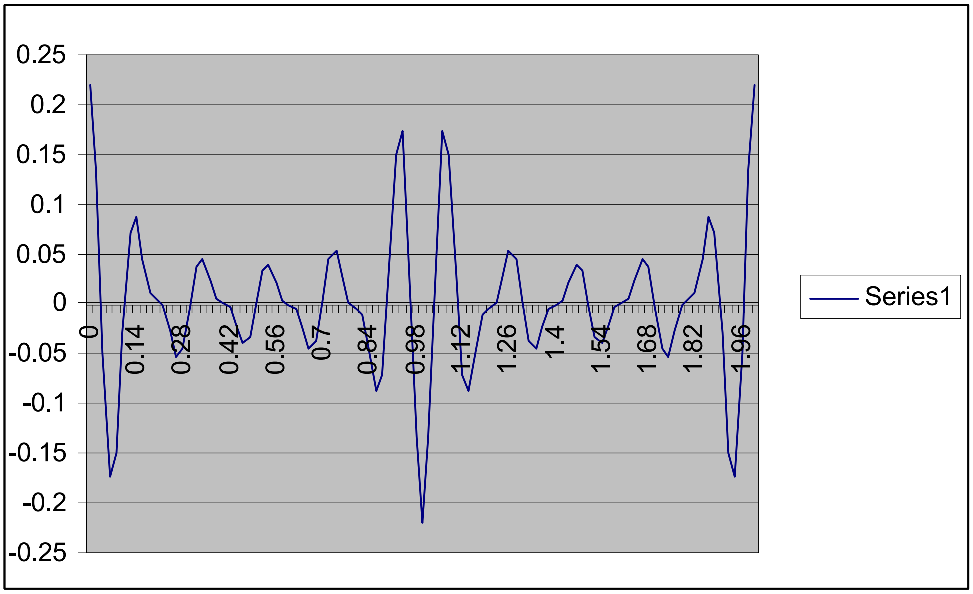


If the lowpass filter has bandwidth 3 Hz, then the first two components are passed. The resulting signal is shown below:



b. Suppose that the signal is applied to a bandpass filter which passes frequencies from 5 to 9 Hz. Plot the output from the filter and compare to the original signal.

In this case the filter passes the components corresponding to k = 5, 6, 7, and 8. The resulting signal is shown below:



While the three signals in part (a) are seen to be converging to a given signal, the signal obtained in part (b) shows no clear relationship to the signals in part (a)

15. Suppose that the 8 kbps periodic signal in Figure 3.15 is transmitted over a system that has an attenuation function that is equal to 1 for all frequencies, and a phase function that is equal to -90^o for all frequencies. Plot the signal that comes out of this system. Does it differ in shape from the input signal?

Solution:

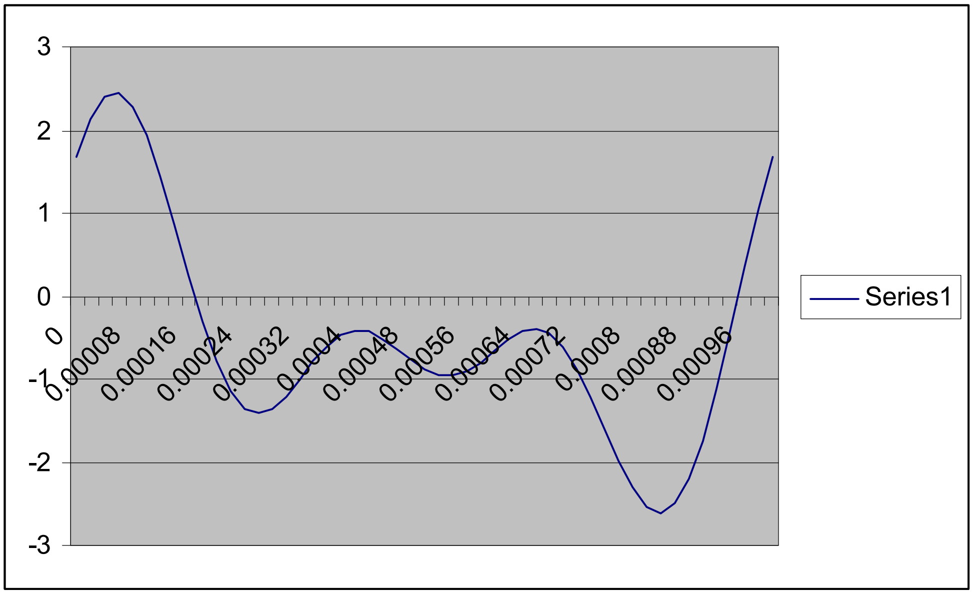
The signal in Figure 3.15 has Fourier series:

$-0.5 + (4/\pi) \{ \sin(\pi/4) \cos(2\pi 1000t) + \sin(2\pi/4) \cos(2\pi 2000t) + \sin(3\pi/4) \cos(2\pi 3000t) + \ldots\}$

The signal that results from transmission over the above system changes the phase of each function by -90^o or equivalently $-\pi/4$ radians. The series for this signal is:

$-0.5 + (4/\pi) \{ \sin(\pi/4) \cos(2\pi 1000t - \pi/4) + \sin(2\pi/4) \cos(2\pi 2000t - \pi/4) + \sin(3\pi/4) \cos(2\pi 3000t - \pi 4) + \ldots\}$

The plot of the signal is shown below. It can be seen that the signal differs considerably from that in Figure 3.15.



16. A 10 kHz baseband channel is used by a digital transmission system. Ideal pulses are sent at the Nyquist rate and the pulses can take 16 levels. What is the bit rate of the system?

Solution:

Nyquist pulses can be sent over this channel at a rate of 20000 pulses per second. Each pulse carries $\log_2 16 = 4$ bits of information, so the bit rate is 80000 bits per second.

17. Suppose a baseband transmission system is constrained to a maximum signal level of $\pm$ 1 volt and that the additive noise that appears in the receiver is uniformly distributed between [–1/16, 1/16]. How many levels of pulses can this transmission system use before the noise starts introducing errors?

Solution:

If two adjacent signal levels are separated by more than 2/16 then the noise cannot translate one adjacent signal into the next. The maximum range that the signal can span is +1 – (–1) = 2, so the maximum number of levels is 2/(1/8) = 16.

18. What is the maximum reliable bit rate possible over a telephone channel with the following parameters?

Solutions follow questions:

a. $W = 2.4$ kHz SNR = 20 dB

An SNR of 20 dB corresponds to a value of 100. The channel capacity formula then gives

$C = 2400 \log_2 (1 + 100) = 15979$ bps.

b. $W = 2.4$ kHz SNR = 40 dB

$C = 2400 \log_2 (1 + 10000) = 31890$ bps.

c. $W = 3.0$ kHz SNR = 20 dB

$C = 3000 \log_2 (1 + 100) = 19974$ bps.

d. $W = 3.0$ kHz SNR = 40 dB

C = 3000 $\log_2$ (1 + 10000) = 39863 bps.

19. Suppose we wish to transmit at a rate of 64 kbps over a 3 kHz telephone channel. What is the minimum SNR required to accomplish this?

Solution:

We know that R = 64 kbps and W = 3 kHz. What we need to find is SNR_{min}. The channel capacity is:

$C = W \log_2 (1 + SNR)$, $C \geq \Rightarrow C_{min} = 64$ kbps

$C_{min} = W \log_2 (1 + SNR_{min}) \Rightarrow \log_2 (1 + SNR_{min}) = 64/3 \Rightarrow 1 + SNR_{min} = 2^{64/3}$

$\Rightarrow SNR_{min} = 2.64 \times 10^6$

in dB: $SNR_{min} = 10 \log_{10} (2.64 \times 10^6) = 64.2$ dB $\therefore$ a very clean channel

20. Suppose that a lowpass communications system has a 1 MHz bandwidth. What bit rate is attainable using 8-level pulses? What is the Shannon capacity of this channel if the SNR is 20 dB? 40 dB?

Solution:

Nyquist pulses can be sent over this system at a rate of 2 million pulses per second. Eight-level signaling carries 3 bits per pulse, so the bit rate is 6 Mbps.

The Shannon capacities are:

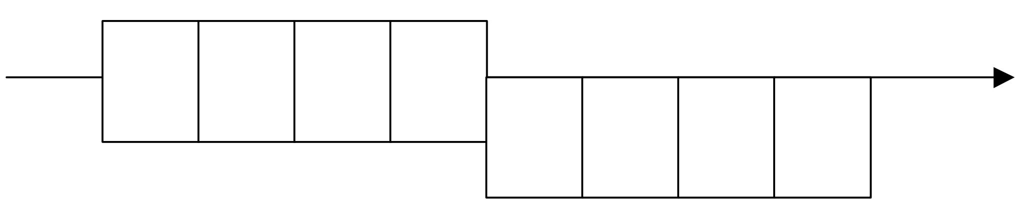
C = 1000000 $\log_2$ (1 + 100) = 6.6 Mbps.

C = 1000000 $\log_2$ (1 + 10000) = 13.3 Mbps.

21. Most digital transmission systems are "self-clocking" in that they derive the bit synchronization from the signal itself. To do this the systems use the transitions between positive and negative voltage levels. These transitions help define the boundaries of the bit intervals.

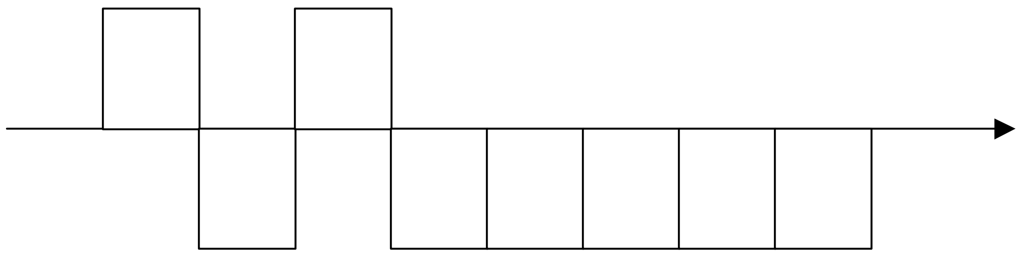
Solutions follow questions:

a. The nonreturn-to-zero (NRZ) signaling method transmits a 0 with a +1 voltage of duration T, and a 1 with a −1 voltage of duration T. Plot the signal for the sequence n consecutive 1s followed by n consecutive 0s. Explain why this code has a synchronization problem.



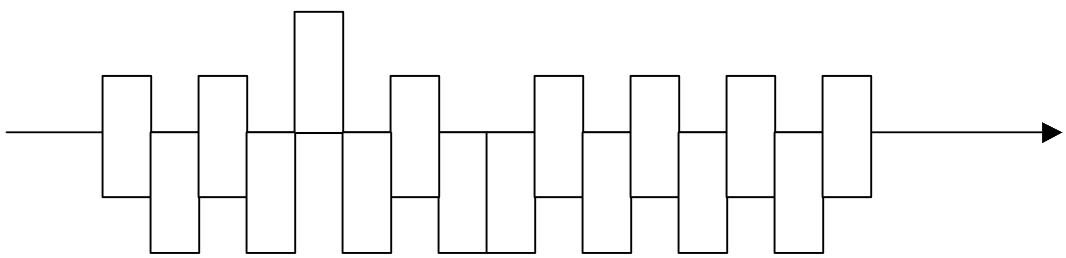
The above figure shows a sequence of 4 1s followed by 4s. A long sequence of 1s or a long sequence of 0s produces a long period during which there is no change in the signal level. Consequently, there are no transitions ("zero crossings") that help a synchronization circuit determine where the boundary of each signaling interval is located.

b. In differential coding the sequence of 0s and 1s induces changes in the polarity of the signal; a binary 0 results in no change in polarity, and a binary 1 results in a change in polarity. Repeat part (a). Does this scheme have a synchronization problem?



The occurrence of a "1" induces a transition and helps synchronization. However sequences of "0s" still result in periods with no transitions.

c. The Manchester signaling method transmits a 0 as a +1 voltage for $T/2$ seconds followed by a −1 for $T/2$ seconds; a 1 is transmitted as a −1 voltage for $T/2$ seconds followed by a +1 for $T/2$ seconds. Repeat part (a) and explain how the synchronization problem has been addressed. What is the cost in bandwidth in going from NRZ to Manchester coding?



Every T-second interval now has a transition in the middle, so synchronization is much simpler. However, the bandwidth of the signal is doubled, as pulses now are essentially half as wide, that is, $T/2$ seconds.

22. Consider a baseband transmission channel with a bandwidth of 10 MHz. What bit rates can be supported by the bipolar line code and by the Manchester line code?

Solution:

From Figure 3.26 we see that a bipolar code with pulses T-seconds wide occupies a bandwidth of $W = 1/T$ Hz. Therefore a 10 MHz bandwidth allows a signaling rate of 10 megabits/second.

From the figure it can also be seen that a Manchester code occupies twice the bandwidth. Hence a 10 MHz bandwidth allows a signaling rate of 5 megabits/second.

23. The impulse response in a T-1 copper-wire transmission system to the input pulse has the idealized form given below, where the initial pulse is of amplitude 1 and duration 1, and the afterpulse is of amplitude –0.1 and of duration 10.

Solutions follow questions:

a. Let $\delta(t)$ be the narrow input pulse in Figure 3.18a. Suppose we use the following signaling method: Every second, the transmitter accepts an information bit; it the information bit is 0, then

$-\delta(t)$ is transmitted, and if the information bit is 1, then $\delta(t)$ is transmitted. Plot the output of the channel for the sequence 1111000. Explain why the system is said to have "dc" or baseline wander.

"DC" stands for direct current and indicates a constant signal level. In a sequence of pulses of the same polarity we would expect to obtain a constant signal level. Instead we get a level in which the signal level drifts in the direction of opposite polarity. For this reason, the term "dc wander" is used.

b. The T-1 transmission system uses bipolar signaling in the following fashion: If the information bit is a 0, then the input to the system is $0^*\delta(t)$; if the information bit is a 1, then the input is $\delta(t)$, for an even occurrence of a 1, and $-\delta(t)$ for an odd occurrence of a 1. Plot the output of the channel for the sequence 1111000. Explain how this signaling has solved the "dc" or baseline wander problem.

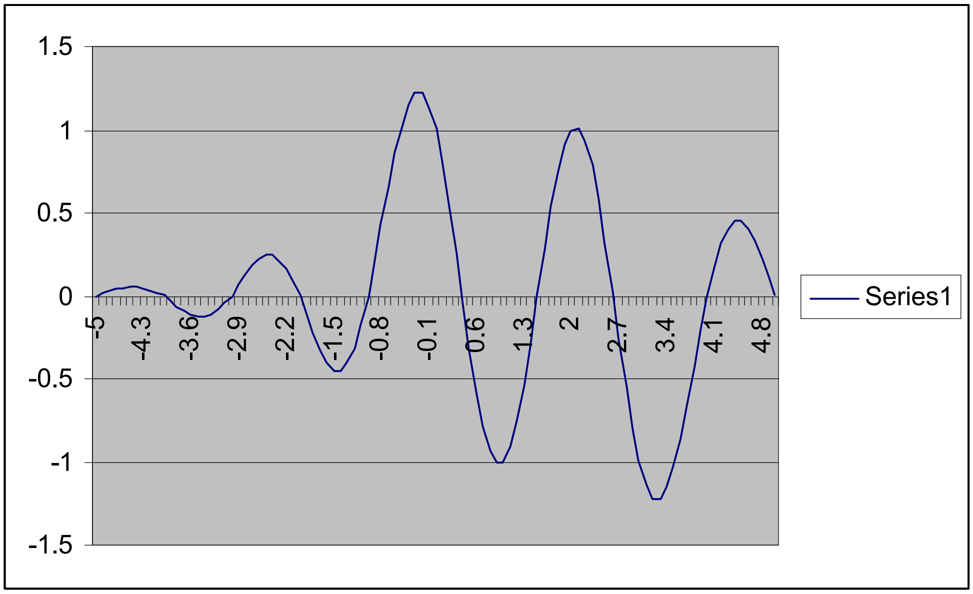
Each pair of "1" produce "tails" of opposite polarity that cancel out.

24. The raised cosine transfer function, shown in Figure 3.21, has a corresponding impulse response given by:

$$p(t) = \frac{\sin(\pi t / T)}{\pi t / T} \frac{\cos(\pi \alpha t / T)}{1-(2\alpha t / T)^2}$$

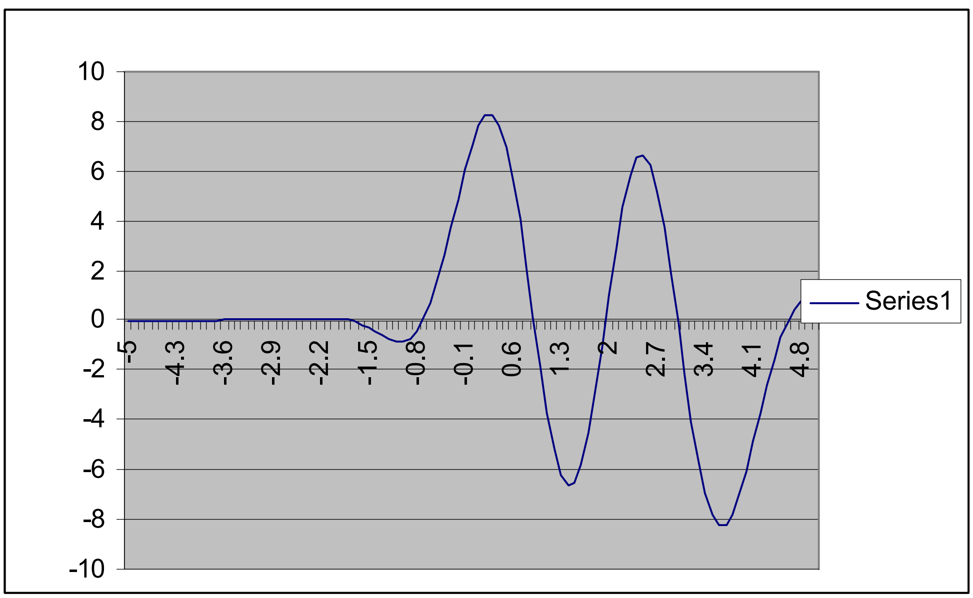
Solutions follow questions:

a. Plot the response of the information sequence 1010 for α = 1/2; α = 1/8.



b. Compare this plot to the response, using the pulse in Figure 3.17.

The plot above has the sequence 1010 occur at times 0, 1, 2, and 3 for α = 1/8. It can be seen that the signal levels are 1 at these time instants. The figure below is for α = 1/2. It can be seen to have fewer oscillations relative to the figure above.



25. Suppose a CATV system uses coaxial cable to carry 100 channels, each of 6 MHz bandwidth. Suppose that QAM modulation is used.

Solutions follow questions:

QAM modulation with 2^m point ⇒ *m* bits/pulse

T = pulse duration = $1/W$ ⇒ pulse rate = baud rate = $W = 6 \times 10^6$

a. What is the bit rate/channel if a four-point constellation is used? eight-point?

$R = mW$, if $m = 4 \Rightarrow R = 24$ Mbps. If $m = 8$, then $R = 48$ Mbps

b. Suppose a digital TV signal requires 4 Mbps. How many digital TV signals can each channel handle for the two cases in part (a)?

$R_{DTV} = 4$ Mbps $\Rightarrow N_{DTV} = R/R_{DTV}$, where N is the number of digital TV signals per channel

If $m = 4 \Rightarrow N_{DTV} = 6$; if $m = 8 \Rightarrow N_{DTV} = 12$.

26. Explain how ASK was used in radio telegraphy. Compare the use of ASK to transmit Morse code with the use of ASK to transmit text using binary information.

Solution:

In ASK, the amplitude of the transmitted sinusoidal signal corresponds to the signal value. In binary amplitude shift keying to transmit text, the presence or absence of the sinusoidal signal corresponds to the transmission of a "1" or a "0". In this approach the duration of a "1" pulse and a "0" pulse are the same.

Morse code also uses the presence of absence of a tone, but in addition also uses the length of the tone to convey a "dot" (short tone) and a "dash" (long tone). The absence of tone amounts to having a third symbol, "space".

27. Suppose that a modem can transmit 8 distinct tones at distinct frequencies. Every T seconds the modem transmits an arbitrary combination of tones (that is, some are present, and some are not present).

Solutions follow questions:

a. What bit rate can be transmitted using this modem?

Each tone is either present or absent, hence there are 2^8 possible combinations of tones that can be transmitted every T seconds. The corresponding transmitted bit rate is $8/T$ bps.

b. Is there are relationship between T and the frequency of the signals?

Yes, there is a relationship. T must be long enough that enough of each sinusoid can be observed to determine its frequency. This implies that the periods of all the sinusoids must be less than T.

28. A phase modulation system transmits the modulated signal $A\cos(2\pi f_c\, t + \phi)$ where the phase ϕ is determined by the 2 information bits that are accepted every T-second interval:

for 00 $\phi = 0$; for 01 $\phi = \pi/2$; for 10 $\phi = \pi$; for 11 $\phi = 3\pi/2$.

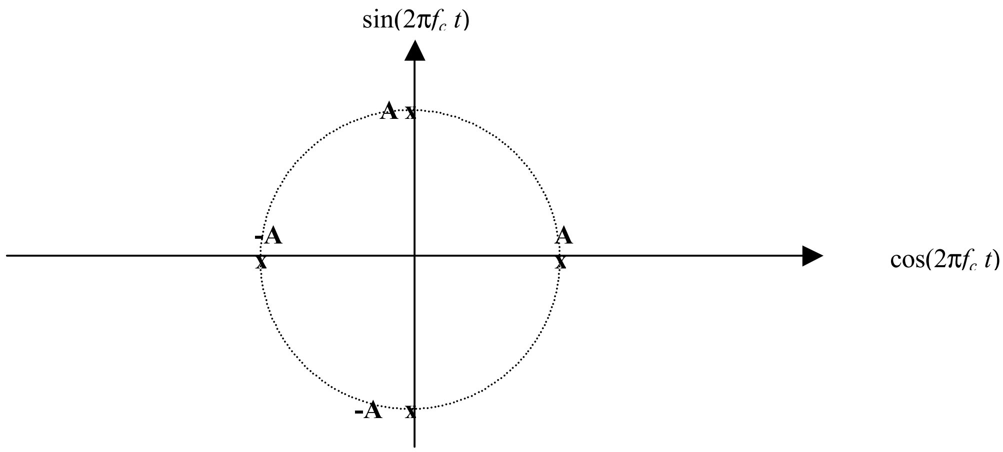
Solutions follow questions:

a. Plot the signal constellation for this modulation scheme.

The transmitted signals corresponding to the phase values are as follows:

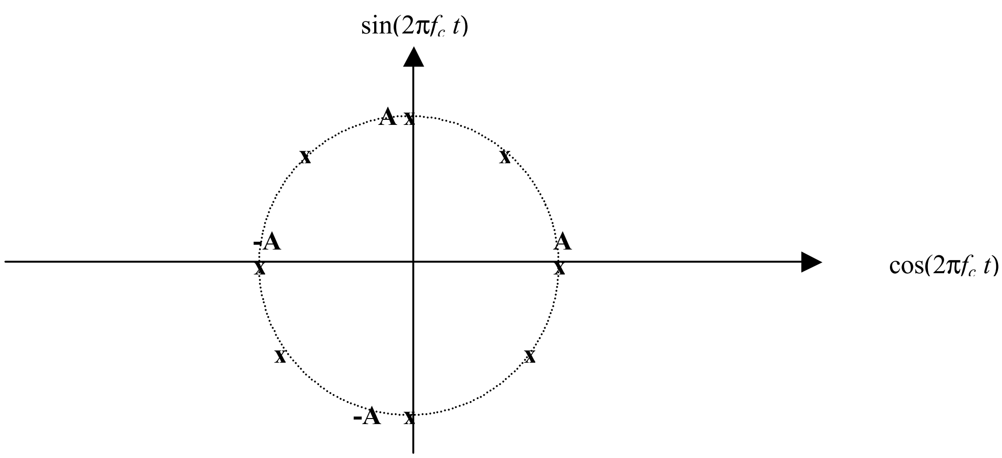
for 00 $\phi = 0$, so $x(t) = A\cos(2\pi f_c\, t)$
for 01 $\phi = \pi/2$, so $x(t) = A\cos(2\pi f_c\, t + \pi/2) = -A\sin(2\pi f_c\, t)$
for 10 $\phi = \pi$, so $x(t) = A\cos(2\pi f_c\, t + \pi) = A\cos(2\pi f_c\, t + \phi)$
for 11 $\phi = 3\pi/2$, so $x(t) = A\cos(2\pi f_c\, t - \pi/2) =- A\sin(2\pi f_c\, t)$

The signal constellation is shown below:



b. Explain how an eight-point phase modulation scheme would operate.

The generalization to an eight-point constellation is straightforward. In the above figure we can see that the four constellation points are placed at equidistant points in a circle about the origin. The figure below shows how eight points can be placed in a circle with angle $\pi/4$ between them.



29. Suppose that the receiver in a QAM system is not perfectly synchronized to the carrier of the received signal; that is, it multiplies the received signal by $2\cos(2\pi f_c t + \phi)$ and by $2\sin(2\pi f_c t + \phi)$ where ϕ is a small phase error. What is the output of the demodulator?

Solution:

The transmitted signal in QAM is $y(t) = A_k \cos(2\pi f_c t) + B_k \sin(2\pi f_c t)$

The upper multiplier in Figure 3.32 computes the following:

$$y(t)\, 2\cos(2\pi f_c t + \phi) = 2\cos(2\pi f_c t + \phi)\ A_k \cos(2\pi f_c t) + 2\cos(2\pi f_c t + \phi)\ \ B_k \sin(2\pi f_c t)$$

$$= A_k \{\cos(\phi) + \cos(4\pi f_c t + \phi)\} + B_k \{\sin(\phi) - \sin(4\pi f_c t + \phi)\}$$

The lowpass filter removes the double-frequency component, so the output of the upper demodulator circuit is: $A_k \cos(\phi) + B_k \sin(\phi)$. When the phase error ϕ is small, then $\cos \phi$ is approximately 1, and sin ϕ is approximately 0, so the phase error causes a small error in the demodulator output. As the phase error increases however, the desired signal A_k becomes harder to discern because the cosine term decreases and the sine term increases.

It can be similarly shown that the output of the upper demodulator is: $B_k \cos(\phi) - A_k \sin(\phi)$.

30. In differential phase modulation the binary information determines the *change* in the phase of the carrier signal $\cos(2\pi f_c t)$. For example, if the information bits are 00, the phase change is 0; if 01, it is $\pi/2$; for 10, it is π; and for 11, it is $3\pi/2$.

Solutions follow questions:

a. Plot the modulated waveform that results from the binary sequence 01100011. Compare it to the waveform that would be produced by ordinary phase modulation as described in problem 28.

Assume that the initial phase is 0, so the initial signal is $\cos(2\pi f_c t)$.

Bit pair	phase change	net phase	modulated signal
01	$\pi/2$	$\pi/2$	$\cos(2\pi f_c t+\pi/2) = -\sin(2\pi f_c t)$
10	π	$3\pi/2 = -\pi/2$	$\cos(2\pi f_c t-\pi/2) = \sin(2\pi f_c t)$
00	0	$-\pi/2$	$\cos(2\pi f_c t-\pi/2) = \sin(2\pi f_c t)$
11	$3\pi/2 = -\pi/2$	$-\pi$	$\cos(2\pi f_c t-\pi) = -\cos(2\pi f_c t)$

The modulated signal looks the same as the signal produced by the scheme in problem 28. The difference is in the manner in which the information bits determine the transmitted signal.

b. Explain how can differential phase modulation be demodulated.

The received signal is demodulated by finding the phase difference between the signals in two consecutive intervals. For example, we can use a delayed version of the signal in an interval (say $\cos(2\pi f_c t + \pi/2)$ in the first interval) and use it to generate two reference signals that multiply the signal in the second interval ($\cos(2\pi f_c t - \pi/2)$) we obtain:

$$\cos(2\pi f_c t + \pi/2) \cos(2\pi f_c t - \pi/2) = 1/2 \{\cos\pi + \cos(4\pi f_c t) \}$$

$$\sin(2\pi f_c t + \pi/2) \cos(2\pi f_c t - \pi/2) = 1/2 \{\sin\pi + \sin(4\pi f_c t) \}$$

The pair of values $\cos\pi = -1$ and $\sin\pi = 0$ uniquely identify the phase difference as being π. The demodulator can then decode the binary pair 10. To check that you understand the solution try demodulating the fourth interval.

31. A new broadcast service is to transmit digital music using the FM radio band. Stereo audio signals are to be transmitted using a digital modem over the FM band. The specifications for the system are the following: Each audio signal is sampled at a rate of 40 kilosamples/second and quantized using 16 bits; the FM band provides a transmission bandwidth of 200 kiloHertz.

Solutions follow questions:

a. What is the total bit rate produced by each stereo audio signal?

The bit rate for each signal is 40 ksamples/sec x 16 bits/sample = 640 kbps. The bit rate for the pair of signals is then 1.28 Mbps.

b. How many points are required in the signal constellation of the digital modem to accommodate the stereo audio signal?

A transmission bandwidth of 200 kHz allows 200 kilopulses/second. To obtain a bit rate of 1.28 Mbps, we need to send 1280/200 = 6.4 bits/pulse. If we use a 27 = 128 point constellation, we can then meet the desired bit rate.

32. A twisted-wire pair has an attenuation of 0.7 dB/kilometer at 1 kHz.

Solutions follow questions:

a. How long can a link be if an attenuation of 20 dB can be tolerated?

20 db / (0.7 db/km) = 28 km

b. A twisted pair with loading coils has an attenuation of 0.2 dB/kilometer at 1 kHz. How long can the link be if an attenuation of 20 dB can be tolerated?

20 db / (0.2 db/km) = 100 km.

33. Use Figure 3.37 and Figure 3.40 to explain why the bandwidth of twisted-wire pairs and coaxial cable decreases with distance.

Solution:

The bandwidth is the range of frequencies where the channel passes a significant proportion of the power in the input signal. Both figures show that the attenuation when measured in dB/km increases with higher frequency. For example, the attenuation for 19-gauge wire at 100 kHz is about 5 dB/km and at 1MHz it is about 15 dB/km. This implies that the relative attenuation between a lower frequency and a higher frequency increases with distance. Using the same example, we have that at 1 km and at 10 km, the attenuation at 100 kHz is 5 dB and 50 dB respectively, but at 1 MHz it is 15 dB and 150 dB respectively. Thus at longer distances higher frequencies are attenuated much more severely and consequently the bandwidth decreases with increasing distance.

34. Calculate the bandwidth of the range of light covering the range from 1200 nm to 1400 nm. Repeat for 1400 nm to 1600 nm. Keep in mind that the speed of light in fiber is approximately 2 x 10^8 m/sec.

Solution:

Frequency and wavelength are related by the expression: $f = v/\lambda$, where $v = 2\text{x}10^8$. The frequencies corresponding to 1200 nm and 1400 nm are $f_1 = v/\lambda_1 = 2(10^8)/1.2(10^{-6})$ and $f_2 = v/\lambda_2 = 2(10^8)/1.4(10^{-6})$ so the bandwidth is $BW = f_1 - f_2 = 23.8$ THz. For the band from 1400 nm to 1600 nm we get $BW = f_1 - f_2 = 2(10^8)(1/1.4 - 1/1.6)10^6 = 17.8$ THz.

35. Compare the attenuation in a 100 km link for optical fibers operating at 850 nm, 1300 nm, and 1550 nm.

Solution:

At 850 nm the attenuation is about 2 dB per km, so the attenuation in 100 km is 200 dB. At 1300 nm, the attenuation is 0.5(100) = 50 dB. At 1500 nm, the attenuation is 0.2(100) = 20 dB.

36. A satellite is stationed approximately 36,000 km above the equator. What is the attenuation due to distance for the microwave radio signal?

Solution:

The attenuation for free space is proportional to $2(10)\log_{10}d = 2\log_{10}36000 = 80\log_{10}3.6 = 44.5$ dB.

37. Suppose a transmission channel operates at 3 Mbps and that it has a bit error rate of 10^{-3}. Bit errors occur at random and independent of each other. Suppose that the following code is used. To transmit a 1, the codeword 111 is sent; To transmit a 0, the codeword 000 is sent. The receiver takes the three received bits and decides which bit was sent by taking the majority vote of the three bits. Find the probability that the receiver makes a decoding error.

Solution:

The receiver makes a decoding error if two or more out of the three bits are in error. Therefore,

$$P_{error} = 3p^2(1 - p) + p^3 = 3(10^{-3})^2(1\text{-}10^{-3}) + (10^{-3})^3 \approx 3(10^{-6})$$

38. An early code used in radio transmission involved using codewords that consist of binary bits and contain the same number of 1s. Thus, the 2-out-of-5 code only transmits blocks of 5 bits in which 2 bits are 1 and the others 0.

Solutions follow questions:

a. List the valid codewords.

11000
10100
10010
10001
01100
01010
01001
00110
00101
00011

b. Suppose that the code is used to transmit blocks of binary bits. How many bits can be transmitted per codeword?

There are 10 possible codewords. Three bits per codeword can be transmitted if eight codewords are used.

c. What pattern does the receiver check to detect errors?

Each received codeword should have exactly two bits that are ones and three bits that are zeros to be a valid codeword.

d. What is the minimum number of bit errors that cause a detection failure?

A valid codeword can be changed into another valid codeword by changing a 1 to a 0 and a 0 to a 1 in the first codeword. Therefore, two bit errors can cause a detection failure.

39. Find the probability of error-detection failure for the code in the previous problem for the following channels:

Solutions follow questions:

a. The random error vector channel.

The only way to convert a valid codeword into another valid codeword is to change a 1 to a 0 and a 0 to a 1. There are 10 patterns that have two 1s and three 0s, but the pattern that corresponds to our first valid codeword must be excluded because it would leave to the all 0s pattern. Therefore there are exactly 9 error patterns that cause error-detection failure. Therefore 9/32 is the fraction of error vectors that cause detection failure.

b. The random bit error channel.

The set of error patterns that cause detection failure are the same as in part a. The probability of occurrence of each error pattern is $p^2(10p)^3$ and there are 9 such patterns, so the probability of detection failure is $9p^2(10p)^3$.

40. Suppose that two check bits are added to a group of $2n$ information bits. The first check bit is the parity check of the first n bits, and the second check bit is the parity check of the second n bits.

Solutions follow questions:

a. Characterize the error patterns that can be detected by this code.

If we rearrange the $2n$ information bits and the 2 parity bits into two codewords, each consisting of n information bits and a parity bit, we see that in effect we have divided the overall codeword into two subcodewords of length $n + 1$. An error pattern is detectable if the error pattern in the first codeword and in the second codeword are both detectable. An error pattern in each subcodeword is detectable if the number of errors in the subcodeword is an even. Therefore an error pattern is detectable if the number of errors in both subcodewords is even.

b. Find the error detection failure probability in terms of the error-detection probability of the single parity check code.

An error detection failure occurs is either the first subcodeword or the second subcodeword or both fail. Let $P_{\text{detect}}(n + 1)$ be the probability of successful error detection in the single parity code of length $n + 1$, then

P[detection failure in code of length $2n + 2$] =

= 1 – P[successful detection in both codes of length $n + 1$]

$= 1 - P_{detect}(n + 1)\, P_{detect}(n + 1)$

c. Does it help to add a third parity check bit that is the sum of the all the information bits?

Note that the above scheme fails to detect the case where there is an odd number of errors in the first subcodeword and an odd number of errors in the second subcodeword. An overall parity bit would detect this error pattern, and so would help.

41. Let $g(x)=x^3+x+1$. Consider the information sequence 1001.

Solutions follow questions:

a. Find the codeword corresponding to the preceding information sequence.

Using polynomial arithmetic we obtain:

```
             1010
1011   | 1001000
       | 1011
          01000
           1011
           0110
```

Codeword = 1001110

b. Suppose that the codeword has a transmission error in the first bit. What does the receiver obtain when it does its error checking?

```
                0001
1011   | 0001110
           1011
            101
```

CRC calculated by Rx = 101 ➔ error

42. ATM uses an eight-bit CRC on the information contained in the header. The header has six fields:
First 4 bits: GFC field
Next 8 bits: VPI field
Next 16 bits: VCI field
Next 3 bits: Type field
Next 1 bit: CLP field
Next 8 bits: CRC

Solutions follow questions:

a. The CRC is calculated using the following generator polynomial: $x^8 + x^2 + x + 1$. Find the CRC bits if the GFC, VPI, Type, and CLP fields are all zero, and the VCI field is 00000000 00001111. Assume the GFC bits correspond to the highest-order bits in the polynomial.

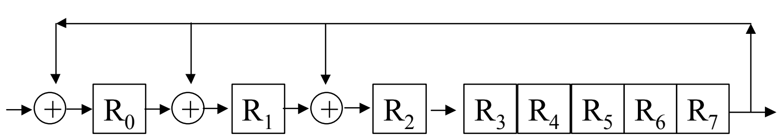
```
                            1111 0010
100000111  | 1111 0000 0000 0000
             1000 0011 1
               111 0011 10
               100 0001 11
                 11 0010 010
                 10 0000 111
                   1 0010 1010
                   1 0000 0111
                         10 1101 000
                         10 0000 111
                              1101 1110
```

CRC = 11011110

b. Can this code detect single errors? Explain why.

Yes, because *g(x)* has more than one term.

c. Draw the shift register division circuit for this generator polynomial.



43. Suppose a header consists of four 16-bit words: (11111111 11111111, 11111111 00000000, 11110000 11110000, 11000000 11000000). Find the internet checksum for this code.

Solution:

b_0 = 11111111 11111111 = $2^{16} - 1$ = 65535

b_1 = 11111111 00000000 = 65280

b_2 = 11110000 11110000 = 61680

b_3 = 11000000 11000000 = 49344

$x = b_0 + b_1 + b_2 + b_3$ modulo 65535 = 241840 modulo 65535 = 45235

$b_4 = -x$ modulo 65535 = 20300

So the internet checksum = 01001111 01001100

44. Let $g_1(x) = x + 1$ and let $g_2(x) = x^3 + x^2 + 1$. Consider the information bits (1,1,0,1,1,0).

Solutions follow questions:

a. Find the codeword corresponding to these information bits if $g_1(x)$ is used as the generating polynomial.

```
       100100
11  | 1101100
      11
       0011
         11
         0000
```

Codeword = 1101100

b. Find the codeword corresponding to these information bits if $g_2(x)$ is used as the generating polynomial.

```
          100011
1101  | 110110000
        1101
         01000
           1101
           1010
           1101
            111
```

Codeword = 110110111

c. Can $g_2(x)$ detect single errors? double errors? triple errors? If not, give an example of an error pattern that cannot be detected.

Single errors can be detected since $g_2(x)$ has more than one term. Double errors can be detected since $g_2(x)$ is primitive. Triple errors cannot be detected since $g_2(x)$ has only three terms.

d. Find the codeword corresponding to these information bits if $g(x) = g_1(x)\, g_2(x)$ is used as the generating polynomial. Comment on the error-detecting capabilities of $g(x)$.

```
                  111101
10111    |  1101100000
            10111
             11000
             10111
              11110
              10111
               10010
               10111
                010100
                 10111
                  1011
```

Codeword = 1101101011

The new code can detect all single and double errors. It can also detect all bursts of length $n - k = 4$ or less. All bursts of length 5 are detected except for the burst that equals $g(x)$. The fraction $1/2^{n-k} = 1/16$ of all bursts of length greater than 5 are detectable.

44. Take any binary polynomial of degree 7 that has an even number of nonzero coefficients. Show by longhand division that the polynomial is divisible by $x+1$.

Solution:

Let $p(x) = x^7 + x^5 + x^2 + 1$, then $p(x) = (x + 1)(x^6 + x^5 + x + 1)$.

45. A repetition code is an $(n,1)$ code in which the $n - 1$ parity bits are repetitions of the information bit. Is the repetition code a linear code? What is the minimum distance of the code?

Solution:

Yes this is a linear code in which: $c_2 = c_1$, $c_3 = c_1$, …, $c_n = c_1$, where c_1 is the information bit. This code has two codewords: (0,0,…,0) and (1,1,…,1) so the minimum distance is $d_{min} = n$.

47. A transmitter takes groups of K groups of k information bits and appends a single parity bit to each group. It then appends a block parity check word in which the jth bit in the check word is the modulo 2 sum of the jth components in the K codewords.

Solutions follow questions:

a. Explain why this is a $((K + 1)(k + 1), Kk)$ linear code.

Every binary linear code is represented by two parameters n and k. n is the total number of bits transmitted (codeword), and k is the number of information bits. The transmitter takes K groups of k information bits, having a total of Kk information bits (k). The code appends one check bit to each row of k information bits, and appends one check bit to each $k + 1$ column, transmitting a total of $(k + 1)(K + 1)$ codeword bits (n). The result is a $[(k + 1)(K + 1), Kk]$ code.

b. Write the codeword as a $(k + 1)$ row by $(K + 1)$ column array in which the first K columns are the codewords and the last column is the block parity check. Use this code to show how the code can detect all single, double, and triple errors. Give an example of a quadruple error that cannot be detected.

The $(k + 1)(K + 1)$ encoded matrix satisfies the pattern that all the rows and columns have even parity. If one, two, or three bit errors occur anywhere in the matrix of bits, then at least one row or column parity check will fail to detect the errors. A set of four errors placed so that they form a rectangle in the array will result in column check bits and row check bits that are zero and hence fail to be detected.

c. Find the minimum distance of the code. Can it correct all single errors? If so explain how the decoding can be done.

The minimum distance is $d_{min} = 4$. Yes, the code can correct all single errors. If a single error occurs in row i and column j, then the check bits in row i and column j will fail, indicating that the bit in that location should be complemented.

d. Find the probability of error-detection failure for the random bit error.

A pattern with exactly k bit errors occurs with probability $p^k(1-p)^{n-k} = (1-p)^n\,(p/(1-p))^k$. For $p < 1/2$, $p\,/\,(1-p) < 1$, so the above probability decreases with increasing value of k. Therefore the error terms with $k = d_{min}$ errors will have the largest probability. We can therefore estimate the probability of error-detection failure by the probability that all rectangular error patterns occur. We first count the number of possible rectangular error patterns:

The number of ways of picking the two rows is $\binom{k+1}{2} = (k+1)k/2$.

The number of ways of picking the two columns is $\binom{K+1}{2} = (K+1)K/2$.

Therefore the total number of rectangular error patterns is $(k+1)\,k\,(K = 1)\,K/4$, and the probability of undetectable error pattern is approximately:

$$P_{detection\ failure} \approx Kk(K+1)(k+1)p^4\,(1-p)^{(K+1)(k+1)-4}$$

48. Consider the $m = 4$ Hamming code.

Solutions follow questions:

a. What is n, and what is k for this code?

$n = 2^m - 1 = 15$; $k = n - m = 11$

(15,11) Hamming code

b. Find parity check matrix for this code.

$$\mathbf{H} = \begin{bmatrix} 0&0&0&0&1&1&1&1&1&1&1&1&0&0&0 \\ 1&1&1&0&0&0&0&1&1&1&1&0&1&0&0 \\ 0&1&1&1&0&1&1&0&0&1&1&0&0&1&0 \\ 1&0&1&1&1&0&1&0&1&0&1&0&0&0&1 \end{bmatrix}$$

c. Give the set of linear equations for computing the check bits in terms of the information bits.

$b_{12} = b_5 + b_6 + b_7 + b_8 + b_9 + b_{10} + b_{11}$
$b_{13} = b_1 + b_2 + b_3 + b_8 + b_9 + b_{10} + b_{11}$
$b_{14} = b_2 + b_3 + b_4 + b_6 + b_7 + b_{10} + b_{11}$
$b_{15} = b_1 + b_3 + b_4 + b_5 + b_7 + b_9 + b_{11}$

d. Write a program to find the set of all codewords. Do you notice anything peculiar about the weights of the codewords?

The minimum weight is 3.

The following is a short program written in C.

```
#include <stdio.h>

main()
{

  int a1=0,a2=0,a3=0,a4=0,a5=0,a6=0,a7=0,a8=0,a9=0,a10=0,a11=0,x,y,u[16],d;
  char yn,enter,yyy='y';

  for (d=0; d<2048; d++) {
    u[0]=0; /*reset matrix values*/
    u[1]=a1;
    u[2]=a2;
    u[3]=a3;
    u[4]=a4;
    u[5]=a5;
    u[6]=a6;
    u[7]=a7;
    u[8]=a8;
    u[9]=a9;
    u[10]=a10;
    u[11]=a11;
    u[12]=0;
    u[13]=0;
    u[14]=0;
    u[15]=0;
     /*operations for Hamming encoding of the word (u)*/
     /*with the Generator matrix (v=uG)*/
    u[12]=u[5]+u[6]+u[7]+u[8]+u[9]+u[10]+u[11];
    u[13]=u[1]+u[2]+u[3]+u[8]+u[9]+u[10]+u[11];
    u[14]=u[2]+u[3]+u[4]+u[6]+u[7]+u[10]+u[11];
    u[15]=u[1]+u[3]+u[4]+u[5]+u[7]+u[9]+u[11];

  for (x=12; x<16; ++x)
    { /*if value =3 then in binary it should be =1*/
      if (u[x]>1) /*if value =2 then in binary it should be =0*/
        {
        if (u[x]==3)
             u[x]=1;
           else
             u[x]=0;
      }
  }
  printf("\nCODEWORD (b1-b11 b12-b15) : %i%i%i%i%i%i%i%i%i%i%i %i%i%i%i",
    u[1],u[2],u[3],u[4],u[5],u[6],u[7],u[8],u[9],u[10],u[11],u[12],u[13],u[14],
    u[15]);
  printf(" w = %i",u[1]+u[2]+u[3]+u[4]+u[5]+u[6]+u[7]+u[8]+u[9]+u[10]+u[11]+
    u[12]+u[13]+u[14]+u[15]);
  a1=a1+1;

  if (a1>1){
    a1=0;
  if (a1==0){
    a2=a2+1;
  if (a2>1){
    a2=0;
  if (a2==0){
    a3=a3+1;
  if (a3>1){
    a3=0;
  if (a3==0){
```

```
    a4=a4+1;
  if (a4>1){
    a4=0;
  if (a4==0){
    a5=a5+1;
  if (a5>1){
    a5=0;
  if (a5==0){
    a6=a6+1;
  if (a6>1){
    a6=0;
  if (a6==0){
    a7=a7+1;
  if (a7>1){
    a7=0;
  if (a7==0){
    a8=a8+1;
  if (a8>1){
    a8=0;
  if (a8==0){
    a9=a9+1;
  if (a9>1){
    a9=0;
  if (a9==0){
    a10=a10+1;
  if (a10>1){
    a10=0;
  if (a10==0){
    a11=a11+1;
  if (a11>1){
    a11=0;
  }}}}}}}}}}}}}}}}}}}}}
 }
}
```

49. Show that an easy way to find the minimum distance is to find the minimum number of columns of **H** whose sum gives the zero vector.

Solution:

In the (7,4) Hamming code, you can see that you need to at least add three columns to obtain the zero vector.

    [1 0 1 1 1 0 0]
H = [1 1 0 1 0 1 0]
    [0 1 1 1 0 0 1]

that is,

(col 1) + (col 2) = [110] + [011] = [101] + (col 3) = [101] + [101] = [000]

50. Suppose we take the (7,4) Hamming code and obtain an (8,4) code by adding an overall parity check bit.

Solutions follow questions:

a. Find the **H** matrix for this code.

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

b. What is the minimum distance?

$d_{\min} = 4$

c. Does the extra check bit increase the error correction capability? the error-detection capability?

The (8,4) extended Hamming code maintains its single error-correcting capability, and increases its error-detection capability to triple-error detecting.

51. A (7,3) linear code has check bits given by

$b_4 = b_1 + b_2$
$b_5 = b_1 \quad + b_3$
$b_6 = \quad b_2 + b_3$
$b_7 = b_1 + b_2 + b_3$

Solutions follow questions:

a. Find the **H** matrix.

$$\mathbf{H} = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

b. Find the minimum distance.

$d_{\min} = 4$

c. Find the set of all codewords. Do you notice anything peculiar about the set of codewords?

000 0000000
001 0010111
010 0101011
011 0111100
100 1001101
101 1011010
110 1100110
111 1110001

Every nonzero codeword has a weight of 4.

52. An error-detecting code takes k information bits and generates a codeword with $2k + 1$ encoded bits as follows:

The first k bits consist of the information bits
The next k bits repeat the information bits
The next bit is the XOR of the first k bits.

Solutions follow questions:

a. Find the check matrix for this code.

$c_{k+1}=c_1,\ c_{k+2}=c_2,\ \ldots,\ c_{2k}=c_k$, and $c_{2k+1}=c_1+\ldots+c_k$

For $k = 4$, we have

$$\mathbf{H} = \begin{bmatrix} 1&0&0&0&1&0&0&0&0 \\ 0&1&0&0&0&1&0&0&0 \\ 0&0&1&0&0&0&1&0&0 \\ 0&0&0&1&0&0&0&1&0 \\ 1&1&1&1&0&0&0&0&1 \end{bmatrix}$$

b. What is the minimum distance of this code?

It can be seen that we need to add three columns to obtain the all zero column, so $d_{min} = 3$.

c. Suppose the code is used on a channel that introduces independent random bit errors with probability 10^{-3}. Estimate the probability that the code fails to detect an erroneous transmission.

The error pattern with three errors has the largest probability, so

$$P_{detection\ failure} \approx \binom{n}{3} p^3 (1-p)^{n-3}$$

53. A (6,3) linear code has check bits given by

$b_4 = b_1 + b_2$
$b_5 = b_1 + b_3$
$b_6 = b_2 + b_3$

Solutions follow questions:

a. Find the check matrix for this code.

$$\begin{bmatrix} 1&1&0&1&0&0 \\ 1&0&1&0&1&0 \\ 0&1&1&0&0&1 \end{bmatrix}$$

b. What is the minimum distance of this code?

$d_{min} = 3$

c. Find the set of all codewords.

000000 100110 010101 001011 110011 101101 011110 111000

54. (Appendix 3A). Consider an asynchronous transmission system that transfers a sequence of N bits between a start bit and a stop bit. What is the maximum value of N if the receiver clock frequency is within 1 percent of the transmitter clock frequency?

Solution:

Let X = transmitter pulse duration and T = receiver pulse duration. Suppose $X = 0.99T$ (transmitter is slower), then we need:

$$1.5T + NT < (N + 2)X = (N + 2).99T$$

which implies $N < 48$.