Water rocket and simulations

Travail de Maturité - Matthias Huguet, Dorian Santa Cruz

Introduction

The goal of this Travail de Maturité (TM) is to compare the trajectory of a real life rocket to a computer simulation, in order to evaluate the reliability of simulations. In order to achieve this goal, we first wrote a basic simulation program in Python, giving an estimation of the rocket's flight altitude in function of time. We will then figure out the possible error sources, design the launchpad, design the rocket, build both, improve the prediction program adding lateral displacement to the altitude prediction, launch the rocket and compare between real life measurement predictions.

Rocket design

Propulsion

We were strongly encouraged by the school administration to do a water rocket instead of a chemically propelled rocket. Thus, for propulsion, we are going to use water, as fuel, and compressed air as a pressurant. This kind of rocket can be compared to a "cold gas thruster" rocket, as no explosive chemical reaction is produced, thrust is given by the simple ejection of mass through the nozzle.[1]

Nose cone

Generally speaking, the flat nose cones are the least aerodynamic, the flat conic are better and the parabolic nose cones are the most aerodynamic. [2]

A short list of different nose cones, sorted in aerodynamic efficiency:
- Flat head
- Pyramid
- Conic
- Bi-conic
- Parabolic
- Power series
- Haack series
- Aerospike

We will go for a simple conic nose as it is relatively simple to make out of paper or cardboard. The other argument is that since we are making a small rocket, the lighter it is, the better and since we could make a nose cone out of reinforced paper, the flat conic type will be best suited.[3]

Body

For the material, we chose cardboard for the body and PET for the pressure chamber as they are both very light and common materials. It would have been better to use fiberglass for the body, but fiberglass is not very suitable for this project due to limited time and resources.

Fig 1, Andros 1.5L juice bottle

The pressure tank will be a 1.5L Andros juice bottle (fig 1) as it has a large throat, allowing us to fit the Gardena threaded hose connector and seal it to the cap. To ensure that the cap will not be a limiting factor in withstanding high pressure, it will be glued and secured to the bottle.

For tank pressure, we will simply use a bike pump with a valve on the pressure tank. The pressure tank will be leak tested underwater, allowing us to easily locate the leaks by looking for bubbles.

Nozzle

As for the nozzle, it could simply be a Gardena Tap Connector (G3/4”)[4], allowing us to make a quick connect-disconnect with the launchpad, having a servo or a solenoid push down on the connector to disconnect it from the launchpad at launch.

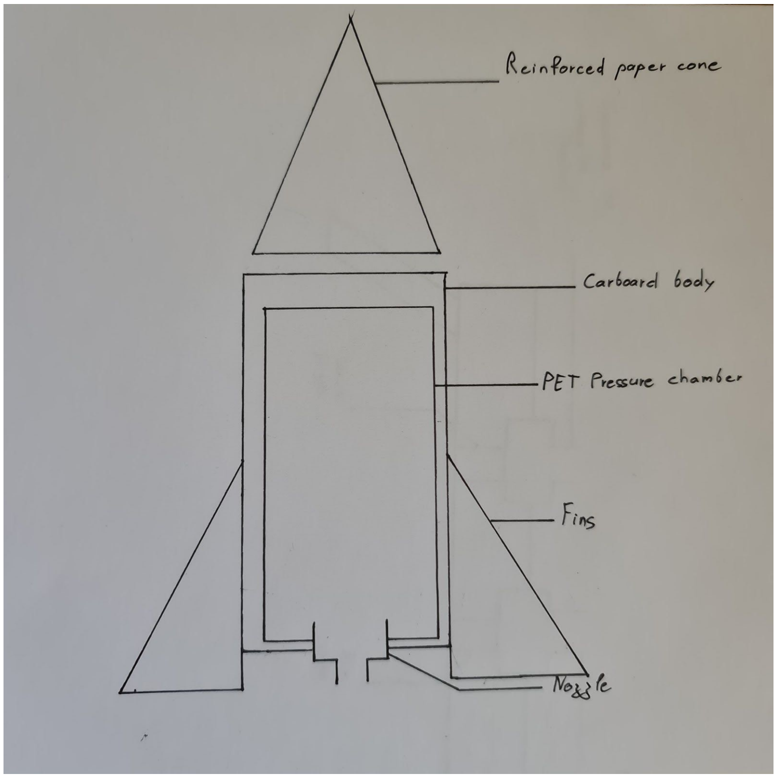


Fig 2, basic rocket sketch

Launchpad

The launchpad will have a Gardena hose connector, controlled by a servomotor, allowing us to release the rocket at a distance. The idea would be to use two valves, one for air and one for water. The structure would be a simple plank of wood, propped up with four adjustable legs. A level meter will be used to determine if the plank is levelled or not. It is crucial that the rocket is levelled as the program does not predict lateral movement yet. As the rocket will be perpendicular to the launchpad, if the launchpad is levelled then the rocket should be levelled as well. The fact that the rocket flies straight up is crucial as the objective of this TM is to evaluate the accuracy of simulation software.

The air will be pumped in with a simple bike pump and the water will be from the garden tap or perhaps from the high pressure cleaner (Eg: Karsher).

Some tests will be conducted with some test water bottles (made to replicate as much as possible the real tank in the rocket) in order to evaluate the maximal air pressure that the tank can withstand given our modifications.

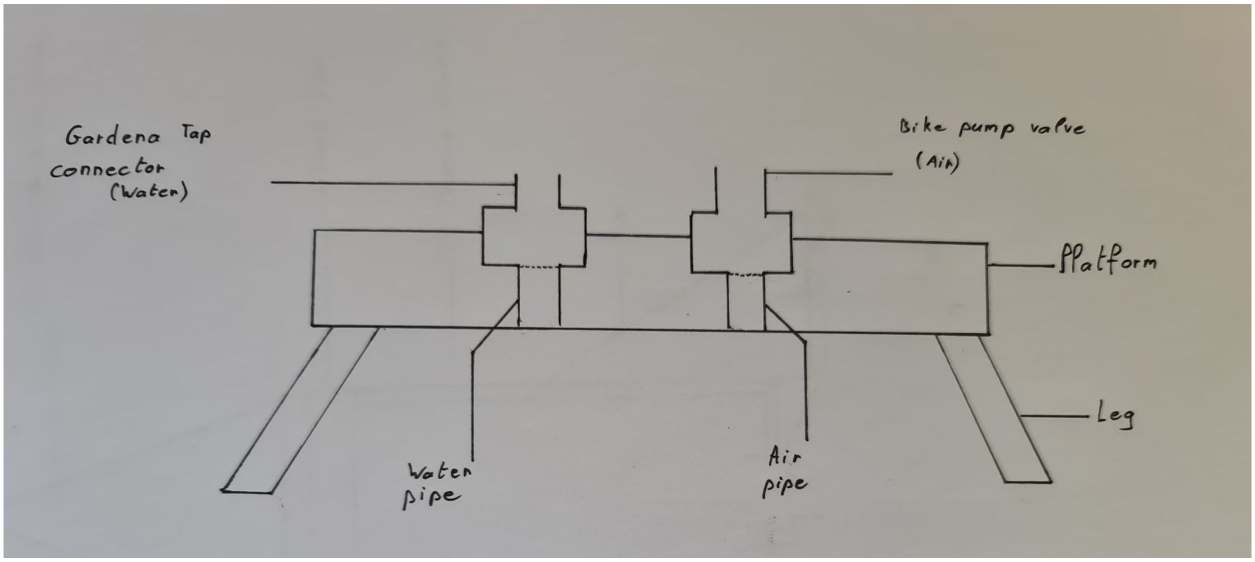


Fig 3, basic launchpad sketch

Program

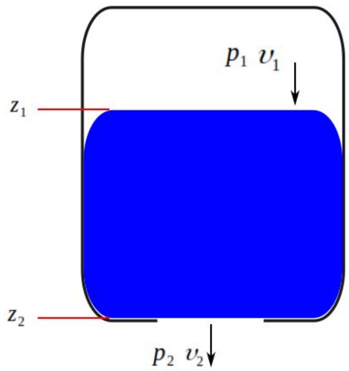
Concept

The idea for the program is to be rather simple and straightforward. It predicts the altitude in function of time, given the tank pressure, the pressurant volume, the water volume, the drag coefficient, the apparent cross-section, the nozzle diameter and the dry mass of the rocket.

Physical basis

Water output velocity

The program calculates the fuel output speed thanks to Bernoulli's principle[5]:



$$\frac{1}{2}\rho v_1{}^2+\rho g z_1+p_1=\frac{1}{2}\rho v_2{}^2+\rho g z_2+p_2$$

Here, p_1 is the tank pressure, p_2 is the atmospheric pressure and ρ is the volumic mass of water. g is the acceleration of gravity.

Fig 4, Water flow diagram

This expression can be simplified as v_1 can be ignored as $v_1 \ll v_2$.

Therefore, we are left with: $\rho g(z_1 - z_2) + p_1 - p_2 = \frac{1}{2}\rho v_2{}^2$

Reorganizing the equation, we get the output velocity: $v_2=\sqrt{\frac{2(\rho g(z_1-z_2)+p_1-p_2)}{\rho}}$

To simplify, since $(z_1 - z_2) \simeq 0$, we finally get: $v_2=\sqrt{\frac{2(p_1-p_2)}{\rho}}$

Thrust

The thrust is calculated in Newtons, given the mass of the water that exited the nozzle and the water's exit velocity. The exit velocity is then derived, giving the ejected water's acceleration. Since we know the mass and the acceleration of the ejected fluid, the force produced by the ejection can be calculated with Newton's second law: *F*=*ma* where *m* is the mass of the ejected fluid, *a* is the acceleration of that fluid and *F* is the force created by ejecting the water.
Given that for every action there is an equal and opposite reaction, the force exerted by the exiting fluid is equal to the thrust exerted on the rocket.

Drag

The drag is given by the drag formula: $F_D = \frac{1}{2}\rho u^2 C_D A$ [6]
Where:

- F_D is the drag force
- ϱ is the volumic mass of the medium (air in our case)
- υ us the velocity of the rocket
- C_D is the drag coefficient (0.75 in our case[7])
- *A* is the apparent cross section of the rocket (surface that is perpendicular to the velocity vector)

Change in air pressure in the tank

The tank pressure over time is calculated with Boyle's law which states that the product of pressure and volume in a gas gives a constant: *PV=k*.[8]
From this law, we can deduce that $P_{t1} V_{t1} = P_{t2} V_{t2}$.
Where P_{t1} is the initial gas pressure, V_{t1} the initial gas volume, P_{t2} is the new gas pressure and V_{t2} the new gas volume.
The change in volume of the fluid gives the change in volume for the pressurant, allowing to calculate the new gas volume. Knowing the previous gas pressure, the previous volume and the new volume, the new pressure can be calculated with: $P_{t2} = \frac{P_{t1} \cdot V_{t1}}{V_{t2}}$

Simulation

The source code for the simulation and optimiser programs can be found here:
https://github.com/DoCloudCompute/TMRocket/tree/master/progs

Inputs

The simulation program has nine variable inputs (default values are indicated in square brackets):

- Dry mass (mass of the rocket without water) [0.2kg]
- Nozzle output diameter (the hole through which water comes out) [22mm]
- Rocket's drag coefficient [0.75]
- Rocket diameter [10cm]
- Tank capacity in liters [1.5L]
- Liquid's volumic mass [998 g/L]
- Amount of fluid in percents of the tank's capacity [49%]
- Compressed air pressure in atm [6 atm]
- Simulation's time step (Δt) [0.00005s]
- Maximum simulation run time [10s]

One important efficiency factor is the balance between air and water.
It is also interesting to discuss other parameters such as the nozzle's output diameter but such debate will remain theoretical as we will not be able to modify it as it is a prefabricated piece.

Water to air ratio

For the water to air ratio, the higher the air pressure in the pressure chamber, the faster the water mass will be ejected giving more thrust to the rocket. The propulsion phase is short in our case but once this phase is finished, the rocket continues its movement because it will have accumulated kinetic energy. However, the disadvantage of consuming all the water faster is that the rocket will go faster, thus increasing the drag dramatically. After our test with the simulator, we deduced that the tank must be filled with 51% of water, giving us an apogee of 32.93m with the default settings.

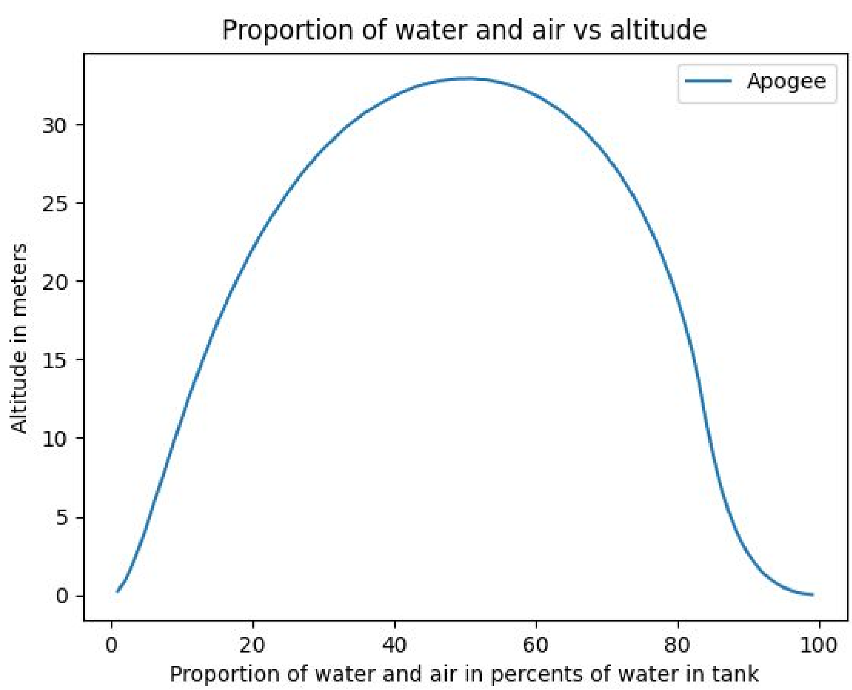
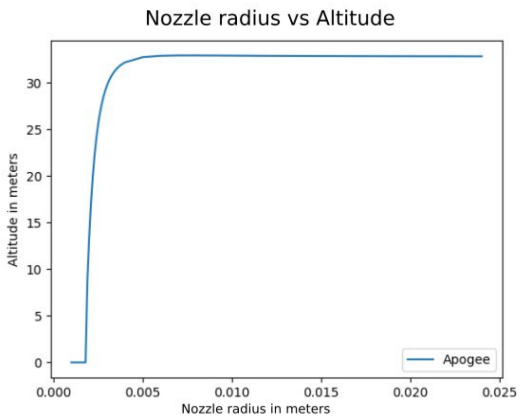


Fig 5, Graph of Proportion of water and air vs altitude

Nozzle radius



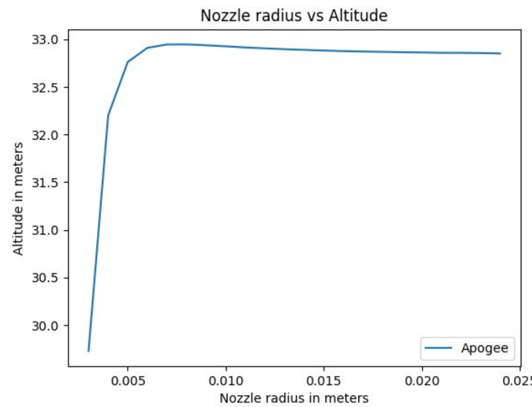


Fig 6, Radius ranging from 1mm to 24mm

Fig 7, Radius ranging from 3mm to 24mm

Note: the altitude in these graphs is the apogee of each simulation where only the nozzle radius is modified

The nozzle radius determines how much mass exits the rocket and at what velocity. Concluding from our graphs, there is an optimal radius as if the radius is too small, the water's velocity is too slow to create any thrust (fig 6). However, the bigger the hole, the higher the acceleration. This high acceleration gives a high initial velocity to the rocket and generates even more drag. This probably explains the drop in apogee as the hole widens (fig 7 and 8), although this is just my uneducated opinion.

The nozzle that we are going to use (Gardena Tap Connector) has a radius of 11mm, which is almost optimal according to the simulations (fig 7)

Measuring the rocket flight

For the trajectory, we will use a camera on land, using triangulation to calculate the height and the camera's frame timestamps for time.

As for the engine burn time the camera will do fine. For the thrust, we will use a test bench with a dynamometer.

For the velocity and acceleration, a raspberry pi zero along with an accelerometer will be used. Although we could use the camera to determine the speed, we decided it would be much more accurate to have an onboard computer instead.

Where to launch

For the launchsite, we need a big field that is far from the airport so as to avoid interfering with air traffic. That being said, the rocket will probably not go higher than 50 meters. We will go to Matthias's house as he lives far from the airport and has lots of ground clearance.

Conclusion

We will build the rocket with materials that are easy to obtain, and will be constructed in a fashion that does not require advanced or non-household tools, as we do not have access to them. We used the simulation in order to evaluate the optimal ratio of water to air, in order to get maximum thrust time as well as minimum useless weight.

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