Lecture2:
Source-free Poisson equation

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Today's goal

- Solve the source-free Poisson equation.
  - The simplest equation in this course.

Source-free Poisson equation

- The Poisson equation
  - The Poisson equation in this context reads
$$\nabla \cdot [-\epsilon(\mathbf{r})\nabla\phi(\mathbf{r})] = \rho(\mathbf{r})$$
  - (It is not a correct term, however, widely adopted in the semiconductor device simulation.)

- Source-free case
  - When the source, $\rho(\mathbf{r})$, vanishes,
$$\nabla \cdot [-\epsilon(\mathbf{r})\nabla\phi(\mathbf{r})] = 0$$
  - In a 1D case,
$$\frac{d}{dx}\left[-\epsilon(x)\frac{d}{dx}\phi(x)\right] = 0$$

Its application

- In the dielectric material, the source vanishes.
- Capacitor with position-dependent material composition
  - By solving the source-free Poisson equation, we can find the potential distribution, $\phi(\mathbf{r})$.
- Solution method
  - Position-dependent permittivity should be taken into account.
  - Integration from $x_{i-0.5}$ to $x_{i+0.5}$ yields

$$-\epsilon(x_{i+0.5}) \left.\frac{d\phi}{dx}\right|_{x_{i+0.5}} + \epsilon(x_{i-0.5}) \left.\frac{d\phi}{dx}\right|_{x_{i-0.5}} = 0$$

  - The first derivative is approximated by

$$\left.\frac{d\phi}{dx}\right|_{x_{i+0.5}} \approx \frac{\phi_{i+1} - \phi_i}{x_{i+1} - x_i}$$

Matrix form

- For $i = 2,3, \ldots N-2, N-1$,
  - The equation reads

$$-\epsilon(x_{i+0.5})\frac{\phi_{i+1}-\phi_i}{x_{i+1}-x_i} + \epsilon(x_{i-0.5})\frac{\phi_i-\phi_{i-1}}{x_i-x_{i-1}} = 0$$

- For $i = 1$ and $i = N$,
  - The equation reads

$$\phi_i = \phi_i^{boundary}$$

  - Here, $\phi_i^{boundary}$'s are given by the user.
- It can be written as the form of $\mathbf{Ax} = \mathbf{b}$.
  - By solving this matrix equation, the electrostatic potential is obtained.

Homework#2

- Due: September 11 (Next Monday)
- Develop a capacitance calculator.
  - Two materials with different permittivity are stacked. Taking the permittivity and thickness of each layer, calculate the capacitance.
  - Compare your result and the analytic expression.