### **Question 2**

a(x,y)	x=0	x=1	x=2	x=3	x=4
y=0	1	2	3	5	13
y=1	2	3	5	13	2^16 - 3
y=2	3	4	7	29	2^(2^16) - 3
y=3	4	5	9	2^6 - 3	
y=4	5	6	11	2^7 - 3	
y=5	6	7	13	2^8 - 3	

NeedlesslyRecursive.a(4, 2) =  $2^{(2^16)}$  - 3

### **Question 3**

(a) worst-case complexity:  $\Theta(2^n)$  (b) worst-case complexity:  $\Theta(n)$ 

### **Question 4**

a)

```
1, 9, 17, 21, 20, 8, 2, 5, 81, 83, 23

1, 2, 17, 21, 20, 8, 9, 5, 81, 83, 23

1, 2, 5, 21, 20, 8, 9, 17, 81, 83, 23

1, 2, 5, 8, 20, 21, 9, 17, 81, 83, 23

1, 2, 5, 8, 9, 21, 20, 17, 81, 83, 23

1, 2, 5, 8, 9, 17, 20, 21, 81, 83, 23

1, 2, 5, 8, 9, 17, 20, 21, 81, 83, 23

1, 2, 5, 8, 9, 17, 20, 21, 81, 83, 23

1, 2, 5, 8, 9, 17, 20, 21, 23, 83, 81

1, 2, 5, 8, 9, 17, 20, 21, 23, 81, 83

1, 2, 5, 8, 9, 17, 20, 21, 23, 81, 83
```

9, 81, 17, 21, 20, 8, 2, 5, 1, 83, 23 9, 17, 81, 21, 20, 8, 2, 5, 1, 83, 23 9, 17, 21, 81, 20, 8, 2, 5, 1, 83, 23 9, 17, 21, 20, 81, 8, 2, 5, 1, 83, 23 9, 17, 20, 21, 81, 8, 2, 5, 1, 83, 23 9, 17, 20, 21, 8, 81, 2, 5, 1, 83, 23 9, 17, 20, 8, 21, 81, 2, 5, 1, 83, 23 9, 17, 8, 20, 21, 81, 2, 5, 1, 83, 23 9, 8, 17, 20, 21, 81, 2, 5, 1, 83, 23 8, 9, 17, 20, 21, 81, 2, 5, 1, 83, 23 8, 9, 17, 20, 21, 2, 81, 5, 1, 83, 23 8, 9, 17, 20, 2, 21, 81, 5, 1, 83, 23 8, 9, 17, 2, 20, 21, 81, 5, 1, 83, 23 8, 9, 2, 17, 20, 21, 81, 5, 1, 83, 23 8, 2, 9, 17, 20, 21, 81, 5, 1, 83, 23 2, 8, 9, 17, 20, 21, 81, 5, 1, 83, 23 2, 8, 9, 17, 20, 21, 5, 81, 1, 83, 23 2, 8, 9, 17, 20, 5, 21, 81, 1, 83, 23 2, 8, 9, 17, 5, 20, 21, 81, 1, 83, 23 2, 8, 9, 5, 17, 20, 21, 81, 1, 83, 23 2, 8, 5, 9, 17, 20, 21, 81, 1, 83, 23 2, 5, 8, 9, 17, 20, 21, 81, 1, 83, 23 2, 5, 8, 9, 17, 20, 21, 1, 81, 83, 23 2, 5, 8, 9, 17, 20, 1, 21, 81, 83, 23 2, 5, 8, 9, 17, 1, 20, 21, 81, 83, 23 2, 5, 8, 9, 1, 17, 20, 21, 81, 83, 23 2, 5, 8, 1, 9, 17, 20, 21, 81, 83, 23 2, 5, 1, 8, 9, 17, 20, 21, 81, 83, 23 2, 1, 5, 8, 9, 17, 20, 21, 81, 83, 23 1, 2, 5, 8, 9, 17, 20, 21, 81, 83, 23 1, 2, 5, 8, 9, 17, 20, 21, 81, 23, 83 1, 2, 5, 8, 9, 17, 20, 21, 23, 81, 83

c)

9, 81, 17, 21, 20, 8, 2, 5, 1, 83, 23 9, 17, 81, 21, 20, 8, 2, 5, 1, 83, 23 9, 17, 21, 81, 20, 8, 2, 5, 1, 83, 23 9, 17, 20, 21, 81, 8, 2, 5, 1, 83, 23 8, 9, 17, 20, 21, 81, 2, 5, 1, 83, 23 2, 8, 9, 17, 20, 21, 81, 5, 1, 83, 23 2, 5, 8, 9, 17, 20, 21, 81, 1, 83, 23 1, 2, 5, 8, 9, 17, 20, 21, 81, 83, 23 1, 2, 5, 8, 9, 17, 20, 21, 81, 83, 23 1, 2, 5, 8, 9, 17, 20, 21, 23, 81, 83

d)

81, 9, 17, 21, 20, 8, 2, 5, 1, 83, 23 23, 9, 17, 21, 20, 8, 2, 5, 1, 83, 81 1, 9, 17, 21, 20, 8, 2, 5, 23, 81, 83 1, 9, 17, 21, 20, 8, 2, 5, 23, 81, 83 1, 9, 5, 21, 20, 8, 2, 17, 23, 81, 83 1, 9, 5, 2, 20, 8, 21, 17, 23, 81, 83 1, 9, 5, 2, 8, 20, 21, 17, 23, 81, 83 1, 8, 5, 2, 9, 20, 21, 17, 23, 81, 83 1, 2, 5, 8, 9, 20, 17, 21, 23, 81, 83 1, 2, 5, 8, 9, 17, 20, 21, 23, 81, 83

e)

9, 81, 17, 21, 8, 20, 2, 5, 1, 23, 83 9, 17, 21, 81, 2, 5, 8, 20, 1, 23, 83 2, 5, 8, 9, 17, 20, 21, 81, 1, 23, 83 1, 2, 5, 8, 9, 17, 20, 21, 23, 81, 83

f)

81, 9, 17, 21, 20, 8, 2, 5, 1, 83, 23 81, 9, 17, 21, 83, 8, 2, 5, 1, 20, 23 81, 9, 17, 21, 83, 8, 2, 5, 1, 20, 23 81, 9, 17, 21, 83, 8, 2, 5, 1, 20, 23 81, 83, 17, 21, 9, 8, 2, 5, 1, 20, 23 81, 83, 17, 21, 23, 8, 2, 5, 1, 20, 9 83, 81, 17, 21, 23, 8, 2, 5, 1, 20, 9 83, 81, 17, 21, 23, 8, 2, 5, 1, 20, 9 9, 81, 17, 21, 23, 8, 2, 5, 1, 20, 83 81, 9, 17, 21, 23, 8, 2, 5, 1, 20, 83 81, 23, 17, 21, 9, 8, 2, 5, 1, 20, 83 81, 23, 17, 21, 20, 8, 2, 5, 1, 9, 83 9, 23, 17, 21, 20, 8, 2, 5, 1, 81, 83 23, 9, 17, 21, 20, 8, 2, 5, 1, 81, 83 23, 21, 17, 9, 20, 8, 2, 5, 1, 81, 83 23, 21, 17, 9, 20, 8, 2, 5, 1, 81, 83 1, 21, 17, 9, 20, 8, 2, 5, 23, 81, 83 21, 1, 17, 9, 20, 8, 2, 5, 23, 81, 83

```
21, 20, 17, 9, 1, 8, 2, 5, 23, 81, 83
5, 20, 17, 9, 1, 8, 2, 21, 23, 81, 83
20, 5, 17, 9, 1, 8, 2, 21, 23, 81, 83
20, 9, 17, 5, 1, 8, 2, 21, 23, 81, 83
2, 9, 17, 5, 1, 8, 20, 21, 23, 81, 83
17, 9, 2, 5, 1, 8, 20, 21, 23, 81, 83
8, 9, 2, 5, 1, 17, 20, 21, 23, 81, 83
9, 8, 2, 5, 1, 17, 20, 21, 23, 81, 83
9, 8, 2, 5, 1, 17, 20, 21, 23, 81, 83
1, 8, 2, 5, 9, 17, 20, 21, 23, 81, 83
8, 1, 2, 5, 9, 17, 20, 21, 23, 81, 83
8, 5, 2, 1, 9, 17, 20, 21, 23, 81, 83
1, 5, 2, 8, 9, 17, 20, 21, 23, 81, 83
5, 1, 2, 8, 9, 17, 20, 21, 23, 81, 83
2, 1, 5, 8, 9, 17, 20, 21, 23, 81, 83
2, 1, 5, 8, 9, 17, 20, 21, 23, 81, 83
1, 2, 5, 8, 9, 17, 20, 21, 23, 81, 83
```

## g)

```
0:
        20
        81, 21, 1
1:
2:
        2
3:
        83, 23
4:
5:
        5
6:
7:
        17
8:
        8
        9
9:
0:
        1, 2, 5, 8, 9
1:
        17
2:
        20, 21, 23
3:
```

4:

5:

6: 7:

8: 81, 83

9:

### **Question 5**

Exchange: comparing the first element with each following element of the array and making any necessary swaps

Insertion: similar to sorting cards where the element is pushed to the correct position

Selection: choosing the smallest element and swapping with the front element

Merge: sorting the elements in sections and combining the results at the end

Distribution: elements are distributed to multiple intermediate structures, which are then gathered and placed in the right order

Hybrid: when two or more different sorting algorithms are being used during the sorting process.

Concurrent: works best on parallel processors when sorting elements in an array

Impractical: algorithms are meant to be bad with a long run time when sorting elements

#### **Question 7**

- a) 16
- b) 15
- c) H, N, F, C, J, O, P, L, M
- d) G
- e) 2
- f) 4
- g) A, D
- h) J, K, L, M, O, P
- i) H, I, J, K, L, M
- j) 4
- k) 6
- 1) 3
- m)  $\langle P K G D A B E \rangle$
- n) 4
- o) A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P
- p) A, B, E, H, I, N, F, C, D, G, J, K, O, P, L, M

# **Questions 8**

