Starboard Matlab Team
Final Presentation

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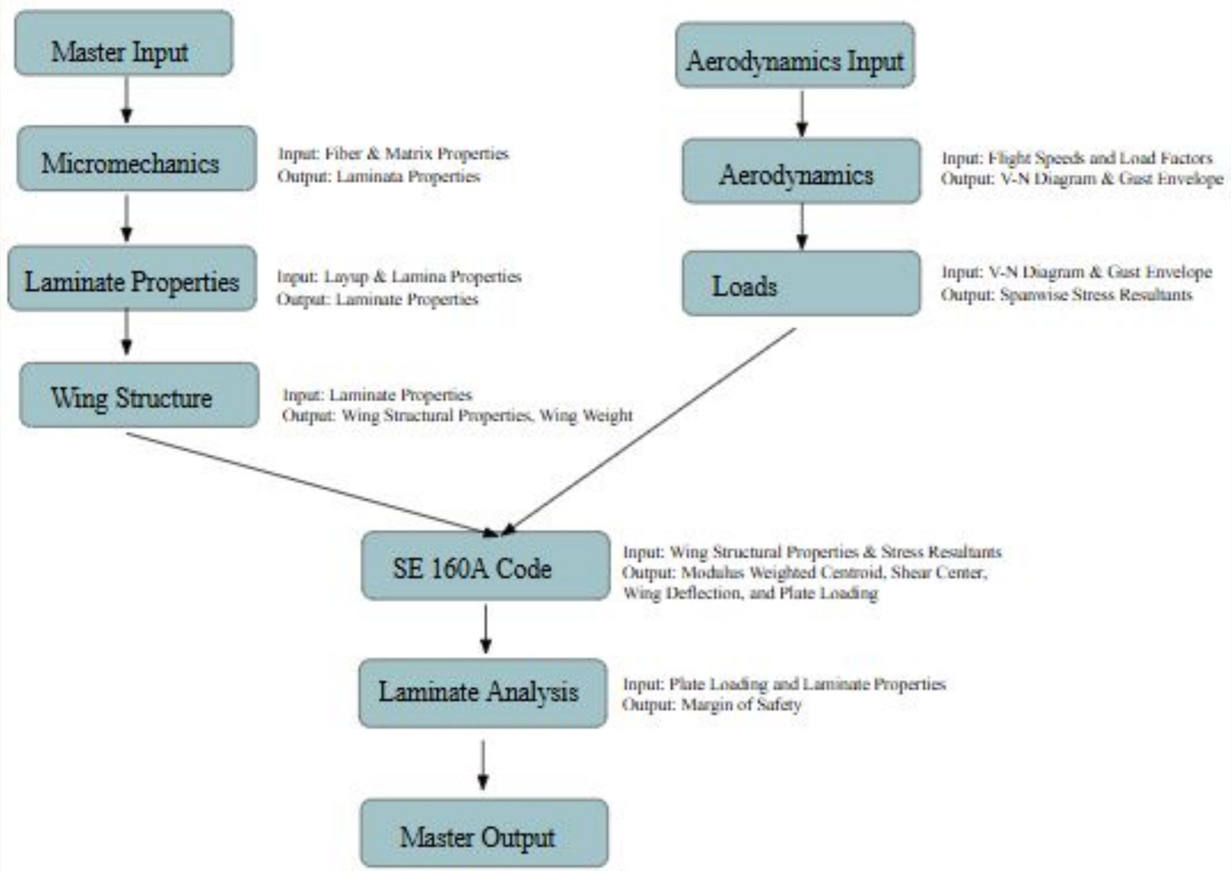
Overview of Matlab Code



Inputs Files:

- Master Input
- Aerodynamics Input

Code Functions

- Laminate Properties
- Wing structure
- Loads
- SE 160A Code (Structural Mechanics)
- Laminate Analysis

Output File

- Master Output

Micromechanics

```
% Convert AFW units
   iAFW = Input_Properties.Input_Units;
   if iAFW == 1
       Input_Properties.AFW = Input_Properties.AFW*4.82253e-5;   % convert to lb/in^2
   elseif iAFW == 2
       Input_Properties.AFW = Input_Properties.AFW*0.001;    % convert to kg/m^2
   end

Vf = 1 / (1+ (Fiber.pf + Resin.pm)) * (Input_Properties.RC / (100 - Input_Properties.RC)); % Fiber Volume Fraction
Vm = 1 - Vf; % Matrix Volume Fraction
% t_ply = (Input_Properties.AFW / Fiber.pf) * ((1 + (Fiber.pf / Resin.pm)) * (Input_Properties.RC / (100 - Input_Properties.RC)));
t_ply = Input_Properties.AFW / (Fiber.pf * Vf); % Lamina (Ply) Thickness
rho_c = Input_Properties.AFW / ((1 - (Input_Properties.RC/100)) * t_ply); % Composite Density
E_L = Fiber.Efl * Vf + Resin.Em * Vm; % Young's Modulus (Longitudinal) E1
E_T = Resin.Em / ((1 - Vf) * (1 - (Resin.Em / Fiber.Eft))); % Young's Modulus (Transverse) E2
G_LT = abs(Resin.Gm / (1 - Vf * ((1 - Resin.Gm) / Fiber.GfLT))); % Shear Modulus (Transverse) G12
v_LT = (Fiber.vFLT * Vf) + (Resin.vm * Vm); % Poisson's Ratio v12
v_TT = (Fiber.vFTT * Vf) + (Resin.vm * Vm); % Poisson's Ratio, Transverse v22
G_TT = E_T / (2 * (1 + v_TT)); % Shear Modulus, Transverse G22
F_fT = Fiber.Efl * Fiber.emt;
F_1T = F_fT * (Vf + (Resin.Em / Fiber.Efl) * Vm); % Fiber Tension Strength
```

Purpose:

- Analyze the properties of a single-layer lamina by combining fiber and resin data.
- Feed effective properties into later laminate-level structural and failure analysis.

Process:

- Import material properties from excel file, containing values for mechanical, thermal, and physical properties.
- User defines material combination and conditions
  - Input: AFW, RC, fiber/matrix properties from student database
- Code then uses Micromechanic equations to blend the fiber and resin contributions into effective lamina properties.
  - Output: Lamina properties are written into Student Database

Variable	Description	Units	Carbon				Boron	Kevlar	Glass
	Material ID Number		101	102	103	104	105	106	107
	Material Name:		T300	IM6	IM7	Ultra Mod Carbon	Boron	Kevlar 49	S2 Glass
	Fiber Diameter	inch	0.0003	0.0003	0.0003	0.0003	0.0056	0.00046	0.00036
E_{fL}	Young's Modulus (Longitudinal)	lb/in^2	33500000	36330000	40030000	70000000	59030400	6300000	12750000
E_{fT}	Young's Modulus (Transverse)	lb/in^2	2175000	2020000	2755700	1100000	5764370	1000000	12750000
G_{fLT}	Shear Modulus (L-T plane)	lb/in^2	4000000	7390000	3916000	2700000	2596100	400000	5170000
G_{fTT}	Shear Modulus (T-T plane)	lb/in^2	1015000	1200000	1015000	700000	2377000	220000	5170000
v_{fLT}	Poisson ratio (L-T plane)		0.2	0.26	0.2	0.2	0.25	0.36	0.2
v_{fTT}	Poisson ratio (T-T plane)		0.2	0.33	0.2	0.25	0.21245	0.36	0.2
ρ_f	Weight Density	lb/in^3	0.06358	0.063	0.0643	0.072	0.09393	0.052	0.089
α_{fL}	Thermal expansion coefficient (Longitidinal)	$(in/in)/^oF$	-0.0000004	-0.000000566	-0.0000001	-0.00000055	0.0000089	-0.0000027	0.0000031
α_{fT}	Thermal expansion coefficient (Tranverse)	$(in/in)/^oF$	0.0000067	0.0000018	0.0000056	0.0000056	0.0000089	0.0000033	0.0000031
K_{fL}	Heat Conduction (Longitudinal)	$BTU/hr/ft^2/^oF/in$	580	580	600	600	22	1.7	21
K_{fT}	Heat Conduction (Transverse)	$BTU/hr/ft^2/^oF/in$	58	58	60	60	22	1.7	21
F_{Ft}	Tension strength	lb/in^2	362590	415000	751295	150000	392900	435000	665720
F_{FC}	Compression strength	lb/in^2	-290000	-367000	-464120	-150000	-936070	-75000	-355340
ε_{mt}	Tension Strain - Ultimate	in/in	0.0109	0.0115	0.0188	0.0022	0.0067	0.075	0.0523
ε_{mc}	Compression Strain - Ultimate	in/in	-0.0087	-0.0102	-0.0116	-0.0022	-0.0159	-0.0042	-0.028

Carbon Properties

Variable	Description	Units	Epoxy			Polyester	Vinyl Ester
	Material ID Number		201	202	203	204	205
	Material Name:		3501-6	8551-7	5250-4 RTM	polyester	vinyl ester
E_m	Young's Modulus	lb/in^2	630000	591700	670000	470000	500000
G_m	Shear Modulus	lb/in^2	231600	214366	248000	170000	170000
v_m	Poisson ratio		0.36	0.38	0.35	0.38	0.38
ρ_m	Weight Density	lb/in^3	0.0469	0.046	0.045	0.042	0.0457
α_m	Thermal expansion coefficient	$(in/in)/^oF$	0.0000309	0.00002	0.00002	0.00006	0.00035
K_1	Heat Conduction	$BTU/hr/ft^2/^oF/in$	1.25	1.25	1.25	1.25	1.25
β	Moisture Absorption	(in/in)/(%)	0.33	0.33	0.33		
F_{mt}	Tension strength	lb/in^2	8085	14358	14900	10400	11000
F_{mc}	Compression strength	lb/in^2	-17180	-18850	-30380	-51080	-54030
F_{ms}	Shear Strength	lb/in^2	12600	16070	17930	21400	22700
ε_{mt}	Tension Strain - Ultimate	in/in	0.04	0.04	0.04	0.04	0.04
ε_{mc}	Compression Strain - Ultimate	in/in	-0.05	-0.05	-0.05	-0.03	-0.03
γ_m	Shear Strain - Ultimate	in/in	0.06	0.09	0.09	0.15	0.15
T_{glass}	Glass Transition Temperature	^oF	420	390	390	150	
T_{max}	Maximum Operating Temperature	^oF	300	300	300		
	Reference						

Resin Properties

	Variable	Description	Units	Carbon/epoxy				Boron/epoxy	Kevlar-49/epoxy	S2-glass/epoxy
1		Material ID Number		301	302	303	304	305	306	307
2		Material Name:		T300/3501-6	uni (IM6/3501-6)	uni (IM7/977-3)	Woven (AGP370-5H/3501-6	uni (Boron/epoxy)	uni (kevlar/epoxy)	uni (Sglass/epoxy)
3	V_f	Fiber Volume Fraction		0.63	0.635 (.66)	0.65	0.62	0.5	0.6	0.5
4	E_1	Young's Modulus (Longitudinal)	lb/in^2	21300000	23300000	27700000	11200000	29200000	11600000	6500000
5	E_2	Young's Modulus (Transverse)	lb/in^2	1500000	1395000	1440000	10900000	3150000	800000	1600000
6	E_3	Young's Modulus (Out-of-Plane)	lb/in^2	1500000	1395000	1440000	2000000	3150000	800000	1600000
7	G_{12}	Shear Modulus (1-2 plane)	lb/in^2	1000000	916000	1130000	940000	780000	310000	660000
8	G_{13}	Shear Modulus (1-3 plane)	lb/in^2	1000000	916000	1130000	740000	780000	310000	660000
9	G_{23}	Shear Modulus (2-3 plane)	lb/in^2	540000	520000	600000	590000	600000	260000	550000
10	v_{12}	Poisson Ratio (1-2 plane)		0.27	0.2965	0.35	0.06	0.17	0.34	0.29
11	v_{13}	Poisson Ratio (1-3 plane)		0.27	0.2965	0.35	0.37	0.17	0.34	0.29
12	v_{23}	Poisson Ratio (2-3 plane)		0.45	0.468	0.45	0.45	0.4	0.4	0.45
13	ρ	Weight Density	lb/in^3	0.058	0.0571	0.058	0.058	0.073	0.05	0.072
14	α_{11}	Thermal Expansion Coefficient (Longiitudinal)	$(in/in)/^oF$	-0.0000005	-0.0000005	-0.0000005	0.0000019	0.0000034	-0.0000011	0.0000039
15	α_{22}	Thermal Expansion Coefficient (Transverse)	$(in/in)/^oF$	0.000015	0.0000124	0.0000124	0.0000021	0.000017	0.000033	0.0000167
16	α_{33}	Thermal Expansion Coefficient (Out-of-Plane)	$(in/in)/^oF$	0.000015	0.0000124	0.0000124	0.000029	0.000017	0.000033	0.0000167

Combination Properties

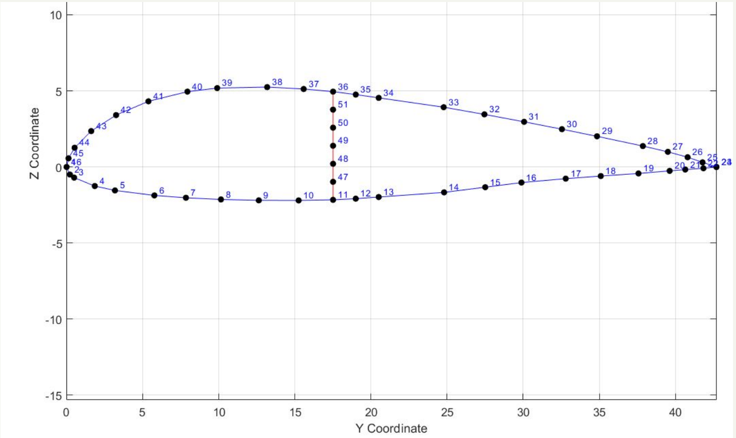
Laminate Layup

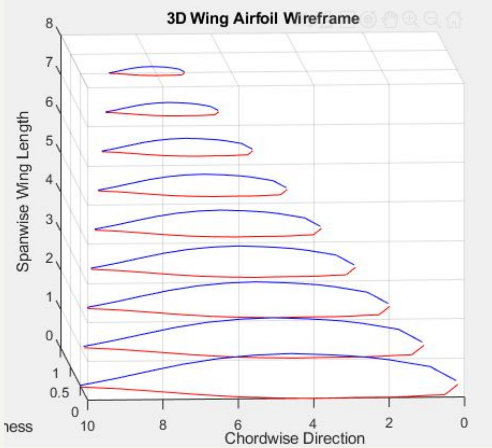
- Defines the stacking sequence, ply thickness, and orientation.
- Each row shows a single ply with:
    - Lamina ID
    - Orientation angle
    - thickness

	Laminate ID:		1
	Total Number of Plies:		4
	Lamina ID	θ_i (degree)	t_{ply} (inch)
12			
11			
10			
9			
8			
7			
6			
5			
4	301	45	0.006
3	301	-45	0.006
2	301	-45	0.006
1	301	45	0.006
tool surface			

Structures

- Assumes Eppler e1230 airfoil
  - Linear interpolation is conducted between three airfoils along the wing to create profiles for the entire wing
- Each segment is assigned an laminate ID which will provide the segment's mechanical properties





Structures

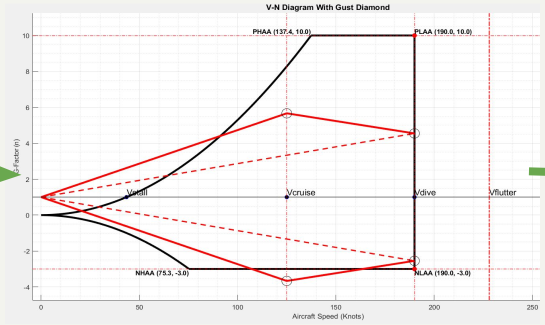
- Inputs:
  - Airfoils along the wing
  - Segment laminate ID assignments
- Outputs:
  - Modulus weighted centroid (MWC)
  - Modulus weighted moment of inertia (MWMI)
  - Area and perimeter of each segment
  - Area encompassed between segment centroids and MWC
    - Relevant for shear flow calculation

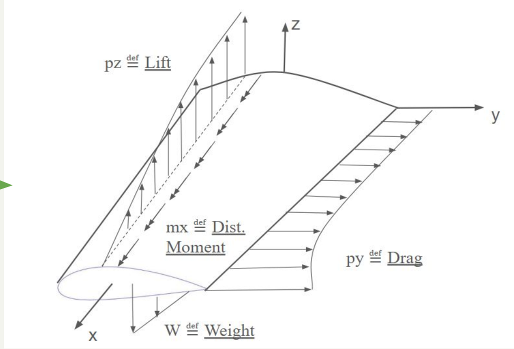
Laminate ID	(#)	node (i)	node (j)	t inch	ρ lb/in^3	Ex Msi	Gxy Msi
1	1	1	2	2	2	5	6
2	2	2	3	2	2	5	6
3	3	3	4	2	2	5	6
4	4	4	5	2	2	5	6
5	5	5	6	2	2	5	6
6	6	6	7	2	2	5	6
1	7	7	8	2		5	6
2	8	8	9	2	2	5	6
3	9	9	10	2	2	5	6

Loads

The loads section of the MATLAB code defines the aerodynamic loading applied to the wing and calculates the corresponding stress resultants.

Variable	Description	Value	Units
dC_L/da	Lift Curve Slope	0.0843	1/degree
a_o	Zero Lift Angle	-2.3	degree
C_{L-max} (+)	Max Lift at Stall (+)	1.61	1
a_{stall} (+)	Stall Angle (+)	16	degree
C_{L-max} (-)	Max Lift at Stall (-)	-1.56	1
a_{stall} (-)	Stall Angle (-)	-20	degree
C_{mo}	Wing Moment Coefficient	-0.045	1
C_{DoA}	Parasitic Drag - Fuselage	0.024	1
C_{DoW}	Parasitic Drag - Wing	0.008	1
C_{DiW}	Induced Drag Coefficient	0.0558	1
d_0	Spanwise drag amplification	1	1
d_{10}	Spanwise drag amplification	0.2	1
Hp	Maximum Motor Power	350	Hp
η	2-Bladed Propeller (75" Dia) Efficiency	90	%





SE 160A Code: Axial Stress

- Inputs:
  - Segment Centroid location
  - Segment Modulus
  - Airfoil EA
  - Segment MWMI
  - Segment Thickness
- Outputs
  - Axial stress/ Nx
  - Bending compliance

$$\sigma_{xx} = E_t\{1, -y, -z\} \begin{bmatrix} \frac{1}{EA} & 0 & 0 \\ 0 & k_{yy} & -k_{yz} \\ 0 & -k_{yz} & k_{zz} \end{bmatrix} \begin{Bmatrix} V_x \\ M_z \\ -M_y \end{Bmatrix}$$

Non Homogeneous Euler Beam Theory Equation for Axial Stress

$$k_{yy} = \frac{EI_{yy}}{EI_{yy}EI_{zz} - EI_{yz}^2}$$

$$k_{zz} = \frac{EI_{zz}}{EI_{yy}EI_{zz} - EI_{yz}^2}$$

$$k_{yz} = \frac{EI_{yz}}{EI_{yy}EI_{zz} - EI_{yz}^2}$$

Bending Compliance Formulas

SE 160A Code: Shear Flow

- Inputs
  - Airfoil MWMI
  - Segment Modulus
  - Segment Centroid
  - Airfoil EA
- Outputs
  - Shear Stress/Flow
  - Shear Center
  - Wing Twist/ Twist Rate
- Note: Airfoil is assumed to have two cells due to the shear web

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ \frac{2A_{12}}{\kappa_{12}} & \frac{2A_{23}}{\overline{\kappa_{23}}} & \frac{2A_{34}}{\overline{\kappa_{34}}} & \frac{2A_{45}}{\overline{\kappa_{45}}} & \frac{2A_{56}}{\kappa_{56}} & \frac{2A_{61}}{\kappa_{61}} & \frac{2A_{25}}{\kappa_{25}+\overline{\kappa_{25}}} \end{bmatrix} \begin{Bmatrix} q_{12} \\ q_{23} \\ q_{34} \\ q_{45} \\ q_{56} \\ q_{61} \\ q_{25} \end{Bmatrix} = \frac{V_y}{EI_{zz}} \begin{Bmatrix} EA_{y_2} \\ EA_{y_3} \\ EA_{y_4} \\ EA_{y_5} \\ EA_{y_6} \\ 0 \end{Bmatrix} + \frac{V_z}{EI_{yy}} \begin{Bmatrix} EA_{z_2} \\ EA_{z_3} \\ EA_{z_4} \\ EA_{z_5} \\ EA_{z_6} \\ 0 \end{Bmatrix} + M_x \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{Bmatrix}$$

Matrix approximate shear flow solution

$$\theta = \sum_{n} \frac{d\theta}{dx} * L$$

Wing Twist Equation

Laminate Analysis

Inputs:

- Ply mat'l ID
- Ply orientations
- Ply thicknesses

Outputs:

- Eff. moduli
- Ply failure num.
- Ply failure type
- Ply failure phase
- Failure ratio/MS

Variable	Description	Value	Units
	Input Units (1 = US, 2 = SI)	1	
	Output Units (1 = US, 2 = SI)	1	
n	Number of plies (12 maximum)		
N_x	Inplane x-direction load	0.00E+00	(lb/in
N_y	Inplane y-direction load	0	(lb/in
N_{xy}	Inplane shear load	0	(lb/in
M_x	Bending Moment (about x-axis	0	(lb-in/
M_y	Bending Moment (about y-axis	0	(lb-in/
M_{xy}	Twisting Moment	0	(lb-in/
T_{REF}	Temperature (Reference)	0	(°F) o
T	Temperature (Operating)	0	(°F) o
SF	Safety Factor:		
F^*_{12}	Tsai-Wu Stress Interaction Factor		
	Laminate Region for analysis	2	

Laminate ID:		2
Total Number of Plies:		8
Lamina ID	θ_i (degree)	t_{ply} (inch)
301	-45	0.006
301	45	0.006
301	-45	0.006
301	45	0.006
301	-45	0.006
301	45	0.006
301	-45	0.006
301	45	0.006

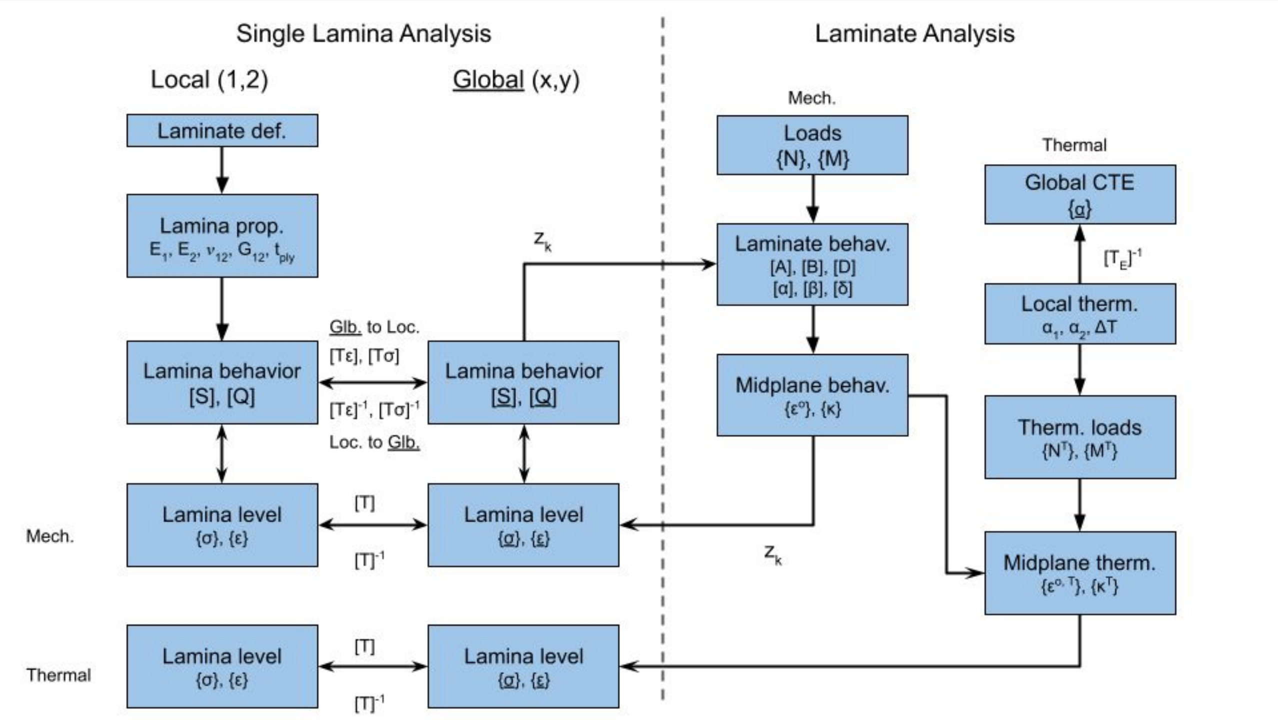
Laminate Analysis cont.

Inputs:

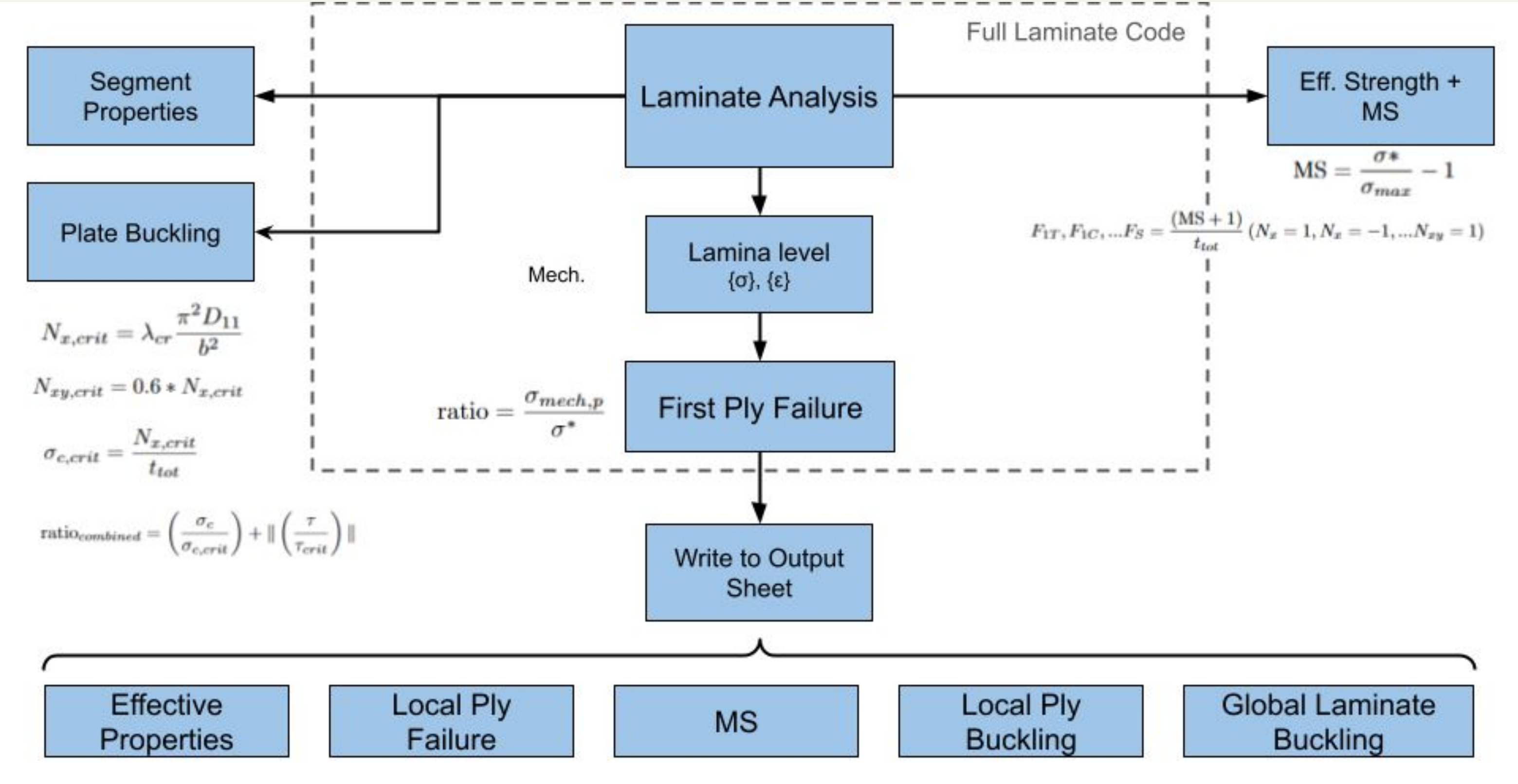
- Ply mat’l ID
- Ply orientations
- Ply thicknesses

Outputs:

- Eff. moduli
- Ply failure num.
- Ply failure type
- Ply failure phase
- Failure ratio/MS



Laminate Analysis cont.



Failure Criteria

- Max Stress

$$\text{ratio} = \frac{\sigma_{mech,p}}{\sigma^*}$$

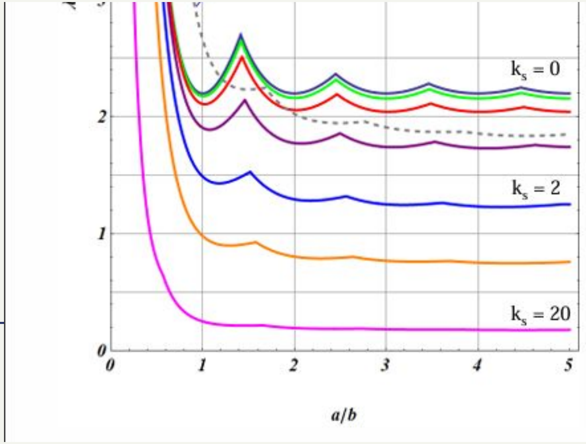
$$\text{MS} = \frac{\sigma*}{\sigma_{max}} - 1$$

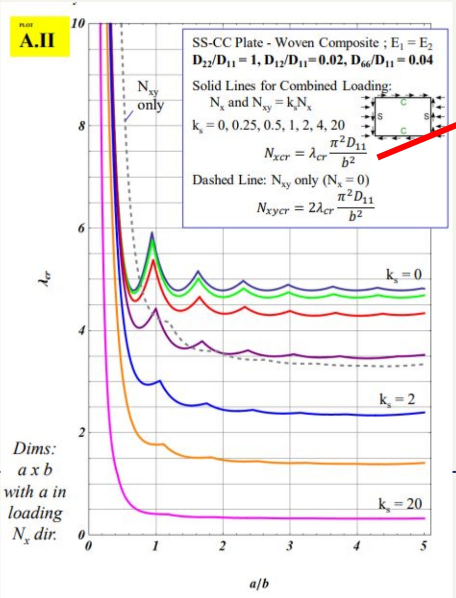
Stress Component	**Failure Phase**	**Failure Type**
σ_x	Fiber	Tension (if positive), Compression (if negative), Evaluate based on F_{1C}: • Fiber compression • Micro-buckling • Delamination
σ_y	Matrix	Tension (if positive), Compression (if negative)
τ_{xy}	Matrix	Shear

Buckling

New formulations (Prof. H. Kim's slides)

- Uses D matrix terms
- For **combined loading**
- Assumes 2 compression loaded SS edges, transverse edges are **clamped**
  - CBF ranges from 0.25 - 6
- 4 SS edges
  - CBF ranges from 0.25 - 3





$$N_{xcr} = \lambda_{cr} \frac{\pi^2 D_{11}}{b^2}$$

$$\sigma_{c,crit} = \frac{N_{x,crit}}{t_{tot}}$$

Failure Criteria (Buckling)

- Max Stress

$$\sigma_{c,crit} = \frac{N_{x,crit}}{t_{tot}}$$

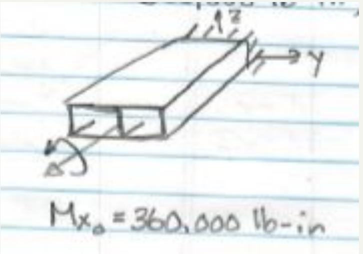
$$\text{ratio}_{combined} = \left(\frac{\sigma_c}{\sigma_{c,crit}}\right) + \|\left(\frac{\tau}{\tau_{crit}}\right)\|$$

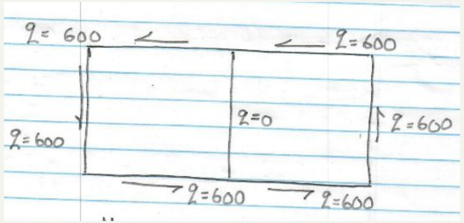
$$\text{ratio} = \frac{\sigma_{mech,p}}{\sigma^*}$$

Validation: Shear Flow

Test Case #1

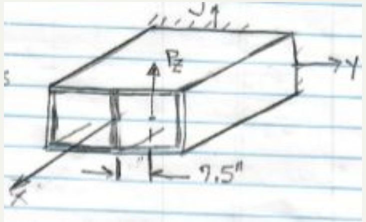
1	600.0000
2	600.0000
3	600.0000
4	600.0000
5	600.0000
6	600.0000
7	600.0000
8	600
9	600
10	600.0000
11	600.0000
12	600.0000
13	600.0000
14	600.0000



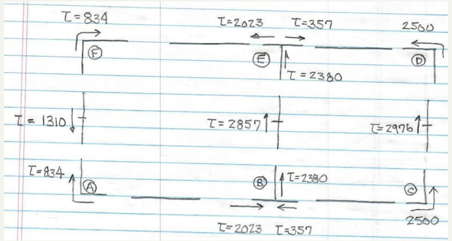


37	-908.0606
38	-1.0962e+03
39	-1.2373e+03
40	-1.3313e+03
41	-1.3784e+03
42	-1.3784e+03
43	-1.3313e+03
44	-1.2373e+03
45	-1.0962e+03
46	-908.0606

Test Case #2



(P_z = 2,000 lb)



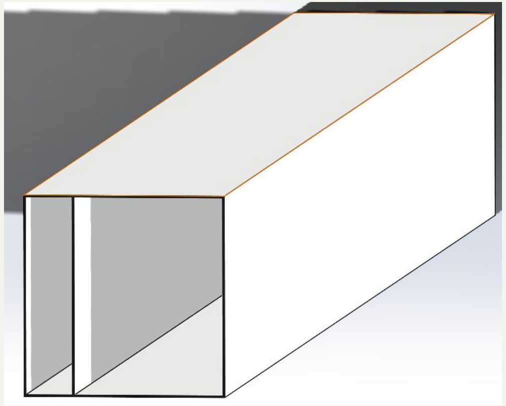
47	2.6125e+03
48	2.7648e+03
49	2.7648e+03
50	2.6125e+03
51	2.3077e+03

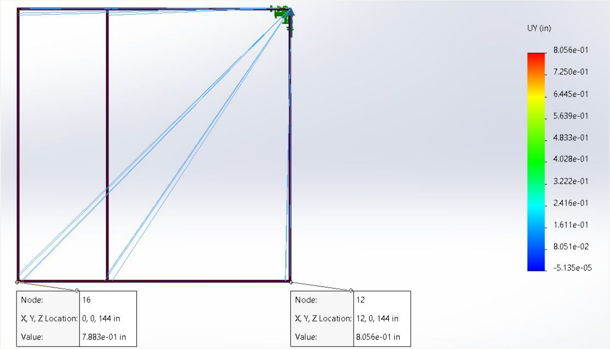
14	2.4985e+03
15	2.6867e+03
16	2.8278e+03
17	2.9218e+03
18	2.9688e+03
19	2.9688e+03
20	2.9218e+03
21	2.8278e+03
22	2.6867e+03

- 4-8% error in approximated values which is reasonable

Validation: Twist and Shear Center

- Geometry: 12 in x 12 x 144 in hollow cube with a shear web at 4 in
  - Wall Thickness = .1 in
- Load Case: Applied point of 1000 lbf upward on Y = 12 in, Z = 0
- Twist: Matlab = .0723 degrees, Hand Calculation = .083, Solidworks = .0826
- Shear center calculated to be Y = 5.39 in, Z = 6 in Matlab and Solidworks



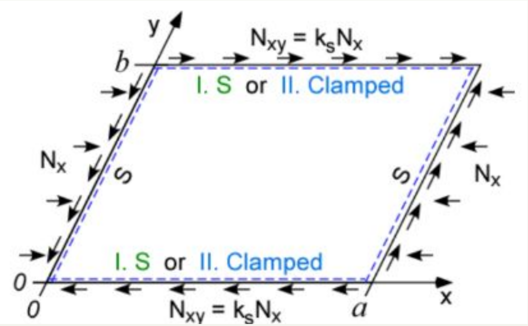


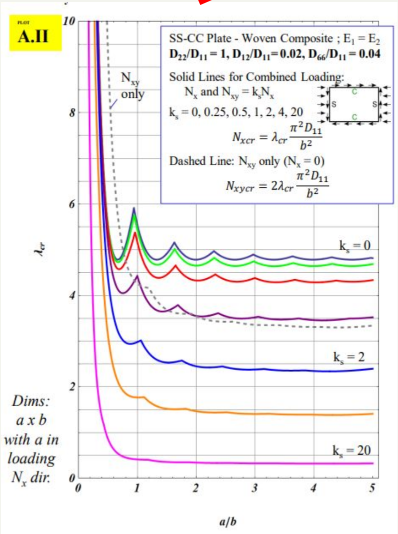
$$\theta_{Tip} = \frac{M_{x0} \cdot l}{GJ}$$

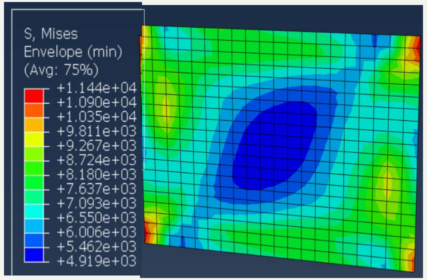
Validation: Buckling Stress

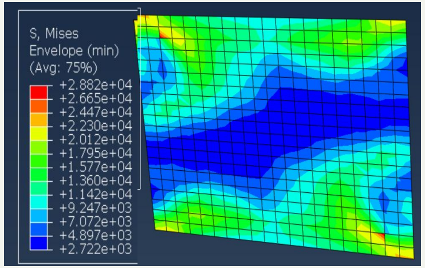
$$\cancel{\tau_{CR} = \frac{k\pi^2 D_{66}}{a^2}; \quad where\ k = 4(\frac{a}{b})}$$

$$N_{xcr} = \lambda_{cr}\frac{\pi^2 D_{11}}{b^2} \qquad D_{ij} = \frac{1}{3}\sum_{k=1}^{n}\bar{Q}_{ij}^{k}(z_k^3 - z_{k-1}^3)$$









sig_cr_comp	4.7378e+03
sig_cr_shear	2.6152e+03

- Critical Buckling Factor Scaled by 2 = 0.5
- Ks = 0.6
- (σcr)compression % error = 3.7%
- (σcr)shear % error = 5.7%

Compared to Abaqus Analysis