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Jam formation in traffic flow on a highway with some slowdown sections

Hirotoshi Hanaura, Takashi Nagatani*, Katsunori Tanaka

Department of Mechanical Engineering, Division of Thermal Science, Shizuoka University, Hamamatsu 432-8561, Japan

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Abstract

We study the traffic jams appearing on a single-lane highway with a few slowdown sections. At low density, the flow (current) increases linearly with density, while it saturates at some values of intermediate density. In such case that some slowdown sections have the same speed limit, when the flow begins to saturate, a single discontinuous front (stationary shock wave) occurs before a slowdown section or some discontinuous fronts appear before some slowdown sections. For the case of different speed limits, the discontinuous front occurs before the section of strongest slowdown. The saturated flow is given by the maximal value of the current of the strongest slowdown section. The relationship between the densities is derived before and after the discontinuity. The dependence of jam lengths on density is derived numerically and analytically.

Keywords: Vehicular dynamics; Traffic jam; Discontinuity; Phase transition

1. Introduction

Traffic flow is a kind of many-body system of strongly interacting vehicles [1–5]. Vehicular traffic has been studied by several traffic models: car-following models, cellular automaton models, gas kinetic models, and hydrodynamic models [6–27]. Mobility is nowadays one of the most significant ingredients of a modern society. The traffic accident often occurs in city traffic networks [22]. Also, traffic networks often exceed the capacity. The city traffic is controlled by speed limit and traffic lights for security and priority for a road [21,23,27,28].

Traffic jams are typical signature of the complex behavior of traffic flow. Traffic jams are classified into two kinds of jams: (1) spontaneous jam (or phantom jam) which propagates backward as the stop- and go-wave and (2) stationary jam which is induced by slowdown or blockage at a section of roadway. If sensitivity of driver is lower than a critical value, the spontaneous jam occurs. The jamming transition is very similar to the conventional phase transitions and critical phenomena [1,26]. When the sensitivity is higher than the critical value, the spontaneous jam does not appear, while the stationary jam induced by slowdown occurs [1,27,28].

*Corresponding author. Fax: +81 53 478 1048.
E-mail address: tmtnaga@ipc.shizuoka.ac.jp (T. Nagatani).

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Such speed limit as slowdown often induces traffic jams when a vehicular density is high. One is interested in the structure and formation of traffic jams induced by slowdown. In the previous paper [27], we have studied the traffic jam induced by slowdown at a section of roadway. We have shown the following. When a density of vehicles is low, vehicles move freely with no jams. If the density is higher than a critical value, the traffic jam is formed just before the section of slowdown. The speed of vehicles within the jam becomes lower than the speed limit of slowdown. The traffic jam ends with forming a queue of slow vehicles. A discontinuous front appears at the end (edge) of traffic jam. The relationships between the headways and velocities before and after the discontinuity have been derived.

In real traffic, some sections of slowdown exist on a highway. When a few sections of distinct slowdowns exist on a highway, where, when, and how do traffic jams occur on a highway? The traffic jams induced by some slowdown sections have little been studied by using modern traffic models.

In this paper, we apply the conventional optimal-velocity model to the traffic flow on a highway with some slowdown sections. We study the traffic states and discontinuous fronts induced by some slowdown sections. We clarify the dynamical states of traffic and the characteristic of discontinuous fronts (or traffic jams). We show where and how the traffic jams occur by increasing the density of vehicles and by varying slowdowns.

2. Model

We consider the vehicular traffic flowing on the single-lane roadway with some sections of slowdown. Vehicles move with no passing on the single-lane roadway under periodic boundary condition. We assume that vehicles are forced to slow down when they enter into a section of the slowdowns. Fig. 1(a) shows the schematic illustration of the traffic model for the single-lane highway with a single section of slowdown. Vehicles move with low speed in the section of slowdown, while they move with the normal velocity except for the section of slowdown. The length of slowdown section is L_S and the length of normal-speed section is L_N where the road length is $L = L_N + L_S$. We extend the traffic flow to that through a few slowdown sections. Fig. 1(b) shows the schematic illustration of the traffic model for the single-lane highway with two slowdown sections. The lengths of slowdown sections are L_{S1} and L_{S2}. The lengths of normal-speed sections are L_{N1} and L_{N2} where the road length is $L = L_{N1} + L_{N2} + L_{S1} + L_{S2}$. Vehicles move with low speed in the slowdown sections, while they move with the normal velocity except for the slowdown sections. We apply the optimal-velocity model to the traffic flow [1,29]. The optimal-velocity model is described by the following

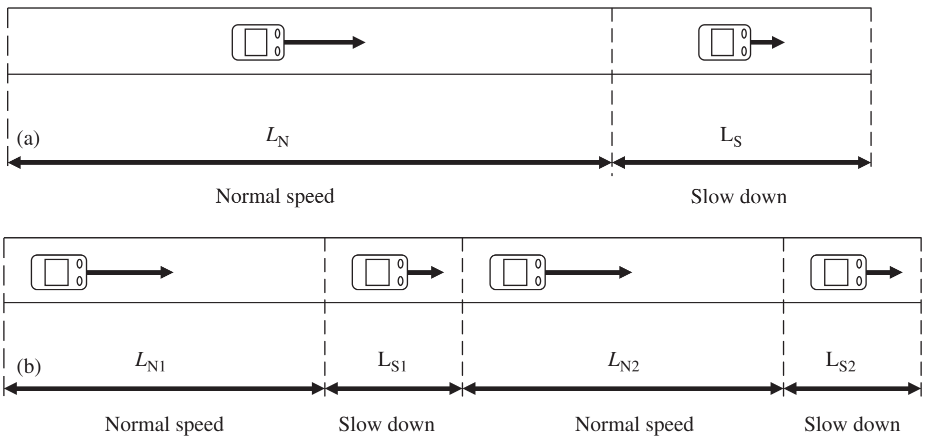


Fig. 1. (a) Schematic illustration of the traffic model for the single-lane highway with a single section of slowdown. Vehicles move with low speed in the section of slowdown, while they move with the normal velocity except for the section of slowdown. The length of slowdown section is L_S and the length of normal-speed section is L_N where the road length is $L = L_N + L_S$. (b) Traffic model for the single-lane highway with two sections of slowdown. The lengths of slowdown sections are L_{S1} and L_{S2}. The lengths of normal-speed sections are L_{N1} and L_{N2} where the road length is $L = L_{N1} + L_{N2} + L_{S1} + L_{S2}$.

equation of motion of vehicle i:

$$\frac{d^2x_i}{dt^2} = a\left\{V(\Delta x_i) - \frac{dx_i}{dt}\right\}, \tag{1}$$

where $V(\Delta x_i)$ is the optimal-velocity function, $x_i(t)$ is the position of vehicle i at time t, $\Delta x_i(t) = (x_{i+1}(t) - x_i(t))$ is the headway of vehicle i at time t, and a is the sensitivity (the inverse of the delay time) [1].

A driver adjusts the vehicular speed to approach the optimal velocity determined by the observed headway. The sensitivity a allows for the time lag $\tau = 1/a$ that it takes the vehicular speed to reach the optimal velocity when the traffic is varying. Generally, it is necessary that the optimal-velocity function has the following properties: it is a monotonically increasing function and it has an upper bound (maximal velocity). In the region of normal-speed sections, the optimal-velocity function of vehicles is given by

$$V(\Delta x_i) = \frac{v_{f,\max}}{2}\left[\tanh \alpha(\Delta x_i - x_{f,c}) + \tanh \alpha(x_{f,c})\right], \tag{2}$$

where $v_{f,\max}$ is the maximal velocity of vehicles on the roadway except for the slowdown sections and $x_{f,c}$ the position of turning point [1].

In the slowdown sections, vehicles move with the forced speed and the vehicular velocity should be lower than the speed limits of slowdown. When vehicles enter into the first section of slowdown, they obey the conventional optimal-velocity function with speed limit $v_{s1,\max}$:

$$V(\Delta x_i) = \frac{v_{s1,\max}}{2}\left[\tanh(\Delta x_i - x_{s,c}) + \tanh(x_{s,c})\right], \tag{3}$$

where $v_{s1,\max}$ is the speed limit of first slowdown section, $v_{s1,\max} < v_{f,\max}$, and $x_{s,c}$ the position of turning point.

Similarly, when vehicles enter into the second slowdown section, they obey the following optimal-velocity function with speed limit $v_{s2,\max}$:

$$V(\Delta x_i) = \frac{v_{s2,\max}}{2}\left[\tanh(\Delta x_i - x_{s,c}) + \tanh(x_{s,c})\right], \tag{4}$$

where $v_{s2,\max}$ is the speed limit of second slowdown section and $v_{s2,\max} < v_{f,\max}$.

Thus, when the density of vehicles is very low, vehicles move at the maximal velocity $v_{f,\max}$ except for the sections of slowdown, while they move at a speed lower than the forced speed limits $v_{s1,\max}$ or $v_{s2,\max}$ in the slowdown sections. With increasing density, vehicles interact with each other. The dynamics is determined by Eqs. (1)–(4). Then, various dynamic states of traffic appear and traffic jams may occur. We study the dynamic states and traffic jams in the traffic flow described by the model in Fig. 1(b). Furthermore, we study the traffic flow on the highway with more slowdown sections than 2.

For late convenience, we summarize the dynamic states and spontaneous jams for the conventional optimal-velocity model. The conventional optimal-velocity model with a single turning point exhibits two phases and a coexisting state of two phases. Below the critical point $(x_{f,c}, a_c)$ $(a_c = v_{f,\max})$, the phase transition occurs [1].

In this paper, we restrict ourselves to such case that sensitivity is higher than the critical point, $a > a_c$, since we do not investigate the spontaneous jam but study the traffic jams induced by slowdown.

3. Simulation and result

We perform computer simulation for the traffic model shown in Fig. 1(b). We simulate the traffic flow under the periodic boundary condition. The simulation is performed until the traffic flow reaches a steady state. We solve numerically Eq. (1) with optimal-velocity functions (2)–(4) by using fourth-order Runge–Kutta method where the time interval is $\Delta t = 1/128$.

We carry out the simulations by varying the initial headway, slowdown-section lengths L_{S1} and L_{S2}, and slowdown's velocities $v_{s1,\max}$ and $v_{s2,\max}$ for 500 vehicles, sensitivity $a = 2.5$ and maximal velocity $v_{f,\max} = 2.0$. Initially, we put all vehicles on the single-lane highway with the same headway Δx_{int}. The density ρ is given by the inverse of the headway. The length L of highway varies with the initial headway.

First, we study the traffic behavior for such case that the length and speed limit of the first slowdown section agree with those of the second slowdown section: $L_{N1} = L_{N2} = L_{S1} = L_{S2} = L/4$ and $v_{s1,\max} = v_{s2,\max} = 1.0$.

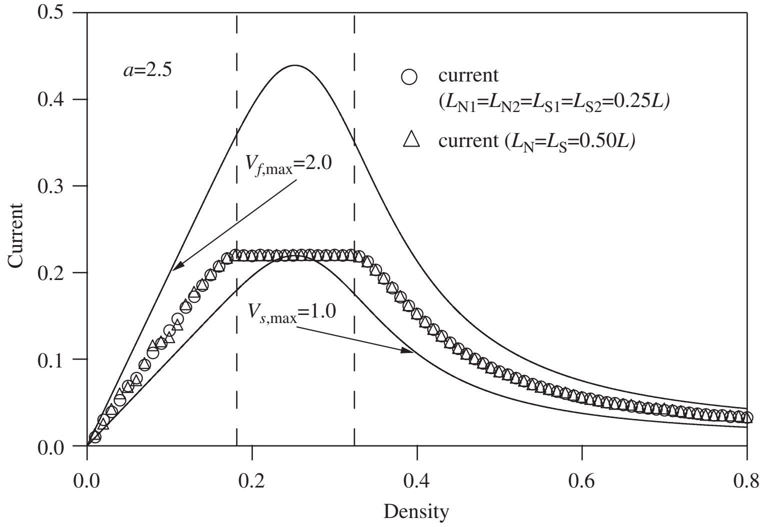


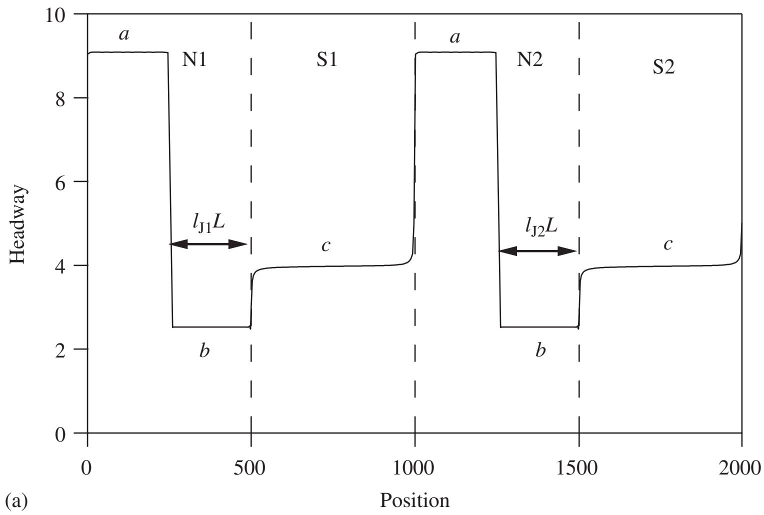
Fig. 2. Plots of traffic currents against density (fundamental diagram). The open circle indicates the traffic current obtained by simulation. The values of traffic current for a single slowdown section ($L_N = L_S = L/2$) in Fig. 1(a) are indicated by open triangles. The traffic currents for two optimal-velocity functions without traffic jams are shown by two solid lines.

Fig. 2 shows the plots of traffic currents against density. The open circle indicates the traffic current obtained by simulation. The current increases linearly with density, then the current saturates and keeps a constant value which agrees with the maximal value of theoretical current curve of $v_{s1,\max} = v_{s2,\max} = 1.0$. When density is higher, the current decreases with increasing density.

For comparison, the values of traffic current obtained by simulation for a single slowdown section ($L_N = L_S = L/2$) in Fig. 1(a) are indicated by open triangles. The traffic current of $L_N = L_S = L/2$ is consistent with that of $L_{N1} = L_{N2} = L_{S1} = L_{S2} = L/4$. Also, the traffic currents for two optimal-velocity functions without traffic jams are shown by two solid lines. The current is called as the theoretical current curve. It is given by $J = V(\Delta x)/\Delta x$ where Δx is the average value of headway and $V(\Delta x)$ is the optimal velocity. The case of no slowdown corresponds to the conventional traffic flow. When sensitivity is higher than critical value 2.0 in traffic flow on a highway without any sections of slowdown, no traffic jams occur and the current agrees with the theoretical current curve.

Figs. 3(a) and (b) show, respectively, the headway and velocity profiles at $\rho = 0.25$ and $t = 50{,}000$ where $L_{N1} = L_{N2} = L_{S1} = L_{S2} = L/4$, $v_{f,\max} = 2.0$, $v_{s1,\max} = v_{s2,\max} = 1.0$, and $a = 2.5$. In the region where the current saturates, two discontinuous fronts appear simultaneously in two sections of normal velocity. Two traffic jams occur simultaneously just before two sections of slowdown and end at the discontinuous fronts. Fig. 4 shows the crossing points at the theoretical current curves through the horizontal line on the maximal current of $v_{s,\max} = 1.0$. The crossing points are indicated by the full circle and the full square. The maximal current of $v_{s,\max} = 1.0$ is indicated by the full triangular point. The densities (inverse of headway) before and after two discontinuous fronts are consistent with those at points a and b in Fig. 4. The density at the sections of slowdown agree with that at point c of the maximal current of $v_{s1,\max} = v_{s2,\max} = 1.0$. The velocities at regions a, b, and c are given, respectively, by the optimal velocities at densities a, b, and c.

The lengths of two jams in Fig. 3(a) increase with density. Fig. 5 shows the plot of jam-length ratio (jam length/L) against density for $L_{N1} = L_{N2} = L_{S1} = L_{S2} = L/4$. Jam-length ratio l_{J1} at section $N1$ and jam-length ratio l_{J2} at section $N2$ are indicated by open circles and crosses, respectively. Jam-length ratio l_{J1} at section $N1$ agrees with jam-length ratio l_{J2} at section $N2$. Two jams occur simultaneously when the density is higher than $\rho = 0.18$. This value of density is consistent with the starting point of current saturation in Fig. 2 and is shown by the vertical dotted line. Two discontinuous fronts of jams disappear simultaneously when the density is higher than $\rho = 0.325$. This value of density agrees with the ending point of current saturation in Fig. 2 and is shown by the dotted line with the arrow. The total account of two jam-length ratios is plotted by open



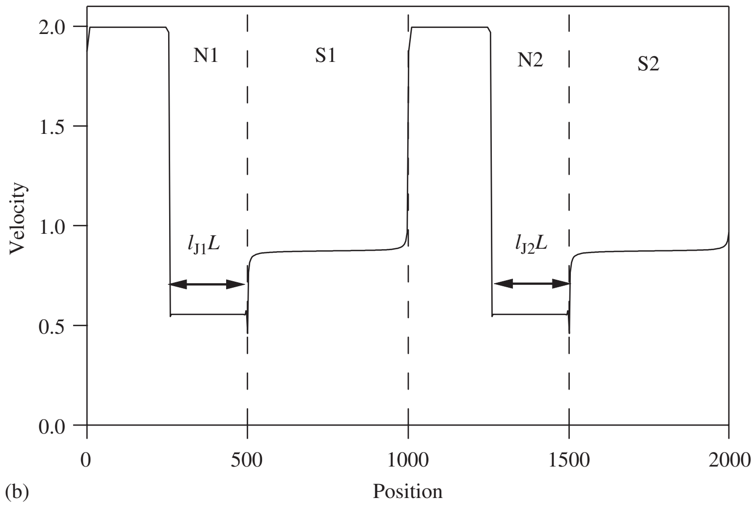


Fig. 3. (a) Headway profile at $\rho = 0.25$ and $t = 50000$ where $L_{N1} = L_{N2} = L_{S1} = L_{S2} = L/4$, $v_{f,\max} = 2.0$, $v_{s1,\max} = v_{s2,\max} = 1.0$, and $a = 2.5$. (b) Velocity profile.

triangles. The total length of two jams agrees with that obtained by the theoretical analysis which is shown by the solid line in Fig. 5.

We study the traffic flow for such case that $L_{N1}(=0.35L) > L_{N2}(=0.15L)$, $L_{S1} = L_{S2} = 0.25L$, and $v_{s1,\max} = v_{s2,\max} = 1.0$. Fig. 6(a) shows the headway profile at $\rho = 0.31$. Fig. 6(b) shows the plot of jam-length ratio (jam length/L) against density. Jam-length ratio l_{J1} at section $N1$ and jam-length ratio l_{J2} at section $N2$ are indicated by open circles and crosses, respectively. Jam-length ratio l_{J1} at section $N1$ is different from jam-length ratio l_{J2} at section $N2$. First, the jam occurs at the first section of normal speed before the first slowdown section when the density is higher than $\rho = 0.18$. This value of density is consistent with the starting point of current saturation in Fig. 2 and is shown by the vertical dotted line. Then, the second jam occurs at the second section of normal speed before the second slowdown section. With increasing density, both jams grow. When the second jam reaches the boundary between the first slowdown section and the second normal-speed section, the second jam does not grow but the first jam continues to grow until the first jam reaches the boundary between the first normal-speed section and the second slowdown section. In Fig. 6(b), the down arrow indicates the point at which the second jam reaches the boundary between $S1$ and $N2$. The second discontinuity stops to move when the density is higher than $\rho = 0.325$. This value of density is shown by the

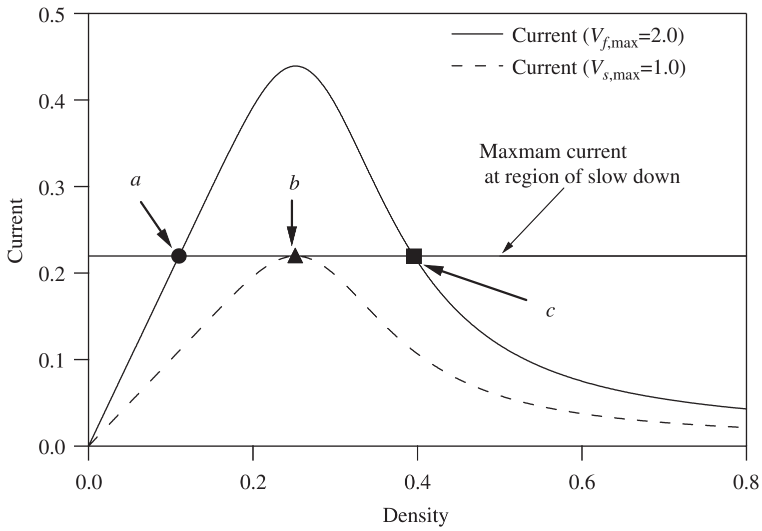


Fig. 4. Crossing points at the theoretical current curve through the horizontal line on the maximal current of $v_{s,\max} = 1.0$. The crossing points are indicated by the full circle and the full square. The maximal current of $v_{s,\max} = 1.0$ is indicated by the full triangular point. The densities (inverse of headway) before and after two discontinuous fronts are consistent with those at points *a* and *b*.

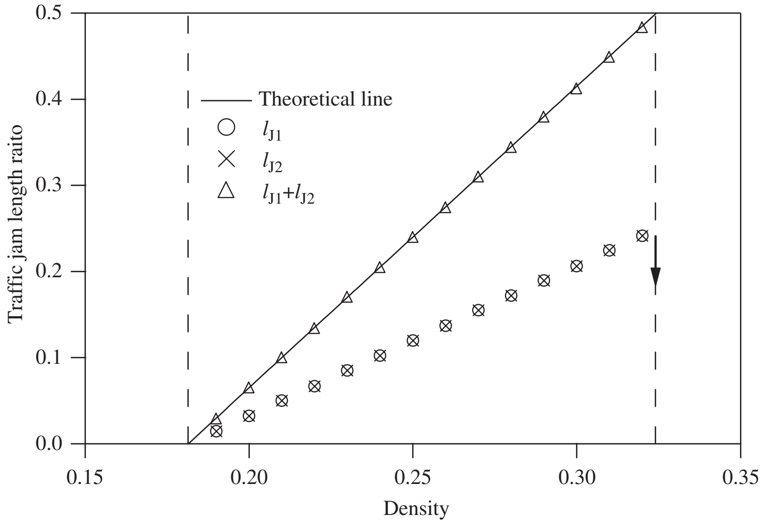
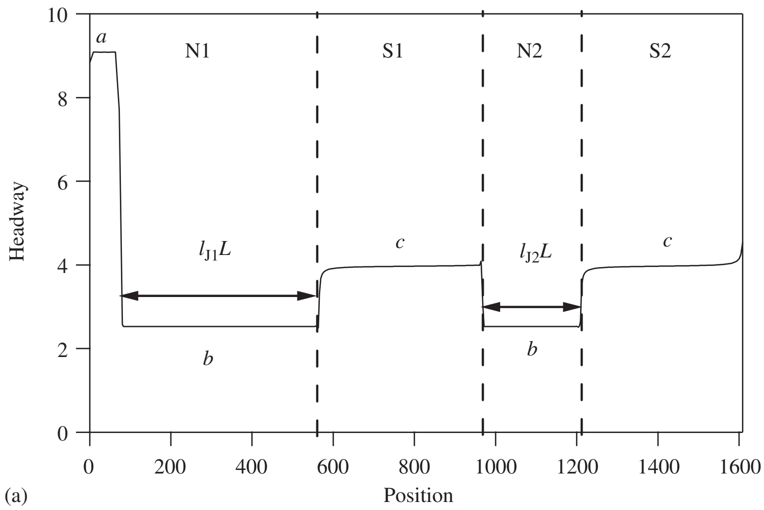


Fig. 5. Plot of jam-length ratio (jam length/L) against density for $L_{N1} = L_{N2} = L_{S1} = L_{S2} = L/4$. Jam-length ratio l_{J1} at section $N1$, and jam-length ratio l_{J2} at section $N2$ are indicated by open circles and crosses, respectively. Jam-length ratio l_{J1} at section $N1$ agrees with jam-length ratio l_{J2} at section $N2$.

dotted line. The total account of two jam-length ratios is plotted by open triangles. The total length of two jams agrees with that obtained by the theoretical analysis which is shown by the solid line in Fig. 6(b). Thus, the occurrence and disappearance of two jams are different from each other when length L_{N1} of the first normal-speed section is different from L_{N2} of the second normal-speed section.

We study the traffic flow for such case that $L_{N1} = L_{N2} = 0.25L$, $L_{S1}(= 0.35L) > L_{S2}(= 0.15L)$, and $v_{s1,\max} = v_{s2,\max} = 1.0$. Fig. 7(a) shows the plots of jam-length ratio (jam length/L) against density. Jam-length ratio l_{J1} at section $N1$ is different from jam-length ratio l_{J2} at section $N2$. First, the jam occurs at first normal-speed section $N1$ when the density is higher than $\rho = 0.18$. Then, the second jam occurs at second



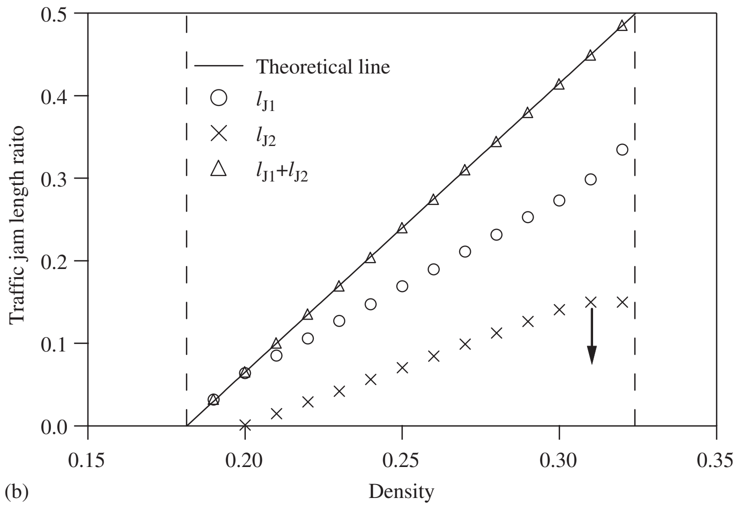
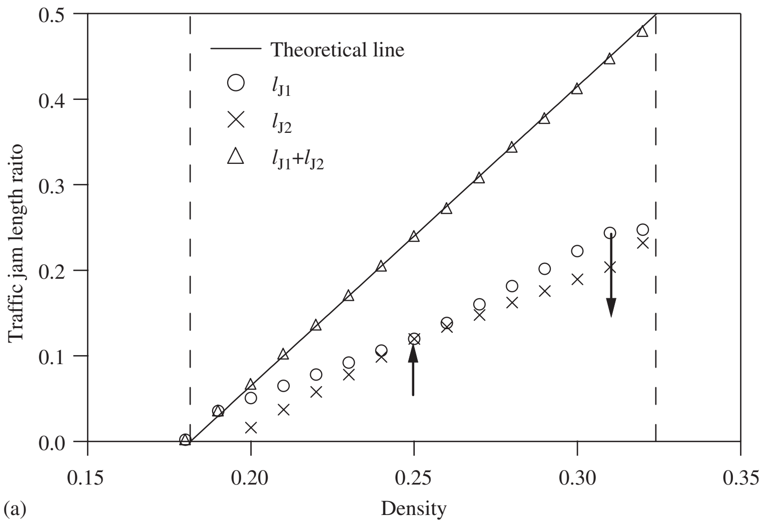


Fig. 6. (a) Headway profile at $\rho = 0.31$. (b) Plots of jam-length ratio (jam length/L) against density. Jam-length ratio l_{J1} at section $N1$ and jam-length ratio l_{J2} at section $N2$ are indicated by open circles and crosses, respectively.

normal-speed section $N2$. With increasing density, both jams grow. Length l_{J1} of the first jam agrees with length l_{J2} of the second jam at $\rho = 0.25$ indicated by the up arrow. At the density indicated by the down arrow, the first jam reaches the boundary between $N1$ and $S2$ and the first discontinuous front stops to move at this density. The total length of two jams agrees with that obtained by the theoretical analysis.

We study the traffic flow for such case that $L_{N1}(=0.15L) < L_{N2}(=0.35L)$, $L_{S1}(=0.15L) < L_{S2}(=0.35L)$, and $v_{s1,\max} = v_{s2,\max} = 1.0$. Fig. 7(b) shows the plots of jam-length ratio (jam length/L) against density. Jam-length ratio l_{J1} at section $N1$ is different from jam-length ratio l_{J2} at section $N2$. First, the jam occurs at second normal-speed section $N2$ when the density is higher than $\rho = 0.18$. Then, the second jam occurs at first normal-speed section $N1$. With increasing density, both jams grow. Length l_{J2} of the first jam is longer than the length l_{J1} of the second jam. At $\rho = 0.25$ indicated by the up arrow, growth rates of both jams change. At the density indicated by the down arrow, both jams reach, simultaneously, the boundaries and both discontinuous fronts stop to move at density $\rho = 0.325$. The total length of two jams agrees with that obtained by the theoretical analysis.

We study the traffic behavior for such case that the length of all the sections is the same and the speed limit of the first slowdown section is higher than that of the second slowdown section: $L_{N1} = L_{N2} = L_{S1} = L_{S2} =$



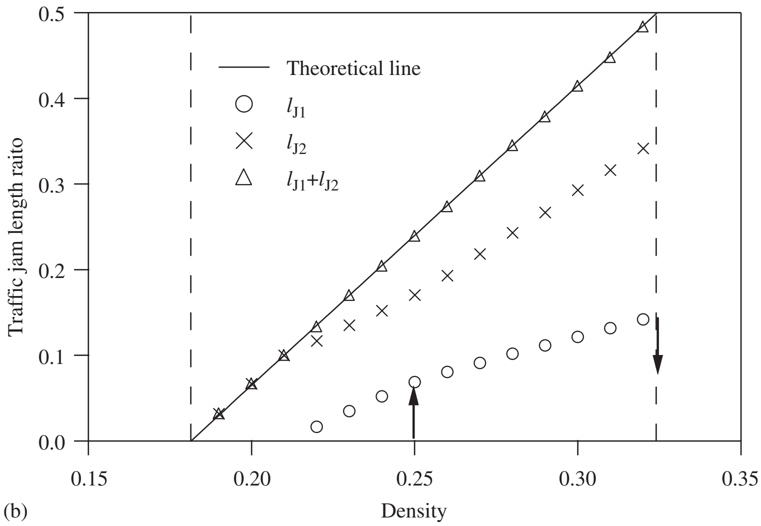


Fig. 7. (a) Plots of jam-length ratio (jam length/L) against density where $L_{N1} = L_{N2} = 0.25L$, $L_{S1}(=0.35L) > L_{S2}(=0.15L)$, and $v_{s1,\max} = v_{s2,\max} = 1.0$. (b) Plots of jam-length ratio (jam length/L) against density where $L_{N1}(=0.15L) < L_{N2}(=0.35L)$, $L_{S1}(=0.15L) < L_{S2}(=0.35L)$, and $v_{s1,\max} = v_{s2,\max} = 1.0$.

$L/4$ and $v_{s1,\max}(=1.5) > v_{s2,\max}(1.0)$. Fig. 8 shows the headway profiles at (a) $\rho = 0.16$, (b) $\rho = 0.25$, and (c) $\rho = 0.33$. Fig. 8(d) shows the plot of jam-length ratio (jam length/L) against density. Fig. 9 shows the theoretical current curves for three optimal-velocity functions. The solid, chain, and dotted curves represent, respectively, the theoretical current curves for $v_{f,\max} = 2.0$, $v_{s1,\max} = 1.5$, and $v_{s2,\max} = 1.0$. Point c indicates the maximal value on the current curve of $v_{s2,\max} = 1.0$. Points a, b, d, and e indicate such points that the horizontal line on point c crosses at the theoretical curves of $v_{f,\max} = 2.0$ and $v_{s1,\max} = 1.5$. First, the jam occurs at section $N2$ before section $S2$ of the strong speed limit. The headway profile at $\rho = 0.16$ is shown in Fig. 8(a). The densities before and after the discontinuity are given by those at points a and b in Fig. 9. With increasing density, the discontinuous front moves to the upstream. If the discontinuity reaches the boundary between $S1$ and $N2$, it goes through the boundary and the discontinuous front is formed at section $S1$. The headway profile at $\rho = 0.25$ is shown in Fig. 8(b). The densities before and after the discontinuity are given by those at points d and e in Fig. 9. When the density increases furthermore, the discontinuous front goes through the boundary between $N1$ and $S1$ and the discontinuity is formed at section $N1$. The densities before and after the discontinuity are given by those at points a and b in Fig. 9. The jam continues to grow with increasing

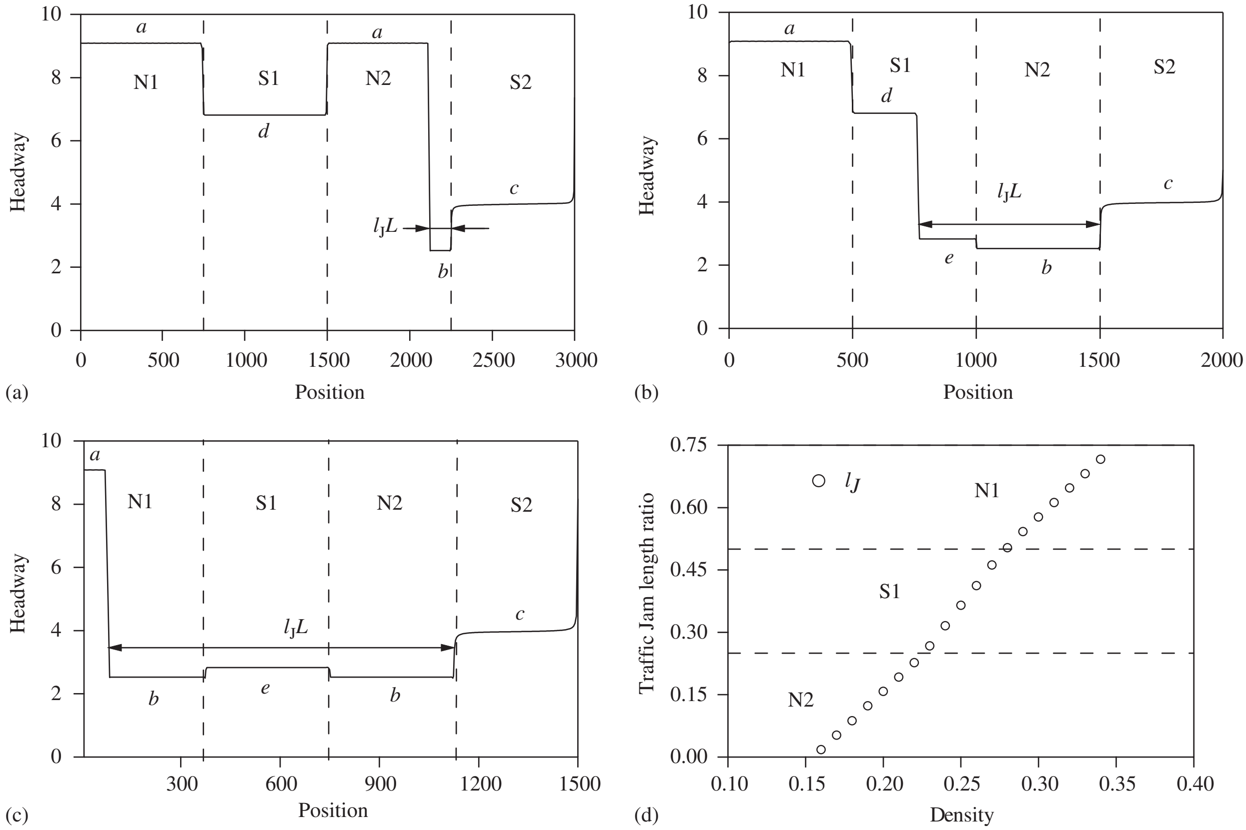


Fig. 8. Headway profiles at (a) $\rho = 0.16$, (b) $\rho = 0.25$, and (c) $\rho = 0.33$. (d) Plot of jam-length ratio (jam length/L) against density.

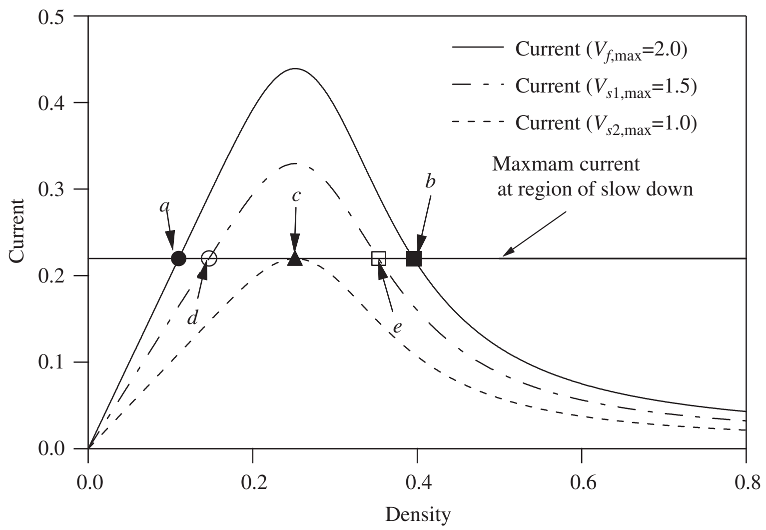
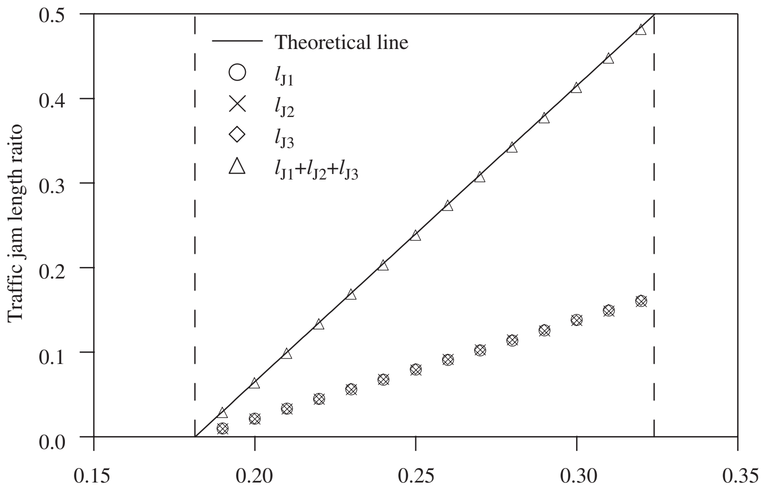


Fig. 9. Theoretical current curves for three optimal-velocity functions. The solid, chain, and dotted curves represent, respectively, the theoretical current curves for $v_{f,\max} = 2.0$, $v_{s1,\max} = 1.5$, and $v_{s2,\max} = 1.0$. Point c indicates the maximal value on the current curve of $v_{s2,\max} = 1.0$. Points a, b, d, and e indicate such points that the horizontal line on point c crosses at the theoretical curves of $v_{f,\max} = 2.0$ and $v_{s1,\max} = 1.5$.



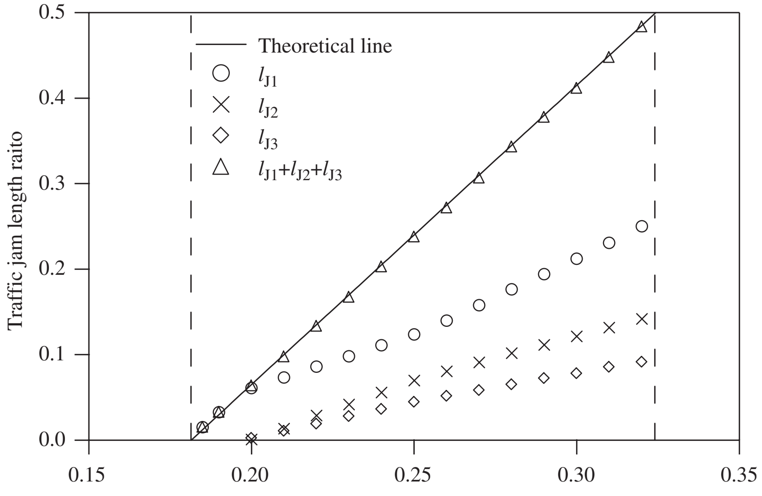


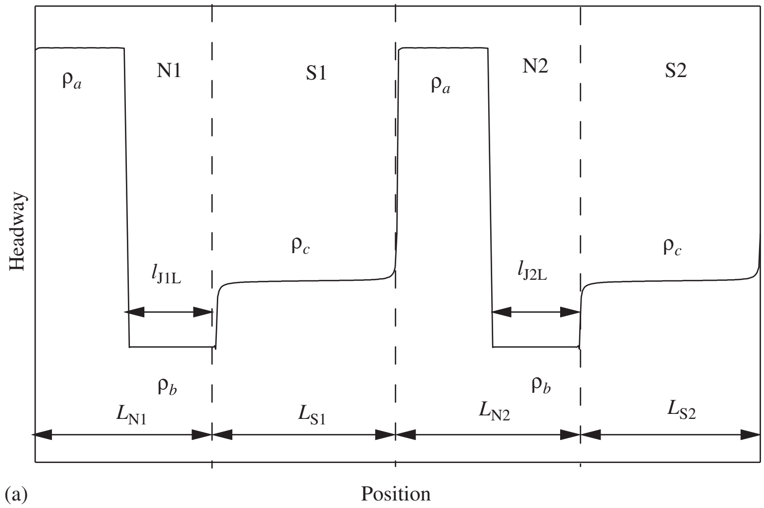
Fig. 10. (a) Plots of jam-length ratio (jam length/L) against density for $L_{N1} = L_{N2} = L_{N3} = L_{S1} = L_{S2} = L_{S3} = L/6$. (b) Jam-length ratios are plotted against density for $L_{N1} = L_{S1} = L/4$, $L_{N2} = L_{S2} = 0.15L$, and $L_{N3} = L_{S3} = L/10$.

density. Jam length l_J is plotted by open circles against density in Fig. 8(d). Thus, the jam occurrence and growth are definitely different from the cases of $v_{s1,\max} = v_{s2,\max} = 1.0$.

We study the traffic flow on a highway with three slowdown sections which are positioned alternately with three normal-speed sections. We take the lengths of sections as $L_{N1} = L_{N2} = L_{N3} = L_{S1} = L_{S2} = L_{S3} = L/6$. Three jams occur simultaneously at normal-speed sections $N1$, $N2$, and $N3$. Fig. 10(a) shows the plots of jam-length ratio (jam length/L) against density. All the jam-length ratios at sections $N1$, $N2$, and $N3$ are the same. For $L_{N1} = L_{S1} = L/4$, $L_{N2} = L_{S2} = 0.15L$, and $L_{N3} = L_{S3} = L/10$, jam-length ratios are plotted against density in Fig. 10(b). The jam occurs first at section $N1$. Then, two jams occur simultaneously at both sections $N2$ and $N3$. The lengths of three jams are different from one another. Thus, the occurrence and growth of jams depend highly on the lengths of normal-speed and slowdown sections.

4. Theory for jam lengths

We derive the relationship between the jam length and density analytically. We consider such case that there exist two slowdown sections on the highway and speed limit $v_{s1,\max}$ at first slowdown section $S1$ is consistent



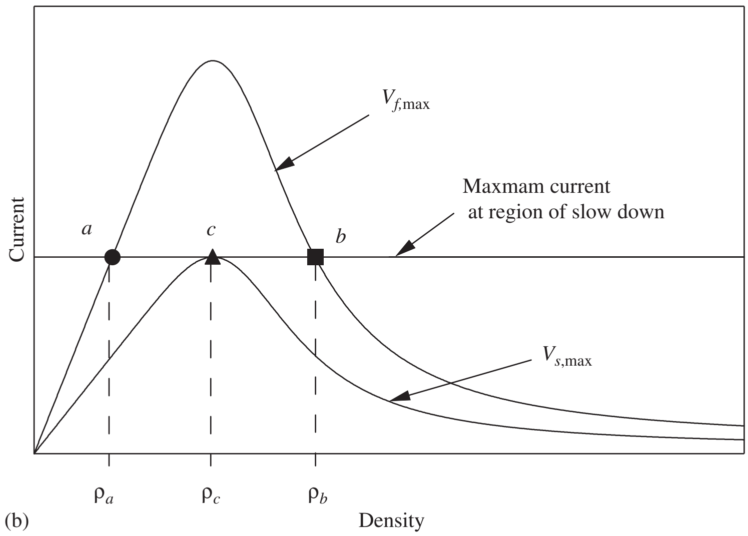


Fig. 11. (a) Schematic headway profile of two jams formed at normal-speed sections $N1$ and $N2$. (b) Two theoretical current curves. Densities ρ_a and ρ_b are at crossing points a and b. Density ρ_c is at point c at the maximal value of the current curve with $v_{s1(\text{or }2),\max}$.

with $v_{s2,\max}$ at second slowdown section $S2$. Either a single jam or two jams are formed at normal-speed sections $N1$ or/and $N2$. Fig. 11(a) shows the schematic illustration of the above case. Then, the amount of vehicular number is conserved and always equals total number N. One obtains the following:

$$N = L\rho = (L_{S1} + L_{S2})\rho_c + (l_{J1} + l_{J2})L\rho_b + \{L - (l_{J1} + l_{J2})L - (L_{S1} + L_{S2})\}\rho_a, \tag{5}$$

where ρ_a and ρ_b are the densities before and after the discontinuous front and ρ_c is the density at the slowdown sections. Densities ρ_a and ρ_b are given by crossing points a and b in Fig. 11(b). Density ρ_c is given by point c at the maximal value of the current curve with $v_{s1(\text{or }2),\max}$.

By solving Eq. (5), one obtains the amount of jam lengths

$$(l_{J1} + l_{J2})L = \frac{1}{\rho_c - \rho_a}\{L\rho - (L_{N1} + L_{N2})\rho_a - (L_{S1} + L_{S2})\rho_b\}. \tag{6}$$

The jam length (6) obtained from the theory is shown by the solid line in Figs. 5, 6(b), 7(a) and (b). The theoretical lines are consistent with the simulation result.

We extend the above theory to such case that there exist more slowdown sections than two on the highway. For m sections of slowdown, one obtains the following:

$$\sum_{i=1}^{m} l_{Ji} L = \frac{1}{\rho_c - \rho_a} \left\{ L\rho - \left(\sum_{i=1}^{m} L_{Ni} \right) \rho_a - \left(\sum_{i=1}^{m} L_{Si} \right) \rho_b \right\}. \tag{7}$$

The theoretical line (7) for $m = 3$ is shown by the solid line in Fig. 10(a). The theoretical result agrees with the simulation result.

5. Summary

We have investigated the occurrence and growth of traffic jams on a single-lane highway with some slowdown sections for high sensitivity, by using the optimal-velocity model. We have derived the position and length of traffic jams. We have clarified where and when the discontinuous fronts (stationary shock wave) occur. We have derived the densities before and after the discontinuous front from the theoretical current curves. We have shown that the discontinuous fronts determine the property of traffic jams. We have found that some traffic jams occur in the normal-speed and/or slowdown sections.

The study will be useful to forecast the emergent jams induced by the slowdowns on a highway.

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