

Basics of modelling the pedestrian flow

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Abstract

For the modelling of pedestrian dynamics we treat persons as self-driven objects moving in a continuous space. On the basis of a modified social force model we qualitatively analyze the influence of various approaches for the interaction between the pedestrians on the resulting velocity–density relation. To focus on the role of the required space and remote force we choose a one-dimensional model for this investigation. For those densities, where in two dimensions also passing is no longer possible and the mean value of the velocity depends primarily on the interaction, we obtain the following result: If the model increases the required space of a person with increasing current velocity, the reproduction of the typical form of the fundamental diagram is possible. Furthermore, we demonstrate the influence of the remote force on the velocity–density relation.

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1. Introduction

Microscopic models are state of the art for computer simulation of pedestrian dynamics. The modelling of the individual movement of pedestrians results in a description of macroscopic pedestrian flow and allows e.g. the evaluation of escape routes, the design of pedestrian facilities and the study of more theoretical questions. For a first overview see [1,2]. The corresponding models can be classified in two categories: the cellular automata models [3–7] and models in a continuous space [8–11]. We focus on models continuous in space. They differ substantially with respect to the ‘interaction’ between the pedestrians and thus to the update algorithms as well. The social force model for example assumes, among other things, a repulsive force with remote action between the pedestrians [8,12–17]. Other models treat pedestrians by implementing a minimum inter-person distance, which can be interpreted as the radius of a hard body [10,11].

One primary test, whether the model is appropriate for a quantitative description of pedestrian flow, is the comparison with the empirical velocity–density relation [18–21]. In this context the fundamental diagram of Weidmann [22] is frequently cited. It describes the velocity–density relation for the movement in a plane without bottlenecks, stairs or ramps. A multitude of causes can be considered which determine this dependency, for instance friction forces, the ‘zipper’ effect [23] and marching in step [24,25]. As shown in

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Ref. [25] the empirical velocity–density relation for the single-file movement is similar to the relation for the movement in a plane in shape and magnitude. This surprising conformance indicates, that lateral interferences do not influence the fundamental diagram at least up to a density-value of 4.5 m^{-2} . This result suggests that it is sufficient to investigate the pedestrian flow of a one-dimensional system without losing the essential macroscopic characteristics. We modify systematically the social force model to achieve a satisfying agreement with the empirical velocity–density relation (fundamental diagram). Furthermore, we introduce different approaches for the interaction between the pedestrians to investigate the influence of the required space and the remote action to the fundamental diagram.

2. Modification of the social force model

2.1. Motivation

The social force model was introduced by Ref. [8]. It models the one-dimensional movement of a pedestrian i at position $x_i(t)$ with velocity $v_i(t)$ and mass m_i by the equation of motion

$$\frac{dx_i}{dt} = v_i, \quad m_i \frac{dv_i}{dt} = F_i = \sum_{j \neq i} F_{ij}(x_j, x_i, v_i). \quad (1)$$

The summation over j accounts for the interaction with other pedestrians. We assume that friction at the boundaries and random fluctuations can be neglected and thus the forces are reducible to a driving and a repulsive term $F_i = F_i^{drv} + F_i^{rep}$. According to the social force model [8] we choose the driving term

$$F_i^{drv} = m_i \frac{v_i^0 - v_i}{\tau_i}, \quad (2)$$

where v_i^0 is the intended speed and τ_i controls the acceleration. In the original model the introduction of the repulsive force F_i^{rep} between the pedestrians is motivated by the observation that pedestrians stay away from each other by psychological reasons, e.g. to secure the private sphere of each pedestrian [8]. The complete model reproduces many self-organization phenomena like e.g. the formation of lanes in bi-directional streams and the oscillations at bottlenecks [8,12–17]. In the publications cited, the exact form of this repulsive interaction changes and the authors note that most phenomena are insensitive to its exact form [16]. We choose the force as in Ref. [13].

$$F_i^{rep} = \sum_{j \neq i} -\nabla A_i(\|x_j - x_i\| - d_i)^{-B_i}. \quad (3)$$

The hard core, d_i , reflects the size of the pedestrian i acting with a remote force on other pedestrians. Without other constraints a repulsive force which is symmetric in space can lead to velocities which are in opposite direction to the intended speed. Furthermore, it is possible that the velocity of a pedestrian can exceed the intended speed through the impact of the forces of other pedestrians. In a two-dimensional system this effect can be avoided through the introduction of additional forces like a lateral friction, together with an appropriate choice of the interaction parameters. In a one-dimensional system, where lateral interferences are excluded, a loophole is the direct limitation of the velocities to a certain interval [8,12].

Another important aspect in this context is the dependency between the current velocity and the space requirement. As suggested by Pauls in the extended ellipse model [26], the area taken up by a pedestrian increase with increasing speed. Thompson also based his model on the assumption, that the velocity is a function of the inter-person distance [10]. Furthermore, Schreckenberg and Schadschneider observed in Refs. [18,19], that in cellular automata model's the consideration, that a pedestrian occupies all cells passed in one time-step, has a large impact on the velocity–density relation. Helbing and Molnár note in Ref. [8] that the range of the repulsive interaction is related to step-length. Following the above suggestion we specify the relation between required space and velocity for a one-dimensional system. In a one-dimensional system the required space changes to a required length d . In Ref. [25] it was shown that for the single-file movement the relation between the required lengths for one pedestrian to move with velocity v and v itself is linear

at least for velocities $0.1 \text{ m/s} < v < 1.0 \text{ m/s}$

$$d = a + bv \quad \text{with} \quad a = 0.36 \text{ m} \quad \text{and} \quad b = 1.06 \text{ s}. \quad (4)$$

Hence it is possible to determine one fundamental microscopic parameter, d , of the interaction on the basis of empirical results. This allows focusing on the question if the interaction and the equation of motion result in a correct description of the individual movement of pedestrians and the impact of the remote action. Summing up, for the modelling of regular motions of pedestrians we modify the reduced one-dimensional social force model in order to meet the following properties: the force is always pointing in the direction of the intended velocity v_i^0 ; the movement of a pedestrian is only influenced by effects which are directly positioned in front; the required length d of a pedestrian to move with velocity v is $d = a + bv$.

2.2. Interactions

To investigate the influence of the remote action both a force which treats pedestrians as simple hard bodies and a force according to Eq. (3), where a remote action is present, will be introduced. For simplicity we set $v_i^0 > 0$, $x_{i+1} > x_i$ and the mass of a pedestrian to $m_i = 1$.

2.2.1. Hard bodies without remote action

$$F_i(t) = \begin{cases} \frac{v_i^0 - v_i(t)}{\tau_i} : x_{i+1}(t) - x_i(t) > d_i(t) \\ -\delta(t)v_i(t) : x_{i+1}(t) - x_i(t) \leq d_i(t) \end{cases} \quad \text{with} \quad d_i(t) = a_i + b_i v_i(t). \quad (5)$$

The force which acts on pedestrian i depends only on the position, its velocity, and the position of the pedestrian $i + 1$ in front. As long as the distance between the pedestrians is larger than the required length, d_i , the movement of a pedestrian is only influenced by the driving term. If the required length at a given current velocity is larger than the distance the pedestrian stops (i.e., the velocity becomes zero). This ensures that the velocity of a pedestrian is restricted to the interval $v_i = [0, v_i^0]$ and that the movement is only influenced by the pedestrian in front. The definition of d_i is such that the required length increases with growing velocity.

2.2.2. Hard bodies with remote action

$$F_i(t) = \begin{cases} G_i(t) : v_i(t) > 0, \\ \max(0, G_i(t)) : v_i(t) \leq 0 \end{cases} \quad (6)$$

with

$$G_i(t) = \frac{v_i^0 - v_i(t)}{\tau_i} - e_i \left(\frac{1}{x_{i+1}(t) - x_i(t) - d_i(t)} \right)^{f_i} \quad \text{and} \quad d_i(t) = a_i + b_i v_i(t).$$

Again the force is only influenced by actions in front of the pedestrian. By means of the required length, d_i , the range of the interaction is a function of the velocity v_i . Two additional parameters, e_i and f_i , have to be introduced to fix the range and the strength of the force. Due to the remote action one has to change the condition for setting the velocity to zero. The above definition assures that the pedestrian i stops if the force would lead to a negative velocity. With the proper choice of e_i and f_i and sufficiently small time steps this condition gets active mainly during the relaxation phase. Without remote action, stops are much more frequent. The pedestrian can proceed when the influence of the driving term is large enough to get positive velocities.

This different formulation of the forces requires different update algorithms, which will be introduced in the next section. A special problem stems from the periodic boundary conditions enforced for the tests of the fundamental diagram, as these destroy the ordering by causality, which otherwise could avoid blocking situations.

2.3. Time stepping algorithm

The social force model gives a fairly large system of second-order ordinary differential equations. For the hard body model with remote action, where the right-hand side of the ODEs is continuous along the solution, an explicit Euler method with a time step of $\Delta t = 0.001$ s was tested and found sufficient. Within that time, the distance between two persons does not change enough to make the explicit scheme inaccurate.

The situation for the hard body model without remote force is more complicated. Here the right-hand side is a distribution, and the position of the Dirac spikes is not known a priori. Hence the perfect treatment is an adaptive procedure, where each global time step is restricted to the interval up to the next contact. Unfortunately, this is a complicated and time consuming process. For a simple time step we choose the following procedure: Each person is advanced one step ($\Delta t = 0.001$ s) according to the local forces. If after this step the distance to the person in front is smaller than the required length, the velocity is set to zero and the position to the old position. Additionally, the step of the next following person is reexamined. If it is still possible, the update is completed. Otherwise, again the velocity is set to zero and the position is set to the old position, and so on. This is an approximation to the exact parallel update. It is not completely correct, however. To test its independence from the ordering of persons, computations using different orders were performed. The differences were minute and not more than expected from reordering of arithmetic operations.

3. Results

To enable a comparison with the empirical fundamental diagram of the single-file movement [25] we choose a system with periodic boundary conditions and a length of $L = 17.3$ m. For both interactions we proofed that for system-sizes of $L = 17.3, 20.0, 50.0$ m finite size effects have no notable influence on the results. The values for the intended speed v_i^0 are distributed according to a normal-distribution with a mean value of $\mu = 1.24$ m/s and $\sigma = 0.05$ m/s. In a one-dimensional system the influence of the pedestrian with the smallest intended speed masks jamming effects which are not determined by individual properties. Thus we choose a σ which is smaller than the empirical value and verified with $\sigma = 0.05, 0.1, 0.2$ m/s, that a greater variation has no influence to the mean velocities at larger densities.

In reality the parameters τ, a, b, e and f are different for every pedestrian i and correlated with the individual intended speed. But we know from experiment [25] that the movement of pedestrians is influenced by phenomena like marching in step, and in reality the action of a pedestrian depends on the entire situation in front and not only on the distance to the next person. Therefore, it is no point to attempt to give fully accurate values of this parameter and we may choose identical values for all pedestrians. We tested variations of the parameters and found that the behavior changes continuously. According to Ref. [17], $\tau = 0.61$ s is a reliable value.

For every run we set at $t = 0$ all velocities to zero and distribute the persons randomly with a minimal distance of a in the system. After 3×10^5 relaxation-steps we perform 3×10^5 measurements-steps. At every step we determine the mean value of the velocity over all particles and calculate the mean value over time. The following figures present the dependency between mean velocity and density for different approaches to the interaction introduced in Section 2.2. To demonstrate the influence of a required length dependent on velocity we choose different values for the parameter b . With $b = 0$ one gets simple hard bodies.

Fig. 1 shows the relation between the mean values of walking speed and density for hard bodies with $a = 0.36$ m and without remote action, according to the interaction introduced in Eq. (5). If the required length is independent of the velocity, one gets a negative curvature of the function $v = v(\rho)$. The velocity-dependence controls the curvature and $b = 0.56$ s results in a good agreement with the empirical data. With $b = 1.06$ s we found a difference between the velocity–density relation predicted by the model and the empirical fundamental diagram. The reason for this discrepancy is that the interaction and equation of motion do not describe the individual movement of pedestrian correctly. To illustrate the influence of the remote force, we fix the parameter $a = 0.36$ m, $b = 0.56$ s and set the values which determine the remote force to $e = 0.07$ N and $f = 2$.

The fundamental diagram for the interaction with remote action according to Eq. (6) is presented in Fig. 2. The influence is small if one considers the velocity-dependence of the required length. But with $b = 0$ one gets

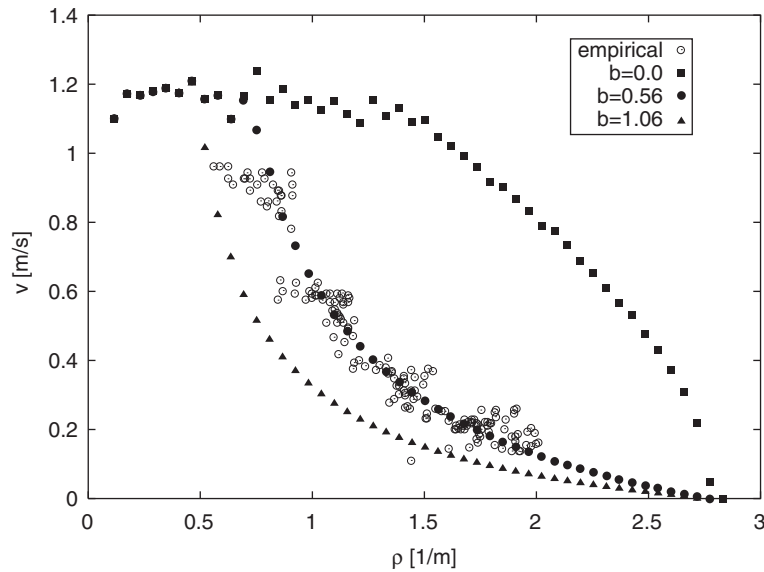


Fig. 1. Velocity–density relation for hard bodies with $a = 0.36$ m and without a remote action in comparison with empirical data from Ref. [25]. The filled squares result from simple hard bodies. The introduction of a required length with $b = 0.56$ s leads to a good agreement with the empirical data.

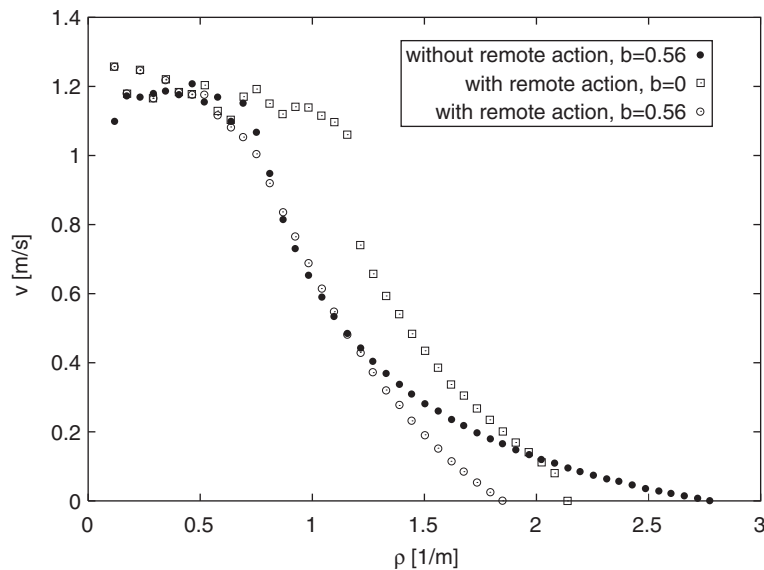


Fig. 2. Velocity–density relation for hard bodies with remote action in comparison with hard bodies without a remote action (filled circles). Again we choose $a = 0.36$ m and $b = 0.56$ s. The parameter $e = 0.07$ N and $f = 2$ determine the remote force. With $b = 0$ one gets a qualitative different fundamental diagram and a gap for the resulting velocities.

a qualitative different fundamental diagram. The increase of the velocity can be expected due to the effective reduction of the required length. The gap at $\rho \approx 1.2 \text{ m}^{-1}$ is surprising. It is generated through the development of distinct density waves, see Fig. 3, as are well known from highways. From experimental view we have so far no hints to the development of strong density waves for pedestrians [25]. The width of the gap can be changed by variation of the parameter f which controls the range of the remote force. Near the gap the occurrence of the density waves depends on the distribution of the individual velocities, too.

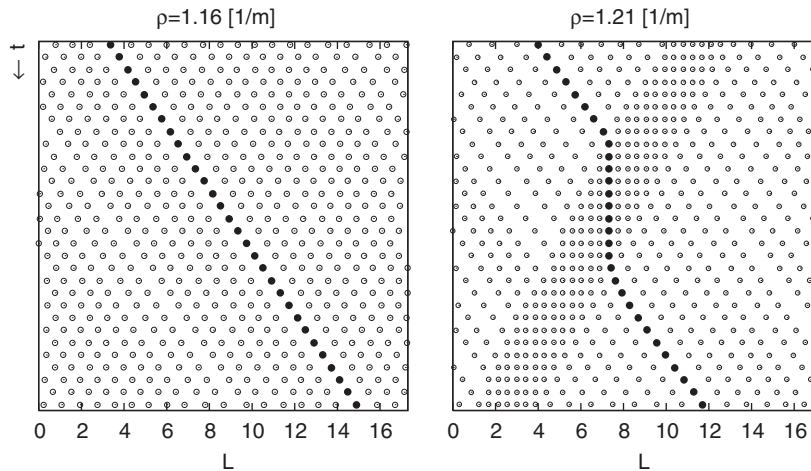


Fig. 3. Time-development of the positions for densities near the velocity-gap, see Fig. 2. For $\rho > 1.2 \text{ m}^{-1}$ density waves are observable. Some individuals leave much larger than average gaps in front.

4. Discussion and summary

For the investigation of the influence of the required space and remote action on the fundamental diagram we have introduced a modified one-dimensional social force model. The modifications warrant that in the direction of intended speed negative velocities do not occur and that the motion of the pedestrians is influenced by objects and actions directly in front only. If one further takes into account that the required length for moving with a certain velocity is a function of the current velocity the model-parameter can be adjusted to yield a good agreement with the empirical fundamental diagram. This holds for hard bodies with and without remote action. The remote action has a sizeable influence on the resulting velocity–density relation only if the required length is independent of the velocity. In this case one observes distinct density waves, which lead to a velocity gap in the fundamental diagram.

Thus we showed that the modified model is able to reproduce the empirical fundamental diagram of pedestrian movement for a one-dimensional system, if it considers the velocity-dependence of the required length. For the model parameter b which correlates the required length with the current velocity, we have found that without remote action the value $b = 0.56 \text{ s}$ results in a velocity–density relation which is in a good agreement with the empirical fundamental diagram. However, from the same empirical fundamental diagram one determines $b = 1.06 \text{ s}$, see [25]. We conclude that a model which reproduces the right macroscopic dependency between density and velocity does not necessarily describe correctly the microscopic situation, and the space requirement of a person at average speed is much less than the average space requirement. This discrepancy may be explained by the ‘short-sightedness’ of the model. Actually, pedestrians adapt their speed not only to the person immediately in front, but to the situation further ahead, too. This gives a much smoother movement than the model predicts.

The above considerations refer to the simplest system in equilibrium and with periodic boundary conditions. In a real life scenario like a building evacuation, where one is interested in estimates of the time needed for the clearance of a building and the development of the densities in front of bottlenecks, one is confronted with open boundaries and conditions far from equilibrium. We assume that a consistency on a microscopic level needs to be achieved before one can accurately describe real life scenarios. The investigation presented provides a basis for a careful extension of the modified social force model and an upgrade to two dimensions including further interactions.

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