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The night driving behavior in a car-following model

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Abstract

In this paper, we have studied the night driving behaviors in the car-following model in periodic boundary conditions. The evolution of uniform traffic under both small and large perturbations is investigated. The simulations show that the traffic is always unstable when $V'<0$ with V the optimal velocity. The traffic clusters, the kink–antikink waves, and the unstable clusters are observed under different sensitivity parameters. Even more interesting phenomenon is observed when the randomness effect is considered. Under large perturbations, it is shown that the traffic will be unstable if its density is smaller than a threshold. The density corresponding to the threshold increases with the decrease of the sensitivity parameter values.

Keywords: Full velocity difference model; Night drive; Car-following model

In the last decades, the traffic flow problems attracted the interests of scientists and engineers [1–7]. To describe the properties of real traffic flow, many traffic flow models, including microscopic ones (such as car-following models and cellular automata models) and macroscopic ones are proposed.

In the literature, the research focused on normal driving behaviors, i.e., with the increase of the density, the average speed of traffic flow decreases. From the point of view of the speed density relationship, it is a non-increasing function.

Recently Leveque studied the night-time traffic behaviors where he considered traffic traveling on an unfamiliar mountain road at night [8]. In this situation it is often easier to drive quickly if there are other cars ahead on the road, since their tail lights indicate how the road twists and turns. When faced with empty road ahead, on the other hand, the driver's speed should be limited by the distance the headlights can illuminate. So a reasonable model for the average speed as a function of density might look something like Fig. 1, with the velocity constant at some value for low density, then increasing for some range of density, and finally decreasing as in classical models if the density is sufficiently large.

Leveque investigated the situation where a set of cars are uniformly spaced on an otherwise empty road. Using the car-following model, he pointed out the clustering occurs. Furthermore, he showed that the

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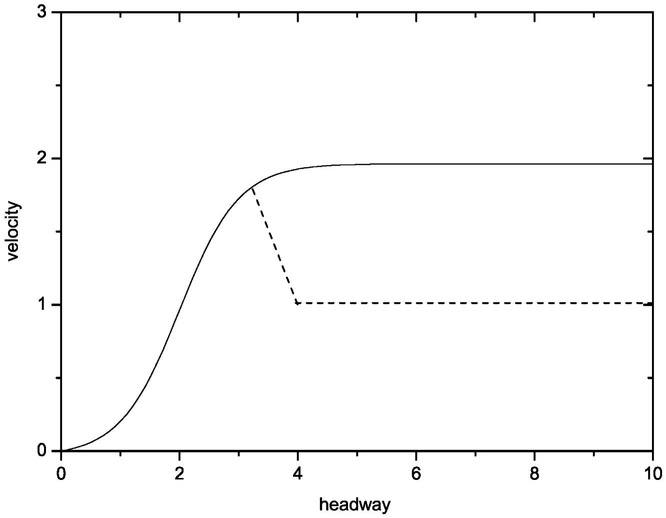

Fig. 1. The normal optimal velocity function (5) (solid line) and the night traffic optimal velocity function (6) (dashed line).

standard Oleinik entropy condition is not the correct admissibility condition for the night-time traffic if the LWR model is used.

In this paper, the night driving behaviors are further investigated. Different from Leveque's work, periodic boundary conditions are adopted here instead of open boundary conditions. The traffic stability under small perturbations is studied. It is shown that for some sensitivity parameters, the unstable clusters appear instead of the kink–antikink wave. Moreover, the traffic stability under large perturbations is investigated as well.

We carry out the simulation with a car-following model, the full velocity difference (FVD) model, which is an improvement over the previous ones in that it not only predicts correct start wave speed but also does not lead to unrealistically high acceleration [9]. In the model, the motion of car $n+1$ following car n is given by

$$\frac{\mathrm{d}v_{n+1}}{\mathrm{d}t} = \kappa[V(\Delta x) - v_{n+1}] + \lambda(v_n - v_{n+1}), \tag{1}$$

where $\Delta x = x_n - x_{n+1}$, x_n and x_{n+1} are the positions of cars n and $n+1$, v_n and v_{n+1} are the velocities of cars n and $n+1$, κ and λ are sensitivity parameters, and $V(\Delta x)$ is the optimal velocity function.

To rewrite Eq. (1) and to integrate it by the Euler scheme, we have

$$\frac{\mathrm{d}v_{n+1}(t)}{\mathrm{d}t} = \kappa[V(x_n(t) - x_{n+1}(t)) - v_{n+1}(t)] + \lambda(v_n(t) - v_{n+1}(t)), \tag{2}$$

$$v_{n+1}(t+\Delta t) = v_{n+1}(t) + \frac{\mathrm{d}v_{n+1}(t)}{\mathrm{d}t}\Delta t. \tag{3}$$

We update the position of the vehicle as follows:

$$x_{n+1}(t+\Delta t) = x_{n+1}(t) + v_{n+1}(t)\Delta t + \frac{1}{2}\frac{\mathrm{d}v_{n+1}(t)}{\mathrm{d}t}(\Delta t)^2. \tag{4}$$

In simulations, the calculation time interval $\Delta t = 0.1$ and periodic boundary conditions are applied.

For the normal traffic condition, we choose the optimal velocity function

$$V(\Delta x) = \tanh(\Delta x - x_c) + \tanh(x_c). \tag{5}$$

However, for the night traffic condition, the optimal velocity function is

$$V(\Delta x) = \begin{cases} \tanh(\Delta x - x_c) + \tanh(x_c), & \Delta x < x_{c1}, \\ a - \Delta x & \text{for } x_{c1} < \Delta x < x_{c2}, \\ b, & \Delta x > x_{c2}. \end{cases} \quad (6)$$

In the simulations, the parameters are: $x_c = 2$, $x_{c1} = 3.2$, $x_{c2} = 4$, $a = 5$, $b = 1$. The system length $L = 500$.

In Fig. 1, the normal optimal velocity function (5) and the night traffic optimal velocity function (6) are shown. According to the linear stability analysis, the FVD model is linear stable provided that $V'(\Delta x) < \kappa/2 + \lambda$. In Fig. 2, the curve of $V'(\Delta x)$ is plotted for both the normal optimal velocity function (5) and the night traffic optimal velocity function (6).

Next we investigate the stability of the night driving behavior under perturbations. The perturbations are exerted as follows. In the initial homogeneous traffic, one vehicle decelerates with constant deceleration $a = -1$ within n_{dec} time steps. Then it moves according to the FVD model. If the vehicle reaches zero velocity

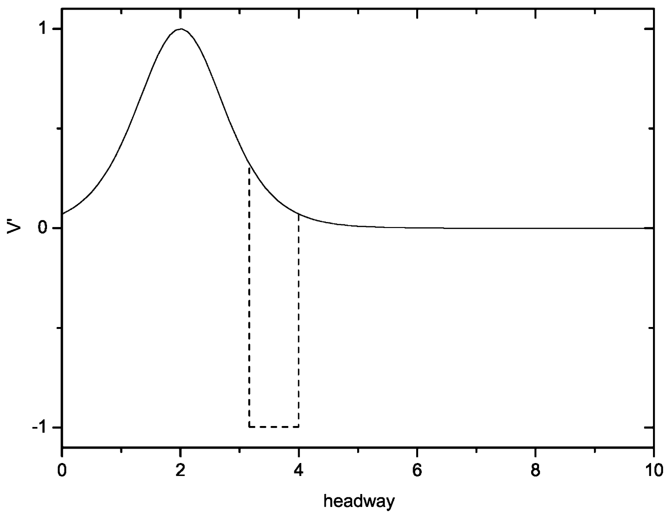

Fig. 2. The curve of $V'(\Delta x)$ for both the normal optimal velocity function (5) (solid line) and the night traffic optimal velocity function (6) (dashed line).

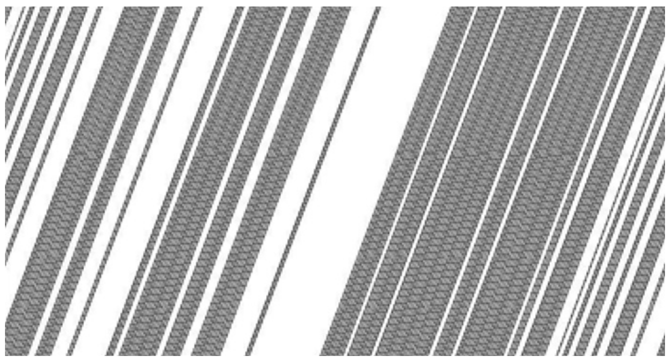

Fig. 3. The traffic pattern in the range $x_{c1} < \Delta x < x_{c2}$. Here $N = 150$, $\kappa = 1.0$, $\lambda = 0.5$. The vehicles driver from left to right and time is increasing in up direction.

within m_{dec} time steps ($m_{dec} < n_{dec}$), the vehicle stops for the remaining $n_{dec} - m_{dec}$ time steps [11]. In this way, the magnitude of the perturbations is quantified: a small n_{dec} corresponds to small perturbations, a large n_{dec} corresponds to large perturbations.

We consider small perturbations firstly ($n_{dec} = 1$). When $\kappa = 1.0$, $\lambda = 0.5$ is used, the linear stable condition is always met. The simulations show that the traffic does be stable in the range $\Delta x < x_{c1}$ and $\Delta x > x_{c2}$. However, in the range $x_{c1} < \Delta x < x_{c2}$, the traffic is unstable. This is due to that the value of V' is negative in this range. In Fig. 3, we show the traffic pattern in this range. One can see that the vehicles are in clusters. The headway of the leading vehicle of each cluster is larger than x_{c2}. As a result, the leading vehicles move with velocity 1. Consequently, all the vehicles move with velocity 1.

In Fig. 4, we show the fundamental diagram of the night driving vehicles under small perturbations for $\kappa = 1.0$ and $\lambda = 0.2$. For this case, the traffic is unstable in the density range $k_{c3} < k < k_{c4}$ from the stability condition. Moreover, it is also unstable in the density range $k_{c2} < k < k_{c1}$. In the density range $k_{c2} < k < k_{c1}$, the traffic pattern is like that in Fig. 3. In the density range $k_{c3} < k < k_{c4}$, the traffic evolves into kink–antikink waves (Fig. 5).

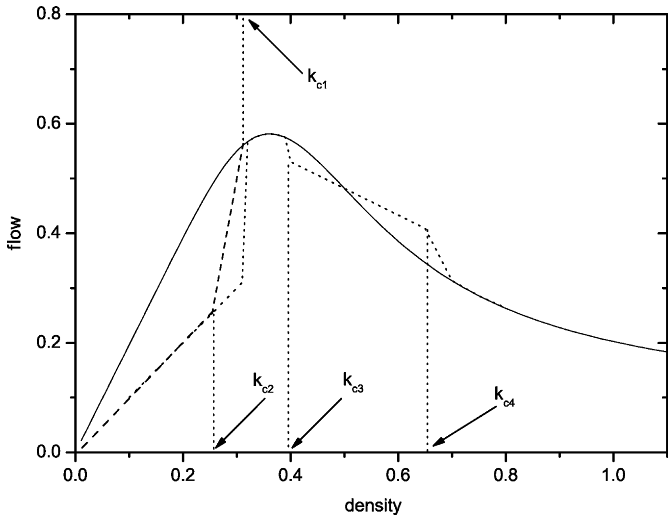

Fig. 4. The fundamental diagram of the night driving vehicles. The solid line corresponds to the normal driving vehicles, the dashed line to the night driving vehicles. The dotted line to the night driving vehicles under small perturbations ($n_{dec} = 1$). Here k_{c1} and k_{c2} correspond to x_{c1} and x_{c2}. $\kappa = 1.0$, $\lambda = 0.2$.

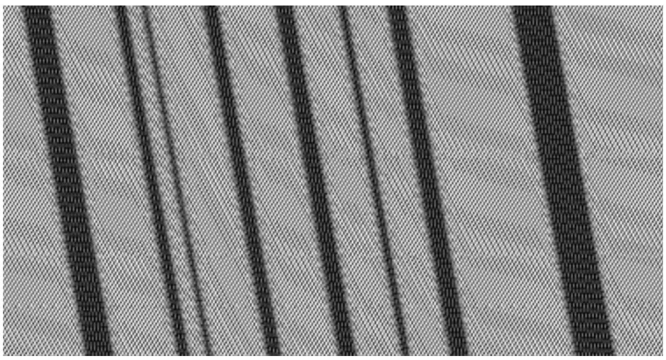

Fig. 5. The traffic pattern in the range $k_{c3} < k < k_{c4}$. Here $N = 250$, $\kappa = 1.0$, $\lambda = 0.2$.

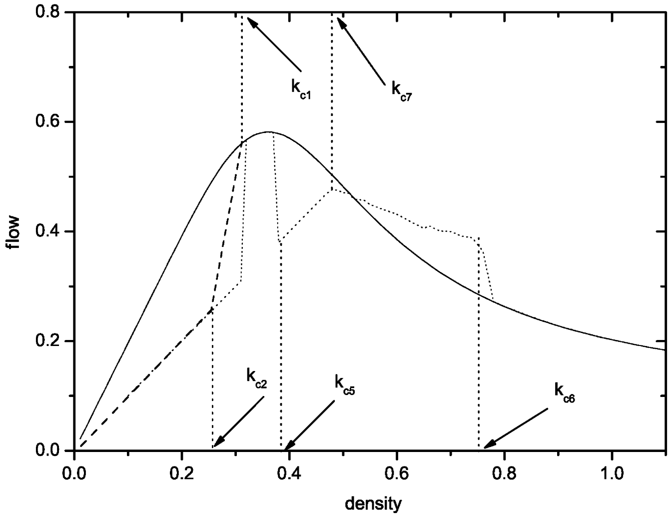

Fig. 6. The fundamental diagram of the night driving vehicles under small perturbations. $\kappa = 1.0$, $\lambda = 0.1$.

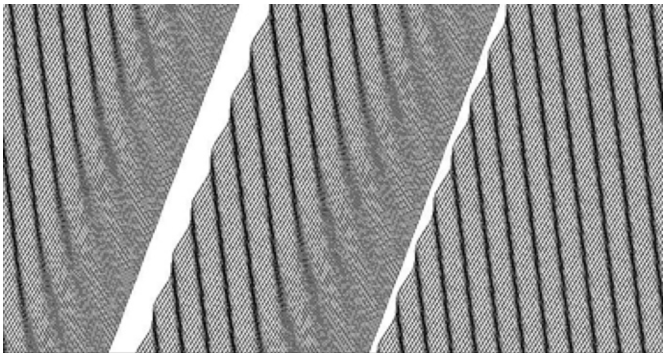

Fig. 7. The traffic pattern in the range $k_{c5} < k < k_{c7}$. Here $N = 230$, $\kappa = 1.0$, $\lambda = 0.1$.

In Fig. 6, we show the fundamental diagram of the night driving vehicles under small perturbations for $\kappa = 1.0$ and $\lambda = 0.1$. For this case, the traffic is unstable in the density range $k_{c5} < k < k_{c6}$ from the stability condition. Moreover, it is also unstable in the density range $k_{c2} < k < k_{c1}$. In the density range $k_{c2} < k < k_{c1}$, the traffic pattern is still like that in Fig. 3. However, in the density range $k_{c5} < k < k_{c6}$, the traffic does not evolve into kink–antikink waves. Instead, in the density range $k_{c5} < k < k_{c7}$, the traffic evolves into stable clusters (Fig. 7). For the leading vehicle, its headway is larger than x_{c2}, so it moves with velocity 1. Consequently, all the vehicles move with average velocity 1. Moreover, different from Fig. 3, the density wave can be seen in Fig. 7. With the increase of density, the clusters involve more and more vehicles. When $k > k_{c7}$, the space between the clusters is exhausted. The traffic pattern becomes unstable clusters (Fig. 8(a)). Here the clusters will disappear after some time and new clusters will form from time to time.

The unstable clusters can also be reproduced by optimal velocity model (Fig. 8(b)) [12], which is a special version of FVD model by setting $\lambda = 0$. However, for such models as Intelligent Driver model [10], the optimal velocity function is not explicitly included in the model equations, and it is not so straightforward to simulate the night drive behaviors. How to describe the night drive behavior in such models is out of the scope of the present paper and will be discussed elsewhere.

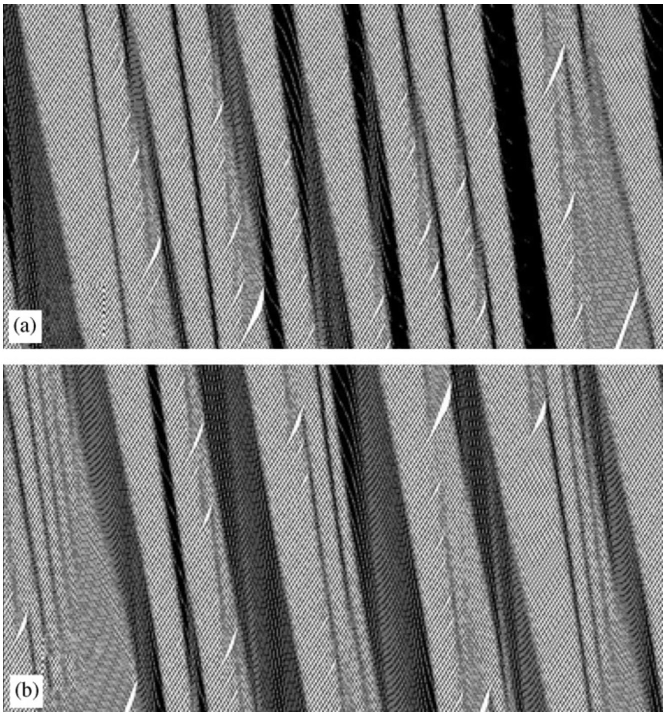

Fig. 8. The traffic pattern in the range $k_{c7} < k < k_{c6}$. Here $N = 300$: (a) FVD model $\kappa = 1.0$, $\lambda = 0.1$; (b) optimal velocity model $\kappa = 1.2$.

Even more interesting phenomenon arises when the randomness is introduced. When we consider the stochastic factor, Eqs.(3) and (4) change into

$$v_{n+1}(t + \Delta t)^{*} = v_{n+1}(t) + \frac{dv_{n+1}(t)}{dt}\Delta t + \mathrm{rand}() \times A, \tag{7}$$

$$v_{n+1}(t + \Delta t) = \min(\max(0, v_{n+1}(t + \Delta t)^{*}), v_{max}), \tag{8}$$

$$x_{n+1}(t + \Delta t) = x_{n+1}(t) + \tfrac{1}{2}(v_{n+1}(t) + v_{n+1}(t + \Delta t))\Delta t. \tag{9}$$

Here rand() is a uniformly distributed random number between -0.5 and 0.5, $v_{max} = V(3.2)$ is the maximum velocity of vehicle, A is magnitude of stochastic factor.

Fig. 9 shows the space-time plots at different values of A. One can see that when the magnitude of stochastic factor is small (Fig. 9(a)), the clusters become narrower than in the case without randomness (c.f. Fig. 8(a)). Moreover, the clusters are essentially stable. With the increase of A, the unstable clusters reappear (Fig. 9(b)). When $A = 0.1$, the unstable clusters are very narrow, but a macroscopic high density region appears (Fig. 9(c)). Further investigations on the randomness effect will be presented in the future work.

Next we investigate the stability of the night driving behavior under large perturbations ($n_{\mathrm{dec}} = 80$). Firstly, $\kappa = 1.0$, $\lambda = 0.5$ is used. In Fig. 10, we show the fundamental diagram of the night driving vehicles under such perturbations. One can see that the traffic is nonlinear unstable for $k < k_{c7}$. For this case, the traffic evolves into one cluster with the leading vehicle having the velocity 1 (Fig. 11). There is no density wave appearing in the cluster.

In Fig. 12, we show the fundamental diagram of the night driving vehicles under large perturbations for $\kappa = 1.0$, $\lambda = 0.2$. For this case, the traffic is unstable for $k < k_{c8}$. For $k_{c7} < k < k_{c8}$, the traffic will evolves into kink–antikink waves as shown in Fig. 5. For $k < k_{c7}$, the traffic evolves into one cluster with the leading vehicle having the velocity 1 and the density wave appearing in the cluster (Fig. 13).

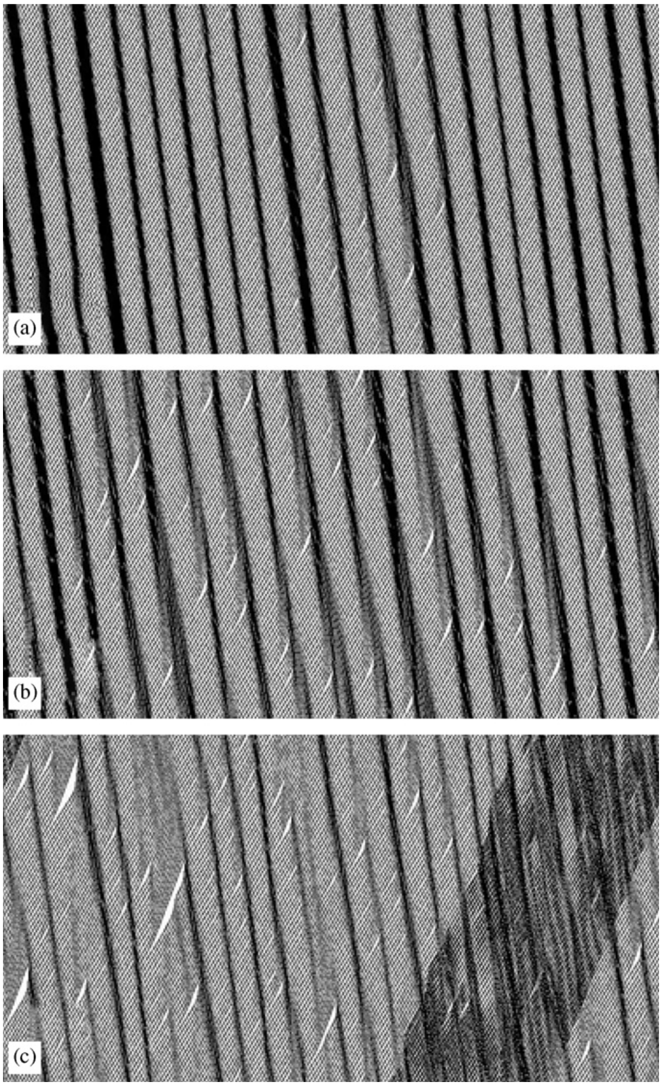

Fig. 9. The traffic pattern when the randomness effect is considered. Here $N = 300$, $\kappa = 1.0$, $\lambda = 0.1$: (a) $A = 0.01$; (b) $A = 0.05$; (c) $A = 0.1$.

In Fig. 14, we show the fundamental diagram of the night driving vehicles under large perturbations for $\kappa = 1.0$, $\lambda = 0.1$. For this case, the traffic is unstable for $k < k_{c9}$. For $k_{c7} < k < k_{c9}$, the traffic will evolve into unstable clusters as shown in Fig. 8. For $k < k_{c7}$, the traffic evolves into the pattern as shown in Fig. 13.

In summary, we have studied the night driving behaviors in the FVD model in periodic boundary conditions. The traffic stability under both small and large perturbations are investigated. The simulations show that the traffic is always unstable when $V' < 0$. However, for density in the density range calculated from $V' > \kappa/2 + \lambda$, different patterns may occur under different parameter values. For relatively large sensitivity parameters, the unstable traffic evolves into kink–antiknik waves; while for small sensitivity parameters, it may evolve into stable clusters (Fig. 7) or unstable clusters (Fig. 8). With the introduction of randomness, the interesting phenomenon appears: the clusters become narrow and essentially stable when the magnitude of randomness is small; with the increase of randomness magnitude, the clusters become unstable again; when the randomness magnitude is even larger, the clusters become very narrow and the macroscopic high density region appears.

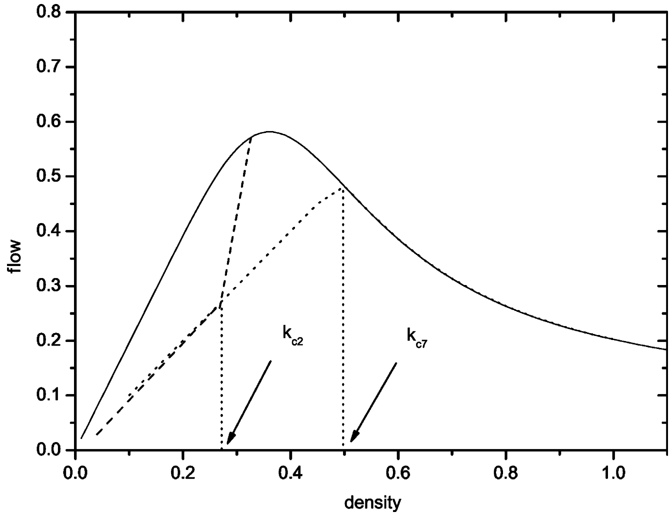

Fig. 10. The fundamental diagram of the night driving vehicles under large perturbations. The parameter $n_{\text{dec}} = 80$, $\kappa = 1.0$, $\lambda = 0.5$.

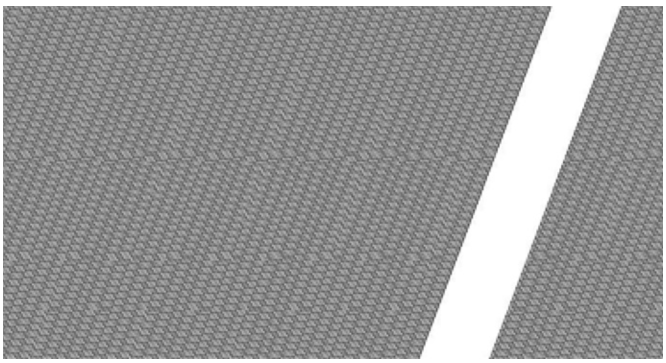

Fig. 11. The traffic pattern in the range $k < k_{c7}$. Here $N = 220$, $\kappa = 1.0$, $\lambda = 0.5$.

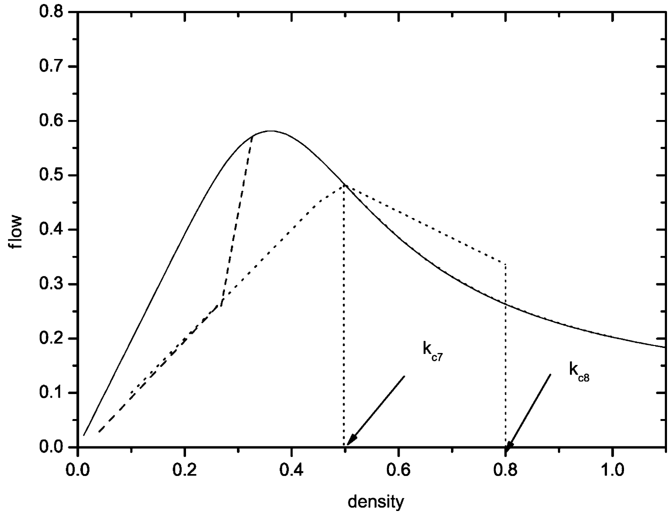

Fig. 12. The fundamental diagram of the night driving vehicles under large perturbations. $\kappa = 1.0$, $\lambda = 0.2$.

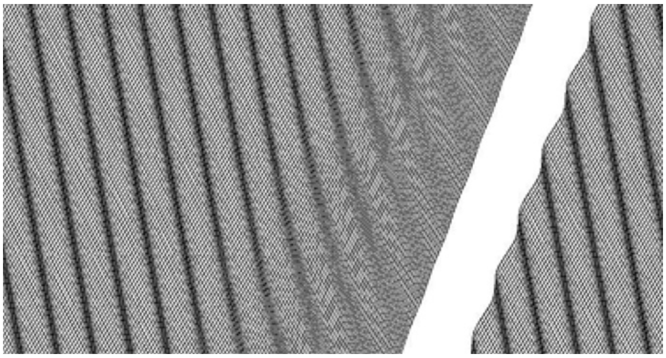

Fig. 13. The traffic pattern in the range $k < k_{c7}$. Here $N = 220$, $\kappa = 1.0$, $\lambda = 0.2$.

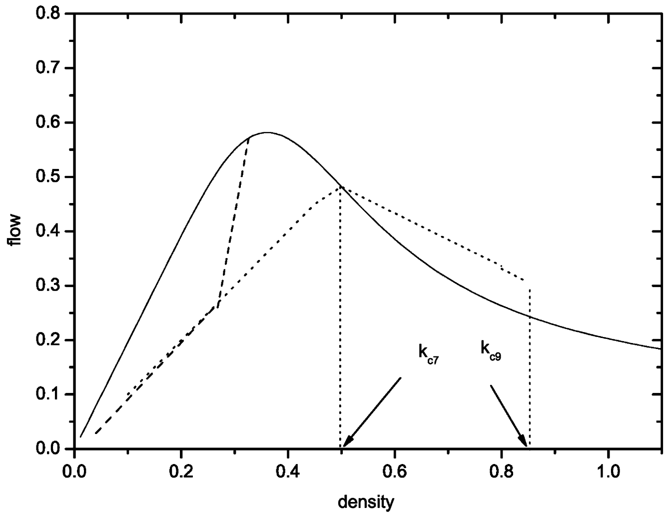

Fig. 14. The fundamental diagram of the night driving vehicles under large perturbations. $\kappa = 1.0$, $\lambda = 0.1$.

It is also shown that under large perturbations, the traffic will be unstable if its density is smaller than a threshold. The density corresponding to the threshold increases with the decrease of the sensitivity parameter values.

To our knowledge, the unstable clusters have not been found in traffic flow study. In our future work, the properties of the unstable clusters will be further investigated.

Acknowledgment

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