Adaptive dual control methods: An overview

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Abstract: This paper gives an overview of different techniques for solving the dual control problem. The optimal solution can quite straightforwardly be characterized. The solution is, however, numerically demanding. A large number of suboptimal dual controllers have been developed over the years and the paper gives an overview and discussion of different approaches.

Keywords: Adaptive control, Optimal adaptive control, dual control, self-tuning control.

1. INTRODUCTION

Feldbaum's seminal work, Feldbaum (1960) and Feldbaum (1961), gave the foundation for the dual control problem. The main idea is that in controlling an unknown system it is necessary to let the controller have dual goals. First, the controller must control the process as well as possible. Second, the controller must inject a probing signal to get more information about the unknown system. By gaining more information better control can be achieved in future steps. The main problem is that to get a better model it is necessary to disturb the process, which is in conflict with the goal of the control purpose. The compromise leads to a controller with dual features.

Feldbaum showed that a functional equation gives the solution to the dual control problem. The derivation is based on dynamic programming and the resulting functional equation is often called the Bellman equation. The numerical problems when solving the functional equation are very large and only a few very simple examples have been solved. There is thus a great need for looking at different approximations that can lead to simpler solutions with dual features. Wittenmark (1975) gives a survey of different ways of approaching the dual control problem. Since then there have been many new attempts to derive suboptimal dual controllers. The paper gives an updated survey.

In the (suboptimal) dual controllers it is necessary to have both cautious and probing features. Both parts of the control action can be obtained in numerous ways and we will try to classify the proposed schemes into a handful of principles.

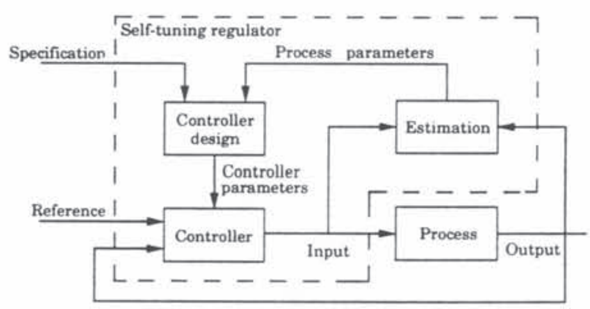

Figure 1. Self-tuning adaptive control system.

2. ADAPTIVE CONTROL

Most adaptive controllers typically have the structure shown in Fig. 1. The information transferred from the estimator to the design part is usually only the estimates of the parameters in the process. This is the case, for instance, in the self-tuning regulators, see Åström and Wittenmark (1995). The uncertainty of the parameter estimates are not used in the design part of the algorithm. This type of controllers is said to be based on the *certainty equivalence principle*. The process parameters are estimated and then used in the design as if they are the true ones. The control action determined in the design block do not try to take any actions that will influence the uncertainty. Any improvement of the parameter estimates is then only a consequence of the new measurements of the inputs and outputs of the process. The controllers do not take any active measures to improve the estimates.

In connection with analysis of different adaptive algorithms it has turned out to be advantageous to introduce nonlinearities in the estimation algorithm. Examples of this are limitations on the residuals or changes in the estimation algorithm when the residuals are small, see, for instance, Egardt (1979). It is also important to match the criteria for identification and control, see Åström (1993).

In many adaptive schemes perturbation signals are introduced see, for instance, Anderson and Johnstone (1985) and Praly (1988). The excitation is in these cases used to guarantee the convergence of the parameters in the controller. These algorithms show that it may be advantageous to introduce probing signals into the process. There is, however, no guidelines how to choose the perturbation signal and especially when to use probing. The probing should not be used all the time. In the dual control formulation the probing is introduced by minimizing a loss function.

The different modifications in the adaptive algorithms that are discussed above indicate that it from theoretical points of view is good to introduce different types of nonlinearities in the algorithms. These experiences lead to some of the questions discussed in connection with dual control. It is, therefore, of interest to make a survey of different ideas in the area of dual control.

A general survey of adaptive control is found in Åström (1987). Robustness of adaptive controllers, mainly based on the certainty equivalence principle, is surveyed in Ortega and Tang (1989).

3. CLASSIFICATION OF CONTROLLERS

Adaptive controllers for stochastic systems can be divided in different ways. In this survey it is convenient to divide the controllers into

- Non-dual controllers
- Controllers with dual features

Within these two classes the controllers can be classified in the ways that they determine the control signal. In the non-dual controllers the learning is "accidental" or "passive". I.e., there is no intentional probing signal introduced. The certainty equivalence controllers are typical for this class. Optimal dual controllers are difficult to compute except in very simple situations. It is thus of interest to to look at different ways of making suboptimal dual controllers or at different ways to introduce probing or active learning in adaptive algorithms. These controllers will be the main interest in this survey.

Many of the controllers are based on the *separation principle*. This implies that the unknown parameters are estimated separately from the design part. The estimation must, of course, be done as well as possible. The design part is then using the result from the estimator to determine the control signal, with or without any intentions to introduce dual features. A recent survey of estimation methods for time-varying systems is Ljung and Gunnarsson (1990). They distinguish between four different descriptions of parameter changes

- Random walk
- Random walk with local and global trends
- Jump changes
- Markov chain

In the Markov chain model the parameters are changing between a finite number of possible outcomes.

4. NON-DUAL ADAPTIVE CONTROLLERS

Most adaptive controllers discussed in the literature are based on the certainty equivalence principle and are thus classified as non-dual controllers. Some exceptions and modifications are discussed below.

When a loss function is minimized one step ahead based on the current estimates we will get a controller that also takes the parameter uncertainties into consideration. This type of controller is called a *cautious controller*. This controller hedges against poor process knowledge. A consequence of this caution is that the gain in the controller decreases. With small control signals less information will be gained about the process and the parameter uncertainties will increase and even smaller control signals will be generated. This vicious circle may lead to ***turn-off*** of the control. This problem has mainly been reported for systems with strongly time-varying parameters, see Åström and Wittenmark (1971) and Wieslander and Wittenmark (1971). In Veres (1994) an adaptive control scheme is denoted ***weakly dual*** if it uses the model uncertainties when deriving the control signal.

To illustrate the different methods we introduce the following notations: $y(t)$ is the process output, $u(t)$ is the control signal, $\theta(t)$ is the possibly unknown parameters of the process, $\hat{\theta}(t)$ is the parameter estimate, and $P(t)$ is the parameter uncertainty. Old inputs and outputs are collected into the vector

$$\mathcal{Y}_{t-1} = [\ y(t-1) \quad u(t-1) \quad \dots \quad y(0) \quad u(0)\ ]$$

If the parameters are known it is assumed that the optimal controller is

$$u(t) = f_{known}(y(t), \mathcal{Y}_{t-1}, \theta(t))$$

The controller based on the certainty equivalence principle is

$$u(t)_{CE} = f_{known}(y(t), \mathcal{Y}_{t-1}, \hat{\theta}(t))$$

and the cautious controller has the information structure

$$u_{cautious}(t) = f_{cautious}(y(t), \mathcal{Y}_{t-1}, \hat{\theta}(t), P(t))$$

EXAMPLE

Consider an integrator in which the unknown gain is changing, i.e. we have the model

$$y(t) - y(t-1) = \theta(t)u(t-1) + e(t)$$

where $e(t)$ is white noise. The minimum variance certainty equivalence controller is given by

$$u(t) = -\frac{1}{\hat{\theta}(t+1)}y(t)$$

and the cautious controller is

$$u(t) = -\frac{\hat{\theta}(t+1)}{\hat{\theta}^2(t+1) + p_\theta(t+1)}y(t)$$

where p_θ is the uncertainty of the estimate $\hat{\theta}$. □

A development of the cautious controller is found in Moore *et al.* (1989) and Ryall and Moore (1989). In these papers a central tendency controller is derived. The key idea is to avoid ill-conditioned calculations and to improve transient performance. The controller maximizes the probability of achieving the control objective given process parameter uncertainty information.

5. OPTIMAL DUAL CONTROLLERS

To understand some of the difficulties of calculating the optimal dual controller we will give the functional equation that can be used to calculate the dual controller. Assume that the loss function to be minimized is

$$J_N = E\left\{\frac{1}{N}\sum_{t=1}^{N}(y(t) - y_r(t))^2\right\}$$

where y is the process output, y_r is the reference signal, and E denotes mathematical expectation. This is called an *N-stage criterion*. The loss function should be minimized with respect to the control signal $u(0), u(1), \ldots, u(N-1)$. The controller obtained for $N = 1$ is a cautious controller and is sometimes called a *myopic controller*, since it is short-sighted and looks only one step ahead. The minimizing controller will be very different if $N = 1$ or if N is large. Define

$$V\ (\xi(t),\, t) = \min_{u(t-1)\ldots u(N-1)} E\left\{\sum_{k=t}^{N}(y(k) - y_r(k))^2\,|\mathcal{Y}_{t-1}\right\}$$

where $V(\xi(t), t)$ can be interpreted as the minimum expected loss for the remaining part of the control horizon given data up to $t-1$, $\mathcal{Y}_{t-1}$. Further ξ is the so called hyperstate of the problem. This includes the parameter estimates, their accuracy, and old inputs and outputs of the system. Using dynamic programming it can be shown that the optimal dual controller satisfy the *Bellman equation*

$$V\,(\xi(t),\,t) \;=\; \min_{u(t-1)} E\Big\{\,(y(t) - y_r(t))^2 + V\,(\xi(t+1),\,t+1)\,|\mathcal{Y}_{t-1}\Big\}$$

The difficulty with this equation is the nested minimization and mathematical expectation. Even in the simplest cases the Bellman equation has to be solved numerically. Since both V and u have to be discretized it follows that the storage requirements increases drastically with decreasing grid size. The resulting control law will be nonlinear in the parameter estimates and the covariance matrix of the parameter estimates. The problem to find the optimal solution has made it interesting to find approximations to the loss function or to find other ways to change the controller such that a dual feature is introduced.

There are some simple examples where the dual controllers have been obtained numerically, see, for instance, Bohlin (1969), Åström and Wittenmark (1971), Sternby (1976), Åström and Helmersson (1986), and Bernhardsson (1989)

6. SUBOPTIMAL DUAL CONTROLLERS

The problem with the turn-off and the numerical problems with the optimal dual controller have lead to different suggestions how to derive controllers that are simple, but still possess some dual features. Some ways are:

- Adding perturbation signals to the cautious controller
- Constraining the variance of the parameter estimates
- Using serial expansion of the loss function
- Modifications of the loss function
- Finite parameter sets
- Using ideas from robust control design

6.1 *Perturbation signals*

The turn-off phenomenon is due to lack of excitation. The addition of a perturbation signal is one way to increase the excitation of the process and to increase the accuracy of the estimates. Typical added signals are pseudo-random binary sequences, square-waves, and white noise signals. The perturbation can be added all the time or only when the variance is exceeding some limit. The controller may have the form

$$u_{perturb}(t) = u_{cautious} + v(t)$$

where v is the intentional perturbation signal. The addition of the extra signal will naturally increase the probing loss but may make it possible to improve the total performance by decreasing the control loss in future steps. See Wieslander and Wittenmark (1971) and Jacobs and Patchell (1972).

As mentioned above perturbation signals have been used in "conventional" adaptive controllers. In these cases the perturbation is introduced to be able to guarantee convergence of the parameter estimates to the true process parameters. A drawback of with the introduction of the perturbation signal is that there is no systematic way of deciding when and how large the signal should be. The introduction of perturbation signals in adaptive controllers is also discussed in Lozano-Leal and Zhao (1994).

6.2 *Constrained one step ahead minimization*

Another class of suboptimal dual controllers is obtained by minimizing the loss function one step ahead under certain constraints. The constraints are used to guarantee a certain level of accuracy of the parameter estimates. Suggested constraints are

- Limitation of the minimum value of the control signal, Hughes and Jacobs (1974).
- Limitation of the variance, Alster and Bèlanger (1974).

6.3 *Serial expansion of the loss function*

One approach to obtain a suboptimal dual controller is to make a serial expansion of the loss function in the Bellman equation. The expansion can be done around the certainty equivalence or the cautious controllers. The numerical computations are, however, quite complex and the approach has mainly been used when the control horizon is short. See Tse and Bar-Shalom (1973) and Bar-Shalom *et al.* (1982).

Another approximation is to try to solve the two-step minimization problem ($N = 2$), see, for instance, Mookerjee and Bar-Shalom (1989). In Mookerjee and Bar-Shalom (1989) an adaptive dual controller is presented for a multi-input multi-output autoregressive moving average system. The plant is assumed to have constant but unknown parameters. A suboptimal dual controller with a two-step horizon is determined. The suboptimal dual control modifies the cautious control design by numerator and denominator correction terms which depend upon the sensitivity functions of the expected future cost and avoids the turn-off and slow convergence. A two-step-ahead loss function is also considered in Sternby (1977). In Birmiwal and Bar-Shalom (1985) the functional equation is approximated by evaluating the value of the future information gathering. This is done through the use of preposterior analysis. The approximate prior probability densities are obtained and used to describe the future learning and control.

6.4 *Modifications of the loss function*

An approach that is similar to constrained minimization is to extend the one step ahead loss function. The idea is to add terms in the loss function that are reflecting the quality of the parameter estimates. This will prevent the cautious controller from turning off the control. One possibility is to use the covariance matrix of the parameter estimates and add a function of the covariance matrix to the loss function. See Goodwin and Payne (1977) and Wittenmark and Elevitch (1985). In the latter the loss function is

$$E\left\{(y(t+1) - y_r(t+1))^2 \,|\mathcal{Y}_t\right\} + \lambda f(P(t+2))$$

where λ is a weighting factor and $P(t+2)$ is the first time at which the covariance is influenced by $u(t)$.

In Milito *et al.* (1982) the quadratic loss function is extended with a part that reflects the need to gather as much information as possible about the unknown parameters. Their loss function is

$$\begin{aligned} J \;=\; E\Big\{ & (y(t+1) - y_r(t+1))^2 \\ & -\lambda(t+1)\nu^2(t+1)|\mathcal{Y}_t\Big\} \end{aligned}$$

where ν is the innovation or prediction error, i.e.

$$\nu(t+1) = y(t+1) - \hat{y}(t+1, \hat{\theta}(t+1))$$

This leads to a closed form solution for the control signal. A similar approach is used in Yame (1987) where the use of an information-theoretical viewpoint and the optimization of the loss function give to cubic form for determining the control signal. The optimal control signal is obtained using a root-finding algorithm.

Chan and Zarrop (1985) uses a generalized loss function which includes the variance of an auxiliary output and its prediction error. This results in a optimization problem which has an analytical solution. A weighting or learning coefficient makes it possible to obtain the cautious or the certainty equivalence controller as special cases. In Radenkovic (1988) the convergence properties of the suboptimal dual controller in Chan and Zarrop (1985) are investigated. To avoid lack of excitation a dither or perturbation signal is introduced.

An one-step-ahead controller where the loss function is supplemented with two sensitivity functions to capture the parameter uncertainties is found in Padilla *et al.* (1980).

6.5 *Finite parameter sets*

When the parameter set contains a finite number of elements it is easier to numerically solve the dual control problem. The mathematical expectation in the Bellman equation is then replaced by a summation. In Casiello and Loparo (1989) an extended loss function is used for the case when the parameters belong to a finite set. Using a standard quadratic loss and the calculation of a posteriori probabilities the optimal controller is derived. The case where the gain of a first-order system may take only two values is numerically solved in Bernhardsson (1989).

6.6 *Robust design methods*

Much research has been devoted to find controllers that are robust against plant uncertainties. Notice that in robust design methods hard bounds are required on the uncertainties. In the discussion above the uncertainties have been given in probabilistic terms. One drawback of the robust controllers is that they by nature must be conservative, since any combination of the assumed uncertainties may occur. So far very few of these methods have been used for adaptive control. The main reason is that it is difficult to find estimation methods that on-line can update the hard bounds on the uncertainties. There are, however, a few attempts in this direction. In Polak *et al.* (1987) on-line tuning is done based on worst-case design using semi-infinite optimization together with a plant uncertainty identification scheme. Similar approaches are taken in Veres and Norton (1993) and Veres (1994).

7. WHEN TO USE DUAL CONTROL?

Many non-dual adaptive controllers are successfully used today in many applications. When may it be advantageous to use a controller with dual features? One obvious situation is when the time horizon is short and when the initial estimates are poor. It is then necessary to rapidly find good estimates before reaching the end of the control horizon. In several papers it is suggested that dual controllers are suited for economic systems. The reason is the short time horizon and the highly stochastic parameters in the processes, see Bar-Shalom and Wall (1980), Kendrick (1982).

Another situation to use dual control is when the parameters of the process are changing very rapidly. This is a situation that is not very common in practice. There are, however, processes where the parameters are changing fairly rapidly and the gain is also changing sign. This is situation when the process has an even nonlinearity and it is desired to operate the process close to the extremum point. The gain of the linearized model will then change sign and at the same time some of the parameters may be small. One such example reported in the literature is grinding processes in the pulp industry discussed in Dumont and Åström (1988) and Allison (1994). The application in Allison (1994) is probably the first application of dual control to process control. The controller is an active adaptive controller, which consists of a constrained certainty equivalence approach coupled with an extended output horizon and a cost function modification to get probing.

8. SUMMARY

There are many ways to make suboptimal dual controllers. Many of the approximations use the cautious controller as a starting point and introduce different active learning features. This can be done by including a term in the loss function that reflects the quality of the estimates. To introduce a dual feature this term must be a function of the control signal that is going to be determined and it should also contain information about the quality of the parameter estimate. The suboptimal controllers should also be such that they easily can be used for higher-order systems. The paper have given an overview of different ways to construct suboptimal dual controllers.

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