Home Assignment 5

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I.
$$\Pi_{t}(i) = \left[P_{t}(i) - \frac{W_{t}}{At}\right] Y_{t}(i) - \frac{\gamma^{2}}{2} \left[\frac{P_{t}(i)}{P_{t-1}(i)} - 1\right]^{2} P_{t} Y_{t}$$

$$Y_{t}(i) = \left[\frac{P_{t}(i)}{P_{t}}\right]^{-\epsilon} C_{t}, C_{t} = Y_{t}$$

$$Y_{t}(t) = P_{t}(t) = P_{t}$$

$$\begin{array}{l} \text{ $\forall i,t$, $P_{t}\left(i\right)=P_{t}$} \\ \text{Firm i, max $V_{t}\left(i\right)=\sum\limits_{k=0}^{+\infty}\mathbb{E}_{t}\Big[\Lambda_{t},t+k\frac{\prod_{t+k}\left(i\right)}{P_{t+k}}\Big]$, $\Lambda_{t},t+k=\beta^{k}$ $U_{c},t+k/U_{c},t$} \end{array}$$

$$\frac{\partial V_{t(i)}}{\partial P_{t(i)}} = 0 \Rightarrow \Lambda_{t,t} \frac{1}{P_{t}} \frac{\partial \Pi_{t(i)}}{\partial P_{t(i)}} + \mathbb{E}_{t} \left[\Lambda_{t,t+1} \frac{1}{P_{t+1}} \frac{\partial \Pi_{t+1}(i)}{\partial P_{t(i)}} \right] = 0$$

$$\frac{\partial V_{t(i)}}{\partial P_{t(i)}} = 0 \Rightarrow \Lambda_{t,t} \frac{1}{P_{t}} \frac{\partial \Pi_{t+1}(i)}{\partial P_{t(i)}} + \mathbb{E}_{t} \left[\Lambda_{t,t+1} \frac{1}{P_{t+1}} \frac{\partial \Pi_{t+1}(i)}{\partial P_{t(i)}} \right] = 0$$

$$\frac{\partial V_{t(i)}}{\partial P_{t(i)}} = 0 \Rightarrow \Lambda_{t,t} \frac{1}{P_{t}} \frac{\partial \Pi_{t+1}(i)}{\partial P_{t(i)}} + \mathbb{E}_{t} \left[\Lambda_{t,t+1} \frac{1}{P_{t+1}} \frac{\partial \Pi_{t+1}(i)}{\partial P_{t(i)}} \right] = 0$$

$$\frac{\partial V_{t(i)}}{\partial P_{t(i)}} = 0 \Rightarrow \Lambda_{t,t} \frac{1}{P_{t}} \frac{\partial \Pi_{t+1}(i)}{\partial P_{t(i)}} + \mathbb{E}_{t} \left[\Lambda_{t,t+1} \frac{1}{P_{t+1}} \frac{\partial \Pi_{t+1}(i)}{\partial P_{t(i)}} \right] = 0$$

$$\Rightarrow \frac{1}{P_{t}} \frac{\partial \overline{\Pi_{t}(i)}}{\partial P_{t}(i)} + P_{t} \underbrace{\frac{U_{c,t+1}}{U_{c,t}}}_{Q_{c,t}} \frac{1}{P_{t+1}} \frac{\partial \overline{\Pi_{t+1}(i)}}{\partial P_{t}(i)} = 0 \quad (#)$$

According to (A)
$$\frac{\partial Y_{t}(i)}{\partial P_{t}(i)} = -\varepsilon C_{t} \left(\frac{P_{t}(i)}{P_{t}}\right)^{-(\varepsilon+1)} \frac{1}{P_{t}} = -\frac{\varepsilon}{P_{t}(i)} Y_{t}(i)$$

$$\frac{\partial Y_{t}(i)}{\partial P_{t}(i)} = -\varepsilon C_{t} \left(\frac{P_{t}(i)}{P_{t}}\right)^{-\varepsilon} C_{t} = 0$$
In a symmetric equilibrium, $P_{t}(i) = P_{t}$; $Y_{t}(i) = \left(\frac{P_{t}(i)}{P_{t}}\right)^{-\varepsilon} C_{t} = 0$

$$\frac{\partial \mathcal{T}_{t}(i)}{\partial P_{t}(i)} = -\mathcal{E} C_{t} \left(\frac{\mathcal{T}_{t}(i)}{P_{t}} \right) \quad P_{t} \qquad P_{t}(i)$$

$$= -\mathcal{E} C_{t} \left(\frac{\mathcal{T}_{t}(i)}{P_{t}} \right) \quad P_{t}(i) = P_{t}; \quad \mathcal{T}_{t}(i) = \left(\frac{P_{t}(i)}{P_{t}} \right)^{-\mathcal{E}} C_{t} = C_{t} = \mathcal{T}_{t}$$

$$= \mathcal{T}_{t}(i) \qquad \qquad \mathcal{T}_{t}(i) \qquad \mathcal{T}_{t}(i) = \mathcal{T}_{t}(i) \qquad \mathcal{T}_{t}(i) \qquad \mathcal{T}_{t}(i)$$

In a symmetric equilibrium,
$$T(t)$$

$$\frac{\partial T(t(i))}{\partial P_{t}(i)} = Y_{t}(i) + \left(P_{t}(i) - \frac{W_{t}}{At}\right) \frac{\partial Y_{t}(i)}{\partial P_{t}(i)} - \gamma \left(\frac{P_{t}(i)}{P_{t-1}(i)} - 1\right) \frac{P_{t}}{P_{t-1}(i)} Y_{t}$$

$$= Y_{t} - \left(P_{t} - \frac{W_{t}}{At}\right) \frac{\varepsilon}{P_{t}} Y_{t} - \gamma \left(\frac{P_{t}}{P_{t-1}} - 1\right) \frac{P_{t}}{P_{t-1}} Y_{t}$$

$$= \Upsilon_{t} - (P_{t} - \frac{W_{t}}{A_{t}}) P_{t} \qquad (P_{t-1})$$

$$= \Upsilon_{t} \left(1 - \varepsilon \left(1 - \frac{W_{t}}{A_{t}P_{t}} \right) - 7 \pi_{t} (1 + \pi_{t}) \right)$$

$$= -\left[\varepsilon\left(\mathcal{M} - \frac{Wt}{AtPt}\right) + r'Tt\left(1+TTt\right)\right]Tt \quad (where \mathcal{M} = \frac{\varepsilon^{-1}}{\varepsilon})$$

$$= -\left[\varepsilon\left(\mathcal{M} - \frac{Wt}{AtPt}\right) + r'Tt\left(1+TTt\right)\right]Tt \quad (where \mathcal{M} = \frac{\varepsilon^{-1}}{\varepsilon})$$

$$\frac{\partial \operatorname{Th}_{t+1}(i)}{\partial \operatorname{Pt}(i)} = \frac{\operatorname{Pt}_{t+1}(i)}{\operatorname{Pt}_{t+1}(i)} \left(\frac{\operatorname{Pt}_{t+1}(i)}{\operatorname{Pt}_{t}(i)} - 1\right) \frac{\operatorname{Pt}_{t+1}(i)}{\operatorname{Pt}_{t}(i)} \\
= \frac{\operatorname{Pt}_{t+1}(i)}{\operatorname{Pt}_{t+1}(i)} = \operatorname{Pt}_{t+1}(1 + \operatorname{Tt}_{t+1})^{2} \operatorname{Tt}_{t+1}(1 +$$

3. Fron (#)

3. From (#)
$$0 = -\left[\mathcal{E}\left(M - \frac{Wt}{AtPt}\right) + r \Pi_{t} \left(1 + \Pi_{t}\right)\right] \frac{rt}{Pt} + \beta E_{t} \left\{\frac{U_{e,t+1}}{U_{e,t}} r' \left(1 + \Pi_{t+1}\right)^{2} \Pi_{t+1} \frac{I_{t+1}}{I_{t+1}}\right\}$$

$$\Rightarrow \mathcal{E}\left(M - \frac{Wt}{AtPt}\right) + r \Pi_{t} \left(1 + \Pi_{t}\right) = \beta E_{t} \left\{\frac{U_{e,t+1}}{I_{t+1}} \frac{I_{t+1}}{r'} r' \left(1 + \Pi_{t+1}\right) \Pi_{t+1}\right\}$$

$$\Rightarrow \mathcal{E}(M - \frac{Wt}{At Pt}) + \gamma \mathcal{T}_{t}(1+\mathcal{T}_{t}) = \beta \mathcal{E}_{t} \left\{ \frac{U_{c,t+1}}{U_{c,t}} \frac{\gamma_{t+1}}{\gamma_{t}} \gamma' (1+\mathcal{T}_{t+1}) \mathcal{T}_{t+1} \right\}$$

Linearize (usig
$$M - \frac{W^{+}}{A+P+} \simeq - (\sigma + \varphi) \mathcal{J}_{t}$$

$$-\varepsilon \left(\sigma + \varphi\right) \widetilde{y_t} + r \pi_t = \beta E_t r \pi_{t+1}$$

$$\Rightarrow \pi_t = \beta \mathbb{E}_t \pi_{t+1} + \frac{\varepsilon}{r} (\sigma + \varphi) \tilde{y_t}$$

Text book:

$$Tt = \beta E t T_{t+1} + (\sigma + \rho) \frac{(1-\theta)(1-\beta\theta)}{\theta} y_t$$

we require $r = \frac{\epsilon \theta}{(1-\theta)(1-\beta\theta)}$ to make the two Phillips Curve (1-\theta) (1-\beta\theta)

equivalent

5.
$$\gamma \approx 104.85$$

Cost of charging the price for firm i as a share of the firm's output

(ost of charging the price for firm i as a share of the firm's output

(This) $\frac{\gamma}{2} \pi_1(i)^2$ Quardratic increase!

(1% 0.52%) Seems plausible according to

1% 13.12%.

Not very plausible

(0%. 52.42%) > Not very plausible

Richter, Alexander W., and Nathaniel A. Throckmorton. "Is Rotemberg pricing justified by macro data?." Economics Letters 149 (2016): 44-48. $\rightarrow r \approx 968^{\circ}$