Home Assignment 5

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I. $\Pi_t(i) = \left[P_t(i) - \frac{W_t}{A_t}\right] Y_t(i) - \frac{\gamma}{2}\left[\frac{P_t(i)}{P_{t-1}(i)} - 1\right]^2 P_t Y_t$ (#)

$Y_t(i) = \left[\frac{P_t(i)}{P_t}\right]^{-\varepsilon} C_t$, $C_t = Y_t$

$\forall i, t$, $P_t(i) = P_t$

Firm i: $\max V_t(i) = \sum_{k=0}^{+\infty} \mathbb{E}_t\left[\Lambda_{t,t+k} \frac{\Pi_{t+k}(i)}{P_{t+k}}\right]$, $\Lambda_{t,t+k} = \beta^k U_{c,t+k} / U_{c,t}$

1. $\frac{\partial V_t(i)}{\partial P_t(i)} = 0 \Rightarrow \Lambda_{t,t} \frac{1}{P_t} \frac{\partial \Pi_t(i)}{\partial P_t(i)} + \mathbb{E}_t\left[\Lambda_{t,t+1} \frac{1}{P_{t+1}} \frac{\partial \Pi_{t+1}(i)}{\partial P_t(i)}\right] = 0$

$\Rightarrow \frac{1}{P_t} \frac{\partial \Pi_t(i)}{\partial P_t(i)} + \beta \mathbb{E}_t\left\{\frac{U_{c,t+1}}{U_{c,t}} \frac{1}{P_{t+1}} \frac{\partial \Pi_{t+1}(i)}{\partial P_t(i)}\right\} = 0$ (#)

2. According to (#)

$\frac{\partial Y_t(i)}{\partial P_t(i)} = -\varepsilon C_t \left(\frac{P_t(i)}{P_t}\right)^{-(\varepsilon+1)} \frac{1}{P_t} = -\frac{\varepsilon}{P_t(i)} Y_t(i)$

In a symmetric equilibrium, $P_t(i) = P_t$; $Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} C_t = C_t = Y_t$

$\frac{\partial \Pi_t(i)}{\partial P_t(i)} = Y_t(i) + \left(P_t(i) - \frac{W_t}{A_t}\right) \frac{\partial Y_t(i)}{\partial P_t(i)} - \gamma\left(\frac{P_t(i)}{P_{t-1}(i)} - 1\right) \frac{P_t}{P_{t-1}(i)} Y_t$

$= Y_t - \left(P_t - \frac{W_t}{A_t}\right) \frac{\varepsilon}{P_t} Y_t - \gamma\left(\frac{P_t}{P_{t-1}} - 1\right) \frac{P_t}{P_{t-1}} Y_t$

$= Y_t\left(1 - \varepsilon\left(1 - \frac{W_t}{A_t P_t}\right) - \gamma \Pi_t(1 + \Pi_t)\right)$

$= -\left[\varepsilon\left(\mathcal{M} - \frac{W_t}{A_t P_t}\right) + \gamma \Pi_t(1 + \Pi_t)\right] Y_t$ (where $\mathcal{M} = \frac{\varepsilon - 1}{\varepsilon}$)

$\frac{\partial \Pi_{t+1}(i)}{\partial P_t(i)} = \gamma P_{t+1} Y_{t+1} \left(\frac{P_{t+1}(i)}{P_t(i)} - 1\right) \frac{P_{t+1}(i)}{P_t^2(i)}$

$= \gamma \frac{P_{t+1}^2}{P_t^2} Y_{t+1} \left(\frac{P_{t+1}}{P_t} - 1\right) = \gamma (1 + \Pi_{t+1})^2 \Pi_{t+1} Y_{t+1}$

3. From (#)

$0 = -\left[\varepsilon\left(\mathcal{M} - \frac{W_t}{A_t P_t}\right) + \gamma \Pi_t(1 + \Pi_t)\right] \frac{Y_t}{P_t} + \beta \mathbb{E}_t\left\{\frac{U_{c,t+1}}{U_{c,t}} \gamma (1 + \Pi_{t+1})^2 \Pi_{t+1} \frac{Y_{t+1}}{P_{t+1}}\right\}$

$\Rightarrow \varepsilon\left(\mathcal{M} - \frac{W_t}{A_t P_t}\right) + \gamma \Pi_t(1 + \Pi_t) = \beta \mathbb{E}_t\left\{\frac{U_{c,t+1}}{U_{c,t}} \frac{Y_{t+1}}{Y_t} \gamma (1 + \Pi_{t+1}) \Pi_{t+1}\right\}$

Linearize (using $\mathcal{M} - \frac{W_t}{A_t P_t} \simeq -(\sigma + \varphi)\tilde{y}_t$

$-\varepsilon(\sigma + \varphi)\tilde{y}_t + \gamma \Pi_t = \beta \mathbb{E}_t \gamma \Pi_{t+1}$

$\Rightarrow \Pi_t = \beta \mathbb{E}_t \Pi_{t+1} + \frac{\varepsilon}{\gamma}(\sigma + \varphi)\tilde{y}_t$

4. Text book:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + (\sigma+\varphi)\frac{(1-\theta)(1-\beta\theta)}{\theta}\tilde{y}_t$$

we require $\psi = \dfrac{\varepsilon\theta}{(1-\theta)(1-\beta\theta)}$ to make the two Phillips Curve equivalent

5. $\psi \approx 104.85$

cost of changing the price for firm i as a share of the firm's output

$\pi_t(i)$	$\frac{\psi}{2}\pi_t(i)^2$	Quardratic increase!
1%	0.52 %	seems plausible. according to
5%	13.12 %	
10%	52.42%	$\longrightarrow$ Not very plausible

Richter, Alexander W., and Nathaniel A. Throckmorton. "Is Rotemberg pricing justified by macro data?." Economics Letters 149 (2016): 44-48. $\to \psi \approx 96.80$

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