Preference extraction: Modeling voter intent in ranked choice elections

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Abstract

During tabulation, ambiguity of **voter intent** (`undervote`, `overvote`) in cast vote records (*CVR*), otherwise readily resolved in plurality voting elections, requires additional consideration in the tabulation of *ranked choice* contests. A preference extraction model provides the factorization of ranked choice ballot processing as preference extraction followed by preference aggregation. Suggestions for the potential improvement to the clarity of specifications, implementations, and validations of RC/IR systems are derived from the model.

1 Introduction

Ranked choice voting (RC) is catching on in many US jurisdictions and as one might expect, not without controversy. For certain offices, the State of Maine has adopted a style of ranked choice voting (*RC*) commonly called instant runoff (*IR*). In November 2018 Maine became the first state to implement Ranked Choice/Instant Runoff (*RC/IR*) for a seat in the United States Congress.

The use of *RC* in 2018 was decisive in Maine's Second Congressional District (CD2), with incumbent Bruce Poliquin receiving the plurality (43.66%) of first round votes, but lacking a majority, losing in the second round of tabulation [TS18].

Congressman Poliquin and others subsequently filed a lawsuit against Maine Secretary of State Matthew Dunlap, seeking to be declared the CD2 winner by virtue of a plurality in the first round tabulations [Mis18b; She18].

While unsuccessful, the Poliquin lawsuit brought attention to the tabulation of *RC/IR ballots*. Public criticism included references to a "proprietary algorithm" used by the State to mechanize runoff tabulations [Sha18]. The algorithm in question is implemented in proprietary software, and as such, is not available for public scrutiny [Mus18].

Nonetheless, shortly after the public release of cast vote records [Wac+19] on November 21, 2018, multiple parties reported successful independent duplication of official State tabulations. As reported in the *Maine Public Radio Podcast 'It's Just Math' — Mainers Recreate Ranked-Choice Voting Results To Test The Process* [Mis18a]:

> "Now, it's true that the software the state uses is proprietary. But the rules that are used by that software are just as public as the election results."

But what about the math? Maine's recent ranked choice implementation has motivated the development of an **election model** generalizing the tabulation of grid layout ballots as **preference extraction** followed by **preference aggregation**. In turn, preference extraction is factored into **adjudication**, **marking**, **trimming** and **ranking**. The mathematical model provides a mechanism to examine the relationship between the legal model and the implementation model. Rendering Maine RC/IR in the language of the model serves to address the following issues raised in the introduction:

1. What are the origins of the rules (axioms)?
2. Can the rules be modeled using elementary mathematics?
3. Are the rules complete, consistent, and independent?
4. Do implementation considerations constrain the rules?

The validation of voting systems is an extraordinary complex enterprise and has been carefully considered elsewhere [Com19b; Com19a]. The aim here is to render mathematical models of the rules.

2 Election model

2.1 Preference aggregation

The mathematical foundation of ranked choice voting finds a home in social choice theory, where rankings are modeled as **partial preference orders** and RC/IR is a particular **preference aggregation rule**. Preference aggregations are an active area of research, with an amazing variety of applications, especially in data rich environments [LWX17].

2.1.1 Social choice model

Definition 1. Preference aggregation

1. A **preference order** is a permutation of the elements of A and a **partial preference order** is a permutation of $S \subseteq A$.
2. Given an ordering of finite E, a **profile** $\mathcal{P}(E, A) = R_1, \cdots, R_{|E|}$ is a sequence of partial preference orders[1].
3. A **preference aggregation rule** $\mathcal{F}$ maps profiles to preference orders[2].

2.1.2 Paper ballots in ranked choice

Maine uses paper grid layout ballots for ranked choice elections[3]. While social choice theory defines a rigorous model of ranked choice tabulation, model inputs (preference orders) must be extracted from ballots subsequent to tabulation (aggregation). In the following section a mathematical model of extraction is developed, in essence factoring RC/IR ballot processing as preference extraction followed by preference aggregation.

2.2 Preference extraction

Preference extraction is modeled as the composition of adjudication, marking, trimming, and ranking functions defined in the subsections that follow.

[1] A pre-ordering of alternatives is adopted to facilitate vector notation. In general, preference aggregations are assumed to satisfy *Neutrality* (independence of alternative ordering) and *anonymity* (independence of elector ordering), as defined by May [May52].

[2] Defining preference orders as permutations is one of several equivalent formulations of the social choice model. Among others, strings of distinct characters and sequences of non-repeated elements are obvious equivalencies.

[3] For a discussion of ballot design alternatives see [QR17].

2.2.1 Ballot adjudication

Consider paper ballots supporting *FPTP* voting[4], including the *hanging chad* variety made famous by the 2000 United States Presidential Election[5].

Candidate	Mark one choice with ×
Amadeus Raschad	O
Chad Downs	O
Charles Chadwick	O
Chadley Wickham	O

Figure 1: FPTP paper ballot

This **column-style** ballot is intended to capture **voter intent** encoded as the presence or absence of a mark (chad) indicating that a particular candidate or alternative has been selected. In election vocabulary, **adjudication** refers to the process of *"resolving flagged cast ballots to reflect voter intent."* [ST] Hanging chads resolved, encoded column-style ballots can be modeled as a binary vector. The corresponding ranked choice equivalent grid layout ballot is readily modeled as a binary matrix.

Definition 2. Consider a ballot with n alternatives and $k \leq n$ ranks. The function c is an **adjudication function** if

$$\begin{aligned} c : E &\to \{0,1\}^{n\times k}, \\ c(e) &= (c_{ij}(e)), \end{aligned}$$

and $c_{ij}(e) = 1$ if and only if elector e has marked alternative a_i as choice j.

Thus adjudication can be thought of as producing an $n \times k$-bit encoding of each ballot, including potential discussions among humans as to the value of any given bit (chad).

2.2.2 Ballot marking

Columns on ranked choice ballots correspond to alternatives on FPTP ballots. In the FPTP setting, an elector is expected to mark a unique alternative but may very well mark more than one alternative (an **overvote**)

[4] *First-Past-The-Post.*

[5] Maine uses paper ballots designed and produced by the Office of the Secretary of State, Bureau of Corporations, Elections & Commissions [Maic; Maid].

or none at all (an **undervote**). Overvotes and undervotes model digital ambiguity in voter intent. In the ranked choice case, adjudication provides a digitized version of each rank (column) and consequently each alternative (row) motivating the following definitions.

Definition 3. Consider an $n \times k$ binary matrix $c = (c_{ij})$, with column vector c_j and row vector c_i.

1. If $|c_j| \neq 1$ we say that c_j has a **digital ambiguity in alternative**.
2. If $|c_i| \neq 1$ we say that c_i has a **digital ambiguity in rank**.

Definition 4. Consider a set of marks (characters) $M = A \cup \mathcal{A},\ A \cap \mathcal{A} = \emptyset$ representing alternatives and ambiguities. Define **marking function**

$$m : c(E) \to M^k,$$

where $m(e) = m_1, \cdots, m_k$ is the **marking sequence** of $e \in E$.

The marking function is a general device, serving to reduce $c(E)$ to the same dimension as the preference order needed for tabulation.

2.2.3 Resolving digital ambiguity

A rule for resolving digitally ambiguous FPTP ballots is both obvious and compelling. Ambiguous ballots are often referred to as **exhausted** as opposed to **continuing** (to be tabulated) [Maie]. Continuing ballots reveal a preference ranking ($k = 1$) and are ready for tabulation.

Now, for $|A| = n$, there are 2^n unique possibilities for adjudication of a FPTP ballot. Of these, one represents an undervote, exactly n are unambiguous, and the remaining $2^n - n - 1$ are various combinations of overvoting. Correspondingly, $|m(e)| = n + h$, where h is the number of ambiguity marks. Thus, literal tabulation of *FPTP* contests can be thought of as placing each ballot in one of the $n + h$ piles on a table. In *FPTP* contests wherein $h = 2$ and n rarely exceeding 4, the limited number of piles preserves the metaphor nicely.

In the ranked choice case with n alternatives and k choices, there are $2^{n \times k}$ unique possibilities for adjudication and $(n + h)^k$ unique marking sequences. In most implementations voters are free to rank fewer choices than allowed so undervotes are to be expected. Unlike their FPTP vector-style counterparts, the grid layout ranked choice ballot affords multiple opportunities for overvoting[6]. Consequently, the trivial process of resolving digital ambiguities in FPTP ballots (`continuing vs. exhausted`) is insufficient for RC/IR ballots. Additional rules are required.

Following the vocabulary of FPTP, rules for RC/IR implementations typically define a round-specific state of continuation or exhaustion for each ballot and alternative[7]. Ballots are said to be **exhausted by alternative** if there is no **continuing alternative**. Ballots are otherwise exhausted when certain subsequences of ambiguity marks appear in the marking sequence prior to any continuing alternative. Terminating subsequences are defined individually as **exhaustion by undervote** or **exhaustion by overvote**, rather than a combination of the two. Common practice is to identify some number of either consecutive or sequential appearances of each particular ambiguity in the marking sequence.

2.2.4 Trimming marking sequences

It is convenient to remodel marking sequences as words from a language $\mathcal{L}$ on the alphabet $M = A \cup \mathcal{A}$ of characters representing alternatives and ambiguities. This formulation affords the pattern matching power of regular expressions[8].

Definition 5. Let $S \subseteq A$ be a subset of alphabet A defining words in a language $\mathcal{L}$. A **trimming rule for** S **on** $\mathcal{L}$ is a regular expression with literals confined to $\mathcal{S}$.

Trimming rules model the loss in confidence in the determination of voter intent generated by ignoring ambiguities and reckoning subsequent alternatives as ranked higher than they appear on the ballot. A regular expression can be used to identify and extract an initial subsequence of a marking sequence beyond which no alternative would be considered.

Definition 6. Let $\mathcal{T}$ be the regular sets generated by $T = \cup t_i$, where $t_1, \cdots, t_r$ are trimming rules for $\mathcal{A}$ on $\mathcal{L}$. The **trimming function** f **defined by** T is

$$f : \mathcal{L} \to \mathcal{T},$$

[6] Of the 10^8 possibilities in the Maine governor's primary, 132250 ballots realized 17513 unique marking sequences, and only 4744 were without ambiguity marks. Just 2273 provided a complete preference order that included a write-in.

[7] Typically, `exhaustion` is an absorbing state for each.

[8] The regular expressions identified initially by Stephen Kleene and subsequently exploited by Chomsky, Thompson and others. [Kle56; CS63; Tho68].

where $f(x)$ is the initial substring of x matching T.

2.2.5 Ranking trimmed ballots

Voters are unconstrained from choosing a particular alternative for multiple rankings. Unless masked by an overvoting in those rankings, the alternative will have multiple appearances in the marking sequence, perhaps even after it has been trimmed. Maine's new RC/IR law provides the following useful definition[Maif]:

> "Highest continuing ranking" means the highest ranking on a voter's ballot for a continuing candidate.

The **highest ranking** for each alternative can be modeled as a subsequence of the marking sequence determined prior to preference aggregation. Furthermore, the preference order described in the law is modeled by the subsequence of highest ranking of alternatives in the clipped marking sequence. Additional notation is required to render such models.

Definition 7. Let $m = m_1, \cdots, m_k$ be a sequence of elements of X.

1. For $a \in X$, we say that m **supports** a if for some $1 \leq i \leq k,\ a = m_i$.
2. Consider $Y \subseteq X$, a set elements in X supported by m. Define the **ranking function** f

$$f : Y \rightarrow \mathbb{N},$$
$$f(y) = min\{j \mid m_j = y\}.$$

3. The **ranking sequence** $\{r_i\}$ is a subsequence of m ordered by the ranking function. That is, for all $1 \leq i < j \leq k,\ f(r_i) < f(r_j)$.

Example 1. Ranking function
Consider a library shelving individual editions of a collection of titles, placing each on a shelf ordered by the printing date of edition. For convenience, assume the dates are distinct. For any title in the library, the ranking function locates the first edition. The ranking sequence reveals the order of appearance of first editions.

For $y \in Y$ supported by m, $f(y)$ provides the index of the first appearance in y in m.

Theorem 1. If $m = m_1, \cdots, m_k$ is a sequence of elements of X and $Y \subseteq X$, with m supporting all $y \in Y$, then the ranking sequence r is a preference order on Y.

Proof. It must be shown that $\forall\ y \in Y,\ \exists!\ i \ \ni \ r_i = y$. For $y \in Y$, let $S(y) = \{i \mid r_i = y\}$[9]. Since m supports y, $S(y) \neq \emptyset$. Existence and uniqueness follow from the corresponding properties of the minimum of a finite set of real numbers. $\square$

Since the ranking function can be restricted to alternatives, the ranking sequence steps over (ignores) ambiguities in the marking sequence.

Corollary 1. Consider $M = A \cup \mathcal{A}$ a set of characters partitioned by alternatives and ambiguities, and let m be a sequence of elements of M. Consider $Y \subseteq A$, the alternatives in A supported by m. The ranking sequence r ordered by the ranking function $f : A \to \{1, \cdots, |Y|\}$ is equivalent to a preference order on Y and a partial preference order on A.

[9]Think of $S(y)$ as enumerating the levels of support enjoyed by alternative y from elector e.

3 Maine ranked choice

3.1 Origins of the rules

The case study follows from the author's effort to independently and unofficially validate RC/IR tabulations presented by Secretary Dunlap for the fall CD2 contest. The goal was to use only official and publicly available sources to identify RC/IR tabulation requirements and to implement an algorithm defined completely and consistently by these sources. To that end, others with potential knowledge of the rules or any implementation of the rules were not consulted. The assumption being:

> *The rules ought to be sufficient to guide a software implementation of an* ***independent tabulator*** *capable of reproducing Maine RC/IR tabulations.*

Requirements for the *independent tabulator* are derived from a hierarchy of sources, flowing down from the top of the hierarchy, with possible increased specificity as they flow down. For the Maine RC/IR requirements flow:

$$\textit{Referenda} \Rightarrow \textit{Legislation} \Rightarrow \textit{Administrative Rule}$$

3.1.1 The Citizen's Initiative

The adoption of RC for Maine elections began with a successful **Citizen's Initiative**, the mechanism available to Maine Citizens for bringing an issue to a referendum vote. In October 2015 Maine election reformers presented a sufficient number of petition signatures (70,000) to Secretary Dunlap, mandating what became *Question 5* in a November 2016 statewide referendum:

> "Do you want to allow voters to rank their choices of candidates in elections for U.S. Senate, Congress, Governor, State Senate, and State Representative, and to have ballots counted at the state level in multiple rounds in which last-place candidates are eliminated until a candidate wins by majority?"

Question 5 passed on 8 November 2016 with 52% approval and RC/IR became law under the title *IB 2015, c.3, "An Act to Establish Ranked-choice Voting"* (*LD1557*)[Maia].

In early 2017 the Maine State Senate sought an advisory opinion from the Maine Supreme Judicial Court as to the appropriateness of RC for certain State offices. In their ruling on May 23, 2017 the Court held that LD1557 violated provisions of the Maine Constitution and could not be used for state offices[10]. Subsequently, *P.L. 2017, ch. 316, "An Act to Implement Ranked-choice Voting in 2021"* (*LD1646*) was adopted by the Maine Legislature and became Maine law on November 4, 2017 without the signature of the Governor[Maib]. The act disallowed implementation of RC for the 2018 and 2020 elections, and repealed RC entirely unless the Maine Constitution were amended prior to December 1, 2021.

3.1.2 The People's Veto

As well as the *Citizen's Initiative* empowering Maine voters to directly enact legislation, Maine provides the ***People's Veto*** for direct repeal of legislation. The forces behind Question 5 were quick to exercise this power, gathering more than 80 thousand signatures in only 88 cold Maine winter days. Certification of the petition occurred on March 5, 2018, effectively suspending LD1646 and forcing the use RC in June 12, 2018 primaries.

Procedures for statewide Maine elections are set forth in *Title 21-A. M.R.S. ELECTIONS* [Mai19]. Responsibilities are shared by the *Office of the Secretary of State* and Maine municipal election officials. Certification of the People's Veto occurred 100 days before the June 12 primary. Ballots needed to be designed, reviewed and distributed. State and local election officials needed training. Detailed procedures needed to be developed, documented and approved. The successful implementation of ranked choice voting under the most difficult and conspicuous of circumstances is a testimony to the power of Yankee ingenuity, and to the quality of service provided to Maine voters by Secretary Dunlap, the Division of Elections, and election officials in 503 Maine towns and cities [Dun18].

3.1.3 Implementation considerations

One component of Maine's remarkable success was the reuse and adaptation of plurality voting protocols and infrastructure. Important features of the strategy include:

i. Municipalities tally the first choice column of grid layout ballots using protocols for plurality contests.

ii. If runoff rounds are required, they are conducted by the Division of Elections at a central location in Augusta.

[10] In particular, the Maine Constitution specifies *"plurality of/the votes"* for elections for governor and legislative seats. US Congressional seats and primaries are not so specified. [Cou].

iii. Grid layout ballots are compatible with the *DS200* model scanner/tabulator used in 263 of Maine's municipalities, including all with more than 1000 registered voters[11]. The DS200 has a thumb-drive interface and associated secure software protocols for physically transferring ballot data to the central location for subsequent tabulation[12].

iv. Thumb-drives and ballots from municipalities without scanners are delivered to the central location via bonded courier service.

v. A model *DS850* high speed scanner/tabulator is used to process previously unscanned ballots.

vi. Cast vote records from thumb-drives and DS850 are loaded for input to proprietary tabulation software[13].

3.1.4 Maine ranked choice rules

Provisions for ranked choice are provided in *21-A. M.R.S. §723-A Determination of winner in election for an office elected by ranked-choice voting* [Maif]. In *§723-A(5A)* the statute calls for the Secretary of State to

> " ... adopt rules for the proper and efficient administration of elections determined by ranked-choice voting."

An emergency version of ***29-250 Code of Maine Rules Chapter 535****: Rules Governing The Administration of Election Determined By Ranked-Choice Voting* was developed by the Office of the Secretary of State, Division of Elections and certified on May 11, 2018[14].

> "SUMMARY: This rule sets forth the procedures for the administration of elections determined by ranked-choice voting, including collection, security and handling of ballots and memory devices between the municipal offices and the central counting facility; aggregating and counting the cast vote records; administering the rounds of ranked-choice counting until only two candidates remain, and the candidate with the most votes in the final round is the winner; and reporting the results."

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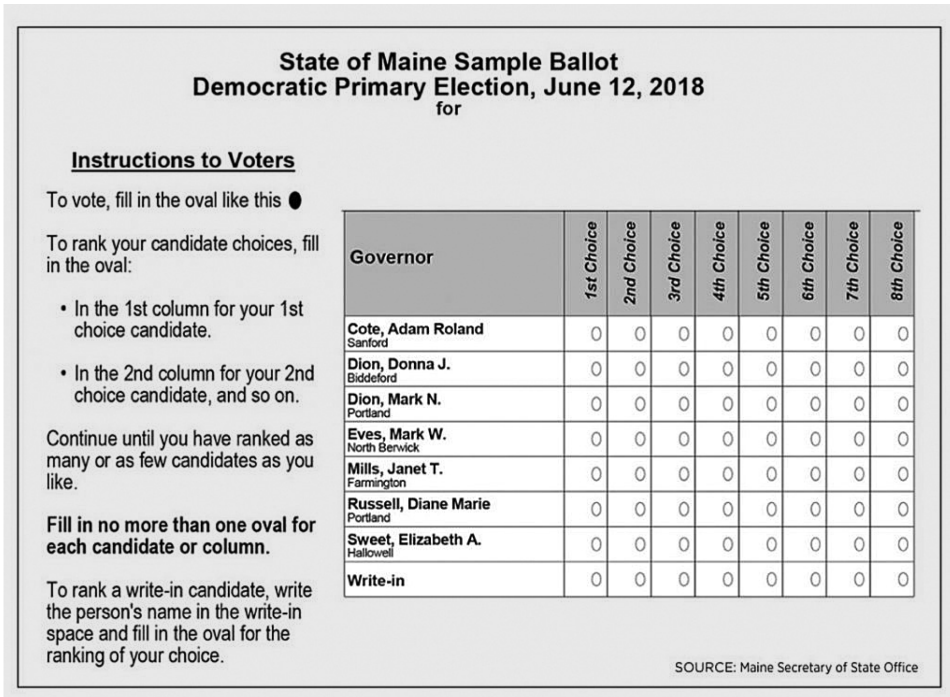
[11] Leased by the municipalities from *Elections Systems & Software* under a 2014 contract with the SoS.

[12] Paper ballots are typically scanned (adjudicated and marked) by certified voting equipment in preparation for tabulation [USE15].

[13] In some isolated cases where paper ballots resist tabulation, election officials my enter cast vote records by hand.

[14] A revised version was certified on November 2, 2018. [Ele18].



State of Maine Sample Ballot
Democratic Primary Election, June 12, 2018
for

Instructions to Voters

To vote, fill in the oval like this ●

To rank your candidate choices, fill in the oval:

- In the 1st column for your 1st choice candidate.
- In the 2nd column for your 2nd choice candidate, and so on.

Continue until you have ranked as many or as few candidates as you like.

Fill in no more than one oval for each candidate or column.

To rank a write-in candidate, write the person's name in the write-in space and fill in the oval for the ranking of your choice.

Governor	1st Choice	2nd Choice	3rd Choice	4th Choice	5th Choice	6th Choice	7th Choice	8th Choice
Cote, Adam Roland Sanford	O	O	O	O	O	O	O	O
Dion, Donna J. Biddeford	O	O	O	O	O	O	O	O
Dion, Mark N. Portland	O	O	O	O	O	O	O	O
Eves, Mark W. North Berwick	O	O	O	O	O	O	O	O
Mills, Janet T. Farmington	O	O	O	O	O	O	O	O
Russell, Diane Marie Portland	O	O	O	O	O	O	O	O
Sweet, Elizabeth A. Hallowell	O	O	O	O	O	O	O	O
Write-in	O	O	O	O	O	O	O	O

SOURCE: Maine Secretary of State Office

Figure 2: Maine 2018 Democratic Gubernatorial Primary

The *Summary* seems to suggest that *Chapter 535* is sufficient for a complete specification of an *independent tabulator*.

3.2 Modeling Maine ranked choice rules

3.2.1 Can the rules be modeled using elementary mathematics?

Mathematical constructions used in the election model are commonly taught in undergraduate courses in mathematics and computer science. Considering such to define elementary mathematics, the question is answered in the affirmative by identifying specific Maine election rules specifying the particulars.

3.2.2 Adjudication

Adjudication for all Maine statewide elections is addressed in *21-A. M.R.S. §696(4) Determination of choice possible*

> "If a voter marks the voter's ballot in a manner that differs from the instructions at the top of the ballot but in such a manner that it is possible to determine the voter's choice, then the vote for the office or question concerned must be counted."

Formally, §696(4) reflects the binary nature of ballot adjudication, an expression of the *"one person, one vote"* principle rejecting **plural voting** and considering ballots to be indivisible.[15] This core principle of American representational equality, cannot be found in the *United States Constitution*. British M.P. George Howell coined the gender-biased version of the phrase as the title of an 1880 campaign pamphlet **One man, One vote**[16]. Perhaps a corollary of *"all men are created equal"*, Chief Justice Earl Warren's United States Supreme Court adopted "one person, one vote" as an organizing principle in the *Apportionment Cases* of the 1960s [How80; Imb17].

3.2.3 Marking and key provisions

A preference extraction model follows from a handful of **key provisions** in *29-250 C.M.R. ch.535 §4 Vote Counting Procedures*. Ballot marking is defined by the overvote and skipped ranking provisions found below[17].

i. (Highest ranking): Each continuing ballot counts as one vote for its highest-ranked continuing candidate for that round[18].

ii. *First overvote*. An overvote occurs when a voter marks more than one candidate for the same ranking (i.e., in the same ranking column). An overvote invalidates the overvoted rankings and all subsequent rankings marked for that contest on the ballot.

iii. *Two consecutive skipped rankings*. When a voter does not mark (i.e., skips or leaves blank) two or more consecutive rankings, then the ballot is deemed exhausted for that contest, and no subsequent candidate rankings marked on that ballot are counted[19].

[15] Unlike fan balloting for MLB All Stars. Technical details are found in ***29-250 Code of Maine Rules Chapter 550****: Rules for Determining Voter Intent* [Ele10].

[16] One person, one vote is a commonly expressed objection to ranked choice voting [MV19; Fai19].

[17] *Highest ranking:* label added for reference and is not part of the original text.

[18] A sentence in the middle of *29-250 C.M.R. ch.535 §4.2 General Procedures*.

[19] Overvotes and skipped rankings (undervotes) are described in *29-250 C.M.R. ch.535 §4.2 Handling of certain ballot rankings*.

The marking function implied by Chapter 35 rules is column-oriented, effectively vectorizing plurality marking as

$$m : c_j(E) \to A \cup \mathcal{A},$$
$$m(c_j(e)) = \begin{cases} U, & \text{if } |c_j(e)| = 0, \\ a_i, & \text{if } |c_j(e)| = 1 = c_{ij}(e), \\ O, & \text{if } |c_j(e)| > 1. \end{cases}$$

We note that m does not consider digital ambiguities in rank. Marking sequences model the rank fields of a **cast vote record** (CVR)[20]. The State of Maine publishes cast vote record marking sequence equivalents for every ranked choice ballot in which a runoff round is required[21].

3.2.4 Trimming and ranking

Trimming rules follow directly from key provisions of *29-250 C.M.R. ch.535 §4*:

- **First overvote**: $\wedge(.*?)O\{1\}$,
- **Two consecutive skipped rankings**: $\wedge(.*?)(U)\{2\}$.

It is worth noting that the later provision is inconsistent with language found in *21-A. M.R.S. §723-A(1)(D)*:

> "Exhausted ballot" means a ballot that does not rank any continuing candidate, contains an overvote at the highest continuing ranking or contains 2 or more sequential skipped rankings before its highest continuing ranking.

The statute references **sequential** skipped rankings whereas the administrative rule refers to **consecutive** skipped rankings. The regular expression implementing a trimming rule for sequential rankings:

- **Two sequential skipped rankings**: $\wedge(.*?U)\{2\}$.

Note that the highest ranking provision allows trimming marked ballots to occur prior to tabulation as a preference extraction step. The highest ranking provision also allows for the application of the ranking function to complete preference extraction.

[20] The US Election Assistance Commission has created voluntary voting system guidelines, and with the help of NIST, a UML specification for cast vote records, including JSON and XML schema implementations [Com19b; Wac+19; Com19a].

[21] Available here [Maig].

3.3 Discussion

3.3.1 Are the rules complete, consistent, and independent?

It appears that the administrative rules defined by *29-250 C.M.R. ch. 550* are complete and sufficient for the specification of a proprietary software implementation.

While internally consistent (non-contradictory), the administrative rules are not in alignment with statute *21-A. M.R.S. §723-A* with regard to skipped rankings. In fact, parties reporting duplicating November CD2 tabulations were unable to duplicate results for the June Gubernatorial primary use the same algorithm[22]. Three idiosyncratic ballots from the June 2018 gubernatorial primary have been identified.

1. File: govd-1.xlsx, Row: 27513, Code: GDUBUFEH, Cast Vote Record: 485831, Precinct: Dixmont
2. File: govd-2.xlsx, Row: 63456, Code: BGDCUFUE, Cast Vote Record: 265479, Precinct: Porter
3. File: govd-2.xlsx, Row: 82459, Code: GUBFDUEA, Cast Vote Record: 335281: Precinct: Portland W2 P2

The first three ballots would be awarded to candidate E (Janet Mills) in the fourth round using the "two consecutive" trimming rule. Independent implementations report 3 fewer votes for Janet Mills in the final round than the official tabulation.

The results generated by the proprietary algorithm can be achieved by trimming each ballot after the second skipped ranking, **except** where the first skipped ranking is the first choice ranking. There were three such exceptional ballots

- File: govd-2.xlsx, Row: 1315, Code: UGUBAEDC, Cast Vote Record: 13216: Precinct: Old Orchard Beach
- File: govd-2.xlsx, Row: 33534, Code: UGUECGAF, Cast Vote Record: 141592: Precinct: Eastport
- File: govd-2.xlsx, Row: 41151, Code: UCUAUUUU, Cast Vote Record: 173959: Precinct: Harpswell W1

[22] Personal communication with Nathan Tefft.

The clarity of the rules suffers a lack of independence among the provisions. In particular, the key provision imply all of the following paragraphs[23].

- "**Single skipped ranking**. A single skipped ranking occurs when a voter does not mark (i.e., skips or leaves blank) a ranking but marks the subsequent ranking for a candidate. The single skipped ranking is ignored, and the subsequent ranking is counted in the current round, as long as that ranking is for a continuing candidate. For example, if the voter did not mark any candidate for the first ranking, but marked a continuing candidate for the second ranking, then the second ranked choice is counted in the first round of the RCV count."

- "**Duplicate ranking for the same candidate**. A duplicate ranking occurs when a voter marks more than one ranking column for the same candidate. If a voter marks a duplicate ranking for one candidate and ranks no other candidates, then the ballot will be counted for the highest ranking of that candidate. If the candidate with the duplicate ranking is defeated, the ballot is deemed exhausted for that contest and no subsequent candidate rankings marked on that ballot are counted. If a voter marks a duplicate ranking for one candidate but also ranks other candidates, and if the candidate with the duplicate ranking is defeated, then the vote for the next continuing candidate ranked by that voter will be counted in the next round."

- "**No ranking for a continuing candidate**. In any round, if a voter has not ranked any continuing candidate, the ballot is deemed exhausted for that contest, and no subsequent candidate rankings marked on that ballot are counted."

3.3.2 Key provisions and voter intent

The complexity of *29-250 C.M.R. ch.535 §4* is driven by the need to extract voter intent from grid layout ballots. The rules assume that columns of the grid layout ballot will be processed and marked using the plurality scheme. But unlike the plurality case, ranked choice demands greater sophistication in the resolution of digital ambiguity to determine voter intent. In fact, the explicit determination of voter intent expressed as a partial preference order, is not an artifact of the processing described by the administrative rule.

[23]First noticed in personal communication by Nathan Tefft. See his Python implementation here.

Subsequent to Maine's well-defined adjudication process, the key provisions are sufficient to define the resolution of digital ambiguities required to determine voter intent. Consequently, administrative rules equivalent to *29-250 C.M.R. ch.535 §4* could be factored to reflect the election model.

- *Preference extraction*: Language embodying the key provisions to define a process to specifically determine voter intent.
- *Preference aggregation*: Language to describe tabulation of partial preference orders.

References

[Com19a] United States Election Assistance Commission. *Draft Recommendations for Requirements for the Voluntary Voting System Guidelines 2.0*. Sept. 9, 2019. URL: https://collaborate.nist.gov/voting/pub/Voting/VVSG20DraftRequirements/vvsg-2.0-2019-09-09-DRAFT-requirements.pdf (visited on 12/18/2019).

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