Vectors, Matricies, and Operations

Computational Mathematics and Statistics Camp
University of Chicago
September 2018

1. Perform the following vector multiplication operations:
   a. $\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} a & b & c \end{bmatrix}'$
   b. $\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} a & b & c \end{bmatrix}$
   c. $\begin{bmatrix} -1 & 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 4 & 3 & 12 \end{bmatrix}'$
   d. $\begin{bmatrix} -1 & 1 & -1 \end{bmatrix} \times \begin{bmatrix} 4 & 3 & 12 \end{bmatrix}$
   e. $\begin{bmatrix} 0 & 9 & 0 & 11 \end{bmatrix} \cdot \begin{bmatrix} 123.98211 & 6 & -6392.38743 & -5 \end{bmatrix}'$
   f. $\begin{bmatrix} 123.98211 & 6 & -6392.38743 & -5 \end{bmatrix} \cdot \begin{bmatrix} 0 & 9 & 0 & 11 \end{bmatrix}'$

2. Find the length of the following vectors:
   a. $(3, 4)$
   b. $(0, -3)$
   c. $(1, 1, 1)$
   d. $(3, 3)$
   e. $(-1, -1)$
   f. $(1, 2, 3)$
   g. $(2, 0)$
   h. $(1, 2, 3, 4)$
   i. $(3, 0, 0, 0, 0)$

3. Recall a property of the **law of cosines**:

$$\cos(\theta) = \frac{\mathbf{v} \cdot \mathbf{w}}{\|\mathbf{v}\| \, \|\mathbf{w}\|}$$

   where θ is the angle from $\mathbf{w}$ to $\mathbf{v}$ measured in radians. Of importance, arccos() is the inverse of cos():

$$\theta = \arccos\left(\frac{\mathbf{v} \cdot \mathbf{w}}{\|\mathbf{v}\| \, \|\mathbf{w}\|}\right)$$

   For each of the following pairs of vectors, calculate the angle between them. Report your answers in both radians and degrees. To convert between radians and degrees:

$$\text{Degrees} = \text{Radians} \times \frac{180^o}{\pi}$$

   a. $\mathbf{v} = (1, 0), \quad \mathbf{w} = (2, 2)$
   b. $\mathbf{v} = (4, 1), \quad \mathbf{w} = (2, -8)$
   c. $\mathbf{v} = (1, 1, 0), \quad \mathbf{w} = (1, 2, 1)$
   d. $\mathbf{v} = (1, -1, 0), \quad \mathbf{w} = (1, 2, 1)$
   e. $\mathbf{v} = (1, 0, 0, 0, 0), \quad \mathbf{w} = (1, 1, 1, 1, 1)$

4. Using the matrices below, calculate the following. Some may not be defined; if that is the case, say so.

$$\mathbf{A} = \begin{bmatrix} 3 \\ -2 \\ 9 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 8 \\ 0 \\ -1 \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} 7 & -1 & 5 \\ 0 & 2 & -4 \end{bmatrix} \quad \mathbf{D} = \begin{bmatrix} 3 & 1 \\ 3 & 4 \\ 3 & -7 \end{bmatrix} \quad \mathbf{E} = \begin{bmatrix} 5 & 2 & 3 \\ 1 & 0 & -4 \\ -2 & 1 & -6 \end{bmatrix}$$

$$\mathbf{F} = \begin{bmatrix} 4 & 1 & -5 \\ 0 & 7 & 7 \\ 2 & -3 & 0 \end{bmatrix} \quad \mathbf{G} = \begin{bmatrix} 2 & -8 & -5 \\ -3 & 7 & -4 \\ 1 & 0 & 3 \\ 1 & 2 & 6 \end{bmatrix} \quad \mathbf{K} = \begin{bmatrix} 9 \\ -2 \\ -1 \\ 0 \end{bmatrix} \quad \mathbf{L} = \begin{bmatrix} 5 & 0 & 3 & 1 \end{bmatrix}$$

a. $\mathbf{A} + \mathbf{B}$
b. $-\mathbf{G}$
c. $\mathbf{D}'$
d. $\mathbf{C} + \mathbf{D}$
e. $3\mathbf{C} - 2\mathbf{D}'$
f. $\mathbf{A}' \cdot \mathbf{B}$
g. $\mathbf{CB}$
h. $\mathbf{BC}$
i. $\mathbf{FB}$
j. $\mathbf{EF}$
k. $\mathbf{K} \cdot \mathbf{L}'$
l. $\|\mathbf{K}\|$
m. $\mathbf{G}'$
n. $\mathbf{E} - 5\mathbf{I_3}$

5. Prove the additive property of matrix transposition:

$$(\mathbf{X} + \mathbf{Y})' = \mathbf{X}' + \mathbf{Y}'$$

6. For two vectors in $\Re^3$ using

$$1 = \cos^2\theta + \sin^2\theta$$

$$\|\mathbf{u} \times \mathbf{v}\|^2 = \|\mathbf{u}\|^2 \|\mathbf{v}\|^2 - (\mathbf{u} \cdot \mathbf{v})^2$$

show that the norm of the cross product between two vectors, $\mathbf{u}$ and $\mathbf{v}$, is:

$$\|\mathbf{u} \times \mathbf{v}\| = \|\mathbf{u}\| \, \|\mathbf{v}\| \sin(\theta)$$