PSET 7, General R.V.

Monday, September 8, 2025 23:13

2 $X \sim U(150{,}000,\ 300{,}000)$

$E[X] = (150{,}000 + 300{,}000)/2 = 225{,}000$

$Var[X] = (300{,}000 - 150{,}000)^2/12 = 1{,}875{,}000{,}000$

$\sigma = \sqrt{Var[X]} = \sqrt{1{,}875{,}000{,}000} \approx 43{,}301.27$

3 A)

Think about commuting to SSRB every time, some days the traffic is light, some days it's heavy. But if we tracked our travel time across many days, the expected value would be the reasonable average travel time we could plan for.

Different levels of moments are like different ways of understanding the time we need to use - the average time, how consistent it is, whether it tends to run over or under, and how extreme the surprises get.

B)

The first moment (mean) is like the balancing point on a seesaw: if we placed all our travel times as weights, the mean is the one point where the plank balances. We care because it gives us the simplest, most useful number to summarize the entire distribution.

C)

Now imagine two friends who also commute the same route and have the same average travel time as us. One drives a well-maintained car, and it takes almost exactly 30 minutes every day. The other drives a beat-up car that breaks down or stalls often - some days the trip takes just 10 minutes, other days it stretches to an hour.

Even though both friends share the same average (first moment), their experiences feel very different. That difference is what the second moment (variance) captures: it tells us not just where the center is, but how steady or volatile the outcomes are around that center.

4 A

$E(U(L)) = E[-(L-v)^2] = -E[(L-v)^2]$

$\because Var(L) = E[(L-\mu_L)^2]$

$E[(L-v)^2] = (E[L]-v)^2 + Var(L) \leftarrow (L-v)^2 = ((L-\mu_L)+(\mu_L-v))^2$

$= (\mu_L - v)^2 + Var(L)$

$$= \underbrace{(L-\mu_L)^2}_{E[(L-\mu_L)^2] = Var(L)} + 2\underbrace{(L-\mu_L)}_{E[L-\mu_L] = 0}(\mu_L-v) + \underbrace{(\mu_L-v)^2}_{\text{constant term}}$$

$\therefore E(U(L)) = -(\mu_L-v)^2 - Var(L)$

Interpretation:

An's expected utility is reduced by two separate factors:

1. Policy distance: the squared difference $(\mu_L - v)^2$ measures how far the candidate's average policy position is from An's ideal point. The further away, the less happy An is.
2. Uncertainty penalty: the variance Var(L) represents the unpredictability of the candidate's policies. Even if the mean lines up with An's ideal, greater uncertainty lowers An's expected utility.

Together, this shows that An is risk averse: not only does distance from their ideal reduce happiness, but uncertainty itself is a source of dissatisfaction.

B
$$\begin{cases} D.S. & \mu_S = 1,\ Var(L_S) = 6 \\ L.K. & \mu_K = 3,\ Var(L_K) = 1 \\ An & v = 1 \end{cases}$$

$E(U(L_S)) = -(1-1)^2 - 6 = -6$

$E(U(L_K)) = -(3-1)^2 - 1 = -5$

$\because -5 > -6$

$\therefore$ An prefer L.K.

This result shows that a voter may choose a candidate farther from their ideal policy if that candidate's outcome is more predictable. Stability can overweight closeness to ideals.

5 a. $\int_0^4 kx^4\,dx = 1$

$\frac{k}{5}x^5\Big|_0^4 = 1$

$\frac{k}{5}(4^5 - 0^5) = 1$

$\therefore k = \frac{5}{1024}$

b. $\int \frac{5}{1024}x^4\,dx = \frac{x^5}{1024}$

$$F(x) = \begin{cases} 0 & , x \le 0 \\ \frac{x^5}{1024} & , 0 < x < 4 \\ 1 & , x \ge 4 \end{cases}$$

c. $E[X] = \int_0^4 x f(x)\,dx = \frac{5}{1024}\int_0^4 x^5\,dx$

$= \frac{5}{1024}\cdot\frac{4^6}{6} = \frac{10}{3}$

$E[X^2] = \int_0^4 x^2 f(x)\,dx = \frac{5}{1024}\int_0^4 x^6\,dx$

$= \frac{5}{1024}\cdot\frac{4^7}{7} = \frac{80}{7}$

$Var[X] = E[X^2] - E[X]^2 = \frac{80}{7} - \left(\frac{10}{3}\right)^2 = \frac{20}{63} \approx 0.3175$

d. $\frac{x^5}{1024} = \frac{1}{2} \quad \therefore x = \sqrt[5]{512} \approx 3.48$

e. $Pr(X=5) = \int_5^5 f(x)\,dx = 0$

f. $Pr(3 < X < 5) = Pr(3 < X < 4) = F(4) - F(3)$

$= \frac{4^5 - 3^5}{1024} = \frac{781}{1024} \approx 0.7627$

g. $Pr(X < 3 \text{ or } X > 5) = Pr(X < 3) = F(3)$

$= \frac{3^5}{1024} = \frac{243}{1024} \approx 0.2373$

6 a. $\int_0^z re^{-rt}\,dt = r\int_0^z e^{-rt}\,dt$

$= r\left(-\frac{1}{r}e^{-rt}\right)\Big|_{t=0}^{z}$

$= r\cdot\left(-\frac{1}{r}e^{-rz}\right) - r(-1)$

$= 1 - e^{-rz}$

$$\therefore F(x) = \begin{cases} 0 & , z < 0 \\ 1 - e^{-rz} & , z \ge 0 \end{cases}$$

b. $Pr(7 \le Z \le 12) = F(12) - F(7)$

$= (1 - e^{-r\cdot 12}) - (1 - e^{-r\cdot 7})$

$= e^{-7r} - e^{-12r}$

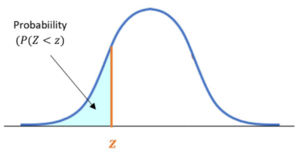
c. $F(q) = 0.1$

$\therefore 1 - e^{-3q} = 0.1 \quad \therefore q = -\frac{1}{3}\ln(0.9) \approx 0.0351$

d. $Pr(Z < 0.5) = F(0.5) = 1 - e^{-3 \cdot 0.5} \approx 0.7769$

$X \sim N(0,1)$, $Y \sim N(1,4)$,

7 a.



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.00	0.5000	0.4960	0.4920	0.4880	0.4841	0.4801	0.4761	0.4721	0.4681	0.4641
-0.10	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.20	0.4207	0.4168	0.4129	0.4091	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.30	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.40	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.50	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.60	0.2743	0.2709	0.2676	0.2644	0.2611	0.2579	0.2546	0.2514	0.2483	0.2451
-0.70	0.2420	0.2389	0.2358	0.2327	0.2297	0.2266	0.2236	0.2207	0.2177	0.2148
-0.80	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.90	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-1.00	0.1587	0.1563	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-1.10	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.20	0.1151	0.1131	0.1112	0.1094	0.1075	0.1057	0.1038	0.1020	0.1003	0.0985
-1.30	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.40	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0722	0.0708	0.0694	0.0681
-1.50	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.60	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.70	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.80	0.0359	0.0352	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.90	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-2.00	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-2.10	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.20	0.0139	0.0136	0.0132	0.0129	0.0126	0.0122	0.0119	0.0116	0.0113	0.0110
-2.30	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.40	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0070	0.0068	0.0066	0.0064
-2.50	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.60	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.70	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.80	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.90	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-3.00	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-3.10	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.20	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.30	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.40	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002	0.0002

$\rightarrow Pr(X \leq -1) = 0.1587$

$\rightarrow Pr(X \leq 1.5) = 1 - Pr(X \leq -1.5)$

$= 1 - 0.0668$

$= 0.9332$

b. Let $W = \frac{Y-1}{2}$

$\because Y \sim N(1,4) \quad \therefore W \sim N(\frac{1-1}{0}, 4/(2^2)) \rightarrow N(0,1)$

$f_W(w) = \frac{1}{\sqrt{2\pi}} e^{-\frac{w^2}{2}}, \quad -\infty < w < \infty$

8 a. $\int_0^4 x^2 dx = \frac{x^3}{3}\Big|_0^4 = \frac{64}{3} \neq 1$, not pdf $\quad$ > Neither

$\lim_{x\to\infty} f(x) = 0 \neq 1$, not CDF

b. $\int_{-\infty}^{\infty} f(x)dx = \int_0^4 0.25\, dx = 4 \cdot 0.25 = 1$, pdf $\quad$ > pdf

$\lim_{x\to\infty} f(x) = 0 \neq 1$, not CDF

c. $\int_{-\infty}^{\infty} f(x)dx = \int_0^4 0.25x\, dx = 0.25 \cdot \frac{4^2}{2} = 2 \neq 1$, not pdf $\quad$ > Neither

$\lim_{x\to\infty} f(x) = 0 \neq 1$, not CDF

d. $\int_{-\infty}^{\infty} f(x)dx = \int_0^1 x^2 dx = \frac{1}{3} \neq 1$, not pdf $\quad$ > Neither

$\lim_{x\to\infty} f(x) = 0 \neq 1$, not CDF

9

A) Exponential: Right-skewed monotone decreasing on [0,∞) with mean 1/γ

B) Normal: Symmetric bell curve on (−∞,∞) with mean μ

C) Student's t: Symmetric, bell-shaped with heavier tails than normal with mean 0 or non-exist(when tails are too heavy)