Linear equations, inequalities, sets and functions, quadratics

Simplify expressions

- 1. Simplify the following expressions as much as possible:¹
- a. $(-x^2y^3)^3$
- b. $9(3^0)$
- c. $(3^2a^2)^2(6a^4)$
- d. $\left(\frac{x^3}{x^4}\right)^3$
- e. $(-2)^{(4-9)}$
- f. $\left(\frac{1}{27b^4}\right)^{1/3}$
- g. $y^5y^6y^5y^2$
- h. $\frac{13a/7b}{13b/2a}$

Simplify a (more complex) expression

2. Simplify the following expression:²

$$(a+b)^2 + (a-b)^2 + 2(a+b)(a-b) - 3a^2$$

Graph sketching

3. Let the functions f(x) and g(x) be defined for all $x \in \Re$ by

$$f(x) = \begin{cases} |x-2| & \text{if } x < 1\\ 1 & \text{if } x \ge 1 \end{cases}, \quad g(x) = \begin{cases} x^3 & \text{if } x < 2\\ 4 & \text{if } x \ge 2 \end{cases}$$

Sketch the graphs of: 3

- 1. y = f(x)
- 2. y = g(x)
- 3. y = f(g(x))
- 4. y = g(f(x))

 $^{^{1}}$ inspired by Gill 1.1

²inspired by Gill 1.2

³inspired by Pemberton and Rau problem 3-1

Root finding

4. Find the roots (solutions) to the following quadratic equations.⁴

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- a. $9x^2 3x 12 = 0$
- b. $x^2 2x 16 = 0$
- c. $6x^2 6x 6 = 0$

Systems of linear equations

- 5. Solve the following systems of equations for their unknown values. If there is no solution, indicate as such.
- a. Two unknowns⁵

$$3x - 2y = 17$$
$$5x - 10y = -10$$

b. Three unknowns⁶

$$5x - 2y + 3z = 9$$
$$2x - 4y - 3z = -9$$
$$x + 6y - 8z = 24$$

c. An animal shelter has a total of 124 animals comprised of cats, dogs, and rabbits. If the number of rabbits is 4 less than twice the number of dogs, and there are 76 more cats than dogs, how many of each animal are at the shelter?⁷

Work with sets

6. Using the sets

$$A = \{2, 3, 7, 9, 12, 13\}$$

$$B = \{x : 6 \le x \le 12 \text{ and } x \text{ is an even integer}\}$$

$$C = \{x : 2 < x < 25 \text{ and } x \text{ is prime}\}$$

$$D = \{1, 4, 9, 16, 25, \ldots\}$$

identify the following:⁸

- 1. $A \cup B$
- 2. $(A \cup B) \cap C$
- 3. $C \cap D$

⁴Gill 1.25

 $^{^5 {\}rm inspired}$ by OpenStax Algebra ex 7.1.12

 $^{^6}$ inspired by OpenStax Algebra ex 7.2.12

⁷inspired by OpenStax Algebra 7.2.54

⁸inspired by Grimmer HW1.1

AI and Resources statement

• Please list (in detail) all resources you used for this assignment. If you worked with people, list them here as well. It is not enough to say that you used a resource for help, you need to be specific on the link and how it was helpful. W/R/T gen AI tools (including GPT, etc.) you cannot use them to do work on your behalf – you cannot put in any of the questions, etc. You can ask for help on logic / sample problems. If you do use GPT or other AI tools, you need to provide a link to your chat transcript. Any suspected academic integrity violations will be immediately reported.