**Problem Set 3** ANSWER KEY

1. Derivative Foundations

   (a) What is a derivative? Why might we find it useful? A derivative enables us to do many different things, primarily understand rates of change at particular points and patterns of change overall. It also enables us to find maxima and minima.

   (b) Give an example of a derivative we might care about (think about the education and salary graphs from lecture) Many examples work: Arrival/exit in the workforce and national GDP, disease rates, vaccination rates, just about anything that you can look at over time.

2. Derivatives by hand : Compute manually, using the formula for a derivative (i.e. $lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$).

   (a) $f(x) = x^3$
   $lim_{h\to 0}\frac{(x+h)^3-x^3}{h} = lim_{h\to 0}\frac{x^3+3x^2h+3xh^2+h^3-x^3}{h} = lim_{h\to 0}\frac{3x^2h+3xh^2+h^3}{h} = lim_{h\to 0}3x^2 + 3xh + h^2 = 3x^2$

   (b) $f(x) = 2x^2 + 4$
   $lim_{h\to 0}\frac{2(x+h)^2+4-2x^2-4}{h} = lim_{h\to 0}\frac{2x^2+4xh+2h^2-2x^2}{h} = lim_{h\to 0}\frac{2h(2x+2h)}{h} = lim_{h\to 0}4x+h = 4x$

   (c) $f(x) = x - 9x^2$
   $lim_{h\to 0}\frac{(x+h)-9(x+h)^2-x+9x^2}{h} = lim_{h\to 0}\frac{(x+h)-9(x^2+2xh+h^2)-x+9x^2}{h} = lim_{h\to 0}\frac{h-9x^2-18xh-9h^2+9x^2}{h} = lim_{h\to 0}\frac{h(1-18x-9h)}{h} = lim_{h\to 0}(1-18x-9h) = 1-18x$

3. Derivatives using the formulas: Compute these using the rules from class.

   (a) $f(x) = 2x^2 + 7x + 9$.
   $f'(x) = 4x + 7$

   (b) $f(x) = 3x^2$
   $f'(x) = 6x$

   (c) $f(x) = x^3$
   $f'(x) = 3x^2$

   (d) $f(x) = 2x^2 + 4$
   $f'(x) = 4x$

   (e) $f(x) = x - 9x^2$ $f'(x) = 1 - 18x$

   (f) $f(x) = ln(x)$
   $f'(x) = \frac{1}{x}$

   (g) $f(x) = e^{3x}$
   $f'(x) = 3e^{3x}$

   (h) $f(x) = 2e^{-2x} - x^{0.5}$
   $f'(x) = -4e^{-2x} - \frac{1}{2}x^{-\frac{1}{2}}$

   (i) $f(x) = (3x^4 - 6x + 2)(x^2 - 4)$
   $f'(x) = (12x^3 - 6)(x^2 - 4) + (3x^4 - 6x + 2)(2x)$

(j) $f(x) = lne^x$ $f'(x) = 1$

(k) $f(x) = (3x^4 - 6x + 2)(x^2 - 4)^{-1}$ $f'(x) = \frac{(12x^3-6)(x^2-4)-(3x^4-6x+2)(2x)}{(x^2-4)^2} = \frac{6x^5-48x^3+6x^2-4x+24}{(x^2-4)^2}$

4. Partial Derivatives: Calculate each derivative first with respect to x, then with respect to y.

   (a) $f(x, y) = x^2 + 3xy - 4$ $\frac{d}{dx} = 2x + 3y$ $\frac{d}{dy} = 3x$

   (b) $f(x, y) = x^3y^2 - x - y$ $\frac{d}{dx} = 3x^2y^2 - 1$ $\frac{d}{dy} = 2x^3y - 1$