Logistic regression

Computational Mathematics and Statistics Camp
University of Chicago
September 2018

1. On January 28, 1986, a routine launch was anticipated for the Challenger space shuttle. Seventy-three seconds into the flight, disaster happened: the shuttle broke apart, killing all seven crew members on board. An investigation into the cause of the disaster focused on a critical seal called an O-ring, and it is believed that damage to these O-rings during a shuttle launch may be related to the ambient temperature during the launch. The table below summarizes observational data on O-rings for 23 shuttle missions, where the mission order is based on the temperature at the time of the launch. *Temp* gives the temperature in Fahrenheit, *Damaged* represents the number of damaged O-rings, and *Undamaged* represents the number of O-rings that were not damaged.

Shuttle Mission	1	2	3	4	5	6	7	8	9	10	11	12
Temperature	53	57	58	63	66	67	67	67	68	69	70	70
Damaged	5	1	1	1	0	0	0	0	0	0	1	0
Undamaged	1	5	5	5	6	6	6	6	6	6	5	6

Shuttle Mission	13	14	15	16	17	18	19	20	21	22	23
Temperature	70	70	72	73	75	75	76	76	78	79	81
Damaged	1	0	0	0	0	1	0	0	0	0	0
Undamaged	5	6	6	6	6	5	6	6	6	6	6

a. Each column of the table above represents a different shuttle mission. Examine these data and describe what you observe with respect to the relationship between temperatures and damaged O-rings.

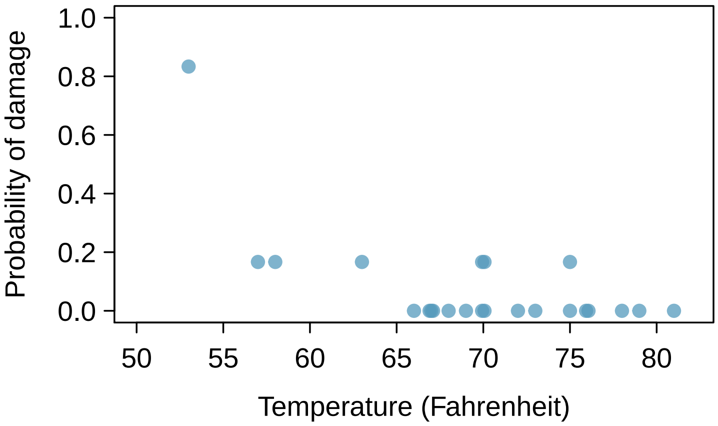
b. Failures have been coded as 1 for a damaged O-ring and 0 for an undamaged O-ring, and a logistic regression model was fit to these data. A summary of this model is given below. Describe the key components of this summary table in words.

	Estimate	Std. Error	z value	Pr(>|z|)
(Intercept)	11.6630	3.2963	3.54	0.0004
Temperature	-0.2162	0.0532	-4.07	0.0000

c. Write out the logistic model using the point estimates of the model parameters.

d. Based on the model, do you think concerns regarding O-rings are justified? Explain.

2. Continuing with the investigation above:



a. The data provided in the previous exercise are shown in the plot. The logistic model fit to these data may be written as

$$\log\left(\frac{\hat{p}}{1-\hat{p}}\right) = 11.6630 - 0.2162 \times Temperature$$

where $\hat{p}$ is the model-estimated probability that an O-ring will become damaged. Use the model to calculate the probability that an O-ring will become damaged at each of the following ambient temperatures: 51, 53, and 55 degrees Fahrenheit. The model-estimated probabilities for several additional ambient temperatures are provided below, where subscripts indicate the temperature:

$$\hat{p}_{57} = 0.341 \quad \hat{p}_{59} = 0.251 \quad \hat{p}_{61} = 0.179 \quad \hat{p}_{63} = 0.124$$
$$\hat{p}_{65} = 0.084 \quad \hat{p}_{67} = 0.056 \quad \hat{p}_{69} = 0.037 \quad \hat{p}_{71} = 0.024$$

b. Add the model-estimated probabilities from part (a) on the plot, then connect these dots using a smooth curve to represent the model-estimated probabilities.

c. Describe any concerns you may have regarding applying logistic regression in this application, and note any assumptions that are required to accept the model's validity.

3. An important question in American politics is why do some people participate in the political process, while others do not? Participation has a direct impact on outcomes – if you fail to participate in politics, the government and political officials are less likely to respond to your concerns. Typical explanations focus on a resource model of participation – individuals with greater resources, such as time, money, and civic skills, are more likely to participate in politics. One area of importance is understanding voter turnout, or why people participate in elections. Using the resource model of participation as a guide, we can develop several expectations. First, women, who more frequently are the primary caregiver for children and earn a lower income, are less likely to participate in elections than men. Second, older Americans, who typically have more time and higher incomes available to participate in politics, should be more likely to participate in elections than younger Americans. Finally, individuals with more years of education, who are generally more interested in politics and understand the value and benefits of participating in politics, are more likely to participate in elections than individuals with fewer years of education.

While these explanations have been repeatedly tested by political scientists, an emerging theory assesses an individual's mental health and its effect on political participation.[1] Depression increases individuals' feelings of hopelessness and political efficacy, so depressed individuals will have less desire to participate in politics. More importantly to our resource model of participation, individuals with depression suffer physical ailments such as a lack of energy, headaches, and muscle soreness which drain an individual's energy and requires time and money to receive treatment. For these reasons, we should expect that individuals with depression are less likely to participate in election than those without symptoms of depression.

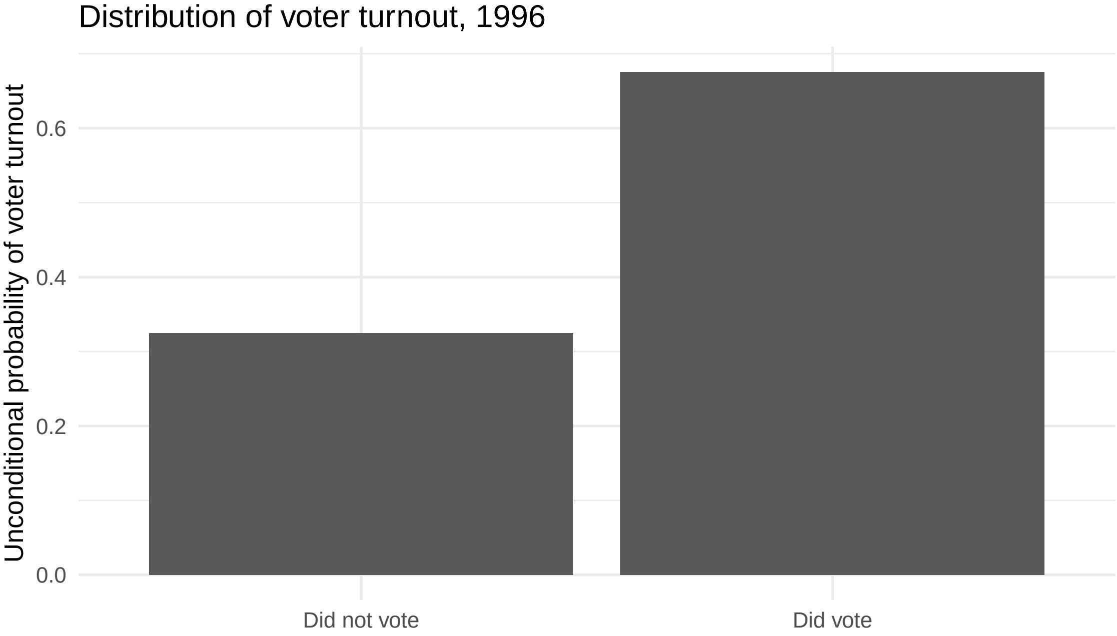
The 1998 General Social Survey included several questions about the respondent's mental health, including:

- `vote96` - 1 if the respondent voted in the 1996 presidential election, 0 otherwise
- `mhealth_sum` - index variable which assesses the respondent's mental health, ranging from 0 (an individual with no depressed mood) to 9 (an individual with the most severe depressed mood)[2]
- `black` - 1 if the respondent is black, 0 otherwise
- `female` - 1 if the respondent is female, 0 if male

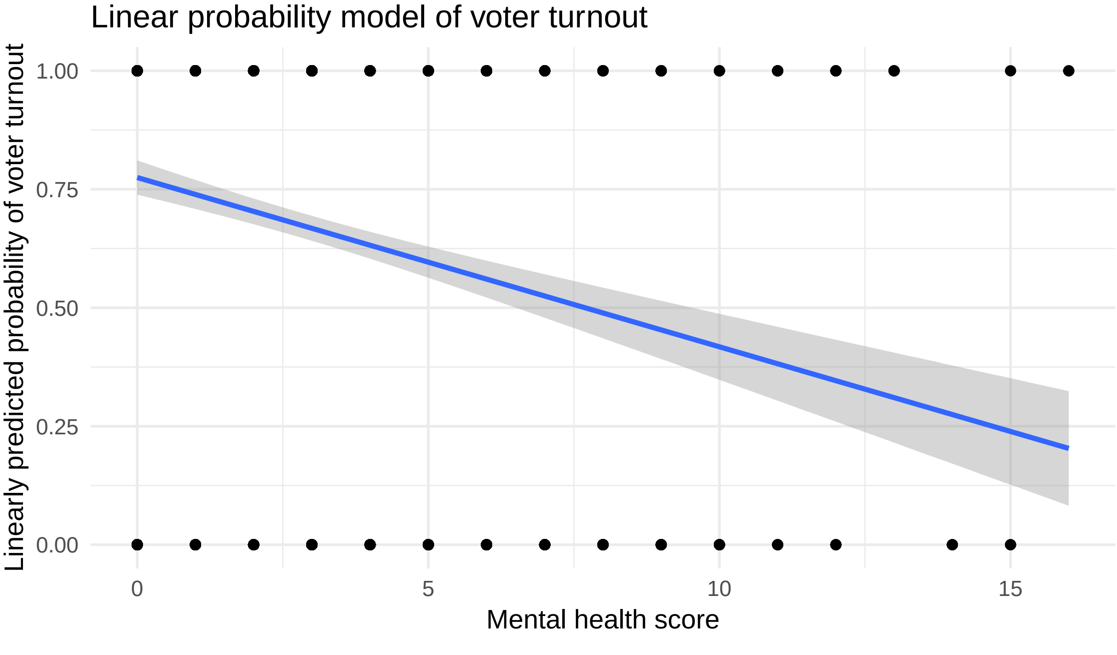
[1] Ojeda, C. (2015). Depression and political participation. *Social Science Quarterly*, 96(5), 1226-1243.

[2] The variable is an index which combines responses to four different questions: "In the past 30 days, how often did you feel: 1) so sad nothing could cheer you up, 2) hopeless, 3) that everything was an effort, and 4) worthless?" Valid responses are none of the time, a little of the time, some of the time, most of the time, and all of the time.

a. Below is a histogram of voter turnout. What is the approximate unconditional probability of a given individual turning out to vote?



b. Below is an OLS model of the relationship between mental health score and probability of voter turnout. What information does this tell us? What is problematic about this linear smoothing line?

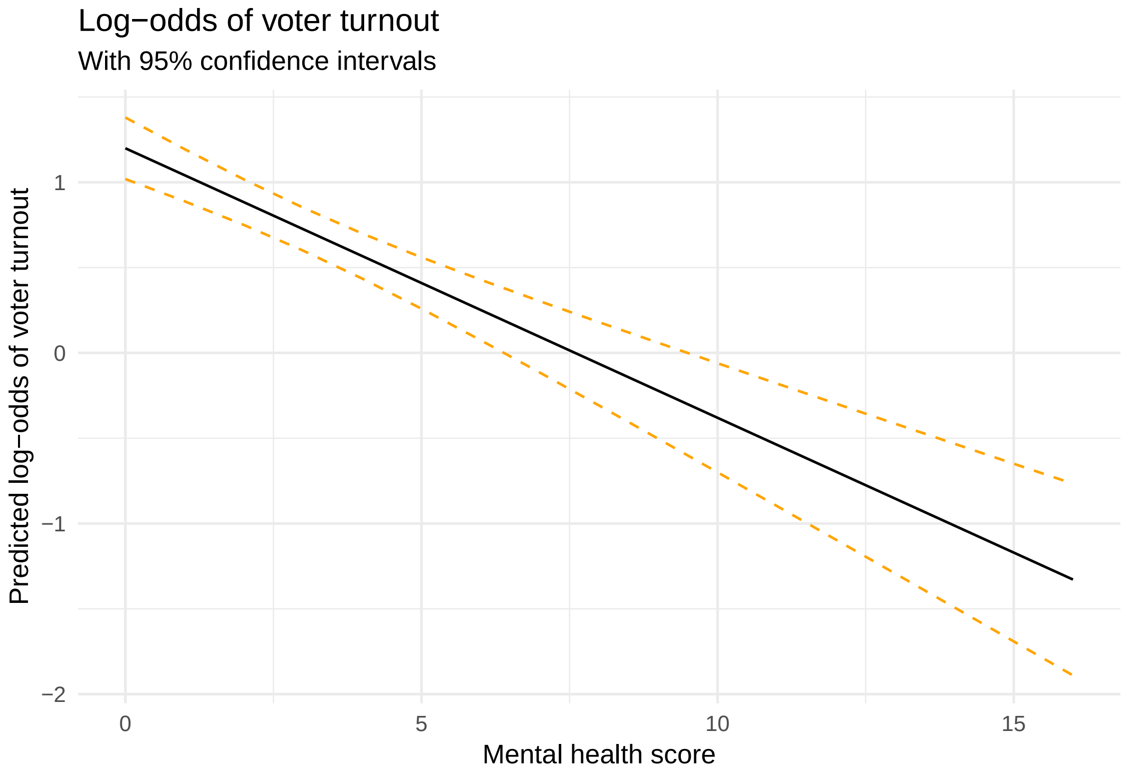


c. Below are the results of a basic logistic regression model of mental health on the probability of voter turnout.

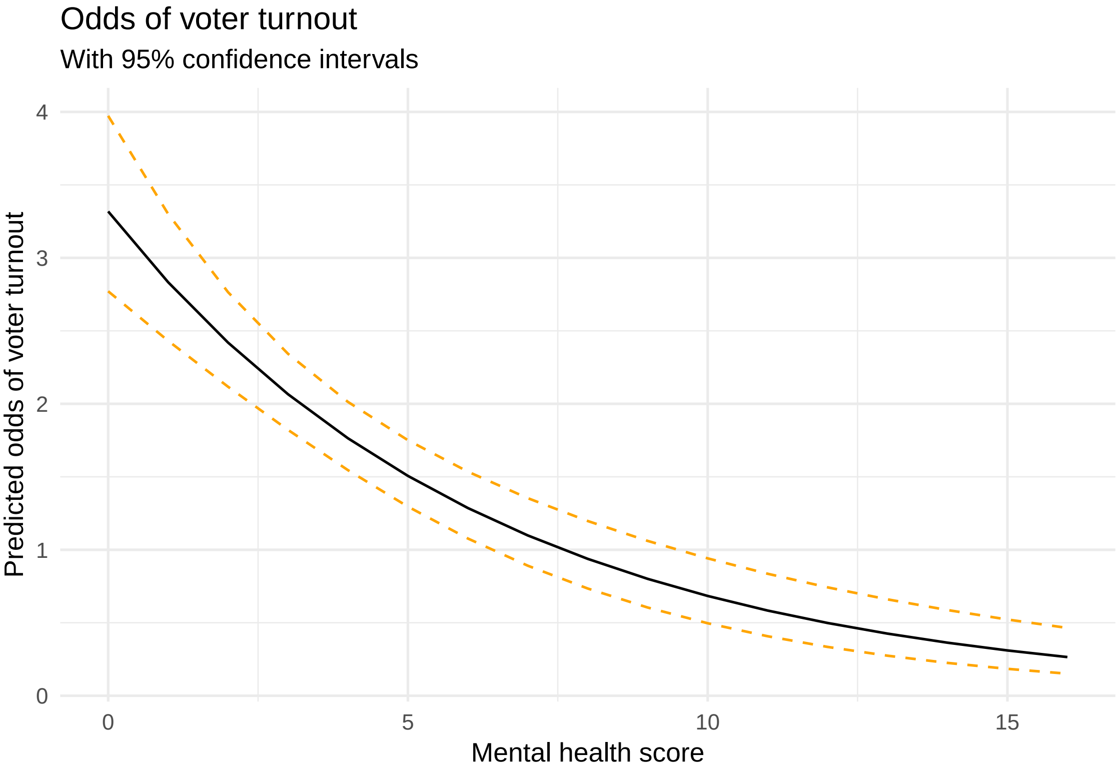
```
##
## Call:
## glm(formula = vote96 ~ mhealth_sum, family = binomial, data = mh)
##
## Deviance Residuals:
##    Min      1Q  Median      3Q     Max
## -1.710  -1.286   0.726   0.832   1.768
##
## Coefficients:
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept)   1.1994     0.0919   13.05  < 2e-16 ***
## mhealth_sum  -0.1580     0.0216   -7.32  2.4e-13 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 1468.3  on 1164  degrees of freedom
## Residual deviance: 1411.8  on 1163  degrees of freedom
## AIC: 1416
##
## Number of Fisher Scoring iterations: 4
```

Is the relationship between mental health and voter turnout statistically and/or substantively significant? Justify your answer.

d. Use the previous results table and this graph to interpret the results in terms of log-odds.



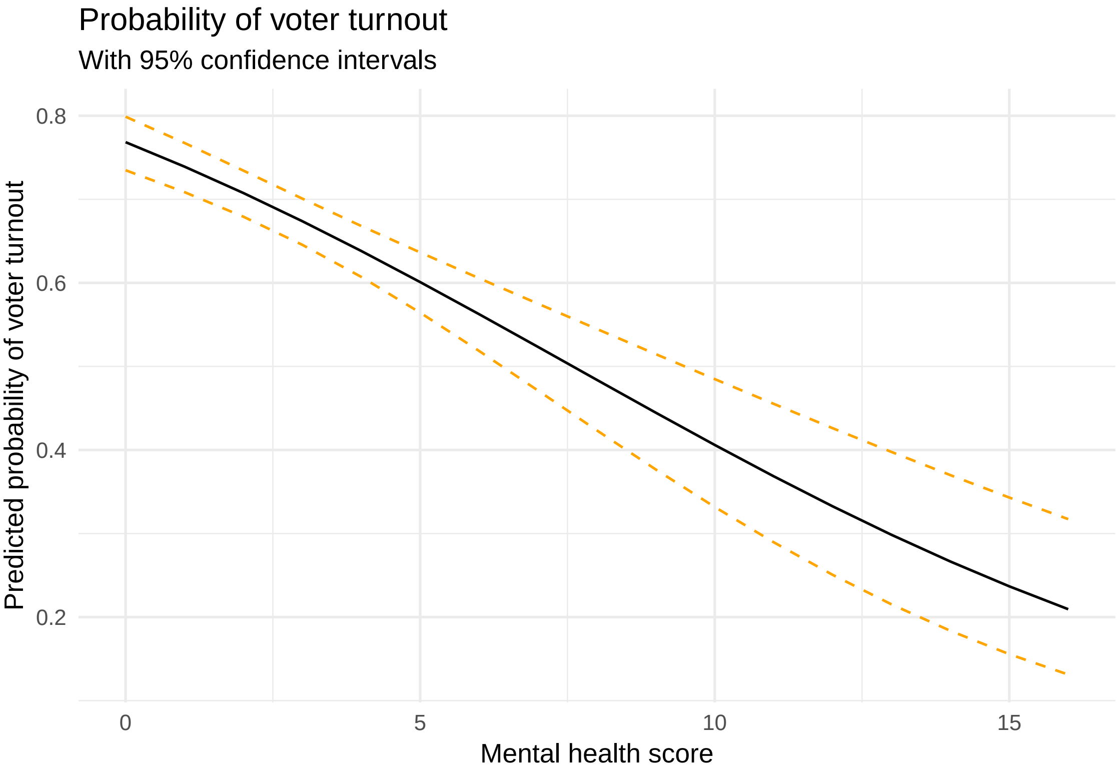
e. Use the previous results table and this graph to interpret the results in terms of odds.



Odds of voter turnout

With 95% confidence intervals

f. Use the previous results table and this graph to interpret the results in terms of probability.

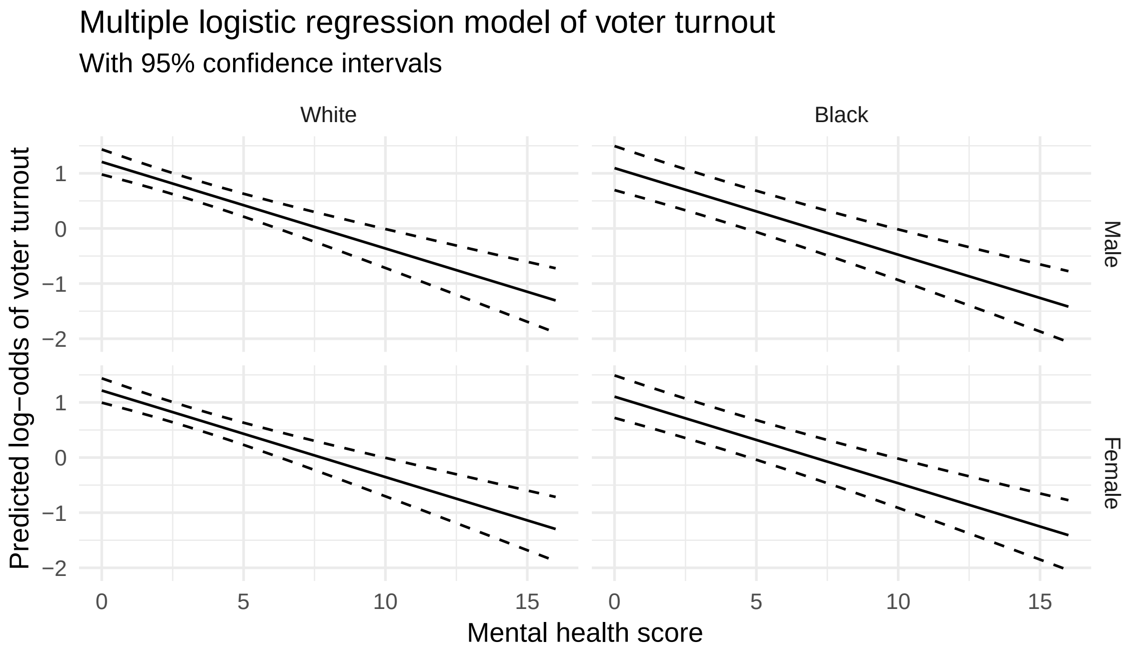


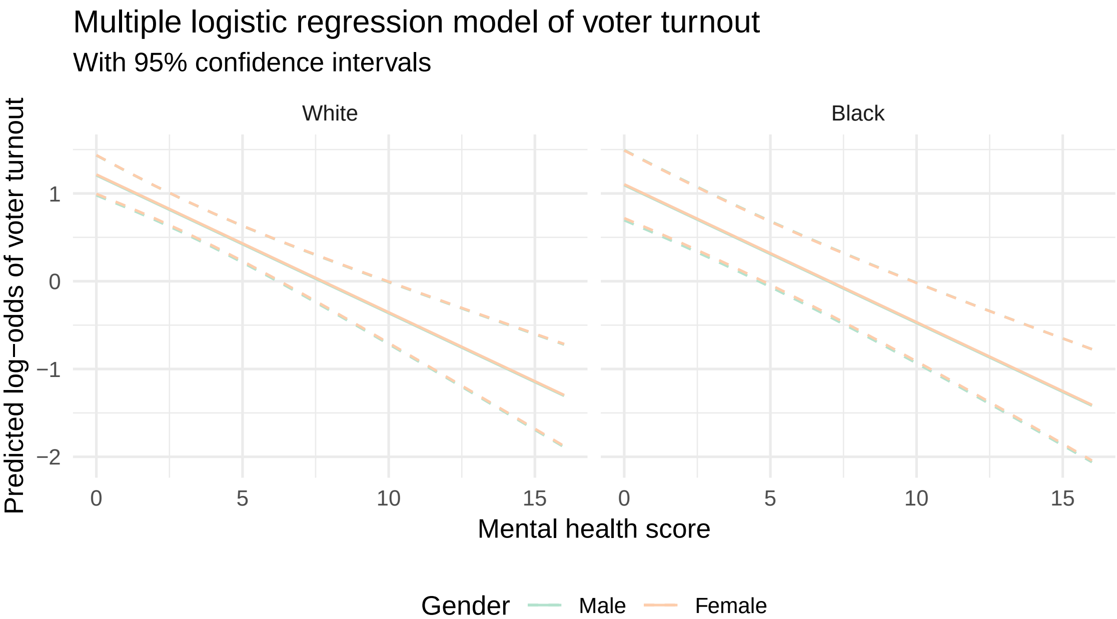
Probability of voter turnout

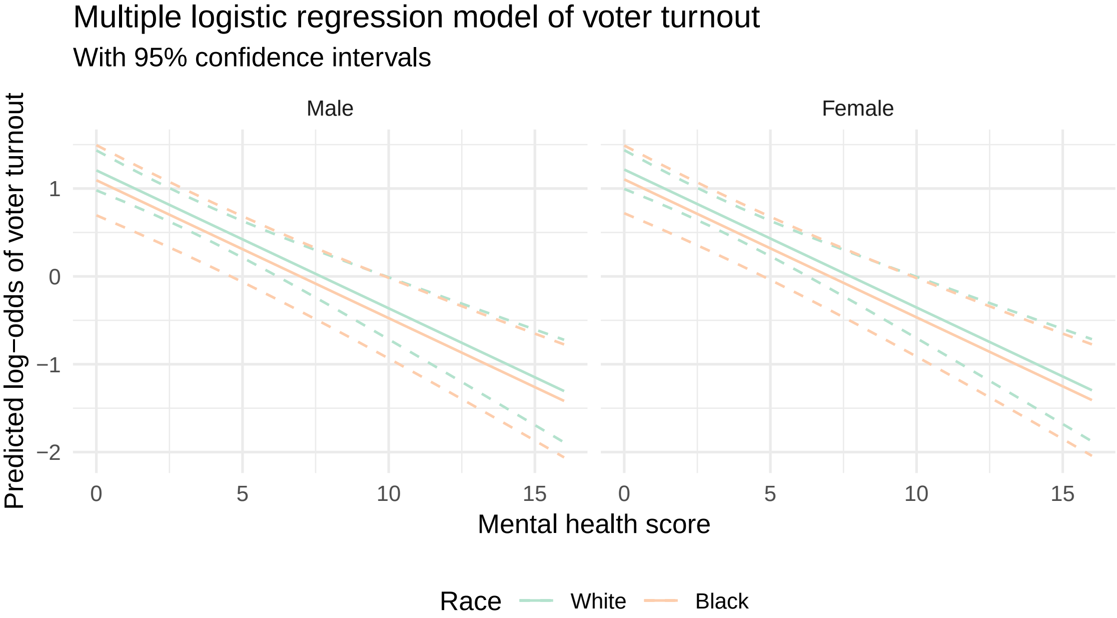
With 95% confidence intervals

g. Here are the results of a multivariate logistic regression model and some related graphs. Interpret the results in paragraph format. This should include a discussion of your results as if you were reviewing them with fellow computational social scientists. Discuss the results using any or all of log-odds, odds, predicted probabilities, and first differences - choose what makes sense to you and provides the most value to the reader.

```
##
## Call:
## glm(formula = vote96 ~ mhealth_sum + female + black, family = binomial,
##     data = mh)
##
## Deviance Residuals:
##    Min      1Q  Median      3Q     Max
## -1.718  -1.292   0.760   0.829   1.808
##
## Coefficients:
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept)  1.20678    0.11590   10.41  < 2e-16 ***
## mhealth_sum -0.15702    0.02162   -7.26  3.8e-13 ***
## female       0.00937    0.12884    0.07     0.94
## black       -0.11189    0.18668   -0.60     0.55
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 1468.3  on 1164  degrees of freedom
## Residual deviance: 1411.4  on 1161  degrees of freedom
## AIC: 1419
##
## Number of Fisher Scoring iterations: 4
```

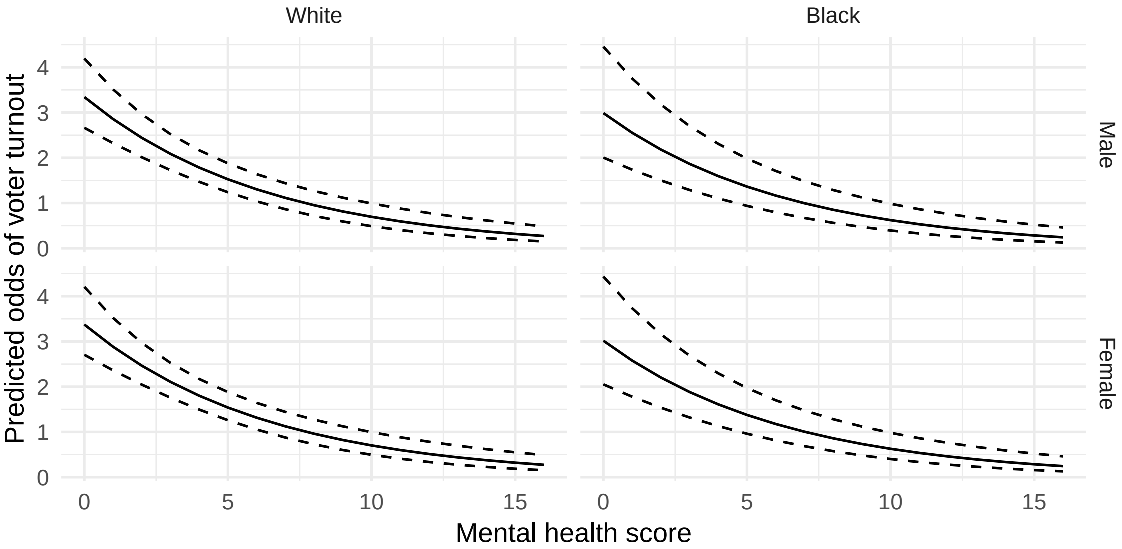



Multiple logistic regression model of voter turnout
With 95% confidence intervals
White
Black
Predicted log−odds of voter turnout
1
0
−1
−2
0
5
10
15
0
5
10
15
Mental health score
Gender
Male
Female


Multiple logistic regression model of voter turnout
With 95% confidence intervals
Male
Female
Predicted log−odds of voter turnout
1
0
−1
−2
0
5
10
15
0
5
10
15
Mental health score
Race
White
Black

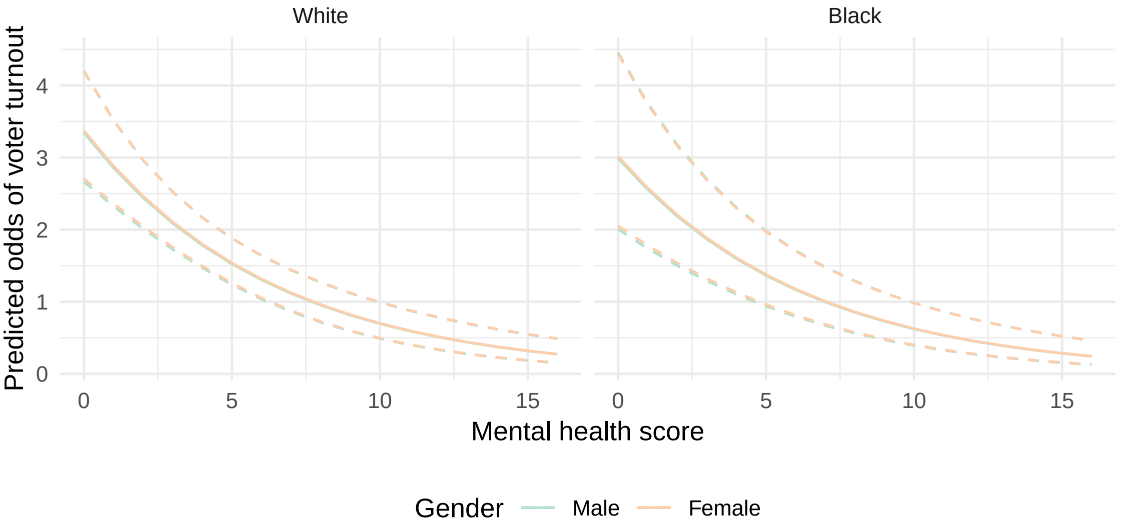
Multiple logistic regression model of voter turnout

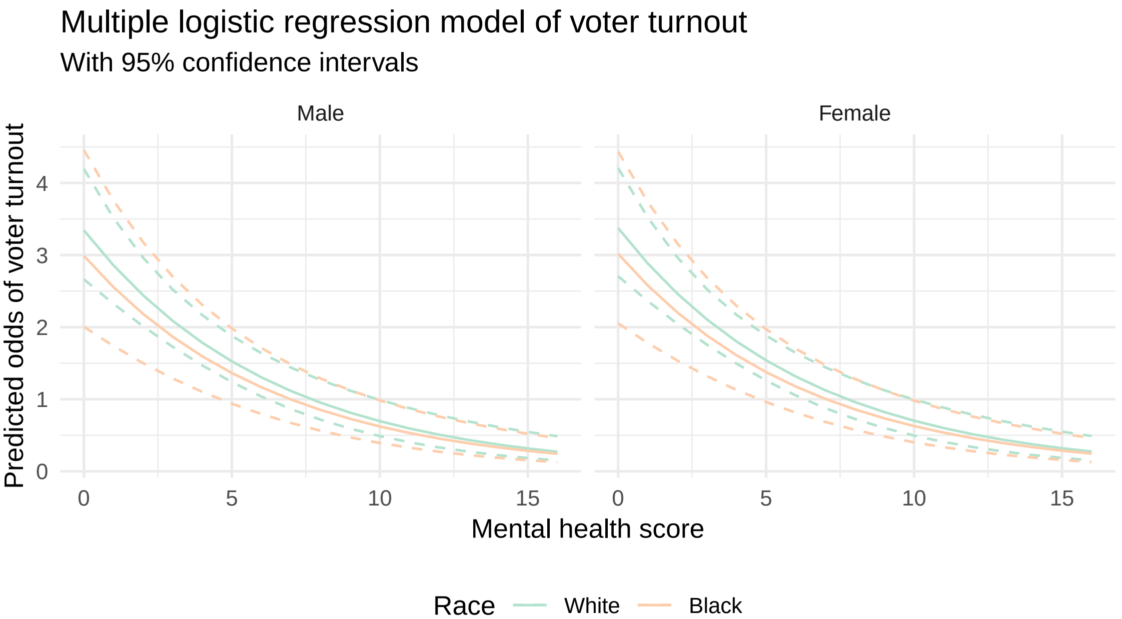
With 95% confidence intervals



Multiple logistic regression model of voter turnout

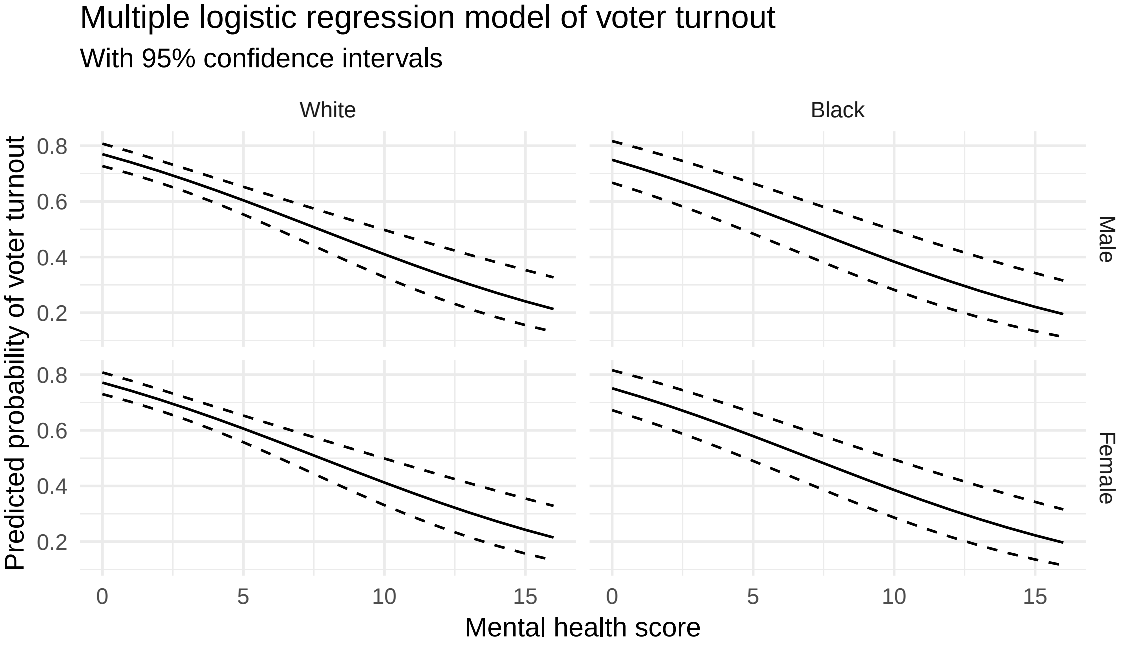
With 95% confidence intervals





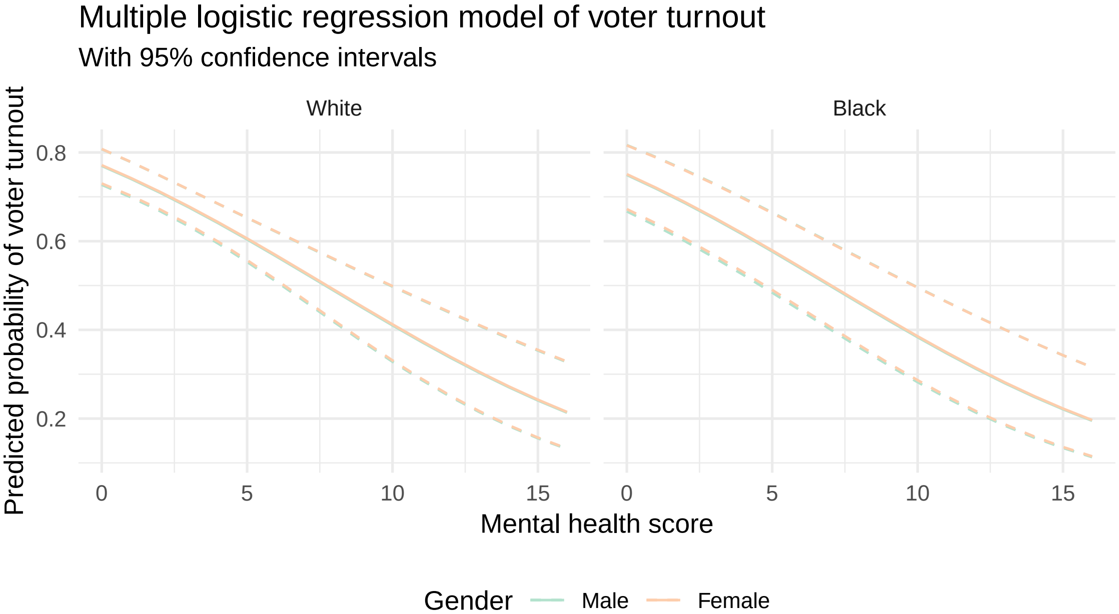
Multiple logistic regression model of voter turnout

With 95% confidence intervals

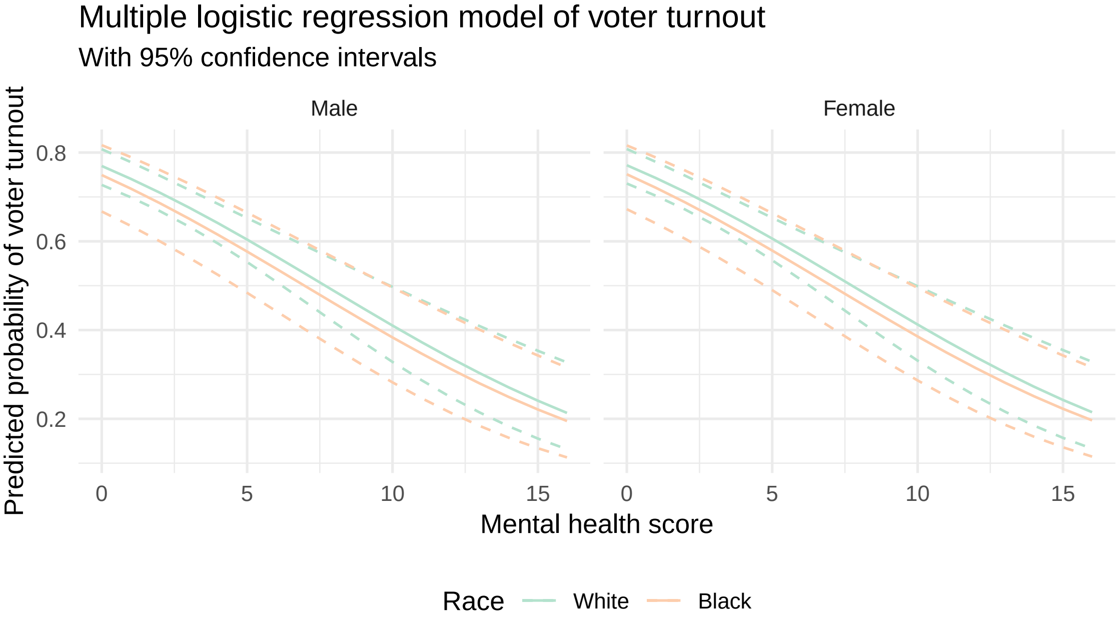


Multiple logistic regression model of voter turnout

With 95% confidence intervals


Multiple logistic regression model of voter turnout

With 95% confidence intervals


Multiple logistic regression model of voter turnout

With 95% confidence intervals