## PSET 8: Multivariate distributions

Note: all homework uploads should be as a PDF or image and have the questions identified. We'll be giving zero credit for submissions that don't follow this protocol as it adds considerable time to grading. Thank you!

- Name
- How long did this problem set take you?
- How difficult was this problem set? very easy 1 2 3 4 5 very challenging

## 1 Calculating conditional PDF

Let 
$$f(x,y) = 5x^2y^2$$
 for  $0 \le x \le y \le 1$ . Find  $f(x|y)$ .

## 2 Properties of a joint PDF

Continuous random variables X and Y have the following joint probability density function (PDF):<sup>2</sup>

$$f_{XY}(x,y) = \begin{cases} kx^3y^2 & \text{where } 0 < x, y < 6\\ 0 & \text{otherwise} \end{cases}$$

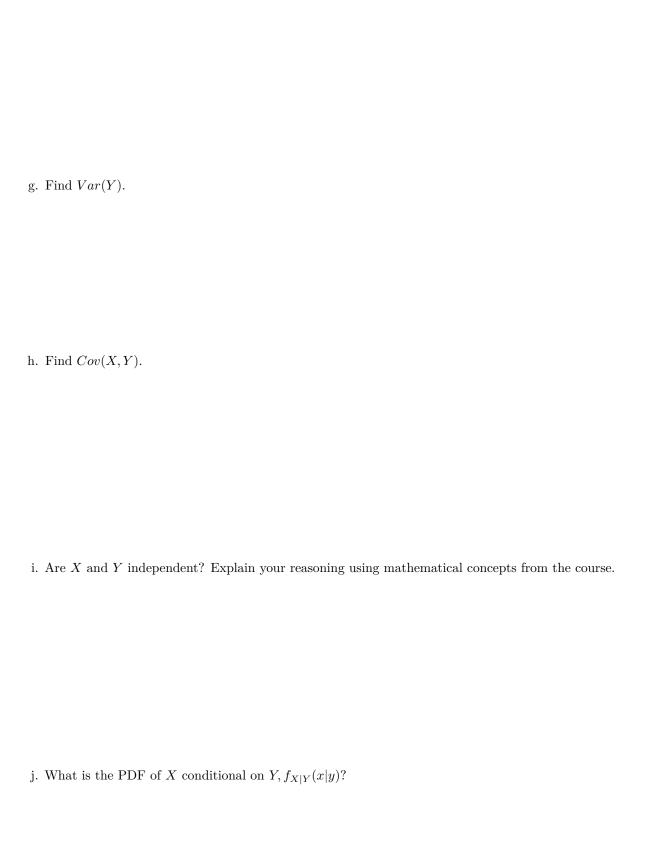
Note: 0 < x, y < 6 means that both x and y are between 0 and 6; it does not mean that x is greater than 0 and y is less than 6.

a. Find k.

 $<sup>^{1}</sup>$ Inspired by Grimmer HW12.4

<sup>&</sup>lt;sup>2</sup>Inspired by Grimmer HW12.1

b.	Find	the marginal PDF of $X$ , $f_X(x)$ .
c.	Find	the marginal PDF of $Y$ , $f_Y(y)$ .
d.	Find	$\mathrm{E}[X].$
e.	Find	$\mathrm{E}[Y].$
f.	Find	Var(X).



k. What is the PDF of Y conditional on  $X, f_{Y\mid X}(y\mid x)?$ 

# 3 Properties of joint random variables<sup>3</sup>

Suppose the following:

- E[D] = 8
- E[F] = 4
- E[DF] = 10
- Var(D) = 30
- Var(F) = 60

a. What is Cov(D, F)?

b. What is the correlation between D and F?

c. Suppose you multiplied F by 2 to generate a new variable, H. What is Cov(D, H)?

 $<sup>^3</sup>$ Inspired by Grimmer HW12.3

d. What is Cor(D, H)? How does this compare to your answer to Part (b) of this question?

e. Suppose instead that Var(D) = 40. How would this change Cor(D, F)?

## 4 Continuous Bayes' theorem

Previously, we used Bayes' theorem to link the conditional probability of discrete events A given B to the probability of B given A. There is an analogous Bayes' theorem that relates the conditional densities of random variables X and  $\theta$  (below) Prove the continuous Bayes' theorem.<sup>4</sup>

$$f(\theta \mid X) = \frac{f(X \mid \theta)f(\theta)}{\int f(X \mid \theta)f(\theta)d\theta}$$

 $<sup>^4</sup>$ Inspired by Grimmer HW12.5

### 5 AI and Resources statement

• Please list (in detail) all resources you used for this assignment. If you worked with people, list them here as well. It is not enough to say that you used a resource for help, you need to be specific on the link and how it was helpful. W/R/T gen AI tools (including GPT, etc.) you cannot use them to do work on your behalf – you cannot put in any of the questions, etc. You can ask for help on logic / sample problems. If you do use GPT or other AI tools, you need to provide a link to your chat transcript. Any suspected academic integrity violations will be immediately reported.

### 5.1 (Optional – complete elsewhere) Submission of practice questions

Submit practice questions for the final exam here: https://forms.gle/CPo9FMQgQRPePDfN7 Note that we need at least 10 people to submit before there's enough to circulate!