PSET 3: Critical points and approximation

1 Assignment Qs

- Name
- How long did this problem set take you?
- How difficult was this problem set? very easy 1 2 3 4 5 very challenging

2 Sketch a function

Sketch the graph of a function (any function you like, no need to specify a functional form) that is:[1]

a. Continuous on $[0, 3]$ and has the following properties: an absolute maximum at 0, an absolute minimum at 3, a local maximum at 1 and a local minimum at 2.
b. Do the same for another function with the following properties: 4 is a **critical number** (i.e. $f'(x) = 0$ or $f'(x)$ is undefined), but there is no local minimum and no local maximum.

3 Find critical values

Find the critical values of these functions:[2]

a. $f(x) = 5x^{2/3} - 4x$
b. $s(t) = 3t^4 - 4t^3 + 6t^2$
c. $f(r) = \dfrac{r}{r^2 + r + 1}$
d. $h(x) = x \ln(x)$

4 Find absolute minimum/maximum values

Find the absolute minimum and absolute maximum values of the functions on the given interval:[3]

a. $f(x) = 3x^2 - 12x + 5, [0, 1]$
b. $f(t) = t^2\sqrt{9 - t^2}, [-1, 4]$
c. $s(x) = x - \ln(x), [1/2, 2]$

5 Approximate root-finding

Show that the equation

$$x^7 + 6x - 4 = 0$$

[1] inspired by Grimmer HW3.1
[2] inspired by Grimmer HW3.2
[3] inspired by Grimmer HW3.3

has a root between 0 and 1.[4]

a. Find an initial approximation by ignoring the term x^7.

b. Use Newton's method to find the root correct to 3 decimal places.

6 Apply the mean value theorem

Does a continuous, differentiable function exist on $[0, 4]$ such that $f(0) = -1$, $f(4) = 4$, and $f'(x) \leq 2 \; \forall x$? Use the mean value theorem to explain your answer.[5]

6.1 Optional!: Finding Max/Min

a. **OPTIONAL** $h(p) = 1 - e^{-p}, [0, 1000]$

b. **OPTIONAL** Demonstrate that the function $f(x) = x^5 + x^3 + x + 1$ has no local maximum and no local minimum.[6]

6.2 AI and Resources statement

- Please list (in detail) all resources you used for this assignment. If you worked with people, list them here as well. It is not enough to say that you used a resource for help, you need to be specific on the link and *how* it was helpful. W/R/T gen AI tools (including GPT, etc.) you cannot use them to do work on your behalf – you cannot put in any of the questions, etc. You can ask for help on logic / sample problems. If you do use GPT or other AI tools, you need to provide a link to your chat transcript. Any suspected academic integrity violations will be immediately reported.

[4] inspired by Pemberton and Rau 10.1.3
[5] inspired by Grimmer HW3.5
[6] inspired by Grimmer HW3.4