# Problem Set 4, MATH PREFRESHER: Integrals

Show work where appropriate. It will be most helpful for you to write your answers as completely as possible.

1. Integrals: Foundations

   (a) What is an integral (e.g. what does it do?) Integrals are ways to sum up the area under the curve – we can use them when we need to essentially add up infinitely many tiny rectangles. They're the sum version for functions.

   (b) Why would we use an integral? It helps us find totals – like the sum function but for functions–any time our question is

   (c) Calculate the area under x^3 on $[1, 4]$ using rectangles.
   See example below–could have selected any rectangle width.
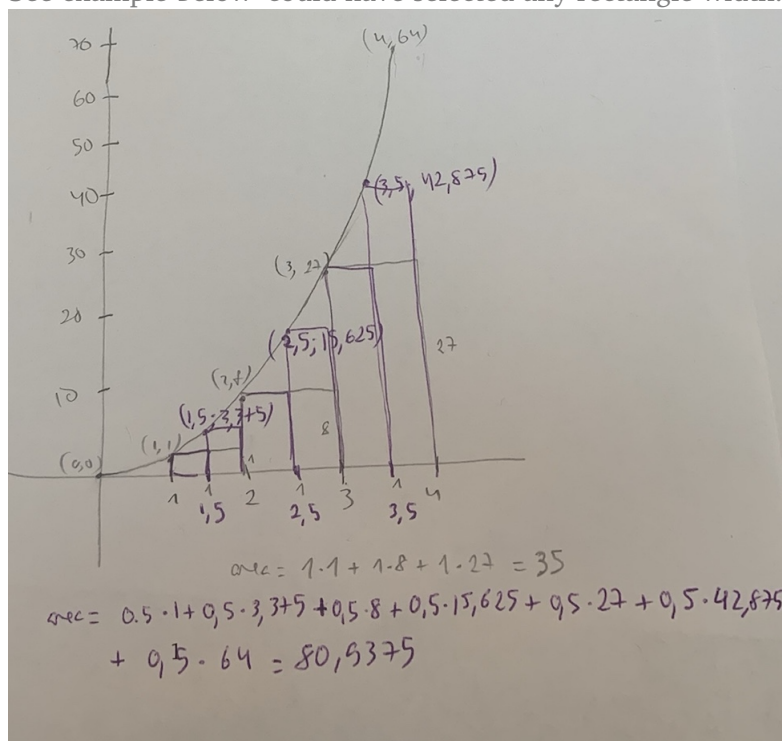
   (d) (Follow up) Now, calculate the area a second time using smaller rectangles. See example above–could have selected any rectangle width. Note that we have one extra term in the example – it should end at 3.5 and exclude the 0.5*64 for a total of approximately 53.

   (e) (Follow up) How do these areas compare? How does your finding here relate to the definition of an integral (above)?The narrower the rectangle width, the closer our estimation becomes to what we'd get using an integral. Note that depending on if you do the 'left' vs 'right' method for rectangles (do you want to over or under-estimate the area?), your estimates may decrease (if you over estimated the area by drawing rectangles such that the right corner touches the curve) or they may increase (if you underestimated the area by drawing rectangles such that the left corner touches the curve, as seen above).

2. Integration Practice: Calculate the definite integrals for the following

   (a) $\int_1^4 x^3 dx = 63.75$

   (b) $\int_0^3 x \, dx = \frac{9}{2}$

   (c) $\int_1^4 (6x^3 - 2) dx = 376.5$

   (d) $\int_4^6 x \, dx = 10$

   (e) $\int_0^y (e^x - 2x^2) dx = e^y - \frac{2y^3}{3} - 1$

3. Iterated Integration Practice: Calculate the following

   (a) $\int_1^4 \int_0^2 (6x^3 - 2y) \, dx \, dy = 42$

   (b) $\int_0^1 \int_1^x 3x - 4 \, dy \, dx = 1.5$

   (c) $\int_0^1 \int_1^y 3x - 4 \, dx \, dy = 1$