PSET 7: General random variables

Note: all homework uploads should be as a PDF *and* have the questions identified. We'll be giving zero credit for submissions that don't follow this protocol as it adds considerable time to grading. Thank you!

1 Background info (GRADED – complete all this section)

- Name
- How long did this problem set take you?
- How difficult was this problem set? very easy 1 2 3 4 5 very challenging

2 Identifying the PDF

A recent college graduate is moving to Des Moines, Iowa to take a new job, and is looking to purchase a home. When searching for properties on the real estate websites, it is possible to select the price range of housing in which one is most interested. Suppose the potential buyer specifies a price range of $150,000 to $300,000, and the result of the search returns a thousand homes with prices distributed uniformly throughout that range. Identify $E[X]$ and σ of the probability density function associated with this random variable.[1]

3 Moments

We discussed moments in class and the slides. Suppose someone were asking you about our class content. How would you explain the following in a non-technical way? Approx 2 sentences each.

a. What is an expected value and how does it relate to a moment?
b. What is the first moment and why do we care about it?
c. What is the second moment and how does it add additional information to the first moment?

4 Calculating ideal points

Suppose An is a voter living in the country of Freedonia, and suppose that in Freedonia, all sets of public policies can be thought of as representing points on a single axis (e.g. a line running from more liberal to more conservative). An has a certain set of public policies that they want to see enacted. This is represented by point v, which we will call An's **ideal point**. The utility, or happiness, that An receives from a set of policies at point l is $U(l) = -(l - v)^2$. In other words, An is happiest if the policies enacted are the ones at their ideal point, and they get less and less happy as policies get farther away from this ideal point. When they vote, An will pick the candidate whose policies will make them happiest. However, An does not know exactly what policies each candidate will enact if elected – they have some guesses, but can't be certain. Each candidate's future policies can therefore be represented by a continuous random variable L with expected value μ_l and variance $Var(L)$.[2]

a. Express $E(U(L))$ as a function of μ_l, $Var(L)$, and v. Why might we say that An is **risk averse** – that is, that An gets less happy as outcomes get more uncertain?

[1] Inspired by Stinerock 6.1
[2] Inspired by Grimmer HW11.2

b. Suppose An is deciding whether to vote for one of two candidates: Dwight Schrute or Leslie Knope Suppose An's ideal point is at 1, Schrute's policies can be represented by a continuous random variable L_S with expected value at 1 and variance equal to 6, and Knope's policies can be represented by a continuous random variable L_K with expected value at 3 and variance equal to 1. Which candidate would An vote for and why? What (perhaps surprising) effect of risk aversion on voting behavior does this example demonstrate?

5 Parliamentary elections

After an election in a parliamentary system, a government (consisting of a prime minister and a cabinet) is formed by gathering the support of a majority of newly elected members of parliament. Typically a government is allowed to remain in power for a certain number of years before new elections must be called. However, elections can be held earlier if the Parliament passes a vote of no confidence or the prime minster decides to dissolve the government. Suppose we are studying Country Z (which uses a parliamentary system) and we are interested in the duration of governments. In Country Z, governments must call elections at least every 5 years, but they could be called sooner if there is a vote of no confidence or the prime minister dissolves the government. Let the continuous random variable X denote the amount of time (measured in years) between the last election and the calling of the next election. X has support on all real numbers between 0 and 5. Suppose we know that X has the probability density function

$$f(x) = \begin{cases} kx^4 & 0 < x < 4 \\ 0 & \text{otherwise} \end{cases}$$

where k is some constant.

a. Find k.

b. Find the CDF of X.

c. Find $E(X)$ and $Var(X)$.

d. Find the median of X (the value of x at which $\Pr(X \leq x) = \frac{1}{2}$).

e. What is the probability that the government remains in power for exactly 5 years? Why?

f. What is the probability that the government remains in power between 3 and 5 years?

g. What is the probability that the government remains in power for less than 3 years or more than 5 years?

6 Calculating the CDF

Z is distributed according to the following PDF

$$f(z) = \begin{cases} \gamma \exp(-\gamma z) & 0 \leq z \\ 0 & \text{otherwise} \end{cases}$$

a. What is $F(z)$, the CDF of this distribution?

b. Using your answer to the previous question, evaluate the CDF for the interval from 7 to 12.
c. Suppose γ is 3. Given this, what is q, the 10th percentile value of Z?
d. We observe a single random draw from Z, what is the probability this observation is less than .5? Again suppose that $\gamma = 3$.

7 Working with normal random variables

Let X and Y be normal random variables with means 0 and 1, respectively, and variances 1 and 4, respectively.[3]

a. Find $\Pr(X \leq 1.5)$ and $\Pr(X \leq -1)$.
b. Find the pdf of $(Y - 1)/2$.

8 Spot the CDF/pdf

From the following, identify functions as either pdfs, CDFs, or neither.

a.

$$f(x) = \begin{cases} x^2 & 0 < x < 4 \\ 0 & \text{otherwise} \end{cases}$$

b.

$$f(x) = \begin{cases} 0.25 & 0 < x < 4 \\ 0 & \text{otherwise} \end{cases}$$

c.

$$f(x) = \begin{cases} 0.25x & 0 < x < 4 \\ 0 & \text{otherwise} \end{cases}$$

d.

$$f(x) = \begin{cases} x^2 & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

9 Functions

We discussed the following functions in class: Uniform Exponential, Gamma, Normal, χ^2, Student's t

Provide a one-sentence summary of the general shape and expected value of:

a. Exponential
b. Normal
c. Student's t

10 AI and Resources statement

Please list (in detail) all resources you used for this assignment. If you worked with people, list them here as well. It is not enough to say that you used a resource for help, you need to be specific on the link and *how* it was helpful. W/R/T gen AI tools (including GPT, etc.) you cannot use them to do work on your behalf – you cannot put in any of the questions, etc. You can ask for help on logic / sample problems. If you do use GPT or other AI tools, you need to provide a link to your chat transcript. Any suspected academic integrity violations will be immediately reported.

[3]Inspired by BT 3.11