

Assignment 2: Sequences, Limits, Derivatives, and Critical Points

Assignment Qs

- Name
- How long did this problem set take you?
- How difficult was this problem set? very easy 1 2 3 4 5 very challenging

Sequences

Write down the first three terms of each of the following sequences. In each case, state whether the sequence is an arithmetic progression, a geometric progression, or neither.¹

a. $u_n = 4 + 3n$

b. $u_n = 4^n$

c. $u_n = n \times 3^n$

Find the limit

In each of the following cases, state whether the sequence $\{u_n\}$ tends to a limit, and find the limit if it exists:²

a. $u_n = 1 + \frac{1}{2}n$

b. $u_n = \left(\frac{1}{2}\right)^n$

c. $\lim_{x \rightarrow -4} \frac{x^2 + 5x + 4}{x^2 + 3x - 4}$

¹Pemberton and Rau 5.1.1

²Pemberton and Rau 5.1.3

Determine convergence or divergence

Determine whether each of the following sequences converges or diverges. If it converges, find the limit.³

a. $a_n = \frac{3+5n^2}{n+n^2}$

b. $a_n = \frac{(-1)^{n-1}n}{n^2+1}$

Find more limits

Given that

$$\lim_{x \rightarrow a} f(x) = -3, \quad \lim_{x \rightarrow a} g(x) = 0, \quad \lim_{x \rightarrow a} h(x) = 8$$

find the limits that exist. If the limit doesn't exist, explain why.⁴

a. $\lim_{x \rightarrow a} [f(x) + h(x)]$

b. $\lim_{x \rightarrow a} \frac{f(x)}{g(x)}$

c. $\lim_{x \rightarrow a} \frac{2f(x)}{h(x)-f(x)}$

Check for discontinuities

Which of the following functions are continuous? If not, where are the discontinuities?⁵

a. $f(x) = \frac{9x^3 - x}{(x-1)(x+1)}$

b. $f(x) = e^{-x^2}$

³Grimmer 2012 HW2.2

⁴Grimmer 2012 HW 2.4

⁵Gill 1.9

Find finite limits

Find the following finite limits:⁶

a. $\lim_{x \rightarrow 1} \left[\frac{x^4 - 1}{x - 1} \right]$

b. $\lim_{x \rightarrow -4} \left[\frac{x^2 + 5x + 4}{x^2 + 3x - 4} \right]$

Find infinite limits

Find the following infinite limits:⁷

Hint: use **L'Hôpital's Rule** to switch from $\lim_{x \rightarrow \infty} \left(\frac{f(x)}{g(x)} \right)$ to $\lim_{x \rightarrow \infty} \left(\frac{f'(x)}{g'(x)} \right)$.

a. $\lim_{x \rightarrow \infty} \left[\frac{9x^2}{x^2 + 3} \right]$

b. $\lim_{x \rightarrow \infty} \left[\frac{3^x}{x^3} \right]$

Assessing continuity and differentiability

For each of the following functions, describe whether it is continuous and/or differentiable at the point of transition of its two formulas.⁸

a.

$$f(x) = \begin{cases} +x^2, & x \geq 0 \\ -x^2, & x < 0 \end{cases}$$

b.

$$f(x) = \begin{cases} x^3, & x \leq 1 \\ x, & x > 1 \end{cases}$$

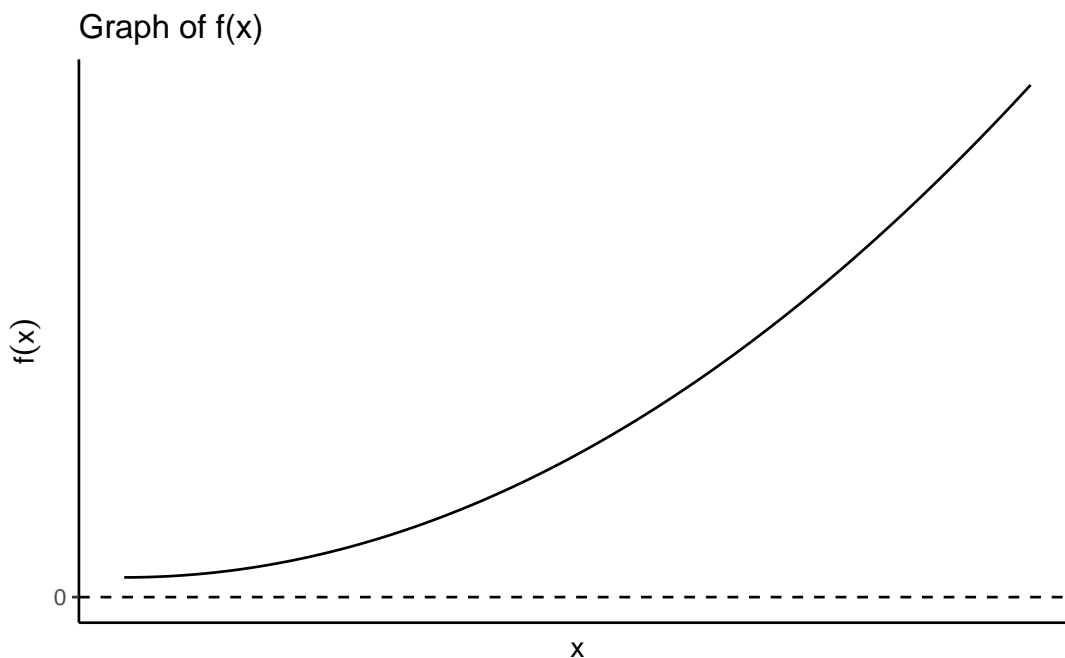
⁶a-d from Gill 5.1. e-h from Grimmer HW2.2

⁷Gill 5.3 and 5.8

⁸Simon and Blume 2.16

Possible derivative

A friend shows you this graph of a function $f(x)$:⁹



Which of the following could be a graph of $f'(x)$? For each graph, explain why or why not it might be the derivative of $f(x)$.

What if the figure below was the graph of $f(x)$? Which of the graphs might potentially be the derivative of $f(x)$ then?

Calculate derivatives

Differentiate the following functions:¹⁰

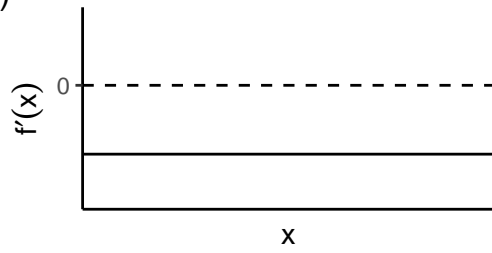
- a. $f(x) = 4x^3 + 2x^2 + 5x + 11$
- b. $y = \sqrt{30}$
- c. $h(t) = \log(9t + 1)$
- d. $f(x) = \log(x^2 e^x)$

⁹Grimmer HW3.6

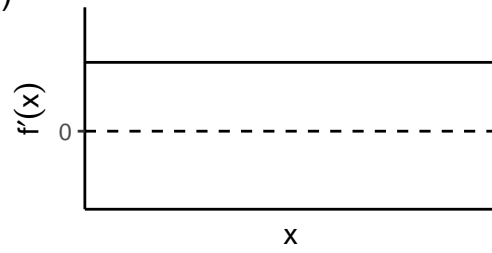
¹⁰Grimmer HW2.3

Potential derivatives

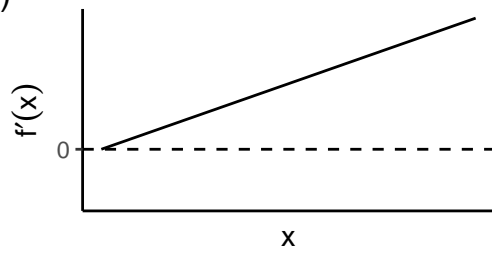
A)



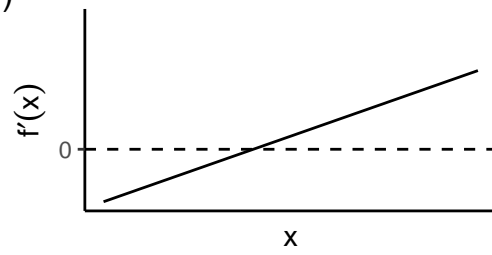
B)



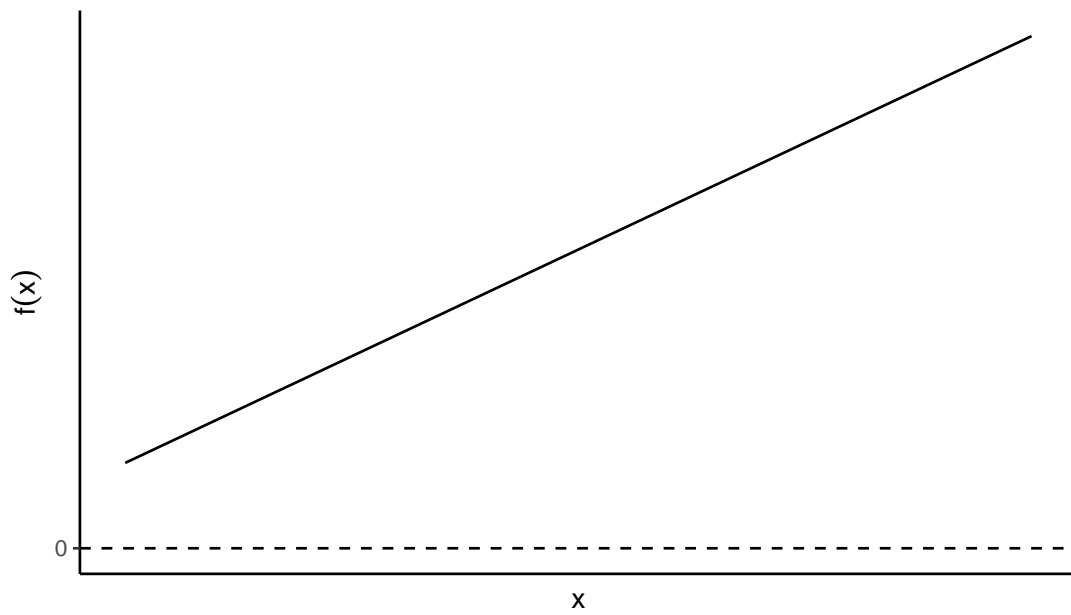
C)



D)



Graph of $f(x)$



e. $h(y) = \left(\frac{1}{y^2} - \frac{3}{y^4} \right) (y + 5y^3)$

f. $h(x) = \frac{x}{\log(x)}$

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Use the product and quotient rules

Differentiate the following function twice – once using the product rule, and once using the quotient rule:¹¹

$$f(x) = \frac{x^2 - 2x}{x^4 + 6}$$

Composite functions

For each of the following pairs of functions $g(x)$ and $h(z)$, write out the composite function $g(h[z])$ and $h(g[x])$. In each case, describe the domain of the composite function.¹²

a. $g(x) = x^2 + 4$, $h(z) = 5z - 1$

b. $g(x) = x^3$, $h(z) = (z - 1)(z + 1)$

Chain rule

Use the chain rule to compute the derivative of the composite functions in the previous section from the derivatives of the two component functions. Then, compute each derivative directly using your expression for the composite function. Simplify and compare your answers.¹³

a. $g(x) = x^2 + 4$, $h(z) = 5z - 1$

b. $g(x) = x^3$, $h(z) = (z - 1)(z + 1)$

c. $g(x) = 4x + 2$, $h(z) = \frac{1}{4}(z - 2)$

¹¹Grimmer HW2.4

¹²Simon and Blume 4.1

¹³Simon and Blume 4.3