

Probability

Computational Mathematics and Statistics Camp

University of Chicago

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1. If you flip a fair coin 10 times, what is the probability of
 - a. getting all tails?
 - b. getting all heads?
 - c. getting at least one tails?
2. If you roll a pair of fair dice, what is the probability of
 - a. getting a sum of 1?
 - b. getting a sum of 5?
 - c. getting a sum of 12?
3. In parts(a) and(b), identify whether the events are disjoint, independent, or neither (events cannot be both disjoint and independent).
 - a. You and a randomly selected student from your class both earn A's in this course.
 - b. You and your class study partner both earn A's in this course.
 - c. If two events can occur at the same time, must they be dependent?
4. Data collected at elementary schools in DeKalb County, GA suggest that each year roughly 25% of students miss exactly one day of school, 15% miss 2 days, and 28% miss 3 or more days due to sickness.
 - a. What is the probability that a student chosen at random doesn't miss any days of school due to sickness this year?
 - b. What is the probability that a student chosen at random misses no more than one day?
 - c. What is the probability that a student chosen at random misses at least one day?
 - d. If a parent has two kids at a DeKalb County elementary school, what is the probability that neither kid will miss any school? Note any assumption you must make to answer this question.
 - e. If a parent has two kids at a DeKalb County elementary school, what is the probability that both kids will miss some school, i.e. at least one day? Note any assumption you make.
 - f. If you made an assumption in part(d) or(e), do you think it was reasonable? If you didn't make any assumptions, double check your earlier answers.
5. $P(A) = 0.3$, $P(B) = 0.7$
 - a. Can you compute $P(A \text{ and } B)$ if you only know $P(A)$ and $P(B)$?
 - b. Assuming that events A and B arise from independent random processes,
 - a. what is $P(A \text{ and } B)$?
 - b. what is $P(A \text{ or } B)$?
 - c. what is $P(A|B)$?
 - c. If we are given that $P(A \text{ and } B) = 0.1$, are the random variables giving rise to events A and B independent?
 - d. If we are given that $P(A \text{ and } B) = 0.1$, what is $P(A|B)$?
6. Suppose 80% of people like peanut butter, 89% like jelly, and 78% like both. Given that a randomly sampled person likes peanut butter, what's the probability that he also likes jelly?

7. A 2010 SurveyUSA poll asked 500 Los Angeles residents, “What is the best hamburger place in Southern California? Five Guys Burgers? In-N-Out Burger? Fat Burger? Tommy’s Hamburgers? Umami Burger? Or somewhere else?” The distribution of responses by gender is shown below.

		<i>Gender</i>		Total
		Male	Female	
<i>Best hamburger place</i>	Five Guys Burgers	5	6	11
	In-N-Out Burger	162	181	343
	Fat Burger	10	12	22
	Tommy’s Hamburgers	27	27	54
	Umami Burger	5	1	6
	Other	26	20	46
	Not Sure	13	5	18
Total		248	252	500

- Are being female and liking Five Guys Burgers mutually exclusive?
 - What is the probability that a randomly chosen male likes In-N-Out the best?
 - What is the probability that a randomly chosen female likes In-N-Out the best?
 - What is the probability that a man and a woman who are dating both like In-N-Out the best?
Note any assumption you make and evaluate whether you think that assumption is reasonable.
 - What is the probability that a randomly chosen person likes Umami best or that person is female?
8. Lupus is a medical phenomenon where antibodies that are supposed to attack foreign cells to prevent infections instead see plasma proteins as foreign bodies, leading to a high risk of blood clotting. It is believed that 2% of the population suffer from this disease. The test is 98% accurate if a person actually has the disease. The test is 74% accurate if a person does not have the disease. There is a line from the Fox television show **House** that is often used after a patient tests positive for lupus: “It’s never lupus.” Do you think there is truth to this statement? Use appropriate probabilities to support your answer.
9. Suppose we pick three people at random. For each of the following questions, ignore the special case where someone might be born on February 29th, and assume that births are evenly distributed throughout the year.
- What is the probability that the first two people share a birthday?
 - What is the probability that at least two people share a birthday?
10. A portfolio’s value increases by 18% during a financial boom and by 9% during normal times. It decreases by 12% during a recession. What is the expected return on this portfolio if each scenario is equally likely?
11. Sally gets a cup of coffee and a muffin every day for breakfast from one of the many coffee shops in her neighborhood. She picks a coffee shop each morning at random and independently of previous days. The average price of a cup of coffee is \$1.40 with a standard deviation of 30¢(\$0.30), the average price of a muffin is \$2.50 with a standard deviation of 15¢, and the two prices are independent of each other.
- What is the mean and standard deviation of the amount she spends on breakfast daily?
 - What is the mean and standard deviation of the amount she spends on breakfast weekly (7 days)?