

PSET 8: Multivariate distributions

Note: all homework uploads should be as a PDF or image *and* have the questions identified. We'll be giving zero credit for submissions that don't follow this protocol as it adds considerable time to grading. Thank you!

- Name
- How long did this problem set take you?
- How difficult was this problem set? very easy 1 2 3 4 5 very challenging

1 Calculating conditional PDF

Let $f(x, y) = 5x^2y^2$ for $0 \leq x \leq y \leq 1$. Find $f(x|y)$.¹

2 Properties of a joint PDF

Continuous random variables X and Y have the following joint probability density function (PDF):²

$$f_{XY}(x, y) = \begin{cases} kx^3y^2 & \text{where } 0 < x, y < 6 \\ 0 & \text{otherwise} \end{cases}$$

Note: $0 < x, y < 6$ means that both x and y are between 0 and 6; it does not mean that x is greater than 0 and y is less than 6.

- a. Find k .

¹Inspired by Grimmer HW12.4

²Inspired by Grimmer HW12.1

b. Find the marginal PDF of X , $f_X(x)$.

c. Find the marginal PDF of Y , $f_Y(y)$.

d. Find $E[X]$.

e. Find $E[Y]$.

f. Find $Var(X)$.

g. Find $Var(Y)$.

h. Find $Cov(X, Y)$.

i. Are X and Y independent? Explain your reasoning using mathematical concepts from the course.

j. What is the PDF of X conditional on Y , $f_{X|Y}(x|y)$?

k. What is the PDF of Y conditional on X , $f_{Y|X}(y|x)$?

3 Properties of joint random variables³

Suppose the following:

- $E[D] = 8$
- $E[F] = 4$
- $E[DF] = 10$
- $Var(D) = 30$
- $Var(F) = 60$

a. What is $Cov(D, F)$?

b. What is the correlation between D and F ?

c. Suppose you multiplied F by 2 to generate a new variable, H . What is $Cov(D, H)$?

³Inspired by Grimmer HW12.3

d. What is $Cor(D, H)$? How does this compare to your answer to Part (b) of this question?

e. Suppose instead that $Var(D) = 40$. How would this change $Cor(D, F)$?

4 Continuous Bayes' theorem

Previously, we used Bayes' theorem to link the conditional probability of discrete events A given B to the probability of B given A . There is an analogous Bayes' theorem that relates the conditional densities of random variables X and θ (below) Prove the continuous Bayes' theorem.⁴

$$f(\theta | X) = \frac{f(X | \theta)f(\theta)}{\int f(X | \theta)f(\theta)d\theta}$$

⁴Inspired by Grimmer HW12.5

5 AI and Resources statement

- Please list (in detail) all resources you used for this assignment. If you worked with people, list them here as well. It is not enough to say that you used a resource for help, you need to be specific on the link and *how* it was helpful. W/R/T gen AI tools (including GPT, etc.) you cannot use them to do work on your behalf – you cannot put in any of the questions, etc. You can ask for help on logic / sample problems. If you do use GPT or other AI tools, you need to provide a link to your chat transcript. Any suspected academic integrity violations will be immediately reported.

5.1 (Optional – complete elsewhere) Submission of practice questions

Submit practice questions for the final exam here: <https://forms.gle/CPo9FMQgQRPePDfN7> Note that we need at least 10 people to submit before there's enough to circulate!