Computational Math Camp

Problem Sets

Benjamin Soltoff

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Contents

Overview

Contains problem sets for the 2019 Computational Math Camp.

Chapter 1

Linear equations, notation, sets, and functions

1.1 Simplify expressions

Simplify the following expressions as much as possible:

a. $(-x^4y^2)^2$
b. $9(3^0)$
c. $(2a^2)(4a^4)$
d. $\dfrac{x^4}{x^3}$
e. $(-2)^{7-4}$
f. $\left(\dfrac{1}{27b^3}\right)^{1/3}$
g. $y^7y^6y^5y^4$
h. $\dfrac{2a/7b}{11b/5a}$
i. $(z^2)^4$

1.2 Simplify a (more complex) expression

Simplify the following expression:

$$(a+b)^2 + (a-b)^2 + 2(a+b)(a-b) - 3a^2$$

1.3 Graph sketching

Let the functions $f(x)$ and $g(x)$ be defined for all $x \in \Re$ by

$$f(x) = \begin{cases} |x| & \text{if } x < 1 \\ 1 & \text{if } x > 1 \end{cases}, \quad g(x) = \begin{cases} x^2 & \text{if } x < 2 \\ 4 & \text{if } x \geq 2 \end{cases}$$

Sketch the graphs of:

1. $y = f(x)$
2. $y = g(x)$
3. $y = f(g(x))$
4. $y = g(f(x))$

1.4 Root finding

Find the roots (solutions) to the following quadratic equations.

Definition 1.1 (The quadratic formula)**.**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

a. $4x^2 - 1 = 17$
b. $9x^2 - 3x - 12 = 0$
c. $x^2 - 2x - 16 = 0$
d. $6x^2 - 6x - 6 = 0$
e. $5 + 11x = -3x^2$

1.5 Work with sets

Using the sets

$$\begin{aligned} A &= \{2, 3, 7, 9, 13\} \\ B &= \{x : 4 \leq x \leq 8 \text{ and } x \text{ is an integer}\} \\ C &= \{x : 2 < x < 25 \text{ and } x \text{ is prime}\} \\ D &= \{1, 4, 9, 16, 25, \ldots\} \end{aligned}$$

identify the following:

1. $A \cup B$
2. $(A \cup B) \cap C$
3. $C \cap D$