PSET 2: Sequence, Limits, Derivatives, Critical Points

2. a. $\log(x) - 2\log(y) + \log(z) = \log(x) - \log(y^2) + \log(z)$

$= \log\left(\frac{xz}{y^2}\right)$

b. $2\log(x) + \log(1) = \log(x^2) + \log(1) = \log(x^2)$

c. $\log(2x) - 2 = \log(2x) - \log(e^2) = \log\left(\frac{2x}{e^2}\right)$

3. a. $u_1 = 13,\ u_2 = 14,\ u_3 = 15 \longrightarrow$ Arithmetic

b. $u_1 = 3,\ u_2 = 18,\ u_3 = 81 \longrightarrow$ Neither

c. $u_1 = 2,\ u_2 = 4,\ u_3 = 8 \longrightarrow$ Geometric

4. a. $\lim_{n\to\infty} u_n = \lim_{n\to\infty} \left(1 + \frac{1}{12}n\right) = \infty$, does not have limit

b. $\lim_{n\to\infty} u_n = \lim_{n\to\infty} \left(\frac{1}{12}\right)^n = 0$

c. $\lim_{x\to 2} \frac{x^2-5x+4}{x^3-3x-4} = \frac{-2}{-2} = 1$

When $x=2$, $x^2-5x-4 = -2 \neq 0$, $x^3-3x-4 = -2 \neq 0$

5. a. Converge to 5

b. Converge to 0, as $a_n = \frac{(-1)^{n-1} n}{n^2+1} \rightarrow \frac{1}{n}$ as n increases

6. a. $\lim_{x\to a}[f(x)+h(x)] = \lim_{x\to a} f(x) + \lim_{x\to a} h(x) = -3 + 8 = 5$

b. $\lim_{x\to a} \frac{f(x)}{g(x)}$ does not have limit as $\lim_{x\to a} g(x) = 0$ and the denominator should not equal 0.

7. a. $\lim_{x\to\infty} \frac{9x^2}{x^2+3} = \lim_{x\to\infty} \frac{18x}{2x} = \lim_{x\to\infty} 9 = 9$

b. $\lim_{x\to\infty} \frac{3^x}{x^3} = \lim_{x\to\infty} \frac{3^x \cdot \ln 3}{3x^2} = \lim_{x\to\infty} \frac{3^x \cdot (\ln 3)^2}{6x}$

$= \lim_{x\to\infty} \frac{3^x \cdot (\ln 3)^3}{6} = \infty$

8. a. $\lim_{x\to 0^+} f(x) = x^2 \big|_{x=0} = 0$

$\lim_{x\to 0^-} f(x) = -x^2 \big|_{x=0} = 0$

They equal to each other

$\therefore$ Continuous.

$\frac{d}{dx}(x^2)\big|_{x\to 0^+} = 2x\big|_{x\to 0^+} = 0$

$\frac{d}{dx}(-x^2)\big|_{x\to 0^-} = -2x\big|_{x\to 0^-} = 0$

Equal

$\therefore$ Differentiable

b. $\lim_{x\to 1^+} f(x) = x^3 \big|_{x=1} = 1$

$\lim_{x\to 1^-} f(x) = x \big|_{x=1} = 1$

Equal

$\therefore$ Continuous

$\frac{d}{dx}(x^3)\big|_{x\to 1^+} = 3x^2\big|_{x\to 1^+} = 3$

$\frac{d}{dx}(x)\big|_{x\to 1^-} = 1\big|_{x\to 1^-} = 1$

Unequal

$\therefore$ Not differentiable

9. C.

B.

10. a. $f(x)=4x^3+2x^2+5x+11$

$f'(x)=12x^2+4x+5$

b. $y=\sqrt{30}$

$y'=0$

c. $h(t)=\log(9t+1)$

$h'(t)=\frac{9}{9t+1}$

d. $f(x)=\log(x^2e^x)=\log(x^2)+\log(e^x)=\log(x^2)+x$

$f'(x)=\frac{2x}{x^2}+1=\frac{2}{x}+1$

e. $h(y)=(\frac{1}{y^2}-\frac{3}{y^4})(y+5y^3)=\frac{1}{y}+5y-\frac{3}{y^3}-\frac{15}{y}$

$=5y-\frac{3}{y^3}-\frac{14}{y}=5y-3y^{-3}-14y^{-1}$

$h'(y)=5+9y^{-4}+14y^{-2}$

f. $h(x)=\frac{x}{\log(x)}$

$h'(x)=\frac{\log(x)-x\cdot\frac{1}{x}}{(\log(x))^2}=\frac{\log(x)-1}{(\log(x))^2}$

11. $f(x) = \frac{x^2-2x}{x^4+6}$

① Production Rule : $f'(x) = u'v + uv'$

where $u = x^2-2x$, $v = \frac{1}{x^4+6}$

$\therefore f'(x) = (2x-2)\frac{1}{x^4+6} + (x^2-2x) \cdot -\frac{4x^3}{(x^4+6)^2}$

$= \frac{2x-2}{x^4+6} - \frac{(x^2-2x)\cdot 4x^3}{(x^4+6)^2}$

$= \frac{-2x^5+6x^4+12x-12}{(x^4+6)^2}$

② Quotation Rule : $f'(x) = \left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$

$\therefore f'(x) = \frac{(2x-2)(x^4+6)-(x^2-2x)\cdot 4x^3}{(x^4+6)^2}$

$= \cdots$

12. $g(h(z)) = ((z-1)(z+1))^3 = (z^2-1)^3$ ~~$z\in$~~

$g(x)$ is for $[-1, \infty)$ when $z \in \mathbb{R}$

$h(g(x)) = (x^3-1)(x^3+1) = x^6-1$

$h(x)$ is for $[-1, \infty)$ when $x \in \mathbb{R}$.

13. a. $g(x)=x^3$. $h(z)=z^2-1$

$g(h(z))=(z^2-1)^3$

Chain Rule: $\frac{d}{dz}g(h(z))=g'(h(z))\cdot h'(z)$

$= 3(h(z))^2\cdot 2z$

$= 6z(z^2-1)^2$

Directly: $\frac{d}{dz}(z^2-1)^3 = 3(z^2-1)^2\cdot 2z$

$= 6z(z^2-1)^2$

all real z.

$h(g(x))=x^6-1$

Chain Rule: $\frac{d}{dx}h(g(x)) = h'(g(x))\cdot g'(x)$

$= 2g(x)\cdot 3x^2$

$= 2x^3\cdot 3x^2 = 6x^5$

Directly: $\frac{d}{dx}(x^6-1) = 6x^5$

all real x

b. $g(x)=4x+2$. $h(z)=\frac{1}{4(z-2)}$

$g(h(z))=4\cdot\frac{1}{4(z-2)}+2=\frac{1}{z-2}+2$

Chain Rule: $g'(x)=4$. $h'(z)=-\frac{1}{4(z-2)^2}$

$\frac{d}{dz}g(h(z))=g'(h(z))\cdot h'(z)=4\cdot\left(-\frac{1}{4(z-2)^2}\right)=-\frac{1}{(z-2)^2}$

Directly: $\frac{d}{dz}\left(\frac{1}{z-2}+2\right)=-\frac{1}{(z-2)^2}$ $z\neq 2$.

$h(g(x))=\frac{1}{4((4x+2)-2)}=\frac{1}{16x}$

Chain Rule: $h'(z)=-\frac{1}{4(z-2)^2}$

$h'(g(x))=-\frac{1}{4(4x)^2}=-\frac{2}{64x}$, $g'(x)=4$

$\therefore \frac{d}{dx}h(g(x))=-\frac{1}{64x^2}\cdot 4=-\frac{1}{16x^2}$

Directly: $\frac{d}{dx}\left(\frac{1}{16x}\right)=-\frac{1}{16x^2}$

$x\neq 0$

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