

Problem Set 2 ANSWER KEY

1. Summation

(a) $\sum_{n=1}^7 3 \cdot 7 \cdot 3 = 21$

(b) $\sum_{n=0}^4 2n + 8 = \sum_{n=0}^4 2n + \sum_{n=0}^4 8 = \frac{2 \cdot 4(5)}{2} + 5 \cdot 8 = 60$ Note that you need to include when $n = 0$

2. Limits and Continuity

(a) What is a limit? A limit is when we focus on what will ultimately happen as a variable (usually x) approaches some pre-determined value.

(b) Why do we care? We care for three reasons: 1) it helps us determine if functions are continuous, 2) we can use limits to understand where a function is headed, even if it's not defined at a certain point, 3) we'll use them for derivatives in particular as we ask about the rate of change at a particular slice of the function, as the slice gets smaller and smaller and smaller.

(c) What is a continuous function? Informally, it is a function that can be drawn without picking a pencil up from the paper. More formally, it is where the function is defined at every point in the domain and where, for any point in that domain, where the limit from the left equals the limit from the right and these equal the value at that point.

3. Set theory

(a) In roster notation, write the set characterized in set-builder notation as $S = \{x \in \mathbb{Z}, 2 < x < 5\}$. $\{3, 4\}$

(b) Graph on the number line the interval $[-3, 2)$. Need open circle

(c) Is the following statement True or False? $\forall x \in S, x \geq 2$ for $S = \{3, 2, 5, 9\}$. T. All values are greater than or equal to 2.

(d) Is the following statement True or False? $\exists x \in S$ s.t. $x \notin \mathbb{Z}$ for $S = \{3, 2, 5, 9\}$. F. All numbers in the set are integers and thus no element is not a member of the set of integers.

(e) Is $\{1, 2, 3, 4\}$ a subset of $\{4, 3, 1, 2\}$? Is it a proper subset? It is a subset, but not proper subset because there are no elements in the second that are not elements of the first set.

(f) Using logical symbols (including \exists and \forall) write the definition of a proper subset. Took most answers as long as they looked 'mathy'. Wanted some version of $A \subset B \leftrightarrow x \in A \rightarrow x \in B \wedge \exists y \in B | y \notin A$

(g) If $A = \{\text{soup}, 8\}$ and $B = \{x, \text{soup}\}$ find $A \cup B$. $A \cup B = \{\text{soup}, 8, x\}$

(h) (Follow-up): Now find $A \cap B$. $A \cap B = \{\text{soup}\}$

(i) (Follow-up): Find the Cartesian Product $A \times B$. $A \times B = \{(\text{soup}, x), (\text{soup}, \text{soup}), (8, x), (8, \text{soup})\}$

4. Review:

(a) Write out '6 choose 3' mathematically and solve. $\binom{6}{3} = \frac{6!}{3!3!} = \frac{6*5*4}{3*2*1} = 20$

(b) Add these two matrices: $\begin{bmatrix} 2 & 4 & 2 \\ 1 & 4 & 0 \\ 2 & 6 & 0 \end{bmatrix} + \begin{bmatrix} 5 & 1 & 1 \\ 2 & 2 & 2 \\ 4 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 7 & 5 & 3 \\ 3 & 6 & 2 \\ 6 & 7 & 3 \end{bmatrix}$

(c) Multiply these two matrices: $\begin{bmatrix} 2 & 4 & 2 \\ 1 & 4 & 0 \\ 2 & 6 & 0 \end{bmatrix} * \begin{bmatrix} 5 & 1 & 1 \\ 2 & 2 & 2 \\ 4 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 26 & 12 & 16 \\ 13 & 9 & 9 \\ 22 & 14 & 14 \end{bmatrix}$

(d) List three things you got wrong on yesterday's assignment. What are they, what is the correct response and how will you address/fix this moving forward? Responses will vary.