

PSET 1: Linear equations, inequalities, sets and functions, quadratics

1 Simplify expressions

1. Simplify the following expressions as much as possible:¹

a. $(-x^2y^3)^3$

b. $9(3^0)$

c. $(3^2a^2)^2(6a^4)$

d. $\left(\frac{x^3}{x^4}\right)^3$

e. $(-2)^{(4-9)}$

f. $\left(\frac{1}{27b^4}\right)^{1/3}$

g. $y^5y^6y^5y^2$

h. $\frac{13a/7b}{13b/2a}$

2 Simplify a (more complex) expression

2. Simplify the following expression:²

$$(a+b)^2 + (a-b)^2 + 2(a+b)(a-b) - 3a^2$$

3 Graph sketching

3. Let the functions $f(x)$ and $g(x)$ be defined for all $x \in \mathfrak{R}$ by

$$f(x) = \begin{cases} |x-2| & \text{if } x < 1 \\ 1 & \text{if } x \geq 1 \end{cases}, \quad g(x) = \begin{cases} x^3 & \text{if } x < 2 \\ 4 & \text{if } x \geq 2 \end{cases}$$

Sketch the graphs of:³

1. $y = f(x)$

2. $y = g(x)$

3. $y = f(g(x))$

4. $y = g(f(x))$

¹inspired by Gill 1.1

²inspired by Gill 1.2

³inspired by Pemberton and Rau problem 3-1

4 Root finding

4. Find the roots (solutions) to the following quadratic equations.⁴

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- a. $9x^2 - 3x - 12 = 0$
- b. $x^2 - 2x - 16 = 0$
- c. $6x^2 - 6x - 6 = 0$

5 Systems of linear equations

5. Solve the following systems of equations for their unknown values. If there is no solution, indicate as such.

- a. Two unknowns⁵

$$\begin{aligned} 3x - 2y &= 17 \\ 5x - 10y &= -10 \end{aligned}$$

- b. Three unknowns⁶

$$\begin{aligned} 5x - 2y + 3z &= 9 \\ 2x - 4y - 3z &= -9 \\ x + 6y - 8z &= 24 \end{aligned}$$

- c. An animal shelter has a total of 124 animals comprised of cats, dogs, and rabbits. If the number of rabbits is 4 less than twice the number of dogs, and there are 76 more cats than dogs, how many of each animal are at the shelter?⁷

6 Work with sets

6. Using the sets

$$\begin{aligned} A &= \{2, 3, 7, 9, 12, 13\} \\ B &= \{x : 6 \leq x \leq 12 \text{ and } x \text{ is an even integer}\} \\ C &= \{x : 2 < x < 25 \text{ and } x \text{ is prime}\} \\ D &= \{1, 4, 9, 16, 25, \dots\} \end{aligned}$$

identify the following:⁸

- 1. $A \cup B$
- 2. $(A \cup B) \cap C$
- 3. $C \cap D$

⁴Gill 1.25

⁵inspired by OpenStax Algebra ex 7.1.12

⁶inspired by OpenStax Algebra ex 7.2.12

⁷inspired by OpenStax Algebra 7.2.54

⁸inspired by Grimmer HW1.1

6.1 AI and Resources statement

- Please list (in detail) all resources you used for this assignment. If you worked with people, list them here as well. It is not enough to say that you used a resource for help, you need to be specific on the link and *how* it was helpful. W/R/T gen AI tools (including GPT, etc.) you cannot use them to do work on your behalf – you cannot put in any of the questions, etc. You can ask for help on logic / sample problems. If you do use GPT or other AI tools, you need to provide a link to your chat transcript. Any suspected academic integrity violations will be immediately reported.