Linear equations, inequalities, sets and functions, quadratics

Simplify expressions

1. Simplify the following expressions as much as possible:[1]

a. $(-x^4y^2)^2$

b. $9(3^0)$

c. $(2a^2)(4a^4)$

d. $\frac{x^4{}^3}{x^3}$

e. $(-2)^{4-7}$

f. $\left(\frac{1}{27b^3}\right)^{1/3}$

g. $y^7y^6y^5y^4$

h. $\frac{2a/7b}{11b/5a}$

i. $(z^2)^4$

Simplify a (more complex) expression

2. Simplify the following expression:[2]

$$(a+b)^2 + (a-b)^2 + 2(a+b)(a-b) - 3a^2$$

Graph sketching

3. Let the functions $f(x)$ and $g(x)$ be defined for all $x \in \Re$ by

$$f(x) = \begin{cases} |x| & \text{if } x < 1 \\ 1 & \text{if } x \geq 1 \end{cases}, \quad g(x) = \begin{cases} x^2 & \text{if } x < 2 \\ 4 & \text{if } x \geq 2 \end{cases}$$

1. Sketch the graphs of:[3]
2. $y = f(x)$
3. $y = g(x)$
4. $y = f(g(x))$
5. $y = g(f(x))$

[1] Gill 1.1
[2] Gill 1.2
[3] Pemberton and Rau problem 3-1

Root finding

4. Find the roots (solutions) to the following quadratic equations.[4]

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

a. $9x^2 - 3x - 12 = 0$

b. $x^2 - 2x - 16 = 0$

c. $6x^2 - 6x - 6 = 0$

Systems of linear equations

5. Solve the following systems of equations for their unknown values. If there is no solution, indicate as such.

a. Two unknowns[5]

$$\begin{aligned} 3x - 2y &= 18 \\ 5x + 10y &= -10 \end{aligned}$$

b. Three unknowns[6]

$$\begin{aligned} 5x - 2y + 3z &= 20 \\ 2x - 4y - 3z &= -9 \\ x + 6y - 8z &= 21 \end{aligned}$$

c. An animal shelter has a total of 350 animals comprised of cats, dogs, and rabbits. If the number of rabbits is 5 less than one-half the number of cats, and there are 20 more cats than dogs, how many of each animal are at the shelter?[7]

Work with sets

6. Using the sets

$$\begin{aligned} A &= \{2, 3, 7, 9, 13, 16\} \\ B &= \{x : 4 \leq x \leq 8 \text{ and } x \text{ is an integer}\} \\ C &= \{x : 2 < x < 25 \text{ and } x \text{ is prime}\} \\ D &= \{1, 4, 9, 16, 25, \ldots\} \end{aligned}$$

identify the following:[8]

1. $A \cup B$
2. $(A \cup B) \cap C$
3. $C \cap D$

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[4] Gill 1.25
[5] OpenStax Algebra ex 7.1.12
[6] OpenStax Algebra ex 7.2.12
[7] OpenStax Algebra 7.2.54
[8] Grimmer HW1.1