

PSET 6: Discrete random variables

SURVEY!! Please fill this out: <https://forms.gle/Kn4iPTpUbfqymErG6>

Note: all homework uploads should be as a PDF *and* have the questions identified. We'll be giving zero credit for submissions that don't follow this protocol as it adds considerable time to grading. Thank you!

- Name
- How long did this problem set take you?
- How difficult was this problem set? very easy 1 2 3 4 5 very challenging

1 Calculate probabilities in a sample space S

Events A and B are contained within a sample space S . Given that $\Pr(A) = 0.65$, $\Pr(B) = 0.3$ and $\Pr(A \cap B) = 0.1$, find:¹

a. $\Pr(A \cup B)$

b. $\Pr(A \cap B^c)$

c. $\Pr[(A \cap B^c) \cup (B \cap A^c)]$

2 Random variables

We have been discussing random variables. Provide the following explanations in your own words:

- a. What is a random variable?

¹Inspired by Grimmer HW 8.4

- What is the difference between upper case and lower case x ? How (if at all) do they matter?
- If I'm thinking about something like $p_X(x_0)$, what do all the parts/pieces mean?

3 Survey Says

A survey has 54% respondents 50 or older and 46% respondents under 50. Within the survey, on a particular question, 9.5% of the 50-plus population agrees strongly while 2.7% of under 50 respondents agree strongly.

1. What is the probability someone selected at random is 50 or older?
2. The selected individual strongly agrees with the survey question. Now what is the likelihood that person is 50 or older? Explain your reasoning and **SHOW ALL YOUR WORK**
3. Are the two answers above the same or different? Explain.
4. (for fun, no points) What is the survey question?

4 PMF vs CMF

Consider the following function: $f(x) = \frac{1}{6}$. Find the pmf and cmf of the function and provide them in a table below.

5 Getting a traffic ticket

You drive to work 5 days a week for a full year (50 weeks), and with probability $p = 0.04$ you get a traffic ticket on any given day, independent of other days. Let X be the total number of tickets you get in the year.²

- a. What is the probability that the number of tickets you get is exactly equal to the expected value of X ?

²Inspired by BT 2.41

- a. Calculate approximately the probability in (a) using a Poisson approximation.

6 Obtaining requests for information

X is a discrete random variable. It takes the value of the number of days required for a governmental agency to respond to a request for information. X is distributed according to the following PMF:³

$$f(x) = e^{-6} \frac{6^x}{x!} \text{ for } X \in \{0, 1, 2, \dots\}$$

- a. Given this information, what is the probability of a response from the agency in 5 days or less?

- b. What is the probability the agency response takes more than 10 but less than 13 days?

- c. What is the probability the agency response takes more than 5 days?

³Inspired by Grimmer HW10.2

d. Suppose using X you generate a new variable, **Responsive**. **Responsive** equals 1 if an agency responds in 5 days or less and 0 otherwise. What is the expected value of **Responsive**?

e. What is the variance of **Responsive**?

7 Modeling electoral outcomes

Suppose we've developed a model predicting the outcome of the upcoming midterm elections in a state with 4 Congressional districts. In each district there are two candidates, a Republican and a Democrat. We have reason to believe the following PMF describes the distribution of potential election results where $K \in \{0, 1, 2, 3, 4\}$ and is the number of seats won by Republican candidates in the upcoming election.

$$\Pr(K = k|\theta) = \binom{4}{k} \theta^k (1 - \theta)^{4-k}$$

Based on polling information, we think the appropriate value for θ is 0.423.⁴

a. What's the expected number of seats Republicans will win in the upcoming election?

⁴Inspired by Grimmer HW10.3. Data from

genericpolling

(<https://www.realclearpolling.com/polls/state-of-the-union/generic-congressional-vote>)

- Given this PMF, what's the probability that no Republican legislators win in the upcoming election?
- What's the probability that Republican legislators win a majority of the seats in this state?
- A prominent political pundit declares they are certain that Republicans will win a majority of seats in the next election and offers the following bet. If Republicans win a majority of the seats, we must pay the pundit \$15.00. If Republican's fail to win a majority of states, we will win \$20.00. Based on our model, should we take this bet? **Hint: Think of the betting outcomes as a random variable. Find the expected value of this random variable.**
- Suppose we are offered a second bet with a more complicated structure. In this case we'll receive \$100 if the Republicans win a majority, \$50 if neither party wins a majority and we'll have to pay \$200 if the Democrats win a majority. Should we take this bet?

8 AI and Resources statement

Please list (in detail) all resources you used for this assignment. If you worked with people, list them here as well. It is not enough to say that you used a resource for help, you need to be specific on the link and *how* it was helpful. W/R/T gen AI tools (including GPT, etc.) you cannot use them to do work on your behalf – you cannot put in any of the questions, etc. You can ask for help on logic / sample problems. If you do use

GPT or other AI tools, you need to provide a link to your chat transcript. Any suspected academic integrity violations will be immediately reported.