Multivariate distributions

Calculating conditional PDF

Let $f(x, y) = 15x^2y$ for $0 \leq x \leq y \leq 1$. Find $f(x|y)$.[1]

Properties of a joint PDF

Continuous random variables X and Y have the following joint probability density function (PDF):[2]

$$f_{XY}(x,y) = \begin{cases} kx^2y^3 & \text{where } 0 < x, y < 6 \\ 0 & \text{otherwise} \end{cases}$$

Note: $0 < x, y < 6$ means that both x and y are between 0 and 6; it does not mean that x is greater than 0 and y is less than 6.

a. Find k.

b. Find the marginal PDF of X, $f_X(x)$.

[1] Grimmer HW12.4
[2] Grimmer HW12.1

c. Find the marginal PDF of Y, $f_Y(y)$.

d. Find E$[X]$.

e. Find E$[Y]$.

f. Find $Var(X)$.

a. Find $Var(Y)$.

b. Find $Cov(X, Y)$.

c. Are X and Y independent? Explain your reasoning using mathematical concepts from the course.

d. What is the PDF of X conditional on Y, $f_{X|Y}(x|y)$?

a. What is the PDF of Y conditional on X, $f_{Y|X}(y|x)$?

Properties of joint random variables[3]

Suppose the following:

- $E[D] = 10$
- $E[F] = 4$
- $E[DF] = 8$
- $Var(D) = 60$
- $Var(F) = 60$

a. What is $Cov(D, F)$?

b. What is the correlation between D and F?

c. Suppose you multiplied F by 2 to generate a new variable, H. What is $Cov(D, H)$?

[3] Grimmer HW12.3

d. What is $Cor(D, H)$? How does this compare to your answer to Part (b) of this question?

e. Suppose instead that $Var(D) = 30$. How would this change $Cor(D, F)$?

Continuous Bayes' theorem

Previously, we used Bayes' theorem to link the conditional probability of discrete events A given B to the probability of B given A. There is an analogous Bayes' theorem that relates the conditional densities of random variables X and θ (below) Prove the continuous Bayes' theorem.[4]

$$f(\theta \mid X) = \frac{f(X \mid \theta)f(\theta)}{\int f(X \mid \theta)f(\theta)d\theta}$$

Submission of practice questions

Submit practice questions for the final exam here: https://forms.gle/CPo9FMQgQRPePDfN7 Note that we need at least 10 people to submit before there's enough to circulate!

[4]Grimmer HW12.5