

## PSET 8: Multivariate distributions

**Note: all homework uploads should be as a PDF or image *and* have the questions identified.** We'll be giving zero credit for submissions that don't follow this protocol as it adds considerable time to grading. Thank you!

### 1 Background info (GRADED – complete all this section)

- Name
- How long did this problem set take you?
- How difficult was this problem set? very easy 1 2 3 4 5 very challenging

### 2 Calculating conditional PDF

Let  $f(x, y) = 5x^2y^2$  for  $0 \leq x \leq y \leq 1$ . Find  $f(x|y)$ .<sup>1</sup>

### 3 Properties of a joint PDF

Continuous random variables  $X$  and  $Y$  have the following joint probability density function (PDF):<sup>2</sup>

$$f_{XY}(x, y) = \begin{cases} kx^3y^2 & \text{where } 0 < x, y < 6 \\ 0 & \text{otherwise} \end{cases}$$

Note:  $0 < x, y < 6$  means that both  $x$  and  $y$  are between 0 and 6; it does not mean that  $x$  is greater than 0 and  $y$  is less than 6.

- a. Find  $k$ .

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<sup>1</sup>Inspired by Grimmer HW12.4

<sup>2</sup>Inspired by Grimmer HW12.1

b. Find the marginal PDF of  $X$ ,  $f_X(x)$ .

c. Find the marginal PDF of  $Y$ ,  $f_Y(y)$ .

d. Find  $E[X]$ .

e. Find  $E[Y]$ .

f. Find  $Var(X)$ .

g. Find  $Var(Y)$ .

h. Find  $Cov(X, Y)$ .

i. Are  $X$  and  $Y$  independent? Explain your reasoning using mathematical concepts from the course.

j. What is the PDF of  $X$  conditional on  $Y$ ,  $f_{X|Y}(x|y)$ ?

k. What is the PDF of  $Y$  conditional on  $X$ ,  $f_{Y|X}(y|x)$ ?

## 4 Properties of joint random variables<sup>3</sup>

Suppose the following:

- $E[D] = 8$
- $E[F] = 4$
- $E[DF] = 10$
- $Var(D) = 30$
- $Var(F) = 60$

a. What is  $Cov(D, F)$ ?

b. What is the correlation between  $D$  and  $F$ ?

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<sup>3</sup>Inspired by Grimmer HW12.3

c. Suppose you multiplied  $F$  by 2 to generate a new variable,  $H$ . What is  $Cov(D, H)$ ?

d. What is  $Cor(D, H)$ ? How does this compare to your answer to Part (b) of this question?

e. Suppose instead that  $Var(D) = 40$ . How would this change  $Cor(D, F)$ ?

## 5 Continuous Bayes' theorem

Previously, we used Bayes' theorem to link the conditional probability of discrete events  $A$  given  $B$  to the probability of  $B$  given  $A$ . There is an analogous Bayes' theorem that relates the conditional densities of random variables  $X$  and  $\theta$  (below) Prove the continuous Bayes' theorem.<sup>4</sup>

$$f(\theta | X) = \frac{f(X | \theta)f(\theta)}{\int f(X | \theta)f(\theta)d\theta}$$

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<sup>4</sup>Inspired by Grimmer HW12.5

## 6 AI and Resources statement

- Please list (in detail) all resources you used for this assignment. If you worked with people, list them here as well. It is not enough to say that you used a resource for help, you need to be specific on the link and *how* it was helpful. W/R/T gen AI tools (including GPT, etc. ) you cannot use them to do work on your behalf – you cannot put in any of the questions, etc. You can ask for help on logic / sample problems. If you do use GPT or other AI tools, you need to provide a link to your chat transcript. Any suspected academic integrity violations will be immediately reported.

### 6.1 (Optional – complete elsewhere) Submission of practice questions

Submit practice questions for the final exam here: <https://forms.gle/CPo9FMQgQRPePDfN7> Note that we need at least 10 people to submit before there's enough to circulate!