Assignment 2: Sequences, Limits, Derivatives, and Critical Points

Assignment Qs

- Name
- How long did this problem set take you?
- How difficult was this problem set? very easy 1 2 3 4 5 very challenging

Simplify logarithms

Express each of the following as a single logarithm:[1]

a. $\log(x) - 2\log(y) + \log(z)$

b. $2\log(x) + log(1)$

c. $\log(2x) - 2$

Sequences

Write down the first three terms of each of the following sequences. In each case, state whether the sequence is an arithmetric progression, a geometric progression, or neither.[2]

a. $u_n = 12 + n$

b. $u_n = n \times 3^n$

c. $u_n = 2^n$

[1] Inspired by Grimmer HW1.3. Assume log is natural log unless specified otherwise.
[2] Pemberton and Rau 5.1.1

Find the limit

In each of the following cases, state whether the sequence $\{u_n\}$ tends to a limit, and find the limit if it exists:[3]

a. $u_n = 1 + \frac{1}{12}n$

b. $u_n = \left(\frac{1}{12}\right)^n$

c. $\lim_{x\to 2} \frac{x^2-5x+4}{x^3-3x-4}$

Determine convergence or divergence

Determine whether each of the following sequences converges or diverges. If it converges, find the limit.[4]

a. $a_n = \frac{3+5n^2}{n+n^2}$

b. $a_n = \frac{(-1)^{n-1}n}{n^2+1}$

Find more limits

Given that

$$\lim_{x\to a} f(x) = -3, \quad \lim_{x\to a} g(x) = 0, \quad \lim_{x\to a} h(x) = 8$$

find the limits that exist. If the limit doesn't exist, explain why.[5]

a. $\lim_{x\to a}[f(x) + h(x)]$

b. $\lim_{x\to a} \frac{f(x)}{g(x)}$

[3] Pemberton and Rau 5.1.3
[4] Grimmer 2012 HW2.2
[5] Grimmer 2012 HW 2.4

Find infinite limits

Find the following infinite limits:[6]

Hint: use **L'Hôpital's Rule** to switch from $\lim_{x\to\infty}\left(\frac{f(x)}{g(x)}\right)$ to $\lim_{x\to\infty}\left(\frac{f'(x)}{g'(x)}\right)$.

a. $\lim_{x\to\infty}\left[\frac{9x^2}{x^2+3}\right]$

b. $\lim_{x\to\infty}\left[\frac{3^x}{x^3}\right]$

Assessing continuity and differentiability

For each of the following functions, describe whether it is continuous and/or differentiable at the point of transition of its two formulas.[7]

a.

$$f(x)=\begin{cases}+x^2, & x\geq 0\\ -x^2, & x<0\end{cases}$$

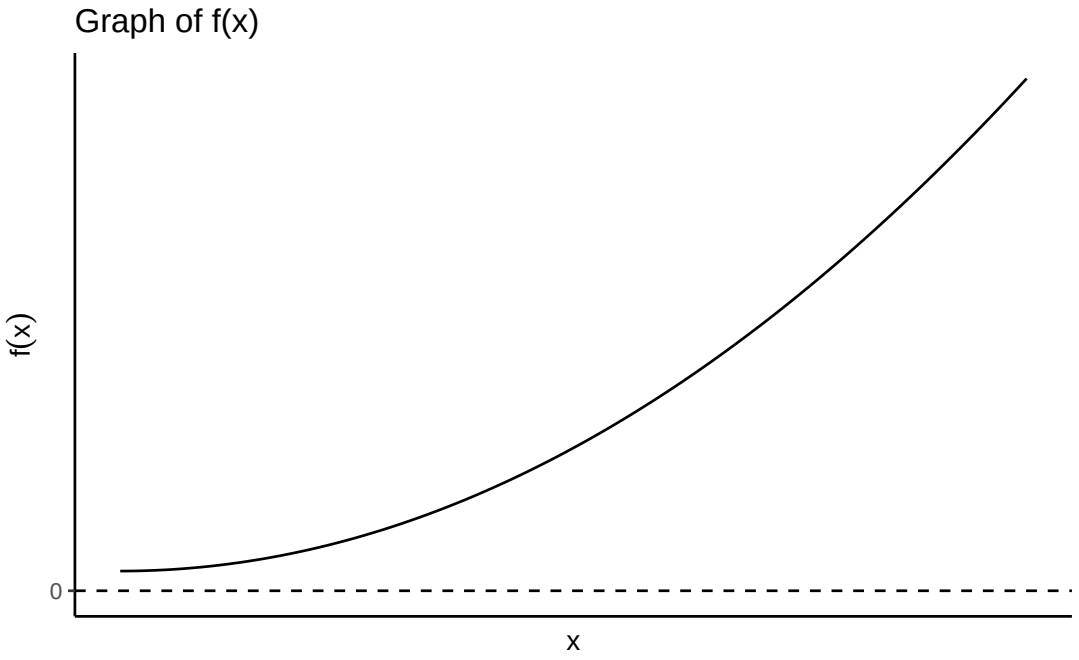
b.

$$f(x)=\begin{cases}x^3, & x\leq 1\\ x, & x>1\end{cases}$$

Possible derivative

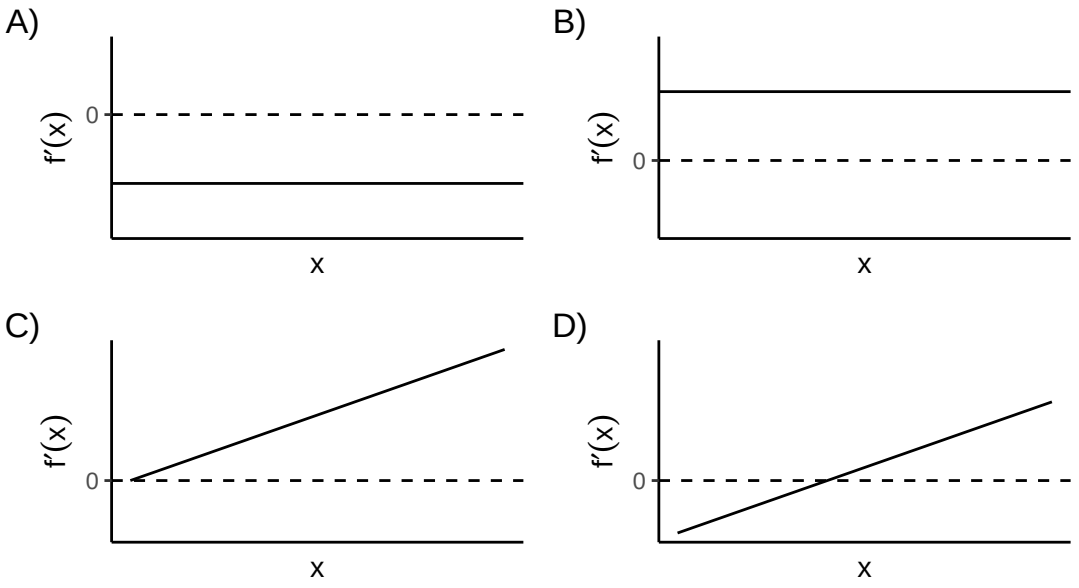
A friend shows you this graph of a function $f(x)$:[8]

[6] Gill 5.3 and 5.8
[7] Simon and Blume 2.16
[8] Grimmer HW3.6

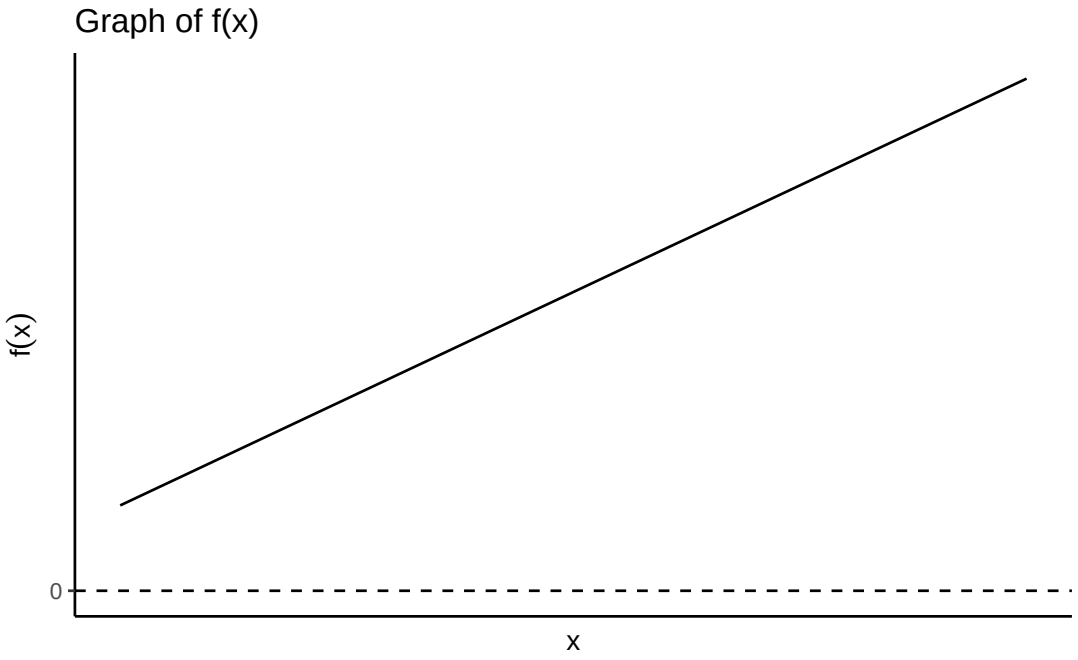


Which of the following could be a graph of $f'(x)$? For each graph, explain why or why not it might be the derivative of $f(x)$.

Potential derivatives



What if the figure below was the graph of $f(x)$? Which of the graphs might potentially be the derivative of $f(x)$ then?



Calculate derivatives

Differentiate the following functions:[9]

a. $f(x) = 4x^3 + 2x^2 + 5x + 11$

b. $y = \sqrt{30}$

c. $h(t) = \log(9t + 1)$

d. $f(x) = \log(x^2 e^x)$

e. $h(y) = \left(\frac{1}{y^2} - \frac{3}{y^4}\right)(y + 5y^3)$

f. $h(x) = \frac{x}{\log(x)}$

Use the product and quotient rules

Differentiate the following function twice – once using the product rule, and once using the quotient rule:[10]

[9] Grimmer HW2.3
[10] Grimmer HW2.4

$$f(x) = \frac{x^2 - 2x}{x^4 + 6}$$

Composite functions

For each of the following pairs of functions $g(x)$ and $h(z)$, write out the composite function $g(h[z])$ and $h(g[x])$. Describe the domain of the composite function.[11]

a. $g(x) = x^3, \quad h(z) = (z-1)(z+1)$

Chain rule

Use the chain rule to compute the derivative of the composite functions in the previous section from the derivatives of the two component functions. Then, compute each derivative directly using your expression for the composite function. Simplify and compare your answers.[12]

a. $g(x) = x^3, \quad h(z) = (z-1)(z+1)$

b. $g(x) = 4x + 2, \quad h(z) = \frac{1}{4}(z-2)$

AI / Resources statement

Please list (in detail) all resources you used for this assignment. If you worked with people, list them here as well. It is not enough to say that you used a resource for help, you need to be specific on the link and *how* it was helpful. W/R/T gen AI tools (including GPT, etc.) you cannot use them to do work on your behalf – you cannot put in any of the questions, etc. You can ask for help on logic / sample problems. If you do use GPT or other AI tools, you need to provide a link to your chat transcript. Any suspected academic integrity violations will be immediately reported.

[11] Simon and Blume 4.1
[12] Simon and Blume 4.3