HW 1 - Due 9/10 wed

Find Cartesian form

$\boxed{1.}$ $(-2+3i)^2 = \cancel{(-2+3i)(-2+3i)}$

$\cancel{(-2+3i)(-2+3i)}$

$\cancel{4-6i-6i-9i}$

$\cancel{4-21i}$

$\cancel{(-2)(-}$

$(-2)(-2)+(-2)(3i)+(3i)(-2)+(3i)(3i)$

$4-6i-6i+9i^2$

$4-12i-9$

$[-12i-5]$ $\quad [-5-12i]$

$\boxed{2.}$ $(-2+3i)^3$

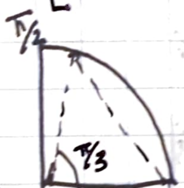
$(-5-12i)(-2+3i)$

$10-15i+24i-36i^2$

$10+9i+36$

$46+9i$ $\quad [46+9i]$

$\boxed{3}$ $e^{i\frac{\pi}{3}}$



$\cos(\pi/3)+i\sin(\pi/3)$

$\boxed{\frac{1}{2}+i\frac{\sqrt{3}}{2}}$

// Cartesian form - seperate real & imaginary parts.

$\boxed{4}$ $\cos(\pi/3)\cdot e^{i\pi/4}$

$\cos(\pi/3)\cdot\cos(\pi/4)+i\sin(\pi/4)$

$\cancel{\frac{1}{2}\cdot\frac{\sqrt{2}}{2}\cdot i\frac{\sqrt{2}}{2}}$ $\quad \frac{1}{2}\left(\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}\right)$

$\cancel{\frac{\sqrt{2}}{4}\cdot i\frac{\sqrt{2}}{2}}$

$\boxed{\frac{\sqrt{2}}{4}+i\frac{\sqrt{2}}{4} \quad // \quad \frac{\sqrt{2}}{4}(1+i)}$

$\boxed{5}$ $e^{i\frac{\pi}{3}} + e^{-i\pi/3}$ $\pi/3$

~~$\cos(\pi/3)$ $\sin($~~

~~$e^{\pi/3}$~~

~~$2\left(\cos(\pi/3) + \sin(\pi/3)\right)$~~

~~$2\left(\frac{1}{2} + \frac{\sqrt{3}}{2}\right) = 2\left(\frac{\sqrt{3}}{2}\right)$~~

$\left(\cos(\pi/3) \cancel{+i\sin(\pi/3)}\right) + \left(\cos(\pi/3) \cancel{- i\sin(\pi/3)}\right)$

$\left[\cos(\pi/3) + \cos(\pi/3) = \frac{1}{2} + \frac{1}{2} = 1\right]$

$\boxed{6}$

$e^{i\frac{3\pi}{4}}$ $\cos\left(\frac{3\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}\right)$

$\left[\frac{-\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right] \parallel \frac{\sqrt{2}}{2}(-1+i)$

$\boxed{7}$ $e^{i\frac{3\pi}{4}} + e^{-i\frac{\pi}{4}}$ $e^{-i\frac{\pi}{4}} = \cos\left(\frac{\pi}{4}\right) - i\sin(\pi/4)$

$\left[\frac{-\sqrt{2}}{2} + \frac{i\sqrt{2}}{2} + \frac{\sqrt{2}}{2} - \frac{i\sqrt{2}}{2} = 0\right]$

$\boxed{8}$ $\sum_{k=0}^{7} e^{ik\frac{\pi}{8}} + \sum_{n=1}^{8} e^{-in\frac{\pi}{8}}$ $\frac{1-r^n}{1-r}$

$\frac{1-\left(e^{ik\frac{\pi}{8}}\right)^8}{1-e^{ik\frac{\pi}{8}}}$ $e^{ik\frac{\pi}{8}}$ $\cos\left(\frac{k\pi}{8}\right) + i\sin\left(\frac{\pi}{8}\right)$

~~$1-\left(\sqrt{\sqrt{2}+2}/2\right) + i\left(\sqrt{2-\sqrt{2}}/2\right)$~~

$1-\left(e^{i\pi/8}\right)^8 = \frac{1-e^{i\pi}}{1-e^{i\pi/8}} = 1-\left(\cos(\pi) + i\sin(\pi)\right)$

dist. Negative

$1-(-1+i0) = \frac{2}{1-e^{i\pi/8}} = \frac{2}{1-\left(\cos(\pi/8) + i\sin(\pi/8)\right)}$

$\frac{2}{1-\cos(\pi/8) - i\sin(\pi/8)} \cdot \frac{1-\cos(\pi/8) + i\sin(\pi/8)}{1-\cos(\pi/8) + i\sin(\pi/8)}$

$\boxed{8}$ $\sum_{k=0}^{7} e^{ik\pi/8}$ $\frac{1-(e^{i\pi/8})^8}{1-e^{i\pi/8}} = \frac{1-e^{i\pi}}{1-e^{i\pi/8}}$

$$\frac{1-(\cos(\pi)+i\sin(\pi))}{1-(\cos(\pi/8)+i\sin(\pi/8))} = \frac{2}{1-(\cos(\pi/8)+i\sin(\pi/8))} \cdot \frac{1-\cos(\pi/8)+i\sin}{1-\cos(\pi/8)+i\sin}$$

$$\frac{2(1-\cos(\pi/8)+i\sin(\pi/8))}{1+2\cos(\pi/8)+\cos^2(\pi/8)+\sin^2(\pi/8)} = \frac{2(1-\cos(\pi/8)+i\sin(\pi/8))}{1+2\cos(\pi/8)+1}$$

$$\frac{2(1-\cos(\pi/8)+i\sin(\pi/8))}{2(1-\cos(\pi/8))} = \frac{1-\cos(\pi/8)}{1-\cos(\pi/8)} + \frac{i\sin(\pi/8)}{1-\cos(\pi/8)}$$

$$\left[1-\frac{i\sin(\pi/8)}{1-\cos(\pi/8)}\right]$$

$$\sum_{n=1}^{8} e^{-in\pi/8} \quad \frac{1-(e^{-i\pi/8})^8}{1-e^{-i\pi/8}} = \frac{1-e^{-i\pi}}{1-e^{-i\pi/8}} = \frac{1-\cos(\pi)-i\sin(\pi)}{1-\cos(\pi/8)-i\sin(\pi/8)}$$

$$\frac{1-(-1)-0}{1-\cos(\pi/8)-i\sin(\pi/8)} \cdot \frac{1-\cos(\pi/8)+i\sin(\pi/8)}{1-\cos(\pi/8)+i\sin(\pi/8)} = \frac{2(1-\cos(\pi/8)+i\sin(\pi/8))}{(1-\cos(\pi/8))^2+(\sin(\pi/8))^2}$$

$$\frac{2(1-\cos(\pi/8)+i\sin(\pi/8))}{1-2\cos(\pi/8)+\cos^2(\pi/8)+\sin^2(\pi/8)} = \frac{2(1-\cos(\pi/8)+i\sin(\pi/8))}{1-2\cos(\pi/8)+1}$$

$$\frac{2(1-\cos(\pi/8)+i\sin(\pi/8))}{2(1-\cos(\pi/8))} = \left[1+\frac{i\sin(\pi/8)}{1-\cos(\pi/8)}\right]$$

$$\sum_{k=0}^{7} e^{ik\pi/8} + \sum_{n=1}^{8} e^{-in\pi/8} = 1+\frac{i\sin(\pi/8)}{1-\cos(\pi/8)} + 1 + \frac{i\sin(\pi/8)}{1-\cos(\pi/8)}$$

$$\boxed{2\left(1+\frac{i\sin(\pi/8)}{1-\cos(\pi/8)}\right)}$$

$\boxed{9}$

$$\sum_{n}^{\infty} \frac{(i\pi/2)^n}{n!} \qquad e^{ix} =$$

$$e^{i\pi/2} = \cos(\pi/2) + i \sin(\pi/2)$$
$$= 0 + i(1) = i$$

$\boxed{10}$ $1 + i\frac{\pi}{2} + i(\pi/2)^2 + i(\pi/2)^3 + (i\pi/2)^4 + (i\pi/2)^5$

$$\sum_{n=0}^{5} e^{i\pi/2^n}$$

$$\frac{1-(i\pi/2)^6}{1-i\pi/2}$$

$$i^6 (\pi/2)^6$$
$$i^1 = i$$
$$i^2 = -1$$
$$i^3 = i(-1) = -i$$
$$i^4 = (-1)(-1) = 1$$
$$i^5 = \cancel{}\, i^4 \cdot i = 1 \cdot i = i$$
$$i^6 = i^4 \cdot i^2 = 1 \cdot (-1) = -1$$

$$\frac{1-(-\pi^6/64)}{1-i\pi/2} = \frac{1+\pi^6/64}{1-i\pi/2} \cdot \frac{1+i\pi/2}{1+i\pi/2}$$

$$\frac{(1+\pi^6/64)(1+i\pi/2)}{(1-i\pi/2)(1+i\pi/2)} = \left[\frac{(1+\pi^6/64)(1+i\pi/2)}{1+\pi^2/4}\right]$$

$$1 + i\,\cancel{\pi/2 - i\pi/2} - i\pi/2$$

$$\frac{1+\pi^6/64}{1+\pi^2/4}$$

$\boxed{11}$ $\frac{1}{2} + \frac{\sqrt{3}}{2} i \qquad r = \sqrt{(1/2)^2 + \sqrt{3}/2^2} \qquad \sqrt{1/4 + 3/4} = 1$

$$\tan^{-1}\left(\frac{\sqrt{3}/2}{1/2}\right) = \pi/3 = \boxed{e^{i\pi/3}}$$

$\boxed{12}$ $1 - \sqrt{3}\, i \qquad \sqrt{1^2 + \sqrt{3}^2} = \sqrt{4} = 2$

$$\tan^{-1}(-\sqrt{3}/1) = -\pi/3 \quad \boxed{2 \cdot e^{-i\pi/3}}$$

$\boxed{13}$ $2i(1+i)$

$$2i + 2i^2 = 2i - 2 = -2 + 2i$$

$$\sqrt{(-2)^2 + (2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

$$\tan^{-1}\left(2/-2\right) = -\pi/4,\ 3\pi/4$$

$$\boxed{2\sqrt{2}\, e^{i3\pi/4}}$$

$\boxed{14}$ $(1+i)^8$ $r = |1+i|$ $\sqrt{1^2+1^2} = \sqrt{2}$

$\left(\sqrt{2}\, e^{i\pi/4}\right)^8$ $\tan^{-1}(1/1) = \pi/4$ $1+i = \sqrt{2}\, e^{i\pi/4}$

$(\sqrt{2})^8 \cdot e^{i 8\pi/4} = 2^{8/2} \cdot e^{i 2\pi} = \boxed{16 \cdot 1}$

$\boxed{15}$ $(-1+i)^8$ $r = |-1+i|$ $\tan^{-1}(1/-1) = 3\pi/4$

$= \sqrt{2} e^{i 3\pi/4}$

$(\sqrt{2})^8 \cdot e^{i 24\pi/4} = 2^{8/2} \cdot e^{i 6\pi} = 2^4 \cdot e^{i \cdot 0} \boxed{= 16}$

$\boxed{16}$ $1 + e^{i\pi/2} + (e^{i\pi/2})^2$

$1 + e^{i\pi/2} + e^{i\pi}$

$1 + \left[\cos(\pi/2) + \sin(\pi/2)\right] + \left[\cos(\pi) + \sin(\pi)\right]$

$1 + [0 + i] + [-1 + 0] = 1 + i + (-1) \boxed{= i} \therefore \boxed{e^{i\pi/2}}$

$\boxed{17}$ $\cos(-\pi/3) + \sin(\pi/3) i$

$1/2 + \sqrt{3}/2\, i$ $\tan^{-1}\left(\frac{\sqrt{3}/2}{1/2}\right) = \pi/3$

$\sqrt{(1/2)^2 + (\sqrt{3}/2)^2} = 1$ $\boxed{e^{i\pi/3}}$

$\boxed{18}$ $\cos(-\pi/3) - \sin(\pi/3) i$

$1/2 - \sqrt{3}/2\, i$ $\tan^{-1}\left(\frac{\sqrt{3}/2}{1/2}\right) = -\pi/3$

$\boxed{e^{-i\pi/3}}$

$\boxed{19}$ $\left(\cos(\pi/3) + \sin(\pi/3) i\right)^3$ $\cos(\pi/3) + i \sin(\pi/3)$

$\theta = \pi/3$

$(e^{i\pi/3})^3 = \boxed{e^{i\pi}}$

$\boxed{20}$ $r = \pi/2$ $\boxed{e^{i\pi/2}}$