Mat 320 - Homework 2

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9/19

1 - Periodic? If true, give smallest period
- Give range f of each

$e^{i\theta}$: $P = 2\pi$ in θ

$e^{i(\theta+2\pi)} = e^{i\theta}$

$\sin\theta : P = 2\pi$ | $\cos\theta : P = 2\pi$

$e^{iat} : P = 2\pi/|a|$, $\theta \neq 0$

Periodic if exists some P: $f(t+P) = f(t)$ for all t

$f(t) = e^{i\omega t}$

$P = 2\pi/\omega$

(a) $f(t) = t + t^2 i$

$t(1+ti)$

$\cancel{f(t+P) = (t+P)(1+(t+P)i)}$

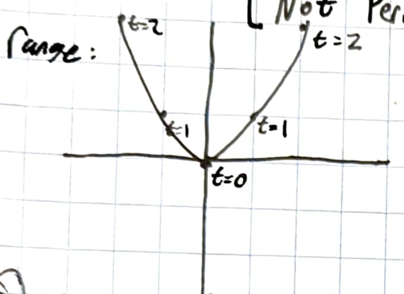
$(t+P) + (t+P)^2 i = t + t^2 i$

$(t+P) + (t^2 + 2tP + 2P) i = t + t^2 t$

$\cancel{t^2 + t + 2P}$

[Not Periodic]

Range:



$f(1) \to x=1 \quad y = \pm 1 i$

$f(2) \to x=2 \quad y = \pm 4 i$

(b) $f(t) = e^{i\pi/3 t}$ $\quad f(t+P) = e^{i\pi/3(t+P)} = e^{i\pi/3 t} \cdot e^{i\pi/3 P}$

$e^{i\pi/3 t} = e^{i\pi/3 t} \cdot e^{i\pi/3 P}$

$1 = e^{i\pi/3 P}$ $\quad 1 = \cos(\pi/3 P) + i\sin(\pi/3 P)$

$2\pi n = \pi/3 P$

[Periodic] - $P = 6$, $t = 1$

$n = \frac{1}{6} P$

$6n = P$

[Range:] $f(1) = e^{i\pi/3}$ $\quad f(2) = e^{i2\pi/3}$ $\quad f(3) = e^{i\pi}$

$f(6) = e^{i2\pi} = 0 = f(0)$

$\{z \in \mathbb{C} : |z| = 1\}$

(c) $f(t) = 3e^{i\pi/4 t}$ $\quad f(t+P) = 3(e^{i\pi/4 t} \cdot e^{i\pi/4 P})$ [Periodic]

$3e^{i\pi/4 t} = 3(e^{i\pi/4 t} \cdot e^{i\pi/4 P})$

$e^{i\pi/4 t} = e^{i\pi/4 t} \cdot e^{i\pi/4 P}$ $\quad 1 = e^{i\pi/4 P}$ $\quad \omega = \frac{2\pi}{\pi/4} = 8$

[Range]: $\{z \in \mathbb{C} : |z| = 3\}$

(d) $3e^{i(\pi/4 t + 4)} = f(t)$

$3(e^{i\pi/4 t} \cdot e^{i4}) = 3e^{i\pi/4(t+P)+4} = 3(e^{i\pi/4 t} \cdot e^{i\pi/4 P} \cdot e^{i4})$

$\cancel{e^{4i} = \cos(4) + i\sin(4)}$ $\quad 3e^4 \cdot e^{i\pi/4 t} = 3e^4(\cos(\pi/4 t) + i\sin(\pi/4 t))$

[Periodic] $\omega = \frac{2\pi}{\pi/4} = 8$

(d) $f(t) = te^{i\pi/3 t}$

$= t\cos(\pi/3 t) + it\sin(\pi/3 t)$

$t(\cos(\pi/3 t) + i\sin(\pi/3 t)) = (t+p)(\cos(\pi/3(t+p)) + i\sin(\pi/3(t+p)))$

$(t+p)[\cos(\pi/3 t)\cos(\pi/3 p) + i\sin(\pi/3 t)$

$(t+p)[\cos(\pi/3 t)\cos(\pi/3 p) - \sin(\pi/3 t)(\sin \pi/3 p)] +$

$[i\sin(\pi/3 t)\cos(\pi/3 p) + \cos(\pi/3 t)\sin(\pi/3 p)]$

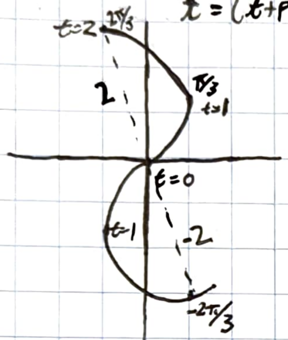
$\cancel{te^{\dots}}$ $(t+p)e^{i\pi/3(t+p)} = (t+p)e^{i\pi/3 t}\cdot e^{i\pi/3 p}$

$te^{i\pi/3 t} = (t+p)e^{i\pi/3 t} e^{i\pi/3 p}$

$t = (t+p)e^{i\pi/3 p}$

[Not Periodic]

Range:



$f(1) = 1\cdot e^{i\pi/3} = \cos(\pi/3) + i\sin(\pi/3$

$f(-1) = -e^{i\pi/3}$

$f(2) = 2e^{i2\pi/3}$

$f(-2) = -2e^{-i2\pi/3}$

(e) $f(t) = i\sin(\pi/3 t) = \cos(\pi/2) + i\sin(\pi/3 t)$

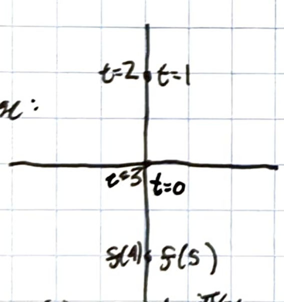
$f(0) = i\sin(0)$ $f(1) = i\sin(\pi/3)$

$f(2) = i\sin(2\pi/3)$ $f(3) = i\sin(\pi)$

$f(4) = i\sin(4\pi/3)$ $f(5) = i\sin(5\pi/3)$

[Periodic]

Range:



$p = 2\pi/\pi/3 = 6$

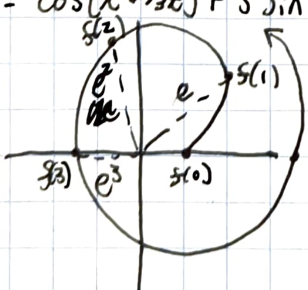
(g) $f(t) = e^{t+i\pi/3 t} = e^t\cdot e^{i\pi/3 t}$

$= \cos(t+\pi/3 t) + i\sin(t+\pi/3 t)$

$f(0) = 1 + 0i$ $f(1) = \cos(1+\pi/3) + i\sin$

$f(2) = 2\pi/3$

Range:

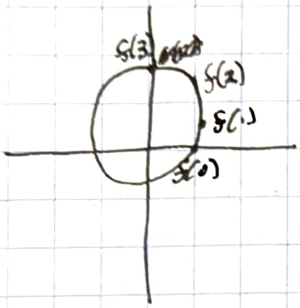


[Not Periodic]

(h) $f(t) = e^{i\pi/2 \sin(\pi/3 t)}$

$P = \cancel{\dfrac{2\pi}{\pi/2 \sin(\pi/8)} = \dfrac{2\pi}{\frac{\pi}{2}\cdot\frac{\sqrt{3}}{2}} = \dfrac{2\pi}{\frac{\sqrt{3}\pi}{4}}}$

Range:



$P(\sin \pi/3 t) = 6$ $\quad P = 8/\sqrt{3}$

$f(0) = e^{i\pi/2 \cdot 0}$ $\quad f(1) = e^{i\pi/2 \cdot \sqrt{3}/2} = e^{i\sqrt{3}\pi/4}$

$f(3) = e^{i\pi/2 \cdot 0} = \cancel{e^{i\pi/2}}\ e^0 = 1$

[Periodic]

// $\pi/2$ is a scalar, doesn't affect period.

$P = \dfrac{2\pi}{\pi/3} = 6$

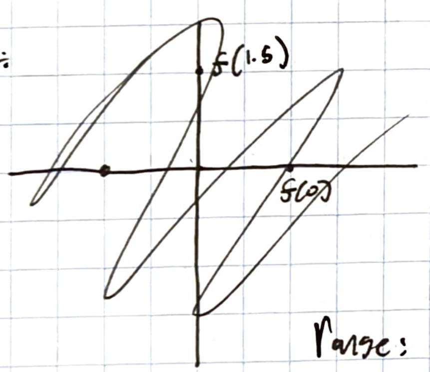
(i) $f(t) = \sin(\pi/3 t)\, e^{i\pi/3 t}$

$f(0) = 0$ $\quad f(1) = \sqrt{3}/2\, e^{i\pi/3}$

$f(2) = -\sqrt{3}/2\, e^{i2\pi/3}$

[periodic]

-~~Range:~~



$P(\sin(\pi/3 t)) = 6$ $\quad P(e^{i\pi/3 t}) = 6$

$P(\sin(\pi/3 t)\, e^{i\pi/3 t}) = 6$

Range: $\{\sin(\pi/3 t) \cdot e^{i\pi/3 t} : t \in \mathbb{R}\}$

$\boxed{2}$ Z_0 (Polar): rotates z by $\pi/6$ & Scales by 6

~~f~~ $f_{z_0}(z) = z_0 z$

Matrix A_z which performs same operation in $\mathbb{R}^2$

$z_0 = 6e^{i\pi/6}$ ~~f~~ $f_{z_0}(z) = 6(\cos(\pi/6\, z) + i\sin(\pi/6\, z))$

$$\begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}\begin{bmatrix} \cos(\pi/6) & -\sin(\pi/6) \\ \sin(\pi/6) & \cos(\pi/6) \end{bmatrix} = \begin{bmatrix} 3\sqrt{3} & -3 \\ 3 & 3\sqrt{3} \end{bmatrix}$$

$\boxed{3}$ w & $z \in \mathbb{C}$, ~~show~~ show linear dependence iff: $w \cdot \bar{z} \in \mathbb{R}$

Assume A is true, implying B, then assume B is true, implying B.

[Scalar Multiple

ⓐ Show: if w & z are linearly dependent, then $w\bar{z}$ is real

ⓑ Show: if $w\bar{z}$ is real, then w & z are linearly dependent

ⓐ w & $z \in \mathbb{R}$ are lin dependent if:

$w = rz,\ r \in \mathbb{R}$ || $w = \vec{0}$ || $z = \vec{0}$

$(rz)\bar{z} = r(z\bar{z}) = r|z|^2$ ✓

$r \in \mathbb{R},\ |z|^2 \in \mathbb{R}$

$[z \cdot \bar{z}]$

$(a+bi)(a-bi)$

$= a^2 - abi + abi - b^2 i^2$

$= a^2 - b^2(-1)$

$= a^2 + b^2 = |z|^2$

ⓑ $w\bar{z} \in \mathbb{R}$ in $\mathbb{C}$: $w = a+bi$ $z = c+di$, $\bar{z} = c - di$

$w\bar{z} = (a+bi)(c-di)$

$ac - adi + cbi - bdi^2$

$ac - adi + cbi + bd =$ ~~$(ac - ad + cb + bd)$~~

$= (ac - bd) + (cbi - adi)$ // cartesian form

$w\bar{z} = (ac - bd) + i(cb - ad)$

$i(cb - ad) = 0$

$\therefore cb - ad = 0$ ✓ (if $z = 0$) // if $z \neq 0$, c or/and d must $\neq 0$

$\therefore cb = ad$

// assume true: $w\bar{z} \in \mathbb{R}$ (no i part)

if $z \neq 0$ & c or d $\neq 0$: $a/c = b/d$

From ⓐ: $w = rz$

$w = a + bi$

$r = a/c = b/d$

$a = rc,\ b = rd \therefore w = rc + rdi = r(c + di)$

$= rz$ ✓