Test ID: TESTID Name: _____

MAT 320 **Quiz 3** Spring 2025

1. Consider the digital filter F_1: $y_t = x_t - \frac{1}{4}y_{t-1}$. What is the transfer function $\mathcal{H}(z)$ for this filter?

   a) $1 - \frac{1}{4z}$ b) $\frac{z}{z+\frac{1}{4}}$ c) $\frac{1}{1+\frac{1}{4}z}$ d) $\frac{z-\frac{1}{4}}{z}$ e) $1 - 4z$

   Correct Answer: $\frac{z}{z+\frac{1}{4}}$

2. Same filter F_1 as in the previous question. This filter has a pole at z equal to:

   a) $-\frac{1}{4}$ b) $\frac{1}{4}$ c) $\frac{1}{4}(1+i)$ d) $-\frac{1}{4}(1+i)$ e) 4

   Correct Answer: $-\frac{1}{4}$

3. Same filter F_1 as in the previous question. What is the frequency response $H(\omega)$ of this filter for $\omega = 0$?

   a) $\frac{3}{5}$ b) $\frac{4}{5}$ c) 1 d) $\frac{\sqrt{3}}{5}(1+i)$ e) $\frac{\sqrt{2}}{5}(1+i)$

   Correct Answer: $\frac{4}{5}$

4. Same filter F_1 as in the previous question. What is the magnitude response $|H(\omega)|$ of this filter for $\omega = \pi/2$?

   a) $\sqrt{\frac{14}{17}}$ b) $\sqrt{\frac{15}{17}}$ c) $\sqrt{\frac{13}{17}}$ d) $\sqrt{\frac{12}{17}}$ e) $\sqrt{\frac{16}{17}}$

   Correct Answer: $\sqrt{\frac{16}{17}}$

5. Same filter F_1 as in the previous question. Which frequency ω has the smallest frequency response?

   a) $5\pi/6$ b) $3\pi/4$ c) $\pi/3$ d) $\pi/2$ e) $2\pi/3$

   Correct Answer: $\pi/3$

6. Same filter F_1 as in the previous question. If the input $\mathbf{x}$ is the unit impulse signal: $(1, 0, 0, 0, \ldots)$ then what is the output value y_2? (Assume values with index less than zero are equal to 0.)

   a) $\frac{1}{4}$ b) $-\frac{1}{4}$ c) 1 d) $\frac{1}{8}$ e) $\frac{1}{16}$

   Correct Answer: $\frac{1}{16}$

7. A reson filter has two poles $Re^{i\theta}$ and $Re^{-i\theta}$. Assume that $\theta = \pi/4$ and $R = 1 - \epsilon$ for some small number ϵ. If the input $\mathbf{x}$ to this reson filter is the unit impulse signal: $(1, 0, 0, 0, \ldots)$ then what is the output value y_2? (Assume values with index less than zero are equal to 0.)

   a) R b) $\sqrt{2}R$ c) ϵ d) 2ϵ e) R^2

   Correct Answer: R^2

8. Same reson filter as in the previous question. What is the approximate half-power bandwidth B for this filter?

   a) R b) $\sqrt{2}R$ c) ϵ d) 2ϵ e) R^2

   Correct Answer: 2ϵ

9. Same reson filter as in the previous question. What is the maximum value of the magnitude response function for this filter (before normalization)?

   a) $1/R$ b) $\frac{\sqrt{2}}{1-R^2}$ c) $\frac{\sqrt{2}}{\epsilon^2}$ d) $\frac{2}{1-\epsilon}$ e) $\sqrt{2}(1-R^2)$

   Correct Answer: $\frac{\sqrt{2}}{1-R^2}$

10. Same reson filter as in the previous question. The value of ω where the magnitude response function obtains its true maximum output, we called ω_m. The cosine of ω_m has what value?

    a) $1/R$ b) $\frac{\sqrt{2}}{1-R^2}$ c) $\frac{1+R^2}{2\sqrt{2}R}$ d) $\frac{\sqrt{2}}{\epsilon^2}$ e) $\frac{2R}{1-\epsilon}$

    Correct Answer: $\frac{1+R^2}{2\sqrt{2}R}$