MAT 320 Homework 5
Fall 2025

Due date: Monday, Nov 17

You can use SciLab, or write a program to help in calculations, for any part of this homework.

Impulse reponse always refers to the output y_t of a filter given input $x_t = \delta_t = (1, 0, 0, \ldots)$. You may also assume that unless otherwise stated, the values of a signal at negative sample indices are always zero.

1. Define an inverse comb filter with equation $y_t = x_t - R^L x_{t-L}$ with $R = 0.999$ and $L = 6$.
   (a) Find the transfer function $\mathcal{H}(z)$.
   (b) Find the frequency response function $H(\omega)$.
   (c) Find the magnitude response function $|H(\omega)|$
   (d) Factor all of these functions. Find the maximum and minimum values of the magnitude response from $\omega = 0$ to $\omega = \pi$, and list them as points (a, b) with a in fractions of sample rate, and b in dB.
   (e) Sketch a graph of the magnitude response function with t axis in fractions of sample rate, and y axis in dB.
   (f) Find the phase response $\theta(\omega)$ for each of the values at which the magnitude response is maximal or minimal.

2. Follow the recipe on page 92, using sampling frequency 44,100 Hz, to design a reson filter with the specified bandwidth B and resonant frequency ψ. Write the filter equation and compute the first three values y_0, y_1, y_2 of the impulse response. Do all of this in two different ways: 1) with $B = 2\phi$, where $\cos\phi = 2 - \frac{1}{2}(R + \frac{1}{R})$ and 2) with $B = 2(1 - R)$. Note: Choose R to be less than 1.
   (a) $B = 10\text{Hz}$, $\psi = 1000\text{Hz}$
   (b) $B = 20\text{Hz}$, $\psi = 5000\text{Hz}$

3. Suppose a reson filter has impulse response given by:

$$y_t = \sqrt{2}(0.99^t)\sin(\frac{\pi}{4}(t+1)).$$

   What is the filter equation? (Hint: see formula 6.6)

4. Chapter 5, Problem 1 (Hint: think power series)

5. Chapter 5, Problem 3a,b (Note: by *peak frequency* the author means the frequency at which the magnitude response reaches its peak. This is the frequency labeled ψ on page 91. In many numerical computational algorithms and software implementations, this would be called “ArgMax”, which means the argument at which the function f achieves its maximum value. If there are mutliple such arguments, then usually the smallest positive one is returned. For example, if $f(x) = \sin(x)$, then $\text{ArgMax}(f) = \frac{\pi}{2}$.)