It's then easy to write out explicitly the real and imaginary parts of the numerator and denominator:

$$
\begin{aligned}
\mathcal{Real}\ \{\text{numerator}\} &= \cos((L+1)\omega) + \cos(L\omega) \\
\mathit{Imag}\ \{\text{numerator}\} &= \sin((L+1)\omega) + \sin(L\omega) \\
\mathcal{Real}\ \{\text{denominator}\} &= 2\cos((L+1)\omega) - R^L \cos\omega - R^L \\
\mathit{Imag}\ \{\text{denominator}\} &= 2\sin((L+1)\omega) - R^L \sin\omega
\end{aligned}
\tag{5.6}
$$

where $\mathcal{Real}$ and *Imag* denote the real and imaginary parts, respectively. I assigned temporary variables for these four components, the real and imaginary parts of the numerator and denominator. The magnitude response is the magnitude of this as a complex function, or

$$
|H(\omega)| = \left[\frac{[\mathcal{Real}\ \{\text{numerator}\}]^2 + [\mathit{Imag}\ \{\text{numerator}\}]^2}{[\mathcal{Real}\ \{\text{denominator}\}]^2 + [\mathit{Imag}\ \{\text{denominator}\}]^2} \right]^{1/2}
\tag{5.7}
$$

I then just evaluated this for ω on a grid in the range from 0 to π radians per sample.

Figure 5.1 shows the result when $L = 32$ and the coefficient $R = 0.999$. Since the round-trip delay of the feedback loop is 32.5 samples, we expect the resonances to occur at integer multiples of $f_s/32.5$, and these frequencies are marked by triangles on the graph.

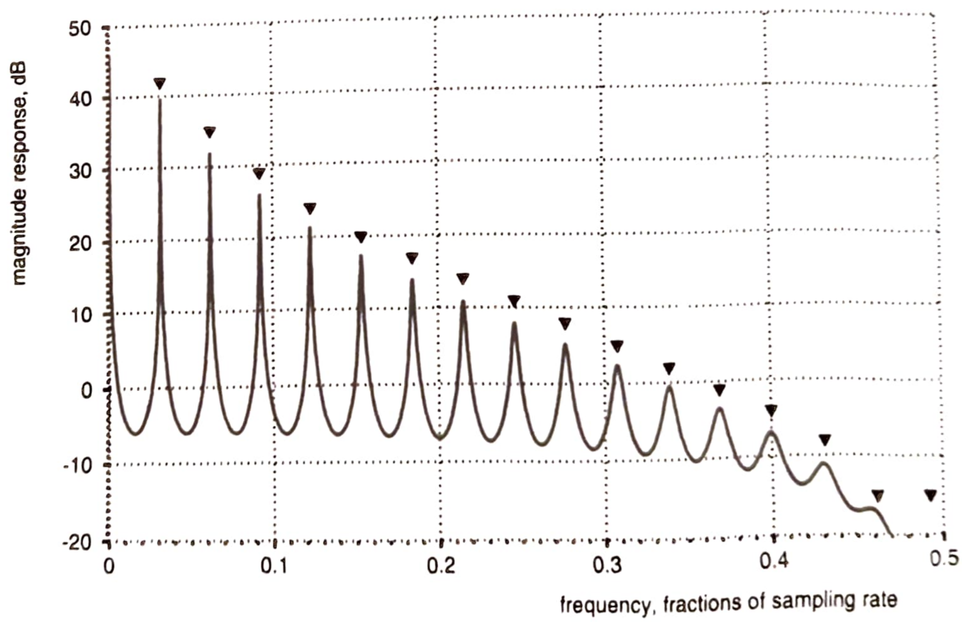

Fig. 5.1 Magnitude response of a plucked-string filter, for the case of a loop length $L = 32$ and pole radius $R = 0.999$. The expected resonant frequencies, integer multiples of $f_s/32.5$, are indicated by triangles.

The first interesting thing to notice in Fig. 5.1 is that the resonance peaks increase in width as frequency increases. This is exactly what we should expect, since the lowpass filter inserted in the loop causes higher frequencies to decay faster. Wider