

# MAT 320 Homework 3

## Fall 2025

Due date: Monday, September 29

You can use SciLab (or other math software) for any part of this homework.

1. Suppose a signal is sampled at the rate  $f_s = 44,100$ . In each part find the smallest positive frequency which is an alias of, but not equal to, the given frequency:
  - (a) 23000 Hz
  - (b) 45000 Hz
  - (c) 1000 Hz
  - (d) 96000 Hz
2. Find the signal to noise ratio SNR for a signal which is sampled with 10 bit values ( $B = 10$ ), assuming that the error is uniformly distributed between 0 and  $1/2$ . Now suppose a signal has values:  $-340, 223.45, 190.6, -48.2$ , which are in the range between  $-2^9$  and  $2^9 - 1$ . Find the RMS value for these four samples (take the average of the squares of the quantization errors, then take the square root).
3. Use a phasor sum to show that

$$2 \cos(20\pi t + \pi/3) + 3 \cos(20\pi t + \pi/4)$$

can be written as:  $A \cos(20\pi t + \phi)$ . Find the constants  $A$  and  $\phi$  as decimal approximations. Use your project or a calculator to compute the answers and any conversions from Cartesian to polar form. Show all work for each step.

4. Let  $B_4 = \{\mathbf{u}_0, \mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$  be the Fourier basis for dimension 4, where  $\mathbf{u}_k$  is the sampled phasor

$$e^{i \frac{2\pi}{4} kt}, \quad t = 0, 1, 2, 3.$$

Let  $A$  be the matrix of column vectors

$$A = (\mathbf{u}_0 \quad \mathbf{u}_1 \quad \mathbf{u}_2 \quad \mathbf{u}_3).$$

- (a) Find the determinant of  $A$ .
- (b) Verify that the determinant of  $A$  equals the product of backward differences from the second column, ie. the product

$$\prod_{0 \leq i < j \leq 3} z_j - z_i$$

where

$$\mathbf{u}_1 = \begin{pmatrix} z_0 \\ z_1 \\ z_2 \\ z_3 \end{pmatrix}.$$

Note: the product notation is like a sum, where we use capital sigma, but now here we use capital pi for product. The product is of all factors  $z_j - z_i$ , where the indexing is over all pairs  $i, j$  with  $i < j$  and  $i$  and  $j$  in the range from 0 to 3.

- (c) Find the inverse matrix  $A^{-1}$ .

- (d) Solve for the coefficients  $a_0, a_1, a_2, a_3$  in the vector equation:

$$\begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} = a_0 \mathbf{u}_0 + a_1 \mathbf{u}_1 + a_2 \mathbf{u}_2 + a_3 \mathbf{u}_3$$

using the inverse matrix.

- (e) Solve for the coefficients  $a_0, a_1, a_2, a_3$  using dot products. (Note: for the complex dot product

$$(c\mathbf{u}) \bullet \mathbf{v} = c(\mathbf{u} \bullet \mathbf{v})$$

however

$$\mathbf{u} \bullet (c\mathbf{v}) = \bar{c}(\mathbf{u} \bullet \mathbf{v}).$$

So, if you multiply both sides of the equation with the dot product by  $\mathbf{u}_i$  on the right, then you can solve for  $a_i$  by one division. But if you multiply with dot product on the left, then you first need to factor out  $\bar{a}_i$  from the dot product, then solve for  $\bar{a}_i$ , and finally take the conjugate to get  $a_i$ . Either way, you should get the same answer for  $a_i$ .)

- (f) Find the DFT of the vector

$$\mathbf{x} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}.$$

- (g) Use these coefficients to write  $\mathbf{x}$  in terms of the Fourier basis.