Q-Algs Quiz 1

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$\boxed{1}$ $e^{i\pi/2}(1+i) = ((\cos(\pi/2) + i\sin(\pi/2))(1+i)$

$\cos(\pi/2) + i\cos(\pi/2) + i\sin(\pi/2) + i^2\sin(\pi/2)$

$0 + i\cos(\pi/2) + i\sin(\pi/2) + i^2\sin(\pi/2)$

$i(\cos(\pi/2) + \sin(\pi/2) + i\sin(\pi/2))$

$i(0 + i + i\sin(\pi/2))$

$\cancel{e^{i\pi/2}}$ $(\cos(\pi/2) + i\sin(\pi/2)) = i$

$i(1+i) = i + i^2$

$|-1+i|$ $\sqrt{(-1)^2+(1)^2} = \sqrt{2}$ (a)

$\boxed{2}$ $A = XY$ $X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$ $Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$ $\begin{bmatrix} 0\cdot 0 + 1\cdot i & 0\cdot(-i) + 1\cdot 0 \\ 1(0) + 0\cdot 0 & 1(-i) + 0\cdot 0 \end{bmatrix} = \begin{bmatrix} i & 1 \\ 1 & -i \end{bmatrix}$

$\cancel{A^* = (\text{conj}).\begin{bmatrix} -i & 1 \\ 1 & i \end{bmatrix} \text{ trans. } \begin{bmatrix} -i & 1 \\ 1 & i \end{bmatrix}}$

$\begin{bmatrix} 0 \\ 1 \end{bmatrix}\begin{bmatrix} 0 \\ i \end{bmatrix} = i$ $\begin{bmatrix} 0 \\ 1 \end{bmatrix}\begin{bmatrix} -i \\ 0 \end{bmatrix} = 0$ $\begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}$

$\begin{bmatrix} 1 \\ 0 \end{bmatrix}\begin{bmatrix} 0 \\ i \end{bmatrix} = 0$ $\begin{bmatrix} 1 \\ 0 \end{bmatrix}\begin{bmatrix} -i \\ 0 \end{bmatrix} = -i$

$A^* = \begin{bmatrix} -i & 0 \\ 0 & i \end{bmatrix}$ (a)

$\boxed{3}$ $\begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}\begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}$ $\begin{bmatrix} i \\ 0 \end{bmatrix}\begin{bmatrix} i \\ 0 \end{bmatrix} = -1$ $\begin{bmatrix} i \\ 0 \end{bmatrix}\begin{bmatrix} 0 \\ -i \end{bmatrix} = 0$

$\begin{bmatrix} 0 \\ -i \end{bmatrix}\begin{bmatrix} i \\ 0 \end{bmatrix} = 0$ $\begin{bmatrix} 0 \\ -i \end{bmatrix}\begin{bmatrix} 0 \\ -i \end{bmatrix} =$

$i^2 = -1$

$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ $\cancel{A}$ e

$\boxed{4}$ $R_{\pi/4}$ $\begin{bmatrix} \cos(\pi/4) & -\sin(\pi/4) \\ \sin(\pi/4) & \cos(\pi/4) \end{bmatrix}^4$ $R = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$

$$\begin{bmatrix} \cos(\pi) & -\sin(\pi) \\ \sin(\pi) & \cos(\pi) \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = -I \quad d$$

$\boxed{5}$ XR^2

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} \cos(2\pi/4) & -\sin(2\pi/4) \\ \sin(2\pi/4) & \cos(2\pi/4) \end{bmatrix} \to \begin{bmatrix} \cos(\pi/2) & -\sin(\pi/2) \\ \sin(\pi/2) & \cos(\pi/2) \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 \\ 1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = 1 \begin{bmatrix} 0 \\ 1 \end{bmatrix}\begin{bmatrix} -1 \\ 0 \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = Z \qquad \begin{bmatrix} 1 \\ 0 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = 0 \begin{bmatrix} 1 \\ 0 \end{bmatrix}\begin{bmatrix} -1 \\ 0 \end{bmatrix} = -1$$

b

$\boxed{6}$ $A = HZH$ $\qquad Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ $H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

$$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \cdot \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} \cancel{\quad} \frac{1}{\sqrt{2}} \quad \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}\begin{bmatrix} 0 \\ -1 \end{bmatrix} = -1/\sqrt{2}$$

$$\begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = 1/\sqrt{2} \quad \begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix}\begin{bmatrix} 0 \\ -1 \end{bmatrix} = 1/\sqrt{2}$$

$$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \qquad \begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix}\begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix} = \cancel{\quad}$$

$$= \frac{1}{2} + \cancel{\quad} - \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} = \frac{1}{-\sqrt{2}^2}$$

$$\frac{1}{2} + (-1/2) 0$$

$\boxed{6}$ $\begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}\begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}$

$\begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix}\begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix} = \frac{1}{2} + (-\frac{1}{2}) = 0$ $\quad \begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix}\begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix} = \frac{1}{2} + \frac{1}{2} = 1$

$\begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}\begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix} = 1$ $\quad \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}\begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix} \frac{1}{2} + (-\frac{1}{2}) = 0$

$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = X \qquad X^* = X$ (a)

$\boxed{7}$ HXH

$\begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}$

$\begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}\begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}$ $\qquad \begin{bmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}\begin{bmatrix} +1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix} = -\frac{1}{2}$

$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = Z$ (d)

$\boxed{8}$ XYZ $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

$\begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix} = iI$ (c)

$\boxed{9}$ $\mathcal{H}_1 : \{|0\rangle, |1\rangle\}$ $X : \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$X\{2|0\rangle, -3|1\rangle\}$ $|0\rangle : \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $|1\rangle : \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\left(2\begin{bmatrix} 0 \\ 1 \end{bmatrix} - 3\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right)$$

$$\left(\begin{bmatrix} 0 \\ 2 \end{bmatrix} - \begin{bmatrix} 3 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} -3 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$$

$-3|0\rangle + 2|1\rangle$ b

$\boxed{10}$

$(|0\rangle + |1\rangle) \otimes (|0\rangle - |1\rangle)$ $\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) \otimes \left(\begin{bmatrix} 0 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \end{bmatrix}\right)$

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix} \otimes \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$|0\rangle \otimes |0\rangle + |0\rangle \otimes -|1\rangle + |1\rangle \otimes |0\rangle + |1\rangle \otimes -|1\rangle$

$|0\rangle \otimes |0\rangle - |0\rangle \otimes |1\rangle + |1\rangle \otimes |0\rangle - |1\rangle \otimes + |1\rangle$

$|00\rangle - |01\rangle + |10\rangle - |11\rangle$

$$\begin{array}{cc} \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix} & \begin{array}{c} 00 \\ 01 \\ 10 \\ 11 \end{array} \end{array}$$

e