10/22

Mat 320 - HW 04

$\boxed{1}$ $y_t = X_t + a_1 X_{t-1}$ Coeffiant of $a = 0.98$ & Delay of 1 sample ([illegible]

ⓐ Graph Magnitude response - freq. in fractions of sampling rate.

$Y_t = e^{j\omega t} |1 + a_1 e^{-j\omega}|$

$H(\omega) = |1 + a_1 e^{-j\omega}|$

$\mathcal{H}(z) = 1 + a_1 z^{-1}$

[Magnitude Response $(H(\omega))$

$|1 + a_1 e^{-j\omega}|$

[Transfer Function $(\mathcal{H}(\omega))$

$1 + a_1 z^{-1}$

$\mathcal{H}(z) =$ ~~a.e98~~ $1 + 0.98 z^{-1}$

$H(\omega) = 1 + 0.98(\cos(\omega) - j \sin(\omega))$

$= 1 + 0.98 \cos(\omega) - j 0.98 \sin(\omega)\}$ [Z-Plane Graphing]

$|H(\omega)| = \sqrt{(1 + 0.98\cos(\omega))^2 + (0.98\sin(\omega))^2}$

↳ Z-Transform:

$\vec{y} = \vec{x} + a_1 z^{-1} \vec{x}$

$= X(1 + a_1 z^{-1})$

$= \sqrt{1 + 0.98\cos(\omega) + 0.98\cos(\omega) + 0.9604\cos^2(\omega) + 0.9604\sin^2(\omega)}$

$= \sqrt{1 + 2(0.98\cos(\omega)) + 0.9604(\cos^2(\omega) + \sin^2(\omega))}$ | $\cos^2(\omega) + \sin^2(\omega) = 1$

~~44 1.92080cos(ω)~~ $+ 0.9604$

$\sqrt{1 + 1.96\cos(\omega) + 0.9604}$

$|H(\omega)| = (1.9604 + 1.96\cos(\omega))^{1/2}$

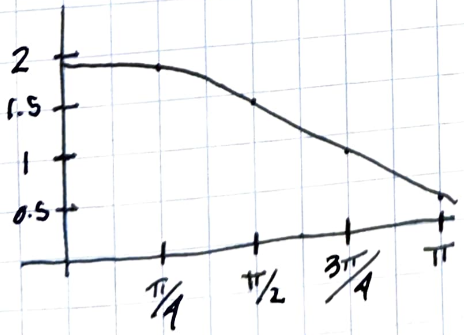
$|H(0)| = 1.98$

$|H(\pi/4)| = 1.83$

$|H(\pi/2)| = 1.4$

$|H(3\pi/4)| = 0.76$

$|H(\pi)| = 0.02$



$\boxed{1}$ ⓑ $H(z) = 1 + a_f z^{-1}$ $|H(\omega)| = (1.9604 + 1.96\cos(\omega))^{1/2}$

Solve for Min & Max values of magnitude & dB

$db = 20\log_{10}(|H(\omega)|)$

$X_t = e^{j\omega}$

- find interval $[0, \pi]$ values
  - $[0.02, 1.98]$ → Magnitude
  - $[-33.98, 5.94]$ dB

ⓒ → See Graph

$\boxed{2}$ $Y_t = X_t - \frac{3}{2}X_{t-1} - X_{t-2}$ $H(z) = 1 - \frac{3}{2}z^{-1} - z^{-2}$

$$Y_t = e^{j\omega t} - \frac{3}{2}e^{j\omega t-1} - e^{j\omega t-2}$$
$$= e^{j\omega t}\left(1 - \frac{3}{2}e^{-j\omega t} - e^{-2j\omega t}\right)$$

ⓐ Impulse Response → output when fed unit impulse $\delta_t \begin{cases} 1, & t=0 \\ 0, & \text{else} \end{cases}$

$t=0$ $X_t = 1$ $X_{t-1} = 0$ $X_{t-2} = 0$

$Y_{t_0} = 1 - \frac{3}{2}(0) - 0$

$\{1, -\frac{3}{2}, -1, 0, \dots\}$

$t=1$ $X_t = 0$ $X_{t-1} = 1$ $X_{t-2} = 0$

$Y_{t_1} = 0 - \frac{3}{2} \cdot 0 = -\frac{3}{2}$

$t=2$ $X_t = 0$ $X_{t-1} = 0$ $X_{t-2} = 1$

$Y_{t_2} = 0 - 0 - 1 = -1$

$t=3$

$Y_{t_3} = 0$

ⓑ $H(z) = 1 - \frac{3}{2}z^{-1} - z^{-2}$

ⓒ Factor $H(z)$

$H_1(z) = \frac{z-2}{2z}$ $H_2(z) = \frac{2z+1}{2z} =$

ⓒ $H(z) = 1 - \frac{3}{2} z^{-1} - z^{-2} \cdot \frac{z^2}{z^2}$

$= \frac{z^2 - \frac{3}{2} z - 1}{z^2} = \frac{(z-2)(z+\frac{1}{2})}{z^2}$ = ~~$(1-2z^{-2})(1+z^{-2}$~~

$= \frac{z-2}{z} \cdot \frac{z+\frac{1}{2}}{z} = (1-2z^{-1})(1+\frac{1}{2}z^{-1})$

$H_1(z) = 1 - 2z^{-1}$ $H_2(z) = 1 + \frac{1}{2} z^{-1}$

ⓓ $H_1(z) = e^{j\omega t}(1 - 2e^{-j\omega}) = e^{j\omega t} - 2e^{j\omega t - 1}$

$Y_{1t} = X_t - 2X_{t-1}$

$H_2(z) = e^{j\omega t}(1 + \frac{1}{2} e^{-j\omega})$

$= e^{j\omega t} + \frac{1}{2} e^{j\omega t - 1}$

$Y_{2t} = X_t + \frac{1}{2} X_{t-1}$

ⓔ Cascade → Combine filters with $H_1(z)$ & $H_2(z)$ X

~~$H_1(z) H_2(z) = (1-2z^{-1})(1+\frac{1}{2}z^{-1})$~~

Get impulse response of each $\vec{\delta}$

$H_1(z)$ $H_2(z)$

$t=0$

$Y_t = 1 - 2 \cdot 0 = 1$ $1 + \frac{1}{2}(0) = 1$

$t=1$

$Y_t = 0 - 2 = -2$ $0 + \frac{1}{2} = \frac{1}{2}$

$> t=2$ 0 0

$\{1, -2, 0\} \cdot \{1, \frac{1}{2}, 0\} =$ ~~$\{1 \cdot 1, -2 \cdot \frac{1}{2}, 0 \cdot 0\}$~~

Convolution :

$$\begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ \frac{1}{2} \\ 0 \end{bmatrix}$$

$$\begin{array}{c|ccc} & 1 & \frac{1}{2} & 0 \\ 1 & & & \\ -2 & & & \\ 0 & & & \end{array} \qquad \begin{array}{c|ccc} & 1 & \frac{1}{2} & 0 \\ -2 & & & \\ 0 & & & \end{array}$$

$$\begin{array}{c|ccc} & 1 & \frac{1}{2} & 0 \\ \hline 1 & 1 & \frac{1}{2} & 0 \\ -2 & -2 & -1 & \\ 0 & & & \end{array} \; H_2 \quad \times \rightarrow [1, -\frac{3}{2}, -1]$$

H_1

$\boxed{3}$ ⓓ $H(z) = 1 - 2z^{-1} + 2z^{-2}$

$H(\omega) = 1 - 2e^{-j\omega} + 2e^{-j2\omega}$

$1 - 2(\cos(\omega) - j\sin(\omega)) + 2(\cos(2\omega) - j\sin(2\omega))$

$1 - 2\cos(\omega) + 2j\sin(\omega) + 2\cos(2\omega) - 2j\sin(2\omega)$

~~$\left(\left[1 - 2\cos(\omega) + 2\cos(2\omega)\right]^2 + \left[2j\sin(\omega) - 2j\sin(2\omega)\right]^2\right)^{1/2}$~~

$|H(\omega)| = \left(\left[1 - 2\cos(\omega) + 2\cos(2\omega)\right]^2 + \left[2\sin(\omega) - 2\sin(2\omega)\right]^2\right)^{1/2}$

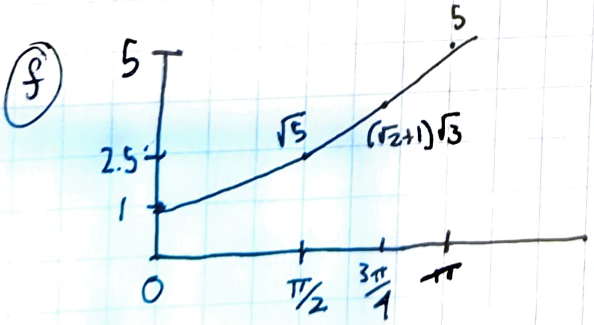
ⓔ $|H(\omega)|$ for $\omega = 0, \pi/2, \pi$

$\omega = 0, \quad |H(\omega)| = 1$

$\omega = \pi/2 \quad \sqrt{5}$

$\omega = \pi \quad 5$

ⓕ



$\boxed{4}$ $Y_t = X_t - 2X_{t-1} + X_{t-2} - 2X_{t-3} + X_{t-4}$ // Palindromic → split via middle term's exp

$e^{-j\theta/2}(e^{j\theta/2} + e^{-j\theta/2})$

$e^{j\theta/2} + e^{-j\theta/2} = 2\cos(\theta/2)$

ⓐ $H(z) = 1 - 2z^{-1} + z^{-2} - 2z^{-3} + z^{-4}$

ⓑ $H(\omega) = 1 - 2e^{-j\omega} + e^{-j2\omega} - 2e^{-j3\omega} + e^{-j4\omega}$

$e^{-j2\omega/2}(e^{j\omega} - 2e^{-j0} + e^{j\omega/2} + e^{-j\omega/2}$

// Middle - $e^{-j2\omega/2} = e^{-j\omega}$

$e^{-j2\omega} = (e^{j\omega} + e^{-j\omega})e^{-j\omega}$

$\downarrow$ only use split if even # of terms (no explicit middle term)

$e^{-j2\omega/2}(e^{j2\omega/2} - 2 + e^{j2\omega/2} + e^{-j2\omega/2} - 2e^{-j2\omega} + e^{-j3\omega})$

$e^{j\omega} - 2 + 2\cos(\omega) - 2e^{-j2\omega} + e^{-j3\omega}$

$e^{-j\omega}(e^{j\omega} - 2 + e^{j\omega} + e^{-j\omega} - 2e^{-j2\omega} + e^{-j3\omega})$

$e^{j\omega} - 2 + 2\cos(\omega)$

ⓒ $e^{-j2\omega}(e^{j2\omega} - 2e^{j\omega} + 1 - 2e^{-j\omega} + e^{-j2\omega})$ $|H(\omega)| = 2\cos(2\omega) - 4\cos(\omega) + 1$

$= e^{-j2\omega}(2\cos(2\omega) - 4\cos(\omega) + 1)$

ⓓ Phase response: -2ω