

# MAT 320 Homework 2

## Fall 2025

Due date: Friday, Sep 19

1. Determine if the following functions are periodic or non-periodic. If the function is periodic, give the smallest possible period. (Note: each function has domain all real numbers, and codomain the complex plane  $\mathbb{C}$ . This means that the range must be a subset of  $\mathbb{C}$ .) In each case, also describe the range of  $f$ . A description can use any combination of: words, formulas, geometry, and pictures.

(a)  $f(t) = t + t^2i$

(b)  $f(t) = e^{i\frac{\pi}{3}t}$

(c)  $f(t) = 3e^{i\frac{\pi}{4}t}$

(d)  $f(t) = 3e^{i(\frac{\pi}{4}t+4)}$

(e)  $f(t) = i \sin(\frac{\pi}{3}t)$

(f)  $f(t) = te^{i\frac{\pi}{3}t}$

(g)  $f(t) = e^{t+i\frac{\pi}{3}t}$

(h)  $f(t) = e^{i\frac{\pi}{2} \sin(\frac{\pi}{3}t)}$

(i)  $f(t) = \sin(\frac{\pi}{3}t)e^{i\frac{\pi}{3}t}$

2. Find the complex number  $z_0$  in polar form which has the property that multiplication by  $z_0$  gives a function which rotates all complex numbers by the angle  $\pi/6$  counterclockwise and also scales them by 6. Call this function  $f_{z_0}$ , so that for any complex number  $z$  the function gives  $f_{z_0}(z) = z_0z$ . Find the cartesian form of  $z_0$ . Find the  $2D$  matrix which performs the same operation on points of the plane  $\mathbb{R}^2$  as  $f_{z_0}$  performs on the complex plane.
3. Show that complex numbers  $w$  and  $z$  are linearly dependent (as real vectors) if and only if  $w\bar{z} \in \mathbb{R}$ . (Note: the linear dependence statement uses only the vector space properties of  $\mathbb{C}$ , but the criterion in this case uses the multiplication of  $\mathbb{C}$ . To show the if and only if statement is true, you need to prove two implications. For example: “A if and only if B” is true if A implies B and B implies A. So, first assume A and show that B is true, then second assume B and show that A is true.)