Quiz 4

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$\boxed{1}$ $f_s = 44100$ $\quad \overset{(f)}{Hz} = 500 \quad \theta(\omega)^{-L} = 0.5$

$L = \quad \alpha = \quad$ delay total $=$

$f_s/f = L + 0.5 + \delta$

$\cancel{Y_t = X_t - R^L X_{t-L}}$

$0 \leq \delta \leq 1$

$88.2 = L + 0.5 + \delta$

$87.7 = L + \delta \longrightarrow L = 87, \ \delta = 0.7$ d

$\boxed{2}$ Comb: $[-6, 24]$ dB $\quad dB = 20 \log_{10}(Hz)$

$\Delta = 30$ dB $\quad 30 = 20 \log_{10}(Hz)$

$1.5 = \log_{10}(Hz)$

$10^{1.5} = Hz = 31.62 \approx 32$ a

$\boxed{3}$ $Y_t = \frac{1}{2}(X_t + X_{t-1}) \quad H(z) = \frac{1}{2} + \frac{1}{2}z^{-1} \quad H(\omega) = \frac{1}{2} + \frac{1}{2}e^{-j\omega}$

$H(z) = \frac{1}{2}(1 + z^{-1}) \quad |H(\omega)| = \frac{1}{2} + \frac{1}{2}(\cos(\omega) - j\sin(\omega))$

$H(\omega) = \frac{1}{2}(1 + e^{-j\omega}) = \frac{1}{2}e^{-j\omega/2}(e^{j\omega/2} + e^{-j\omega/2}) \quad \theta(\omega) = -\omega/2$

$(\cos(\omega/2) + j\sin(\omega/2)) + (\cos(\omega/2) - j\sin(\omega/2))$

$\frac{1}{2}e^{-j\omega/2}(2\cos(\omega/2)) = \frac{2}{2}e^{-j\omega/2}\cos(\omega/2)$

$H(\omega) = e^{-j\omega/2}\cos(\omega/2) \quad \theta(\omega) = -\omega/2$ Linear Phase

$|H(\omega)| = [\cos(\omega/2) - j\sin(\omega/2)] \cdot \cos(\omega/2)]^{1/2} \quad |\cos(\omega/2) - j\sin(\omega/2)| =$

$\cancel{= [2\cos(\omega/2)^2 + \sin(\omega/2)^2]^{1/2}} \quad [\cos^2(\omega/2) \cdot \sin^2(\omega/2)]^{1/2}$

$\sqrt{1} = 1$

$\cancel{= \cos^2(\omega/2) - j\sin(\omega/2)\cos(\omega/2)}$

$|H(\omega)| = |1 \cdot \cos(\omega/2)| \ != \sin(\omega/2) \quad \therefore$ d

$\boxed{4}$ $Y_t = aX_t + X_{t-1} - aY_{t-1}$

$Y_t + aY_{t-1} = aX_t + X_{t-1}$ $\qquad H(z) = \dfrac{a+z^{-1}}{1+az^{-1}}$

$Y(z)(1+az^{-1}) = X(z)(a+z^{-1}) = \dfrac{Y(z)}{X(z)} = \dfrac{a+z^{-1}}{1+az^{-1}}$

Pole:

$0 = 1 + az^{-1} = -1 = a\frac{1}{z} = -\frac{1}{a} = \frac{1}{z} \quad -a = z$ 1 Pole

$H(\omega) = \dfrac{a+e^{-j\omega}}{1+ae^{-j\omega}} \qquad |H(\omega)| = \dfrac{|a+e^{-j\omega}|^2}{|1+ae^{-j\omega}|^2}$

$|a+e^{-j\omega}|^2 = [a + (\cos(\omega)]^2 + [\sin(\omega))]^2$

$= a^2 + 2a\cos(\omega) + \cos^2(\omega) + \sin^2(\omega) = a^2 + 2a\cos(\omega) + 1$

$|1+ae^{-j\omega}|^2 = [1 + a\cos(\omega)]^2 + [a\sin(\omega)]^2$

$\cancel{1^2 + a\cos^2(\omega) + a^2\sin^2(\omega)}$ $1^2 + 2a\cos(\omega) + a^2\cos^2(\omega) + a^2\sin^2(\omega)$

$= 1 + 2a\cos(\omega) + a^2(1)$

$|H(\omega)| = \dfrac{a^2 + 2a\cos(\omega) + 1}{a^2 + 2a\cos(\omega) + 1} = 1$

$H(\omega) = \dfrac{a + \cos(\omega) - j\sin(\omega)}{1 + a[\cos(\omega) - j\sin(\omega)]} \qquad \theta(\omega)_N = \tan^{-1}\dfrac{-\sin(\omega)}{a+\cos(\omega)}$

$\theta(\omega)_D = \tan^{-1}\dfrac{-a\sin(\omega)}{1+a\cos(\omega)}$

Phase control on a ✓

d

$\boxed{5}$ (a) $H(z) = \frac{\frac{1}{2}z+1}{z+1/2}$ $\frac{Y(z)}{X(z)} = \frac{1/2 z+1}{z+1/2}$ allpass: $H(z) = \frac{a+z}{1+az}$

$|H(\omega)| = 1$

$\cancel{Y(z)(z+1/2) = X(z)(X}$ $\cancel{Y(z)(z^{-1}+1/2) =}$

$Y(z)(1/2 z^{-1}+1) = X(z)(z^{-1}+1/2)$

$\frac{1}{2} Y_{t-1} + Y_t = X_{t-1} + X_t$ $Y_t = \frac{1}{2}X_t + X_{t-1} - \frac{1}{2}Y_{t-1}$

$Y_t = 1/2 X_t + X_{t-1} - 1/2 Y_{t-1}$

$\cancel{(b)\ H(z) = z + z^{-1}}$ (a) $H(\omega) = \cancel{\frac{1}{2}(\cos}\ 1/2\cos(\omega) - \frac{1}{2}\sin(\omega) + 1$

$|H(\omega)_N|^2 = [1/2\cos + 1]^2 + [1/2 \sin(\omega)]^2$

$\boxed{6}$ (a) $\cancel{\dfrac{e^{j\omega/2}+3e^{j\omega/2}}{e^{j\omega/2}+\frac{1}{3}e^{-j\omega/2}}}$ $\cancel{|H(\omega)|^2_n = [20\cdot\cos^2(\omega/2)}$

$\cancel{(20\cos(\omega/2))^2 - A + 1/6}$

$\cancel{|H(\omega)| = \dfrac{(e^{-j\omega/2}+3e^{j\omega/2})^2}{(e^{j\omega/2}+\frac{1}{3}e^{-j\omega/2})^2} = 3^2(e^{-j\omega/2}+e^{j\omega/2})^2}$

$\times \quad 9 = \dfrac{\cos(\omega/2) - j\sin(\omega/2)}{\cos(\omega/2)+j\sin(\omega/2)}$

$e^{j\omega/2} + 3e^{j\omega/2} = e^{j\omega/2} + \frac{1}{3}e^{-j\omega/2}$

$3e^{j\omega/2} = \frac{1}{3}e^{-j\omega/2}$ $\qquad 9e^{j\omega/2} = e^{-j\omega/2}$

(b) $\dfrac{e^{j\omega/2}+3e^{j\omega/2}}{e^{j\omega/2}-\frac{1}{3}}$ $\quad e^{j\omega/2} - \frac{1}{3}e^{-j\omega/2} = e^{j\omega/2}+3e^{j\omega/2}$

$-\frac{1}{3}e^{-j\omega/2} = 3e^{j\omega/2}$ $\quad \times \quad \cdot 9 = \dfrac{\cos(\omega/2)-j\sin(\omega/2)}{\cos(\omega/2)+j\sin(\omega/2)}$

(c) $\dfrac{e^{j\omega/2}+2e^{j\omega/2}}{e^{j\omega/2}+2e^{-j\omega/2}}$ $\quad 2e^{-j\omega/2} = 2e^{j\omega/2}$

$\dfrac{2}{2} = \dfrac{e^{j\omega/2}}{e^{j\omega/2}} = 1$ ✓

(d) $\dfrac{e^{j\omega/2}-2e^{j\omega/2}}{e^{-j\omega/2}+2e^{j\omega/2}}$ $\quad e^{j\omega/2}+2e^{-j\omega/2} = e^{j\omega/2}-2e^{j\omega/2}$

C

$\frac{1}{3} - \dfrac{2\cos^2(\omega)}{3} - \dfrac{2\sin(\omega)\cos(\omega)j}{3}$

$\boxed{7}$ $x \approx 0$ $x = \tan(x) = \tan^{-1}(x)$

$x < 0$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \dots$$

$$\tan^{-1}(x) = x - x^3/3 + x^5/5 + \dots$$

Using 2nd order approximation:

$\tan x = x + \frac{1}{3}x^3$ | $\tan^{-1}x = x - \frac{1}{3}x^3$

$x^3 < 0$ | $x^3 < 0$

$\frac{1}{3}x^3 < 0$ | $-\frac{1}{3}x^3 > 0$

$x + \frac{1}{3}x^3 < 0$ $\therefore \tan x < x$ | $x - \frac{1}{3}x^3 > 0$ $\therefore \tan^{-1}x > \cancel{x}$

$\tan x < x < \tan^{-1}x$ (a)

$\boxed{8}$ $L = 9$ $R = 0.9$

$Y_t = X_t + 0.9^9 Y_{t-9}$ $H(z) = 1 + 0.387 z^{-9}$

$\sum_{t=0}^{\infty} h_t$ ~~$\delta_t = \{1, 0, 0, 0, 0, 0 \dots$~~

$h_0 = 1$ $h_{9t} = [0.9^9]^{9/t}$ $\sum_{t=0}^{\infty} h_t = 1 +$

$\sum_{t=0}^{\infty} h_t = \frac{a}{1-r}$ $a = 1$ $r = 0.387$

$$\frac{1}{1-0.387} = 1.6313$$ (c)