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## cf\_callmax

Let

- $T = \text{maturity date} \quad (T > t)$
- K = strike price
- x1 = spot1 price
- x2 = spot 2 price
- t = pricing date
- $\sigma 1 = \text{volatility} 1$
- $\sigma 2 = \text{volatility} 2$
- $\rho = \text{correlation}$
- r = interest rate
- $\delta 1 = \text{dividend 1 yields}$
- $\delta 2 = \text{dividend 2 yields}$
- $\bullet$   $\theta = T t$

Here, closed formulas due to Johnson and Stulz are presented [1],[2]. We set

• 
$$d = \frac{\log \frac{x_1}{x_2} + \left(\delta_2 - \delta_1 + \frac{\sigma^2}{2}\right)\theta}{\sigma\sqrt{\theta}}$$

$$\bullet \quad \sigma = \sqrt{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2}$$

• 
$$d_i = \frac{\log\left(\frac{x_i}{K}\right) + \left(r - \delta_i + \frac{\sigma_i^2}{2}\right)\theta}{\sigma_1\sqrt{\theta}}, \quad i = 1, 2$$

$$\bullet \quad \rho_1 = \frac{\sigma_1 - \rho \sigma_2}{\sigma}$$

$$\bullet \quad \rho_2 = \frac{\sigma_2 - \rho \sigma_1}{\sigma}$$

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and M as the cumulative bivariate normal distribution function:

$$M(a,b;\rho) = \frac{1}{2\pi\sqrt{1-\rho^2}} \int_{-\infty}^{a} \int_{-\infty}^{b} e^{-\frac{x^2-2\rho xy+y^2}{2(1-\rho^2)}} dx dy.$$

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## Call On Maximum Option

Payoff 
$$C_T = (\max(S_T^1, S_T^2) - K)_+$$
Price 
$$C(t, x_1, x_2) = x_1 e^{-\delta_1 \theta} M(d_1, d; \rho_1) + x_2 e^{-\delta_2 \theta} M(d_2, -d + \sigma \sqrt{\theta}; \rho_2)$$

$$-K e^{-r\theta} \left( 1 - M(-d_1 + \sigma_1 \sqrt{\theta}, -d_2 + \sigma_2 \sqrt{\theta}; \rho) \right)$$
Delta 
$$\frac{\partial C(t, x_1, x_2)}{\partial x_1} = e^{-\delta_1 \theta} M(d_1, d; \rho_1)$$

$$\frac{\partial C(t, x_1, x_2)}{\partial x_2} = e^{-\delta_2 \theta} M(d_2, -d + \sigma \sqrt{\theta}; \rho_2)$$

## References

- [1] H.JOHNSON. Options on the maximum of the minimum of several assets. J.Of Finance and Quantitative Analysis, 22:227–283, 1987. 1
- [2] R.STULZ. Options on the minimum or the maximum of two risky assets. J. of Finance, 10:161–185, 1992. 1