Help

```
#ifndef __time_change_levy__
#define __time_change_levy__

#include "pnl/pnl_vector.h"
#include "pnl/pnl_band_matrix.h"
#include "pnl/pnl_tridiag_matrix.h"
#include "pde_tools.h"

extern dcomplex Ctgamma_log(dcomplex z);

/*
  The two following class of model come from :

  @article {MR1995283,
    AUTHOR = {Carr, Peter and Geman, H{{'e}lyette and Madan
    , Dilip B. and
              Yor, Marc},
     TITLE = {Stochastic volatility for {L}{'evy processes}
    ,
   JOURNAL = {Math. Finance},
  FJOURNAL = {Mathematical Finance. An International Journa
    l of Mathematics,
              Statistics and Financial Economics},
    VOLUME = {13},
      YEAR = {2003},
    NUMBER = {3},
     PAGES = {345--382},
      ISSN = {0960-1627},
   MRCLASS = {91B28 (60G51)},
  MRNUMBER = {MR1995283 (2005a:91054)},
  }


  CIR stochastic clock

  the CIR process can be use as rate of time change. it fo
    llows the SDE

  $$ dy_t = {kappa {left( {eta - y_t {right) dt + {lambda {
    sqrt{y_t} d W_t .$$
```

```
*/
typedef struct _CIR_diffusion CIR_diffusion;

struct _CIR_diffusion
{
  double Kappa;
  double Eta;
  double Lambda;
  double Drift; // proportional to Drift correction
  double y0;
  double Kappa_sqr;
  double Lambda_sqr;
  double Kappa_sqr_eta_div_lambda_sqr;
  double Two_kappa_eta_div_lambda_sqr;

  double time;
  double Jump_drift;
  double Jump_drift_psi;
  void * Levy;
  dcomplex (*characteristic_exponent)(dcomplex u,void *
    mod);
};

extern CIR_diffusion * CIR_diffusion_create(double Kappa,
    double Eta,double Lambda,double y0,
                                    void * Levy_,
                                    dcomplex (*charact
    eristic_exponent_)(dcomplex,void *),
                                    double *jump_drift)
    ;
extern dcomplex CIR_diffusion_characteristic_exponent(dcom
    plex u,double t,void * mod);
extern dcomplex CIR_diffusion_ln_characteristic_function(dc
    omplex u,double t,void * mod);
extern double CIR_diffusion_get_sigma_square(CIR_diffusion
    *Process);
extern void CIR_diffusion_fourier_stiffness(CIR_diffusion *
     mod,double hx,double bnd_fourier,int Nw,int kmin,int kmax
    ,int Dupire,PnlVect *row_stiffness);
extern void CIR_diffusion_update(CIR_diffusion *  process,
```

```
    double t);
/*

  Gamma- OU stochastic clock

  the rate of time change is now solution of the SDE

  $$ dy_t = - {lambda y_t  dt +  d z_{{lambda t} .$$

  Choice $z_t$ as a compound poisson process,
  $$ z_t ={sum_{n=1}^N_t x_n$$
  where $N_t$ is a Poisson process with intensity parapmet
    er ${alpha$
  and each $x_n$ follows an exponential law with mean ${
    frac{1}{{beta}$.
*/
  typedef struct _GammaOU_diffusion GammaOU_diffusion;

struct _GammaOU_diffusion
{
  double Lambda;
  double Alpha;
  double Beta;
  double Drift; // proportional to Drift correction
  double y0;

  double Lambda_a;
  double Lambda_b;
  double y0_one_m_el_div_lambda;
  double y0_el;
  double one_m_el_div_lambda;
  double beta_el;

  double time;
  double Jump_drift;
  double Jump_drift_psi;
  void * Levy;
  dcomplex (*characteristic_exponent)(dcomplex u,void *
    mod);

};
```

```
extern GammaOU_diffusion * GammaOU_diffusion_create(double
    Lambda,double Alpha,double Beta,double y0,
                                                  void *
    Levy_,
                                                  dcomplex (*
    characteristic_exponent_)(dcomplex,void *),
                                                  double *
    jump_drift);
extern dcomplex GammaOU_diffusion_characteristic_exponent(
    dcomplex u,double t,void * mod);
extern dcomplex GammaOU_diffusion_ln_characteristic_
    function(dcomplex u,double t,void * mod);
extern double GammaOU_diffusion_get_sigma_square(GammaOU_
    diffusion *Process);
extern void GammaOU_diffusion_fourier_stiffness(GammaOU_dif
    fusion * mod,double hx,double bnd_fourier,int Nw,int kmin,
    int kmax,int Dupire,PnlVect *row_stiffness);
extern void GammaOU_diffusion_update(GammaOU_diffusion *
    process,double t);


extern void test_CIR_diffusion(void );
extern void test_GammaOU_diffusion(void );


typedef struct _Heston_diffusion Heston_diffusion;

struct _Heston_diffusion
{
  double Eta;
  double Kappa;
  double Rho;
  double Theta;
  double Sigma;
  double sigma_sqr;
  double theta_sqr;
  double sigma_sqr_d_eta_kappa;
  double etakappathetam2;
  double rho_theta;
  double Drift;
```

```
};

extern Heston_diffusion * Heston_diffusion_create(double Et
    a_,double Kappa_,double Rho_,
                                                double Th
    eta_,double Sigma_,
                                                double *
    jump_drift);

extern dcomplex Heston_diffusion_characteristic_exponent(dc
    omplex u,double t,void * mod);
extern dcomplex Heston_diffusion_ln_characteristic_
    function(dcomplex u,double t,void * mod);

/*

  dS_t = (r-q-{lambda_y {mu) S_t dt + {sqrt{V_t} S_t dW_t^1
     + J_y S_t dq_y(t)
  dV_t = {kappa_{nu} {left( {eta_{nu} + V_t {right) + {thet
    a_{nu} {sqrt{V_t} dW_t^2
  dW^1 dW^2 = {rho dt

  (1+J_y) is a lognormally distributed with mean ${mu_y$ an
    d variance
  ${sigma_y^2$

  $q_{y}$ is an indenpendent Poisson process with arrival
    rate
  ${lambda_{y}$
  ${mu= {left( {exp{{mu_y+{sigma_y^2/2}-1{right).

*/

typedef struct _Bates_diffusion Bates_diffusion;

struct _Bates_diffusion
{
  double Eta;
  double Kappa;
  double Theta;
  double Rho;
```

```
  double Sigma;

  double mu_J;
  double Sigma_J;
  double Lambda_J;


  double sigma_sqr;
  double theta_sqr;
  double sigma_sqr_d_eta_kappa;
  double etakappathetam2;
  double rho_theta;
  double lnonepmuj;
  double sigmaj_sqr_demi;
  double Drift;
};

extern Bates_diffusion * Bates_diffusion_create(double Eta_
    ,double Kappa_,double Rho_,
                                                double Thet
    a_,double Sigma_,
                                                double mu_
    J_,
                                                double Si
    gma_J_,double Lambda_J_,double *jump_drift);
extern dcomplex Bates_diffusion_characteristic_exponent(dc
    omplex u,double t,void * mod);
extern dcomplex Bates_diffusion_ln_characteristic_function(
    dcomplex u,double t,void * mod);
/*
  @article {MR1841412,
    AUTHOR = {Barndorff-Nielsen, Ole E. and Shephard, Neil}
    ,
     TITLE = {Non-{G}aussian {O}rnstein-{U}hlenbeck-based
    models and some of
              their uses in financial economics},
   JOURNAL = {J. R. Stat. Soc. Ser. B Stat. Methodol.},
  FJOURNAL = {Journal of the Royal Statistical Society. Se
    ries B.
              Statistical Methodology},
    VOLUME = {63},
```

```
    YEAR = {2001},
  NUMBER = {2},
   PAGES = {167--241},
    ISSN = {1369-7412},
 MRCLASS = {62M07 (62M09 62M10 62P20)},
MRNUMBER = {MR1841412 (2002c:62127)},
}

The square volatility follows the SDE of the form :
$$d{sigma^2_t = -{lambda {sigma^2_t dt + d z_{{lambda t }
  $$
where ${lambda >0 $ and $z$ is a subordinator.
The risk neutral dynamic of the log price $x-t= log S_t $
   are given by
$$ dW_t=(r-q-{lambda k(-{rho) -{sigma^2/2 ) dt + {sigma_
  t dW_t + {rho dz_t,
{quad x_0=log(S_0).$$

where $k(u)= {log{{mean{{exp{-u z_1}}}.
Choice $z_t$ as a compound poisson process,
$$ z_t ={sum_{n=1}^N_t x_n$$
where $N_t$ is a Poisson process with intensity parapmet
  er ${alpha$
and each $x_n$ follows an exponential law with mean ${
  frac{1}{{beta}$.
One can show that the process ${sigma^2_t$ is a stationa
  ry process with a
marginal law that follows a Gamma distribution with mean
  ${alpha$ and
variance ${frac{{alpha}{{beta}$. In this case,
$$k(u)={frac{-au}{b+u}.$$

*/

typedef struct _BNS_diffusion BNS_diffusion;

struct _BNS_diffusion
{
  double Lambda;
  double Rho;
  double Beta;
```

```
  double Alpha;
  double Sigma0;

  double Sigma0_sqr ;
  double Lambda_m1;
  double Drift; // proportional to Drift correction
};

extern BNS_diffusion * BNS_diffusion_create(double Lambda_,
    double Rho_,
                                        double Beta_,
    double Alpha_,
                                        double Sigma0_,
    double *jump_drift);
extern dcomplex BNS_diffusion_characteristic_exponent(dcom
    plex u,double t,void * mod);
extern dcomplex BNS_diffusion_ln_characteristic_function(dc
    omplex u,double t,void * mod);


/*
  @article {MR1793362,
    AUTHOR = {Duffie, Darrell and Pan, Jun and Singleton,
    Kenneth},
     TITLE = {Transform analysis and asset pricing for affine
              jump-diffusions},
   JOURNAL = {Econometrica},
  FJOURNAL = {Econometrica. Journal of the Econometric Soc
    iety},
    VOLUME = {68},
      YEAR = {2000},
    NUMBER = {6},
     PAGES = {1343--1376},
      ISSN = {0012-9682},
     CODEN = {ECMTA7},
   MRCLASS = {91B28 (60J60)},
  MRNUMBER = {MR1793362 (2001m:91081)},
}

dS_t = (r-q-{lambda_y {mu) S_t dt + {sqrt{V_t} S_t dW_t^1 +
     J_y S_t dq_y(t) {{
```

```
dV_t = {kappa_{nu} {left( {eta_{nu} + V_t {right) + {theta_
    {nu} {sqrt{V_t}
dW_t^2 + J_V dq_{{nu}(t)

dW^1 dW^2 = {rho dt

(1+J_y) is a lognormally distributed with mean ${mu_y$ and variance
${sigma_y^2$

J_V has an exponential distribution with mean ${mu_{nu}$

$q_{y}$ and $q_{{nu}$ are indenpendent Poisson process wit
    h arrivals rates
${lambda_{y}$ and ${lambda_{nu}$

${mu= {left( {exp{{mu_y+{sigma_y^2/2}-1{right).


*/

typedef struct _DPS_diffusion DPS_diffusion;

struct _DPS_diffusion
{
  double Eta;
  double Kappa;
  double Rho;
  double Theta;
  double Sigma;

  double mu_y;
  double  Sigma_y_sqr_demi;
  double  Lambda_y;

  double  mu_v;
  double  Lambda_v;

  double  sigma_cy_sqr_demi;
  double  mu_cy;
  double  mu_cv;
  double  Lambda_c;
```

```
  double rho_j;


  double  s_lambda;


  double sigma_sqr;
  double theta_sqr;
  double sigma_sqr_d_eta_kappa;
  double etakappathetam2;
  double rho_theta;

  double Drift;
};

extern DPS_diffusion * DPS_diffusion_create(double Eta_,
    double Kappa_,
                                        double Rho_,
    double Theta_,
                                        double Sigma_,
    double mu_y_,
                                        double Sigma_y_
    ,double Lambda_y_,
                                        double mu_v_,
    double Lambda_v_,
                                        double mu_cy_,
    double Sigma_cy_,
                                        double mu_cv_,
    double Lambda_c_,
                                        double rho_j_,
    double *jump_drift);
extern dcomplex DPS_diffusion_characteristic_exponent(dcom
    plex u,double t,void * mod);
extern dcomplex DPS_diffusion_ln_characteristic_function(dc
    omplex u,double t,void * mod);




typedef struct _Levy_diffusion Levy_diffusion;
```

```
struct _Levy_diffusion
{
  void * process;
  dcomplex (*characteristic_exponent)(dcomplex u,double t,
    void * mod);
  dcomplex (*ln_characteristic_function)(dcomplex u,double
    t,void * mod);
  // Arificial volatility term to come back to parabolic
    problem
  double vol_square;
};

extern Levy_diffusion * Levy_diffusion_create(void * proces
    s_, dcomplex (*characretristic_exponent_)(dcomplex u,
    double t,void * mod),
                                              dcomplex (*ln
    _characrteristic_function_)(dcomplex u,double t,void *
    mod));
extern dcomplex Levy_diffusion_characteristic_exponent(dcom
    plex u,double t,Levy_diffusion * mod);
extern double Levy_diffusion_get_sigma_square(
    Levy_diffusion *Levy);
extern void Levy_diffusion_fourier_stiffness(
    Levy_diffusion * mod,double t,double hx,double bnd_fourier,int Nw,int km
    in,int kmax,int Dupire,PnlVect *row_stiffness);
extern dcomplex Levy_diffusion_ln_characteristic_function(
    dcomplex u,double t,void * mod);
extern dcomplex Levy_diffusion_characteristic_function(dcom
    plex u,double t,Levy_diffusion * mod);


#endif
```

# References