ap_lba

Output parameters:

- Price
- Delta

This routine gives either the put price or the call price. The put price is obtained from the call price of a symmetric option by invertion : $K \leftrightarrow x$ and $r \leftrightarrow \delta$. This is the reason why almost all the functions are designed to compute the call option price.

Broadie and Detemple [1] have developped approximations for pricing standard american call options. They consider a european up and out call option with strike K, barriere L and rebate $(L - K)$. They maximise over L the price of this option.

Since the call up and out with rebate $(L - K)$ corresponds to exercise at the minimum of the hitting time of the boundary L and the maturity T, its price is smaller than the price of the american call option. Therefore, $C^l(x) = \max_L C(x, L)$ provides a lower bound for the price of the American call.

From this bound, they obtain the approximation by applying a multiplicative coefficient λ. This multiplicative parameter is obtained by Broadie and Detemple after a linear regression on 2500 options.

/*assign_var_temp*/

This function sets some temporary variables widely used in this program.

/*assign_var_temp_L*/

It sets temporary variables depending on L.

/*call_up_out*/

Gives $C(x,L)$, the price of an up and out european call option with barrier L and rebate (L-K).

$$\begin{aligned} C(x,L) &= (L-K)[\lambda^{\frac{2\phi}{\sigma^2}}N(d_0)+\lambda^{\frac{2\phi}{\sigma^2}}N(d_0+2f\frac{\sqrt{T}}{\sigma})] \\ &\quad +x.e^{-\delta T}[N(d_1^-(L)-\sigma\sqrt{T})-N(d_1^-(K)-\sigma\sqrt{T})] \\ &\quad -\lambda^{-2\frac{r-\delta}{\sigma^2}}L.e^{-\delta T}[N(d_1^+(L)-\sigma\sqrt{T})-N(d_1^+(K)-\sigma\sqrt{T})] \\ &\quad -K.e^{-rT}\quad [N(d_1^-(L))-N(d_1^-(K)) \\ &\qquad\qquad -\lambda^{1-2\frac{r-\delta}{\sigma^2}}[N(d_1^+(L))-N(d_1^+(K))]] \end{aligned}$$

Where : $//b=\delta-r+\frac{1}{2}\sigma^2$

$f=\sqrt{b^2+2r.\sigma^2}$

$\phi=\frac{1}{2}(b-f)$

$\alpha=\frac{1}{2}(b+f)$

$\lambda=\frac{x}{L}$

$d_0=\frac{log(\lambda)-f(T)}{\sigma\sqrt{T}}$

$d_1^+(x)=\frac{log(\lambda)-log(L)+log(x)+b.T}{\sigma\sqrt{T}}$

$d_1^-(x)=\frac{-log(\lambda)-log(L)+log(x)+b.T}{\sigma\sqrt{T}}$

/*dCdl*/

Returns $\frac{\partial C(x,L)}{\partial L}$. This derivative value is necessary for the maximisation. It is based on the closed formula for $\frac{\partial C(x,L)}{\partial L}$.

/*maximise_C*/

Return the value L_{max} for which $C(x,L)$ is a maximum. This result is obtained by a dichotomy research started on the interval $[x, 1000(x+K)]$

/*low_coeff*/

Returns the multiplicative coefficient λ to obtain the approximation from the lower bound. This coefficient is obtained using Broadie and Detemple's formula.

/*call_low_approx*/

Calculates the lower bound applying the L_{max} value to the /*call_up_out*/ function. Then multiplies the result by λ and returns the lowerbound approximation :

$$C_{lba}(x) = C(x, L_{max})$$

/*call_low_delta*/

Calculates the delta for the call option :

$$\frac{C_{lba}(x + 10^{-5}) - C_{lba}(x)}{10^{-5}}$$

/*put_low_delta*/

Calculates the delta for the put option :

$$\frac{P_{lba}(x + 10^{-5}) - P_{lba}(x)}{10^{-5}}$$

P_{lba} is the put price obtained from the symetric call option : C_{lba}.

References

[1] M.BROADIE J.DETEMPLE. American option valuation : new bounds, approximations and a comparison of existing methods. *Review of financial studies, to appear*, 1995. 1