ap_fixedasian_zhang

Output parameters:

- Price
- Delta

Description: By a well chosen change of variables, Zhang [1]ăobtains that the price at time t of the arithmetic average asian call option with fixed strike K, maturity T, spot $S = S_t$ and integral $I = \int_0^t S_u$ is given by $e^{-\delta\tau}\frac{S}{T}f(\xi,\tau)$ where

$$\begin{cases} \tau = T - t \\ \xi = \frac{TK-I}{S}e^{-(r-\delta)\tau} - \frac{\left(1-e^{-(r-\delta)}\right)}{r-\delta} \end{cases}$$

and the function f satisfies the following PDE :

$$\begin{cases} \frac{\partial f}{\partial \tau} - \frac{\sigma^2}{2}\left(\xi + \frac{1-e^{-(r-\delta)\tau}}{r-\delta}\right)^2 \frac{\partial^2 f}{\partial \xi^2} = 0 \quad \forall(\tau,\xi) \in [0,T] \times \mathbb{R} \\ \text{with the initial condition } f(\xi,0) = (-\xi)^+. \end{cases}$$

The analytical approximation is based on the explicit solution f_0 of the same PDE but with diffusion coefficient $\frac{\sigma^2}{2}\left(\frac{1-e^{-(r-\delta)\tau}}{r-\delta}\right)^2$ independent of ξ :

$$f_0(\eta,\xi) = -\xi N\left(-\frac{\xi}{\sqrt{2\eta}}\right) + \sqrt{\frac{\eta}{\pi}}e^{-\frac{\xi^2}{4\eta}}$$

where $\eta = \frac{\sigma^2}{4(r-\delta)^3}(-3 + 2(r-\delta) + 4e^{-(r-\delta)\tau} - e^{-2(r-\delta)\tau})$. The corresponding approximations for the price and the delta of the Asian Option are :

$$C_0(S,I,t) = e^{-\delta\tau}\frac{S}{T}\left(-\xi N\left(-\frac{\xi}{\sqrt{2\eta}}\right) + \sqrt{\frac{\eta}{\pi}}e^{-\frac{\xi^2}{4\eta}}\right)$$

$$\Delta_0 = e^{-\delta\tau}\left(\frac{1-e^{-(r-\delta)\tau}}{(r-\delta)T}N\left(-\frac{\xi}{\sqrt{2\eta}}\right) + \frac{1}{T}\sqrt{\frac{\eta}{\pi}}e^{-\frac{\xi^2}{4\eta}}\right).$$

The PDE satisfied by the correction term $f_1(\xi,\tau) = f(\xi,\tau) - f_0(\xi,\tau)$

$$\begin{cases} \frac{\partial f_1}{\partial \tau} - c(\xi,\tau)\frac{\partial^2 f_1}{\partial \xi^2} = R(\xi,\tau) \\ f_1(\xi,\tau=0) = 0 \end{cases} \tag{1}$$

where

$$c(\xi,\tau) = \frac{\sigma^2}{2}\left(\xi + \frac{1-e^{-(r-\delta)\tau}}{r-\delta}\right)^2 \quad \text{and} \quad R(\xi,\tau) = \frac{\sigma^2 \xi}{4\sqrt{\pi\eta}}\left(\xi + \frac{2}{r-\delta}\left(1-e^{(r-\delta)\tau}\right)\right)$$

is solved by the finite differences method.

/*Computation of the price*/

computes the approximate price and the corrected price.

/*Computation of the delta*/

computes the approximate delta and the corrected delta.

/*Diffusion coefficient C(t,x)*/

computes the diffusion coefficient $C(t,x)$ in (1).

/*right-hand-side R(t,x)*/

computes the right-hand-side $R(t,x)$ in (1).

/*Computation of f_0*/ /*derivative of f0 w.r.t. S*/

compute f_0 and its derivative w.r.t. S.

/*Tridiagonal matrix*/

computes the tridiagonal matrix involved in the finite differences scheme for (1).

/*resolution of the system*/

At each time-step a system is solved by Gauss method.

/*correction delta*/

computes the correction of the delta.

References

[1] J.E.ZHANG. A semy-analtycal method for pricing and hedging continously-sampled arithmetic average rate options. *preprint*, September 2000. 1