fd_explicit_cir1d_capfloor

Input parameters:

- Time StepNumber M

Output parameters:

- Price

The stochastic differential equation representing the short rate is given by

$$dr_t = k(\theta - r_t)dt + \sigma\sqrt{r_t}dW(t)$$

The price of the zero-coupon bond is solution of the following PDE

$$u_t + \frac{1}{2}\sigma^2 r u_{rr} + [k(\theta - r)]u_r - ru = 0u(r,T,T) = 1$$

that we solve using explicit scheme of Hull-White[1]. The price of the option is obtained solving the same PDE with boundary condition at the maturity of the option T, the price of the Zero Coupon Bond. The pricing procedure is in two steps: in the first step(INITPROBA) we compute probabilities associated to the explicit scheme; this is done simply matching the first and the second moment of the change in r over time step Δt. The branching in the lattice is modified at boundary points $r = r_m in$ and $r = r_m ax$ to ensure that the probabilities associated with all three branches remain positive. For this purpose Hull-White[1] propose alternative branching procedures in the explicit finite difference method.
The second step is standard dynamic programming backward pricing algorithm. A cap(floor) is equivalent to a portfolio of European zero-coupon Put(Call)-Options.

References

[1] J.Hull and A.WHITE. Valuing derivative securities using the explicit finite difference method. *Journal of Financial and Quantitative Analysis*, 25:87–100, 1990. 1