Source | Model | Option
| Model_Option | Help on mc methods | Archived Tests

mc_fixedasian_robbinsmoro

Input parameters

- Number of iterations N
- Generator type
- Increment inc
- Confidence Value

Output parameters

- Price P
- Error price σ_P
- Delta δ
- Error delta σ_{delta}
- Price Confidence Interval: ICp [Inf Price,Sup Price]
- Delta Confidence Interval: ICp [Inf Delta,Sup Delta]

Description

Computation of the price of a asian option when the underlying asset follows the Black and Scholes model.
/*The model*/
Under the standard Black and Scholes assumptions the price of the underlying asset is driven by the SDE

$$dS_t = S_t((r-q)dt + \sigma dW_t), \quad S_{T_0} = x, \tag{1}$$

with r the risk-free, continuously compounded interest rate, $\sigma(t, y)$ the asset volatility, W a Brownian motion, and x fixed.
The solution to this equation can be simulated without dicretization error on a discrete grid of points $T_0 < T_1 < \cdots < T_m = T$, by setting

$$S_{T_i} = S_{T_{i-1}} \exp((r - \tfrac{1}{2}\sigma^2)\delta t + \sigma\sqrt{\delta t} Z_i), \quad i = 1, \ldots, m,$$

where $Z = (Z_1, \ldots, Z_m) \sim \mathcal{N}(0, I_m)$ and I_m is the identity matrix of $\mathbb{R}^m$.
/*The option real and approximate prices*/
For arbitrage reasons, the price of an option with payoff $\psi(S_t, t \leq T)$ is given by

$$V_0 = \mathbb{E}[e^{-r(T-T_0)}\psi(S_t, t \leq T)].$$

For a call option we have $\psi(S_t, t \leq T) = \left(\frac{1}{T-T_0}\int_{T_0}^{T} S_t dt - K\right)^+$. which we rewrite

$$G(Z) = e^{-r(T-T_0)}\left(\hat{A}(T_0, T, Z) - K\right)^+,$$

where Z is a random gaussian vector, $\hat{A}(T_0, T, Z)$ is the dicretized mean and G is a function we can compute by using the dicretization of the mean $A(T_0, T) = \frac{1}{T-T_0}\int_{T_0}^{T} S_t dt$ and the payoff function. Thus the approximate price of the option is given by

$$\hat{V}_0 = \mathbb{E}[G(Z)].$$

Importance sampling

We change the law of $Z = (Z_1, \ldots, Z_m)$ by adding a drift vector $\mu = (\mu_1, \ldots, \mu_m)$. An elementary version of Girsanov theorem leads to the following representation of $\hat{V}_0$:

$$\hat{V}_0 = \mathbb{E}[g(\mu, Z)],$$

with

$$g(\mu, Z) = G(Z + \mu)e^{-\mu \cdot Z - \frac{1}{2}\|\mu\|^2}, \tag{2}$$

where $\|x\|$ denotes the Euclidean norm of a vector $x \in \mathbb{R}^m$ and $x \cdot y$ is the inner product of two vectors $x, y \in \mathbb{R}^m$. In (2) the optimal μ solves the problem

$$\min_{\mu} \mathbb{E}[G(Z)^2 e^{-\mu \cdot Z + \frac{1}{2}\|\mu\|^2}].$$

Note that even if the optimal μ can be found, it will not in general provide a zero-variance estimator. In practice, finding the optimal μ exactly is infeasible and some approximation is required. Here the basic idea is to use a

Robbins-Monro algorithm to assess the optimal sampling direction μ^* that minimizes the variance of $g(\mu, Z)$, for $\mu \in \mathbb{R}^m$ or equivalently

$$H(\mu) = \mathbb{E}[g^2(\mu, Z)]. \tag{3}$$

RM algorithms and variance reduction

See [1] ========================.
/*The MC price computation*/
If $(Z^n)_{1 \leq n \leq N}$ is an $i.i.d.$ sample from the gaussian law $\mathcal{N}(0, I_m)$ then the MC price of the option is given by

$$\hat{V}_0 \sim \frac{1}{N} \sum_{n=1}^{N} G(Z^n + \mu^*) e^{-\mu^* \cdot Z^n - \frac{1}{2}\|\mu^*\|^2}.$$

References

[1] B.Arouna. Robbind-monro algorithm and variance reduction. *Journal of Computational Finance*, 7-2:335–362, 2003-04. 3