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Help
/* Black-Scholes model */
#include <stdlib.h>
#include <math.h>
#include <string.h>
#include "pnl/pnl_random.h"
#include "optype.h"
#include "pnl/pnl_mathtools.h"
#include "black.h"
#include "bsnd stdnd.h"
#include "pnl/pnl cdf.h"
static double *VDSC=NULL, *AuxBS=NULL;
static double *Vector_BS_Mean=NULL, *AuxDBS=NULL;
static double *Aux_Stock=NULL;
static PnlMat *Inv Sqrt BS Dispersion=NULL;
static PnlMat *Sigma=NULL, *InvSigma=NULL;
static double *Aux_BS_TD_1=NULL, *Aux_BS_TD_2=NULL;
static double DetInvSigma, Norm BS TD;
int Init_BS(int BS_Dimension, double *BS_Volatility,
            double *BS Correlation, double BS Interest Rate,
            double *BS Dividend Rate)
{
  int i,j,k;
  double aux;
  if (Sigma==NULL)
    {
      Sigma=pnl mat create (BS Dimension, BS Dimension);
      if (Sigma==NULL) return MEMORY ALLOCATION FAILURE;
      /*BlackSholes dispersion matrix*/
      for (i=0;i<BS Dimension;i++){</pre>
        pnl mat set (Sigma, i, i, BS Volatility[i]*BS
    Volatility[i]);
        for (j=i+1; j < BS_Dimension; j++){
          double tmp;
          tmp = BS_Volatility[i]*BS_Volatility[j]*BS_
    Correlation[i*BS_Dimension+j];
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pnl mat set (Sigma, i, j, tmp);
      pnl_mat_set (Sigma, j, i, tmp);
    }
  }
  /*square root of the BlackSholes dispersion matrix*/
  pnl_mat_chol (Sigma);
  if (VDSC==NULL) {
    VDSC=(double*)malloc(BS Dimension*sizeof(double));
    if (VDSC==NULL)
      return MEMORY_ALLOCATION_FAILURE;
    for (i=0;i<BS Dimension;i++){</pre>
      aux=0;
      for (j=0; j<=i; j++)
        aux+=pnl_mat_get (Sigma, i, j) * pnl_mat_get (
Sigma, i, j);
     VDSC[i]=aux*0.5;
    }
  }
  if (AuxBS==NULL){
    AuxBS=(double*)malloc(BS_Dimension*sizeof(double));
    if (AuxBS==NULL)
      return MEMORY ALLOCATION FAILURE;
    for (i=0;i<BS Dimension;i++)</pre>
      AuxBS[i]=VDSC[i]-BS Interest Rate+BS Dividend Ra
te[i];
  }
  if (Aux Stock==NULL){
    Aux Stock=(double*)malloc(BS Dimension*sizeof(
double));
    if (Aux_Stock==NULL)
      return MEMORY ALLOCATION FAILURE;
  }
  if (Vector_BS_Mean==NULL){
    Vector BS Mean=(double*)malloc(BS Dimension*sizeof(
double));
    if (Vector_BS_Mean==NULL)
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return MEMORY ALLOCATION FAILURE;
      }
      if (Inv Sqrt BS Dispersion==NULL){
        Inv_Sqrt_BS_Dispersion=pnl_mat_create (BS_Dimensio
    n, BS Dimension);
        if (Inv_Sqrt_BS_Dispersion==NULL){
          return MEMORY ALLOCATION FAILURE;
      }
      if (AuxDBS==NULL){
        AuxDBS=(double*)malloc(BS_Dimension*BS_Dimension*si
    zeof(double));
        if (AuxDBS==NULL)
          return MEMORY_ALLOCATION_FAILURE;
        for (i=0;i<BS Dimension;i++){</pre>
          for (j=0;j<BS_Dimension;j++){</pre>
            aux=0;
            for (k=0;k<BS Dimension;k++)</pre>
               aux+=pnl_mat_get (Sigma, i, k) * pnl_mat_get
    (Sigma, j, k);
            AuxDBS[i*BS_Dimension+j]=aux*0.5;
          }
        }
      }
    }
  return OK;
void End BS()
  if (Sigma!=NULL){
    pnl mat free (&Sigma);
  if (VDSC!=NULL){
    free(VDSC);
    VDSC=NULL;
  if (AuxBS!=NULL){
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free(AuxBS);
    AuxBS=NULL;
  }
  if (Vector BS Mean!=NULL){
    free(Vector BS Mean);
    Vector BS Mean=NULL;
  }
  if (Inv Sqrt BS Dispersion!=NULL){
    pnl_mat_free (&Inv_Sqrt_BS_Dispersion);
  if (AuxDBS!=NULL){
    free(AuxDBS);
    AuxDBS=NULL;
  if (Aux_Stock!=NULL){
    free(Aux_Stock);
    Aux Stock=NULL;
  }
}
void Init_Brownian_Bridge(double *Brownian_Bridge,long
    MonteCarlo_Iterations,
                           int BS_Dimension, double OP_Matu
    rity, int generator)
{
  int i;
  long j;
  double Sqrt_Maturity;
  /*brownian bridge initialization at the maturity*/
  Sqrt_Maturity=sqrt(OP_Maturity);
  for (j=0;j<MonteCarlo Iterations;j++)</pre>
    for (i=0;i<BS Dimension;i++)</pre>
      Brownian_Bridge[j*BS_Dimension+i]=Sqrt_Maturity*pnl_
    rand normal(generator);
}
void Compute_Brownian_Bridge(double *Brownian_Bridge,
    double Time, double Step,
                              int BS_Dimension, long
    MonteCarlo_Iterations,
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int generator)
  double aux1,aux2,*ad,*admax;
  /*backward computation of the brownian bridge at time "
    Time", knowing its value at time "Time+Step"*/
  aux1=Time/(Time+Step);
  aux2=sqrt(aux1*Step);
  ad=Brownian Bridge;
  admax=Brownian_Bridge+BS_Dimension*MonteCarlo_Iterations;
  for (ad=Brownian Bridge;ad<admax;ad++)</pre>
    *ad=aux1*(*ad)+aux2*pnl rand normal(generator);
}
void Init_Brownian_Bridge_A(double *Brownian_Bridge,long
    MonteCarlo_Iterations,
                             int BS Dimension, double OP
    Maturity,
                             int generator)
{
  int i;
  long j;
  double Sqrt_Maturity,aux;
  /*brownian bridge initialization at the maturity with an
    tithetic values*/
  Sqrt Maturity=sqrt(OP Maturity);
  for (j=0;j<MonteCarlo Iterations/2;j++){</pre>
    for (i=0;i<BS Dimension;i++){</pre>
      aux=pnl rand normal(generator);
      Brownian_Bridge[2*j*BS_Dimension+i]=Sqrt_Maturity*aux
      Brownian Bridge[(2*j+1)*BS Dimension+i]=-Sqrt Maturit
    y*aux;
    }
  if (PNL IS ODD(MonteCarlo Iterations)){
    for (i=0;i<BS_Dimension;i++){</pre>
      Brownian_Bridge[(MonteCarlo_Iterations-1)*BS_Dimensio
    n+i]=Sqrt Maturity*pnl rand normal(generator);
  }
```

```
}
void Compute_Brownian_Bridge_A(double *Brownian_Bridge,
    double Time, double Step,
                                int BS Dimension, long
    MonteCarlo Iterations,
                                int generator)
{
  int i;
  long n;
  double aux,aux1,aux2;
  /*backward computation of the brownian bridge at time "
    Time", knowing its value at time "Time+Step". Antithetic paths
    */
  aux1=Time/(Time+Step);
  aux2=sqrt(aux1*Step);
  for (n=0;n<MonteCarlo_Iterations/2;n++){</pre>
    for (i=0;i<BS_Dimension;i++){</pre>
      aux=pnl rand normal (generator);
      Brownian Bridge[2*n*BS Dimension+i]=aux1*Brownian Br
    idge[2*n*BS_Dimension+i]+aux2*aux;
      Brownian_Bridge[(2*n+1)*BS_Dimension+i]=aux1*Brownia
    n Bridge[(2*n+1)*BS Dimension+i]+aux2*(-aux);
  }
  if (PNL IS ODD(MonteCarlo Iterations)){
    for (i=0;i<BS Dimension;i++){</pre>
      Brownian_Bridge[(MonteCarlo_Iterations-1)*BS_Dimensio
    n+i]=aux1*Brownian_Bridge[(MonteCarlo_Iterations-1)*BS_Dim
    ension+i]+aux2*pnl_rand_normal (generator);
    }
}
void Backward Path(double *Paths, double *Brownian Bridge,
                    double *BS_Spot,
                    double Time,
                    long MonteCarlo Iterations, int BS Dim
    ension)
{
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int j,k;
  long n,auxad;
  double aux;
  /*computation of the BlackScholes paths at time "Time"
    related to the brownian bridge: simply add the sigma*B_t an
    d the drift*/
  auxad=0:
  for (n=0;n<MonteCarlo Iterations;n++){</pre>
    for (j=0;j<BS_Dimension;j++){</pre>
      aux=0.;
      for (k=0; k \le j; k++)
        aux+=pnl_mat_get (Sigma, j, k) * Brownian_Bridge[au
    xad+k];
      aux-=Time*AuxBS[j];
      Paths[auxad+j]=BS_Spot[j]*exp(aux);
    auxad+=BS_Dimension;
  }
}
void BS_Forward_Path(double *Paths, double *Brownian_Paths,
     double *BS Spot, double Time,
            long MonteCarlo_Iterations, int BS_Dimensio
    n)
{
  int j,k;
  long n;
  double aux, aux1;
  /*computation of the BlackScholes paths at time "Time"*/
  for (n=0;n<MonteCarlo Iterations;n++){</pre>
    for (j=0;j<BS_Dimension;j++){</pre>
      aux=0.;
      aux1=Brownian_Paths[n*BS_Dimension+j];
      for (k=0; k \le j; k++)
               aux+=MGET (Sigma, j, k)*aux1;
      aux-=Time*AuxBS[j];
      Paths[n*BS_Dimension+j]=BS_Spot[j]*exp(aux);
    }
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}
double European call price average(PnlVect *BS Spot,
    double Time, double OP Maturity, double Strike,
                                    int BS_Dimension, double
    BS Interest Rate,
                                    PnlVect *BS_Dividend_Ra
    te)
{
  int i;
  double mean, d1, d2, Spot;
  mean=0.;
  for (i=0;i<BS_Dimension;i++)</pre>
    {
      Spot=GET(BS_Spot,i);
      d1=log(Spot/(double)Strike)-AuxBS[i]*(OP_Maturity-
    Time);
      d1/=sqrt(2*VDSC[i]*(OP Maturity-Time));
      d2=d1-sqrt(2*VDSC[i]*(OP_Maturity-Time));
      mean+=Spot*exp(-GET(BS_Dividend_Rate,i)*(OP_Maturity-
    Time))*cdf nor(d1)-Strike*exp(-BS Interest Rate*(OP Maturity-
    Time))*cdf_nor(d2);
    }
  mean/=(double)BS Dimension;
  return(mean);
}
double European_call_put_geometric_mean(PnlVect *BS_Spot,
    double Time, double OP Maturity,
                                         double Strike, int
    BS_Dimension, double BS_Interest_Rate,
                                         PnlVect *BS Divid
    end Rate, double *BS Volatility,
                                         double *BS_
    Correlation, int iscall)
{
  int i,j;
  double r=0.,sig=0.,div=0.,sumsig2=0.,price,d1,d2,Spot=1.;
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```
double S=0.;
Spot = pnl_vect_prod(BS_Spot);
div = pnl vect sum(BS Dividend Rate);
for (i=0;i<BS Dimension;i++)</pre>
  {
    sumsig2+=BS_Volatility[i]*BS_Volatility[i];
  }
Spot=POW(Spot,1./(double)BS_Dimension);
r=BS Interest Rate-div/(double)BS Dimension-sumsig2/(
  double)(2*BS Dimension);
for (i=0;i<BS Dimension;i++)</pre>
   S = 0.;
    for (j=0; j<BS Dimension; j++)</pre>
        S += BS_Volatility[j]*BS_Correlation[i*BS_Dimens
  ion+j];
   sig += S*BS_Volatility[i];
sig=sqrt(sig)/(double)(BS_Dimension);
d1=log(Spot/(double)Strike)+(r+sig*sig/2.)*(OP Maturity-
  Time);
d1/=sig*sqrt(OP_Maturity-Time);
d2=d1-sig*sqrt(OP_Maturity-Time);
if (iscall == TRUE)
    price=Spot*exp((r-BS_Interest_Rate)*(OP_Maturity-
  Time))*cdf nor(d1)-Strike*exp(-BS Interest Rate*(OP Maturity-
  Time))*cdf nor(d2);
  }
else
  {
    price = -Spot*exp((r-BS_Interest_Rate)*(OP_Maturity-
  Time))*cdf_nor(-d1)+Strike*exp(-BS_Interest_Rate*(OP_Maturity-
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Time))*cdf nor(-d2);
  return(price);
void Compute Brownian Paths(double *Brownian Paths, double
    Sqrt_Time,
                             int BS Dimension, long
    MonteCarlo_Iterations,
                             int generator)
{
  int j;
  int n;
  /*computation of the BlackScholes paths at time "Time"*/
  for (n=0;n<MonteCarlo_Iterations;n++){</pre>
    for (j=0;j<BS_Dimension;j++){</pre>
      Brownian_Paths[n*BS_Dimension+j]=Sqrt_Time*pnl_rand_
    normal (generator);
}
void Compute_Brownian_Paths_A(double *Brownian_Paths,
    double Sqrt Time,
                               int BS Dimension, long
    MonteCarlo_Iterations,
                               int generator)
{
  int j,n;
  double aux;
  /*computation of the BlackScholes paths at time "Time"*/
  for (n=0; n < MonteCarlo Iterations/2; n++){</pre>
    for (j=0; j<BS_Dimension; j++){
      aux=Sqrt_Time*pnl_rand_normal (generator);
      Brownian_Paths[2*n*BS_Dimension+j] = aux;
      Brownian Paths[(2*n+1)*BS Dimension+j]=-aux;
    }
  }
  if (PNL IS ODD(MonteCarlo Iterations)) {
    for (j=0;j<BS_Dimension;j++) {</pre>
      Brownian_Paths[(MonteCarlo_Iterations-1)*BS_Dimensio
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n+j]=Sqrt Time*pnl rand normal (generator);
  }
}
static double BS_Mean(int i, double t, const PnlVect *BS_
    Spot, double BS Interest Rate,
                      const PnlVect *BS Dividend Rate)
{
  /*mean of the ith BlackScholes stock at time t*/
  return pnl vect get (BS Spot, i)*exp(-t*(pnl vect get (
    BS_Dividend_Rate,i)-BS_Interest_Rate));
}
static double BS_Dispersion(int i, int j, double t, int BS_
    Dimension, const PnlVect *BS Spot,
                            double BS_Interest_Rate, const
    PnlVect *BS_Dividend_Rate)
{
  /*coefficient (i,j) of the BlackScholes dispersion matrix
     at time t*/
  return pnl_vect_get (BS_Spot,i)*pnl_vect_get (BS_Spot,j)*
    exp(-t*(pnl vect get (BS Dividend Rate,i)+pnl vect get
    (BS_Dividend_Rate,j)-2* BS_Interest_Rate))*(exp(t*AuxDBS[
    i*BS Dimension+j])-1);
}
void Compute_Inv_Sqrt_BS_Dispersion(double time, int BS_Dim
    ension, const PnlVect *BS_Spot,
                                    double BS_Interest_Ra
    te, const PnlVect *BS_Dividend_Rate)
{
  int j,k;
  PnlMat *inv = pnl mat create (0,0);
  /*computation of the inverse of the square root of the
                                                              BlackScholes disper
  for (k=0;k<BS_Dimension;k++){</pre>
    Vector BS Mean[k]=BS Mean(k,time,BS Spot,BS Interest Ra
    te,BS_Dividend_Rate);
  }
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```
for (j=0;j<BS_Dimension;j++){</pre>
    for (k=j;k<BS_Dimension;k++){
      pnl_mat_set (Inv_Sqrt_BS_Dispersion,
                    j,k,BS_Dispersion(j,k,time,BS_Dimension,
    BS Spot,
                                        BS_Interest_Rate,BS_
    Dividend_Rate));
  }
  pnl_mat_chol (Inv_Sqrt_BS_Dispersion);
  pnl mat lower inverse (inv, Inv Sqrt BS Dispersion);
  pnl_mat_clone (Inv_Sqrt_BS_Dispersion, inv);
  pnl_mat_free (&inv);
}
void NormalisedPaths(double *Paths, double *PathsN, long
    MonteCarlo_Iterations,
                       int BS_Dimension)
{
  long i;
  int j,k;
  double aux;
  /*BlackScholes paths normalization (mean 0, variance Id).
    */
  for (i=0;i<MonteCarlo Iterations;i++){</pre>
    for (k=0;k<BS Dimension;k++){</pre>
      {\tt PathsN[i*BS\_Dimension+k]=Paths[i*BS\_Dimension+k]-Vec}
    tor_BS_Mean[k];
  }
  for (i=0;i<MonteCarlo Iterations;i++){</pre>
    for (j=0;j<BS_Dimension;j++){</pre>
      aux=0;
      for (k=0; k \le j; k++) {
        aux+=pnl_mat_get (Inv_Sqrt_BS_Dispersion, j, k)*
    PathsN[i*BS_Dimension+k];
      Aux_Stock[j]=aux;
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for (j=0; j<BS Dimension; j++)</pre>
      PathsN[i*BS Dimension+j]=Aux Stock[j];
  }
}
void ForwardPath(double *Path, double *Initial Stock, int
    Initial_Time,
                  int Number Dates, int BS Dimension,
    double Step, double Sqrt_Step,
                  int generator)
{
  int i,j,k;
  double aux;
  double *SigmapjmBS_Dimensionpk;
  /*computation of a BlackScholes path between times "Ini
    tial Time" and "Initial Time+Number Dates-1" with spot "Ini
    tial_Stoc" at time "Initial_Time"*/
  for (j=0; j<BS_Dimension; j++) Path[Initial_Time*BS_Dimens
    ion+j]=Initial Stock[j];
  for (i=Initial_Time+1;i<Initial_Time+Number_Dates;i++){</pre>
    for (j=0;j<BS_Dimension;j++){</pre>
      Aux Stock[j]=Sqrt Step*pnl rand normal (generator);
    SigmapjmBS Dimensionpk=pnl mat lget (Sigma, 0, 0);
    for (j=0; j<BS Dimension; j++){</pre>
      aux=0.;
      for (k=0; k<=j; k++){
        aux+=(*SigmapjmBS_Dimensionpk)*Aux_Stock[k];
        SigmapjmBS Dimensionpk++;
      SigmapjmBS_Dimensionpk+=BS_Dimension-j-1;
      aux-=Step*AuxBS[j];
      Path[i*BS Dimension+j]=Path[(i-1)*BS Dimension+j]*exp
    (aux);
    }
  }
}
void BlackScholes_Transformation(double Instant, double *
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BS, double* B, int BS_Dimension, double *BS_Spot)
  double aux;
  int j,k;
  /*computation a the BlackScholes stock related to the br
    ownian motion value B*/
  for (j=0;j<BS_Dimension;j++){</pre>
    aux=0.;
    for (k=0; k \le j; k++)
      aux+=pnl_mat_get (Sigma, j, k)*B[k];
    aux-=Instant*AuxBS[j];
    BS[j]=BS Spot[j]*exp(aux);
  }
}
double Discount(double Time, double BS_Interest_Rate)
  /*discounting factor*/
  return exp(-BS_Interest_Rate*Time);
}
int BS_Transition_Allocation(int BS_Dimension, double Step)
{
  int i;
  /*memory allocation of the variables needed by the
    function BS TD*/
  Aux BS TD 1=(double*)malloc(BS Dimension*sizeof(double));
  if (Aux_BS_TD_1==0) return MEMORY_ALLOCATION_FAILURE;
  Aux_BS_TD_2=(double*)malloc(BS_Dimension*sizeof(double));
  if (Aux BS TD 2==0)return MEMORY ALLOCATION FAILURE;
  InvSigma=pnl_mat_create (0, 0);
  if (InvSigma==0)return MEMORY_ALLOCATION_FAILURE;
  /* Sigma is a Cholesky factorisation */
  pnl_mat_lower_inverse (InvSigma, Sigma);
  /* determinant of InvSigma */
  DetInvSigma=1;
  for (i=0;i<BS_Dimension;i++)</pre>
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DetInvSigma*= pnl mat get (InvSigma, i , i);
  /*normalization constant of the density */
  Norm_BS_TD=exp(-BS_Dimension*0.5*log(2.*M_PI*Step));
  return OK;
}
void BS_Transition_Liberation(char *ErrorMessage, int BS_
    Dimension, double Step)
{
  if (Aux_BS_TD_1!=NULL){
    free(Aux BS TD 1);
    Aux_BS_TD_1=NULL;
  if (Aux_BS_TD_2!=NULL){
    free(Aux_BS_TD_2);
    Aux BS TD 2=NULL;
  if (InvSigma!=NULL) pnl_mat_free (&InvSigma);
}
double BS_TD(double *X, double *Z, int BS_Dimension,
    double Step)
{
  int i,j;
  double aux1,aux2;
  /* density function of the BlackScholes stock transition
    kernel at time "Step", knowing X */
  for (i=0;i<BS_Dimension;i++){</pre>
    Aux_BS_TD_1[i]=log(Z[i]/X[i])+Step*AuxBS[i];
  }
  aux1=Z[0];
  for (i=1;i<BS_Dimension;i++){</pre>
    aux1*=Z[i];
  if (aux1==0){
    return -1;
  else {
    for (i=0;i<BS_Dimension;i++){</pre>
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Aux BS TD 2[i]=0;
      for (j=0; j<=i; j++){}
       Aux_BS_TD_2[i]+=pnl_mat_get (InvSigma, i , j) * Aux
    BS TD 1[j];
      }
    }
    aux2=0;
    for (i=0;i<BS Dimension;i++){</pre>
      aux2+=Aux_BS_TD_2[i]*Aux_BS_TD_2[i];
    aux2=exp(-aux2/(2.*Step));
    return Norm BS TD*DetInvSigma*aux2/aux1;
  }
}
void BS_Forward_Step(double *Stock, double *Initial_Stock,
    int BS Dimension,
                     double Step, double Sqrt_Step,
                     int generator)
{
  int j,k;
  double Aux;
  /*BlackScholes stock knowing "Initial Stock" at the prece
    ding time*/
  for (j=0;j<BS Dimension;j++)</pre>
    Aux Stock[j]=Sqrt Step*pnl rand normal (generator);
  for (j=0; j<BS_Dimension; j++)</pre>
      Aux=0.;
      for (k=0; k \le j; k++)
       Aux+=pnl_mat_get (Sigma, j , k)*Aux_Stock[k];
      Aux-=Step*AuxBS[j];
      Stock[j]=Initial_Stock[j]*exp(Aux);
    }
}
*/
          Routines for LS importance sampling
                                                  */
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```
/*array copy*/
void Xcopy(double* Original,double* Copy,int dim)
  int i;
  for(i=0;i<dim;i++){Copy[i]=Original[i];}</pre>
/*initialize RM drift*/
void InitThetasigma(double *theta,double *thetasigma,int
    BS Dimension)
{
  int i,j;
  for (i=0;i<BS_Dimension;i++){</pre>
    thetasigma[i]=0.0;
    for (j=0;j<=i;j++) thetasigma[i]+= pnl_mat_get (Sigma,
    i , j) * theta[j];
  return;
/*build drifted path from undrifted*/
void ThetaDriftedPaths(double *Paths,double *thetasigma,
    double Time, long AL MonteCarlo Iterations, int BS Dimension)
{
  long i;
  int j;
  for(i=0;i<AL_MonteCarlo_Iterations;i++){</pre>
    for(j=0;j<BS Dimension;j++){</pre>
     Paths[i*BS Dimension+j]*=exp(thetasigma[j]*Time);
  }
}
/*RM step*/
void RMsigma(double *sigma,int BS_Dimension)
{
  int j,i;
  for(i=0;i<BS_Dimension;i++){</pre>
    for (j=0; j<BS_Dimension; j++)</pre>
```

References