

[Help](#)

```

#ifndef __levy_diffusion__
#define __levy_diffusion__

#include "pnl/pnl_vector.h"
#include "pnl/pnl_band_matrix.h"
#include "pnl/pnl_tridiag_matrix.h"

extern dcomplex Ctgamma_log(dcomplex z);

typedef struct _Heston_diffusion Heston_diffusion;

struct _Heston_diffusion
{
    double Eta;
    double Kappa;
    double Rho;
    double Theta;
    double Sigma;
    double sigma_sqr;
    double theta_sqr;
    double sigma_sqr_d_eta_kappa;
    double etakappathetam2;
    double rho_theta;
    double Drift;
    int nb_parameters;
};

extern Heston_diffusion * Heston_diffusion_create(double Et
    a_,double Kappa_,double Rho_,
                                double Th
    eta_,double Sigma_,
                                double *
    jump_drift);

extern void Heston_diffusion_list(const Heston_diffusion *
    process);
extern dcomplex Heston_diffusion_characteristic_exponent(dc
    omplex u,double t,void * mod);
extern dcomplex Heston_diffusion_ln_characteristic_
    function(dcomplex u,double t,void * mod);

```

```

/*

dS_t = (r-q-{\lambda_y \mu}) S_t dt + {\sqrt{V_t}} S_t dW_t^1
      + J_y S_t dq_y(t)
dV_t = {\kappa_{\nu}} {\left( {\eta_{\nu}} + V_t \right) + {\theta_t}
      a_{\nu}} {\sqrt{V_t}} dW_t^2
dW^1 dW^2 = {\rho} dt

(1+J_y) is a lognormally distributed with mean {\mu_y} and
variance
{\sigma_y^2}

{q_y} is an independent Poisson process with arrival
rate
{\lambda_y}
{\mu = \left( {\exp\{{\mu_y} + {\sigma_y^2}/2\}} - 1 \right) \cdot}

*/

typedef struct _Bates_diffusion Bates_diffusion;

struct _Bates_diffusion
{
    double Eta;
    double Kappa;
    double Theta;
    double Rho;
    double Sigma;

    double mu_J;
    double Sigma_J;
    double Lambda_J;

    double sigma_sqr;
    double theta_sqr;
    double sigma_sqr_d_eta_kappa;
    double etakappathetam2;
    double rho_theta;
    double lnonepmuj;

```

```

double sigmaj_sqr_demi;
double Drift;
int nb_parameters;
};

extern Bates_diffusion * Bates_diffusion_create(double Eta_
, double Kappa_, double Rho_,
double Thet
a_, double Sigma_,
double mu_
J_,
double Si
gma_J_, double Lambda_J_, double *jump_drift);
extern dcomplex Bates_diffusion_characteristic_exponent(dc
omplex u, double t, void * mod);
extern dcomplex Bates_diffusion_ln_characteristic_function(
dcomplex u, double t, void * mod);
/*
@article {MR1841412,
  AUTHOR = {Barndorff-Nielsen, Ole E. and Shephard, Neil}
,
  TITLE = {Non-{G}aussian {0}rnstein-{U}hlenbeck-based
models and some of
their uses in financial economics},
  JOURNAL = {J. R. Stat. Soc. Ser. B Stat. Methodol.},
  FJOURNAL = {Journal of the Royal Statistical Society. Se
ries B.
Statistical Methodology},
  VOLUME = {63},
  YEAR = {2001},
  NUMBER = {2},
  PAGES = {167--241},
  ISSN = {1369-7412},
  MRCLASS = {62M07 (62M09 62M10 62P20)},
  MRNUMBER = {MR1841412 (2002c:62127)},
}

```

The square volatility follows the SDE of the form :

$$d\sigma_t^2 = -\lambda \sigma_t^2 dt + dZ_{\{\lambda t\}}$$

where  $\lambda > 0$  and  $Z$  is a subordinator.

The risk neutral **dynamic** of the log price  $x_t = \log S_t$  are given by

$$dW_t = (r - q - \frac{1}{2}\sigma^2) dt + \sigma dz_t,$$

$$x_0 = \log(S_0).$$

where  $k(u) = \log\{\mathbb{E}[\exp\{-u z_1\}]\}$ .

Choice  $z_t$  as a compound poisson process,

$$z_t = \sum_{n=1}^{N_t} x_n$$

where  $N_t$  is a Poisson process with intensity parameter  $\alpha$

and each  $x_n$  follows an exponential law with mean  $\frac{1}{\beta}$ .

One can show that the process  $\sigma_t^2$  is a stationary process with a marginal law that follows a Gamma distribution with mean  $\alpha$  and

variance  $\frac{\alpha}{\beta}$ . In this case,

$$k(u) = \frac{-\alpha}{\beta + u}.$$

\*/

```
typedef struct _BNS_diffusion BNS_diffusion;
```

```
struct _BNS_diffusion
```

```
{
```

```
    double Lambda;
```

```
    double Rho;
```

```
    double Beta;
```

```
    double Alpha;
```

```
    double Sigma0;
```

```
    double Sigma0_sqr ;
```

```
    double Lambda_m1;
```

```
    double Drift; // proportional to Drift correction
```

```
    int nb_parameters;
```

```
};
```

```
extern BNS_diffusion * BNS_diffusion_create(double Lambda_,
```

```
        double Rho_,
```

```
        double Beta_,
```

```

double Alpha_,
double Sigma0_,
double *jump_drift);
extern dcomplex BNS_diffusion_characteristic_exponent(dcomplex u, double t, void * mod);
extern dcomplex BNS_diffusion_ln_characteristic_function(dcomplex u, double t, void * mod);
extern void BNS_diffusion_list(const BNS_diffusion * processes);

/*
@article {MR1793362,
  AUTHOR = {Duffie, Darrell and Pan, Jun and Singleton, Kenneth},
  TITLE = {Transform analysis and asset pricing for affine jump-diffusions},
  JOURNAL = {Econometrica},
  FJOURNAL = {Econometrica. Journal of the Econometric Society},
  VOLUME = {68},
  YEAR = {2000},
  NUMBER = {6},
  PAGES = {1343--1376},
  ISSN = {0012-9682},
  CODEN = {ECMTA7},
  MRCLASS = {91B28 (60J60)},
  MRNUMBER = {MR1793362 (2001m:91081)},
}

dS_t = (r-q-{\lambda_y \mu}) S_t dt + {\sqrt{V_t}} S_t dW_t^1 +
J_y S_t dq_y(t) {{
dV_t = {\kappa_{\nu}} \left( {\eta_{\nu}} + V_t \right) + {\theta_{\nu}} {\sqrt{V_t}}
dW_t^2 + J_V dq_{\{\nu\}}(t)

dW^1 dW^2 = {\rho} dt

(1+J_y) is a lognormally distributed with mean $\mu_y$ and variance
$\sigma_y^2$

```

$J_V$  has an exponential distribution with mean  $\mu_{\nu}$

$q_y$  and  $q_{\nu}$  are independent Poisson process with arrivals rates

$\lambda_y$  and  $\lambda_{\nu}$

$\mu = \left( \exp\{\mu_y + \sigma_y^2/2\} - 1 \right)$

\*/

```
typedef struct _DPS_diffusion DPS_diffusion;
```

```
struct _DPS_diffusion
{
    double Eta;
    double Kappa;
    double Rho;
    double Theta;
    double Sigma;

    double mu_y;
    double Sigma_y_sqr_demi;
    double Lambda_y;

    double mu_v;
    double Lambda_v;

    double sigma_cy_sqr_demi;
    double mu_cy;
    double mu_cv;
    double Lambda_c;
    double rho_j;

    double s_lambda;

    double sigma_sqr;
    double theta_sqr;
    double sigma_sqr_d_eta_kappa;
```

```

double etakappathetam2;
double rho_theta;

double Drift;
int nb_parameters;
};

extern DPS_diffusion * DPS_diffusion_create(double Eta_,
double Kappa_,
double Rho_,
double Theta_,
double Sigma_,
double mu_y_,
double Sigma_y_,
double Lambda_y_,
double mu_v_,
double Lambda_v_,
double mu_cy_,
double Sigma_cy_,
double mu_cv_,
double Lambda_c_,
double rho_j_,
double *jump_drift);
extern dcomplex DPS_diffusion_characteristic_exponent(dcomplex u, double t, void * mod);
extern dcomplex DPS_diffusion_ln_characteristic_function(dcomplex u, double t, void * mod);
extern void DPS_diffusion_list(const DPS_diffusion * model)
;

/*
The two following class of model come from :

@article {MR1995283,
  AUTHOR = {Carr, Peter and Geman, H{{'e}}lyette and Madan, Dilip B. and Yor, Marc},
  TITLE = {Stochastic volatility for {L}{{'e}}vy processes}
,
  JOURNAL = {Math. Finance},

```

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FJOURNAL = {Mathematical Finance. An International Journal
  of Mathematics,
            Statistics and Financial Economics},
VOLUME = {13},
YEAR = {2003},
NUMBER = {3},
PAGES = {345--382},
ISSN = {0960-1627},
MRCLASS = {91B28 (60G51)},
MRNUMBER = {MR1995283 (2005a:91054)},
}

```

CIR stochastic clock

the CIR process can be use as rate of time change. it follows the SDE

$$dy_t = \{\kappa \left( \eta - y_t \right) dt + \{\lambda \sqrt{y_t} dW_t\}$$

```

*/
typedef struct _CIR_diffusion CIR_diffusion;

struct _CIR_diffusion
{
    double Kappa;
    double Eta;
    double Lambda;
    double y0;
    double Drift; // proportional to Drift correction
    double Kappa_sqr;
    double Lambda_sqr;
    double Kappa_sqr_eta_div_lambda_sqr;
    double Two_kappa_eta_div_lambda_sqr;

    double time;
    double Jump_drift;
    double Jump_drift_psi;
    int nb_parameters;
    void * Levy;
}

```



```

    dcomplex (*characteristic_exponent)(dcomplex u,void *
        mod);
};

extern CIR_diffusion * CIR_diffusion_create(double Kappa,
    double Eta,double Lambda,double y0,
    void * Levy_,
    dcomplex (*characteristic_exponent_)(dcomplex,void *),
    double *jump_drift);

extern void CIR_diffusion_list(const CIR_diffusion * process);

extern dcomplex CIR_diffusion_characteristic_exponent(dcomplex u,double t,void * mod);
extern dcomplex CIR_diffusion_ln_characteristic_function(dcomplex u,double t,void * mod);
extern double CIR_diffusion_get_sigma_square(CIR_diffusion *Process);
extern void CIR_diffusion_fourier_stiffness(CIR_diffusion * mod,double hx,double bnd_fourier,int Nw,int kmin,int kmax,int Dupire,PnlVect *row_stiffness);
extern void CIR_diffusion_update_time(CIR_diffusion * process,double t);
/*

Gamma- OU stochastic clock

the rate of time change is now solution of the SDE


$$dy_t = - \{\lambda y_t \, dt + d z_{\{\lambda t\}}\}.$$


Choice  $z_t$  as a compound poisson process,

$$z_t = \sum_{n=1}^{N_t} x_n$$

where  $N_t$  is a Poisson process with intensity parameter  $\alpha$ 
and each  $x_n$  follows an exponential law with mean  $\frac{1}{\beta}$ .
*/

```

```

typedef struct _GammaOU_diffusion GammaOU_diffusion;

struct _GammaOU_diffusion
{
    double Lambda;
    double Alpha;
    double Beta;
    double y0;
    double Drift; // proportional to Drift correction

    double Lambda_a;
    double Lambda_b;
    double y0_one_m_el_div_lambda;
    double y0_el;
    double one_m_el_div_lambda;
    double beta_el;

    double time;
    double Jump_drift;
    double Jump_drift_psi;
    int nb_parameters;
    void * Levy;
    dcomplex (*characteristic_exponent)(dcomplex u,void *
        mod);
};

extern GammaOU_diffusion * GammaOU_diffusion_create(double
    Lambda,double Alpha,double Beta,double y0,
                                                    void *
    Levy_,
                                                    dcompl
    ex (*characteristic_exponent_)(dcomplex,void *),
                                                    double
    *jump_drift);
extern dcomplex GammaOU_diffusion_characteristic_exponent(
    dcomplex u,double t,void * mod);
extern dcomplex GammaOU_diffusion_ln_characteristic_
    function(dcomplex u,double t,void * mod);
extern double GammaOU_diffusion_get_sigma_square(GammaOU_
    diffusion *Process);

```

```

extern void GammaOU_diffusion_fourier_stiffness(GammaOU_diffusion * mod, double hx, double bnd_fourier, int Nw, int kmin, int kmax, int Dupire, PnlVect *row_stiffness);
extern void GammaOU_diffusion_update_time(GammaOU_diffusion * process, double t);
extern void GammaOU_diffusion_list(const GammaOU_diffusion * process);

extern void test_CIR_diffusion(void );
extern void test_GammaOU_diffusion(void );

typedef struct _Levy_diffusion Levy_diffusion;

struct _Levy_diffusion
{
    void * process;
    int nb_parameters;
    int type_model;
    dcomplex (*characteristic_exponent)(dcomplex u, double t, void * mod);
    dcomplex (*ln_characteristic_function)(dcomplex u, double t, void * mod);
    // Artificial volatility term to come back to parabolic problem
    double vol_square;
};

extern Levy_diffusion * Levy_diffusion_create(void * process_, dcomplex (*characteristic_exponent_)(dcomplex u, double t, void * mod), dcomplex (*ln_characteristic_function_)(dcomplex u, double t, void * mod));
extern Levy_diffusion * Levy_diffusion_create_from_vect(int model, const double * input);
extern void Levy_diffusion_free(Levy_diffusion ** Levy);
extern dcomplex Levy_diffusion_characteristic_exponent(dcomplex u, double t, Levy_diffusion * mod);
extern double Levy_diffusion_get_sigma_square(

```

```
    Levy_diffusion *Levy);
extern void Levy_diffusion_fourier_stiffness(
    Levy_diffusion * mod,double t,double hx,double bnd_fourier,int Nw,int km
    in,int kmax,int Dupire,PnlVect *row_stiffness);
extern dcomplex Levy_diffusion_ln_characteristic_function(
    dcomplex u,double t,Levy_diffusion * mod);
extern dcomplex Levy_diffusion_ln_characteristic_function_w
    ith_cast(dcomplex u,double t,void * mod);
//extern dcomplex Levy_diffusion_characteristic_function(dc
    omplex u,double t,Levy_diffusion * mod);

extern void Levy_diffusion_constraints(PnlVect *res, const
    Levy_diffusion * Levy);

#endif
```

## References