cf_exchange

Let

- $T =$ maturity date $\quad (T > t)$
- $\lambda =$ exchange ratio
- $x1 =$ spot1 price
- $x2 =$ spot2 price
- $t =$ pricing date
- $\sigma 1 =$ volatility1
- $\sigma 2 =$ volatility2
- $\rho =$ correlation
- $r =$ interest rate
- $\delta 1 =$ dividend1 yields
- $\delta 2 =$ dividend2 yields
- $\theta = T - t$

Here, closed formulas due to Johnson and Stulz are presented [1],[2]. We set

- $d = \dfrac{\log \frac{x_1}{x_2} + \left(\delta_2 - \delta_1 + \frac{\sigma^2}{2}\right)\theta}{\sigma\sqrt{\theta}}$
- $\sigma = \sqrt{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2}$
- $d_i = \dfrac{\log\left(\frac{x_i}{K}\right) + \left(r - \delta_i + \frac{\sigma_i^2}{2}\right)\theta}{\sigma_1\sqrt{\theta}}, \quad i = 1, 2$
- $\rho_1 = \dfrac{\sigma_1 - \rho\sigma_2}{\sigma}$
- $\rho_2 = \dfrac{\sigma_2 - \rho\sigma_1}{\sigma}$

and M as the cumulative bivariate normal distribution function:

$$M(a,b;\rho) = \frac{1}{2\pi\sqrt{1-\rho^2}} \int_{-\infty}^{a} \int_{-\infty}^{b} e^{-\frac{x^2-2\rho xy+y^2}{2(1-\rho^2)}} dxdy.$$

Exchange Option

$$\text{PAYOFF} \qquad E_T = (S_T^1 - \lambda S_T^2))_+$$

$$\text{PRICE} \qquad E(t, x_1, x_2) = x_1 e^{-\delta_1 \theta} N(\widehat{d}_1) - \lambda x_2 e^{-\delta_2 \theta} N(\widehat{d}_2)$$

where

$$\widehat{d}_1 = \frac{\log\left(\frac{x_1}{\lambda x_2}\right) + \left(\delta_2 - \delta_1 + \frac{\sigma^2}{2}\right)\theta}{\sigma\sqrt{\theta}}, \qquad \widehat{d}_2 = \hat{d}_1 - \sigma\sqrt{\theta}$$

References

[1] H.JOHNSON. Options on the maximum ot the minimum of several assets. *J.Of Finance and Quantitative Analysis*, 22:227–283, 1987. 1

[2] R.STULZ. Options on the minimum or the maximum of two risky assets. *J. of Finance*, 10:161–185, 1992. 1