```
Help
/* We need Nd1 here */
#define USE_ND1
#include "bs1d_std.h"
#define AP JU Nmax 3
#define AP_JU_err 1e-7
#define AP JU Infinity 100.0
#define AP_JU_Neginfinity -100.0 /* AP_JU_Neginfinity for -
    infinity */
#define AP_JU_h 1e-4
/*Put Whaley Exponent*/
static double WhaleyPut_Exp(double r,double divid,double si
    gma, double T)
{
  double ratio = 2.0 * (r-divid)/(sigma * sigma);
  double delta = (ratio - 1.0);
  if(r==0.)
    delta=SQR(delta)+4.0*(2.0/(sigma*sigma))/T;
    delta=SQR(delta)+4.0*(2.0*r/(sigma*sigma))/(1.0-exp(-r*
    T));
  return 0.5*(1.-ratio-sqrt(delta));
}
static double Contact_PointPut(double r,double divid,
    double sigma,
             double T,double K, double (*exponent_
    method)(double,double,double,double))
  const double precision = 0.00001;
  double previous;
  double exponent = (*exponent method)(r,divid,sigma,T);
  double current = K;
  double put_price,put_delta;
```

do{

```
previous = current;
    pnl_cf_put_bs(previous,K,T,r,divid,sigma,
            &put_price,&put_delta);
    current=-exponent*(K-put_price)/((1.-exp(-divid*T)
              *Nd1(previous,r,divid,-sigma,T,K))-expon
    ent);
  }while(!(fabs((previous-current)/current)<=precision));</pre>
  return current;
}
double critical price (double r, double divid, double sigma,
           double T, double K)
{
  double x;
  r=(r!=0.?r:1e-6);
  x = Contact PointPut(r,divid,sigma,T,K,WhaleyPut Exp);
  return x;
}
/* Mathematical functions */
/*derivx */
static double deriv_x(double(*f)(double*),double *tab)
{
  double tmp1;
  tab[0]+=AP_JU_h;
  tmp1=(*f)(tab);
  tab[0]-=AP JU h;
  return (tmp1-(*f)(tab))/AP_JU_h;
}
/*derivy*/
static double deriv_y(double(*f)(double *),double *tab)
  double tmp1;
```

```
tab[1]+=AP JU h;
  tmp1=(*f)(tab);
  tab[1]-=AP_JU_h;
  return (tmp1-(*f)(tab))/AP_JU_h;
}
/*function d1*/
static double ap_ju_d1(double x, double y, double t,double
    r, double divid, double sigma)
{
  if (t!=0.)
    {
      return (log(x/y)+(r-divid)*t)/(sigma*sqrt(t))+sigma*
    sqrt(t)/2.;
    }
  else
      if (x==y)
    return 0.;
      else if ( x>y )
  {
    return AP_JU_Infinity;
    /* we take 100 for AP JU Infinity because N(100)=1=N(
    AP_JU_Infinity)*/
  }
      else
  {
    return AP JU Neginfinity;
    /* we take -100 for AP_JU_Neginfinity because N(-100)=
    O=N(-AP_JU_Infinity)*/
  }
    }
}
/* function I */
static double ap_ju_I(double t1, double t2, double x,
```

```
double y, double z, double Phi, double Nu, double r, double div
         id, double sigma)
double z1 = (r-divid-z+0.5*Phi*sigma*sigma)/sigma;
double z2 = log(x/y)/sigma;
double z3 = sqrt(z1*z1+2.*Nu);
double res;
double sqrtt1,sqrtt2;
sqrtt1=sqrt(t1);
sqrtt2=sqrt(t2);
if (t1!=0.)
         {
                 /* case t1 different of 0*/
                  res = exp(-Nu*t1)*cdf_nor(z1*sqrtt1+z2/sqrtt1)-exp(-
         Nu*t2)*cdf nor(z1*sqrtt2+z2/sqrtt2)+0.5*(z1/z3+1)*exp(z2*(z3
         -z1))*(cdf nor(z3*sqrtt2+z2/sqrtt2)-cdf nor(z3*sqrtt1+z2/
         sqrtt1)+0.5*(z1/z3-1)*exp(-z2*(z3+z1))*(cdf_nor(z3*sqrtt2-z2*(z3+z1))*(cdf_nor(z3*sqrtt2-z2*(z3+z1))*(cdf_nor(z3*sqrtt2-z2*(z3+z1))*(cdf_nor(z3*sqrtt2-z2*(z3+z1))*(cdf_nor(z3*sqrtt2-z2*(z3+z1))*(cdf_nor(z3*sqrtt2-z2*(z3+z1))*(cdf_nor(z3*sqrtt2-z2*(z3+z1))*(cdf_nor(z3*sqrtt2-z2*(z3+z1))*(cdf_nor(z3*sqrtt2-z2*(z3+z1))*(cdf_nor(z3*sqrtt2-z2*(z3+z1))*(cdf_nor(z3*sqrtt2-z2*(z3+z1))*(cdf_nor(z3*sqrtt2-z2*(z3+z1))*(cdf_nor(z3*sqrtt2-z2*(z3+z1))*(cdf_nor(z3*sqrtt2-z2*(z3+z1))*(cdf_nor(z3*sqrtt2-z2*(z3+z1))*(cdf_nor(z3*sqrtt2-z2*(z3+z1))*(cdf_nor(z3*sqrtt2-z2*(z3+z1))*(cdf_nor(z3*sqrtt2-z2*(z3+z1))*(cdf_nor(z3*sqrtt2-z2*(z3+z1))*(cdf_nor(z3*sqrtt2-z2*(z3+z1))*(cdf_nor(z3*sqrtt2-z2*(z3+z1))*(cdf_nor(z3*sqrtt2-z2*(z3+z1))*(cdf_nor(z3*sqrtt2-z2*(z3+z1))*(cdf_nor(z3*zqrtt2-z2*(z3+z1))*(cdf_nor(z3*zqrt)-z2*(z3+z1))*(cdf_nor(z3*zqrt)-z2*(z3+z1))*(cdf_nor(z3*zqrt)-z2*(z3+z1))*(cdf_nor(z3*zqrt)-z2*(z3+z1))*(cdf_nor(z3*zqrt)-z2*(z3+z1))*(cdf_nor(z3*zqrt)-z2*(z3+z1))*(cdf_nor(z3*zqrt)-z2*(z3+z1))*(cdf_nor(z3*zqrt)-z2*(z3+z1))*(cdf_nor(z3*zqrt)-z2*(z3+z1))*(cdf_nor(z3*zqrt)-z2*(z3+z1))*(cdf_nor(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*zqrt)-z2*(z3*
         z2/sqrtt2)-cdf nor(z3*sqrtt1-z2/sqrtt1));
         } else
         {
                  if (x==y)
         /* case
                                                      x=y ( i.e. z2=0 ) and t1=0 */
         res = 0.5-\exp(-Nu*t2)*cdf nor(z1*sqrtt2)+0.5*(z1/z3+1
         *(cdf nor(z3*sqrtt2)-0.5)+0.5*(z1/z3-1)*(cdf nor(z3*sqrtt))*(cdf nor(z3*sqrtt))*(cdf
         2)-0.5);
}
                  else if (x > y)
         /* case
                                                      x>y ( i.e. z2>0 ) and t1=0*/
         res = 1-\exp(-Nu*t2)*cdf nor(z1*sqrtt2+z2/sqrtt2)+0.5
         (z1/z3+1)*exp(z2*(z3-z1))*(cdf nor(z3* sqrtt2+z2/sqrtt2)-
         1)+0.5*(z1/z3-1)*exp(-z2*(z3+z1))*cdf nor(z3* sqrtt2-z2/
         sqrtt2);
}
                 else
{
         /* case x < y (i.e. z2 < 0) and t1 = 0 * /
        res = -\exp(-Nu*t2)*cdf_nor(z1*sqrtt2+z2/sqrtt2)+0.5
```

```
(z_1/z_3+1)*\exp(z_2*(z_3-z_1))*cdf nor(z3* sqrtt2+z2/ sqrtt2)+
    0.5*(z1/z3-1)*exp(-z2*(z3+z1))*(cdf nor(z3* sqrtt2-z2/sq
   rtt2)-1);
  }
  /*printf("%f %f{n",res,z1);*/
  return res;
}
/* function IS*/
static double ap_ju_IS(double t1, double t2, double x,
    double y, double z, double Phi, double Nu, double r, double div
    id, double sigma)
{
  double z1 = (r-divid-z+0.5*Phi*sigma*sigma)/sigma;
  double z2 = log(x/y)/sigma;
  double z3 = sqrt(z1*z1+2.*Nu);
  double res;
  double sqrtt1,sqrtt2;
  sqrtt1=sqrt(t1);
  sqrtt2=sqrt(t2);
  if (t1!=0.)
    {
      /* case t1 different of 0 */
      res = (exp(-Nu*t1)*pnl normal density(z1*sqrtt1+z2/sq
    rtt1)/sqrtt1-exp(-Nu*t2)*pnl_normal_density(z1*sqrtt2+z2/sq
    rtt2)/sqrtt2)/(sigma*x)+0.5*(z3-z1)*(z1/z3+1)*exp(z2*(z3-z1)
    )*(cdf_nor(z3*sqrtt2+z2/sqrtt2)-cdf_nor(z3*sqrtt1+z2/sqrtt
    1))/(sigma*x)+0.5*exp(z2*(z3-z1))*(z1/z3+1)*(pnl normal de
    nsity(z3*sqrtt2+z2/sqrtt2)/sqrtt2-pnl normal density(z3*sq
    rtt1+z2/sqrtt1)/sqrtt1)/(sigma*x)-0.5*exp(-z2*(z3+z1))*(z1/
    z3-1)*(cdf nor(z3*sqrtt2-z2/sqrtt2)-cdf nor(z3*sqrtt1-z2/sq
    rtt1)*(z3+z1)/(sigma*x)-0.5*exp(-z2*(z3+z1))*(z1/z3-1)*(pn
    1 normal density(z3*sqrtt2-z2/sqrtt2)/sqrtt2-pnl normal de
    nsity(z3*sqrtt1-z2/sqrtt1)/sqrtt1)/(sigma*x);
    }
  else
      if (x==y)
```

```
{
         /* case x=y ( i.e. z2=0 ) and t1=0 */
         res = -exp(-Nu*t2)*pnl_normal_density(z1*sqrtt2)/sqrtt
         2/(sigma*x)+0.5*(z3-z1)*(z1/z3+1)*(cdf nor(z3*sqrtt2)-1)/(
         sigma*x)+0.5*(z1/z3+1)*pnl normal density(z3*sqrtt2)/sqrtt2
         /(sigma*x)-0.5*(z1/z3-1)*cdf nor(z3*sqrtt2)*(z3+z1)/(sigma*x)
         *x)-0.5*(z1/z3-1)*pnl normal density(z3*sqrtt2)/sqrtt2/(si
         gma*x);
    }
              else if (x > y)
    {
         /* case x>y ( i.e. z2>0 ) and t1=0*/
         res = -exp(-Nu*t2)*pnl normal density(z1*sqrtt2+z2/sq
         rtt2)/sqrtt2/(sigma*x)+0.5*(z3-z1)*(z1/z3+1)*exp(z2*(z3-z1))
         *(cdf_nor(z3*sqrtt2+z2/sqrtt2)-1)/(sigma*x)+0.5*exp(z2*(z3))
         -z1))*(z1/z3+1)*pnl normal density(z3*sqrtt2+z2/sqrtt2)/sq
         rtt2/(sigma*x)-0.5*(z1/z3-1)*exp(-z2*(z3+z1))*cdf nor(z3*sq
         rtt2-z2/sqrtt2)*(z3+z1)/(sigma*x)-0.5*(z1/z3-1)*exp(-z2*(z3+z1)/sigma*x)-0.5*(z1/z3-1)*exp(-z2*(z3+z1)/sigma*x)-0.5*(z1/z3-1)*exp(-z2*(z3+z1)/sigma*x)-0.5*(z1/z3-1)*exp(-z2*(z3+z1)/sigma*x)-0.5*(z1/z3-1)*exp(-z2*(z3+z1)/sigma*x)-0.5*(z1/z3-1)*exp(-z2*(z3+z1)/sigma*x)-0.5*(z1/z3-1)*exp(-z2*(z3+z1)/sigma*x)-0.5*(z1/z3-1)*exp(-z2*(z3+z1)/sigma*x)-0.5*(z1/z3-1)*exp(-z2*(z3+z1)/sigma*x)-0.5*(z1/z3-1)*exp(-z2*(z3+z1)/sigma*x)-0.5*(z1/z3-1)*exp(-z2*(z3+z1)/sigma*x)-0.5*(z1/z3-1)*exp(-z2*(z3+z1)/sigma*x)-0.5*(z1/z3-1)*exp(-z2*(z3+z1)/sigma*x)-0.5*(z1/z3-1)*exp(-z2*(z3+z1)/sigma*x)-0.5*(z1/z3-1)*exp(-z2*(z3+z1)/sigma*x)-0.5*(z1/z3-1)*exp(-z2*(z3+z1)/sigma*x)-0.5*(z1/z3-1)*exp(-z2*(z3+z1)/sigma*x)-0.5*(z1/z3-1)*exp(-z2*(z3+z1)/sigma*x)-0.5*(z1/z3-1)*exp(-z2*(z3+z1)/sigma*x)-0.5*(z1/z3-1)*exp(-z2*(z3+z1)/sigma*x)-0.5*(z1/z3-1)*exp(-z2*(z3+z1)/sigma*x)-0.5*(z1/z3-1)*exp(-z2*(z3+z1)/sigma*x)-0.5*(z1/z3-1)*exp(-z2*(z3+z1)/sigma*x)-0.5*(z1/z3-1)*exp(-z2*(z3+z1)/sigma*x)-0.5*(z1/z3-1)*exp(-z2*(z3+z1)/sigma*x)-0.5*(z1/z3-1)*exp(-z2*(z3+z1)/sigma*x)-0.5*(z1/z3-1)*exp(-z2*(z1/z3-1)/sigma*x)-0.5*(z1/z3-1)*exp(-z2*(z1/z3-1)/sigma*x)-0.5*(z1/z3-1)*exp(-z2*(z1/z3-1)/sigma*x)-0.5*(z1/z3-1)*exp(-z2*(z1/z3-1)/sigma*x)-0.5*(z1/z3-1)*exp(-z2*(z1/z3-1)/sigma*x)-0.5*(z1/z3-1)*exp(-z2*(z1/z3-1)/sigma*x)-0.5*(z1/z3-1)*exp(-z2*(z1/z3-1)/sigma*x)-0.5*(z1/z3-1)*exp(-z2*(z1/z3-1)/sigma*x)-0.5*(z1/z3-1)*exp(-z2*(z1/z3-1)/sigma*x)-0.5*(z1/z3-1)*exp(-z2*(z1/z3-1)/sigma*x)-0.5*(z1/z3-1)*exp(-z2*(z1/z3-1)/sigma*x)-0.5*(z1/z3-1)*exp(-z2*(z1/z3-1)/sigma*x)-0.5*(z1/z3-1)*exp(-z1/z3-1)*exp(-z1/z3-1)*exp(-z1/z3-1)*exp(-z1/z3-1)*exp(-z1/z3-1)*exp(-z1/z3-1)*exp(-z1/z3-1)*exp(-z1/z3-1)*exp(-z1/z3-1)*exp(-z1/z3-1)*exp(-z1/z3-1)*exp(-z1/z3-1)*exp(-z1/z3-1)*exp(-z1/z3-1)*exp(-z1/z3-1)*exp(-z1/z3-1)*exp(-z1/z3-1)*exp(-z1/z3-1)*exp(-z1/z3-1)*exp(-z1/z3-1)*exp(-z1/z3-1)*exp(-z1/z3-1)*exp(-z1/z3-1)*exp(-z1/z3-1)*exp(-z1/z3-1)*exp(-z1/z3-1)*exp(-z1/z3-1)*exp(-z1/z3-1)*exp(-z1/z3-1)*exp(-z1/z3-1)*exp(-z1/z3-1)*exp(-z1/z3-1)*exp(-z1/z3-1)*exp(-z1/z3-1)*exp(-z1/z3-1)*
         z1))*pnl_normal_density(z3*sqrtt2-z2/sqrtt2)/sqrtt2/(sigma*
         x);
    }
              else
         /* case x < y (i.e. z2 < 0) and t1 = 0 * /
         res = -exp(-Nu*t2)*pnl normal density(z1*sqrtt2+z2/sq
         rtt2)/sqrtt2/(sigma*x)+0.5*(z1/z3+1)*(z3-z1)*exp(z2*(z3-z1))
         *cdf nor(z3*sqrtt2+z2/sqrtt2)/(sigma*x)+0.5*(z1/z3+1)*exp(
         z2*(z3-z1))*pnl normal density(z3*sqrtt2+z2/sqrtt2)/sqrtt2/
          (sigma*x)-0.5*(z1/z3-1)*exp(-z2*(z3+z1))*(cdf nor(z3*sqrtt))
         2-z2/sqrtt2)-1)*(z3+z1)/(sigma*x)-0.5*(z1/z3-1)*exp(-z2*(
         z3+z1))*pnl normal density(z3*sqrtt2-z2/sqrtt2)/sqrtt2/(si
         gma*x);
    }
    return res;
/* det*/
static int det(double(*f1)(double *),double(*f2)(double *),
         double *tab,double *d)
```

}

{

```
if (deriv x(f1,tab)*deriv y(f2,tab)-deriv x(f2,tab)*deriv
    _y(f1,tab)==0)
      return WRONG;
    }
  else
    {
      *d=deriv_x(f1,tab)*deriv_y(f2,tab)-deriv_x(f2,tab)*de
    riv_y(f1,tab);
      return OK;
    }
}
/* coefficients of the inverse of the jacobian matrix */
/* coefficient 00 */
static double InvJ_00(double(*f1)(double *),double(*f2)(
    double *),double *tab)
  double d;
  if (det(f1,f2,tab,&d)!=WRONG)
    return deriv y(f2,tab)/d;
  else return 0.;
}
/* coefficient 01 */
static double InvJ 01(double(*f1)(double *),double(*f2)(
    double *),double *tab)
  double d;
  if (det(f1,f2,tab,&d)!=WRONG)
    return -(deriv_y(f1,tab))/d;
  else return 0.;
/* coefficient 10 */
static double InvJ 10(double(*f1)(double *),double(*f2)(
    double *),double *tab)
{
  double d;
  if (det(f1,f2,tab,&d)!=WRONG)
    return -(deriv_x(f2,tab))/d;
  else return 0.;
```

```
}
/* coefficient 11 */
static double InvJ 11(double(*f1)(double *),double(*f2)(
    double *),double *tab)
{
  double d;
  if (det(f1,f2,tab,&d)!=WRONG)
    return deriv x(f1,tab)/d;
  else return 0.;
}
/* inverse of the jacobian matrix */
static void create InvJac(double(*InvJac[2][2])(double(*)(
    double*),double(*)(double*),double *))
  InvJac[0][0]=&InvJ 00;
  InvJac[0][1]=&InvJ_01;
  InvJac[1][0]=&InvJ_10;
  InvJac[1][1]=&InvJ 11;
}
/* method of Newton-Raphson */
static int Newton Raphson(double(*f1)(double *),double(*f2)
    (double *), double S, double K, double T, double r, double div
    id, double sigma, double *coeff B, double *coeff b, int type,
    double *x1,double *x2)
{
  double x[AP JU Nmax];
  double tab1[8];
  double tab2[10];
  double tab3[12]; /*=\{x[1],x[2],S,K,T,r,divid,sigma,x1[0],x\}
    1[0],x2[0],x2[0]};*/
  double d;
  double *adresse;
  double(*InvJac[2][2])(double(*)(double*),double(*)(
    double*),double *);
  double(*f[2])(double *);
  double first term, second term, f0 ad, f1 ad;
  x[0]=0.;
```

```
x[1]=x1[0];
x[2]=x2[0];
tab1[0]=x[1];
tab1[1]=x[2];
tab1[2]=S;
tab1[3]=K;
tab1[4]=T;
tab1[5]=r;
tab1[6]=divid;
tab1[7]=sigma;
tab2[0]=x[1];
tab2[1]=x[2];
tab2[2]=S;
tab2[3]=K;
tab2[4]=T;
tab2[5]=r;
tab2[6]=divid;
tab2[7]=sigma;
tab2[8]=x1[0];
tab2[9]=x2[0];
tab3[0]=x[1];
tab3[1]=x[2];
tab3[2]=S;
tab3[3]=K;
tab3[4]=T;
tab3[5]=r;
tab3[6]=divid;
tab3[7]=sigma;
tab3[8]=x1[0];
tab3[9]=x1[0];
tab3[10]=x2[0];
tab3[11]=x2[0];
create_InvJac(InvJac);
f[0]=f1;
f[1]=f2;
if(type==1)
```

```
{
    adresse=tab1;
else if(type==2)
    adresse=tab2;
  }
else
  {
   x[1]=x2[0];
   x[2]=x2[1];
    tab3[0]=x[1];
    tab3[1]=x[2];
    tab3[9]=x1[1];
    tab3[11]=x2[1];
    adresse=tab3;
  }
if(det(f1,f2,adresse,&d) == WRONG)
    return WRONG;
  }
else
  {
    f0_ad=f[0](adresse);f1_ad=f[1](adresse);
    first_term=InvJac[0][0](f1,f2,adresse)*f0_ad+InvJac[0
  ][1](f1,f2,adresse)*f1 ad;
    second_term=InvJac[1][0](f1,f2,adresse)*f0_ad+InvJac[
  1][1](f1,f2,adresse)*f1_ad;
    while ((fabs(first_term)>AP_JU_err) || (fabs(second_
  term)>AP_JU_err))
{
  x[1]-=first term;
  x[2]-=second_term;
  adresse[0]=x[1];
  adresse[1]=x[2];
  f0 ad=f[0](adresse);f1 ad=f[1](adresse);
  first_term=InvJac[0][0](f1,f2,adresse)*f0_ad+InvJac[0]
  [1](f1,f2,adresse)*f1_ad;
```

```
second term=InvJac[1][0](f1,f2,adresse)*f0 ad+InvJac[1
    ][1](f1,f2,adresse)*f1 ad;
  }
      *coeff B=x[1];
      *coeff b=x[2];
      return OK;
    }
}
/* APPROXIMATION BY ONE EXPONENTIAL */
/* Functions for which (B11,b11) is solution */
static double f1_11(double *tab)
 double B11=tab[0];
  double b11=tab[1];
  /* double S=tab[2]; */
 double K=tab[3];
  double T=tab[4];
  double r=tab[5];
  double divid=tab[6];
  double sigma=tab[7];
  double put_price, put_delta;
 pnl_cf_put_bs(B11,K,T,r,divid,sigma,&put_price,&put_delt
    a);
  return K-B11-put_price-K*(1-exp(-r*T))+B11*(1-exp(-divid*
    T))+K*ap_ju_I(0,T,B11,B11,b11,-1,r,r,divid,sigma)-B11*
    ap ju I(0,T,B11,B11,b11,1,divid,r,divid,sigma);
}
static double f2 11(double *tab)
  double B11=tab[0];
  double b11=tab[1];
  /* double S=tab[2]; */
  double K=tab[3];
  double T=tab[4];
```

```
double r=tab[5];
  double divid=tab[6];
  double sigma=tab[7];
  double exp_minus_divid_T=exp(-divid*T);
  return -1.+exp_minus_divid_T*cdf_nor(-ap_ju_d1(B11,K,T,r,
    divid,sigma))+(1.-exp_minus_divid_T)+K*ap_ju_IS(0,T,B11,B11,
    b11,-1,r,r,divid,sigma)-ap ju I(0,T,B11,B11,b11,1,divid,r,
    divid, sigma) -B11*ap_ju_IS(0,T,B11,B11,b11,1,divid,r,divid,si
    gma);
}
/*P1*/
static int ap_ju_pricing1(double S,double K,double T,
    double r,double divid,double sigma,double *P1)
  double B11,b11;
  double put_price,put_delta;
  double temp1[1];
  double temp2[1];
  temp1[0]=critical_price(r,divid,sigma,T,K);
  temp2[0]=0.;
  pnl_cf_put_bs(S,K,T,r,divid,sigma,&put_price,&put_delta);
  Newton Raphson(&f1 11,&f2 11,S,K,T,r,divid,sigma,&B11,&b1
    1,1,temp1,temp2);
  if (S<=B11)
    *P1=K-S;
  else
    *P1=put price+K*(1-exp(-r*T))-S*(1-exp(-divid*T))-K*
    ap_ju_I(0,T,S,B11,b11,-1,r,r,divid,sigma)+S*ap_ju_I(0,T,S,B11,b1
    1,1,divid,r,divid,sigma);
  return OK;
}
/* APPROXIMATION BY TWO EXPONENTIAL PIECES */
/* functions which (B21,b21) is solution */
```

```
static double f1 21(double *tab)
{
 double B21=tab[0];
  double b21=tab[1];
  /* double S=tab[2]; */
  double K=tab[3];
  double T=tab[4]:
  double r=tab[5];
  double divid=tab[6];
  double sigma=tab[7];
  double put price, put delta;
  double B21_exp=B21*exp(b21*T/2.);
 pnl_cf_put_bs(B21_exp,K,T/2,r,divid,sigma,&put_price,&
    put_delta);
 return K-B21 exp-put price-K*(1-exp(-r*T/2))+B21 exp*(1-
    \exp(-\text{divid}*T/2)) + K*ap_ju_I(0,T/2.,B21_exp,B21_exp,b21,-1,r,
    r,divid,sigma)-B21_exp*ap_ju_I(0,T/2,B21_exp,B21_exp,b21,1,
    divid,r,divid,sigma);
}
static double f2_21(double *tab)
 double B21=tab[0];
  double b21=tab[1];
  /* double S=tab[2];*/
  double K=tab[3];
  double T=tab[4];
  double r=tab[5];
  double divid=tab[6];
  double sigma=tab[7];
  double exp minus divid ToverTwo=exp(-divid*T/2.);
  double B21_exp=B21*exp(b21*T/2.);
  return -1.+exp_minus_divid_ToverTwo*cdf_nor(-ap_ju_d1(B21
    _exp,K,T/2.,r,divid,sigma))+(1.-exp_minus_divid_ToverTwo)+
    K*ap ju IS(0,T/2,B21 exp,B21 exp,b21,-1,r,r,divid,sigma)-
    ap ju I(0,T/2,B21*exp(b21*T/2),B21*exp(b21*T/2),b21,1,divid,r,
    divid, sigma) -B21_exp*ap_ju_IS(0,T/2.,B21_exp,B21_exp,b21,1,
```

```
divid, r, divid, sigma);
}
/* functions for which (B22,b22) is solution */
static double f1_22(double *tab)
  double B22=tab[0];
  double b22=tab[1];
  /* double S=tab[2]; */
  double K=tab[3];
  double T=tab[4];
  double r=tab[5];
  double divid=tab[6];
  double sigma=tab[7];
  double B21=tab[8];
  double b21=tab[9];
  double put_price, put_delta,value;
  pnl_cf_put_bs(B22,K,T,r,divid,sigma,&put_price,&put_delt
    a);
  value=K-B22-put_price-K*(1-exp(-r*T))+B22*(1-exp(-divid*
    T))+K*ap_ju_I(0,T/2.,B22,B22,b22,-1,r,r,divid,sigma)-B22*
    ap ju I(0,T/2,B22,B22,b22,1,divid,r,divid,sigma)+K*ap ju <math>I(T/2,B22,B22,b22,1,divid,r,divid,sigma)
    T,B22,B21,b21,-1,r,r,divid,sigma)-B22*ap_ju_I(T/2.,T,B22,B2)
    1,b21,1,divid,r,divid,sigma);
  return value;
}
static double f2 22(double *tab)
  double B22=tab[0];
  double b22=tab[1];
  /* double S=tab[2]; */
  double K=tab[3];
  double T=tab[4];
  double r=tab[5];
  double divid=tab[6];
  double sigma=tab[7];
```

```
double B21=tab[8];
  double b21=tab[9];
  double exp_minus_divid_T=exp(-divid*T);
  return -1.+exp minus divid T*cdf nor(-ap ju d1(B22,K,T,r,
    divid, sigma))+(1.-exp minus divid T)+K*ap ju IS(0,T/2.,B22,
    B22,b22,-1,r,r,divid,sigma)-ap_ju_I(0,T/2,B22,B22,b22,1,div
    id,r,divid,sigma)+K*ap ju IS(T/2,T,B22,B21,b21,-1,r,r,divid
    ,sigma)-B22*ap_ju_IS(0,T/2,B22,B22,b22,1,divid,r,divid,si
    gma)-ap_ju_I(T/2,T,B22,B21,b21,1,divid,r,divid,sigma)-B22*
    ap_ju_IS(T/2.,T,B22,B21,b21,1,divid,r,divid,sigma);
}
/*P2*/
static int ap_ju_pricing2(double S,double K,double T,
    double r,double divid,double sigma,double *P2)
{
  double B11,b11;
  double B21,b21;
  double B22,b22;
  double BT=MIN(K,divid!=0.?K*r/divid:K);
  double temp1[1];
  double temp2[1];
  double put_price,put_delta;
  temp1[0]=critical price(r,divid,sigma,T,K);
  temp2[0]=0.;
  if (fabs(BT-(*temp1))<0.05*BT)
    {
      Newton Raphson(&f1 11,&f2 11,S,K,T,r,divid,sigma,&B11
    ,&b11,1,temp1,temp2);
      Newton_Raphson(&f1_21,&f2_21,S,K,T,r,divid,sigma,&B21
    ,&b21,1,&B11,0);
      Newton Raphson(&f1 22,&f2 22,S,K,T,r,divid,sigma,&B22
    ,&b22,2,&B21,0);
    }
  else
    {
```

```
Newton Raphson(&f1 11,&f2_11,S,K,T,r,divid,sigma,&B11
             ,&b11,1,temp1,temp2);
                   Newton_Raphson(&f1_21,&f2_21,S,K,T,r,divid,sigma,&B21
             ,&b21,1,&B11,&b11);
                   Newton Raphson(&f1 22,&f2 22,S,K,T,r,divid,sigma,&B22
             ,&b22,2,&B21,&b21);
            }
      pnl_cf_put_bs(S,K,T,r,divid,sigma,&put_price,&put_delta);
      if (S<=B22)
            *P2=K-S;
      else
            *P2=put_price+K*(1-exp(-r*T))-S*(1-exp(-divid*T))-K*
            ap_{ju}I(0,T/2.,S,B22,b22,-1,r,r,divid,sigma)+S*ap_{ju}I(0,T/2.,S,
            B22,b22,1,divid,r,divid,sigma)-K*ap ju I(T/2,T,S,B21,b21,-1
             ,r,r,divid,sigma)+S*ap_ju_I(T/2,T,S,B21,b21,1,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,r,divid,
            id,sigma);
      return OK;
}
/* APPROXIMATION BY THREE EXPONENTIAL PIECES*/
/*functions for which (B31,b31) is solution */
static double f1 31(double *tab)
      double B31=tab[0];
      double b31=tab[1];
      /* double S=tab[2];*/
      double K=tab[3];
      double T=tab[4];
      double r=tab[5];
      double divid=tab[6];
      double sigma=tab[7];
      double put_price, put_delta,value;
      double B31_exp=B31*exp(2.*b31*T/3.);
      value=pnl_cf_put_bs(B31_exp,K,T/3,r,divid,sigma,&put_
```

```
price,&put delta);
  value=K-B31 exp-put price-K*(1.-exp(-r*T/3.))+B31 exp*(1-
    \exp(-\text{divid}*T/3.))+K*ap_ju_I(0,T/3.,B31_exp,B31_exp,b31,-1,r,
    r,divid,sigma)-B31_exp*ap_ju_I(0,T/3.,B31_exp,B31_exp,b31,1
    ,divid,r,divid,sigma);
  return value;
}
static double f2 31(double *tab)
  double B31=tab[0];
  double b31=tab[1];
  /* double S=tab[2]; */
  double K=tab[3];
  double T=tab[4];
  double r=tab[5];
  double divid=tab[6];
  double sigma=tab[7];
  double value;
  double exp_minus_divid_ToverThree=exp(-divid*T/3.);
  double B31 exp=B31*exp(2.*b31*T/3.);
  value=-1.+exp minus divid ToverThree*cdf nor(-ap ju d1(B3
    1 exp,K,T/3.,r,divid,sigma))+(1.-exp minus divid ToverThr
    ee)+K*ap_ju_IS(0,T/3.,B31_exp,B31_exp,b31,-1,r,r,divid,si
    gma)-ap_ju_I(0,T/3.,B31_exp,B31_exp,b31,1,divid,r,divid,si
    gma)-B31_exp*ap_ju_IS(0,T/3.,B31_exp,B31_exp,b31,1,divid,r,
    divid, sigma);
  return value;
}
/* functions for which (B32,b32) is solution */
static double f1_32(double *tab)
{
```

```
double B32=tab[0];
  double b32=tab[1];
  /* double S=tab[2];*/
  double K=tab[3];
  double T=tab[4];
  double r=tab[5];
  double divid=tab[6];
  double sigma=tab[7];
  double B31=tab[8];
  double b31=tab[9];
  double put_price, put_delta,value;
  double B31 exp=B31*exp(b31*T/3.);
  double B32 exp=B32*exp(b32*T/3.);
  double twoT_over_three=2*T/3;
  pnl_cf_put_bs(B32_exp,K,twoT_over_three,r,divid,sigma,&
    put price,&put delta);
  value=K-B32_exp-put_price-K*(1-exp(-r*twoT_over_three))+
    B32_exp*(1-exp(-divid*twoT_over_three))+K*ap_ju_I(0,T/3.,B3
    2 exp,B32 exp,b32,-1,r,r,divid,sigma)-B32 exp*ap ju I(0,T/
    3,B32 exp,B32 exp,b32,1,divid,r,divid,sigma)+K*ap ju I(T/3
    ,2*T/3,B32_exp,B31_exp,b31,-1,r,r,divid,sigma)-B32_exp*
    ap_ju_I(T/3,twoT_over_three,B32_exp,B31_exp,b31,1,divid,r,divid,
    sigma);
  return value;
}
static double f2_32(double *tab)
 double B32=tab[0];
  double b32=tab[1];
  /* double S=tab[2]; */
  double K=tab[3];
  double T=tab[4];
  double r=tab[5];
  double divid=tab[6];
  double sigma=tab[7];
  double B31=tab[8];
  double b31=tab[9];
```

```
double value;
  double exp minus divid twoToverThree=exp(-divid*2.*T/3.);
  double B31 exp=B31*exp(b31*T/3.);
  double B32 exp=B32*exp(b32*T/3.);
  double twoT over three=2*T/3;
  value=-1.+exp minus divid twoToverThree*cdf nor(-ap ju d1
    (B32_exp,K,twoT_over_three,r,divid,sigma))+(1.-exp_minus_
    divid_twoToverThree)+K*ap_ju_IS(0,T/3,B32_exp,B32_exp,b32,-1
    ,r,r,divid,sigma)-ap_ju_I(0,T/3,B32_exp,B32_exp,b32,1,div
    id,r,divid,sigma)+K*ap ju IS(T/3,2*T/3,B32 exp,B31 exp,b31,
    -1,r,r,divid,sigma)-B32_exp*ap_ju_IS(0,T/3,B32_exp,B32_exp
    ,b32,1,divid,r,divid,sigma)-ap_ju_I(T/3.,twoT_over_three,
    B32_exp,B31_exp,b31,1,divid,r,divid,sigma)-B32_exp*ap_ju_I
    S(T/3.,twoT_over_three,B32_exp,B31_exp,b31,1,divid,r,divid,
    sigma);
  return value;
}
/* functions for which (B33,b33) is solution */
static double f1 33(double *tab)
  double B33=tab[0];
  double b33=tab[1];
  /* double S=tab[2]; */
  double K=tab[3];
  double T=tab[4];
  double r=tab[5];
  double divid=tab[6];
  double sigma=tab[7];
  double B31=tab[8];
  double b31=tab[9];
  double B32=tab[10];
  double b32=tab[11];
  double put_price, put_delta,value;
  pnl_cf_put_bs(B33,K,T,r,divid,sigma,&put_price,&put_delt
    a);
```

```
value=K-B33-put price-K*(1-exp(-r*T))+B33*(1-exp(-divid*))
    T))+K*ap ju I(0,T/3,B33,B33,b33,-1,r,r,divid,sigma)-B33*
    ap_ju_I(0,T/3,B33,B33,b33,1,divid,r,divid,sigma)+K*ap_ju_I(T/3,2
    *T/3,B33,B32,b32,-1,r,r,divid,sigma)-B33*ap ju I(T/3,2*T/3
    ,B33,B32,b32,1,divid,r,divid,sigma)+K*ap ju I(2*T/3,T,B33,
    B31,b31,-1,r,r,divid,sigma)-B33*ap ju I(2*T/3,T,B33,B31,b31
    ,1,divid,r,divid,sigma);
  return value;
}
static double f2 33(double *tab)
  double B33=tab[0];
  double b33=tab[1];
  /* double S=tab[2];*/
  double K=tab[3];
  double T=tab[4];
  double r=tab[5];
  double divid=tab[6];
  double sigma=tab[7];
  double B31=tab[8];
  double b31=tab[9];
  double B32=tab[10];
  double b32=tab[11];
  double value:
  double exp minus divid T=exp(-divid*T);
  double twoT over three=2.*T/3.;
  value =-1.+exp minus divid T*cdf nor(-ap ju d1(B33,K,T,r,
    divid, sigma))+(1.-exp minus divid T)+K*ap ju IS(0,T/3.,B33,
    B33,b33,-1,r,r,divid,sigma)+K*ap ju IS(T/3.,twoT over three
    ,B33,B32,b32,-1,r,r,divid,sigma)+K*ap_ju_IS(twoT_over_th
    ree, T, B33, B31, b31, -1, r, r, divid, sigma) -ap ju I(0, T/3., B33, B33
    ,b33,1,divid,r,divid,sigma)-ap ju I(T/3.,twoT over three,
    B33,B32,b32,1,divid,r,divid,sigma)-ap_ju_I(twoT_over_three,
    T,B33,B31,b31,1,divid,r,divid,sigma)-B33*ap_ju_IS(0,T/3.,B3
    3,B33,b33,1,divid,r,divid,sigma)-B33*ap ju IS(T/3.,twoT ov
    er three, B33, B32, b32, 1, divid, r, divid, sigma) -B33*ap ju IS(tw
    oT_over_three, T, B33, B31, b31, 1, divid, r, divid, sigma);
```

```
return value;
}
/*P3*/
static int ap ju pricing3(double S, double K, double T,
    double r, double divid, double sigma, double *P3)
  double B11,b11;
  double B31,b31;
  double B32,b32;
  double B33,b33;
  double BT=MIN(K,divid!=0?K*r/divid:K);
  double temp1[1];
  double temp2[1];
  double temp3[2];
  double temp4[2];
  double put_price,put_delta;
  temp1[0]=critical price(r,divid,sigma,T,K);
  temp2[0]=0.;
  if (fabs(BT-(*temp1))<0.05*BT)
    {
      Newton Raphson(&f1 11,&f2 11,S,K,T,r,divid,sigma,&B11
    ,&b11,1,temp1,temp2);
      Newton Raphson(&f1 31,&f2 31,S,K,T,r,divid,sigma,&B31
    ,&b31,1,&B11,0);
      Newton_Raphson(&f1_32,&f2_32,S,K,T,r,divid,sigma,&B32
    ,&b32,2,&B31,0);
      temp3[0]=B31;
      temp3[1]=b31;
      temp4[0]=B32;
      temp4[1]=0;
      Newton_Raphson(&f1_33,&f2_33,S,K,T,r,divid,sigma,&B33
    , &b33,3,temp3,temp4);
  else
    {
      Newton_Raphson(&f1_11,&f2_11,S,K,T,r,divid,sigma,&B11
    ,&b11,1,temp1,temp2);
```

```
Newton Raphson(&f1 31,&f2 31,S,K,T,r,divid,sigma,&B31
    ,&b31,1,&B11,&b11);
     Newton_Raphson(&f1_32,&f2_32,S,K,T,r,divid,sigma,&B32
    ,&b32,2,&B31,&b31);
     temp3[0]=B31;
     temp3[1]=b31;
     temp4[0]=B32;
     temp4[1]=b32;
     Newton_Raphson(&f1_33,&f2_33,S,K,T,r,divid,sigma,&B33
    ,&b33,3,temp3,temp4);
   }
 pnl_cf_put_bs(S,K,T,r,divid,sigma,&put_price,&put_delta);
  if (S<=B33)
   *P3=K-S;
  else
   *P3=put_price+K*(1.-exp(-r*T))-S*(1.-exp(-divid*T))-K*
   3,b33,1,divid,r,divid,sigma)-K*ap_ju_I(T/3,2*T/3,S,B32,b32
    ,-1,r,r,divid,sigma)+S*ap_ju_I(T/3,2*T/3,S,B32,b32,1,divid)
    ,r,divid,sigma)-K*ap_ju_I(2*T/3,T,S,B31,b31,-1,r,r,divid,
   sigma)+S*ap ju I(2*T/3,T,S,B31,b31,1,divid,r,divid,sigma);
  return OK;
}
/*PRICING*/
static int PutAmer_Ju(double S,NumFunc_1 *p,double T,
   double r,double divid,double sigma,double *put price,double *
   put delta)
  double P1, P2, P3, K;
 double P1 h,P2 h,P3 h;
 K=p->Par[0].Val.V_DOUBLE;
  ap_ju_pricing1(S,K,T,r,divid,sigma,&P1);
  ap_ju_pricing2(S,K,T,r,divid,sigma,&P2);
  ap_ju_pricing3(S,K,T,r,divid,sigma,&P3);
```

```
ap_ju_pricing1(S+AP_JU_h,K,T,r,divid,sigma,&P1_h);
  ap_ju_pricing2(S+AP_JU_h,K,T,r,divid,sigma,&P2_h);
  ap_ju_pricing3(S+AP_JU_h,K,T,r,divid,sigma,&P3_h);
  /*Price*/
  *put_price=4.5*P3-4.*P2+0.5*P1;
  /**put price=2.*P2-P1;*/
  /*Delta*/
  *put_delta=((4.5*P3_h-4*P2_h+0.5*P1_h)-(*put_price))/
    AP JU h;
  /**put_delta=(2.*P2_h-P1_h-*put_price)/AP_JU_h;*/
 return OK;
}
int CALC(AP_Ju_PutAmer)(void *Opt,void *Mod,PricingMethod *
    Met)
{
  TYPEOPT* ptOpt=(TYPEOPT*)Opt;
  TYPEMOD* ptMod=(TYPEMOD*)Mod;
 double r, divid;
 r=log(1.+ptMod->R.Val.V DOUBLE/100.);
  divid=log(1.+ptMod->Divid.Val.V DOUBLE/100.);
  return PutAmer Ju(ptMod->SO.Val.V PDOUBLE,
        ptOpt->PayOff.Val.V_NUMFUNC_1,
        ptOpt->Maturity.Val.V_DATE-ptMod->T.Val.V_DATE,r,
    divid,
        ptMod->Sigma.Val.V PDOUBLE,
        &(Met->Res[0].Val.V DOUBLE),&(Met->Res[1].Val.V
    DOUBLE));
}
static int CHK_OPT(AP_Ju_PutAmer)(void *Opt, void *Mod)
  if (strcmp( ((Option*)Opt)->Name, "PutAmer")==0)
    return OK;
 return WRONG;
```

```
}
static int MET(Init)(PricingMethod *Met,Option *Opt)
  if ( Met->init == 0)
   {
      Met->init=1;
    }
 return OK;
PricingMethod MET(AP_Ju_PutAmer)=
  "AP_Ju_PutAmer",
 {{" ",PREMIA_NULLTYPE,{0},FORBID}},
  CALC(AP_Ju_PutAmer),
  {{"Price",DOUBLE,{100},FORBID},{"Delta",DOUBLE,{100},FORB
    ID} ,{" ",PREMIA_NULLTYPE,{0},FORBID}},
  CHK_OPT(AP_Ju_PutAmer),
 CHK_ok ,
 MET(Init)
};
```

References