```
Help
#include <stdlib.h>
#include "nig1d_pad.h"
#include "pnl/pnl_vector.h"
#include "pnl/pnl matrix.h"
#include "pnl/pnl_complex.h"
#include "pnl/pnl_mathtools.h"
#include "pnl/pnl fft.h"
#include "math/ap_fusai_levy/DiscreteAsianFMM.h"
#if defined(PremiaCurrentVersion) && PremiaCurrentVersion <</pre>
    (2012+2) //The "#else" part of the code will be freely av
   ailable after the (year of creation of this file + 2)
int CALC(AP_Asian_FMMNIG)(void *Opt,void *Mod,Pricing
   Method *Met)
return AVAILABLE IN FULL PREMIA;
}
#else
//-----
   _____
static dcomplex cfNIG(double dt, dcomplex g, PnlVect *Para
   meters)
  // NIG Characteristic Function
  //----
   ____
  double alpha, beta, delta;
  dcomplex term1, term2;
  dcomplex charexp;
  alpha=pnl_vect_get(Parameters,0);
  beta=pnl_vect_get(Parameters,1);
  delta=pnl_vect_get(Parameters,2)*dt;
  term1=Cmul(g,Complex(0,1));
  term1=Cadd(Complex(beta,0),term1);
  term1=Cpow real(term1,2);
  term1=Csub(Complex(SQR(alpha),0),term1);
```

```
term1=Csqrt(term1);
 term2=Complex(sqrt(SQR(alpha)-SQR(beta)),0);
 term1=Csub(term1,term2);
 term1=RCmul(-delta,term1);
 charexp=Cexp(term1);
 return charexp;
}
//----
   -----
static double MomentsNIG(int moment, double rf, double dt,
    PnlVect *Parameters)
{
 //----
 // compute moments of the NIG model
 //-----
 double alpha, beta, delta;
 double mom=0.;
 alpha=pnl_vect_get(Parameters,0);
 beta=pnl_vect_get(Parameters,1);
 delta=pnl_vect_get(Parameters,2);
 if(moment==1){
     mom=(dt*(beta*delta + (-SQR(alpha)+SQR(beta)+sqrt((
   SQR(alpha)-SQR(beta))*(SQR(alpha)-(1+beta)*(1+beta))))*delt
   a+sqrt(SQR(alpha)-SQR(beta))*rf))/sqrt(SQR(alpha)-SQR(beta)
   );
 }
 if(moment==2){
     mom=(SQR(beta)*delta*dt)/(POW(SQR(alpha)-SQR(beta),1
   .5))+(delta*dt)/sqrt(SQR(alpha)-SQR(beta)) + SQR(dt*((bet
   a*delta)/sqrt(SQR(alpha)-SQR(beta))-sqrt(SQR(alpha)-SQR(bet
   a))*delta+sqrt(SQR(alpha)-SQR(1+beta))*delta+rf));
 }
 return mom;
}
//-----
```

```
_____
static dcomplex charfunction(double r, double divid,
  double dt, dcomplex g, PnlVect *Parameters)
{ //-----
 // Levy Characteristic Function
 //----
 double m;
 dcomplex result, mdtg, temp;
 temp=cfNIG(dt,Complex(0.,-1.), Parameters);
 m = Creal(Csub(Complex((r-divid)*dt, 0.), Clog(temp)));
 mdtg = Cmul(Complex(0., m), g);
 temp=cfNIG(dt, g, Parameters);
 result=Cmul(Cexp(mdtg), temp);
 return result;
}//-----
  _____
static double BoundUpperTailLevy(double x, double rf,
  double divid, double dt, int maxmoment, PnlVect *Parameters)
 //----
 // compute upper truncation
 //----
 double minup, bound;
 int i;
 minup = 1.0;
 for(i = 1; i < maxmoment + 1; i++){
    bound = Creal(charfunction(rf, divid, dt, Complex(
  0,-i), Parameters))/exp(x*i);
    minup = MIN(minup, bound);
 return minup;
//-----
```

```
static double BoundLowerTailLevy(double x, double rf,
   double divid, double dt, int maxmoment, PnlVect *Parameters)
 //-----
 // compute lower truncation
 //-----
 double minlow, bound;
 int i;
 minlow = 1.0;
 for(i = 1; i < maxmoment + 1; i++){
    bound = Creal(charfunction(rf, divid, dt, Complex(
   0,i), Parameters))/exp(x*i);
    minlow = MIN(minlow, bound);
 return minlow;
}
   _____
static double findlowuplimit(double rf, double dt, PnlVect
   *Parameters)
{
 //----
 // Truncate the transition density domain
 //-----
 double mom1, mom2, levylow, levyup, bound;
 int maxnummoments, lowfactor, upfactor;
   maxnummoments=10;
 lowfactor=5;
 upfactor=5;
 mom1=MomentsNIG(1, rf, dt, Parameters);
 mom2=MomentsNIG(2, rf, dt, Parameters);
```

```
levylow=mom1-lowfactor*POW(mom2-mom1*mom1,0.5);
   bound=BoundLowerTailLevy(-levylow, rf, 0., dt, maxnumm
   oments, Parameters);
 while(bound>POW(10.0, -8.0))
   lowfactor=lowfactor+1;
   levylow=mom1-lowfactor*POW(mom2-mom1*mom1,0.5);
   bound=BoundLowerTailLevy(-levylow, rf, 0.,dt, maxnumm
   oments, Parameters);
 }
 levyup=mom1+upfactor*POW(mom2-mom1*mom1,0.5);
   bound=BoundUpperTailLevy(levyup, rf, 0., dt, maxnummom
   ents, Parameters);
 while(bound>POW(10.0, -8.0))
   upfactor=upfactor+1;
   levyup=mom1+upfactor*POW(mom2-mom1*mom1,0.5);
   bound=BoundUpperTailLevy(levyup, rf, 0., dt, maxnumm
   oments, Parameters);
 }
 return MAX(ABS(levylow),levyup);
static double truncate(double r, double divid, double dt,
   PnlVect *Parameters)
{
 //-----
 // find u for which cf(u)<10^-10
 //-----
 double abs cf, step, umax;
 dcomplex cf;
 step = 1.5;
 umax = 5.0;
 cf = charfunction(r, divid,dt, Complex(umax, 0), Para
```

```
meters);
 abs_cf = sqrt(Creal(cf)*Creal(cf) + Cimag(cf)*Cimag(cf))
 while (abs cf > POW(10., -10.))
 {
   umax = umax * step;
   cf = charfunction(r, divid, dt, Complex(umax, 0),
   Parameters);
   abs_cf = sqrt(Creal(cf)*Creal(cf) + Cimag(cf)*Cimag(
   cf)); //compute abs error
 }
 return (umax + umax / step) / 2;
}
//-----
   _____
static void kernel(double r, double divid, double dt, lon
   g N, double b, PnlVect *Parameters, PnlVect *inv, PnlVect *
   logk)
{
 //-----
   _____
 // Compute the transition density function by using the
   Fractional Fourier Transform
 //-----
   _____
 int j;
 double wj,eta,alpha,dx,umax;
 dcomplex term1,ft,aa,a,cgyz;
 PnlVectComplex *y1, *y2;
 y1=pnl_vect_complex_create(2*N);
 y2=pnl_vect_complex_create(2*N);
 // bound of characteristic function grid
 umax = truncate(r, divid, dt, Parameters);
 // bound of the density function grid
 b=b*1.25;
```

```
// grids'steps
eta = umax / N;
dx = 2 * b / N;
alpha = eta * dx / (2 * M PI);
for (j=0; j<=N-1; j++)
      // trapezoidal quadrature weights
      if((j == 0) || j == (N - 1)){}
            wj = 0.5*eta;
      }
      else {
            wj = eta;
      a = Complex(cos(SQR(j) * alpha * M_PI), sin(SQR(j) *
      alpha * M_PI));
      pnl_vect_complex_set(y2,j,a);
      aa = Complex(cos(SQR(N-j) * alpha * M_PI), sin(SQR(N-j) * alpha 
      j) * alpha * M_PI));
      pnl vect complex set(y2, j+N,aa);
      ft = charfunction(r, divid, dt, Complex(j * eta, 0),
      Parameters);
      term1 = Cexp(Complex(0,b*eta* j));
      ft = Cmul(term1, ft);
      pnl_vect_complex_set(y1,j,Cmul(RCdiv(wj,a),ft));
      pnl_vect_complex_set(y1,j+N,CZERO);
}
pnl_fft_inplace(y1);
pnl_fft_inplace(y2);
pnl_vect_complex_mult_vect_term(y1,y2);
//FFT inversion
pnl_ifft_inplace(y1);
for( j = 0; j \le N - 1; j++)
      a = Complex(cos(SQR(j) * alpha * M_PI), sin(SQR(j) *
      alpha * M_PI));
      cgyz = Cdiv(pnl vect complex get(y1,j),CRmul(a,M PI));
      pnl_vect_set(logk,j,-b+j*dx);
      pnl_vect_set(inv,j,Creal(cgyz));
```

```
pnl vect complex free(&y1);
 pnl_vect_complex_free(&y2);
//-----
   -----
static int FMMNIG Asian(double pseudo stock, double pseudo
   strike, NumFunc 2 *po, double t, double r, double divid, double
   sigma, double theta, double kappa, int Mdates, int N, double *pt
   price,double *ptdelta)
{
 int flagCP=0,asian type=0;
 //----
 //Compute price and delta of an Asian call option under
   the NIG process
 // RECURSIVE PROCEDURE
 //----
   _____
 //Recursive approach proposed in
 //Fusai, Marazzina, Marena, SIAM JOURNAL OF FINANCIAL
   MATHEMATICS, 2011
 //-----
 int c,i,j,Ni,start,count,flag,startcol,max len b;
 long nfft;
 double dt,low,up,b,x,y,xy,h,price,delta;
 double alphaNIG,betaNIG,deltaNIG,temp;
 PnlVect *CoeffLambda, *abscissa, *weights, *a_temp, *w_
   temp, *xdens, *dens, *Parameters, *vector, *vector1;
 PnlMat *Kmatrix;
 //-----
   _____
//Call Fixed
 if((po->Compute) == &Call_OverSpot2)
   {flagCP=0;
    asian type=0;
 //Put Fixed
```

```
else if((po->Compute) == &Put_OverSpot2)
  { flagCP=1;
    asian_type=0;
//Call Floating
else if((po->Compute) == &Call StrikeSpot2)
  { flagCP=0;
    asian_type=1;
 //Put Floating
else if((po->Compute) == &Put_StrikeSpot2)
  { flagCP=1;
    asian_type=1;
  }
 temp=1/(kappa*SQR(sigma));
betaNIG=theta*kappa*temp;
alphaNIG=sqrt(SQR(betaNIG)+temp);
deltaNIG=1/(kappa*sqrt(temp));
Parameters=pnl_vect_create_from_list(3,alphaNIG,betaNIG,
  deltaNIG);
// STEP O: PREPARE GRID AND COEFFICIENTS
//-- payoff coefficients
if (asian type==0) {
   CoeffLambda=pnl vect create from double(2,1./(Mdates+1
  ));
   pnl vect set(CoeffLambda,0,pnl vect get(CoeffLambda,0)
  -pseudo strike/pseudo stock);
   c=0;
}
else{
   CoeffLambda=pnl_vect_create_from_double(2,-1./(Mdates+
  1));
   c=-1;
}
//-- price grid
low=-(3./2+30./Mdates); //lower bound
up=pnl_vect_get(CoeffLambda,1); //upper bound
//-- generate abscissa and weights for quadrature
if (asian_type==0){
Ni=N; //number of nodes for x<0
```

```
N=Ni+N; //total number of nodes
abscissa=pnl vect create from zero(N);
weights=pnl_vect_create_from_zero(N);
a temp=pnl vect create from zero(Ni);
w temp=pnl vect create from zero(Ni);
gauleg pn(low,0,a temp,w temp,Ni);
for (i=0;i<Ni;i++){
  x=pnl vect get(a temp,i);
  y=pnl_vect_get(w_temp,i);
  pnl_vect_set(abscissa,i,x);
  pnl vect set(weights,i,y);
  //--- we consider the same number of nodes
  pnl vect set(abscissa,i+Ni,(x-low)*up/(-low));
  pnl_vect_set(weights,i+Ni, y*up/(-low));
}
pnl_vect_free(&a_temp);
pnl_vect_free(&w_temp);
}
else{
abscissa=pnl vect create from zero(N);
weights=pnl vect create from zero(N);
gauleg_pn(low,up,abscissa,weights,N);
Ni=N;
}
//-- time grid
dt=t/Mdates;
//-- compute the transition density
b=findlowuplimit(r,dt,Parameters);
nfft=32768;
dens=pnl vect create from zero(nfft);//contains the dens
xdens=pnl_vect_create_from_zero(nfft);//contains the ab
  scissa of the density
kernel(r,divid,dt,nfft,b,Parameters,dens,xdens);
// STEP 2: CREATE MATRICES AND VECTORS
vector=pnl vect create from zero(N);
for (j=0; j<=N-1; j++) {
  x=pnl_vect_get(abscissa,j)-(double)c;
```

```
if (x>0.0)
    pnl vect set(vector, j, x);
}
// MATRIX
startcol=0; max len b=0; count=0;
Kmatrix=pnl_mat_create_from_double(Ni,N+2,0.0);
for (i=0; i<=Ni-1; i++){
  flag=0; start=0;
  x=pnl_vect_get(abscissa,i);
        for (j=startcol; j<=N-1; j++){
    y=pnl vect get(abscissa,j);
    xy=log(x/(y-pnl_vect_get(CoeffLambda,1)));
    if (ABS(xy) \le b){
      if (flag==0){
        flag=1; // start to fill the row
        pnl mat set(Kmatrix,i,0,j); // first element
  of the band
        startcol=j;
        count=1;
      }
      count=count+1;
      xy=MAX(interp_lin(xy, nfft, &start, xdens, dens)
  ,0.0);
      pnl_mat_set(Kmatrix,i,count,-exp(-r*dt)*xy*(x/SQ
  R(y-pnl vect get(CoeffLambda,1))));
      if (j==N-1){ // stop to fill the row (since it
  is finished)
      pnl mat set(Kmatrix,i,1,(N-1)-(int)pnl mat get(
  Kmatrix,i,0)+1); // length of the band
      max_len_b=MAX(max_len_b,(int)pnl_mat_get(Kmatr
  ix,i,1));
    }else if (flag==1){ // stop to fill the row
      pnl mat set(Kmatrix,i,1,(j-1)-(int)pnl mat get(
  Kmatrix,i,0)+1); // length of the band
      max_len_b=MAX(max_len_b,(int)pnl_mat_get(Kmatrix
  ,i,1));
     break;
   }
   }
```

```
}
 // STEP 3: RECURSIVE APPROACH
 vector1=pnl vect create from zero(Ni);
 if (asian type==0){
   for(i=0;i<Mdates;i++){</pre>
     pnl vect mult vect term(vector, weights);
     bmat mult vect(Kmatrix, vector, vector1, Ni, N+2);
     for (j=0;j<Ni;j++)
        pnl_vect_set(vector,j,pnl_vect_get(vector1,j));
     xy=pnl vect get(CoeffLambda,1)*exp((r-divid)*dt)*(1
   -\exp((i+1)*(r-divid)*dt))/(1-\exp((r-divid)*dt));
     for (j=Ni;j<N;j++)</pre>
           pnl_vect_set(vector,j,exp(-r*(i+1)*dt)*(pnl_vec
   t_get(abscissa,j)+xy));
   }
}else{
   for(i=0;i<Mdates;i++){</pre>
     pnl vect mult vect_term(vector, weights);
     bmat mult vect(Kmatrix, vector, vector1, Ni, N+2);
         pnl_vect_clone(vector, vector1);
   }
 }
 xy=interp lin1(pnl vect get(CoeffLambda, 0), N, abscissa, vec
price=pseudo stock*xy;
 if (asian type==0){
   h=exp((up-low)/N);
   x=(pseudo stock-h)*interp lin1(pnl vect get(CoeffLambd
   a,1)-pseudo strike/(pseudo stock-h),N,abscissa,vector);
   y=(pseudo stock+h)*interp lin1(pnl vect get(CoeffLambd
   a,1)-pseudo_strike/(pseudo_stock+h),N,abscissa,vector);
  xy=(y-x)/(2*h);
 }
 delta=xy;
 if (flagCP==1){
   price=price-pseudo_stock*(pnl_vect_get(CoeffLambda,0)*
   exp(-r*Mdates*dt)-c);
```

```
delta=delta-(pnl vect get(CoeffLambda,1)*exp(-r*Mdates*
   dt)-c):
 for(i=0;i<Mdates;i++)</pre>
   price=price-pseudo stock*pnl vect get(CoeffLambda,1)*
   exp(-r*i*dt);
   delta=delta-pnl_vect_get(CoeffLambda,1)*exp(-r*i*dt);
 }
 *ptprice=price;
 *ptdelta=delta;
 //----DESTROY-----
   _____
 pnl_vect_free(&xdens);
 pnl_vect_free(&dens);
 pnl vect free(&Parameters);
 pnl_vect_free(&vector);
 pnl_vect_free(&vector1);
 pnl mat free(&Kmatrix);
 pnl_vect_free(&abscissa);
 pnl_vect_free(&weights);
 pnl_vect_free(&CoeffLambda);
 return OK;
int CALC(AP_Asian_FMMNIG)(void *Opt, void *Mod, Pricing
   Method *Met)
 TYPEOPT* ptOpt=(TYPEOPT*)Opt;
 TYPEMOD* ptMod=(TYPEMOD*)Mod;
 int return_value;
 double r, divid, time spent, pseudo spot, pseudo strike;
 double t 0, T 0;
 r=log(1.+ptMod->R.Val.V_DOUBLE/100.);
 divid=log(1.+ptMod->Divid.Val.V DOUBLE/100.);
 T_0 = ptMod->T.Val.V_DATE;
```

}

```
t O= (ptOpt->PathDep.Val.V NUMFUNC 2)->Par[0].Val.V PDOUB
   LE:
 if(T 0 < t 0)
   {
     Fprintf(TOSCREEN, "T_0 < t_0, untreated case{n\{n\{n''\}\};
     return value = WRONG;
 /* Case t_0 <= T_0 */
 else
   {
     time_spent=(ptMod->T.Val.V_DATE-(ptOpt->PathDep.Val.
   V_NUMFUNC_2)->Par[0].Val.V_PDOUBLE)/(ptOpt->Maturity.Val.V_
   DATE-(ptOpt->PathDep.Val.V_NUMFUNC_2)->Par[0].Val.V_PDOUB
     pseudo spot=(1.-time spent)*ptMod->SO.Val.V PDOUBLE;
     pseudo_strike=(ptOpt->PayOff.Val.V_NUMFUNC_2)->Par[0]
    .Val.V_PDOUBLE-time_spent*(ptOpt->PathDep.Val.V_NUMFUNC_2)
   ->Par[4].Val.V PDOUBLE;
 return_value= FMMNIG_Asian(pseudo_spot,pseudo_strike,pt
   Opt->PayOff.Val.V_NUMFUNC_2,ptOpt->Maturity.Val.V_DATE-ptMod-
   >T.Val.V_DATE,r,divid,ptMod->Sigma.Val.V_PDOUBLE,ptMod->Th
   eta.Val.V_DOUBLE,ptMod->Kappa.Val.V_SPDOUBLE,Met->Par[0].
   Val.V INT2,Met->Par[1].Val.V INT2,&(Met->Res[0].Val.V
   DOUBLE),&(Met->Res[1].Val.V DOUBLE));
 return return_value;
}
#endif //PremiaCurrentVersion
static int CHK_OPT(AP_Asian_FMMNIG)(void *Opt, void *Mod)
   if ((strcmp(((Option*)Opt)->Name, "AsianCallFixedEuro")
   ==0) || (strcmp( ((Option*)Opt)->Name, "AsianPutFixedEuro")
   return OK;
 return WRONG;
}
```

```
static int MET(Init)(PricingMethod *Met,Option *Opt)
{
   if ( Met->init == 0)
      Met->init=1;
      Met->Par[0].Val.V_INT2=12;
      Met->Par[1].Val.V INT2=3000;
  return OK;
}
PricingMethod MET(AP_Asian_FMMNIG)=
  "AP_Asian_FMM_NIG",
  {{"Nb.of Monitoring Dates",INT2,{2000},ALLOW },
   {"Nb.of Integration Points ",INT2,{1000},ALLOW},
   {" ",PREMIA_NULLTYPE, {O}, FORBID}},
  CALC(AP_Asian_FMMNIG),
    {{"Price",DOUBLE,{100},FORBID},{"Delta",DOUBLE,{100},FO
    RBID} ,{" ",PREMIA_NULLTYPE,{0},FORBID}},
  CHK_OPT(AP_Asian_FMMNIG),
  CHK_ok,
  MET(Init)
};
```

References