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```
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# tr\_ritchken\_out

#### Input parameters:

• StepNumber N

### Output parameters:

- Price
- Delta

This routine is taken from [1]. It is a modification of the Down Out Ritchken trinomial tree (cf Routine tr\_ritchken\_downout.c) designed to handle an upper and lower barrier. As described in the article, the idea is the following: choose first the stretch parameter within the tree to reach exactly one of the barrier (here we choose the upper barrier). Then the other (lower here) barrier will in general cross one level of meshes of the tree. For the meshes at this level, move the son node near the barrier on it. The mesh is modified, therefore it is necessary to ensure the local consistency conditions to modify also the risk-neutral probability (if you don't do that, then it is as if you were changing the volatility and/or dividend rate of the underlying at this level) The correction factor is given in the article: assume the three nodes at the critical mesh are given (in log scale) by  $\lambda \sigma \sqrt{h}$ 

$$\begin{cases} \lambda \sigma \sqrt{h} \\ 0 \\ -\gamma \lambda \sigma \sqrt{h} \end{cases}$$

where  $1 \le \gamma < 2$  Then the corresponding probability is

$$\begin{cases} p_{du} = \frac{b+a\gamma}{1+\gamma} \\ p_{dm} = 1. - p_d - p_u \\ p_{dd} = \frac{b-a}{\gamma(1+\gamma)} \end{cases}$$

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with 
$$a=\frac{\left(r-divid-\frac{\sigma^2}{2}\right)\sqrt{h}}{\lambda\sigma}$$
 and  $b=\frac{1}{\lambda^2}$ .  
The convergence follows from Kusnher's theorem, numerical results are good.

The convergence follows from Kushher's theorem, numerical results are good. The same restriction holds as in the single barrier Ritchken algorithm: the case of a Barrier too close to the underlying at a given N is not handled.

/\*return values: 0-ok 1-unable to allocate memory 2-barrier l to close to s\*/

/\*Price, intrinsic value arrays\*/

Since this is a flat tree we store the intrinsic values in an array  ${\tt iv}$ . The array  ${\tt G}$  will hold the values of the price at the next (forward) time step.

/\*ASSUMES THE BARRIER ARE CONSTANTS\*/
Here we get the Barriers and rebate values from the parameters.

/\*Up and Down factors\*/

These are the Rithken up and down factors computed to hit exactly the upper barrier.

/\*Discounted Probability\*/

The corresponding Ritchken trinomial risk-neutral probability (it is computed there).

They are denoted here  $p_{uu}, p_{um}, p_{ud}$ .

/\*Stretching factor gamma\*/

The factor  $\gamma$  for the lower node above the lower barrier to be on the barrier.

/\*One of the Barrier is too close (ie breached at the first mesh)\*/
Then the routine returns 2.

/\*Corrected Discounted Probability for the breaching downard mesh\*/
The above  $p_{du}, p_{dm}, p_{dd}$ .

/\*Intrinsic value initialization and terminal values\*/
The lower Barrier is at AO down moves from SO, the upper Barrier at etaO
up moves. We start the indexing from below, so npoints=etaO+AO is the
index of the upper Barrier.

/\*Backward Resolution\*/
Two parts:

/\*First Part: At least one of the Barrier is active\*/

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/\*First Case: the first (forward) active Barrier is the lower Barrier\*/ This corresponds to A0<=eta0 (all right?). Then we begin with the time region where:

/\*The two Barriers are active\*/

In this area the tree grid is rectangular, so **npoints** is not decreased at each time step backward. The index 0 is for the node just above the barrier.

#### Then:

/\*Only the Lower Barrier is active\*/

The same as above, but there is one point less at each time step backward.

/\*Second Case: the first (forward) active Barrier is the upper Barrier\*/
This corresponds to A0>eta0 (still arguing?).

/\*The two Barriers are active\*/
Same as above.

/\*Only the upper Barrier is active\*/

No special treatment for the lower node is needed any longer, so we don't need the auxiliary array G. This is the standard Backard Cycle in Routine tr\_ritchken\_downout.c.

/\*Second Part: None of the Barriers are active\*/
This is from now on a standard Kamrad-Ritchken tree, cf
Routine tr\_kamradritchken\_bs.c.

```
/*Delta*/
/*First time step*/
/*Price*/
/*Memory Desallocation*/
```

## References

[1] P.RITCHKEN. On pricing barrier options. *Journal Of Derivatives*, pages 19–28, Winter 95 1995. 1