Source | Model | Option
| Model_Option | Help on mc methods | Archived Tests

mc_antithetic

Input parameters:

- Number of iterations N
- Generator_Type
- Increment inc
- Confidence Value

Output parameters:

- Price P
- Error Price σ_P
- Delta δ
- Error delta σ_δ
- Price Confidence Interval: IC_P =[Inf Price, Sup Price]
- Delta Confidence Interval: IC_δ =[Inf Delta, Sup Delta]

Description:
Computation for a Call - Put - CallSpread or Digit European Option of its Price and its Delta with the Antithetic Variables method with Monte Carlo or Quasi-Monte Carlo simulation. In the case of Monte Carlo simulation, the method also provides an estimation for the integration error and a confidence interval.

The underlying asset price evolves according to the Black and Scholes model, that is:

$$dS_u = S_u((r-d)du + \sigma dB_u), \quad S_{T-t} = s$$

then

$$S_T = s \exp\left((r - d - \frac{\sigma^2}{2})t\right) \exp(\sigma B_t)$$

where S_T denotes the spot at maturity T, s is the initial spot, t is the time to maturity.

The Price of an option at $T - t$ is:

$$P = E\left[\exp(-rt) f(K, S_T, R)\right]$$

where f denotes the payoff of the option, K the strike and R the rebate (for Digit option only).
The Delta is given by:

$$\delta = \frac{\partial}{\partial s} E[\exp(-rt) f(K, S_T, R)]$$

Estimators with antithetic variables are expressed as:

$$\widetilde{P} = \frac{1}{2N} \exp(-rt) \sum_{i=1}^{N} (P(i) + P'(i))$$

$$\widetilde{\delta} = \frac{1}{N} \exp(-rt) \sum_{i=1}^{N} \frac{\partial}{\partial s} P(i) = \frac{1}{2N} \exp(-rt) \sum_{i=1}^{N} (\delta(i) + \delta'(i))$$

$P(i)$ and $P'(i)$ are antithetic payoffs obtained from $S_T(i)$ and $S'_T(i)$ given by:

$$\begin{cases} S_T(i) = s \exp\left[\left(r - d - \frac{\sigma^2}{2}\right) t\right] \exp(\sigma B_t(i)) \\ S'_T(i) = s \exp\left[\left(r - d - \frac{\sigma^2}{2}\right) t\right] \exp(\sigma B'_t(i)) \end{cases}$$

where $B_t(i) = \sqrt{t} g_i$ and $B'_t(i) = -\sqrt{t} g_i$ are antithetic brownian motions. g_i is a standard gaussian variable.
Values for $P(i)$ and $\delta(i)$ are detailed for each option in the following points.
Values for $P'(i)$ and $\delta'(i)$ are obtained in the same way.

- **Put**: The payoff is $(K - S_T)^+$. We have:

$$P(i) = (K - S_T(i))^+$$

$$\delta(i) = \begin{cases} -\frac{\partial S_T(i)}{\partial s} = -\frac{S_T(i)}{s} & \text{if} \quad P(i) > 0 \\ 0 & \text{otherwise} \end{cases}$$

- **Call**: The payoff is $(S_T - K)^+$.
  The *Call-Put Parity* relations for price and delta are expressed by:

  $$C = P + s\exp(-dt) - K\exp(-rt)$$

  $$\delta_C = \delta_P + \exp(-dt)$$

  where C and P respectively denotes the Call and the Put prices.
  They will be used for a Call simulation (which corresponds to a method of control variate and leads generally to a reduced variance for the estimator).

- **CallSpread**: The payoff is $(S_T - K_1)^+ - (S_T - K_2)^+$.
  We have:

  $$P(i) = \left[(S_T(i) - K_1)^+ - (S_T(i) - K_2)^+\right]$$

  $$\delta(i) = \begin{cases} \frac{\partial S_T(i)}{\partial s} = \frac{S_T(i)}{s} & \text{if} \quad S_T(i) > K_1 \quad \text{and} \quad S_T(i) < K_2 \\ -\frac{\partial S_T(i)}{\partial s} = -\frac{S_T(i)}{s} & \text{if} \quad S_T(i) > K_2 \quad \text{and} \quad S_T(i) < K_1 \\ 0 & \text{otherwise} \end{cases}$$

- **Digit**: The payoff is $R1_{\{S_T - K \geq 0\}}$.
  We have:

  $$P(i) = R1_{\{S_T(i) - K \geq 0\}}$$

  To have an estimation of the Delta in the case of a Digit option, we need to use the increment value *inc* at each iteration i as:

  $$\delta_i = \begin{cases} \frac{R}{2s \cdot inc} & \text{if} \quad S_T(i)(s(1+inc)) > K \quad \text{and} \quad S_T(i)(s(1-inc)) < K \\ 0 & \text{otherwise} \end{cases}$$

  $S_T(i)(s(1+inc))$ is the spot value at T with initial value $s(1+inc)$.
  $S_T(i)(s(1+inc))$ and $S_T(i)(s(1-inc))$ are computed with the same brownian motion for each iteration. Thus we always have $S_T(i)(s(1+inc)) > S_T(i)(s(1-inc))$ and there is only one case for which $delta_i > 0$.

Algorithm:

/* Value to construct the confidence interval */
For example if the confidence value is equal to 95% then the value z_α used to construct the confidence interval is 1.96. This parameter is taken into account only for MC simulation and not for QMC simulation.
/*Initialization*/
/*Call-Spread*/

Strike K_1 and K_2 used for a Call-Spread option.
/*Median forward stock and delta values*/
Computation of intermediate values we use several times in the program.
/* Change a Call into a Put to apply the Call-Put parity */
In case of Call, we modify parameters of the option; they will be reinitialized at the end of the simulation program. Simulation will be done as for a put.

- /*MC sampling*/

Initialization of the simulation: generator type, dimension, size N of the sample
/* Test after initialization for the generator */
Test if the dimension of the simulation is compatible with the selected generator. (See remarks on QMC simulation, especially on dimension of low-discrepancy sequences). For standard Monte Carlo in the one-dimensional Black and Scholes model, we never have any problem with the dimension, fixed to 1 at the beginning of the programm.
Definition of a parameter which exprimes if we realize a MC or QMC simulation. Some differences then appear in the algorithm: for simulation of a gaussian variable and in results in the simulation.

/* Begin N iterations */

- /* Simulation of a gaussian variable according to the generator type, that is Monte Carlo or Quasi Monte Carlo. */
Call to the appropriate function to generate a standard gaussian variable. See the part about simulation of random variables for explanations on this point. We just recall that for a MC simulation, we use the Gauss-Abramovitz algorithm, and for a QMC simulation we use an inverse method.

- /*Price*/
At the iteration i, we obtain $S_T(i)$ and $S'_T(i)$ from a simulation according to the selected generator of antithetics brownian motions $B_t(i)$ as $\sqrt{t}g_i$ and $B'_t(i)$ as $-\sqrt{t}g_i$ where g_i is a standard gaussian variable.

$$P(i) = \text{Payoff}(S_T(i), K)$$

$$P'(i) = \text{Payoff}(S'_T(i), K)$$

- /*Delta*/
Calculation of Delta δ_i and δ'_i with formula previously detailed for each option.

/*Digit*/
/*CallSpread*/
/*Call-Put*/

/*Sum*/
Computation of the sums $\sum \frac{1}{2}(P(i)+P'(i))$ and $\sum \frac{1}{2}(\delta(i)+\delta'(i))$ for the mean price and the mean delta.

/*Sum of squares*/
Computation of the sums $\sum(P(i)+P'(i))^2$ and $\sum(\delta(i)+\delta'(i))^2$ necessary for the variance price and the variance delta estimations. (finally only used for MC estimation)

/* End N iterations */

- /*Price*/

The price estimator is:

$$P = \frac{1}{N}\exp(-rt)\sum_{i=1}^{N}\frac{1}{2}\left(P(i)+P'(i)\right)$$

The error estimator is σ_P with :

$$\sigma_P^2 = \frac{1}{N-1}\left(\frac{1}{N}\exp(-2rt)\sum_{i=1}^{N}\frac{1}{4}\left(P(i)+P'(i)\right)^2 - P^2\right)$$

- /*Delta*/

$$\delta = \frac{1}{N}\exp(-rt)\sum_{i=1}^{N}\frac{1}{2}\left(\delta(i)+\delta'(i)\right)$$

The error estimator is σ_δ with:

$$\sigma_\delta^2 = \frac{1}{N-1}\left(\frac{1}{N}\exp(-2rt)\sum_{i=1}^{N}\frac{1}{4}\left(\delta(i)+\delta'(i)\right)^2 - \delta^2\right)$$

- /* Call Price and Delta with the Call Put Parity */

We now compute the price and the delta in case of a call, because call was considered as a put until now.
Parameters of the option are reinitialized. This step is necessary: if you want to begin an other call simulation just after a first one with the antithetic

method, the parameters must have been modified to specify that we really consider a Call and not a Put.

- /* Price Confidence Interval */

The confidence interval is given as:

$$IC_P = [P - z_\alpha \sigma_P ; P + z_\alpha \sigma_P]$$

with z_α computed from the confidence value.

- /* Delta Confidence Interval */

The confidence interval is given as:

$$IC_\delta = [\delta - z_\alpha \sigma_\delta ; \delta + z_\alpha \sigma_\delta]$$

with z_α computed from the confidence value.

Confidence intervals are always computed, but for a QMC simulation they don't work, thus they don't appear in the results.

References