```
Help
#include <stdlib.h>
#include "hk1d stdi.h"
#include "mathsb.h"
#include "currentzcb.h"
#include "hktree.h"
static char init[]="initialyield.dat";
/*Swaption=Option on Coupon-Bearing Bond*/
/*All details comments for the functions used here are mai
   nly in "hwtree1dincludes.h" and partially in this file*/
static void HK_iterations( int flat_flag, double r_flat,
   char* init, double a, double sigma_HW,
                      double TO, double per, int m,
   double KO, int xnumber, discrete_fct *N);
// computes V_O(K), the current Hull-White-price of the
   digital (T,S)-caplet with strike K
/*static double HW DigitalCaplet(double a0, double sigma0,
   double T, double S, double tau0, double POT, double POS, double
   K)
 double sigma P, log term;
 sigma_P = sigma0 * (exp(-a0*T) - exp(-a0*S))/a0 * sqrt( (
   \exp(2*a0*T)-1)/(2*a0));
 log term = log(POT / POS / (tau0*K+1));
 return tau0 * POS * cdf_nor( log_term/sigma_P - sigma_P/2
    );
 }*/
```

```
#if defined(PremiaCurrentVersion) && PremiaCurrentVersion <</pre>
    (2007+2) //The "#else" part of the code will be freely av
   ailable after the (year of creation of this file + 2)
static int CHK OPT(TR SWAPTION)(void *Opt, void *Mod)
 return NONACTIVE;
int CALC(TR SWAPTION)(void *Opt, void *Mod, PricingMethod *
 return AVAILABLE IN FULL PREMIA;
#else
// solves V O(K)=x, where V O(K) is the current Hull-White-
   price of the digital (T,S)-caplet with strike K
static double Inv HW DigitalCaplet(double a0, double sigma0
   , double T, double S, double tau0, double POT, double POS,
    double x)
 double tau0 inv, sigma P, exp term, K;
 tau0_inv = 1/tau0;
 sigma_P = sigma0 * (exp(-a0*T) - exp(-a0*S))/a0 * sqrt( (
   \exp(2*a0*T)-1)/(2*a0));
 exp term = exp(-SQR(sigma P)/2 - sigma P * pnl inv cdf
   nor( x / (tau0*POS) ));
 K = tau0_inv*POT/POS * exp_term - tau0_inv;
 /*if (K<O) printf("Inv HW DigitalCaplet returns the negat</pre>
   ive strike K = %f\{n'', K\}; */
```

```
return K;
// solves V 0^{m-1},HK(K) = V 0^{m-1},HW(K) in terms of si
  gma_HK, where V_0^{m-1},HK(K) resp. V_0^{m-1},HK(K)
// is the current Hunt-Kennedy-price resp. Hull-White
  price of the digital (T {m-1},T m)-caplet with strike K
static double ComputeSigma HK(double a, double sigma HW,
  double Tm 1, double Tm, double tau, double POTm 1, double POTm,
  double K)
{
 double sigma_HK, ROm_1, p, q, sigma_P;
 ROm 1 = (POTm 1/POTm - 1.) / tau;
 sigma_P = sigma_HW * (exp(-a*Tm_1) - exp(-a*Tm))/a * sq
  rt( (\exp(2.*a*Tm 1)-1)/(2.*a));
 p = 2. * log((ROm 1+tau)/(K+tau)) / sigma P - sigma
 q = -2. * log(ROm 1/K);
 sigma HK = (sqrt(SQR(p)/4. - q) - p/2.) / sqrt((exp(2.
  *a*Tm 1)-1)/(2.*a));
 return sigma HK;
// functional form of the INVERSE of the numeraire at T[m-1
  ], i.e. of 1/P(T_{m-1}, T_m)
static double N mminus1(double x, double C 2)
{
```

```
return 1 + C 2*exp(x);
// functional form of the INVERSE of the numeraire at T[m-2]
   ], i.e. of 1/P(T_{m-2}, T_m)
/*static double N_mminus2(double a_HW, double sigma_HW,
   double T_mminus2, double T_mminus1, double tau_mminus2, double
   POT mminus2,
 double POT_mminus1, double POT_m, double C_0, double C_1,
   double Sig, double x)
 {
 double result, J_term, P_term, V0_market_inv;
 P term = 1 + C 0 * exp(x);
 J_term = POT_m * tau_mminus2 * ( cdf_nor(-x/Sig) + C_1*
   cdf nor(-x/Sig+Sig) );
 VO_market_inv = Inv_HW_DigitalCaplet( a_HW, sigma_HW, T_
   mminus2, T_mminus1, tau_mminus2, POT_mminus2, POT_mminus1,
   J term);
 result = P term * (1 + tau mminus2 * V0 market inv);
 return result;
 */
///
// returns the variance of the HK-process x t given x s
///
static double SigmaSqr( double t, double s, double sigma,
  double a)
{
 return SQR(sigma) * (exp(2.*a*t) - exp(2.*a*s))/(2*a);
```

```
}
/////
// returns (U \{t,s\}f)(x), where U is the semigroup of operators
// corresponding to the HK-process
//////
static double U(double t, double s, discrete fct *f,
   double x, double sigma, double a)
{
 return NormalTab( x, SigmaSqr(t,s,sigma,a), f);
static void SetUf(discrete fct *g, double t, double s, dis
   crete fct *f, double sigma, double a)
    // Sets g = U_{t,s}f in a reasonable way
 SetNf( g, SigmaSqr(t,s,sigma,a), f);
static double UfUpBound (discrete_fct *f, double t, double
   s, double vmax, double sigma, double a)
    // returns the minimum of all x \ge f.x  such that U {
   t,s{f*1 {(x,infty)})(0) < vmax
 return NfUpBound ( f, SigmaSqr(t,s,sigma,a), vmax);
}
static double UfLoBound (discrete_fct *f, double t, double
   s, double vmin, double sigma, double a)
```

// returns the maximum of all x<f.right such that $U_{\{}$

t,s{ $(f*1_{(x,infty)})(0) > vmin$

```
return NfLoBound ( f, SigmaSqr(t,s,sigma,a), vmin);
static void HK iterations (int flat flag, double r flat,
    char* init, double a, double sigma HW,
                           double TO, double per, int m,
    double KO, int xnumber, discrete fct *N)
                       : flag to decide wether initial yi
// flat flag
    eld curve is flat at r_flat (0) or read from the file init (
    1)
// a, sigma HW
                      : parameters of the HW-model repres
    enting the market ("a" is common with the HK-process)
// TO
                       : first HK-date
// per
                       : HK-periodicity
// m
                       : number of HK-dates
// KO
                       : calibration strike (for the compu
   tation of sigma_HK)
// N
                       : functional forms of the INVERSE of
    the numeraire at T[i], i.e. of 1/P(T i, T m), for i=0,...,
   m-1
// xnumber
                       : parameter for the discretization
    of the functional forms
{
  double *T;
                                /* HK-dates */
                                 /* tau[i] = year fraction
  double *tau;
    from T[i] to T[i+1] */
  double *P0;
                                /* PO[i] = P(0,T[i]) (ini
    tial zcb prices) */
  double **Sigma;
                                 /* corresponds to Sigma {
    T[i],T[j-1], where T[-1] denotes 0 */
  /* here SQR(Sigma_{t,s}) is the variance of x_t given x_s
    s */
  double C 2;
                                 /* corresponds to the cons
    tant C 2 in the formula for N[m-1] */
  double x, s, result, J term, P term, VO market inv;
  double xle, xste, xri, eps;
  double sigma_HK;
                          /* parameter of the HK-proces */
```

```
double xleft,xstep;
                        /* parameters for the discretiz
  ation of the functional forms */
int i,j,k;
discrete fct Ptilde i, Ptilde ix;
// initialisation of the main variables //
T = malloc((m+1)*sizeof(double));
for (i=0; i \le m; i++) T[i] = i * per + T0;
// T[0]=T0
                 : first resetting date of the swap
// T[1],...,T[m] : payment dates of the swap
// T[0],...,T[n-1] : exercise dates of the swaption
  ---> We suppose m>=n !!
tau = malloc(m*sizeof(double));
for(i=0; i<m; i++) tau[i]=per;</pre>
P0 = malloc((m+1)*sizeof(double));
for(i=0; i<=m; i++) PO[i] = CurrentZCB(T[i], flat_flag,</pre>
  r flat, init);
// computation of sigma_HK
sigma_HK = ComputeSigma_HK(a, sigma_HW, T[m-1], T[m], ta
  u[m-1], PO[m-1], PO[m], KO);
Sigma = malloc(m*sizeof(double*));
for(i=0; i<m; i++)
  {
    Sigma[i] = malloc((i+1)*sizeof(double));
    for(j=0; j<=i; j++)
     {
       if (j==0) s=0; else s=T[j-1];
```

```
Sigma[i][j] = sigma HK * sqrt((exp(2*a*T[i])-exp)
 (2*a*s)) / (2*a) );
    }
 }
// constant in the formula for N[m-1]
C = (P0[m-1]/P0[m] - 1) * exp(-SQR(Sigma[m-1][0])/2);
// initialization of N[m-1] (for which we have an explic
 it formula !) //
xleft = -SQR(Sigma[m-1][0]) - Sigma[m-1][0]*sqrt(40.);
xstep = 2*fabs(xleft)/(double)xnumber;
Set discrete fct( &N[m-1], xleft, xstep, xnumber);
for(j=0; j<N[m-1].xnumber; <math>j++)
   x = N[m-1].xleft + j*N[m-1].xstep;
   N[m-1].val[j] = N mminus1(x, C2);
/*
  printf("N[%d]{n",m-1); ShowDiscreteFct( &N[m-1] );
  sprintf(filename, "N[%d].txt", m-1);
  SaveDiscreteFctToFile( &N[m-1], filename);
  printf("{ninitial discounted ZCB price for maturity T[
 d= f : \{n', m-1, T[m-1]\};
  printf("HK: %f{n", U(T[m-1], 0., &N[m-1], 0., sigma
 HK, a));
  printf("HW: %f{n{n", PO[m-1]/PO[m]);
  */
// iterative computation of the N[m-2], ..., N[0] //
```

```
for(i=m-2; i>=0; i--)
   // printf("beginning for i=%d{n", i); // scanf("%d",
 &j);
   // setting of P^tilde_i := U_{T[i+1],T[i]} N[i+1] //
   SetUf( &Ptilde_i, T[i+1], T[i], &N[i+1], sigma_HK, a)
   // sets Ptilde i such that domain( Ptilde i ) = [ U {
 T[i+1], T[i] > N[i+1] > 0
   /*
     printf("Ptilde i{n"); ShowDiscreteFct( &Ptilde i )
      sprintf(filename, "Ptilde[%d].txt", i);
      SaveDiscreteFctToFile( &Ptilde_i, filename);
      printf("{ninitial discounted ZCB price for maturit
 y T[%d] = %f : {n'', i+1, T[i+1]};
      printf("HK: %f{n", U(T[i], 0., &Ptilde i, 0., si
 gma_HK, a) );
      printf("HW: %f{n{n", P0[i+1]/P0[m]);
      */
   // setting of N[i] //
   eps = 0.000001;
   xle = UfUpBound( &Ptilde i, T[i], 0., P0[i+1]/P0[m]-
 eps, sigma HK, a);
   xri = UfLoBound( &Ptilde_i, T[i], 0., eps, sigma_HK,
 a);
   xste=(xri-xle)/(double)(xnumber-1);
   Set discrete fct( &N[i], xle, xste, xnumber);
   // printf("N[%d]{n",i); ShowDiscreteFct( &N[i] );
```

```
// initialization of P^tilde_{i,x} as a (restricted)
copy of P^tilde i //
 Set_discrete_fct( &Ptilde_ix, N[i].xleft, N[i].xstep,
N[i].xnumber+1);
 for(j=0; j<Ptilde ix.xnumber; j++)</pre>
    x = Ptilde_ix.xleft + j*Ptilde_ix.xstep;
    Ptilde ix.val[j] = U( T[i+1], T[i], &N[i+1], x,
sigma HK, a);
   }
 // evaluation of N i in its discretizing points N[i].
xleft + j*N[i].xstep //
 for(j=0; j<N[i].xnumber; j++)</pre>
    x = N[i].xleft + j*N[i].xstep;
                             // observe: x =
Ptilde ix.xleft + j*Ptilde ix.xstep
    P_term = Ptilde_ix.val[j];
                             // hence: P ter
m = P i^tilde(x) !!!
    Ptilde ix.val[j] = 0;
    // VERY IMPORTANT: now Ptilde ix corresponds rea
lly to P^{tilde}_{i,x} := P^{tilde}_{i} * 1_{(x,infty)} !!!
    J_{term} = PO[m] * tau[i] * U(T[i], O., &Ptilde_ix
, 0., sigma HK, a);
    if (J_term==0) {printf("At j=%d: J_term=0 !{n",
j); scanf("%d", &k); ShowDiscreteFctVal( &Ptilde ix ); }
    if (J term/(tau[i]*P0[i+1])>=1)
      {printf("At j=%d: J_term too large !{n", j);
```

```
scanf("%d", &k);}
         V0_market_inv = Inv_HW_DigitalCaplet( a, sigma_
   HW, T[i], T[i+1], tau[i], PO[i], PO[i+1], J term);
         result = P_term * ( 1 + tau[i] * V0_market_inv);
         // now we have: result = N i(x)
         N[i].val[j] = result;
       }
     /*
        printf("eval. of N[%d] in its discret. points fini
   shed{n", i);}
        sprintf(filename, "N[%d].txt", i);
        SaveDiscreteFctToFile( &N[i], filename);
        printf("end for i=%d{n{n",i);
        */
     Delete discrete fct(&Ptilde i);
     Delete_discrete_fct(&Ptilde_ix);
   } // end of i-loop
 // free the variables
 free(T);
  free(tau);
  free(P0);
 for(i=0; i<m; i++) free(Sigma[i]);</pre>
 free(Sigma);
 // end of: free the variables //
  static int swaption_hk1d(int flat_flag,double a,double t0,
```

```
double sigma HW, double r flat, double T final, double TO, NumFunc 1
     *p,int am,double Nominal,double K,double per,long N step,
    int xnumber,double *price/*,double *delta*/)
{
  // flat_flag
                         : flag to decide wether initial yi
    eld curve is flat at r flat (0) or read from the file init (
    1)
  // a, sigma HW
                         : parameters of the HW-model rep
    resenting the market ("a" is common with the HK-process)
  // TO
                         : first reset date of the swap (=
    first HK-date)
  // per
                         : reset period of the swap (= HK-
   period)
  // n
                         : number of exercise dates of the
    swaption
                         : number of payment dates of the
    swap (= number of HK-dates)
  // t0
                         : time for which the swaption
    price is computed
  // payer
                         : payer swaption (1) or receiver
    swaption (0)
  // K
                         : strike of the swaption
  // Nominal
                         : nominal value of the swap
                         : number of time steps in the tre
  // N step
    e for the HK-process
  // xnumber
                         : parameter for the discretization
     of the functional forms
  int n;
  double *T;
                                 /* reset/payment dates of
    the swap; exercise dates of the Bermudan swaption; HK-da
    tes */
                                 /* tau[i] = year fraction
  double *tau;
    from T[i] to T[i+1] */
  double **Sigma;
                                 /* corresponds to Sigma {
    T[i],T[j-1], where T[-1] denotes 0 */
  /* here SQR(Sigma_{t,s}) is the variance of x_t given x_
    s */
  discrete fct *N;
                                 /* functional forms of th
    e INVERSE of the numeraire at T[i], */
```

```
/* i.e. of 1/P(T i, T m), for i=0,...,m-1 */
                              /* tree for the HK-proces
struct Tree Tr;
 s */
double x, s, **disc payoff, PtildeTiTk, calib strike, dis
  c_price;
                                      /* parameter of
double sigma HK;
  the HK-proces */
int i,j,k,*ind,*Size,payer_sign;
int m,payer;
/* n ==1 for European case*/
m=(int)((T_final-T0)/per);
if ((p->Compute) ==&Put)
 payer=1;
else
  /*if ((p->Compute) == &Call) */
 payer=0;
// initialisation of the main variables //
T = malloc((m+1)*sizeof(double));
for (i=0; i \le m; i++) T[i] = i * per + T0;
            : first resetting date of the swap
// T[0]=T0
// T[1],...,T[m] : payment dates of the swap
// T[0],...,T[n-1] : exercise dates of the swaption
  ---> We suppose m>=n !!
tau = malloc(m*sizeof(double));
for(i=0; i<m; i++) tau[i]=per;</pre>
// comput. of sigma_HK via calibr. to the digital (T[m-1]
  ,T[m])-caplet
// calib strike = (P0[m-1]/P0[m]-1)/tau[m-1]; // at the
  money case
calib_strike = K;
```

```
sigma HK = ComputeSigma HK(a, sigma HW, T[m-1], T[m], ta
 u[m-1],
                        CurrentZCB(T[m-1], flat_flag,
 r flat, init),
                        CurrentZCB(T[m] , flat flag,
 r flat, init), calib strike);
Sigma = malloc(m*sizeof(double*));
for(i=0; i<m; i++)
 {
   Sigma[i] = malloc((i+1)*sizeof(double));
   for(j=0; j<=i; j++)
     {
       if (j==0) s=0; else s=T[j-1];
       Sigma[i][j] = sigma_HK * sqrt( (exp(2*a*T[i])-exp
  (2*a*s)) / (2*a) );
     }
 }
// functional forms of the INVERSE of the numeraire at T[
 i], i.e. of 1/P(T_i,T_m), for i=0,...,m-1
N = malloc(m*sizeof(discrete fct));
// end of: initialisation of the main variables //
///////
// construction of a trinomial tree for the HK-process
       //
// the last exercise date T[n-1] is the final time of th
 e tree, N_{step} is the number of time steps
SetTimegrid( &Tr, T[n-1], N step );
```

```
// add (if necessary) the exercise dates T[0], ..., T[n-2]
  to the time grid of the tree
for (i=0; i<n-1; i++) AddTime( &Tr, T[i] );
// construct a tree for the HK-process (x_t) given by: dx
  _{t} = sigma*exp(a*t) dW_{t}, x_{0}=0
SetHKtree( &Tr, a, sigma HK );
// end of: construction of a trinomial tree for the HK-
  process //
///////
ind = malloc(n*sizeof(int));
for (i=0; i<n; i++)
  ind[i] = indiceTime( &Tr, T[i] );
// we have: Tr.t[ ind[i] ] = T[i]
Size = malloc(n*sizeof(int));
for (i=0; i<n; i++)
  Size[i] = Tr.TSize[ind[i]];
// at T[i], the tree has Size[i] nodes
disc_payoff = malloc(n*sizeof(double*));
// disc_payoff[i] will represent the discounted payoff of
   the payer swaption at T[i] !!
for (i=0; i<n; i++)
  {
    disc payoff[i] = malloc(Size[i]*sizeof(double));
    for (j=0; j<Size[i]; j++)</pre>
     disc_payoff[i][j] = -1 - K*tau[m-1];
// for the moment, disc_payoff[i] represents the constant
   payoff -1-K*tau[m-1]
```

```
// Construct the functional forms N[0], ..., N[m-1]
HK_iterations( flat_flag, r_flat, init, a, sigma_HW,
               TO, per, m, K, xnumber, N);
// Complete the discounted payoff of the payer swaption
  in disc payoff
for(i=0; i<n; i++)
  for(j=0; j<Size[i]; j++)</pre>
    {
      x = Tr.pLRij[ind[i]][j];
      disc_payoff[i][j]+=InterpolDiscreteFct( &N[i], x );
      for(k=i+1; k<m; k++)
          PtildeTiTk = NormalTab( x, SQR(Sigma[k][i+1]),
  &N[k]);
          disc_payoff[i][j]-=K * tau[k-1] * PtildeTiTk;
    }
// now disc_payoff[i] represents 1/P(T[i],T[m]) - (1+K*ta
  u[m-1]) - K*sum_{k=i+1}^{m-1} tau[k-1] *
// P^tilde(T[i],T[k]) which is the correct discounted
  payoff at T[i] of the payer swaption !!
if (payer) payer_sign=1; else payer_sign=-1;
initPayoff1(&Tr, T[n-1]);
for (i=0; i<n; i++)
    for (j=0; j<Size[i]; j++)</pre>
        Tr.Payoffunc[ind[i]][j] = MAX( payer sign * disc
  payoff[i][j] , 0 );
      }
  }
```

```
// Compute the swaption from the last exercise date T[n-1]
  ] to 0 in Tr.plQij
Computepayoff1(&Tr, T[n-1]);
// return plQij[0][1] as discounted price of the swapt
  ion
if (t0==0)
  {
   disc_price = Nominal * Tr.pLQij[0][1];
   //
           printf("disc. price = %e{n", disc_price);
           *price = PO[m] * disc_price;
    *price = CurrentZCB(T[m], flat_flag, r_flat, init) *
  disc_price;
   // printf("price = %e{n", *price);
else printf("Evaluation in t>0 is not implemented.{n");
// free the variables
free(T);
free(tau);
// free(P0);
for(i=0; i<m; i++) free(Sigma[i]);</pre>
free(Sigma);
for(i=0; i<m; i++) Delete_discrete_fct(&N[i]);</pre>
free(N);
DeletePayoff1(&Tr, T[n-1]);
DeleteTree(&Tr);
for(i=0; i<n; i++) free(disc payoff[i]);</pre>
free(disc_payoff);
free(ind);
free(Size);
// end of: free the variables //
```

```
return OK;
}
int CALC(TR_SWAPTION)(void *Opt,void *Mod,PricingMethod *
  TYPEOPT* ptOpt=(TYPEOPT*)Opt;
  TYPEMOD* ptMod=(TYPEMOD*)Mod;
  return swaption_hk1d(ptMod->flat_flag.Val.V_INT,
                      ptMod->a.Val.V_DOUBLE,
                      ptMod->T.Val.V_DATE,
                      ptMod->Sigma.Val.V_PDOUBLE,
                      MOD(GetYield)(ptMod),
                      ptOpt->BMaturity.Val.V_DATE,
                      ptOpt->OMaturity.Val.V DATE,
                      ptOpt->PayOff.Val.V_NUMFUNC_1,
                      ptOpt->EuOrAm.Val.V_BOOL,
                      ptOpt->Nominal.Val.V PDOUBLE,
                      ptOpt->FixedRate.Val.V_PDOUBLE,
                      ptOpt->ResetPeriod.Val.V DATE,
                      Met->Par[0].Val.V LONG,
                      Met->Par[1].Val.V_INT,
                      &(Met->Res[0].Val.V_DOUBLE));
}
static int CHK_OPT(TR_SWAPTION)(void *Opt, void *Mod)
{
  if ((strcmp(((Option*)Opt)->Name, "PayerSwaption")==0) ||
    (strcmp(((Option*)Opt)->Name, "ReceiverSwaption")==0))
    return OK;
  else
    return WRONG;
}
```

```
#endif //PremiaCurrentVersion
static int MET(Init)(PricingMethod *Met,Option *Opt)
  if ( Met->init == 0)
    {
      Met->init=1;
      Met->Par[0].Val.V LONG=140;
      Met->Par[1].Val.V_INT=1000;
    }
  return OK;
}
PricingMethod MET(TR_SWAPTION)=
  "TR_HK1d_SWAPTION",
  {{"TimeStepNumber",LONG,{100},ALLOW},
      {"Parameter for the discretization of the functional
    forms",INT,{100},ALLOW},
      {" ",PREMIA_NULLTYPE, {0}, FORBID}},
  CALC(TR_SWAPTION),
  {{"Price",DOUBLE,{100},FORBID}/*,{"Delta",DOUBLE,{100},FO
    RBID\*/ ,{" ",PREMIA_NULLTYPE,{0},FORBID}},
  CHK OPT (TR SWAPTION),
  CHK ok,
  MET(Init)
} ;
```

References