cf_put_merhes

The evolution process of the Heston model, for the stochastic volatility, and Merton model, for the jumps, is:

$$\begin{cases} \frac{dS_t}{S_t} & = & (r-d)dt + \sqrt{V_t}dW_t^1 + (e^J - 1)dN_t \\ dV_t & = & \kappa(\theta - V_t)dt + \sigma_v\sqrt{V_t}dW_t^2 \\ S(t=0) & = & S_0 \\ V(t=0) & = & V_0 \end{cases}$$

where $d < W^1, W^2 >_t = \rho dt$ and $J \sim N(m, v)$.

For European options, two pricing formula are giving based on the Fourier transform method [1]. In this document, we use the following notaton: $\varphi = +1$ for a call and $\varphi = -1$ for a put; $\tau = T - t$; $x_t = \ln(S_t)$ and $X = ln(S_t/K) + (r-d)\tau$.

1 The characteristic formula

The price $F(x_t, t)$ is given by:

$$F(x_t, t) = \frac{1+\varphi}{2}e^{x_t-(T-t)d} + \frac{1-\varphi}{2}e^{1-(T-t)r}K - e^{-(T-t)r}f(x, V, \lambda, t)$$

where

$$f(x, V, \lambda, \tau) = \frac{K}{\pi}\int_0^\infty \Re\left[\frac{Q(k, x, V, \lambda, \tau)}{k^2 + 1/4}\right]dk$$

with

$$Q(k, x, V, \lambda, \tau) = e^{(-ik+1/2)X + A(k,\tau) + B(k,\tau)V_0 + C(k,\tau) + D(k,\tau)\lambda}$$

The hedge δ is given by:

$$\delta = \frac{1+\varphi}{2}e^{-(T-t)d} - e^{-(T-t)r}\frac{K}{\pi}\int_0^\infty \Re\left[\frac{(1/2 - ik)}{S_t}\cdot\frac{Q(k, x, V, \lambda, \tau)}{k^2 + 1/4}\right]dk$$

The coefficients $A(k, \tau)$, $B(k, \tau)$, $C(k, \tau)$ and $D(k, \tau)$ are specified as follows:

1. Volatility:

   - Constant volatility :
     $A(k,\tau)=0,\ B(k,\tau)=-1/2(k^2+1/4)\tau.$
   - Stochastic volatility (Heston) :
     $A(k,\tau)=-\frac{\kappa\theta}{\sigma_v^2}\left[\psi_+\tau+2ln\left(\frac{\psi_-+\psi_+e^{-\tau\zeta}}{2\zeta}\right)\right]$
     $B(k,\tau)=-(k^2+1/4)\frac{1-e^{-\tau\zeta}}{\psi_-+\psi_+e^{-\tau\zeta}}$
     where $\psi_\pm=\mp(u+ik\rho\sigma_v)+\zeta$, $\zeta=\sqrt{k^2\sigma_v^2(1-\rho^2)+2ik\rho\sigma_v u+u^2+\sigma_v^2/4}$
     and $u=\kappa-\rho\sigma_v/2$.

2. Jumps :

   - Merton model : constant jump rate intensity and log-normal jump size distribution
     $C(k,\tau)=0,\ D(k,\tau)=\tau\Lambda(k)$ where
     $\Lambda(k)=e^{-ik(m+v^2/2)-(k^2-1/4)v^2/2+1/2m)}-1-(-ik+1/2)(e^{m+v^2/2}-1).$

2 The Black-Scholes-style fomula

The price $F(x_t,t)$ is given by

$$F(x_t,t)=\varphi\left(e^{-d(T-t)}S_tP_1(\varphi)-e^{-r(T-t)}KP_2(\varphi)\right)$$

and the hedge δ by

$$\delta=\varphi e^{-d(T-t)}P_1(\varphi)$$

where $P_j(\varphi)=\frac{1-\varphi}{2}+\varphi\Pi_j$ for $j\in\{1,2\}$ with

$$\Pi_j=\frac{1}{2}+\frac{1}{\pi}\int_0^\infty\Re\left[\frac{\phi_j(k)}{ik}\right]dk$$

where the characteristic functions ϕ_j, for $j\in\{1,2\}$, are given by:

$$\phi_j(k)=e^{ikX+A(k,\tau)+B(k,\tau)V_0+C(k,\tau)+D(k,\tau)\lambda}]$$

Using the notations :

$$\begin{cases} u=+1, & I=1, & b=\kappa-\rho\sigma_v & if & j=1\\ u=-1, & I=0, & b=\kappa & if & j=2\end{cases}$$

the coefficients $A(k,\tau)$, $B(k,\tau)$, $C(k,\tau)$ and $D(k,\tau)$ are given as follows:

1. Volatility:

   - Constant volatility :
     $A(k,\tau)=0,\ B(k,\tau)=-1/2(k^2-uik)\tau.$
   - Stochastic volatility (Heston) :

     $A(k,\tau)=-\frac{\kappa\theta}{\sigma_v^2}\left[\psi_+\tau+2ln\left(\frac{\psi_-+\psi_+e^{-\tau\zeta}}{2\zeta}\right)\right]$
     $B(k,\tau)=-(k^2-uik)\frac{1-e^{-\tau\zeta}}{\psi_-+\psi_+e^{-\tau\zeta}}$
     where $\psi_\pm=\mp(b-\rho\sigma_v ik)+\zeta$ and $\zeta=\sqrt{(b-\rho\sigma_v ik)^2+\sigma_v^2(k^2-iuk)}.$

2. Jumps :

   - Merton model : constant jump rate intensity and log-normal jump size distribution
     $C(k,\tau)=0,\ D(k,\tau)=\tau\Lambda(k)$ where
     $\Lambda(k)=e^{(m+Iv^2)ik-v^2k^2/2+I(m+v^2/2)}-1+(ik+I)(e^{m+v^2/2}-1).$

3 Numerical integration

In order to compute the infinite integrals, needed in the pricing formulas, we use the approximation:

$$\int_0^\infty f(x)dx \simeq \sum_{j=0}^{N}\int_{jh}^{(j+1)h} f(x)dx.$$

The number N of the sub-integrals used is determinated when the contribution of the last strip $[jh,(j+1)h]$ is smaller than a given tolerance ϵ. Each sub-integral $\int_{jh}^{(j+1)h} f(x)dx$ is computed using a Gaussian quadrature.

4 Implementation of the pricing routines

4.1 The svj.c file

This file contains the pricing routines, giving the price of an european call or put in the Merton/Heston/Merton+Heston models. Any of the two pricing formulas presented in Sections 1 and 2 can be used. *Opt* is the option type, *call* or *put*. *Model* is the model used, *merton* for the Merton model, *heston* for the Heston model and *hestmert* for the combined model Heston+Merton.

In each file, we set the option type and the model parameters, next, we call the `calc_price_svj` rountine from `svj.c` file. The default pricing method used is the Black-Scholes like formula given in 2.

References

[1] A.Sepp. Pricing european-style options under jump diffusion processes with stochastic volatility: Applications of fourier transform. *Proceedings of the 7th Tartu Conference on Multivariate Statistics*, 2004. 1