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We assume that the underlying S_t evolves according to Merton's model:

$$S_t = S_0(e^{\sigma W_t + (\mu - \delta - \frac{\sigma^2}{2})t} \prod_{j=1}^{N_t} e^{m+\sqrt{v}g_j})$$

where
$\cdot W_t$ is a brownian motion.
$\cdot N_t$ is the counting function of an independent Poisson process with parameter λ.
$\cdot$The $(g_j)_j$ are independent normal variables with mean 0 and variance 1, independent of both W and N.
$\cdot\sigma$: volatility.
$\cdot\mu$: trend.
$\cdot\delta$: dividend rate.
$\cdot(m, v)$: parameters of the jump law.

In other words, the spot evolves according to the Black-Scholes model between the jump times (τ_j) of the Poisson process, and jumps at times τ_j:

$$S_{\tau_j} = S_{\tau_{j-}}(e^{m+\sqrt{v}g_j})$$

We choose Merton's risk neutral probability measure in order to price the call option with maturity T and strike K. Therefore, we define the price of this option at time t by:

$$C_t = \mathbf{E}^*(e^{-r(T-t)}(S_T - K)_+|\mathcal{F}_t)$$

where $\frac{d\mathbf{P}^*}{d\mathbf{P}} = e^{\int_0^T \frac{r-\lambda e^{m+\frac{v}{2}} - \mu}{\sigma}\, dW_t - \frac{1}{2}\int_0^T (\frac{r-\lambda e^{m+\frac{v}{2}} - \mu}{\sigma})^2\, dt}$.

[1].

References

[1] P. CARR D.B.MADAN. Option valuation using the fast fourier transform. *Journal of Computational Finance*, 2(2):61–73, 1998. 1