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   Help
#include <stdio.h>
#include <stdlib.h>
#include <math.h>

#include "pnl/pnl_vector.h"
#include "pnl/pnl_matrix.h"
#include "pnl/pnl_mathtools.h"
#include "pnl/pnl_interpolation.h"
#include "pnl/pnl_integration.h"

#include "lmm_stochvol_piterbarg.h"


static int _n_swap;
static int _m_swap;


double ParametricForm(PnlMat *Params, double t, double Tn,
    int k)
{
    return (MGET(Params, 0, k)*(Tn-t) + MGET(Params,1,k))*
    exp(-MGET(Params,2,k)*(Tn-t)) + MGET(Params,3,k);
}
// Libor instantaneous volatility functions.
// t: time. Tn:maturity of Libor. k: index of the diffusio
    n factor
double LiborRate_vol(StructLmmPiterbarg *LmmPiterbarg,
    double t, double Tn, int k)
{
    if (t<=Tn)
    {
        return ParametricForm(LmmPiterbarg->VolsParams, t,
    Tn, k);
    }
    else return 0.;
}

// Libor instantaneous skew functions.
double LiborRate_skew(StructLmmPiterbarg *LmmPiterbarg,
    double t, double Tn)
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{
    if (t<=Tn) return ParametricForm(LmmPiterbarg->Skew
    sParams, t, Tn, 0);
    else return 0.;
}

// This function create a structure StructLmmPiterbarg th
    at contains info about the model i.e. market data, paramete
    rs and time grid.
StructLmmPiterbarg* SetLmmPiterbarg(int InitYieldCurve_fla
    g, double R_flat, double period, double T_last, double
    Var_SpeedMeanReversion, double Var_Volatility, int NbrVol
    Factors, PnlMat* SkewsParams, PnlMat* VolsParams)
{
    int N = intapprox(T_last/period)-1;
    int i;
    StructLmmPiterbarg *LmmPiterbarg = malloc(sizeof(
    StructLmmPiterbarg));

    LmmPiterbarg->ZCMarket = malloc(sizeof(ZCMarketData));
    SetInitYieldCurve(InitYieldCurve_flag, R_flat, LmmP
    iterbarg->ZCMarket);

    LmmPiterbarg->TimeDates = pnl_vect_create(N+1);

    LmmPiterbarg->SkewsParams = pnl_mat_new();
    pnl_mat_clone(LmmPiterbarg->SkewsParams, SkewsParams);
    LmmPiterbarg->VolsParams = pnl_mat_new();
    pnl_mat_clone(LmmPiterbarg->VolsParams, VolsParams);

    LmmPiterbarg->NbrVolFactors = NbrVolFactors;

    LmmPiterbarg->Var_SpeedMeanReversion = Var_SpeedMeanReversion;
    LmmPiterbarg->Var_Volatility = Var_Volatility;

    for (i=0; i<=N; i++) LET(LmmPiterbarg->TimeDates, i) =
    (i+1)*period;

    return LmmPiterbarg;
}
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// Free StructLmmPiterbarg
void FreeLmmPiterbarg(StructLmmPiterbarg **LmmPiterbarg)
{
    pnl_vect_free(&((*LmmPiterbarg)->TimeDates));

    DeleteZCMarketData((*LmmPiterbarg)->ZCMarket);
    free((*LmmPiterbarg)->ZCMarket);

    pnl_mat_free(&((*LmmPiterbarg)->SkewsParams));
    pnl_mat_free(&((*LmmPiterbarg)->VolsParams));

    free(*LmmPiterbarg);
    LmmPiterbarg=NULL;
}

// T_{i-1} < s <= T_{i}
int indiceTimeGrid(PnlVect *TimeGrid, double s)
{
    int i=0, N=TimeGrid->size-1;

    while (i<=N && s-GET(TimeGrid, i)>1e-10) i++;

    return i;
}


double SwapRate_vol_k(StructLmmPiterbarg *LmmPiterbarg,
    double t, int n_swap, int m_swap, int k)
{
    int i;
    double Ti1, Ti2, Tn, Tm, P0_Tn, P0_Tm, q_i_n_m, swp_    vol_k_n_m;

    Tn = GET(LmmPiterbarg->TimeDates, n_swap);
    Tm = GET(LmmPiterbarg->TimeDates, m_swap);

    P0_Tn = BondPrice(Tn, LmmPiterbarg->ZCMarket);
    P0_Tm = BondPrice(Tm, LmmPiterbarg->ZCMarket);

    swp_vol_k_n_m = 0.;
    for (i=n_swap; i<m_swap; i++)
    {
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        Ti1 = GET(LmmPiterbarg->TimeDates, i);
        Ti2 = GET(LmmPiterbarg->TimeDates, i+1);
        q_i_n_m = BondPrice(Ti1, LmmPiterbarg->ZCMarket)-Bo
    ndPrice(Ti2, LmmPiterbarg->ZCMarket);

        swp_vol_k_n_m += q_i_n_m * LiborRate_vol(LmmPiterb
    arg, t, Ti1, k);
    }
    swp_vol_k_n_m /= (P0_Tn-P0_Tm);

    return swp_vol_k_n_m;
}

double SwapRate_vol(StructLmmPiterbarg *LmmPiterbarg,
    double t, int n_swap, int m_swap)
{
    int k;
    double swp_vol_n_m=0.;

    for (k=0; k<LmmPiterbarg->NbrVolFactors; k++)
    {
        swp_vol_n_m += pow(SwapRate_vol_k(LmmPiterbarg, t,
    n_swap, m_swap, k), 2);
    }

    return sqrt(swp_vol_n_m);
}

double SwapRate_skew(StructLmmPiterbarg *LmmPiterbarg,
    double t, int n_swap, int m_swap)
{
    int i, k;
    double Ti, sum_vol_skew, swp_skew_n_m, sqr_swp_vol_n_m,
     swp_vol_k_n_m;

    swp_skew_n_m = 0.;
    sqr_swp_vol_n_m = 0.;

    for (k=0; k<LmmPiterbarg->NbrVolFactors; k++)
    {
        swp_vol_k_n_m = SwapRate_vol_k(LmmPiterbarg, t, n_
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    swap, m_swap, k);
        sqr_swp_vol_n_m += pow(swp_vol_k_n_m, 2);

        sum_vol_skew = 0.;
        for (i=n_swap; i<m_swap; i++)
        {
            Ti = GET(LmmPiterbarg->TimeDates, i);
            sum_vol_skew += LiborRate_vol(LmmPiterbarg, t,
    Ti, k)*LiborRate_skew(LmmPiterbarg, t, Ti);
        }

        swp_skew_n_m += swp_vol_k_n_m * sum_vol_skew;
    }

    return swp_skew_n_m/(sqr_swp_vol_n_m*(m_swap-n_swap));
}


static double func_to_intg_1(double t, void *LmmPiterbarg)
{
    double tmp = SwapRate_vol(LmmPiterbarg, t, _n_swap, _m_
    swap);
    return tmp * tmp;
}

static double func_to_intg_2(double s, void *LmmPiterbarg)
{
    double theta = ((StructLmmPiterbarg*)LmmPiterbarg)->
    Var_SpeedMeanReversion;
    return pow(SwapRate_vol(LmmPiterbarg, s, _n_swap, _m_
    swap), 2)*(exp(theta*s)-exp(-theta*s))/(2*theta);
}

static double func_to_intg_3(double t, void *LmmPiterbarg)
{
    PnlFunc func;
    int NbrPts = 20;
    double v1, v2, v_nm2;
    double theta=((StructLmmPiterbarg*)LmmPiterbarg)->
    Var_SpeedMeanReversion, eta=((StructLmmPiterbarg*)LmmPiterbarg)
    ->Var_Volatility;
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    func.params = LmmPiterbarg;
    func.function = func_to_intg_1;
    v1 = pnl_integration(&func, 0.0, t, NbrPts, "simpson");

    func.function = func_to_intg_2;
    v2 = pnl_integration(&func, 0.0, t, NbrPts, "simpson");

    v_nm2 = v1 + SQR(eta)*exp(-theta*t)*v2;

    return v_nm2*pow(SwapRate_vol(LmmPiterbarg, t, _n_swap,
     _m_swap),2);
}

static double func_to_intg_4(double t, void *LmmPiterbarg)
{
    return func_to_intg_3(t, LmmPiterbarg)*SwapRate_skew(
    LmmPiterbarg, t, _n_swap, _m_swap);
}

double SwapRate_skew_avg(StructLmmPiterbarg *LmmPiterbarg,
    int n_swap, int m_swap)
{
    PnlFunc func;
    int NbrPts = 30;
    double result, sumw, Tn=GET(LmmPiterbarg->TimeDates, n_
    swap);

    _n_swap = n_swap;
    _m_swap = m_swap;

    func.params = LmmPiterbarg;
    func.function = func_to_intg_4;
    result = pnl_integration(&func, 0.0, Tn, NbrPts, "simp
    son");

    func.function = func_to_intg_3;
    sumw = pnl_integration(&func, 0.0, Tn, NbrPts, "simpson
    ");

    return result/sumw;
```

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}


static double log_LapTransf_intg_v0(StructLmmPiterbarg *
    LmmPiterbarg, double u, int n_swap, int m_swap)
{
    double Tn, gamma, exp_g_Tn, B_0, A_0;
    double theta=LmmPiterbarg->Var_SpeedMeanReversion, eta=
    LmmPiterbarg->Var_Volatility;

    Tn = GET(LmmPiterbarg->TimeDates, n_swap);
    gamma = sqrt(SQR(theta) + 2*SQR(eta)*u);
    exp_g_Tn = exp(-gamma*Tn);

    B_0 = 2*u*(1-exp_g_Tn)/( (theta+gamma)*(1-exp_g_Tn) + 2
    *gamma*exp_g_Tn);
    A_0 = 2*theta/SQR(eta) * log(2*gamma/((theta + gamma)*(
    1-exp_g_Tn) + 2*gamma*exp_g_Tn)) - 2*theta*u*Tn/(theta+gam
    ma);

    return (A_0 - B_0);
}

static double LapTransf_F(StructLmmPiterbarg *LmmPiterbarg,
     double t, double y, double u, int n_swap, int m_swap)
{
    double theta=LmmPiterbarg->Var_SpeedMeanReversion, eta=
    LmmPiterbarg->Var_Volatility;

    return -theta*y-0.5*SQR(eta*y)+u*pow(SwapRate_vol(LmmP
    iterbarg, t, n_swap, m_swap), 2);
}

static double log_LapTransf_intg_v1(StructLmmPiterbarg *
    LmmPiterbarg, double u, int n_swap, int m_swap)
{
    double B_i, A_i, h, ti, k1, k2, k3, k4;
    int i, n_step=2*n_swap;
    double theta=LmmPiterbarg->Var_SpeedMeanReversion;

    B_i = 0.;
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```
    A_i = 0.;
    ti = 0.;
    h = GET(LmmPiterbarg->TimeDates, n_swap)/n_step;

    for (i=0; i<n_step; i++)
    {
        k1 = LapTransf_F(LmmPiterbarg, ti, B_i, u, n_swap,
    m_swap);
        k2 = LapTransf_F(LmmPiterbarg, ti+0.5*h, B_i+0.5*h*
    k1, u, n_swap, m_swap);
        k3 = LapTransf_F(LmmPiterbarg, ti+0.5*h, B_i+0.5*h*
    k2, u, n_swap, m_swap);
        k4 = LapTransf_F(LmmPiterbarg, ti+h, B_i+h*k3, u,
    n_swap, m_swap);
        B_i = B_i + h/6.*(k1+2*k2+2*k3+k4); // Runge-Kutta
    of order 4

        A_i -= theta*B_i*h;
        ti += h;
    }

    A_i += 0.5*theta*B_i*h;

    return (A_i - B_i);
}

double Func_zero(StructLmmPiterbarg *LmmPiterbarg, double
    lambda, int n_swap, int m_swap, double skew_n_m, double c,
    double phi_c)
{
    return log_LapTransf_intg_v0(LmmPiterbarg, c*SQR(lambd
    a), n_swap, m_swap) - phi_c;
}

double SwapRate_vol_avg(StructLmmPiterbarg *LmmPiterbarg,
    int n_swap, int m_swap, double skew_n_m)
{
    int i;
    double c, phi_c, intg_sigma_n_m, lambda, lambda_sup=0.,
     lambda_inf=0., f_lambda;
    double Tn=GET(LmmPiterbarg->TimeDates, n_swap), precisi
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```
    on=1e-7;
    PnlFunc func;
    int NbrPts = 2*n_swap;

    _n_swap = n_swap;
    _m_swap = m_swap;

    func.params = LmmPiterbarg;
    func.function = func_to_intg_1;
    intg_sigma_n_m = pnl_integration(&func, 0.0, Tn, NbrPts
    , "simpson");

    c = (4. + SQR(skew_n_m)*intg_sigma_n_m)/(8.*intg_sigma_
    n_m);
    phi_c = log_LapTransf_intg_v1(LmmPiterbarg, c, n_swap,
    m_swap);

    if (LmmPiterbarg->Var_Volatility==0) return sqrt(intg_
    sigma_n_m/Tn);

    for (i=0; i<=n_swap; i++)
    lambda_inf = MAX(lambda_inf, SwapRate_vol(LmmPiterbarg,
     GET(LmmPiterbarg->TimeDates, i), n_swap, m_swap));

    i=0;
    do
    {
        lambda= 0.5*(lambda_inf+lambda_sup);
        f_lambda = Func_zero(LmmPiterbarg, lambda, n_swap,
    m_swap, skew_n_m, c, phi_c);

        if (f_lambda<0) lambda_inf = lambda;
        else lambda_sup = lambda;
        i++;
    }
    while (fabs(f_lambda)>precision && fabs(lambda_sup-lam
    bda_inf)>precision);

    return lambda;
}
```

References