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mc_robbinsmoro_hes

Input parameters

- Number of iterations N
- Generator type
- Increment inc
- Confidence Value
- Volatility of volatility

Output parameters

- Price P
- Error price σ_P
- Delta δ
- Error delta σ_{delta}
- Price Confidence Interval: ICp [Inf Price, Sup Price]
- Delta Confidence Interval: ICp [Inf Delta, Sup Delta]

Description

Computation of a european option in the Heston stochastic volatility model.

This model is given by,

$$\begin{aligned} dS_t &= (r - q)S_t dt + \sqrt{v_t}S_t dW_t^1, \\ dv_t &= k(\theta - v_t)dt + \sigma\sqrt{v_t}dW_t^2, \end{aligned}$$

where W^1 and W^2 are two correlated brownian motions with $\langle W^1, W^2 \rangle_t = \rho t$, and k , θ and σ are constants.

/*The Heston closed formula*/

In the case of a European call option, Heston [1] guessed a solution of the form

$$C(S, v, t) = Se^{-q(T-t)}P_1 - Ke^{-r(T-t)}P_2,$$

by analogy with the Black-Scholes formula. The first term in this formula is the present value of the spot price while the second term is the present value of the strike-price payment.

Using this model, Heston has given a closed form solution to the pricing of a European call option by the characteristic functions technique.

MC pricing Algorithm:

/*The price*/

The objective is to compute $V_0 = \mathbb{E}[\phi(S_T)]$ where $(S_t)_{t \leq T}$ is the Heston model.

/*Simulate the discretized underlying*/

Discretizing with an *Euler scheme* leads to

$$\begin{aligned} S_{T_{i+1}} &= S_{T_i}(1 + (r - q)\Delta t + \sqrt{\sigma_i \Delta t}Z_i), \\ v_{T_{i+1}} &= v_{T_i} + k(\theta - v_{T_i})\Delta t + \sigma\sqrt{\Delta t v_{T_i}}(\rho Z_i + \sqrt{1 - \rho^2}Z_{m+i}), \end{aligned}$$

where $(Z_i)_{i \geq 1}$ is a sequence of independent Gaussian variables with mean 0 and variance 1. In our implementations we have taken the stochastic input to model to be the single vector (Z_1, \dots, Z_{2m}) . In some respects, it might be more natural to think of two separate vectors, each of length m . So at each iterations generate a random gaussian vector of size $2 \times N$ where N is the total number of MC iterations. Then separate this vector in two vectors of same size N to simulate both the underlying asset and the volatility.

/*Importance sampling*/

In the discretized problem we have to evaluate $\hat{V}_0 = \mathbb{E}[\hat{\phi}(Z)]$ where $Z = (Z_1, \dots, Z_m)$ is a standard gaussian vector. Using an elementary version of Girsanov theorem leads to the following representation of \hat{V}_0 :

$$\hat{V}_0 = \mathbb{E}[g(\mu, Z)], \tag{1}$$

with

$$g(\mu, Z) = \hat{\phi}(Z + \mu) e^{-\mu \cdot Z - \frac{1}{2} \|\mu\|^2}, \quad (2)$$

where $\|x\|$ denotes the Euclidean norm of a vector $x \in \mathbb{R}^m$ and $x \cdot y$ is the inner product of two vectors $x, y \in \mathbb{R}^m$.

/*Variance reduction*/

The idea is then to make use of a Robbins and Monro algorithm to assess the optimal μ^* that minimizes the variance of $g(\mu^*, Z)$.

/*The price computation and confidence interval*/

By rebalancing this optimal μ^* in the MC computation of the price we reduce the variance by a factor of 5 and more. Finally by the use of the central limit theorem we get a confidence interval with a length equal to the length of the MC standard confidence interval by a factor of 2.5 to 3 and even more for options that are far from the money.

References

- [1] S.L.HESTON. A closed-form solution for options with stochastic volatility with applications to bond and currency options. *Review of Financial Studies*, 6(2):327–343, 1993. 2