Source | Model | Option
| Model_Option | Help on mc methods | Archived Tests

mc_fixedasian_glassermann

Input parameters

- Number of iterations N
- Generator type
- Increment inc
- Confidence Value

Output parameters

- Price P
- Error price σ_P
- Delta δ
- Error delta σ_{delta}
- Price Confidence Interval: ICp [Inf Price,Sup Price]
- Delta Confidence Interval: ICp [Inf Delta,Sup Delta]

Description

Computation of the price of a asian option when the underlying asset follows the Black and Scholes model.
/*The model*/
Under the standard Black and Scholes assumptions the price of the underlying asset is driven by the SDE

$$dS_t = S_t((r-q)dt + \sigma dW_t), \quad S_{T_0} = x, \tag{1}$$

with r the risk-free, continuously compounded interest rate, $\sigma(t, y)$ the asset volatility, W a Brownian motion, and x fixed.
The solution to this equation can be simulated without dicretization error on a discrete grid of points $T_0 < T_1 < \cdots < T_m = T$, by setting

$$S_{T_i} = S_{T_{i-1}} \exp((r - \tfrac{1}{2}\sigma^2)\delta t + \sigma\sqrt{\delta t} Z_i), \quad i = 1, \ldots, m,$$

where $Z = (Z_1, \ldots, Z_m) \sim \mathcal{N}(0, I_m)$ and I_m is the identity matrix of $\mathbb{R}^m$.
/*The option real and approximate prices*/
For arbitrage reasons, the price of an option with payoff $\psi(S_t, t \leq T)$ is given by

$$V_0 = \mathbb{E}[e^{-r(T-T_0)}\psi(S_t, t \leq T)].$$

For a call option we have $\psi(S_t, t \leq T) = \left(\frac{1}{T-T_0}\int_{T_0}^{T} S_t dt - K\right)^+$. which we rewrite

$$G(Z) = e^{-r(T-T_0)}\left(\hat{A}(T_0, T, Z) - K\right)^+,$$

where Z is a random gaussian vector, $\hat{A}(T_0, T, Z)$ is the dicretized mean and G is a function we can compute by using the dicretization of the mean $A(T_0, T) = \frac{1}{T-T_0}\int_{T_0}^{T} S_t dt$ and the payoff function. Thus the approximate price of the option is given by

$$\hat{V}_0 = \mathbb{E}[G(Z)].$$

Importance sampling

We change the law of $Z = (Z_1, \ldots, Z_m)$ by adding a drift vector $\mu = (\mu_1, \ldots, \mu_m)$. An elementary version of Girsanov theorem leads to the following representation of $\hat{V}_0$:

$$\hat{V}_0 = \mathbb{E}[g(\mu, Z)],$$

with

$$g(\mu, Z) = G(Z + \mu)e^{-\mu \cdot Z - \frac{1}{2}\|\mu\|^2}, \tag{2}$$

where $\|x\|$ denotes the Euclidean norm of a vector $x \in \mathbb{R}^m$ and $x \cdot y$ is the inner product of two vectors $x, y \in \mathbb{R}^m$. In (2) the optimal μ solves the problem

$$\min_{\mu} \mathbb{E}[G(Z)^2 e^{-\mu \cdot Z + \frac{1}{2}\|\mu\|^2}].$$

Note that even if the optimal μ can be found, it will not in general provide a zero-variance estimator. In practice, finding the optimal μ exactly is infeasible and some approximation is required. In their paper the authors of the method have shown that this optimal μ maximizes the function $F(z) - \frac{1}{2}z \cdot z$ with $F(z) = \log(G(z))$. That is equivalent to finding the solution of the fixed point problem

$$\nabla F(z) = z.$$

It is proved that the solution to this problem is (asymptotically) optimal in some sens.

Asian option

In the sequel we will restrict our attention to the case of a *Riemanian* (or *Euler*) discretization of the mean $A(T_0, T) = \frac{1}{T-T_0}\int_{T_0}^{T} S_t dt$.
Due to the structure of the asian options, we can find particularly efficient solution of this optimization problem.
Consider the discretized payoff $G(z) = (\hat{A}(T_0, T, Z) - K)^+$, it clearly suffices to consider the points z at which $G(z) \neq 0$ and thus G and F are differentiable.
/*The algorithm*/
The first-order conditions for optimality become

$$z_j = \frac{\sigma\sqrt{\Delta t}\sum_{i=j}^{m} S_i}{mG(z)}, \quad j = 1, ..., m,$$

where we S_i for $S_{i\Delta t}$. This implies that

$$z_1 = \frac{\sigma\sqrt{\Delta t}[G(z) + K]}{G(z)}, \quad z_{j+1} = z_j - \frac{\sigma\sqrt{\Delta t} S_j}{mG(z)}, \quad j = 1, ..., m-1. \tag{3}$$

Given a value $G(z) \equiv y$, equation (3) determines z together with

$$S_j = S_{j-1}e^{(r-q-\frac{1}{2}\sigma^2)\Delta t + \sigma\sqrt{\Delta t} z_j}, \quad j = 1, ..., m. \tag{4}$$

Subject to the first-order conditions, we may therefore view the S_i as functions of the scalar y rather than the vector z. The optimization problem thus reduces to finding the y that indeed produced a payoff of y at $S_1(y), ..., S_m(y)$; that is, finding the root of the equation

$$g(y) \equiv \frac{1}{m}\sum_{j=1}^{m} S_i(y) - K - y = 0.$$

There is no proof that this equation has a unique root, but numerically this appears to be the case. Bisections find the root very quickly, and given

this scalar y, equations (3) and (4) recover z efficiently. We denote this vector by μ^*.
/*The MC price computation*/
If $(Z^n)_{1 \leq n \leq N}$ is an $i.i.d.$ sample from the gaussian law $\mathcal{N}(0, I_m)$ then the MC price of the option is given by

$$\hat{V}_0 \sim \frac{1}{N} \sum_{n=1}^{N} G(Z^n + \mu^*) e^{-\mu^* \cdot Z^n - \frac{1}{2} \|\mu^*\|^2}.$$

See [1] for more details.

References

[1] P.GLASSERMAN P.HEIDELBERGER P.SHAHABUDDIN. Asymptotically optimal importance sampling and stratification for prcing path-dependent options. *Mathematical Finance*, 2,April:117–152, 1999. 4