cf_putdownout

Reiner-Rubinstein [1] have developed formulas for pricing standard barrier options:
Let

- $T =$ maturity date $\quad (T > t)$
- $K =$ strike price
- $L =$ down barrier
- $R =$ rebate
- $x =$ spot price
- $t =$ pricing date
- $\sigma =$ volatility
- $r =$ interest rate
- $\delta =$ dividend yields
- $\theta = T - t$
- $b = r - \delta$

We set here:

- $A = \phi x e^{-\delta\theta} N(\phi x_1) - \phi K e^{-r\theta} N(\phi x_1 - \phi\sigma\sqrt{\theta})$
- $B = \phi x e^{-\delta\theta} N(\phi x_2) - \phi K e^{-r\theta} N(\phi x_2 - \phi\sigma\sqrt{\theta})$
- $C = \phi x (\frac{L}{x})^{2(\mu+1)} e^{-\delta\theta} N(\eta y_1) - \phi K e^{-r\theta} (\frac{L}{x})^{2\mu} N(\eta y_1 - \eta\sigma\sqrt{\theta})$
- $D = \phi x (\frac{L}{x})^{2(\mu+1)} e^{-\delta\theta} N(\eta y_2) - \phi K e^{-r\theta} (\frac{L}{x})^{2\mu} N(\eta y_2 - \eta\sigma\sqrt{\theta})$
- $E = R e^{-r\theta} \left[N(\eta x_2 - \eta\sigma\sqrt{\theta}) - (\frac{L}{x})^{2\mu} N(\eta y_2 - \eta\sigma\sqrt{\theta}) \right]$
- $F = R(\frac{L}{x})^{\mu+\lambda} N(\eta z) + (\frac{L}{x})^{\mu-\lambda} N(\eta z - 2\eta\lambda\sigma\sqrt{\theta}))$

and

- $dA = \phi e^{-\delta\theta} N(\phi x_1)$
- $dB = \phi e^{-\delta\theta} N(\phi x_2) + e^{-\delta\theta} \frac{n(x_2)}{\sigma\sqrt{\theta}}(1 - \frac{k}{l})$
- $dC = \frac{1}{x}\phi 2\mu \left[\left(\frac{L}{x}\right)^{2\mu} \left(xe^{-\delta\theta}\frac{L^2}{x^2}N(\eta y_1) - Ke^{-r\theta}N(\eta y_1 - \eta\sigma\sqrt{\theta}) -\right.\right.$
  $\left.\phi\left(\frac{L}{x}\right)^{2(\mu+1)} e^{-b\theta}N(\eta y_1)\right]$
- $dD = -2\mu\frac{\phi}{x}\left[\left(\frac{L}{x}\right)^{2\mu}\left(se^{-\delta\theta}\frac{L^2}{x^2}N(\eta y_2) - Ke^{-r\theta}N(\eta y_2)\times\right.\right.$
  $\times N\left(\eta y_2 - \eta\sigma\sqrt{\theta} - \phi\left(\frac{L}{x}\right)^{2\mu+2}e^{-\delta\theta}\right)N(\eta y_2) - \phi\eta\left(\frac{L}{x}\right)^{2\mu+2}e^{-\delta\theta}\Big] \times$
  $\times \frac{n(y_2)}{\sigma\sqrt{\theta}}\left(1 - \frac{k}{l}\right)$
- $dE = 2\frac{R}{x}e^{-r\theta}(\frac{L}{x})^{2\mu}\left[N(\eta y_2 - \eta\sigma\sqrt{\theta})\mu + \eta\frac{n(\eta y_2 - \sigma\sqrt{\theta})}{\sigma\sqrt{\theta}}\right]$
- $dF = -\frac{R}{x}(\frac{L}{x})^{\mu+\lambda}\left[(\mu+\lambda)N(\eta z) + (\mu-\lambda)(\frac{L}{x})^{2\lambda}\right] - 2\eta R(\frac{L}{x})^{\mu+\lambda}\frac{n(z)}{x\sigma\sqrt{\theta}}$

where

- $x_1 = \dfrac{\log(x/K)}{\sigma\sqrt{\theta}} + (1+\mu)\sigma\sqrt{\theta}$ $\qquad x_2 = \dfrac{\log(x/L)}{\sigma\sqrt{\theta}} + (1+\mu)\sigma\sqrt{\theta}$
- $y_1 = \dfrac{\log(L^2/xK)}{\sigma\sqrt{\theta}} + (1+\mu)\sigma\sqrt{\theta}$ $\qquad y_2 = \dfrac{\log(L/x)}{\sigma\sqrt{\theta}} + (1+\mu)\sigma\sqrt{\theta}$
- $z = \dfrac{\log(L/x)}{\sigma\sqrt{\theta}} + \lambda\sigma\sqrt{\theta}$
- $\mu = \dfrac{b - \sigma^2/2}{\sigma^2}$ $\qquad \lambda = \sqrt{\mu^2 + \dfrac{2r}{\sigma^2}}$

and η, ϕ belong to $\{-1, 1\}$ and will be fixed later.

Let

$$M_{t,T} = \sup_{t\le\tau\le T} S_\tau \qquad \text{and} \qquad m_{t,T} = \inf_{t\le\tau\le T} S_\tau$$

Down-Out Put Option

$$\textsc{Payoff} \qquad P_T = \begin{cases} (K - S_T)_+ & \text{if} \quad m_{t,T} > L, \\ R & \text{otherwise} \end{cases}$$

$$\textsc{Price} \qquad P(t,x) = \begin{cases} A - B + C - D + F & \text{if} \quad K > L, \\ F & \text{otherwise} \end{cases}$$

$$\textsc{Delta} \qquad \frac{\partial P(t,x)}{\partial x} = \begin{cases} dA - dB + dC - dD + dF & \text{if} \quad K > L, \\ dF & \text{otherwise} \end{cases}$$

and take

$$\eta = 1 \qquad \phi = -1$$

References

[1] E.REINER M.RUBINSTEIN. Breaking down the barriers. *Risk*, 4:28–35, 191. 1