Sensitivity analysis in a market with jumps using Premia

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Abstract

We expose a simplified review of Malliavin calculus on Poisson space and an application to sensitivity analysis for Asian options in a market with jumps as in [5], using the methods of [8]. A simulation graph resulting from an implementation in Premia of the algorithm is presented.

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1 Introduction

Sensitivity analysis consists in the computation of partial derivatives (also called Greeks) of the form

$$\text{Delta} = \frac{\partial C(x)}{\partial x}, \qquad \text{Gamma} = \frac{\partial^2 C(x)}{\partial x^2},$$
$$\text{Rho} = \frac{\partial C(r)}{\partial r}, \qquad \text{Vega} = \frac{\partial C(\sigma)}{\partial \sigma},$$

where $C(\zeta) = E\left[f\left(F^{\zeta}\right)\right]$ is an option price and ζ is a given parameter (volatility, initial condition, interest rate ...). The Greeks can be computed by permutation of partial derivative and expectation:

$$\frac{\partial}{\partial_\zeta} E\left[f\left(F^\zeta\right)\right] = E\left[f'\left(F^\zeta\right)\frac{\partial}{\partial_\zeta}F^\zeta\right] \tag{1.1}$$

however this does not apply to a non-differentiable f, for example in the case of digital options $[f(x) = 1_{[K,\infty)}(x)]$, or for the second derivative (Gamma) of a European option $[f(x) = (x-K)^+]$. Finite differences of the form

$$\text{Delta} = \frac{C(x(1+\varepsilon)) - C(x(1-\varepsilon))}{2\varepsilon}, \quad \text{Gamma} = \frac{C(x(1+\varepsilon)) - 2C(x) + C(x(1-\varepsilon))}{\varepsilon^2},$$

or more generally

$$\frac{1}{2\varepsilon}\left(E\left[f\left(F^{\zeta(1+\varepsilon)}\right)\right] - E\left[f'\left(F^{\zeta(1-\varepsilon)}\right)\right]\right),$$

are also inefficient from the point of view of numerical computations, since the error in the difference $E\left[f\left(F^{\zeta(1+\varepsilon)}\right)\right] - E\left[f'\left(F^{\zeta(1-\varepsilon)}\right)\right]$ can not always be neglected in front of ε when the expectations are computed by Monte-Carlo methods. In [8], a method relying on a Malliavin calculus argument has been introduced, using a gradient operator D_w having the derivation property and acting on random functionals. The chain rule of derivation

$$D_w f\left(F^\zeta\right) = f'(F^\zeta) D_w F^\zeta$$

allows to express $f'(F^\zeta)$ as

$$f'(F^\zeta) = \frac{D_w f\left(F^\zeta\right)}{D_w F^\zeta}.$$

Thus (1.1) can be transformed as

$$\frac{\partial}{\partial_\zeta} E\left[f\left(F^\zeta\right)\right] = E\left[f'\left(F^\zeta\right)\frac{\partial}{\partial_\zeta}F^\zeta\right] = E\left[\frac{\partial_\zeta F^\zeta}{D_w F^\zeta} D_w f\left(F^\zeta\right)\right].$$

Assuming further that D_w has an adjoint operator δ we get

$$\frac{\partial}{\partial_\zeta} E\left[f\left(F^\zeta\right)\right] = E\left[f\left(F^\zeta\right)\delta\left(w\frac{\partial_\zeta F^\zeta}{D_w F^\zeta}\right)\right].$$

The interest in this formula is that it no longer involves the derivative f' of f and thus it yields more efficient computational results. In view of the above, we will essentially need an operator D which satisfies a chain rule of derivation and has an adjoint operator δ.

2 Integration by parts

In this section we review several integration by parts formulas on the Wiener and Poisson spaces. We start by chaos decompositions and multiple stochastic integrals methods in the setting of normal martingales, which include Brownian motion and compensated Poisson processes.

2.1 Normal martingales

Let $(M_t)_{t\in\mathbb{R}_+}$ be a square-integrable martingale, and further assume that $(M_t)_{t\in\mathbb{R}_+}$ is a normal martingale, i.e. $d\langle M_t, M_t\rangle = dt$, see [4], [7]. We denote by $I_1(f) = \int_0^\infty f(t)dM_t$ the single stochastic integral of $f \in L^2(\mathbb{R}_+)$, and by $I_n(f_n)$ the iterated Itô integral of the symmetric function $f_n \in L^2(\mathbb{R}_+^n)$:

$$I_n(f_n) = n! \int_0^\infty \int_0^{t_n} \cdots \int_0^{t_2} f_n(t_1, \ldots, t_n) dM_{t_1} \cdots dM_{t_n},$$

with the isometry formula:

$$E[I_n(f_n)I_m(g_m)] = n! 1_{\{n=m\}} \langle f_n, g_m\rangle_{L^2(\mathbb{R}_+^n)}. \tag{2.1}$$

The gradient operator D and the Skorohod integral operator δ are defined by linearity as

$$D_t I_n(f_n) = n I_{n-1}(f_n(*, t)), \qquad t \in \mathbb{R}_+, \tag{2.2}$$

and

$$\delta(I_n(f_{n+1}(*, \cdot))) = I_{n+1}(\tilde{f}_{n+1}). \tag{2.3}$$

They are closable operators, extended to their respective domains Dom (D), Dom (δ), and from (2.1) they are mutually adjoint in the following sense:

$$E[\langle DF, u\rangle_{L^2(\mathbb{R}_+)}] = E[F\delta(u)], \qquad F \in \text{Dom } (D),\ u \in \text{Dom } (\delta).$$

Moreover, δ satisfies the following relation with the stochastic integral:

$$\delta(u) = \int_0^\infty u(t) dM_t \tag{2.4}$$

provided u is an adapted square-integrable process.

2.2 Brownian case

Let $(B_t)_{t\in\mathbb{R}_+}$ be a standard Brownian motion, which is in particular a normal martingale. As in Section 2.1, we denote by $I_n(f_n)$ the iterated Wiener integral of $f_n \in L^2(\mathbb{R}_+^n)$. Itô calculus shows that

$$I_1(v)I_n(u^{\circ n}) = I_{n+1}(u^{\circ n} \circ v) + n\langle u, v\rangle_{L^2(\mathbb{R}_+)} I_{n-1}(u^{\circ(n-1)}). \tag{2.5}$$

Let H_n denote the Hermite polynomial of degree n and parameter $\sigma^2 > 0$ defined by $H_0(x, \sigma^2) = 1$, $H_1(x, \sigma^2) = x$, and the recurrence relation

$$H_{n+1}(x, \sigma^2) = xH_n(x, \sigma^2) - n\sigma^2 H_{n-1}(x, \sigma^2), \qquad n \geq 0. \tag{2.6}$$

The comparison of (2.5) and (2.6) shows by induction that $I_n(e_k^{\circ n}) = H_n(I_1(e_k))$, where $(e_n)_{n\in\mathbb{N}}$ denotes a Hilbert basis of $L^2(\mathbb{R}_+)$, and more generally:

$$I_n(e_{k_1}^{\circ n_1} \circ \cdots \circ e_{k_d}^{\circ n_d}) = H_{n_1}(I_1(e_{k_1})) \cdots H_{n_d}(I_1(e_{k_d})).$$

In other terms, the multiple stochastic integral of a function f_n given by its development

$$f_n = \sum_{\substack{n_1+\cdots+n_d=n \\ k_1,\ldots,k_d\geq 0}} a_{k_1,\ldots,k_d}^{n_1,\ldots,n_d} e_{k_1}^{\circ n_1} \circ \cdots \circ e_{k_d}^{\circ n_d},$$

is a series of polynomials of degree n:

$$I_n(f_n) = \sum_{\substack{n_1+\cdots+n_d=n \\ k_1,\ldots,k_d\geq 0}} a_{k_1,\ldots,k_d}^{n_1,\ldots,n_d} H_{n_1}(I_1(e_{k_1})) \cdots H_{n_d}(I_1(e_{k_d})).$$

Similarly, any orthogonal decomposition of the form

$$F = E[F] + \sum_{n=1}^{\infty} \sum_{\substack{n_1+\cdots+n_d=n \\ k_1,\ldots,k_d\geq 0}} a_{k_1,\ldots,k_d}^{n_1,\ldots,n_d} H_{n_1}(I_1(e_{k_1})) \cdots H_{n_d}(I_1(e_{k_d}))$$

can be rewritten using multiple stochastic integrals as

$$F = E[F] + \sum_{n=1}^{\infty} I_n(f_n).$$

The chaos representation property of Brownian motion states that every square-integrable Wiener functional admits such a decomposition. Using this expansion, the gradient and Skorohod integral operators D, δ, can be defined as in (2.2), (2.3), with the same properties as in the general case, in particular δ extends the stochastic integral with respect to Brownian motion as in (2.4). The derivation rule

$$H_n'(x) = nH_{n-1}(x) \tag{2.7}$$

of Hermite polynomials implies the relation

$$D_t f(I_1(e_1),\dots,I_1(e_n)) = \sum_{k=0}^{\infty} e_k(t)\frac{\partial}{\partial x_i)} f(I_1(e_1),\dots,I_1(e_n)), \qquad t \geq 0,$$

hence D has the derivation property. On the Wiener space, the operator D has been used in [8] for the computation of Greeks in continuous markets.

2.3 Poisson case

Consider a standard Poisson process $(N_t)_{t\in\mathbb{R}_+}$, and let $\{N(A) \ : \ A \in \mathcal{B}(\mathbb{R}_+)\}$ denote the random measure associated to $(N_t)_{t\in\mathbb{R}_+}$ from

$$N((s,t]) = N_t - N_s, \qquad 0 \leq s \leq t.$$

We denote by $I_n(f_n)$ the iterated integral of the symmetric function $f_n \in L^2(\mathbb{R}_+^n)$ with respect to the normal martingale $\tilde{N}_t = N_t - t$, $t \in \mathbb{R}_+$, with the relation

$$I_1(u)I_n(v^{\circ n}) = I_{n+1}(v^{\circ n} \circ u) + nI_n((uv)\circ v^{\circ(n-1)}) + n\langle u, v\rangle_{L^2(\mathbb{R}_+,\sigma)} I_{n-1}(v^{\circ(n-1)}), \tag{2.8}$$

where σ is the Lebesgue measure. The Charlier polynomials of parameter $t > 0$ are defined as $C_0(x,t) = 1$, $C_1(x,t) = x - t$, and from the recurrence relation

$$C_{n+1}(x,t) = (x-n-t)C_n(x,t) - ntC_{n-1}(x,t), \qquad n \geq 1. \tag{2.9}$$

By comparison of (2.8) with (2.9) we get

$$I_n(1_A^{\circ n}) = C_n(N(A), \sigma(A)), \qquad A \in \mathcal{B}(\mathbb{R}_+), \quad n \geq 0.$$

More generally, if f_n has the decomposition

$$f_n = \sum_{\substack{n_1+\cdots+n_d=n \\ k_1,\dots,k_d\geq 0}} a_{k_1,\dots,k_d}^{n_1,\dots,n_d} 1_{A_{k_1}^n}^{\circ n_1} \circ \cdots \circ 1_{A_{k_d}^n}^{\circ n_d},$$

we have

$$I_n(f_n) = \sum_{\substack{n_1+\cdots+n_d=n \\ k_1,\dots,k_d\geq 0}} a_{k_1,\dots,k_d}^{n_1,\dots,n_d} C_{n_1}(N(A_{k_1}^n), \sigma(A_{k_1}^n)) \cdots C_{n_d}(N(A_{k_d}^n), \sigma(A_{k_d}^n)), \qquad n \geq 1.$$

Hence the expansion

$$F = E[F] + \sum_{n=1}^{\infty} \sum_{\substack{n_1+\cdots+n_d=n \\ k_1,\ldots,k_d\geq 0}} a_{k_1,\ldots,k_d}^{n_1,\ldots,n_d} C_{n_1}(N(A_{k_1}^n), \sigma(A_{k_1}^n) \cdots C_{n_d}(N(A_{k_d}^n), \sigma(A_{k_d}^n))$$

becomes

$$F = E[F] + \sum_{n=1}^{\infty} I_n(f_n),$$

and the Poisson process also has the chaos representation property. The gradient and Skorohod integral operators D, δ, are also defined as in (2.2), (2.3), with the same properties as in the general setting of normal martingales, i.e. they are mutually adjoint and they satisfy (2.4). However, the Charlier polynomials satisfy the finite difference rule

$$C_n(k+1,t) - C_n(k,t) = nC_{n-1}(k,t),$$

from which it can be shown that

$$D_t F(N_\cdot) = F(N_\cdot + 1_{[t,\infty)}) - F(N_\cdot),$$

cf. [9], [11]. Hence in the Poisson case, the use of chaos decompositions does not yield a derivation operator, and therefore it is not suitable for the computation of Greeks.

2.4 Derivation operator on Poisson space

The use of infinitesimal perturbations in the Malliavin calculus on Poisson space has been introduced in [2], [1], [3], [6]. In this section we give a presentation of the Malliavin calculus on Poisson space of [3], [12], [13], adapted to our framework. Let $(T_k)_{k\geq 1}$ denote the jump times of $(N_t)_{t\in[0,T]}$, on a probability space $(\Omega, \mathcal{F}_T, P)$. Let $\mathcal{C}_c((0,T))$, resp. $\mathcal{C}_c^1((0,T))$, denote the space of continuous, resp. continuously differentiable, functions on $[0,T]$ with support in the open interval $(0,T)$.

Definition 2.1. *Given $T > 0$, let $\mathcal{S}_T$ denote the set of smooth functionals of the form*

$$F = f_0 \mathbf{1}_{\{N_T=0\}} + \sum_{n=1}^{m} \mathbf{1}_{\{N_T=n\}} f_n(T_1, \ldots, T_n), \tag{2.10}$$

where $f_0 \in \mathbb{R}$ and $f_n \in \mathcal{C}^1([0,T]^n)$, $1 \leq n \leq m$, is symmetric in n variables, $m \geq 1$.

Recall that we have under P, for all $F \in \mathcal{S}_T$ of the form (2.10):

$$E[F] = e^{-\lambda T} f_0 + e^{-\lambda T} \sum_{n=1}^{m} \frac{1}{n!} \int_0^T \cdots \int_0^T f_n(t_1, \ldots, t_n) dt_1 \cdots dt_n.$$

Definition 2.2. *Given $w \in \mathcal{C}_c((0,T))$, let D_w denote the gradient operator defined on $F \in \mathcal{S}_T$ of the form (2.10) by*

$$D_w F = -\sum_{n=1}^{m} \mathbf{1}_{\{N_T = n\}} \sum_{k=1}^{k=n} w(T_k) \partial_k f_n(T_1, \ldots, T_n),$$

where $\partial_k f_n$ denotes the partial derivative of f_n with respect to its k-th variable.

The next proposition relies on the finite-dimensional integrations by parts:

$$\begin{aligned} E[D_w F] &= -e^{-\lambda T} \sum_{n=1}^{m} \frac{1}{n!} \int_0^T \cdots \int_0^T \sum_{k=1}^{k=n} w(t_k) \partial_k f_n(t_1, \ldots, t_n) dt_1 \cdots dt_n \\ &= e^{-\lambda T} \sum_{n=1}^{m} \frac{1}{n!} \int_0^T \cdots \int_0^T f_n(t_1, \ldots, t_n) \sum_{k=1}^{k=n} w'(t_k) dt_1 \cdots dt_n \\ &= E\left[F \sum_{k=1}^{k=N_T} w'(T_k) \right] = E\left[F \int_0^T w'(t) dN_t \right], \end{aligned}$$

where $w'(t)$ denotes the derivative of w with respect to t and where we used the boundary conditions $w(0) = w(T) = 0$.

Proposition 2.1. *Let $w \in \mathcal{C}_c^1((0,T))$.*
a) The operator D_w is closable and admits a closable adjoint D_w^* such that

$$E[G D_w F] = E[F D_w^* G], \quad F, G \in \mathcal{S}_T. \tag{2.11}$$

b) For all $F \in \mathrm{Dom}\ (D_w) \bigcap L^4(\Omega)$ we have $F \in \mathrm{Dom}\ (D_w^*)$ and:

$$D_w^* F = F \int_0^T w'(t) dN_t - D_w F. \tag{2.12}$$

Proof. Relation (2.11) is proved above when $G = 1$. Now, define $D_w^* G$, $G \in \mathcal{S}_T$, by (2.12), with for all $F \in \mathcal{S}_T$:

$$E[G D_w F] = E[D_w(FG) - F D_w G] = E\left[F \left(G \int_0^T w'(t) dN_t - D_w G \right) \right] = E[F D_w^* G],$$

which proves (2.11). From this relation we have

$$|E[UG]| \;\leq\; |E[F_n D_w^* G] - E[UG]| + |E[F_n D_w^* G]|$$

$$\begin{aligned} &= |E[(D_wF_n - U)G]| + |E[F_nD_w^*G]| \\ &\le \|D_wF_n - U\|_{L^2(\Omega)}\|G\|_{L^2(\Omega)} + \|F_n\|_{L^2(\Omega)}\|D_w^*G\|_{L^2(\Omega)}, \end{aligned}$$

from which the closability of D_w follows: if $F_n \to 0$ in $L^2(\Omega)$ and $DF_n \to U$ in $L^2(\Omega)$, then $U = 0$. For all F in the space Dom (D_w) of functionals $F \in L^2(\Omega)$ for which there exists a sequence $(F_n)_{n\in\mathbb{N}} \subset \mathcal{S}_T$ converging to F such that $(D_wF_n)_{n\in\mathbb{N}}$ converges in $L^2(\Omega)$, we let $D_wF = \lim_{n\to\infty} D_wF_n$, and D_wF is well-defined. A similar argument applies to D_w^* and finally, (2.12) is extended to $F \in$ Dom $(D_w)\bigcap L^4(\Omega)$. □

In particular, $D_w^*\mathbf{1}_\Omega$ coincides with the Poisson stochastic integral of w':

$$D_w^*\mathbf{1}_\Omega = \int_0^T w'(t)dN_t.$$

The following proposition provides derivation rules for Poisson deterministic and stochastic integrals.

Proposition 2.2. *(i) Assume that $F(t,k) \in$ Dom D_w, $t \in \mathbb{R}_+$, $k \in \mathbb{N}$. We have*

$$D_w \int_0^T F(t,N_t)dt = \int_0^T w_t\nabla_k F(t,N_t)dN_t + \int_0^T [D_wF](t,N_t)dt.$$

where $\nabla_k f(t,k) = f(t,k) - f(t,k-1)$.

(ii) Assume that $F(t,k) \in$ Dom D_w, $t \in \mathbb{R}_+$, $k \in \mathbb{N}$, and $F(\cdot,k) \in \mathcal{C}_c^1(\mathbb{R}_+)$. Then

$$D_w \int_0^T F(t,N_t)dN_t = -\int_0^T w_t\partial_t F(t,N_t)dN_t + \int_0^T [D_wF](t,N_t)dN_t.$$

3 Computations of sensitivities

The main tool for the computation of sensitivities is presented in the next proposition. It follows from a classical Malliavin calculus argument applied to the derivation operator D_w. Let $I = (a,b)$ be an open interval of $\mathbb{R}$.

Proposition 3.1. *Let $(F^\zeta)_{\zeta\in I}$ be a family of random functionals, continuously differentiable in* Dom (D_w) *in the parameter $\zeta \in I$. Let $w \in \mathcal{C}_c^1((0,T))$ such that*

$$D_wF^\zeta \neq 0, \quad a.s.\ on\ \left\{\frac{\partial}{\partial_\zeta}F^\zeta \neq 0\right\}, \quad \zeta \in I,$$

and such that $\partial_\zeta F^\zeta / D_w F^\zeta$ *is continuous in* ζ *in* $\mathrm{Dom}\,(D_w) \bigcap L^4(\Omega)$. *We have for any function* f *such that* $f(F^\zeta) \in L^2(\Omega)$, $\zeta \in I$:

$$\frac{\partial}{\partial \zeta} E\left[f(F^\zeta)\right] = E\left[W_\zeta f(F^\zeta)\right], \tag{3.1}$$

where the weight W_ζ *is given on* A *by*

$$W_\zeta = \frac{\partial_\zeta F^\zeta}{D_w F^\zeta}\left(\int_0^T w'(t)dN_t + \frac{D_w D_w F^\zeta}{D_w F^\zeta}\right) - \frac{D_w \partial_\zeta F^\zeta}{D_w F^\zeta}, \qquad \zeta \in I.$$

Proof. Assuming that $f \in \mathcal{C}_b^\infty(\mathbb{R})$, we have

$$\begin{aligned} \frac{\partial}{\partial \zeta} E\left[f(F^\zeta)\right] &= E\left[f'\left(F^\zeta\right)\frac{\partial}{\partial \zeta}F^\zeta\right] \\ &= E\left[\frac{\partial_\zeta F^\zeta}{D_w F^\zeta} D_w f(F^\zeta)\right] \\ &= E\left[f(F^\zeta) D_w^*\left(\frac{\partial_\zeta F^\zeta}{D_w F^\zeta}\right)\right]. \end{aligned}$$

Using (2.12) and the chain rule of derivation for D_w, the weight $D_w^*\left(\frac{\partial_\zeta F^\zeta}{D_w F^\zeta}\right)$ can be computed using Poisson stochastic integrals:

$$\begin{aligned} D_w^*\left(\frac{\partial_\zeta F^\zeta}{D_w F^\zeta}\right) &= \frac{\partial_\zeta F^\zeta}{D_w F^\zeta}\int_0^T w'(t)dN_t - D_w\left(\frac{\partial_\zeta F^\zeta}{D_w F^\zeta}\right) \\ &= \left(\frac{\partial_\zeta F^\zeta}{D_w F^\zeta}\int_0^T w'(t)dN_t - \frac{D_w \partial_\zeta F^\zeta}{D_w F^\zeta} + \frac{\partial_\zeta F^\zeta}{\left(D_w F^\zeta\right)^2} D_w D_w F^\zeta\right). \end{aligned}$$

The extension to square-integrable f is obtained as in [8], [5], using an approximating sequence $(f_n)_{n \in \mathbb{N}}$ of smooth functions. □

For the Delta we have $F^x = xF$ and

$$W_\zeta = \frac{1}{x}\left(\frac{F}{D_w F}\int_0^T w_t' dN_t - \frac{D_w F}{D_w F} + \frac{F}{\left(D_w F\right)^2} D_w D_w F\right).$$

4 Market model and simulations

Consider $(S_t)_{t \in [0,T]}$ an underlying asset price

$$dS_t = r_t S_t dt + \sigma_t S_{t^-} dN_t, \qquad S_0 = x.$$

We have

$$S_t = F(t, N_t),$$

with

$$F(t,k) = xe^{\int_0^t r_s(N_s)ds} \prod_{i=0}^{i=k} \left(1 + \sigma_{T_i}(i)\right).$$

The weight corresponding to the Delta of an Asian option with price

$$C(x) = e^{-\int_0^T r_s ds} E\left[f\left(\int_0^T S_u^x du\right)\right],$$

is equal to:

$$\frac{1}{x\sigma}\left(\frac{\int_0^T S_t dt \int_0^T w_t' dN_t}{\int_0^T w_t S_{t^-}\beta_{N_{t^-}} dN_t} - 1 - \frac{\int_0^T S_t dt \int_0^T w_t \left(w_t' + \alpha w_t\right) S_{t^-}\beta_{N_{t^-}} dN_t}{\left(\int_0^T w_t S_{t^-}\beta_{N_{t^-}} dN_t\right)^2}\right),$$

and the finite difference method gives Delta as

$$\text{Delta} = \frac{C(x(1+\epsilon)) - C(x(1-\epsilon))}{2\epsilon}.$$

We take $T = 1$, $x = 100$, $r = 0.095310$, $\sigma = 0.008944$, $\epsilon = 0.001$, and a Poisson process intensity of 500. For the Malliavin approach we choose $w_t = \sin(\pi t/T)$ as weight function. In the following graph, realized with Premia, the Delta of a binary Asian option with Strike K and price

$$C(x) = e^{-rT} E\left[1_{[K,\infty[}\left(\frac{1}{T}\int_0^T S_t^x dt\right)\right]$$

is represented as a function of the Strike K with 10000 iterations for each value of K for both the Malliavin and finite difference methods.

The simulation graph shows a better convergence of the Greeks obtained from the Malliavin method on Poisson space for Asian options in a market with jumps, when compared to the finite difference approximation.

In [5] a more general model has been considered, for an underlying asset price given under the risk-neutral probability by the linear equation

$$dS_t = r_t(N_t)S_t dt + \sigma_t(N_{t^-})S_{t^-}(\beta_{N_{t^-}} dN_t - \nu dt), \tag{4.1}$$

where $(\beta_k)_{k\in\mathbb{N}}$ is a discrete-time process independent of $(N_t)_{t\in\mathbb{R}_+}$, which models jumps of different heights. This method can be extended to a model with state-dependent coefficients given by a nonlinear equation of the form

$$dS_t = \alpha_t(S_t)dt + \sigma_t(S_{t^-})\beta_{N_{t^-}} dN_t, \quad S_0 = x, \tag{4.2}$$

where the computations of $\int_0^T S_t dt$ and its derivatives can still be performed by induction. An independent diffusion term may also be introduced in the driving stochastic differential equation as in the complete market model of [10]:

$$dS_t = r_t S_t dt + \sigma_t S_{t^-} \left(1_{\{\phi_t=0\}} dB_t + \phi_t(\beta_{N_{t^-}} dN_t - \nu_t dt) \right), \quad t \in \mathbb{R}_+,$$

where $\phi : \mathbb{R}_+ \longrightarrow \mathbb{R}$ is a deterministic bounded functions satisfying $1 + \sigma_t \beta_{N_{t^-}} \phi_t > 0$, $t \in \mathbb{R}_+$, and $(B_t)_{t\in\mathbb{R}_+}$ is a Brownian motion independent of $(N_t)_{t\in\mathbb{R}_+}$.

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