

```

    Help
#include <stdlib.h>
#include "hk1d_std.h"
#include "mathsb.h"
#include "currentzcb.h"
#include "hktree.h"

#if defined(PremiaCurrentVersion) && PremiaCurrentVersion <
    (2007+2) //The "#else" part of the code will be freely av
    ailable after the (year of creation of this file + 2)
static int CHK_OPT(TR_BERMUDANSWAPTION)(void *Opt, void *
    Mod)
{
    return NONACTIVE;
}
int CALC(TR_BERMUDANSWAPTION)(void *Opt,void *Mod,Pricing
    Method *Met)
{
    return AVAILABLE_IN_FULL_PREMIA;
}
#else

static char init[]="initialyield.dat";

/*Swaption=Option on Coupon-Bearing Bond*/
/*All details comments for the functions used here are mai
    nly in "hwtree1dincludes.h" and partially in this file*/
static void HK_iterations( int flat_flag, double r_flat,
    char* init, double a, double sigma_HW,
                        double T0, double per, int m,
    double K0, int xnumber, discrete_fct *N);

////////////////////////////////////
    //////////////////////////////////
// computes V_0(K), the current Hull-White-price of the
    digital (T,S)-caplet with strike K
////////////////////////////////////
    //////////////////////////////////

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/*static double HW_DigitalCaplet(double a0, double sigma0,
    double T, double S, double tau0, double POT, double POS, double
    K)
{
    double sigma_P, log_term;

    sigma_P = sigma0 * (exp(-a0*T) - exp(-a0*S))/a0 * sqrt( (
        exp(2*a0*T)-1)/(2*a0) );

    log_term = log( POT / POS / (tau0*K+1) );

    return tau0 * POS * cdf_nor( log_term/sigma_P - sigma_P/2
        );
}
*/

////////////////////////////////////
////////////////////////////////////
// solves V_0(K)=x, where V_0(K) is the current Hull-White-
// price of the digital (T,S)-caplet with strike K
////////////////////////////////////
////////////////////////////////////
static double Inv_HW_DigitalCaplet(double a0, double sigma0
    , double T, double S, double tau0, double POT, double POS,
    double x)
{
    double tau0_inv,sigma_P,exp_term,K;

    tau0_inv = 1/tau0;

    sigma_P = sigma0 * (exp(-a0*T) - exp(-a0*S))/a0 * sqrt( (
        exp(2*a0*T)-1)/(2*a0) );

    exp_term = exp( -SQR(sigma_P)/2 - sigma_P * pnl_inv_cdf
        nor( x / (tau0*POS) ) );

    K = tau0_inv*POT/POS * exp_term - tau0_inv;
    /*if (K<0) printf("Inv_HW_DigitalCaplet returns the negat
        ive strike K = %f\n", K); */
}

```

```

    return K;
}

////////////////////////////////////
////////////////////////////////////
// solves  $V_0^{\{m-1, HK\}}(K) = V_0^{\{m-1, HW\}}(K)$  in terms of si
// gma_HK, where  $V_0^{\{m-1, HK\}}(K)$  resp.  $V_0^{\{m-1, HK\}}(K)$ 
// is the current Hunt-Kennedy-price resp. Hull-White
// price of the digital  $(T_{\{m-1\}}, T_m)$ -caplet with strike K
////////////////////////////////////
////////////////////////////////////
static double ComputeSigma_HK(double a, double sigma_HW,
    double Tm_1, double Tm, double tau, double POTm_1, double POTm,
    double K)
{
    double sigma_HK, R0m_1, p, q, sigma_P;

    R0m_1 = (POTm_1/POTm - 1.) / tau;
    sigma_P = sigma_HW * (exp(-a*Tm_1) - exp(-a*Tm))/a * sq
        rt( (exp(2.*a*Tm_1)-1)/(2.*a) );
    p = 2. * log( (R0m_1+tau)/(K+tau) ) / sigma_P - sigma
        _P;
    q = -2. * log( R0m_1/K );
    sigma_HK = (sqrt( SQR(p)/4. - q ) - p/2.) / sqrt( (exp(2.
        *a*Tm_1)-1)/(2.*a) );

    return sigma_HK;
}

////////////////////////////////////
////////////////////////////////////
// functional form of the INVERSE of the numeraire at T[m-1
// ], i.e. of  $1/P(T_{\{m-1\}}, T_m)$ 
////////////////////////////////////
////////////////////////////////////
static double N_mminus1(double x, double C_2)
{

```

```

    return 1 + C_2*exp(x);
}

////////////////////////////////////
////////////////////////////////////
// functional form of the INVERSE of the numeraire at T[m-2
// ], i.e. of 1/P(T_{m-2}, T_m)
////////////////////////////////////
////////////////////////////////////
/*static double N_mminus2(double a_HW, double sigma_HW,
    double T_mminus2, double T_mminus1, double tau_mminus2, double
    POT_mminus2,
    double POT_mminus1, double POT_m, double C_0, double C_1,
    double Sig, double x)
{
    double result, J_term, P_term, V0_market_inv;

    P_term = 1 + C_0 * exp(x);
    J_term = POT_m * tau_mminus2 * ( cdf_nor(-x/Sig) + C_1*
        cdf_nor(-x/Sig+Sig) );
    V0_market_inv = Inv_HW_DigitalCaplet( a_HW, sigma_HW, T_
        mminus2, T_mminus1, tau_mminus2, POT_mminus2, POT_mminus1,
        J_term);

    result = P_term * ( 1 + tau_mminus2 * V0_market_inv);

    return result;
}*/

////////////////////////////////////
////////////////////////////////////
// returns the variance of the HK-process x_t given x_s
////////////////////////////////////
////////////////////////////////////
static double SigmaSqr( double t, double s, double sigma,
    double a)
{
    return SQR(sigma) * ( exp(2.*a*t) - exp(2.*a*s) )/(2*a);
}

```

```
}

```

```

////////////////////////////////////
/////
// returns (U_{t,s}f)(x), where U is the semigroup of operators
// corresponding to the HK-process
////////////////////////////////////
/////
static double U(double t, double s, discrete_fct *f,
    double x, double sigma, double a)
{
    return NormalTab( x, SigmaSqr(t,s,sigma,a), f);
}

```

```

static void SetUf(discrete_fct *g, double t, double s, discrete_fct *f, double sigma, double a)
    // Sets g = U_{t,s}f in a reasonable way
{
    SetNf( g, SigmaSqr(t,s,sigma,a), f);
}

```

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static double UfUpBound (discrete_fct *f, double t, double s, double vmax, double sigma, double a)
    // returns the minimum of all x>=f.xleft such that U_{t,s}(f*1_{(x,infty)})(0) < vmax
{
    return NfUpBound ( f, SigmaSqr(t,s,sigma,a), vmax);
}

```

```

static double UfLoBound (discrete_fct *f, double t, double s, double vmin, double sigma, double a)
    // returns the maximum of all x<f.right such that U_{t,s}(f*1_{(x,infty)})(0) > vmin

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{
    return NfLoBound ( f,  SigmaSqr(t,s,sigma,a), vmin);
}

static void HK_iterations( int flat_flag, double r_flat,
    char* init, double a, double sigma_HW,
    double T0, double per, int m,
    double K0, int xnumber, discrete_fct *N)
// flat_flag          : flag to decide wether initial yi
    eld curve is flat at r_flat (0) or read from the file init (
    1)
// a, sigma_HW        : parameters of the HW-model repres
    enting the market ("a" is common with the HK-process)
// T0                  : first HK-date
// per                  : HK-periodicity
// m                    : number of HK-dates
// K0                   : calibration strike (for the compu
    tation of sigma_HK)
// N                    : functional forms of the INVERSE of
    the numeraire at T[i], i.e. of 1/P(T_i,T_m), for i=0,...,
    m-1
// xnumber              : parameter for the discretization
    of the functional forms
{

    double *T;           /* HK-dates */
    double *tau;          /* tau[i] = year fraction
        from T[i] to T[i+1] */
    double *P0;           /* P0[i] = P(0,T[i]) (ini
        tial zcb prices) */
    double **Sigma;       /* corresponds to Sigma_{
        T[i],T[j-1]}, where T[-1] denotes 0 */
    /* here SQR(Sigma_{t,s}) is the variance of x_t given x_
        s */
    double C_2;           /* corresponds to the cons
        tant C_2 in the formula for N[m-1] */

    double x, s, result, J_term, P_term, V0_market_inv;
    double xle, xste, xri, eps;
    double sigma_HK;      /* parameter of the HK-proces */

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```

double xleft,xstep;      /* parameters for the discretiz
    ation of the functional forms */
int i,j,k;
discrete_fct Ptilde_i, Ptilde_ix;

////////////////////////////////////
// initialisation of the main variables //

T = malloc((m+1)*sizeof(double));
for (i=0; i<=m; i++)  T[i] = i * per + T0;
// T[0]=T0           : first resetting date of the swap
// T[1],...,T[m]      : payment dates of the swap
// T[0],...,T[n-1]    : exercise dates of the swaption
    ---> We suppose m>=n !!

tau = malloc(m*sizeof(double));
for(i=0; i<m; i++)  tau[i]=per;

P0 = malloc((m+1)*sizeof(double));
for(i=0; i<=m; i++)  P0[i] = CurrentZCB(T[i], flat_flag,
    r_flat, init);

// computation of sigma_HK
sigma_HK = ComputeSigma_HK(a, sigma_HW, T[m-1], T[m], ta
    u[m-1], P0[m-1], P0[m], K0);

Sigma = malloc(m*sizeof(double*));
for(i=0; i<m; i++)
{
    Sigma[i] = malloc((i+1)*sizeof(double));
    for(j=0; j<=i; j++)
    {
        if (j==0) s=0; else s=T[j-1];
        Sigma[i][j] = sigma_HK * sqrt( (exp(2*a*T[i]))-exp

```

[illegible]


```

{
    // printf("beginning for i=%d\n", i); // scanf("%d",
    &j);

    //////////////////////////////////////
    // setting of  $P_{i+1} := U_{T[i+1], T[i]} N[i+1]$  //
    //////////////////////////////////////
    SetUf( &Ptilde_i, T[i+1], T[i], &N[i+1], sigma_HK, a)
;
    // sets Ptilde_i such that domain( Ptilde_i ) = [ U_{T[i+1], T[i]} N[i+1] > 0 ]

    /*
        printf("Ptilde_i\n"); ShowDiscreteFct( &Ptilde_i )
;
        sprintf(filename, "Ptilde[%d].txt", i);
        SaveDiscreteFctToFile( &Ptilde_i, filename);
        printf("\ninitial discounted ZCB price for maturity T[%d]=%f :{n", i+1, T[i+1]);
        printf("HK: %f{n", U(T[i], 0., &Ptilde_i, 0., sigma_HK, a) );
        printf("HW: %f{n{n", P0[i+1]/P0[m]);
    */

    //////////////////////////////////////
    // setting of N[i] //
    //////////////////////////////////////
    eps = 0.000001;
    xle = UfUpBound( &Ptilde_i, T[i], 0., P0[i+1]/P0[m]-eps, sigma_HK, a);
    xri = UfLoBound( &Ptilde_i, T[i], 0., eps, sigma_HK, a);
    xste=(xri-xle)/(double)(xnumber-1);
    Set_discrete_fct( &N[i], xle, xste, xnumber);

    // printf("N[%d]{n",i); ShowDiscreteFct( &N[i] );

    //////////////////////////////////////

```

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//////////
// initialization of P~tilde_{i,x} as a (restricted)
copy of P~tilde_i //
//////////
//////////
Set_discrete_fct( &Ptilde_ix, N[i].xleft, N[i].xstep,
N[i].xnumber+1);
for(j=0; j<Ptilde_ix.xnumber; j++)
{
    x = Ptilde_ix.xleft + j*Ptilde_ix.xstep;
    Ptilde_ix.val[j] = U( T[i+1], T[i], &N[i+1], x,
sigma_HK, a);
}

//////////
//////////
// evaluation of N_i in its discretizing points N[i].
xleft + j*N[i].xstep //
//////////
//////////
for(j=0; j<N[i].xnumber; j++)
{
    x = N[i].xleft + j*N[i].xstep;    // observe: x =
Ptilde_ix.xleft + j*Ptilde_ix.xstep
    P_term = Ptilde_ix.val[j];        // hence: P_ter
m = P_i~tilde(x) !!!

    Ptilde_ix.val[j] = 0;
    // VERY IMPORTANT: now Ptilde_ix corresponds rea
lly to P~tilde_{i,x} := P~tilde_i * 1_{(x,infty)} !!!

    J_term = P0[m] * tau[i] * U( T[i], 0., &Ptilde_ix
, 0., sigma_HK, a);

    if (J_term==0) {printf("At j=%d: J_term=0 !\n",
j); scanf("%d", &k); ShowDiscreteFctVal( &Ptilde_ix ); }
    if (J_term/(tau[i]*P0[i+1])>=1)
        {printf("At j=%d: J_term too large !\n", j);
scanf("%d", &k);}
}

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        VO_market_inv = Inv_HW_DigitalCaplet( a, sigma_
HW, T[i], T[i+1], tau[i], P0[i], P0[i+1], J_term);

        result = P_term * ( 1 + tau[i] * VO_market_inv);
        // now we have: result = N_i(x)

        N[i].val[j] = result;
    }

    /*
        printf("eval. of N[%d] in its discret. points fini
shed\n", i);
        sprintf(filename, "N[%d].txt", i);
        SaveDiscreteFctToFile( &N[i], filename);
        printf("end for i=%d\n",i);
    */

    Delete_discrete_fct(&Ptilde_i);
    Delete_discrete_fct(&Ptilde_ix);

} // end of i-loop

////////////////////////////////////
// free the variables          //

free(T);
free(tau);
free(P0);
for(i=0; i<m; i++) free(Sigma[i]);
free(Sigma);

// end of: free the variables //
////////////////////////////////////

}

static int bermudanswaption_hk1d(int flat_flag,double a,
```

```

double t0, double sigma_HW,double r_flat,double T_final,double
T0,NumFunc_1 *p,int am,double Nominal,double K,double per/*
,int n*/,long N_step,int xnumber,double *price*/,double *
delta*/)
{
// flat_flag          : flag to decide wether initial yi
// eld curve is flat at r_flat (0) or read from the file init (
// 1)
// a, sigma_HW        : parameters of the HW-model rep
// resenting the market ("a" is common with the HK-process)
// T0                  : first reset date of the swap (=
// first HK-date)
// per                 : reset period of the swap (= HK-
// period)
// n                   : number of exercise dates of the
// swaption
// m                   : number of payment dates of the
// swap (= number of HK-dates)
// t0                  : time for which the swaption
// price is computed
// payer               : payer swaption (1) or receiver
// swaption (0)
// K                   : strike of the swaption
// Nominal              : nominal value of the swap
// N_step               : number of time steps in the tre
// e for the HK-process
// xnumber              : parameter for the discretization
// of the functional forms

double *T;                /* reset/payment dates of
the swap; exercisedates of the Bermudan swaption; HK-da
tes */
double *tau;               /* tau[i] = year fraction
from T[i] to T[i+1] */
double **Sigma;            /* corresponds to Sigma_{
T[i],T[j-1]}, where T[-1] denotes 0 */
/* here SQR(Sigma_{t,s}) is the variance of x_t given x_
s */
discrete_fct *N;          /* functional forms of th
e INVERSE of the numeraire at T[i], */
/* i.e. of 1/P(T_i,T_m), for i=0,...,m-1 */

```

```

struct Tree Tr;                                /* tree for the HK-proces
s */

double x, s, **disc_payoff, PtildeTiTk, calib_strike, disc_price;
double sigma_HK;                               /* parameter of
the HK-proces */
int i,j,k,*ind,*Size,payer_sign;
int m,n,payer;

m=(int)((T_final-T0)/per);
n=m;

if ((p->Compute)==&Put)
    payer=1;
else
    /*if ((p->Compute)==&Call)*/
    payer=0;
//////////
// initialisation of the main variables //

T = malloc((m+1)*sizeof(double));
for (i=0; i<=m; i++) T[i] = i * per + T0;
// T[0]=T0          : first resetting date of the swap
// T[1],...,T[m]    : payment dates of the swap
// T[0],...,T[n-1]  : exercise dates of the swaption
---> We suppose m>=n !!

tau = malloc(m*sizeof(double));
for(i=0; i<m; i++) tau[i]=per;

// comput. of sigma_HK via calibr. to the digital (T[m-1]
,T[m])-caplet
// calib_strike = (P0[m-1]/P0[m]-1)/tau[m-1]; // at the
money case
calib_strike = K;
sigma_HK = ComputeSigma_HK(a, sigma_HW, T[m-1], T[m], ta

```

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    u[m-1],
                                CurrentZCB(T[m-1], flat_flag,
    r_flat, init),
                                CurrentZCB(T[m]   , flat_flag,
    r_flat, init), calib_strike);

Sigma = malloc(m*sizeof(double*));
for(i=0; i<m; i++)
{
    Sigma[i] = malloc((i+1)*sizeof(double));
    for(j=0; j<=i; j++)
    {
        if (j==0) s=0; else s=T[j-1];
        Sigma[i][j] = sigma_HK * sqrt( (exp(2*a*T[i])-exp
(2*a*s)) / (2*a) );
    }
}

// functional forms of the INVERSE of the numeraire at T[
i], i.e. of 1/P(T_i,T_m), for i=0,...,m-1
N = malloc(m*sizeof(discrete_fct));

// end of: initialisation of the main variables //
////////////////////////////////////

////////////////////////////////////
//////////
// construction of a trinomial tree for the HK-process
//

// the last exercise date T[n-1] is the final time of th
e tree, N_step is the number of time steps
SetTimegrid( &Tr, T[n-1], N_step );

// add (if necessary) the exercise dates T[0],...T[n-2]

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    to the time grid of the tree
    for (i=0; i<n-1; i++) AddTime( &Tr, T[i] );

// construct a tree for the HK-process (x_t) given by: dx
//_t = sigma*exp(a*t) dW_t , x_0=0
SetHKtree( &Tr, a, sigma_HK );

// end of: construction of a trinomial tree for the HK-
// process //
////////////////////////////////////
/////////

ind = malloc(n*sizeof(int));
for (i=0; i<n; i++)
    ind[i] = indiceTime( &Tr, T[i] );
// we have: Tr.t[ ind[i] ] = T[i]

Size = malloc(n*sizeof(int));
for (i=0; i<n; i++)
    Size[i] = Tr.TSize[ind[i]];
// at T[i], the tree has Size[i] nodes

disc_payoff = malloc(n*sizeof(double*));
// disc_payoff[i] will represent the discounted payoff of
// the payer swaption at T[i] !!
for (i=0; i<n; i++)
{
    disc_payoff[i] = malloc(Size[i]*sizeof(double));

    for (j=0; j<Size[i]; j++)
        disc_payoff[i][j] = -1 - K*tau[m-1];
}
// for the moment, disc_payoff[i] represents the constant
// payoff -1-K*tau[m-1]

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// Construct the functional forms N[0],...,N[m-1]
HK_iterations( flat_flag, r_flat, init, a, sigma_HW,
               T0, per, m, K, xnumber, N);

// Complete the discounted payoff of the payer swaption
// in disc_payoff
for(i=0; i<n; i++)
    for(j=0; j<Size[i]; j++)
    {
        x = Tr.pLRij[ ind[i] ][j];
        disc_payoff[i][j] += InterpolDiscreteFct( &N[i], x );
        for(k=i+1; k<m; k++)
        {
            PtildeTiTk = NormalTab( x, SQR(Sigma[k][i+1]),
            &N[k] );
            disc_payoff[i][j] -= K * tau[k-1] * PtildeTiTk;
        }
    }
// now disc_payoff[i] represents  $1/P(T[i],T[m]) - (1+K\tau_{u[m-1]}) - K\sum_{k=i+1}^{m-1} \tau[k-1] * P^{\text{tilde}}(T[i],T[k])$  which is the correct discounted
// payoff at T[i] of the payer swaption !!

if (payer) payer_sign=1; else payer_sign=-1;

initPayoff1(&Tr, T[n-1]);
for (i=0; i<n; i++)
{
    for (j=0; j<Size[i]; j++)
    {
        Tr.Payofffunc[ind[i]][j] = MAX( payer_sign * disc_
        payoff[i][j] , 0 );
    }
}

// Compute the swaption from the last exercise date T[n-1

```



```

    ] to 0 in Tr.plQij
    Computepayoff1(&Tr, T[n-1]);

// return plQij[0][1] as discounted price of the swapt
ion
if (t0==0)
{
    disc_price = Nominal * Tr.plQij[0][1];
    //      printf("disc. price = %e\n", disc_price);
    //      *price = P0[m] * disc_price;
    *price = CurrentZCB(T[m], flat_flag, r_flat, init) *
disc_price;
    // printf("price = %e\n", *price);
}
else printf("Evaluation in t>0 is not implemented.\n");

////////////////////////////////////
// free the variables          //

free(T);
free(tau);
// free(P0);
for(i=0; i<m; i++) free(Sigma[i]);
free(Sigma);
for(i=0; i<m; i++) Delete_discrete_fct(&N[i]);
free(N);

DeletePayoff1(&Tr, T[n-1]);
DeleteTree(&Tr);

for(i=0; i<n; i++) free(disc_payoff[i]);
free(disc_payoff);
free(ind);
free(Size);

// end of: free the variables //
////////////////////////////////////

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    return OK;
}

int CALC(TR_BERMUDANSWAPTION)(void *Opt,void *Mod,Pricing
    Method *Met)
{
    TYPEOPT* ptOpt=(TYPEOPT*)Opt;
    TYPEMOD* ptMod=(TYPEMOD*)Mod;

    return bermudanswaption_hk1d(ptMod->flat_flag.Val.V_INT,
                                ptMod->a.Val.V_DOUBLE,
                                ptMod->T.Val.V_DATE,
                                ptMod->Sigma.Val.V_PDOUBLE,
                                MOD(GetYield)(ptMod),
                                ptOpt->BMaturity.Val.V_DATE,
                                ptOpt->OMaturity.Val.V_DATE,
                                ptOpt->PayOff.Val.V_NUMFUNC_
1,
                                ptOpt->EuOrAm.Val.V_BOOL,
                                ptOpt->Nominal.Val.V_PDOUB
LE,
                                ptOpt->FixedRate.Val.V_PDOUB
LE,
                                ptOpt->ResetPeriod.Val.V_DA
TE,
                                Met->Par[0].Val.V_LONG,
                                Met->Par[1].Val.V_INT,
                                &(Met->Res[0].Val.V_DOUBLE))
    ;
}

static int CHK_OPT(TR_BERMUDANSWAPTION)(void *Opt, void *
    Mod)
{
    if ((strcmp(((Option*)Opt)->Name,"PayerBermudanSwaption")
        ==0) || (strcmp(((Option*)Opt)->Name,"
        ReceiverBermudanSwaption")==0))
        return OK;
    else

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```

        return WRONG;
    }

#endif //PremiaCurrentVersion
static int MET(Init)(PricingMethod *Met,Option *Opt)
{
    if ( Met->init == 0)
    {
        Met->init=1;

        Met->Par[0].Val.V_LONG=140;
        Met->Par[1].Val.V_INT=1000;

    }
    return OK;
}

PricingMethod MET(TR_BERMUDANSWAPTION)=
{
    "TR_HK1d_BERMUDANSWAPTION",
    {"TimeStepNumber",LONG,{100},ALLOW},
    {"Parameter for the discretization of the functional
    forms",INT,{100},ALLOW},
    {" ",PREMIA_NULLTYPE,{0},FORBID}},
    CALC(TR_BERMUDANSWAPTION),
    {"Price",DOUBLE,{100},FORBID}/*,{"Delta",DOUBLE,{100},FORBID}*/ ,{" ",PREMIA_NULLTYPE,{0},FORBID}},
    CHK_OPT(TR_BERMUDANSWAPTION),
    CHK_ok,
    MET(Init)
} ;

```

References