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#include <cmath>
#include <iostream>
#include "config.h"

#include "numerics.h"

const double SQ2P = sqrt(2.0*M_PI);
const double FPMIN = 1.0e-100;
const double EPS=0.00000001;
const double ITMAX=100;

double gammp(double a, double x){
    //Returns P(a,x)
    double gamser, gammcf, gln;
    if (x < 0.0 || a <= 0.0) myerror("Invalid arguments in routine gammp");
    if (x < (a + 1.0)) {
        gser(&gamser, a, x, &gln);
        return gamser;
    } else {
        gcf(&gammcf, a, x, &gln);
        return 1.0 - gammcf;
    }
}

double gammq(double a, double x){
    //Returns Q(a,x)
    double gamser, gammcf, gln;
    if (x < 0.0 || a <= 0.0) myerror("Invalid arguments in routine gammq");
    if (x < (a + 1.0)) {
        gser(&gamser, a, x, &gln);
        return 1.0 - gamser;
    } else {
        gcf(&gammcf, a, x, &gln);
        return gammcf;
    }
}
```

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void gser(double *gamser, double a, double x, double *gln)
//low-level routine for calculating Q(a; x)
{
    int n;
    double sum,del,ap;

    *gln=lgamma(a);
    if (x <= 0.0) {
        if (x < 0.0) myerror("x less than 0 in routine gser");
        *gamser=0.0;
        return;
    } else {
        ap=a;
        del=sum=1.0/a;
        for (n=1;n<=ITMAX;n++) {
            ++ap;
            del *= x/ap;
            sum += del;
            if (fabs(del) < fabs(sum)*EPS) {
                *gamser=sum*exp(-x+a*log(x)-(*gln));
                return;
            }
        }
        myerror("a too large, ITMAX too small in routine gser")
        ;
        return;
    }
}

void gcf(double *gammcf, double a, double x, double *gln) {
    //low-level routine for calculating Q(a; x)
    double an,b,c,d,del,h;
    *gln=lgamma(a);
    b=x+1.0-a;
    c=1.0/FPMIN;
    d=1.0/b;
    h=d;
    int i;
    for (i=1;i<=ITMAX;i++) {
        an = -i*(i-a);
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    b += 2.0;
    d=an*d+b;
    if (std::fabs(d) < FPMIN) d=FPMIN;
    c=b+an/c;
    if (std::fabs(c) < FPMIN) c=FPMIN;
    d=1.0/d;
    del=d*c;
    h *= del;
    if (std::fabs(del-1.0) < EPS) break;
}
if (i > ITMAX) myerror("a too large, ITMAX too small
    in routine gcf");
*gammcfc = std::exp(-x+a*std::log(x)-(*gln))*h;
}

double normPDF(double x) {
    return std::exp(-x*x/2)/SQ2P;
}

double normCDF(double x) {
    if(x > 0) return 1.0 - normCDF(-x);
    if(x * x < 3.0) return 0.5 * (1 - gammp(0.5, 0.5 * x * x)
    );
    else return 0.5 * gammq(0.5, 0.5 * x * x);
}

double expint(int n, double x) {
    int nm1;
    double a,b,c,d,del,fact,h,psi,ans=0;

    nm1=n-1;
    if (n<0 || x<0.0 || (x==0.0 && (n==0 || n==1)))
        myerror("bad argument in expint");
    else {
        if (n==0) ans=exp(-x)/x; //special case
        else {
            if (x==0.0) ans=1.0/nm1; //another special case
            else {
                if (x>1.0) {

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    b=x+n;
    c=1.0/FPMIN;
    d=1.0/b;
    h=d;
    for (int i=1; i<ITMAX; i++) {
        a = -i*(nm1+i);
        b += 2.0;
        d=1.0/(a*d+b);
        c=b+a/c;
        del=c*d;
        h *= del;
        if (fabs(del-1.0) < EPS) {
            ans = h*exp(-x);
            return ans;
        }
    }
    myerror("continued fraction failed in exprint");
} else {
    ans = (nm1 != 0 ? 1.0/nm1 : -log(x)-EULER);
    fact=1.0;
    for (int i=1; i<= ITMAX; i++) {
        fact *= -x/i;
        if (i != nm1) del = -fact/(i-nm1);
        else {
            psi = -EULER;
            for (int ii=1; ii<=nm1; ii++) psi += 1.0/ii;
            del=fact*(-log(x)+psi);
        }
        ans += del;
        if (fabs(del) < fabs(ans)*EPS) return ans;
    }
    myerror("series failed in exprint");
}
}
}
}
return ans;
}

double bessil(double x){
    double ax,ans;

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double y; //Accumulate polynomials in double precision

if ((ax=fabs(x)) < 3.75){ //Polynomial fit
    y = x/3.75;
    y *= y;
    ans = ax*(0.5 + y*(0.87890594 + y*(0.51498869 + y*(0.15
084934 + y*(0.2658733e-1
                                + y*(0.301532e-2 + y*0.32411
                                e-3))))));
} else{
    y = 3.75/ax;
    ans = 0.2282967e-1 + y*(-0.2895312e-1 + y*(0.1787654e-1
- y*0.420059e-2));
    ans = 0.39894228 + y*(-0.3988024e-1 + y*(-0.362018e-2+
y*(0.163801e-2
                                + y*(-0.1031555e-1 + y*ans))));
    ans *= (exp(ax)/sqrt(ax));
}
return x < 0.0 ? -ans : ans;
}

double bessk1(double x){
    double y,ans; //Accumulate polynomials in double precisi
on

    if(x <= 2.0){ //Polynomial fit
        y = x*x/4;
        ans = (log(x/2)*bessi1(x)) + (1./x)*(1+y*(0.15443144 +
y*(-0.67278579
                                +y*(-0.18156897 + y*(-0.1919402
e-1 + y*(-0.110404e-2 + y*(-0.4686e-4))))));
    } else{
        y = 2./x;
        ans = (exp(-x)/sqrt(x))*(1.25331414+y*(0.23498619 + y*(
-0.3655620e-1 + y*(0.1504268e-1
                                +y*(-0.780353e-2+y*(0.3256
14e-2+y*(-0.68245e-3))))));
    }
    return ans;
}
}

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//-----
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void polint(double xa[], double ya[], int n, double x,
           double *y, double *dy)
//Given arrays xa[1..n] and ya[1..n], and given a value x,
//this routine returns a value y, and
//an error estimate dy. If P(x) is the polynomial of degree
//N - 1 such that P(xai) = yai, i =
//1, . . . , n, then the returned value y = P(x).
{
    int ns=1;
    double den,dif,dift,ho,hp,w;
    double* c = new double[n];
    double* d = new double[n];
    c--; //for c and range between 1 and n
    d--; //instead of 0 and n-1
    dif=fabs(x-xa[1]);
    for (int i=1;i<=n;i++) { //Here we find the index ns of the
        //closest table entry,
        if ( (dift=fabs(x-xa[i])) < dif) {
            ns=i;
            dif=dift;
        }
        c[i]=ya[i]; //and initialize the tableau of c's and d's.
        d[i]=ya[i];
    }
    *y=ya[ns--]; //This is the initial approximation to y.
    for (int m=1;m<n;m++) { //For each column of the tableau,
        for (int i=1;i<=n-m;i++) { //we loop over the current
            //c's and d's and update them.
            ho=xa[i]-x;
            hp=xa[i+m]-x;
            w=c[i+1]-d[i];
            if ( (den=ho-hp) == 0.0) myerror("Error in routine polint");
            //This error can occur only if two input xa's are (
            //to within roundoff) identical.
            den=w/den;
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    d[i]=hp*den; //Here the c's and d's are updated.
    c[i]=ho*den;
}
*y += (*dy=(2*ns < (n-m) ? c[ns+1] : d[ns--]));
//After each column in the tableau is completed, we decide which correction, c or d,
//we want to add to our accumulating value of y, i.e., which path to take through the
//tableau-forking up or down. We do this in such a way as to take the most "straight
//line" route through the tableau to its apex, updating ns accordingly to keep track of
//where we are. This route keeps the partial approximations centered (insofar as possible)
//on the target x. The last dy added is thus the error indication.
}
c++;
d++;
delete[] d;
delete[] c;
}

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References