```
   Help
#include  "bs2d_std2d.h"
#include "error_msg.h"
#define PRECISION 1.0e-7 /*Precision for the localization
    of FD methods*/



static int howard_amer2(double s1,double s2,NumFunc_2  *p,
    double t,double r,double divid1,double divid2,double sigma1,
    double sigma2,double rho,int N, int M,double epsilon,double *pt
    price,double *ptdelta1,double *ptdelta2)
{
  double k,h,x1,x2,sigma11,sigma21,sigma22,m1,m2,trend1,tre
    nd2,limit,aa,bb,error,error2,temp,g0,g1;
  double **P,**Obst,**G,**R,**A,**B,**Q;
  int Index,TimeIndex,i,j;
  int **pp;

  /*Memory Allocation*/
  P=(double **)calloc(N+1,sizeof(double *));
  if (P==NULL)
    return MEMORY_ALLOCATION_FAILURE;
  for (i=0;i<N+1;i++)
    {
      P[i]=(double *)calloc(N+1,sizeof(double));
      if (P[i]==NULL)
        return MEMORY_ALLOCATION_FAILURE;
    }

  R=(double **)calloc(N+1,sizeof(double *));
  if (R==NULL)
    return MEMORY_ALLOCATION_FAILURE;
  for (i=0;i<N+1;i++)
    {
      R[i]=(double *)calloc(N+1,sizeof(double));
      if (R[i]==NULL)
        return MEMORY_ALLOCATION_FAILURE;
    }
```

```
Obst=(double **)calloc(N+1,sizeof(double *));
if (Obst==NULL)
  return MEMORY_ALLOCATION_FAILURE;
for (i=0;i<N+1;i++)
  {
    Obst[i]=(double *)calloc(N+1,sizeof(double));
    if (Obst[i]==NULL)
      return MEMORY_ALLOCATION_FAILURE;
  }


pp=(int **)calloc(N+1,sizeof(int *));
if (pp==NULL)
  return MEMORY_ALLOCATION_FAILURE;
for (i=0;i<N+1;i++)
  {
    pp[i]=(int *)calloc(N+1,sizeof(int));
    if (pp[i]==NULL)
      return MEMORY_ALLOCATION_FAILURE;
  }

G=(double **)calloc(N+1,sizeof(double *));
if (G==NULL)
  return MEMORY_ALLOCATION_FAILURE;
for (i=0;i<N+1;i++)
  {
    G[i]=(double *)calloc(N+1,sizeof(double));
    if (G[i]==NULL)
      return MEMORY_ALLOCATION_FAILURE;
  }


A=(double **)calloc(N+1,sizeof(double *));
if (A==NULL)
  return MEMORY_ALLOCATION_FAILURE;
for (i=0;i<N+1;i++)
  {
    A[i]=(double *)calloc(N+1,sizeof(double));
    if (A[i]==NULL)
      return MEMORY_ALLOCATION_FAILURE;
  }
```

```
B=(double **)calloc(N+1,sizeof(double *));
if (B==NULL)
  return MEMORY_ALLOCATION_FAILURE;
for (i=0;i<N+1;i++)
  {
    B[i]=(double *)calloc(N+1,sizeof(double));
    if (B[i]==NULL)
      return MEMORY_ALLOCATION_FAILURE;
  }

Q=(double **)calloc(N+1,sizeof(double *));
if (Q==NULL)
  return MEMORY_ALLOCATION_FAILURE;
for (i=0;i<N+1;i++)
  {
    Q[i]=(double *)calloc(N+1,sizeof(double));
    if (Q[i]==NULL)
      return MEMORY_ALLOCATION_FAILURE;
  }



/*Covariance Matrix*/
sigma11=sigma1;
//sigma12=0.0;
sigma21=rho*sigma2;
sigma22=sigma2*sqrt(1.0-SQR(rho));

m1=(r-divid1)-SQR(sigma11)/2.0;
m2=(r-divid2)-(SQR(sigma21)+SQR(sigma22))/2.0;



/*Space Localisation*/
limit=sqrt(t)*sqrt(log(1/PRECISION));
h=2*limit/(double)N;

/*Time Step*/
k=t/(double)M;
```

```
/*Terminal Values*/
x1=log(s1);
x2=log(s2);
trend1=exp(x1+m1*t);
trend2=exp(x2+m2*t);

for (i=1;i<N;i++)
  for (j=1;j<N;j++)
    {
P[i][j]=(p->Compute)(p->Par,trend1*exp(sigma11*(-limit+
  h*(double)j)),trend2*exp(sigma21*(-limit+h*(double)j)+sigma
  22*(limit-h*(double)i)));
    }



/*Homegenous Dirichlet Conditions*/
for(i=0;i<=N;i++)
  {
    P[i][0]=0.;
    P[i][N]=0.;
    P[0][i]=0.;
    P[N][i]=0.;
  }

/*Factor*/
aa=1+2.*k/(h*h)+r*k;
bb=-0.5*k/(h*h);


/*Finite Difference Cycle*/
for (TimeIndex=1;TimeIndex<M+1;TimeIndex++)
  {
    trend1=exp(x1+m1*(t-TimeIndex*k));
    trend2=exp(x2+m2*(t-TimeIndex*k));

    for (i=1;i<N;i++)
for (j=1;j<N;j++)
  Obst[i][j]=(p->Compute)(p->Par,trend1*exp(sigma11*(-    limit+h*(double)j)),
  sigma22*(limit-h*(double)i)));
```

```
    /*Init pp and R*/
    for (i=0;i<=N;i++)
for (j=0;j<=N;j++)
  {
    pp[i][j]=0;
    R[i][j]=-P[i][j];
  }
    /*Howard Cycle*/
    do
{
  error=0.;

  for (i=1;i<N;i++)
    for (j=1;j<N;j++)
      {
Q[i][j]=P[i][j];
g0=P[i][j]*aa+(P[i+1][j]+P[i-1][j]+P[i][j+1]+P[i][j-1
])*bb+R[i][j];
g1=P[i][j]-Obst[i][j];
if (g0<g1) pp[i][j]=0;else pp[i][j]=1;
    }

  for (i=1;i<N;i++)
    for (j=1;j<N;j++)
      {
if (pp[i][j]==0)
  {
    G[i][j]=-R[i][j];A[i][j]=aa;B[i][j]=bb;
  }
else {G[i][j]=Obst[i][j];A[i][j]=1;B[i][j]=0.;}
    }

  /*Solve the system*/
  do
    {
      error2=0.;
      for (i=1;i<N;i++)
for (j=1;j<N;j++)
  {
    temp=P[i][j];
```

```
        P[i][j]=(-(P[i+1][j]+P[i-1][j]+P[i][j+1]+P[i][j-1
    ])*B[i][j]+G[i][j])/A[i][j];
        error2+=fabs(P[i][j]-temp);
      }
      }
    while (error2>epsilon);

    for (i=1;i<N;i++)
      for (j=1;j<N;j++)
        error+=fabs(P[i][j]-Q[i][j]);

  }
      while (error>epsilon);
      /*End Howard Cycle*/
    }
  /*End Finite Difference Cycle*/

  Index=(int)((double)N/2.0);

  /*Price*/
  *ptprice=P[Index][Index];

  /*Deltas*/
  *ptdelta2=(P[Index-1][Index]-P[Index+1][Index])/(2.*s2*h*
    sigma22);
  *ptdelta1=((P[Index][Index+1]-P[Index][Index-1])/(2.*s1*
    h)-sigma21*(*ptdelta2))/sigma11;


  /*Memory desallocation*/
  for (i=0;i<N+1;i++)
    free(P[i]);
  free(P);

  for (i=0;i<N+1;i++)
    free(R[i]);
  free(R);


  for (i=0;i<N+1;i++)
    free(Obst[i]);
```

```
  free(Obst);

  for (i=0;i<N+1;i++)
    free(pp[i]);
  free(pp);


  for (i=0;i<N+1;i++)
    free(G[i]);
  free(G);

  for (i=0;i<N+1;i++)
    free(Q[i]);
  free(Q);

  for (i=0;i<N+1;i++)
    free(A[i]);
  free(A);

  for (i=0;i<N+1;i++)
    free(B[i]);
  free(B);

  return OK;

}




int CALC(FD_Howard)(void *Opt,void *Mod,PricingMethod *Met)
{
  TYPEOPT* ptOpt=(TYPEOPT*)Opt;
  TYPEMOD* ptMod=(TYPEMOD*)Mod;
  double r,divid1,divid2;

  r=log(1.+ptMod->R.Val.V_DOUBLE/100.);
  divid1=log(1.+ptMod->Divid1.Val.V_DOUBLE/100.);
  divid2=log(1.+ptMod->Divid2.Val.V_DOUBLE/100.);

  return howard_amer2(ptMod->S01.Val.V_PDOUBLE,ptMod->S02.
```

```
    Val.V_PDOUBLE,ptOpt->PayOff.Val.V_NUMFUNC_2,
          ptOpt->Maturity.Val.V_DATE-ptMod->T.Val.V_DATE,
    r,divid1,divid2,
          ptMod->Sigma1.Val.V_PDOUBLE,ptMod->Sigma2.Val.
    V_PDOUBLE,ptMod->Rho.Val.V_RGDOUBLE,
          Met->Par[0].Val.V_INT,Met->Par[1].Val.V_INT,
    Met->Par[2].Val.V_RGDOUBLE,
          &(Met->Res[0].Val.V_DOUBLE),&(Met->Res[1].Val.
    V_DOUBLE),&(Met->Res[2].Val.V_DOUBLE) );
}

static int CHK_OPT(FD_Howard)(void *Opt, void *Mod)
{
  Option* ptOpt=(Option*)Opt;
  TYPEOPT* opt=(TYPEOPT*)(ptOpt->TypeOpt);

  if ((opt->EuOrAm). Val.V_BOOL==AMER)
    return OK;

  return WRONG;
}



static int MET(Init)(PricingMethod *Met,Option *Opt)
{
  if ( Met->init == 0)
    {
      Met->init=1;

      Met->Par[0].Val.V_INT2=80;
      Met->Par[1].Val.V_INT2=80;
      Met->Par[2].Val.V_RGDOUBLE=0.000001;


    }

  return OK;
}
```

```
PricingMethod MET(FD_Howard)=
{
  "FD_Howard2d",
  {{"SpaceStep",INT2,{100},ALLOW},{"TimeStep",INT2,{100},
    ALLOW}, {"Epsilon",RGDOUBLE,{100},ALLOW} ,{" ",PREMIA_NULLT
    YPE,{0},FORBID}},
  CALC(FD_Howard),
  {{"Price",DOUBLE,{100},FORBID},{"Delta1",DOUBLE,{100},FO
    RBID} ,{"Delta2",DOUBLE,{100},FORBID} ,
   {" ",PREMIA_NULLTYPE,{0},FORBID}},
  CHK_OPT(FD_Howard),
  CHK_ok,
  MET(Init)
};
```

References