Pricing of Exotic Options under Infinite Activity Lévy Model

DIA El Hadj Aly *

An infinite activity Lévy process can be approximated by a Lévy process with finite activity. The resulting errors can be controlled. In this note we will see how, once the approximation made, we can evaluate the prices of lookback and Asian options.

Premia 14

1 Preliminaries

A real Lévy process X is characterized by its generating triplet (γ, σ^2, ν). Where $(\gamma, \sigma) \in \mathbb{R} \times \mathbb{R}^+$, and ν is a Radon measure satisfying

$$\int_{\mathbb{R}} \left(1 \wedge x^2\right) \nu(dx) < \infty$$

By Lévy-Itô decomposition X can be written in this form

$$X_t = \gamma t + \sigma B_t + X_t^l + \lim_{\epsilon \downarrow 0} \widetilde{X}_t^\epsilon \tag{1.1}$$

With

$$X_t^l = \int_{|x|>1, s\in[0,t]} x J_X(dx \times ds) \equiv \sum_{0\le s\le t}^{|\Delta X_s|\ge 1} \Delta X_s$$

$$\begin{aligned}
\widetilde{X}_t^\epsilon &= \int_{\epsilon\le|x|\le 1, s\in[0,t]} x(J_X(dx\times ds) - \nu(dx)dt) \\
&\equiv \int_{\epsilon\le|x|\le 1, s\in[0,t]} x\widetilde{J}_X(dx\times ds) \\
&\equiv \sum_{0\le s\le t}^{\epsilon\le|\Delta X_s|<1} \Delta X_s - t\int_{\epsilon\le|x|\le 1} x\nu(dx)
\end{aligned}$$

Where J is a Poisson measure on $\mathbb{R} \times [0, \infty)$ with rate $\nu(dx)dt$ and B is a standard Brownian motion. In Lévy-Khinchine representation X, we characterize X by its characteristic function. That means

$$\mathbb{E}e^{iuX_t} = e^{t\varphi(u)} \quad \forall u \in \mathbb{R}$$

where φ is given by

$$\varphi(u) = i\gamma u - \frac{\sigma^2 u^2}{2} + \int_{\mathbb{R}} (e^{iux} - 1 - iux\mathbb{1}_{|x|\le 1})\nu(dx) \tag{1.2}$$

*INRIA Paris-Rocquencourt, domaine de Voluceau, BP 105, 78153 Le Chesnay Cedex France (dia.eha@gmail.com).

For any $\epsilon \in (0,1)$ we define the process R^ϵ by

$$R_t^\epsilon = -\widetilde{X}_t^\epsilon + \lim_{\delta \downarrow 0} \widetilde{X}_t^\delta \tag{1.3}$$

and X^ϵ by

$$X_t^\epsilon = \gamma t + \sigma B_t + X_t^l + \widetilde{X}_t^\epsilon \tag{1.4}$$

Then

$$X_t = X_t^\epsilon + \mathbb{R}_t^\epsilon \tag{1.5}$$

We set

$$\begin{aligned} M_t &= \sup_{0 \le s \le t} X_s \\ M_t^{\epsilon,X} &= \sup_{0 \le s \le t} X_s^\epsilon \\ m_t^{\epsilon,X} &= \inf_{0 \le s \le t} X_s^\epsilon \\ \hat{M}_t^\epsilon &= \sup_{0 \le s \le t} (X_s^\epsilon + \sigma_\epsilon W_s) \end{aligned}$$

Where W is a standard Brownian motion independent of X, and $\sigma(\epsilon) = \sqrt{\int_{|x|<\epsilon} x^2 \nu(dx)}$.

2 Simulation method

We focus on the simulation of a lookback option with maturity T, where the Levy process is infinite activity without Brownian part. Our goal is to simulate M_T. In fact we can not simulate M_T, we will then approximated by M_T^ϵ or $\hat{(M)}_T^\epsilon$. This introduces a bias. Denote by J the Poisson measure on $\mathbb{R} \times [0,\infty)$ of intensity $\nu(dx)dt$, then for $t \ge 0$, we have

$$\begin{aligned} X_t^\epsilon &= X_t - R_t^\epsilon \\ &= \gamma t + \int_{|x|>1, s \in [0,t]} x J_X(dx \times ds) + \int_{\epsilon \le |x| \le 1, s \in [0,t]} x J_X(dx \times ds) \\ &= \left(\gamma - \int_{\epsilon \le |x| \le 1} x \nu(dx)\right) t + \int_{|x|>\epsilon, s \in [0,t]} x J_X(dx \times ds) \\ &= \left(\gamma - \int_{\epsilon \le |x| \le 1} x \nu(dx)\right) t + \int_{x>\epsilon, s \in [0,t]} x J_X(dx \times ds) \\ &\quad + \int_{x<-\epsilon, s \in [0,t]} x J_X(dx \times ds) \\ &= \gamma_0^\epsilon t + \sum_{i=1}^{N_t^+} Y_i^+ - \sum_{i=1}^{N_t^-} Y_i^- \end{aligned}$$

Where

$$\gamma_0^\epsilon = \gamma - \int_{\epsilon \le |x| \le 1} x \nu(dx), \tag{2.6}$$

the r.v. $\left(Y_i^+\right)_{i \ge 1}$ are i.i.d. with common law $\frac{\nu_\epsilon^+(dx)}{\nu(\epsilon,+\infty)}$, the r.v. $\left(Y_i^-\right)_{i \ge 1}$ are i.i.d. with common law $\frac{\nu_\epsilon^-(-dx)}{\nu(-\infty,\epsilon)}$. The measures ν_ϵ^+ and ν_ϵ^- correspond respectively to ν restricted on $(0,+\infty)$ and on $(-\infty,0)$.

The process X^ϵ is a compound Poisson process. So to simulate M_T^ϵ, it suffices to simulate the instants of jump of X^ϵ and the corresponding jump. The random variable $\hat{(M)}_T^\epsilon$ must be approximated by its discrete version in the case of lookback options. The number of discretization points in this case is greater than in the case of classic jump-diffusion model. The Probem that arises is because the numbers of jumps on $[0, T]$ is relatively large, how to quickly simulate the size of the jumps. The simulation of the instants of jump is relatively simple. We will focus on the simulation of of the positive jumps. The simulation of $\left(Y_i^-\right)_{i\geq 1}$ will be identical. Let $\lambda_+^\epsilon = \nu(\epsilon, \infty)$. The cumulative distribution function of Y_1^+ cannot be determined explicitly, and so the inverse of the cdf either. So one way to simulate Y_1^+ is to use rejection sampling. This is time consuming, especially since it will make on average $\lambda_+^\epsilon T$ simulations. The alternative is to make a *discrete inversion* of the cdf, F_+, of Y_1^+. We have, for all $x > \epsilon$

$$F_+(x) = \frac{1}{\lambda_+^\epsilon}\int_\epsilon^x \nu(dx)$$

We will define a positive real A in order to have $\nu(A, +\infty)$ very small, in order of 10^{-16} for example (that is what we choose in our simulations). We suppose then that the r.v. Y_1^+ is in $[\epsilon, A]$. Set for any $k \in \{0, \dots, n\}$

$$\begin{aligned} x_k &= k\frac{A-\epsilon}{n} + \epsilon \\ y_k &= \frac{F_+(x_k)}{F_+(A)} \end{aligned}$$

Where n is the number of the discretization points on $[\epsilon, A]$. Note that $y_0 = 0$. How do we compute $(F_+(x_k))_{1\leq k\leq n}$? Notice that for any $k \in \{1, \dots, n\}$, we have

$$F_+(x_k) = \sum_{j=1}^{k} \left(F_+(x_j) - F_+(x_{j-1})\right)$$

with

$$\left(F_+(x_j) - F_+(x_{j-1})\right) = \int_{x_{j-1}}^{x_j} \nu(dx)$$

Depending on the Lévy measure, we will define an approximation method for the integrale $\int_{x_{j-1}}^{x_j} \nu(dx)$. We define the function G_+ by, for any $y \in [0, 1]$

$$G_+(y) = x$$

where x is the unique real satisfying $\frac{F_+(x)}{F_+(A)} = y$. Let $y \in [0, 1]$, to compute $G_+(y)$, we use the following method. We have to find first the integer $k > 1$ satifying $y_{k-1} \leq y < y_k$. Then we have

$$yF_+(A) = y_{k-1} + \int_{x_{k-1}}^{G_+(y)} \nu(dy)$$

We must approximate the above integrale depending on $G_+(y)$, and express the latter as a function of y. We will call G_+, the *discrete inverse function* of F_+. When n and A go to the infinity, we will get the inverse function of F_+. For our simulations, we suppose that Y_1^+ is equal in distribution to $G_+(U)$, where U is a uniform r.v. on $[0, 1]$.

3 Estimates of the inverse cdf of the jumps

We will, for some popular models, estimate the function G_+. The models that we consider in this section are VG, CGMY and NIG. Nonetheless, using the same methodology we can estimate the function G_+ for any other model.

3.1 The Variance-Gamma case

Let G be a gamma process with de parameters $(\mu, \kappa) \in \mathbb{R}_+^* \times \mathbb{R}_+^*$ (see [8]), satisfying $G_0 = 0$ and for any $t \geq 0$ and $h > 0$, $G_{t+h} - G_t$ have a gamma distribution with parameters $\left(h\frac{\mu^2}{\kappa}, \frac{\kappa}{\mu}\right)$. In fact in financial applications $\mu = 1$, and the process $(W_{G_t})_{t\geq 0}$ is a VG processus VG with parameter (θ, σ, κ). Its characteristic exponent is given by

$$\varphi(u) = \log\left(\left(1 - i\theta\kappa u + \frac{\sigma^2}{2}\kappa u^2\right)^{-\frac{1}{\kappa}}\right)$$

The process $(W_{G_t})_{t\geq 0}$, can be defined by its Lévy measure ν. Indeed

$$\nu(dx) = C\frac{e^{-Mx}}{x}\mathbb{1}_{x>0}dx + C\frac{e^{-G|x|}}{|x|}\mathbb{1}_{x<0}dx$$

Where

$$\begin{aligned}
C &= \frac{1}{\kappa} \\
M &= \frac{1}{\sigma}\sqrt{\frac{2}{\kappa} + \frac{\theta^2}{\sigma^2}} - \frac{\theta}{\sigma^2} \\
G &= \frac{1}{\sigma}\sqrt{\frac{2}{\kappa} + \frac{\theta^2}{\sigma^2}} + \frac{\theta}{\sigma^2}
\end{aligned}$$

This is a particular case of the CGMY process (by taking $Y = 0$, see [4]). The probability density function of Y_1^+ is then

$$f_+(x) = \frac{C}{\lambda_+^\epsilon}\frac{e^{-Mx}}{x}, \ x > \epsilon$$

Then for any $x > \epsilon$

$$F_+(x) = \frac{C}{\lambda_+^\epsilon}\int_\epsilon^x \frac{e^{-My}}{y}dy$$

Hence

$$F_+(x_k) - F_+(x_{k-1}) = \frac{C}{\lambda_+^\epsilon}\int_{x_{k-1}}^{x_k} \frac{e^{-My}}{y}dy$$

We approximate this integrale by

$$\frac{C}{\lambda_+^\epsilon}e^{-Mx_{k-1}}\int_{x_{k-1}}^{x_k} \frac{dy}{y}dy = \frac{C}{\lambda_+^\epsilon}e^{-Mx_{k-1}}\log\left(\frac{x_k}{x_{k-1}}\right)$$

The function G_+ satisfy

$$yF_+(A) = y_{k-1} + \frac{C}{\lambda_+^\epsilon}\int_{x_{k-1}}^{G_+(y)} \frac{e^{-My}}{y}dy$$

As previously the above integrale is approximated by

$$\frac{C}{\lambda_+^\epsilon}e^{-Mx_{k-1}}\log\left(\frac{G_+(y)}{x_{k-1}}\right)$$

Hence $G_+(y)$ can be approximated by

$$x_{k-1}\exp\left[\frac{\lambda_+^\epsilon}{C}\left(yF_+(A) - y_{k-1}\right)e^{-Mx_{k-1}}\right] \tag{3.7}$$

3.2 The CGMY case

It is a pure jump Lévy process (see [8]), with Lévy measure

$$\nu(dx) = C\frac{e^{-Mx}}{x^{1+Y}}\mathbb{1}_{x>0}dx + C\frac{e^{-G|x|}}{|x|^{1+Y}}\mathbb{1}_{x<0}dx$$

Where C, G and M are positive, and $Y \in (0,2)$. When $Y = 0$, we get the Variance-Gamma model. Its characteristic exponent is given by

$$\varphi(u) = \begin{cases} C\left((M-iu)\log\left(1-\frac{iu}{M}\right) + (G+iu)\log\left(1+\frac{iu}{G}\right)\right), & if\ Y=1 \\ C\Gamma(-Y)\left[M^Y\left(\left(1-\frac{iu}{M}\right)^Y - 1 + \frac{iuY}{M}\right) + G^Y\left(\left(1+\frac{iu}{G}\right)^Y - 1 - \frac{iuY}{G}\right)\right], & if\ Y \neq 1 \end{cases}$$

In the CGMY model, the probability density function of Y_1^+ is

$$f_+(x) = \frac{C}{\lambda_+^\epsilon}\frac{e^{-Mx}}{x^{1+x}},\ x > \epsilon$$

Then its cdf is

$$F_+(x) = \frac{C}{\lambda_+^\epsilon}\int_\epsilon^x \frac{e^{-My}}{y^{1+Y}}dy$$

Hence

$$F_+(x_k) - F_+(x_{k-1}) = \frac{C}{\lambda_+^\epsilon}\int_{x_{k-1}}^{x_k} \frac{e^{-My}}{y^{1+Y}}dy$$

Then we approximate $F_+(x_k) - F_+(x_{k-1})$ by

$$\frac{C}{\lambda_+^\epsilon}e^{-Mx_{k-1}}\int_{x_{k-1}}^{x_k}\frac{dy}{y^{1+Y}} = \frac{C}{\lambda_+^\epsilon Y}e^{-Mx_{k-1}}\left(\frac{1}{x_{k-1}^Y} - \frac{1}{x_k^Y}\right)$$

So G_+ is solution of the equation

$$yF_+(A) = y_{k-1} + \frac{C}{\lambda_+^\epsilon}\int_{x_{k-1}}^{G_+(y)}\frac{e^{-My}}{y^{1+Y}}dy$$

We approximate the above integrale by

$$\frac{C}{\lambda_+^\epsilon Y}e^{-Mx_{k-1}}\left(\frac{1}{x_{k-1}^Y} - \frac{1}{(G_+(y))^Y}\right)$$

Hence $G_+(y)$ can be approximated by

$$\left[\frac{1}{x_{k-1}^Y} - \frac{\lambda_+^\epsilon Y}{C}e^{Mx_{k-1}}\left(yF_+(A) - y_{k-1}\right)\right]^{-\frac{1}{Y}} \tag{3.8}$$

3.3 The NIG case

Like the VG model, the NIG (Normal Inverse Gaussian) model (see [2]) is a particular case of the hyperbolic models. It is charterized by four parameters : α, β, $\hat{\delta}$ and μ. Where $0 \leq |\beta| \leq \alpha$, $\hat{\delta} > 0$ and $\mu \in \mathbb{R}$. Its generating triplet are $(\gamma, 0, \nu)$, where

$$\begin{aligned} \gamma &= \mu + 2\frac{\alpha\hat{\delta}}{\pi}\int_0^1 \sinh(\beta x) K_1(\alpha x) \\ \nu(dx) &= \frac{\alpha\hat{\delta}}{\pi|x|} K_1(\alpha|x|) e^{\beta x} dx \end{aligned}$$

with

$$K_\lambda(z) = \frac{1}{2}\int_{\mathbb{R}^+} y^{\lambda-1} \exp\left(-\frac{1}{2} z \left(y + \frac{1}{y}\right)\right) dy$$

In financial applications we set $\mu = 0$. Then the NIG is represented by three parameters : $(\alpha, \beta, \hat{\delta})$. The cdf of Y_1^+ is

$$f_+(x) = \frac{\alpha\hat{\delta}}{\pi x} K_1(\alpha x) e^{\beta x}, \; x > \epsilon$$

And then its cdf is given by

$$F_+(x) = \frac{\alpha\hat{\delta}}{\pi}\int_\epsilon^x \frac{K_1(\alpha y)}{y} e^{\beta y} dy$$

Therefore

$$F_+(x_k) - F_+(x_{k-1}) = \frac{\alpha\hat{\delta}}{\pi}\int_{x_{k-1}}^{x_k} \frac{K_1(\alpha y)}{y} e^{\beta y} dy$$

To approximate the above integrale, we need to study the asymptotic behaviour of K_1. We have (see [1], Formula 9.7.2 and Formula 9.8.7)

$$\begin{aligned} K_1(x) &\underset{x \downarrow 0}{\sim} \frac{C}{x}, \text{ for a given C>0} \\ K_1(x) &\underset{x \to +\infty}{\sim} \sqrt{\frac{\pi}{2x}} e^{-x} \end{aligned}$$

Hence the following approximation

$$\frac{\alpha\hat{\delta}}{\pi} x_{k-1} K_1(\alpha x_{k-1}) e^{\beta x_{k-1}} \int_{x_{k-1}}^{x_k} \frac{dy}{y^2} = \frac{\alpha\hat{\delta}}{\pi} x_{k-1} K_1(\alpha x_{k-1}) e^{\beta x_{k-1}} \left(\frac{1}{x_{k-1}} - \frac{1}{x_k}\right)$$

In NIG case G_+ satisfy

$$yF_+(A) = y_{k-1} + \frac{\alpha\hat{\delta}}{\pi}\int_{x_{k-1}}^{G_+(y)} \frac{K_1(\alpha y)}{y} e^{\beta y} dy$$

So we approximate $G_+(y)$ by

$$\left(\frac{1}{x_{k-1}} - \frac{\pi}{\alpha\hat{\delta}} \frac{yF_+(A) - y_{k-1}}{x_{k-1} K_1(\alpha x_{k-1})} e^{-\beta x_{k-1}}\right)^{-1} \tag{3.9}$$

The Y_1^- case is treated in the same way, we only need to substitute β by $-\beta$.

4 Asian option valuation

We will focus on the fixed strike Asian put option. The call case can be easily deduced. Floating Asian options, can be valuated using fixed strike options and symmetry. Consider the following payoff:

$$\left(K - \frac{1}{T}\int_0^T S_0 e^{X_s} ds\right)^+, \text{ fixed strike Asian put option}$$

We set

$$V_a = e^{-rT}\mathbb{E}\left(K - \frac{1}{T}\int_0^T S_0 e^{X_s} ds\right)^+$$

The generating triplet of X is $(\gamma, 0, \nu)$. In fact we will estimate the quantities V_a^ϵ and $\hat{V}_a^\epsilon$ obtained by replacing X by X^ϵ or $\hat{X}^\epsilon$. Let $\left(T_j^\epsilon\right)_{j\geq 1}$ be arrival times of X^ϵ. Note

$$\begin{aligned} T_0^\epsilon &= 0 \\ T_j^\epsilon &= T_j^\epsilon \wedge T \end{aligned}$$

We have

$$\begin{aligned} V_a^\epsilon &= e^{-rT}\mathbb{E}\left(K - \frac{1}{T}\int_0^T S_0 e^{X_s^\epsilon} ds\right)^+ \\ &= e^{-rT}\mathbb{E}\left(K - \frac{S_0}{T}\sum_{j=1}^{N_T^\epsilon+1}\int_{\hat{T}_{j-1}^\epsilon}^{\hat{T}_j^\epsilon} e^{X_s^\epsilon} ds\right)^+ \\ &= e^{-rT}\mathbb{E}\left(K - \frac{S_0}{T}\sum_{j=1}^{N_T^\epsilon+1}\int_{\hat{T}_{j-1}^\epsilon}^{\hat{T}_j^\epsilon} e^{\gamma_0^\epsilon s + \sum_{i=1}^{j-1} Y_i^\epsilon} ds\right)^+, \text{ see (2.6).} \end{aligned}$$

So

$$\begin{aligned} V_a^\epsilon &= e^{-rT}\mathbb{E}\left(K - \frac{S_0}{T}\sum_{j=1}^{N_T^\epsilon+1} e^{\sum_{i=1}^{j-1} Y_i^\epsilon}\int_{\hat{T}_{j-1}^\epsilon}^{\hat{T}_j^\epsilon} e^{\gamma_0^\epsilon s} ds\right)^+ \\ &= e^{-rT}\mathbb{E}\left(K - \frac{S_0}{T}\sum_{j=1}^{N_T^\epsilon+1} e^{\sum_{i=1}^{j-1} Y_i^\epsilon}\frac{e^{\gamma_0^\epsilon \hat{T}_j^\epsilon} - e^{\gamma_0^\epsilon \hat{T}_{j-1}^\epsilon}}{\gamma_0^\epsilon}\right)^+ \\ &= e^{-rT}\mathbb{E}\left(K - \frac{S_0}{T}\sum_{j=1}^{N_T^\epsilon+1} \frac{e^{\gamma_0^\epsilon \hat{T}_j^\epsilon + \sum_{i=1}^{j-1} Y_i^\epsilon} - e^{\gamma_0^\epsilon \hat{T}_{j-1}^\epsilon + \sum_{i=1}^{j-1} Y_i^\epsilon}}{\gamma_0^\epsilon}\right)^+. \end{aligned}$$

Hence

$$V_a^\epsilon = e^{-rT}\mathbb{E}\left(K - \frac{S_0}{T}\sum_{j=1}^{N_T^\epsilon+1} \frac{e^{X^\epsilon_{\hat{T}_j^\epsilon -}} - e^{X^\epsilon_{\hat{T}_{j-1}^\epsilon}}}{\gamma_0^\epsilon}\right)^+. \tag{4.10}$$

When we replace X^ϵ by $\hat{X}^\epsilon$), we get

Proposition 4.1. *Let X be an infinite activity Lévy process with generating triplet $(\gamma, 0, \nu)$ and f be a Lipschitz function. We assume that $\mathbb{E}e^{M_T} < \infty$. Then*

$$\mathbb{E}f\left(\frac{1}{T}\int_0^T S_0 e^{\hat{X}_s^\epsilon} ds\right) = \mathbb{E}f\left(\frac{S_0}{T}\sum_{j=1}^{N_T^\epsilon+1} e^{\hat{X}^\epsilon_{\hat{T}^\epsilon_{j-1}}}\left(\frac{e^{\gamma_0^\epsilon\left(\hat{T}_j^\epsilon-\hat{T}_{j-1}^\epsilon\right)}-1}{\gamma_0^\epsilon}+\sigma(\epsilon)g_j^\epsilon\right)\right)+O\left(\sigma(\epsilon)^2\right),$$

with

$$g_j^\epsilon = \int_{\hat{T}_{j-1}^\epsilon}^{\hat{T}_j^\epsilon} e^{\gamma_0^\epsilon\left(s-\hat{T}_{j-1}^\epsilon\right)}\left(W_s - W_{\hat{T}_{j-1}^\epsilon}\right) ds$$

Knowing N_T^ϵ and $\left(\hat{T}_j^\epsilon\right)_{1\leq j\leq N_T^\epsilon}$, the r.v. $\left(g_j^\epsilon\right)_{1\leq j\leq N_T^\epsilon+1}$ are independent and gaussian, and

$$var\left(g_j^\epsilon\right) = \frac{1}{2\left(\gamma_0^\epsilon\right)^3}\left(\left(2\gamma_0^\epsilon\left(\hat{T}_j^\epsilon-\hat{T}_{j-1}^\epsilon\right)-3\right)e^{2\gamma_0^\epsilon\left(\hat{T}_j^\epsilon-\hat{T}_{j-1}^\epsilon\right)}+4e^{\gamma_0^\epsilon\left(\hat{T}_j^\epsilon-\hat{T}_{j-1}^\epsilon\right)}-1\right) \tag{4.11}$$

Furthermore we have

$$cov\left(g_j^\epsilon, W_{\hat{T}_j^\epsilon}-W_{\hat{T}_{j-1}^\epsilon}\right) = \frac{\hat{T}_j^\epsilon-\hat{T}_{j-1}^\epsilon}{\gamma_0^\epsilon}e^{\gamma_0^\epsilon\left(\hat{T}_j^\epsilon-\hat{T}_{j-1}^\epsilon\right)} - \frac{e^{\gamma_0^\epsilon\left(\hat{T}_j^\epsilon-\hat{T}_{j-1}^\epsilon\right)}-1}{\left(\gamma_0^\epsilon\right)^2} \tag{4.12}$$

5 Numerical examples

In the VG model M_T is approximated by M_T^ϵ. In the table 5, we observe the convergence of our method with respect to ϵ. Note that the errors are relative, and the benchmark price is that obtained by

ϵ	price	Monte Carlo error	total error
10^{-1}	7.076	0.05%	24.7%
10^{-2}	9.347	0.08%	0.50%
10^{-3}	9.401	0.08%	0.04%

Table 5.1: Approximation of the continuous call lookback price in VG model. Les parameters are : $S_0 = 100$, $r = 0.0548$, $\delta = 0$, $T = 0.40504$, $S_+ = 100$, $\theta = -0.2859$, $\kappa = 0.2505$, $\sigma = 0.1927$ and $n = 1000000$. The benchmark call price is 9.39827.

[Becker(2008)].

In CGMY model, M_T is approximated by $\hat{M}_T^\epsilon$. In the table 5, we observe the convergence of our method with respect to ϵ. The errors are relative, and the benchmark price is that obtained by *[Feng-*

ϵ	price	Monte-Carlo error	total error
10^{-1}	14.12	0.07%	1.88%
10^{-2}	13.869	0.07%	0.06%
10^{-3}	13.860	0.07%	0.00%

Table 5.2: Approximation of the discrete put lookback price (where the number of discretization points is $N = 252$) in CGMY model. The parameters are : $S_0 = 100$, $r = 0.05$, $\delta = 0.02$, $T = 1$, $S_+ = 100$, $C = 4$, $G = 50$, $M = 60$, $Y = 0.7$ and $n = 1000000$. The benchmark price is 13.8600.

Linetsky(2009)].

ϵ	price	Monte-Carlo error	total error
10^{-1}	12.89	0.0%	5.46%
10^{-2}	12.24	0.08%	0.15%
10^{-3}	12.21	0.08%	0.01%

Table 5.3: Approximation of the discrete put lookback price (where the number of discretization points is $N = 252$) in NIG model. The parameters are : $S_0 = 100$, $r = 0.05$, $\delta = 0.02$, $T = 1$, $S_+ = 100$, $\alpha = 15$, $\beta = -5$,$\hat{\delta} = 0.5$ and $n = 1000000$. The benchmark price is 12.2224.

In NIG model, M_T is approximated by $\hat{M}_T^\epsilon$. In the table 5, we observe the convergence of our method with respect to ϵ. The errors are relative, and the benchmark price is that obtained by *[Feng-Linetsky(2009)]*.

In table 5.4 we have Asian options prices in NIG and CGMY models. Parameters for NIG model are: $\alpha = 6.1882$, $\beta = -3.8941$, $\hat{\delta} = 0.1622$ and $r = 0.0387$. Parameters for CGMY model are: $C = 0.2703$, $G = 17.56$, $M = 54.82$, $Y = 0.8$ and $r = 0.04$. Others parametres are given in the table 5.4. These results

ϵ/Model	NIG	CGMY
10^{-1}	12.624	11.624
10^{-2}	12.673	11.642
10^{-3}	12.675	11.642

Table 5.4: Approximation of a fixed strike Asian call option. Parameters are: $S_0 = 100$, $\delta = 0$, $T = 1$ and $n = 1000000$. Monte-Carlo error is 0.03%.

can be compared with Fusai-Meucci's results(for NIG) and Cerny-Kyriakou (for CGMY).

References

[1] Abramowitz, M. et I. Stegun (1972). Handbook of Mathematical Functions, 9th ed. Dover Publications, New York. 6

[2] Barndorff-Nielsen, 0. E. (1995). Normal Inverse Gaussian Processes and the Modelling of Stock Returns. Research Report 300, University of Aarhus, 1995. 6

[3] Becker, M. (2008). Unbiased Monte Carlo Valuation of Lookback, Swing and Barrier Options Under Variance Gamma Model. Pre-print, May 19, 2008.

[4] Carr, P. P., H. Geman, D. B. Madan, and M. Yor (2002). The fine structure of asset returns: An empirical investigation. Journal of Business, 75 (2002), pp. 305-332. 4

[5] Cerny, A. and I. Kyriakou. An Improved Convolution Algorithm for Discretely Sampled Asian Options. $http://papers.ssrn.com/sol3/papers.cfm?abstract\_id = 1323252$ (2009).

[6] Feng, L. and V. Linetsky (2009). Computing Exponential Moments of the Discrete Maximum of a Lévy process and Lookback Options. Finance and Stochastics 13:4, 501-529.

[7] Fusai, G. and A. Meucci. Pricing discretely monitored Asian options under Lévy processes, Journal of Banking & Finance 32, 2076-2088 (2008).

[8] Madan, D. B., P. P. Carr, and E. C. Chang (1998). The Variance Gamma Process and Option Pricing. European Finance Review, 2(1), pp. 79-105. 4, 5

[9] Madan, D. B. et E. Seneta (1990). The Variance Gamma (V.G.) Model for Share Market Returns. Journal of Business, 63(4), pp. 511-524.