```
   Help
/* Glasserman-Heidelberger-Shahabuddin Algorithm
   Importance Sampling and Stratification Variance Reductio
    n*/

#include <stdlib.h>
#include  "bs1d_pad.h"
#include "enums.h"

#define FACTOR  1.6
#define JMAX     40
#define NTRY     80

static double mu[50000];
static double t,sig, ri, dvd, S0, strike, step_nb;

/* Find the domain containg the zero of the function*/
static int zbrac(double(*func)(double),double *xmin,double
    *xmax)
{
  int j;
  double f1,f2;

  if(*xmin==*xmax)
    printf("mauvais depart dans la fonction zbrac()");

  f1=(*func)(*xmin);
  f2=(*func)(*xmax);

  for(j=1;j<=NTRY;j++)
    {
      {
  if(f1*f2<0.0)
    return 1;
      }

      if(fabs(f1)<fabs(f2))
  f1=(*func)(*xmin+=FACTOR*(*xmin-*xmax));
      else
  f2=(*func)(*xmax+=FACTOR*(*xmax-*xmin));
    }
```

```
  return 0; /*envoie 0 si [xmin,xmax] devient trop large*/
}

/*-------------------------------------------------------
    -----------*/
/* Methode de dichotomies permet de trouver un zero d'une
    fonction*/
/* sachant que ce zero se trouve entre x1 et x2. Precision
    = xacc*/
/*-------------------------------------------------------
    ----------*/
static double rtbis(double (*func)(double),double x1,
    double x2,double xacc)
{
  int j;
  double dx,f,fmid,xmid,rtb;

  f=(*func)(x1);
  fmid=(*func)(x2);


  if(f*fmid>=0.0){
    printf("La racine ne se trouve pas dans [x1,x2]");
    exit(-1);
  }

  rtb=f<0.0?(dx=x2-x1,x1):(dx=x1-x2,x2); /* oriente la rech
    erche*/

  for(j=1;j<=JMAX;j++){
    fmid=(*func)(xmid=rtb+(dx*=0.5));
    if(fmid<=0.0)rtb=xmid;
    if(fabs(dx)<xacc||fmid==0.0)return rtb;
  }

  return 0.0;
}

/*-------------------------------------------------------
    ---------------*/
/*Premiere partie : recherche du mu optimal*/
```

```
/*La fonction ci-dessous est celle qu'il faut appeller pour
     trouver le mu */
/*optimal. On cherche d'abord son unique racine qu'on reinj
    ecte ensuite*/
/*dans les z[1..PAS] et s[1..PAS]; le dernier z[] est alo
    rs le mu optimal.*/
static double ghscall(double g)
{
  int i;
  double z=0.0;
  double s;
  double dt,ans,s_dt,trend;

  s=S0;
  dt=t/step_nb;s_dt=sig*sqrt(dt);
  trend=(ri-dvd-0.5*sig*sig)*dt;

  if(g!=0)
    {
      ans=0;
      z=s_dt*(g+strike)/g;
      for(i=1;i<step_nb;i++)
  {
    s=s*exp(trend+s_dt*z);
    z=z-s_dt*s/(step_nb*g);
    ans+=s;
  }

      ans/=step_nb;
      return (ans=(ans-strike-g));
    }
  return 0.0;
}
/*------------------------------*/
static double ghsput(double g)
{
  int i;
  double z=0.0;
  double s;
  double dt,ans,s_dt,trend;
```

```
  s=S0;
  dt=t/step_nb;s_dt=sig*sqrt(dt);
  trend=(ri-dvd-0.5*sig*sig)*dt;

  if(g!=0){
    ans=s;
    z=s_dt*(g-strike)/g;
    for(i=1;i<step_nb;i++){
      s=s*exp(trend+s_dt*z);
      z=z+s_dt*s/(step_nb*g);
      ans+=s;
    }

    ans/=step_nb;
    return (ans=(strike-ans-g));
  }
  else{
    printf("problem at line 138 of Pricin_util.h ...{n");
    exit(-1);
  }
}


/* ------------------------------------------------------
    -------------- */
/* Computation of drift correction
                     */
/* ------------------------------------------------------
    -------------- */

static void Drift_Computation(int generator, int step_numb
    er, double T,  double x, double r, double divid, double si
    gma, NumFunc_2 *p, double K)
{
  double   S_t;
  double h = T / step_number;
  /* double sqrt_h = sqrt(h);*/
  double trend= (r -divid)- 0.5 * SQR(sigma);
  double ss_dt=sigma*sqrt(h);
  double *xmin,*xmax,x_min,x_max,dot2;
  int i;
```

```
double g;

t=T;ri=r;
S0=x;strike=K;
sig=sigma;
dvd=divid;
step_nb=step_number;

for(i=0;i<step_number;i++)
  mu[i]=0.;

if((p->Compute) == &Call_OverSpot2)
  {
    x_min=2.5*t;x_max=5.0*t;
    xmin=&x_min;xmax=&x_max;
    /*trouve le bon intervalle [xmin,xmax]*/
    zbrac(ghscall,xmin,xmax);
    /*resoud l equation ghs(x)=0*/
    g=rtbis(ghscall,(*xmin),(*xmax),1e-8);
    mu[0]=ss_dt*(g+K)/g;
    dot2=SQR(mu[0]);S_t=1.0;
    for(i=1;i<step_number;i++)
{
  mu[i]=mu[i-1]-ss_dt*S0*S_t/(step_number*g);
  S_t=S_t*exp(trend*h+ss_dt*mu[i]);
  dot2+=SQR(mu[i]);
}
  }
else if((p->Compute) == &Put_OverSpot2)
  {
    x_min=-5.0;x_max=-0.1;
    xmin=&x_min;xmax=&x_max;
    /*trouve le bon intervalle [xmin,xmax]*/
    zbrac(ghsput,xmin,xmax);
    /*resoud l equation ghs(x)=0*/
    g=rtbis(ghsput,(*xmin),(*xmax),1e-8);
    mu[0]=ss_dt*(g-K)/g;
    dot2=SQR(mu[0]);S_t=1.0;
    for(i=1;i<step_number;i++)
{
  mu[i]=mu[i-1]+ss_dt*S0*S_t/(step_number*g);
```

```
    S_t=S_t*exp(trend*h+ss_dt*mu[i]);
    dot2+=SQR(mu[i]);
  }
    }

  return;
}

/* ------------------------------------------------------
    ----------*/
/* Pricing of a asian option by the Monte Carlo GHS method
   Estimator of the price and the delta.
   s et K are pseudo-spot and pseudo-strike. */
/* ------------------------------------------------------
    --------- */
static int  FixedAsian_Stratification(double s, double K,
    double time_spent, NumFunc_2 *p, double t, double r, double div
    id, double sigma, long nb,int nb_strat, int M, int     generator, double con
    double *pterror_price, double *pterror_delta, double *inf_price,
     double *sup_price, double *inf_delta, double *sup_delta)
{
  long i,ipath;
  double price_sample, delta_sample, mean_price, mean_delt
    a, var_price, var_delta,mean_price_strata, mean_delta_stra
    ta, var_price_strata, var_delta_strata;
  int init_mc;
  int simulation_dim;
  double alpha, z_alpha,dot1,dot2; /* inc=0.001;*/
  double integral, S_t, g1;
  double h = t /(double)M;
  double sqrt_h = sqrt(h);
  double trend= (r -divid)- 0.5 * SQR(sigma);
  int step_number=M;
  double norme_mu,uniform,Xi,dot3,val,temp;
  int i_strat;
  double *Y_t,*u_t,*gauss_vect;
  Y_t = malloc(M*sizeof(double));
  u_t = malloc(M*sizeof(double));
  gauss_vect = malloc(M*sizeof(double));
```

```
/* Value to construct the confidence interval */
alpha= (1.- confidence)/2.;
z_alpha= pnl_inv_cdfnor(1.- alpha);

/*Initialisation*/
mean_price= 0.0;
mean_delta= 0.0;
var_price= 0.0;
var_delta= 0.0;

/* Size of the random vector we need in the simulation */
simulation_dim= M;

/* MC sampling */
init_mc= pnl_rand_init(generator, simulation_dim,nb);
/* Test after initialization for the generator */
if(init_mc == OK)
  {


    /* Price  */
    (void)Drift_Computation(generator, M, t, s,r, divid,
  sigma, p, K);

    dot2=0;
    for(i=0;i<step_number;i++)
dot2+=mu[i]*mu[i];

    norme_mu=sqrt(dot2);

    for(i=0;i<M;i++)
u_t[i]=mu[i]/norme_mu;

    for(i_strat=0;i_strat<nb_strat;i_strat++)
{
  mean_price_strata= 0.0;
  mean_delta_strata= 0.0;
  var_price_strata= 0.0;
  var_delta_strata= 0.0;
```

```
for(ipath= 1;ipath<= nb;ipath++)
  {
    /* Begin of the N iterations */
    g1= pnl_rand_gauss(step_number, CREATE, 0,     generator);
    uniform=pnl_rand_uni(generator);
    val=(i_strat+uniform)/nb_strat;
    Xi=pnl_inv_cdfnor(val);

    /*Simulation of Conditional Gaussian Law*/

    dot3=0.0;
    for(i=0 ; i< step_number ; i++)
{
  g1= pnl_rand_gauss(step_number, RETRIEVE, i,     generator);
  gauss_vect[i]=g1;
  dot3+=u_t[i]*g1;
}

    dot1=0.;
    for(i=0 ; i< step_number ; i++)
{
  temp=(Xi-dot3)*u_t[i]+gauss_vect[i];
  Y_t[i]=temp;
  dot1+=temp*mu[i];
}

    /*Simulation of Stock and Average*/
    integral=0.0;
    S_t=s;
    for(i=0 ; i< step_number ; i++)
{
  S_t *=exp(trend *h +sigma*sqrt_h*(Y_t[i]+mu[i]));
  integral+=S_t;
}

    price_sample=(p->Compute)(p->Par, s,integral/(
double)step_number)*exp(-dot1-0.5*dot2);

    /* Delta */
    if(price_sample >0.0)
```

```
    delta_sample=(1-time_spent)*(integral/(s*(double)step
    _number))*exp(-dot1-0.5*dot2);
        else  delta_sample=0.;

        /* Sum */
        mean_price_strata+= price_sample;
        mean_delta_strata+= delta_sample;

        /* Sum of squares */
        var_price_strata+= SQR(price_sample);
        var_delta_strata+= SQR(delta_sample);

      }/*End of MonteCarlo*/

    var_price_strata/=nb;
    var_delta_strata/=nb;
    mean_price_strata/=nb;
    mean_delta_strata/=nb;

    var_price_strata-= SQR(mean_price_strata);
    var_delta_strata-= SQR(mean_delta_strata);

    /* Sum */
    mean_price+= mean_price_strata;
    mean_delta+= mean_delta_strata;

    /* Sum of squares */
    var_price+=var_price_strata;
    var_delta+=var_delta_strata;

  }
      /* End of the N iterations */

      /* Price estimator */
      *ptprice=(mean_price/(double)nb_strat);
      *pterror_price= exp(-r*t)*sqrt(var_price/(double)(nb*
    SQR(nb_strat)));
      *ptprice= exp(-r*t)*(*ptprice);

      /* Price Confidence Interval */
      *inf_price= *ptprice - z_alpha*(*pterror_price);
```

```
      *sup_price= *ptprice + z_alpha*(*pterror_price);


      /* Delta estimator */
      *ptdelta=exp(-r*t)*(mean_delta/(double)nb_strat);
      if((p->Compute) == &Put_OverSpot2)
  *ptdelta *= (-1);
      *pterror_delta= sqrt(exp(-2.0*r*t)*(var_delta/((
    double)nb*SQR(nb_strat))));

      /* Delta Confidence Interval */
      *inf_delta= *ptdelta - z_alpha*(*pterror_delta);
      *sup_delta= *ptdelta + z_alpha*(*pterror_delta);
    }
  free(Y_t);
  free(u_t);
  free(gauss_vect);
  return init_mc;
}


int CALC(MC_FixedAsian_Stratification)(void *Opt,void *Mod,
    PricingMethod *Met)
{
  TYPEOPT* ptOpt=(TYPEOPT*)Opt;
  TYPEMOD* ptMod=(TYPEMOD*)Mod;

  double T, t_0, T_0;
  double r, divid, time_spent, pseudo_strike, true_strike,
    pseudo_spot;
  int return_value;

  r=log(1.+ptMod->R.Val.V_DOUBLE/100.);
  divid=log(1.+ptMod->Divid.Val.V_DOUBLE/100.);

  T= ptOpt->Maturity.Val.V_DATE;
  T_0 = ptMod->T.Val.V_DATE;
  t_0= (ptOpt->PathDep.Val.V_NUMFUNC_2)->Par[0].Val.V_PDOUB
    LE;
  time_spent= (T_0-t_0)/(T-t_0);
```

```
if(T_0 < t_0)
  {
    Fprintf(TOSCREEN,"T_0 < t_0, untreated case{n{n{n");
    return_value = WRONG;
  }

/* Case t_0 <= T_0 */
else
  {
    pseudo_spot= (1.-time_spent)*ptMod->S0.Val.V_PDOUBLE;
    pseudo_strike= (ptOpt->PayOff.Val.V_NUMFUNC_2)->Par[0
  ].Val.V_PDOUBLE-time_spent*(ptOpt->PathDep.Val.V_NUMFUNC_2
  )->Par[4].Val.V_PDOUBLE;


    true_strike= (ptOpt->PayOff.Val.V_NUMFUNC_2)->Par[0].
  Val.V_PDOUBLE;

    (ptOpt->PayOff.Val.V_NUMFUNC_2)->Par[0].Val.V_PDOUB
  LE= pseudo_strike;

    if (pseudo_strike<=0.)
{
  Fprintf(TOSCREEN,"FORMULE ANALYTIQUE{n{n{n");
  return_value= Analytic_KemnaVorst(pseudo_spot,
            pseudo_strike,
            time_spent,
            ptOpt->PayOff.Val.V_NUMFUNC_2,
            T-T_0,
            r,
            divid,
            &(Met->Res[0].Val.V_DOUBLE),
            &(Met->Res[1].Val.V_DOUBLE));

}
    else
return_value= FixedAsian_Stratification(pseudo_spot,
          pseudo_strike,
          time_spent,
          ptOpt->PayOff.Val.V_NUMFUNC_2,
          T-T_0,
```

```
            r,
            divid,
            ptMod->Sigma.Val.V_PDOUBLE,
            Met->Par[3].Val.V_LONG,
            Met->Par[1].Val.V_INT2,
            Met->Par[0].Val.V_INT2,
            Met->Par[2].Val.V_ENUM.value,
            Met->Par[4].Val.V_DOUBLE,
            &(Met->Res[0].Val.V_DOUBLE),
            &(Met->Res[1].Val.V_DOUBLE),
            &(Met->Res[2].Val.V_DOUBLE),
            &(Met->Res[3].Val.V_DOUBLE),
            &(Met->Res[4].Val.V_DOUBLE),
            &(Met->Res[5].Val.V_DOUBLE),
            &(Met->Res[6].Val.V_DOUBLE),
            &(Met->Res[7].Val.V_DOUBLE));

      (ptOpt->PayOff.Val.V_NUMFUNC_2)->Par[0].Val.V_PDOUB
    LE=true_strike;
    }
  return return_value;
}



static int CHK_OPT(MC_FixedAsian_Stratification)(void *Opt,
     void *Mod)
{
  if ( (strcmp( ((Option*)Opt)->Name,"AsianCallFixedEuro")=
    =0) || (strcmp( ((Option*)Opt)->Name,"AsianPutFixedEuro")=
    =0) )
    return OK;

  return WRONG;
}



static int MET(Init)(PricingMethod *Met,Option *Opt)
{
  int type_generator;
```

```
  if ( Met->init == 0)
    {
      Met->init=1;

      Met->Par[0].Val.V_INT2= 360;
      Met->Par[1].Val.V_INT2= 100;
      Met->Par[2].Val.V_ENUM.value=0;
      Met->Par[2].Val.V_ENUM.members=&PremiaEnumRNGs;
      Met->Par[3].Val.V_LONG= 200;
      Met->Par[4].Val.V_DOUBLE= 0.95;


    }

  type_generator= Met->Par[2].Val.V_ENUM.value;

  if(pnl_rand_or_quasi(type_generator)==PNL_QMC)
    {
      Met->Res[2].Viter=IRRELEVANT;
      Met->Res[3].Viter=IRRELEVANT;
      Met->Res[4].Viter=IRRELEVANT;
      Met->Res[5].Viter=IRRELEVANT;
      Met->Res[6].Viter=IRRELEVANT;
      Met->Res[7].Viter=IRRELEVANT;

    }
  else
    {
      Met->Res[2].Viter=ALLOW;
      Met->Res[3].Viter=ALLOW;
      Met->Res[4].Viter=ALLOW;
      Met->Res[5].Viter=ALLOW;
      Met->Res[6].Viter=ALLOW;
      Met->Res[7].Viter=ALLOW;
    }

  return OK;
}
```

```
PricingMethod MET(MC_FixedAsian_Stratification)=
{
  "MC_FixedAsian_Stratification_GHS",
  {{"TimeStepNumber",INT2,{100},ALLOW},
   {"Number of Strata",INT2,{100},ALLOW},
   {"RandomGenerator",ENUM,{100},ALLOW},
   {"N iterations",LONG,{100},ALLOW},
   {"Confidence Value",DOUBLE,{100},ALLOW},
   {" ",PREMIA_NULLTYPE,{0},FORBID}},
  CALC(MC_FixedAsian_Stratification),
  {{"Price",DOUBLE,{100},FORBID},
   {"Delta",DOUBLE,{100},FORBID} ,
   {"Error Price",DOUBLE,{100},FORBID},
   {"Error Delta",DOUBLE,{100},FORBID} ,
   {"Inf Price",DOUBLE,{100},FORBID},
   {"Sup Price",DOUBLE,{100},FORBID} ,
   {"Inf Delta",DOUBLE,{100},FORBID},
   {"Sup Delta",DOUBLE,{100},FORBID} ,
   {" ",PREMIA_NULLTYPE,{0},FORBID}},
  CHK_OPT(MC_FixedAsian_Stratification),
  CHK_ok,
  MET(Init)
};
```

References