Robust approximations for pricing Asian options and volatility swaps under stochastic volatility

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Most of what is presented here is taken from [1].

According to Forde and Jacquier (cf [1]), if we consider S is a non-negative, continuous martingale defined as

$$\begin{cases} S_t = e^{\int_0^t r_u du}\tilde{S}_t, \\ d\tilde{S}_t = \tilde{S}_t \sigma_t dW_t, \end{cases} \tag{0.1}$$

where W is a standard Brownian motion, σ_t is a progressively measurable non-negative volatility process and r_t is a time-dependent interest rate with $0 \leq r_t < r_m ax < +\infty$ for all t, then we have the following relationship between the second moments of S_t and the second moment of $I_t := \int_0^t S_u du$

$$\mathbb{E}\, I_t^2 = 2\int_0^t B_u \int_0^s B_u^{-1}\mathbb{E}(S_u^2) du ds \tag{0.2}$$

where $B_t = e^{\int_0^t r_u du}$.

A model-independent upper bound for the price of an Asian call option can be derived from this equality, given by

$$P = B_t^{-1}\mathbb{E}\left(I_t - K\right)_+ \leq \frac{B_t^{-1}}{4(K-L^*)}\left[\mathbb{E}\, I_t^2 - 2L^*\mathbb{E}\, I_t + L^{*2}\right], \tag{0.3}$$

where

$$L^* = K - \left[\mathrm{Var}(I_t) + (K - \mathbb{E}\ I_t)^2\right]^{\frac{1}{2}} \tag{0.4}$$

Under Black-Scholes model, with $r_t \equiv r$ we have

$$\mathbb{E}\ I_t^2 = \frac{2S_t^2}{t^2}\left(\frac{e^{(2r+\sigma^2)t}}{(r+\sigma^2)(2r+\sigma^2)} + \frac{1}{r}\left(\frac{1}{2r+\sigma^2} - \frac{e^{rt}}{r+\sigma^2}\right)\right) \tag{0.5}$$

Under Heston model, where the forward price process S_t is defined by the following stochastic differential equations

$$\begin{cases} \frac{dS_t}{S_t} = \sqrt{V_t}dW_t^1, \\ dV_t = (a - bV_t)\,dt + \sigma\sqrt{V_t}dW^2,\ d\langle W^1, W^2\rangle_t = \rho dt, \end{cases} \tag{0.6}$$

We have

$$\mathbb{E}\ I_t^2 = S_0^2 \int_0^t e^{rs} \int_0^s e^{-ru} \frac{e^{\frac{(\kappa-2\rho\sigma)\kappa\theta u}{\sigma^2}}\ e^{\frac{2V_0}{\kappa-2\rho\sigma+\gamma\coth(\frac{1}{2}\gamma u)}}}{(\cosh(\frac{1}{2}\gamma u) + \frac{\kappa-2\rho\sigma}{\gamma}\sinh(\frac{1}{2}\gamma u))^{\frac{2\kappa\theta}{\sigma^2}}} duds \tag{0.7}$$

where $\gamma = \sqrt{(\kappa - 2\rho\sigma)^2 - 2\sigma^2}$

References

[1] Forde, M. and Jaquier, A. Robust approximations for pricing Asian options and volatility swaps under stochastic volatility, Applied Mathematical Finance, 17 (3): 241-259, 2010. 1

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