

$$\frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

4.2  $\begin{matrix} x_0 & y_0 & x_1 & y_1 & x_2 & y_2 \\ (-1, 1) & (1, 2) & (2, 3) \end{matrix}$

$$P(x) = \sum_{i=0}^n y_i \prod_{j \neq i} \frac{x - x_j}{x_i - x_j}$$

$$y_0 \left[ \frac{x - x_1}{x_0 - x_1} \right] \left[ \frac{x - x_2}{x_0 - x_2} \right] + y_1 \left[ \frac{x - x_0}{x_1 - x_0} \right] \left[ \frac{x - x_2}{x_1 - x_2} \right] + y_2 \left[ \frac{x - x_0}{x_2 - x_0} \right] \left[ \frac{x - x_1}{x_2 - x_1} \right]$$

$$= 1 \left[ \frac{x-1}{-1-1} \right] \left[ \frac{x-2}{-1-2} \right] + 2 \left[ \frac{x-1}{1-1} \right] \left[ \frac{x-2}{1-2} \right] + 3 \left[ \frac{x-1}{2-1} \right] \left[ \frac{x-1}{2-1} \right]$$

$$= \frac{x-1}{-2} \cdot \frac{x-2}{-3} + \frac{2x+2}{2} \cdot \frac{x-2}{-1} + \frac{3x+3}{3} \cdot \frac{x-1}{1}$$

$$\frac{(x-1)(x-2)}{6} + \frac{-(x^2-2x-2)}{2} + \frac{(x+1)(x-1)}{1}$$

$$= \frac{x^2-2x-1x+2}{6} + \frac{-x^2+2x-x+2}{-1} + x^2-x+1$$

$$\frac{x^2-3x+2}{6} = \frac{x^2}{6} + \frac{-3x}{6} + \frac{2}{6}$$

$$\frac{x^2-3x+2}{6} + x+1 = \frac{x^2-3x+2}{6} + \frac{6x}{6} + \frac{6}{6}$$

$$= \frac{x^2-3x+2+6x+6}{6}$$

$$= \frac{x^2+3x+8}{6}$$

4.3.1 Due to rounding error, I would think it is not advantageous to pick  $h$  close to 0. Floating point error will cause bad results.