1、本题可由复合Poisson过程结论得

设 X 为营业额

则 $EX = \lambda t E\eta$

$DX = \lambda t E\eta^2$

$\therefore EX = 3\times60\times60\times10\times0.15 = 16200$

$DX = 3\times60\times60\times[10\times0.15\times0.85+(10\times0.15)^2] = 38070$

则误差为195.12

2. 设总陨石数强度为λ

由Poisson过程的分流不变性可得，

落地陨石为强度为λp的Poisson过程

$\therefore \quad EW = DW = 0.0001\times10000\times\frac{1}{12} = \frac{1}{12}$

$P(W\geq2) = 1-P(W=1)-P(W=0)$

$= 1-\frac{(\frac{1}{12})^0 e^{-\frac{1}{12}}}{0!} - \frac{(\frac{1}{12})^1 e^{-\frac{1}{12}}}{1!}$

$= 1-\frac{13}{12}e^{-\frac{1}{12}}$

3. 由PDF的要求

$\int_{-1}^{1}\frac{c}{\sqrt{1-x^2}}dx = 1$

设 $x = \cos\theta$

$= \int_{\pi}^{0}\frac{c}{\sin\theta}\,-\sin\theta\, d\theta$

$= \int_{0}^{\pi} c\, d\theta = \pi c = 1$

$\therefore c = \frac{1}{\pi}$

(b) $\int_{-\frac{1}{2}}^{\frac{1}{2}}\frac{1}{\pi}\frac{1}{\sqrt{1-x^2}}dx$

$= \frac{1}{\pi}\int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} c\, d\theta = \frac{1}{3}$

4. 首先 $c>0$

且 $\int_0^c \frac{1}{\theta}e^{-\frac{x}{\theta}}dx$

$= -[e^{-\frac{c}{\theta}}-1]$

$= 1-e^{-\frac{c}{\theta}} = \frac{1}{2}$

$\therefore c = \theta\ln 2$

5. 设 $X_1: x<a$

$X_2: x \geq a$

则 $E|x-a| = \int_{-\infty}^a f(x)(a-x)dx + \int_a^\infty f(x)(x-a)dx$

$= a\left[\int_{-\infty}^a f(x)dx - \int_a^\infty f(x)dx\right] - \int_{-\infty}^a x f(x)dx + \int_a^\infty f(x)x\,dx$

则 $h'(a) = \int_{-\infty}^a f(x)dx - \int_a^\infty f(x)dx + a[f(a)+f(a)] - af(a) - af(a)$

$= 2F(a)-1$

且 $h''(a) \geq 0$

$\therefore$ 当 $F(a)=\frac{1}{2}$ 时，即 $P(x\leq a)=\frac{1}{2}$ 时，$h(a)$ 最小

6. Poisson 分布：

偏度系数：$E\,X^{*3}$

$= E\left(\frac{X-\mu}{\sigma}\right)^3 = \frac{1}{\sigma^3}E[X^3-3X^2\mu+3\mu^2X-\mu^3]$

$= \frac{1}{\sigma^3}[EX^3 - 3\mu EX^2 + 3\mu^2 EX - \mu^3]$

而 $M_X(t) = e^{\lambda(e^t-1)}$ 可得 $M'''(t) = \lambda^3 e^{\lambda(e^t-1)+3t} + 3\lambda^2 e^{\lambda(e^t-1)+2t} + \lambda e^{\lambda(e^t-1)+t}$

$M''''(t) = \lambda^4 e^{\lambda(e^t-1)+4t} + 6\lambda^3 e^{\lambda(e^t-1)+3t} + 7\lambda^2 e^{\lambda(e^t-1)+2t} + \lambda e^{\lambda(e^t-1)+t}$

$\therefore EX = \lambda$

$EX^2 = \lambda^2+\lambda$

$EX^3 = \lambda^3+3\lambda^2+\lambda$

$EX^4 = \lambda^4+6\lambda^3+7\lambda^2+\lambda$

$\therefore$ 偏度：$\frac{\lambda^3+3\lambda^2+\lambda-3\lambda(\lambda^2+\lambda)+3\lambda^3-\lambda^3}{\lambda^{\frac{3}{2}}} = \frac{1}{\sqrt{\lambda}}$

峰度：$\frac{EX^4-4\lambda(\lambda^3+3\lambda^2+\lambda)+6\lambda^2(\lambda^2+\lambda)-3\lambda^4}{\lambda^2} - 3$

$= \frac{1}{\lambda}$

7. $N_t, t\geq 0$ 为密度为 $\lambda>0$ 的 Poisson 过程

$$X_t = \sum_{i=1}^{N_t} Y_i$$

则 $X_{m+n} - X_m$

$$= \sum_{i=1}^{N_{m+n}} Y_i - \sum_{i=1}^{N_m} Y_i$$

$$= \sum_{i=N_{m+1}}^{N_{m+n}} Y_i$$

$\because$ 由于 Y_i 独立同分布

$\therefore X_t$ 是时齐的独立增量过程