7.1.

4. $\because X_{(n)}$ 密度函数为

$$f_n(x)=\begin{cases}\dfrac{nx^{n-1}}{\theta^n}, & 0<x<\theta\\ 0, & otherwise\end{cases}$$

$\therefore$ 检验犯第一类错误的概率为

$$\alpha(\theta)=P(X_{(n)}\le 2.5\mid H_0)=\left(\frac{2.5}{\theta}\right)^n$$

最大值 $\alpha=\alpha(3)=\left(\frac{2.5}{3}\right)^n$

7. 由 $\alpha=P_0(X\ge C)=1-C$,

$\beta=1-P_1(X\ge C)=C^2$ 可得

$\alpha+2\beta=1-C+2C^2$

$\therefore$ 当 $C=\frac{1}{4}$ 时，$\alpha+2\beta$ 取到最小值 $\frac{7}{8}$

7.2.

5. $\because \alpha=0.05$ 且 $u_{0.05}=-1.65$

$\mu\le 13$ 时,

$\beta=P(u>-1.65\mid \mu\le 13)\le 0.05$

$=1-\Phi\left(-1.65+\frac{15-\mu}{\sqrt{2.5/n}}\right)\le 0.05$

$\therefore n\ge 7$

6. (1) $\alpha=0.05$, $u_{0.975}=1.96$

$\bar{x}=100.104$

$u=\frac{\sqrt{10}(\bar{x}-100)}{0.5}=0.6578$

未落入拒绝域，应接受原假设.不能认为 $\mu>100$

(2) σ 未知时，应用 t 检验,

拒绝域为 $\{|t|\ge t_{1-\frac{\alpha}{2}}(n-1)\}$

$\alpha=0.05$ $t_{0.975}(9)=2.2622$

由样本观测值 $S=0.4760$ $t=\sqrt{10}\frac{100.104-100}{0.4760}=0.6909<2.2622$ $\therefore$ 接受原假设

13. $H_0: \mu_1=\mu_2 \quad H_1: \mu_1 \neq \mu_2$

这是一个双侧检验问题,因而拒绝域为$\{|t| \geq t_{1-\frac{\alpha}{2}}(m+n-2)\}$

而$S_w = 0.3903$

$t = -0.2056$

而当$\alpha = 0.05$时, $t_{0.975}(15) = 2.1314 > 0.2056$

$\therefore$ 接受H_0

15: $H_0: \mu_1 = 2\mu_2$

$H_1: \mu_1 > 2\mu_2$

$\because \bar{x} - 2\bar{y} \sim N(\mu_1 - 2\mu_2, \frac{\sigma_1^2}{n} + \frac{4\sigma_2^2}{m})$, 且$\sigma_1^2$、$\sigma_2^2$已知

$\therefore u$检验

$$u = \frac{\bar{x} - 2\bar{y}}{\sqrt{\frac{\sigma_1^2}{n} + \frac{4\sigma_2^2}{m}}}$$

当H_0成立时, $u \sim N(0,1)$

而$W = \{u \geq u_{1-\alpha}\}$

20. 设冬、夏两季新生女婴体重分别服从$N(\mu_1, \sigma_1^2)$

$N(\mu_2, \sigma_2^2)$

考虑: $H_0: \sigma_1^2 = \sigma_2^2$

VS

$H_1: \sigma_1^2 < \sigma_2^2$

因而,考虑检验统计量 $F = \frac{S_1^2}{S_2^2} \sim F(n_1-1, n_2-1)$

$n_1 = 6, \ n_2 = 10, \ \alpha = 0.05$

$S_1^2 = 241666.667$

$S_2^2 = 93955.556$

$$F_\alpha(n_1-1, n_2-1) = F_{0.05}(5,9) = \frac{1}{F_{0.95}(9,5)} = \frac{1}{4.77} = 0.2096$$

$$F = \frac{241666.667}{93955.556} = 2.572 > 0.2096$$

不拒绝原假设

21. 关于正态总体方差的双侧检验问题：

$H_0: \sigma^2 = 0.048^2 \quad H_1: \sigma^2 \neq 0.048^2$

$n=5$ 取 $\alpha=0.05$

$\because \chi^2_{0.025}(4) = 0.4844$

$\chi^2_{0.975}(4) = 11.1433$

拒绝域为 $W=\{\chi^2 \le 0.4844 \cup \chi^2 \ge 11.1433\}$

$\because \chi^2 = \frac{(n-1)S^2}{\sigma_0^2} = \frac{0.03112}{0.048^2} = 13.5069 > 11.1433$

$\therefore$ 拒绝 H_0

26.

(1) 使用F检验

$\bar{x} = 0.1407 \quad \bar{y} = 0.1385 \quad S_x = 0.0028 \quad S_y = 0.0027$

取 $\alpha = 0.05$

则 $F_{0.975}(5,5) = 7.15$

$F_{0.025}(5,5) = \frac{1}{F_{0.975}(5,5)} = 0.1399$

拒绝域为 $W=\{F \le 0.1399$ 或 $F \ge 7.15\}$，而

$F = \frac{S_x^2}{S_y^2} = 1.0754$

没有落入拒绝域内，可认为总体的方差相等

(2) $\because$ (1)中已接受两总体方差一致

$\therefore$ 可使用两样本t检验

当 $\alpha=0.05$ 时 $t_{0.975}(10) = 2.2281$，故拒绝域为 $\{|t| > 2.2281\}$

而 $S_w = 0.00275$

$t = \frac{0.1407 - 0.1385}{0.00275\sqrt{\frac{1}{6}+\frac{1}{6}}} = 1.3856 < 2.2281$