- 例1：比较$\sqrt{2}-1$和$1/(\sqrt{2}+1)$的误差，其中$\sqrt{2}$取1.414

$x=\sqrt{2}, x^*=1.414$, 舍入误差$0.5*10^{-3}$

$$f_1(x)=x-1,\ f_1'(x)=1$$

$$f_2(x)=\frac{1}{x+1}, f_2'(x)=-\frac{1}{(1+x)^2}$$

$$|f_1(x)-f_1(x^*)|=|f_1'(\xi_1)|\cdot|x-x^*|\le 0.5*10^{-3}$$

$$|f_2(x)-f_2(x^*)|=|f_2'(\xi_2)|\cdot|x-x^*|$$

$$=\frac{1}{(1+\xi_2)^2}\cdot 0.5*10^{-3}\le\frac{1}{(1+1.414)^2}*0.5*10^{-3}$$

- 例2：取$\sqrt{3}$ =1.732，试分析f($\sqrt{3}$)的误差上限。当$f(x)=x^2$时如何？

$x=\sqrt{3}, x^*=1.732$, 舍入误差$0.5*10^{-3}$

$$|f(x)-f(x^*)|=|f'(\eta)||x-x^*|$$

$$\le\max|f'(x)|\cdot 0.5*10^{-3}$$

$$f(x)=x^2, f'(x)=2x, x\in(1.7315, 1.7325)$$

$$|f(x)-f(x^*)|\le 2*1.7325*0.5*10^{-3}$$

- 例3：分析$L_1(x)=\frac{(x-x_1)}{(x_0-x_1)}y_0+\frac{(x-x_0)}{(x_1-x_0)}y_1$中观测误差$\Delta y_0, \Delta y_1$的影响。（假设$\Delta y_0=\Delta y_1$）

$$|\Delta L|\le\max\left|\frac{\partial L}{\partial y_0}\right|\Delta y_0+\max\left|\frac{\partial L}{\partial y_1}\right|\Delta y_1$$

$$=\left(\left|\frac{x-x_0}{x_1-x_0}\right|+\left|\frac{x-x_1}{x_0-x_1}\right|\right)\Delta y_0=\Delta y_0$$

- 例4：分析$y_{n+1}=\frac{1}{2}y_n+\frac{1}{n+1}$中观测误差的累积结果

初始误差Δy_0

$$\Delta y_1=\frac{1}{2}\Delta y_0$$

$$\Delta y_2=\frac{1}{2}\Delta y_1=\frac{1}{4}\Delta y_0$$

$$\cdots$$

$$\Delta y_n=\frac{1}{2^n}\Delta y_0$$