P42 2.

证明：$|P(AB)-P(AC)| \le 1-P(BC)$

由对称性：不妨设 $P(AB) \ge P(AC)$

$P(AB)-P(AC) \le 1-P(BC)$

即 $P(AB)+P(BC)-P(AC) \le 1$

而 $P(AB)+P(BC)-P(AC)$

$= P(A\cap B\cap C^c)+P(B\cap C\cap A^c)-P(A\cap C\cap B^c)$

三个事件空间互不重叠，故其和小于等于 $P(\Omega)$，即为 1

再讨论取等：举一特例，当 $A=B=\Omega$，$C=\emptyset$ 时取到等号

P43 13.

$$P=\frac{2^3-1}{4^3}=\frac{7}{64}$$

P43 14.

$$\frac{1}{11}\times\frac{2}{10}\times\frac{2}{9}\times\frac{1}{8}\times\frac{1}{7}\times\frac{1}{6}\times\frac{1}{5}$$

$$=\frac{1}{415800}$$

P44. 21.

$$1-(0.7)^3=0.657$$

P44 23.

(a)

$$\frac{b}{b+r}\cdot\frac{b+d}{b+r+d}=\frac{b(b+d)}{(b+r)(b+r+d)}$$

(b)

$$\frac{b}{b+r}\cdot\frac{r}{b+r+d}\cdot\frac{r+d}{b+r+2d}$$

$$=\frac{br(r+d)}{(b+r)(b+r+d)(b+r+2d)}$$

(c) $C_n^{n_1}\dfrac{\prod_{i=0}^{n_1-1}(b+id)\prod_{i=0}^{n_2-1}(r+id)}{\prod_{i=0}^{n}(b+r+id)}$

P44. 24.

设第n次抽中白球的概率为 $P_n$，可知此时白球有 $(a+b)P_n$ 个

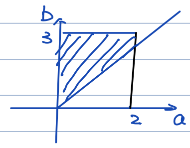
故 $P_{n+1} = \dfrac{P_n\cdot(a+b)P_n + (1-P_n)[(a+b)P_n+1]}{a+b} = \dfrac{(a+b)P_n+1-P_n}{a+b} = \left(1-\dfrac{1}{a+b}\right)P_n + \dfrac{1}{a+b}$

补充题

故 $P_n = 1-\dfrac{b}{a+b}\left(\dfrac{a+b-1}{a+b}\right)^n$

1、(A) $\dfrac{1+2+3}{4\times3} = \dfrac{1}{2}$

(B)



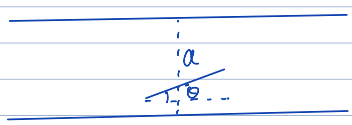

没有实根：

$4a^2-4b^2<0$

即 $a<b$

概率为 $\dfrac{2}{3}$

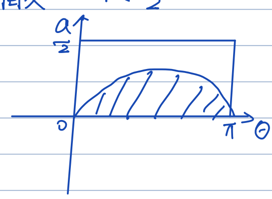
2.




设针中心距线最近距离为x，夹角为θ

则 $0\le x\le \dfrac{a}{2}$，取 $0\le\theta\le\pi$

相交：$x\le \dfrac{l}{2}\sin\theta$




$P=\dfrac{2\cdot\frac{l}{2}}{\frac{a}{2}\pi} = \dfrac{2l}{a\pi}$

3.

$P(A)=\dfrac{n!}{N^n}$

$P(B)=C_N^n\dfrac{n!}{N^n}$

4. $P\cdot\dfrac{7}{50}+3P\cdot\dfrac{3}{50}$

$=\dfrac{16}{50}P=\dfrac{8}{25}P$

由于 $P+3P=1$ 故 $P=\dfrac{1}{4}$ $\therefore P(A)=\dfrac{2}{25}$

5.

(a) $P(A)=\left(\frac{1}{2}\right)^3+C_3^1\left(\frac{1}{2}\right)^3=\frac{1}{2}$

$P(B)=1-2\left(\frac{1}{2}\right)^3=\frac{3}{4}$

$P(AB)=3\left(\frac{1}{2}\right)^3=\frac{3}{8}$

$P(AB)=P(A)P(B)$ 故A.B独立

(b) $P(A) = \left(\frac{1}{2}\right)^4 + C_4^1 \left(\frac{1}{2}\right)^4 = \frac{5}{16}$

$P(B) = 1 - 2\left(\frac{1}{2}\right)^4 = \frac{7}{8}$

$P(AB) = 4 \cdot \left(\frac{1}{2}\right)^4 = \frac{1}{4}$

$P(AB) \neq P(A)P(B)$

故不独立