习题14:

5.

$E(G)=10$ $\lambda(4)=\lambda(3)=2$, 其余顶点最大度数为2. 求顶点最少, 则令其余顶点度数均为2.

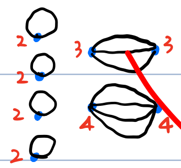
设最少有n个顶点. 则 $2m=20=d(v_1)+\cdots+d(v_n)=4\times2+3\times2+2\times(n-4) \Rightarrow n=7$

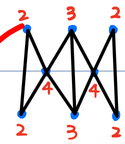
即G中至少有7个顶点, 顶点最少时度数列: $d=(4,4,3,3,2,2,2)$, $\Delta(G)=4$, $\delta(G)=2$

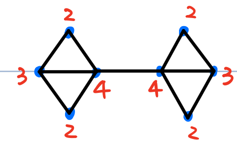
14.

(1) $d(v_1)+d(v_2)+\cdots+d(v_n)\neq 0 \pmod 2$ 故非可图化

(2) 可图化

① 

② 

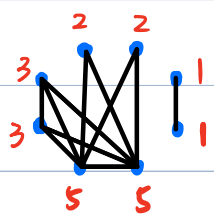
③ 

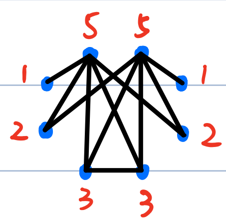
15-(1).

$(6,6,5,5,3,3,2) \Rightarrow (5,4,4,2,2,1) \Rightarrow (3,3,1,1,0) \Rightarrow (2,0,0,0)$ 不可简单图化

15-(2)

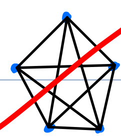
$(5,5,3,3,2,2,1,1) \Rightarrow (4,2,2,1,1,1,1) \Rightarrow (1,1,0,0,1,1) \Rightarrow (1,1,1,1,0,0)$ 可简单图化



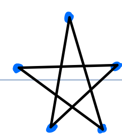


补1: 讨论K_5有几个非同构的生成子图为正则图.

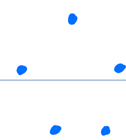
五阶完全图有3个非同构生成子图为正则图.



4-正则图



2-正则图



0-正则图.

设每个顶点度数为k, 则 $5k=2m$, k必为偶数, 则 $k=0,2,4$

$k=0$或4 显然无同构. $K=2$时: $\delta(G)=2$, 则必有$|\Gamma|\geq3$的圈.

若$|\Gamma|=3$, 则剩余两点无法达到$d=2$, 若$|\Gamma|=4$, 则剩余一点无法达到$d=2$,

故$|\Gamma|=5$, 即为一个长度为5的圈.

补2：在无向完全图 K_n $(n\geq 2)$ 中，寻找边数最多的生成子图，使其成为完全二部图 $K_{s,t}$.

所有 n 阶简单图都是 n 阶无向完全图的生成子图.

$s+t=n$，而完全二部图有 $s\cdot t$ 边，$s+t\geq 2\sqrt{st}$　$s\cdot t\leq (\frac{s+t}{2})^2=\frac{n^2}{4}$

若 n 为偶数，则 $s=\frac{n}{2}$，$t=\frac{n}{2}$ 的完全二部图满足条件.

若 n 为奇数，则 $s=\frac{n-1}{2}$，$t=\frac{n+1}{2}$ 的完全二部图满足条件.