

25.

28.

递推

$$P_{n+1} = P_n \cdot 0 + (1 - P_n) \frac{1}{3}$$

$$= -\frac{1}{3} P_n + \frac{1}{3}$$

$$\therefore P_{n+1} - \frac{1}{4} = -\frac{1}{3} (P_n - \frac{1}{4})$$

$$\therefore P_n = (P_0 - \frac{1}{4}) \cdot (-\frac{1}{3})^n + \frac{1}{4}$$

$$\text{且 } P_0 = 1$$

$$\therefore P_n = \frac{3}{4} \cdot (-\frac{1}{3})^n + \frac{1}{4}$$

$$\text{将 } n=7 \text{ 代入, 得: } P_7 = \frac{182}{729}$$

31.

设 A = 取出全是白球

B = 摸出 3 点

$$P(A) = \frac{1}{6} \frac{C_4^1}{C_{10}^1} + \frac{1}{6} \frac{C_4^2}{C_{10}^2} + \frac{1}{6} \frac{C_4^3}{C_{10}^3} + \frac{1}{6} \frac{C_4^4}{C_{10}^4}$$

$$= \frac{1}{6} \times \frac{4}{10} + \frac{1}{6} \times \frac{6}{45} + \frac{1}{6} \times \frac{4}{120} + \frac{1}{6} \times \frac{1}{210} = \frac{2}{21}$$

$$P(AB) = \frac{1}{6} \times \frac{C_4^3}{C_{10}^3} = \frac{1}{180}$$

$$P(B|A) = \frac{P(AB)}{P(A)} = \frac{7}{120}$$

35.

设 A = A 写上加号

B = 主持人看到加号

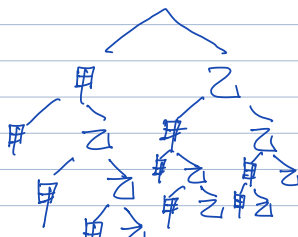
$$P(AB) = \frac{1}{3} \times [\frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} + \frac{1}{3} \times \frac{2}{3} \times \frac{2}{3} + \frac{2}{3} \times \frac{2}{3} \times \frac{1}{3} + \frac{2}{3} \times \frac{1}{3} \times \frac{2}{3}]$$

$$= \frac{1}{3} \times [\frac{1}{27} + \frac{4}{27} + \frac{4}{27} + \frac{4}{27}]$$

$$= \frac{13}{81}$$

$$P(B) = \frac{2}{3} \times \frac{14}{27} + \frac{13}{81} = \frac{41}{81}$$

$$P(A|B) = \frac{13}{41}$$



36.

$$P(\text{甲得冠军}) = \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{8} + \frac{1}{16} + \frac{1}{16} = \frac{11}{16}$$

$$P(\text{乙得冠军}) = \frac{5}{16}$$

37.

$$P = 0.8 \times 0.2 \times 0.2 \times 0.3 + 2 \times 0.8 \times 0.8 \times 0.2 \times (1-0.7 \times 0.7) + 0.8 \times 0.8 \times 0.8 \times (1-0.7 \times 0.7 \times 0.7) \\ = 0.476544$$

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2.

$$\therefore \text{成反比} \quad \therefore \text{设 } P(X=k) = \frac{a}{k(k+1)}$$

$$\text{则 } \sum_{k=1}^{\infty} P(X=k) = a \\ \therefore a = 1$$

$$\text{而 } P(X \leq k) (k \in \mathbb{Z} \text{ 且 } k \geq 1) = \frac{k}{k+1}$$

3.

$$P(X=1) = \frac{3}{C_4^1} = \frac{1}{2}$$

$$P(X=2) = \frac{2}{C_4^2} = \frac{1}{3}$$

$$P(X=3) = \frac{1}{C_4^3} = \frac{1}{4}$$

$$\therefore \begin{array}{c|ccc} X & 1 & 2 & 3 \\ \hline P & \frac{1}{2} & \frac{1}{3} & \frac{1}{6} \end{array}$$

6.

X 可能取值为 0, 1, 2, 3

$$P(X=0) = \frac{9}{12} = \frac{3}{4}$$

$$P(X=1) = \frac{3}{12} \times \frac{9}{11} = \frac{9}{44}$$

$$P(X=2) = \frac{3}{12} \times \frac{2}{11} \times \frac{9}{10} = \frac{9}{220}$$

$$P(X=3) = \frac{3}{12} \times \frac{2}{11} \times \frac{1}{10} \times 1 = \frac{1}{220}$$

$$\therefore \begin{array}{c|cccc} X & 0 & 1 & 2 & 3 \\ \hline P & \frac{3}{4} & \frac{9}{44} & \frac{9}{220} & \frac{1}{220} \end{array}$$

12.

掷三颗骰子点数和为奇数的概率为 $\frac{1}{2}$

属于几何分布, 由期望可知, 平均需要 2 次

16. 本题即为求二项分布中奇项减偶项的概率

$$\text{令 } q = 1-p$$

$$\text{则 } (p+q)^n = C_n^0 q^n + C_n^1 p q^{n-1} + \dots + C_n^n p^n$$

$$(q-p)^n = C_n^0 q^n - C_n^1 p q^{n-1} + \dots + (-1)^n C_n^n p^n$$

$$\text{可得 } F_Y = (1-2p)^n$$

补充

共 7 次试验, 最后一次一定是失败的

$$\therefore C_6^2 \cdot (0.4)^3 \times (0.6)^4$$

$$= 0.124416$$