清華大學
Tsinghua University

常微分方程数值求解应用

自动化系 赵虹

应用实例：洛伦兹吸引子

Edward Lorenz：美国著名数学家和气象学家，混沌理论创始人

大气流体动力学的简化模型，模拟大气的对流，可用于天气预报

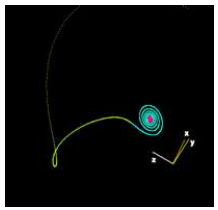
$$\frac{dx}{dt} = \sigma(y - x)$$
$$\frac{dy}{dt} = x(\rho - z) - y$$
$$\frac{dz}{dt} = xy - \beta z$$

x：与大气对流相关

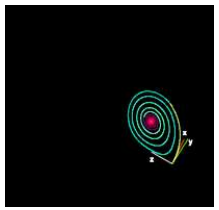
y：温度水平变化

z：温度竖直变化

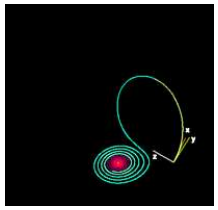
参数对洛伦兹系统解的影响



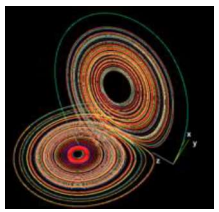
$\rho = 14, \sigma = 10, \beta = 8/3$



$\rho = 13, \sigma = 10, \beta = 8/3$



$\rho = 15, \sigma = 10, \beta = 8/3$

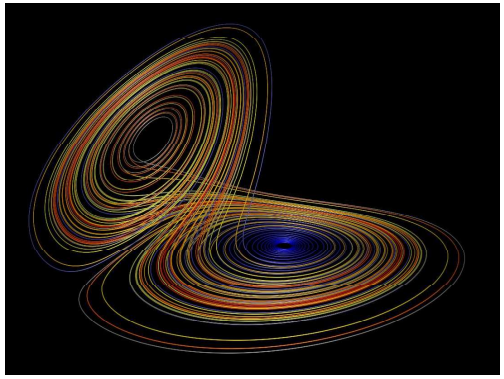


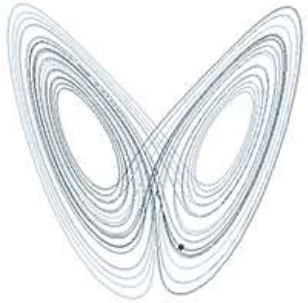
$\rho = 28, \sigma = 10, \beta = 8/3$

For small values of ρ, the system is stable and evolves to one of two fixed point attractors. When ρ is larger than 24.74, the fixed points become repulsors and the trajectory is repelled by them in a very complex way.

http://en.wikipedia.org/wiki/Lorenz_system

洛伦兹吸引子



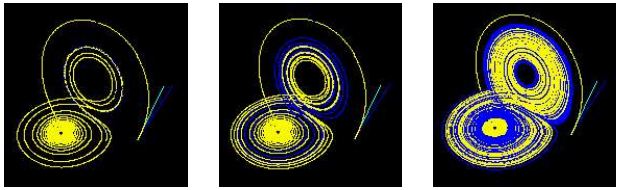


A sample solution in the Lorenz attractor when ρ = 28, σ = 10, and β = 8/3

http://paulbourke.net/fractals/lorenz/new1.png
https://upload.wikimedia.org/wikipedia/commons/1/13/A_Trajectory_Through_Phase_Space_in_a_Lorenz_Attractor.gif

初始值对洛伦兹系统解的影响

Time $t = 1$ Time $t = 2$ Time $t = 3$



These figures — made using $\rho = 28$, $\sigma = 10$ and $\beta = 8/3$ — show three time segments of the 3-D evolution of two trajectories (one in blue, the other in yellow) in the Lorenz attractor starting at two initial points that differ only by 10^{-5} in the x-coordinate. Initially, the two trajectories seem coincident (only the yellow one can be seen, as it is drawn over the blue one) but, after some time, the divergence is obvious.

http://en.wikipedia.org/wiki/Lorenz_system

微分方程应用

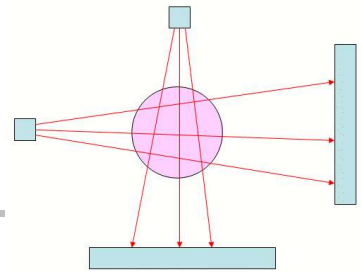
- One thing that will never change is the fact that the world is constantly changing.
- If you try and use math to describe the world around you — say the growth of a plant, the fluctuations of the stock market, the spread of diseases, or physical forces acting on an object — you soon find yourself dealing with derivatives of functions.
- The way they inter-relate and depend on other mathematical parameters is described by differential equations.

https://plus.maths.org/content/teacher-package-differential-equations

微分方程应用

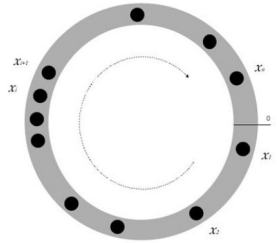
Saving lives: tomography

A CAT scan of the inside of a head



微分方程应用

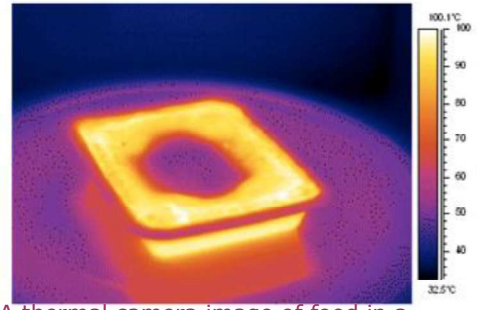
Unjamming Traffic



Mathematicians from the Universities of Exeter, Bristol and Budapest have developed a model of traffic behaviour that explains how an unexpected event as simple as a car changing lanes, or a van braking suddenly, can bring traffic to a grinding halt kilometres behind the incident.
Their model suggests that when reacting to an unexpected event, drivers may slow down to below a critical speed, which then forces the car behind it to slow down further still. Eventually, cars further back in the queue must stop. This produces a wave travelling backwards from the point of disturbance.

微分方程应用

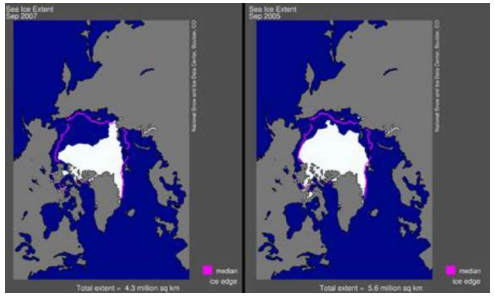
Eat, drink and be merry: making sure it is safe

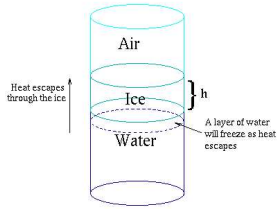


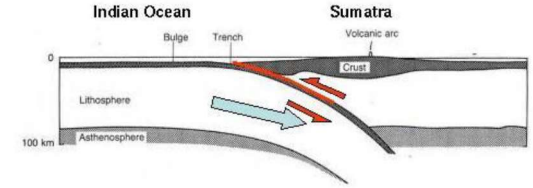
A thermal camera image of food in a domestic turntable oven, showing a distinct cold spot in the centre caused by a local minimum in the radiant electro magnetic field. This can leave to very uneven cooking with potentially dangerous consequences.

微分方程应用

Math and Climate Change:







2005 Abel Prize Laureate: Peter D. Lax

The Abel Prize Laureate 2005

The Norwegian Academy of Science and Letters has decided to award the Abel Prize for 2005 to

Peter D. Lax

Courant Institute of Mathematical Sciences, New York University

"for his groundbreaking contributions to the theory and application of partial differential equations and to the computation of their solutions."

The Abel Prize is an international prize presented by the King of Norway to one or more outstanding mathematicians. Named after Norwegian mathematician Niels Henrik Abel (1802–1829), the Abel Prize and the Fields Medal have often been described as the "Mathematician's Nobel Prizes".

https://www.abelprize.no/c53864/seksjon/vis.html?tid=53872&strukt_tid=53864