

1. a) 变量: $X = \{X_1, X_2, \dots, X_6\} = \{A_1, A_2, A_3, A_4, B_1, B_2, B_3, B_4, C_1, C_2, C_3, C_4, D_1, D_2, D_3, D_4\}$

值域: $D = \{D_1, D_2, \dots, D_6\}$, $X_i \in D_i = \{1, 2, 3, \dots, 15, 16\}$

约束: ① $\forall x_i, x_j \in X (i \neq j)$ 有 $x_i \neq x_j$

$$(2) A_1 + A_2 + A_3 + A_4 = B_1 + B_2 + B_3 + B_4 = C_1 + C_2 + C_3 + C_4 = D_1 + D_2 + D_3 + D_4 = 34;$$

$$A_1 + B_1 + C_1 + D_1 = A_2 + B_2 + C_2 + D_2 = A_3 + B_3 + C_3 + D_3 = A_4 + B_4 + C_4 + D_4 = 34;$$

$$A_1 + B_2 + C_3 + D_4 = A_4 + B_3 + C_2 + D_1 = 34$$

b) 約束傳播策略:

考虑 C_2 , 根据约束 $9 + C_2 + 7 + 12 = 34$ 得 $C_2 = 6$

考虑 D_1 , 根据约束 $D_1 + C_2 + B_3 + A_4 = D_1 + 6 + 11 + 13 = 34$ 得 $D_1 = 4$

考虑 D_2 , 根据约束 $D_1 + D_2 + D_3 + D_4 = 4 + D_2 + 14 + 1 = 34$ 得 $D_2 = 15$

此时 $A_1, A_2, B_1, B_2 \in \{3, 5, 10, 16\}$, 而根据约束 $A_1 + A_2 = 19$, $B_1 + B_2 = 15$.

$$A_1 + B_1 = 21, A_2 + B_2 = 13, \text{最后得 } A_1 = 16, A_2 = 3, B_1 = 5, B_2 = 10$$

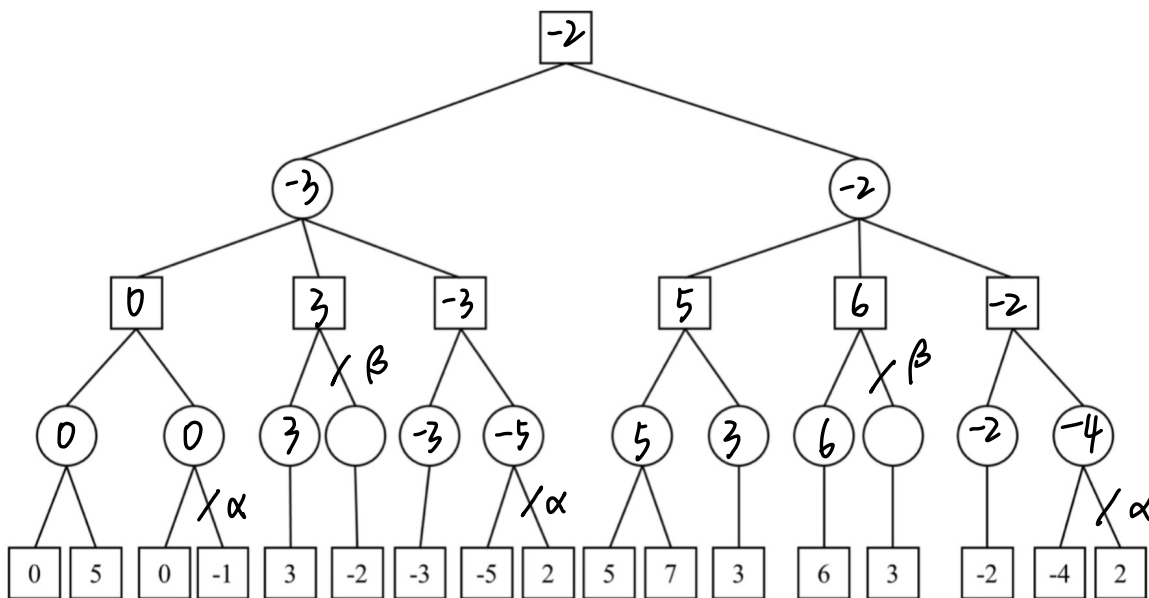
回溯搜索过程求解: $(A_1, A_2, B_1, B_2, C_2, D_1, D_2 \in \{3, 4, 5, 6, 10, 15, 16\})$ 值域确定

$$A_1 = 3 \rightarrow A_2 = 16 \rightarrow B_2 = 23 \notin \text{值域} \times$$
$$A_1=4 \rightarrow A_2=15 \rightarrow B_2=22 \text{ \& 值域 } x$$
$$A_1 = 5 \Rightarrow A_2 = 14 \notin \text{值域} \times$$
$$A_1=6 \rightarrow A_2=13 \notin \text{值域} \times$$
$$A_1 = 10 \rightarrow A_2 = 9 \in \text{值域} x$$
$$A_1=15 \rightarrow A_2=4 \rightarrow B_2=11 \text{ 值域} \times$$
$$A_1=16 \rightarrow A_2=3 \rightarrow B_2=10 \rightarrow B_1=5 \rightarrow C_2=6 \rightarrow A=4 \rightarrow D_2=15$$

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

补全的4阶幻方如右图所示:

3.



修剪叶节点
总数为5个