P97 26.

$$\rho_{(X,Y)} = \frac{Cov(X,Y)}{\sigma_X \sigma_Y} = \frac{\sum_{k=1}^{n} Cov(\xi_k, Y)}{\sqrt{DX\,DY}} = \frac{\sum_{k=1}^{n}\sum_{j=1}^{n} Cov(\xi_k, \xi_{j+m})}{\sqrt{DX\,DY}}$$

由于 $DX = DY = n\,D\xi$

$\therefore$ 原式 $= \frac{n-m}{n}$

P97 27.

由于 $X \sim Binomial(n, \frac{1}{6})$, $Y \sim Binomial(n, \frac{1}{6})$

令

$X_i = \begin{cases} 1 & \text{若第 } i \text{ 次出 1 点} \\ 0 & \text{否则} \end{cases}$ $Y_i = \begin{cases} 1 & \text{若第 } i \text{ 次出 6 点} \\ 0 & \text{否则} \end{cases}$

$\therefore X = \sum_{i=1}^{n} X_i$; $Y = \sum_{i=1}^{n} Y_i$

$\therefore EXY = \frac{n(n-1)}{36}$

且 $EX = \frac{n}{6}$, $EY = \frac{n}{6}$ $\therefore$ 协方差 $Cov(X,Y) = -\frac{n}{36}$

方差: $DX_i = E[X^2] - [EX]^2 = \frac{5}{36}$

$\therefore DX = DY = \frac{5n}{36}$

$\therefore$ 相关系数 $-\frac{1}{5}$

P98 30.

Y	1	2	3
$E(X\|Y)$	$\frac{13}{7}$	$\frac{28}{15}$	$\frac{11}{5}$
P	$\frac{7}{27}$	$\frac{15}{27}$	$\frac{5}{27}$

解得 $EX = \frac{52}{27}$

P98 32.

设首次检修生产的产品为 Y, 其中一等品件数为 X

(a) 由于满足二项分布,
故 n 件中一等品件数的期望为: $(n-1)\frac{P_1}{P_1+P_2}$

(b) $EX = \sum_{k=1}^{n} P_k\, P(X|Y=k)$

$$= \sum_{k=1}^{\infty} (1-p_3)^{k-1} \cdot p_3 \, (k-1) \frac{p_1}{p_1+p_2}$$

$$= \frac{p_1 p_2}{p_1+p_2} \cdot \sum_{j=0}^{\infty} j (1-p_3)^{j}$$

$$= \frac{p_1}{p_3}$$

P98 33.

设第一张抽中号码数为随机变量

$E(X|Y=k) = \frac{1+k}{2}$

则 $EX = E(E(X|Y)) = \frac{1}{2} + \frac{1}{2}\frac{n+1}{2} = \frac{n+3}{4}$

P98 37.

0 \ L	2	3	4
0	$\frac{3}{7}$	0	$\frac{1}{7}$
1	$\frac{1}{7}$	0	0
2	0	$\frac{1}{7}$	$\frac{1}{7}$

由结论，最佳预测值为 $E(L|X)$

而 $E(L|X=0) = \frac{5}{2}$

$E(L|X=1) = 2$

$E(L|X=2) = \frac{7}{2}$

P98 36.

$E[D(X|Y)]$

$= E\{E(X^2|Y) - [E(X|Y)]^2\}$

$= E(X^2) - E\{[E(X|Y)]^2\}$

而 $D[E(X|Y)]$

$= E(E(X|Y)^2) - E(E(X|Y))^2 = E(E(X|Y)^2) - (EX)^2$

两式相加，即可证得

补 1:

$EX = \frac{1}{3}[E(X|Y=1) + E(X|Y=2) + E(X|Y=3)]$

其中 $E(X|Y=1) = 2$

$E(X|Y=2) = 3 + \frac{1}{2} \times 2 + \frac{1}{2} \times (5+2) = 7.5$

$$E(X \mid Y=3) = 5 + \frac{1}{2}\times 2 + \frac{1}{2}\times(3+2) = 8.5$$

$\therefore EX = 6$ 小时

补2：

$Y = n$ 时

$$E(X \mid Y=n) = \frac{n-1}{5}$$

$$EX = E(E(X|Y)) = \sum_{n=1}^{\infty} \left(\frac{5}{6}\right)^{n-1} \cdot \left(\frac{1}{6}\right) \cdot \frac{n-1}{5}$$

$$= 1$$