

## 12. 求前束范式

$$(1) \forall x F(x) \rightarrow \forall y G(x, y)$$

$$\Leftrightarrow \forall x F(x) \rightarrow \forall y G(t, y) \quad \text{换名规则.}$$

$$\Leftrightarrow \exists x (F(x) \rightarrow \forall y G(t, y)) \quad \text{量词辖域扩张}$$

$$\Leftrightarrow \exists x \forall y (F(x) \rightarrow G(t, y)) \quad \text{量词辖域扩张}$$

$$(2) \forall x F(x, y) \leftrightarrow \exists x G(x, y)$$

$$\Leftrightarrow \forall t F(t, y) \leftrightarrow \exists w G(w, y) \quad \text{换名规则.}$$

$$\Leftrightarrow (\neg \forall m F(m, y) \vee \exists n G(n, y)) \wedge (\forall p F(p, y) \vee \neg \exists q G(q, y)) \quad \text{置换}$$

$$\Leftrightarrow \exists m \exists n (\neg F(m, y) \vee G(n, y)) \wedge \forall p \forall q (F(p, y) \vee \neg G(q, y)) \quad \text{量词辖域扩张}$$

$$\Leftrightarrow \exists m \exists n \forall p \forall q ((\neg F(m, y) \vee G(n, y)) \wedge (F(p, y) \vee \neg G(q, y))) \quad \text{量词辖域扩张}$$

$$(3) \exists x_1 F(x_1, x_2) \rightarrow (F(x_1) \rightarrow \neg \exists x_2 G(x_1, x_2))$$

$$\Leftrightarrow \exists x F(x, y) \rightarrow (F(t) \rightarrow \neg \exists w G(m, w)) \quad \text{换名}$$

$$\Leftrightarrow \exists x F(x, y) \rightarrow \forall w (F(t) \rightarrow \neg G(m, w)) \quad \text{置换.}$$

$$\Leftrightarrow \forall x (F(x, y) \rightarrow \forall w (F(t) \rightarrow \neg G(m, w))) \quad \text{量词辖域扩张}$$

$$\Leftrightarrow \forall x \forall w (F(x, y) \rightarrow (F(t) \rightarrow \neg G(m, w))) \quad \text{量词辖域扩张}$$

## 17.

附加前提引入错误, 要证明的结论不为  $\forall x H(x) \rightarrow \forall x \neg F(x)$ .

## 19.

$$(1) \exists x F(x) \rightarrow \forall x G(x) \quad \text{前提引入}$$

$$(2) \exists x F(x) \rightarrow \forall y G(y) \quad \text{换名规则}$$

$$(2) \forall x (F(x) \rightarrow \forall y G(y)) \quad \text{置换}$$

(3)  $\forall x \forall y (F(x) \rightarrow G(y))$  置换.

(4)  $\forall x (F(x) \rightarrow G(y))$   $\forall -$

23.

(1)

$F(x)$ :  $x$  是有理数.  $G(x)$ :  $x$  是实数  $H(x)$ :  $x$  是整数.

前提:  $\forall x (F(x) \rightarrow G(x)), \exists x (F(x) \wedge H(x))$

结论:  $\exists x (G(x) \wedge H(x))$

①  $\exists x (F(x) \wedge H(x))$  前提引入

②  $F(c) \wedge H(c)$   $\exists -$

③  $\forall x (F(x) \rightarrow G(x))$  前提引入

④  $F(c) \rightarrow G(c)$   $\forall -$

⑤  $F(c)$  ② 化简

⑥  $G(c)$  ④⑤ 假言推理

⑦  $H(c)$  ② 化简

⑧  $H(c) \wedge G(c)$  ⑥⑦ 合取

⑨  $\exists x (G(x) \wedge H(x))$   $\exists +$

(2)  $F(x)$ :  $x$  是有理数.  $G(x)$ :  $x$  是无理数  $H(x)$ :  $x$  是实数.  $T(x)$ :  $x$  是虚数.

前提:  $\forall x (F(x) \rightarrow H(x)), \forall x (G(x) \rightarrow H(x)), \forall x (T(x) \rightarrow \neg H(x))$ ,

结论:  $\forall x (T(x) \rightarrow (\neg F(x) \wedge \neg G(x)))$

①  $\forall x (F(x) \rightarrow H(x))$  前提引入

②  $F(x) \rightarrow H(x)$   $\forall -$

③  $\neg H(x) \rightarrow \neg F(x)$  置换, 假言易位

④  $\forall x (G(x) \rightarrow H(x))$  前提引入

⑤  $G(x) \rightarrow H(x)$   $\forall -$

$$\textcircled{6} \quad \neg H(x) \rightarrow \neg G(x)$$

置换, 假言易位.

$$\textcircled{7} \quad \forall x (T(x) \rightarrow \neg H(x))$$

前提引入

$$\textcircled{8} \quad T(x) \rightarrow \neg H(x)$$

$\forall$ -

$$\textcircled{9} \quad T(x) \rightarrow \neg F(x)$$

③⑧ 假言三段论

$$\textcircled{10} \quad T(x) \rightarrow \neg G(x)$$

③⑩ 假言三段论

$$\textcircled{11} \quad T(x) \rightarrow (\neg F(x) \wedge \neg G(x)) \quad \textcircled{9} \textcircled{10} \text{ 合值式.}$$

习题 7.

2.

否, 若  $A = \emptyset$ , 则  $\forall B, C$ , 都有  $A \times B \subseteq A \times C$

3.

$$(1) \quad (A \cap B) \times (C \cap D) = (A \times C) \cap (B \times D)$$

$$\langle x, y \rangle \in (A \cap B) \times (C \cap D)$$

$$\Rightarrow x \in A \cap B \wedge y \in C \cap D$$

$$\Rightarrow x \in A \wedge y \in C \wedge x \in B \wedge y \in D$$

$$\Rightarrow \langle x, y \rangle \in A \times C \wedge \langle x, y \rangle \in B \times D$$

$$\Rightarrow \langle x, y \rangle \in (A \times C) \cap (B \times D)$$

$$\langle x, y \rangle \in (A \times C) \cap (B \times D)$$

$$\Rightarrow \langle x, y \rangle \in A \times C \wedge \langle x, y \rangle \in B \times D$$

$$\Rightarrow x \in A \wedge y \in C \wedge x \in B \wedge y \in D$$

$$\Rightarrow x \in (A \cap B) \wedge y \in (C \cap D)$$

$$\Rightarrow \langle x, y \rangle \in (A \cap B) \times (C \cap D)$$

$$\text{故: } (A \cap B) \times (C \cap D) = (A \times C) \cap (B \times D)$$

(2)

① 不成立  $A = \{1\}, B = \{2\}, C = \{3\}, D = \{4\}$

$$(A \cup B) \times (C \cup D) = \{1, 2\} \times \{3, 4\} = \{\langle 1, 3 \rangle, \langle 1, 4 \rangle, \langle 2, 3 \rangle, \langle 2, 4 \rangle\}$$

$$(A \times C) \cup (B \times D) = \{\langle 1, 3 \rangle\} \cup \{\langle 2, 4 \rangle\} = \{\langle 1, 3 \rangle, \langle 2, 4 \rangle\}$$

> 不并

② 不成立  $A = \{1, 2\}, B = \{2\}, C = \{3\}, D = \{4\}$

$$(A - B) \times (C - D) = \{\langle 1, 3 \rangle\}$$

> 不并

$$(A \times C) - (B \times D) = \{\langle 1, 3 \rangle, \langle 2, 3 \rangle\}$$