数值分析与算法

Numerical Analysis

主讲教师：赵虹

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- 1993.9 – 1999.7 清华大学自动化系，本科、硕士研究生
- 1999.7 – 2006.7 美国马里兰大学电子与计算机工程系，博士、博士后
- 2006.8 – 2016.2 加拿大阿尔伯塔大学电子与计算机工程系，助理教授、副教授
- 2016.5 – 至今，清华大学自动化系，长聘副教授
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教材

教材

- 数值分析，李庆扬、王能超、易大义编，清华大学出版社，2008年第五版

教参

- 数值分析基础，关治、陆金甫，高等教育出版社，2019第三版
- 数值分析与算法，喻文健，清华大学出版社，2019第三版

以上教材及教参在学校教参服务平台均可直接访问。

微信群

请将群昵称改为真实姓名

2022秋数值分析课程群

Valid until 9/18 and will update upon joining group

第一章 绪论

第一节 简介

- 数值分析：Numerical Analysis
  - 研究数学问题的计算机（数值）解
- 工科专业（信息、机械、航空航天、土木水利、电机等）中解决各种问题都需要很多数学手段
  - 需要利用计算机来求解这些数学问题
- 为什么学习过数学课后还需要学习这门课程？

数值分析应用举例




Andy Serkis，英国演员

数值分析应用举例

THE LORD OF THE RINGS

"The character of Gollum is a completely digital creature, but I was determined that I wanted an actor to actually create the character, which in this case is Andy Serkis," says Peter Jackson.

http://www.serkis.com/cinlotr.htm

数值分析应用举例

数值分析应用举例

http://www.movieactors.com/actors/andyserkis.htm

数值分析应用举例

http://filmbuffonline.com/fbolnewsreel/wordpress/tag/academy-awards-2015

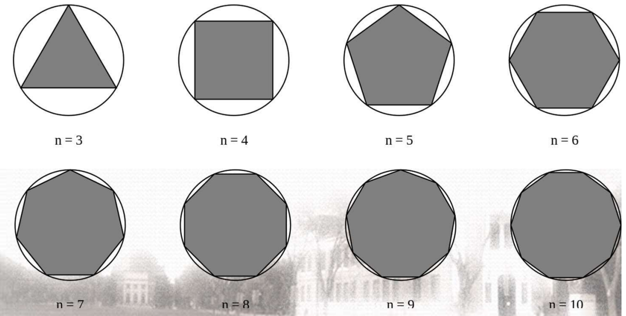
几个数值计算应用的例子

- 例子
  - 水库储水量测算
  - 经济增长预测（从历史数据总结规律）
  - 信号（图像）变换（数据压缩）与反变换
  - 卫星发射设计与控制
- 共同特点　　数学模型↔计算

  　　　　　　连续情形　数字化（离散）
- 产生误差，误差需要控制在应用许可的范围内！

例：求圆的周长（r=1）

- 用内接n等边多边形来近似圆
- 是否收敛到圆
- n=m时误差又多大
- 计算机储存误差的影响



The Butterfly Effect 蝴蝶效应

In 1961, Edward Norton Lorenz, a mathematician and meteorologist, was running a numerical computer model to redo a weather prediction from the middle of the previous run as a shortcut. He entered the initial condition 0.506 from the printout instead of entering the full precision 0.506127 value. The result was a completely different weather scenario. These observations ultimately led him to formulate what became known as the butterfly effect--a term that grew out of an academic paper he presented in 1972 entitled: "Predictability: Does the Flap of a Butterfly's Wings in Brazil Set Off a Tornado in Texas?"

http://news.mit.edu/2008/obit-lorenz-0416
https://belfastchildis.com/2016/05/02/lorenzs-butterfly-effect-whats-it-all-about/

The Patriot Missile (爱国者导弹) Failure

On February 25, 1991, an American Patriot Missile in Saudi Arabia failed to track and intercept an incoming Iraqi Scud missile, killing 28 soldiers and injuring around 100 other people.

Cause: inaccurate calculation of time since boot due to computer arithmetic errors:

The system's internal clock measured time as multiples of 1/10 sec, expressed as a 24-bit integer. This number was multiplied by 1/10 to convert to real number. Error is 0.0000000000000000000000011001100... binary, or about 0.000000095 decimal. The Patriot battery had been up for 100+ hours, accumulated error ~0.34 sec. A Scud travels at about 1,676 m/s, far enough for the scud to be outside the “range gate” the Patriot tracked.

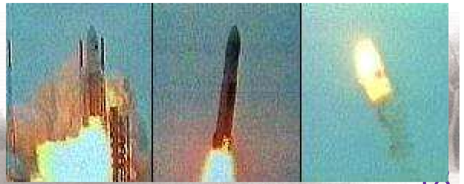
http://www-users.math.umn.edu/~arnold/disasters/patriot.html

The Short Flight of Ariane 5

On June 4,1996, an unmanned Ariane 5 rocket launched by the European Space Agency exploded just 39 seconds after its lift-off. The rocket was on its first voyage, after a decade of development costing $7billion. The destroyed rocket and its cargo were valued at $500 million.

Cause of explosion: a 64 bit floating point number relating to the horizontal velocity was converted to a 16 signed integer. The number was larger than 32,767, the largest integer a 16-bit signed integer can represent, and the conversion failed.

http://www.bbc.com/future/story/20150505-the-numbers-that-lead-to-disaster



The Gangnam Style Glitch

In December 2014, it was reported that Gangnam Style, the most popular video of all time on YouTube had “broken” the website’s view counter. The counter had apparently been programmed to only run to 2,147,483,647 – again, the maximum positive value of a 32-bit signed register. YouTube changed the maximum view limit to 9,223,372,036,854,775,808 (64-bit), or more than nine quintillion.

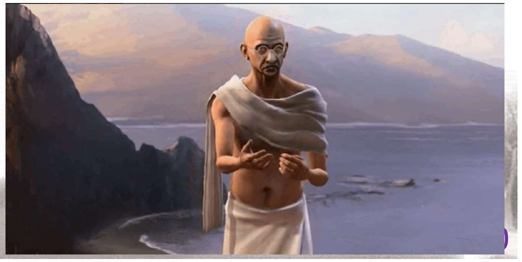
On 1 December, YouTube posted a statement saying: "We never thought a video would be watched in numbers greater than a 32-bit integer... but that was before we met Psy."

https://www.bbc.com/news/world-asia-30288542

The Aggressive Gandhi

In the video game Civilization, an unanticipated bug caused the peaceful character Gandhi to become uncharacteristically hostile. When players chose a certain mode to play in, the value which defined Gandhi’s aggressiveness rolled backwards past zero to the maximum. Consequently, he would threaten players with nuclear weapons at every turn – to the great amusement of many players.

https://kotaku.com/why-gandhi-is-such-an-asshole-in-civilization-1653818245



课程内容

- 函数逼近：插值、最佳逼近、拟合
- 数值积分、微分
- 常微分方程的数值解
- 线性代数方程组求解
- 非线性方程求解
- 矩阵特征值、特征向量计算

- 与Matlab等工具相比，本门课程意义何在？
  - **MATLAB/EDA被禁，工业软件之殇**
  - 知其然，知其所以然
  - 科研、应用中很多问题无法直接用现成工具解决
  - Matlab等工具无法做误差分析

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MATLAB被禁如何影响中国制造

数据堂 数据堂 7/1

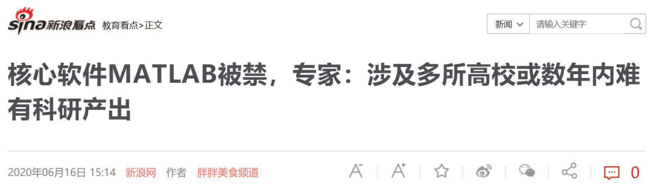
在5月22日美国商务部发布的“实体清单”中，中国两所工科类院校哈尔滨工业大学、哈尔滨工程大学赫然在列。受此“清单”的影响，MATLAB中止了对两所高校的正版授权。

The 24 entities to be added to the Entity List are:

- Beijing Cloudmind Technology Co., Ltd.
- Beijing Computational Science Research Center
- Beijing Jincheng Huanyu Electronics Co., Ltd.
- Center for High Pressure Science and Technology Advanced Research
- Chengdu Fine Optical Engineering Research Center
- China Jiuyuan Trading Corporation
- Cloudminds (Hong Kong) Limited
- Cloudminds Inc.
- Harbin Chuangyue Technology Co., Ltd.
- Harbin Engineering University
- Harbin Institute of Technology
- Harbin Yun Li Da Technology and Development Co., Ltd.
- JCN (HK) Technology Co. Ltd.
- K Logistics (China) Limited
- Kunhai (Yanjiao) Innovation Research Institute

美国商务部5月22日发布的实体清单

该消息一出迅速引起了学术界和科技界的广泛讨论。那么MATLAB被禁究竟有何影响？有无替代方案弥补其被禁带来的损失？



sina新浪看点 教育看点>正文

新闻 请输入关键字

核心软件MATLAB被禁，专家：涉及多所高校或数年内难有科研产出

2020年06月16日 15:14 新浪网 作者 胖胖美食频道

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主要掌握内容

- 误差分析（包括收敛性与稳定性分析）
- 数值计算的基本原理和方法

注重基本原理和概念的掌握，强调理论和实践的结合，锻炼解决实际问题的能力和自学的能力。

要求

- 按时上课，按要求**独立**完成每次大作业和作业，积极参加课堂讨论
  - 按时交作业，**不允许补交作业**！
- 考察方式
  - 作业（约20%）+大作业（项目训练，约30%）期末考试（半开卷，约50%）
  - 可能会视疫情发展及/或学期进展情况而调整

请各位同学务必合理安排时间！

第二节 误差

一、误差来源

- 观测误差
- 模型误差
- 方法误差（截断误差）
- 舍入误差（计算误差、存储误差）

观测误差

舍入误差

对最后结果的影响相似，
通过计算公式累积变化

模型误差

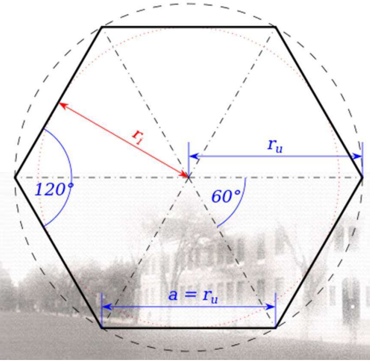
方法误差/
截断误差

从定义上看即为对最后结果的影响值，
二者比较相似

一般来说:
- 前者指将对象进行数学化带来的误差（无法量化分析，利用经验判断分析）
- 后者指进一步写成计算公式时带来的误差

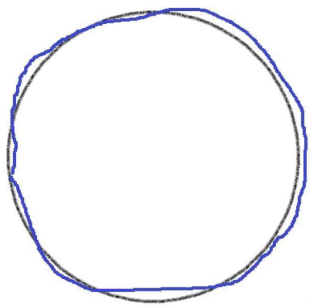
例：用内接多边形周长近似圆周长(r_u=1)

- 观测误差 – 无
- 模型误差 – 无 （可证明可以收敛到圆）
- 方法误差 – 迭代停止后的理想值与2π的差
- 舍入误差 – 每步都有（计算中间结果存储），累积



例：求不规则形状的周长

- 用圆近似，进而用内接n多边形来逼近求周长 – 模型误差



例

- 已知
  sin0.32=0.314567 --观测误差
  sin0.34=0.333487 --观测误差
- 求sin0.3367的近似值和误差界
- 用$L_1(x) = \frac{(x-x_1)}{(x_0-x_1)}y_0 + \frac{(x-x_0)}{(x_1-x_0)}y_1$

例：求e^{-1}

- $e^{-1} = 1 + (-1) + \frac{(-1)^2}{2!} + \cdots + \frac{(-1)^k}{k!} + \cdots$

取 k=7， $e^{-1} \cong 0.3678571 \ldots$

$R_7 \le \frac{1}{8!} < \frac{1}{4} * 10^{-4}$ --截断误差

$e^{-1} \cong 0.3679$ --舍入误差

- 观测误差和舍入误差会随着计算累积变化

$$I_n = 1 - nI_{n-1}, n = 1,2,3,\cdots$$

初始值I_0, 误差ΔI_0

$$|\Delta I_1| = |\Delta I_0|$$

$$|\Delta I_2| = 2|\Delta I_0|$$

$$\vdots$$

$$|\Delta I_n| = n!\,|\Delta I_0|$$

不稳定

二、误差衡量

x：准确值，x*：近似值

绝对误差：$|\Delta x| = |x - x^*|$

相对误差：$\left|\frac{\Delta x}{x}\right| \approx \left|\frac{\Delta x}{x^*}\right|$

三、有效数字（从左至右第一位非零数字开始的所有数字）

x=1.2345678

三位有效数字：x*=1.23， $|\Delta x| \le 0.5 * 10^{-2}$

五位有效数字：x*=1.2346， $|\Delta x| \le 0.5 * 10^{-4}$

四位有效数字：

187.9325	0.0378551	8.000033	8
187.9	0.03786	8.000	

Open Question Points: 10 Setting

思考：二进制时如何考虑有效数字问题？

$(10.0101011)_2$

四位有效数字

五位有效数字

六位有效数字

误差上限？

Open Question is only supported on Version 2.0 or newer.

Answer

四、多元函数的误差估计

可用来分析观测误差和舍入误差对下一步结果的影响

$A = f(x_1, x_2, \cdots, x_n)$：准确值x

$A^* = f(x_1^*, x_2^*, \cdots, x_n^*)$：近似值x*

$\Delta A = A - A^*$

$$\approx \left(\frac{\partial f}{\partial x_1}\right)\Bigg|_{x_1^*}(x_1 - x_1^*) + \cdots + \left(\frac{\partial f}{\partial x_n}\right)\Bigg|_{x_n^*}(x_n - x_n^*)$$

近似有：

$$|\Delta A| \le \left|\left(\frac{\partial f}{\partial x_1}\right)^*\right||\Delta x_1| + \left|\left(\frac{\partial f}{\partial x_2}\right)^*\right||\Delta x_2| + \ldots + \left|\left(\frac{\partial f}{\partial x_n}\right)^*\right||\Delta x_n|$$

以上不等式是近似的；如果要精确分析误差上限，则有

$$|\Delta A| \le max\left|\left(\frac{\partial f}{\partial x_1}\right)\right||\Delta x_1| + max\left|\left(\frac{\partial f}{\partial x_2}\right)\right||\Delta x_2| + \ldots + max\left|\left(\frac{\partial f}{\partial x_n}\right)\right||\Delta x_n|$$

思考：如何推导上述不等式？为什么？

- 例1：比较$\sqrt{2}-1$和$1/(\sqrt{2}+1)$的误差，其中$\sqrt{2}$取1.414
- 例2：取$\sqrt{3}$ =1.732，试分析f($\sqrt{3}$)的误差上限。当$f(x)=x^2$时如何？
- 例3：分析$L_1(x)=\frac{(x-x_1)}{(x_0-x_1)}y_0+\frac{(x-x_0)}{(x_1-x_0)}y_1$中观测误差的影响。
- 例4：分析$y_{n+1}=\frac{1}{2}y_n+\frac{1}{n+1}$中观测误差的累积结果

Open Question Points: 10 Setting

例1：比较$\sqrt{2}-1$和$1/(\sqrt{2}+1)$的误差，其中$\sqrt{2}$取1.414。

Open Question is only supported on Version 2.0 or newer.

Answer

五、数值计算的基本原则

1. 选择数值稳定性好的计算公式

$$I_n=1-nI_{n-1}(|\Delta I_n|=n!\,|\Delta I_0|)$$

$$I_n=\frac{1-I_{n-1}}{n}(|\Delta I_n|=\frac{1}{n!}|\Delta I_0|)$$

2. 防止被除数远大于除数

$$\begin{cases}0.00001x_1+x_2=1\\ 2x_1+x_2=2\end{cases}$$

（当先求取x_2，再待人第一式中求取x_1时）

3. 防止相近的数相减

$$\begin{cases}1-\cos 1^o=1-0.9998=0.0002\\ 2\sin^2 0.5^o=1.523\text{^}10^{-4}\end{cases}$$

4. 防止大数吃掉小数

12345+(0.1+0.2+0.3+…)

-- 先加小括号内小数

5. 简化计算步骤

$$\begin{aligned}p_3(x)&=a_3x^3+a_2x^2+a_1x+a_0\\&=((a_3x+a_2)x+a_1)x+a_0\end{aligned}$$