第4章.

9-(1) $\forall x(G(x,y)\to\exists y F(x,y))$

$\forall x(x<-1\to\exists y\ x=y)$ 假命题.

9-(3) $\exists x G(x,y)\to\forall y F(f(x,y),a)$

$\exists x\ x<-1\to\forall y\ x-y=0$ 假命题.

12-(1) $F(x)\to\forall x F(x)$

解释I_1: $D_1=N^+$ $\bar{F}(x): x>0$, 真命题.

解释I_2: $D_2=R$ $\bar{F}(x): x>0$, 假命题

故为非永真式的可满足式.

12-(2) $\exists x F(x)\to F(x)$

解释I_1: $D_1=R$, $\bar{F}(x): x>0$, 赋值 $\sigma(x)=1$, 真命题

解释I_2: $D_1=R$, $\bar{F}(x): x>0$, 赋值 $\sigma(x)=0$, 假命题

故为非永真式的可满足式.

12-(3): $\forall x(F(x)\to G(x))\to(\forall x F(x)\to\forall x G(x))$

若 $\forall x(F(x)\to G(x))$, 即 $\forall x$, 若 $F(x)$ 为真, 则 $G(x)$ 为真

若 $\forall x F(x)$ 为真, 则 $\forall x G(x)$ 为真. 故命题永真..

12-(4): $(\forall x F(x)\to\forall x G(x))\to\forall x(F(x)\to G(x))$

解释I_1: $D=N$, $\bar{F}(x): x>0$, $\bar{G}(x)$ $x<0$ 真命题

解释I_2: $D=N^+$, $\bar{F}(x): x>0$, $\bar{G}(x)$ $x<0$ 假命题

故为非永真式的可满足式.

第五章：

2.

(1) $\forall x \exists y (F(x) \wedge G(y))$

$\Leftrightarrow \exists y (F(a) \wedge G(y)) \wedge (F(b) \wedge G(y)) \wedge (F(c) \wedge G(y)) \Leftrightarrow \exists y\ F(a) \wedge F(b) \wedge F(c) \wedge G(y)$

$\Leftrightarrow (F(a) \wedge F(b) \wedge F(c) \wedge G(a)) \vee (F(a) \wedge F(b) \wedge F(c) \wedge G(b)) \vee (F(a) \wedge F(b) \wedge F(c) \wedge G(c))$

$\Leftrightarrow F(a) \wedge F(b) \wedge F(c) \wedge (G(a) \vee G(b) \vee G(c))$

(3) $\forall(x) F(x) \rightarrow \forall y G(y)$

$(F(a) \wedge F(b) \wedge F(c)) \longrightarrow (G(a) \wedge G(b) \wedge G(c))$

3.

(1) $\forall x (F(x) \rightarrow G(x)) \Leftrightarrow \forall x (\neg F(x) \vee G(x))$

(2) $\exists x (F(x) \wedge G(x)) \Leftrightarrow \exists x (F(x) \wedge G(x))$

只需 $\forall x(F(x) \wedge G(x))$ 则 (1) (2) 均为真

只需 $\forall x (F(x) \wedge \neg G(x))$ 则 (1) (2) 均为假.

故 I_1: $D=N$, $F(x): x \in R$, $G(x): x \geq 0$

I_2: $D=N$, $F(x): x \in R$, $G(x): x < 0$

8.—(1)

$F(x)$: x 为正数 $G(x)$: x 小于负数.

$\forall x (F(x) \rightarrow \neg G(x))$

$\Leftrightarrow \forall x (\neg F(x) \vee \neg G(x))$ 蕴含等值式

$\Leftrightarrow \forall x \neg (F(x) \wedge G(x))$ 德摩根率.

$\Leftrightarrow \neg \exists x (F(x) \wedge G(x))$ 量词否定等值式.

9.

$\exists x F(x) \wedge \exists x G(x) \Leftrightarrow \exists x (F(x) \wedge G(x))$ 错误.

存在量词对 $\wedge$ 无分配律.