P153 8.

有实根：$\Delta=16k^2-16(k+2)$

$=16k^2-16k-32>0$

$\therefore \quad (k-2)(k+1)>0$

又$\because$ k 服从 $(0,5)$ 上均匀分布

$\therefore$ 概率为 $\frac{3}{5}$

P153 10.

$P(X=k)=\frac{1}{5}e^{-\frac{1}{5}k}$

而 $P(x>10)=e^{-2}$

$\therefore$ 每次离开概率为 e^{-2}

Y:	0	1	2	3	4	5
$P(Y)$	$(1-e^{-2})^5$	$C_5^1(e^{-2})^1(1-e^{-2})^4$	$C_5^2(e^{-2})^2(1-e^{-2})^3$	$C_5^3(e^{-2})^3(1-e^{-2})^2$	$C_5^4(e^{-2})^4(1-e^{-2})$	$(e^{-2})^5$

期望：$5e^{-2}$

$P(Y\geq 1)=1-P(Y=0)$

$=1-(1-e^{-2})^5=0.5167$

P153 11.

$\because$ 正态分布均值为2

$\therefore P(x<0)=P(x>4)=0.2$

P154 14. $E|x-\mu|$

$=\int_{-\infty}^{\mu}(x-\mu)\,p(x)+(\mu-x)\int_{\mu}^{+\infty}p(x)$

$=\int_{-\infty}^{\mu}x\,p(x)-\int_{\mu}^{+\infty}x\,p(x)$

$=\int_{-\infty}^{\mu}x\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(x-\mu)^2}{2\sigma^2}}dx-\int_{\mu}^{+\infty}x\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(x-\mu)^2}{2\sigma^2}}dx$

$=\int_{-\infty}^{0}(x+\mu)\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{x^2}{2\sigma^2}}dx-\int_{0}^{+\infty}(x+\mu)\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{x^2}{2\sigma^2}}dx$

$=\frac{2}{\sqrt{2\pi}\sigma}\int_{-\infty}^{0}x\,e^{-\frac{x^2}{2\sigma^2}}dx$

$=\sigma\sqrt{\frac{2}{\pi}}$

P154 15.

$$h(a) = \int_{-\infty}^{a} (a-x) f(x) + \int_{a}^{+\infty} (x-a) f(x)$$

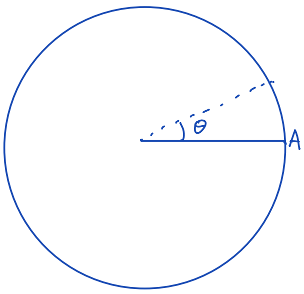
$$h'(a) = af(a) - \int_{-\infty}^{a} f(x) - af(a) + af(a) + \int_{a}^{+\infty} f(x) - af(a)$$

$$= -\int_{a}^{+\infty} f(x) + \int_{-\infty}^{a} f(x) = 2F(a)-1$$

$$h''(a) = 2f(a) > 0$$

$\therefore$ $F(a) = \frac{1}{2}$ 时 $h(a)$ 达到最小

P154. 16.



对与A夹角为θ的点(由对称,只算 $0 \le \theta \le \pi$)

弦长: $2R\cos\frac{\theta}{2}$

密度: $\frac{d\theta}{\pi}$

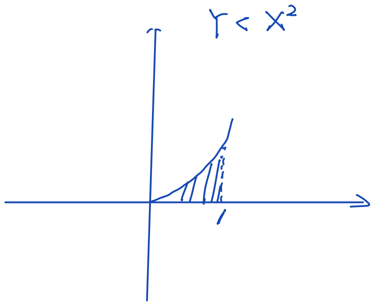
$$\therefore \int_0^{\pi} 2R\cos\frac{\theta}{2} \cdot \frac{1}{\pi} d\theta$$

$$= \frac{4R}{\pi} \int_0^{\frac{\pi}{2}} \cos\theta \, d\theta = \frac{4R}{\pi}$$

P154 22.

有实根: $4X^2 - 4Y > 0$

$Y < X^2$



概率为 $\int_A f(x,y)\,dxdy$

$$= \int_0^1 2x \int_0^{x^2} e^{-y} dy \quad dx$$

$$= \int_0^1 2x\,(1-e^{-x^2})\,dx$$

$$= 1 + e^{-1} - 1 = e^{-1}$$

P154 23. 当 $x \ge 1$ 时

$$f(x) = \int_{\frac{1}{x}}^{x} \frac{1}{2x^2 y} dy$$

$$= \frac{1}{2x^2} \ln y \Big|_{\frac{1}{x}}^{x}$$

$$= \frac{1}{2x^2}\left(\ln x - \ln\frac{1}{x}\right) = \frac{\ln x}{x^2}$$

否则, $f(x) = 0$

$$\therefore f_X(x) = \begin{cases} \frac{1}{x^2}\ln x, & x \ge 1 \\ 0, & 否则 \end{cases}$$

同理,

$$f_Y(y)=\begin{cases}\frac{1}{2}, & 0<y<1\\ \frac{1}{2y^2}, & 1\le y<+\infty\\ 0, & 其他\end{cases}$$

∴不相互独立

P154 25.

(a) $EX=\int_{-\infty}^{+\infty} x\cdot\frac{1}{2}e^{-|x|}=0$

$$EX^2=\int_{-\infty}^{+\infty}x^2\frac{1}{2}e^{-|x|}dx$$

$$=\int_0^{+\infty}x^2e^{-x}dx$$

$$=-x^2e^{-x}\Big|_0^{+\infty}+\int_0^{+\infty}2x\,e^{-x}dx$$

$$=-2xe^{-x}\Big|_0^{+\infty}+\int_0^{+\infty}2\,e^{-x}dx=2$$

$\therefore DX=EX^2-(EX)^2=2$

(b) $EX=0$

$E|X|=\int_0^{+\infty}xe^{-x}=1$

$$E(X\cdot|X|)=\int_{-\infty}^{+\infty}k\cdot|k|\cdot\frac{1}{2}e^{-|k|}\,dk$$

$$=\frac{1}{2}\int_0^{+\infty}k^2e^{-k}dk-\frac{1}{2}\int_{-\infty}^{0}k^2e^{k}\,dk$$

$$=1-0=0$$

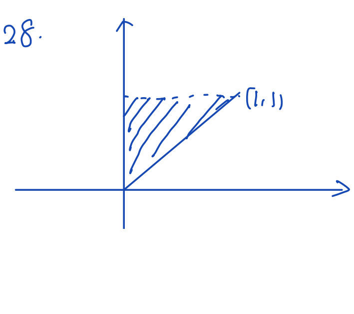
$\therefore Cov(X,|X|)=0$

∴不相关

(c) 由于当X取某值时，|X|唯一确定

∴不独立

P156 28.



$$P(X>\tfrac{1}{2}) = \int_{\frac{1}{2}}^{1} f_Y(y)\int_{\frac{1}{2}}^{y} f_{X|Y}(x|y)\,dx\,dy$$

$$= \int_{\frac{1}{2}}^{1}\left(1-\frac{1}{8y^3}\right)\cdot 5y^4\,dy$$

$$= \int_{\frac{1}{2}}^{1} 5y^4\,dy - \int_{\frac{1}{2}}^{1}\frac{5}{8}y\,dy$$

$$= 1-\frac{1}{2^5} - \frac{5}{16}\times\frac{3}{4}$$

$$= \frac{64-2-15}{64} = \frac{47}{64}$$

$$EX = \int^{1} f_Y(y)\int_0^{y} f_{X|Y}(x|y)\,x\cdot dx\,dy$$

$$= \int_0^1 5y^4 \int_0^y \frac{3x^3}{y^3}\,dx\,dy$$

$$= \int_0^1 \frac{15}{4}y^5\,dy$$

$$= \frac{5}{8}$$

$$f(XY) = 15x^2y$$

$$\therefore E(XY) = \int_D 15x^3Y^2\,dx\,dy$$

$$= \int_0^1 dy\int_0^y 15x^3y^2\,dx$$

$$= \int_0^1 \frac{15}{4}y^6\,dy$$

$$= \frac{15}{28}$$

P156. 30. $EX = \int_{-\pi}^{\pi}\sin\theta\,\frac{d\theta}{2\pi}$

$= 0$

$EY = \int_{-\pi}^{\pi}\cos\theta\,\frac{d\theta}{2\pi} = 0$

$EXY = \int_{-\pi}^{\pi}\sin\theta\cos\theta\,\frac{d\theta}{2\pi} = 0$

∴ 相关系数为0

但是两者不独立

P156 33.

$$E(X \mid a<X<b) = \frac{\int_a^b k \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(k-\mu)^2} dk}{P(a<X<b)}$$

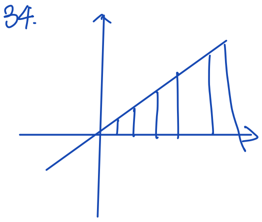
而 $$\int_a^b k\, e^{-\frac{1}{2\sigma^2}(k-\mu)^2} dk = \int_{a-\mu}^{b-\mu} (j+\mu) e^{-\frac{1}{2\sigma^2} j^2} dj$$

$$= \mu \int_{a-\mu}^{b-\mu} e^{-\frac{1}{2\sigma^2} j^2} dj + \frac{1}{2} \int_{a-\mu}^{b-\mu} e^{-\frac{1}{2\sigma^2} j^2} dj^2$$

$$= \mu \int_{a-\mu}^{b-\mu} e^{-\frac{1}{2\sigma^2} j^2} dj + \sigma^2 \left[e^{-\frac{1}{2\sigma^2}(b-\mu)^2} - e^{-\frac{1}{2\sigma^2}(a-\mu)^2} \right.$$

$$\therefore 原式 = \mu + \frac{\sigma\left[e^{-\frac{(a-\mu)^2}{2\sigma^2}} - e^{-\frac{(b-\mu)^2}{2\sigma^2}}\right]}{\sqrt{2\pi}\left[\Phi\left(\frac{b-\mu}{\sigma}\right) - \Phi\left(\frac{a-\mu}{\sigma}\right)\right]}$$

P156 34.



首先 $$\int_D \lambda^2 e^{-\lambda x} dxdy$$

$$= \int_0^{+\infty} dx \int_0^x \lambda^2 e^{-\lambda x} dy$$

$$= \lambda^2 \int_0^{+\infty} x e^{-\lambda x} dx$$

$$= -\lambda \left[x e^{-\lambda x} \Big|_0^{+\infty} - \int_0^{+\infty} e^{-\lambda x} dx \right]$$

$$= \lambda \int_0^{+\infty} e^{-\lambda x} dx = 1$$ 符合题意

则 $$E(X \mid Y=y) = \frac{\int_y^{+\infty} x \cdot \lambda^2 e^{-\lambda x} dx}{P(Y=y)}$$

先求y的边缘分布 $$F_Y(y) = \int_y^{+\infty} \lambda^2 e^{-\lambda x} dx$$

$$= -\lambda(0 - e^{-\lambda y})$$

$$= \lambda e^{-\lambda y}$$

$$\therefore 原式 = \frac{\lambda \int_y^{+\infty} x e^{-\lambda x} dx}{e^{-\lambda y}} = y + \frac{1}{\lambda}$$

类似地，$D(X \mid Y=y) = E(X^2 \mid Y=y) - [E(X \mid Y=y)]^2 = \frac{1}{\lambda^2}$