University of Belgrade
School of Electrical Engineering

PROJECT TASK IN HYDRAULIC AND PNEUMATIC SYSTEMS

Mentor:
prof. dr. Kosta Jovanović

Student:
Cvetković Dositej 2020/0224

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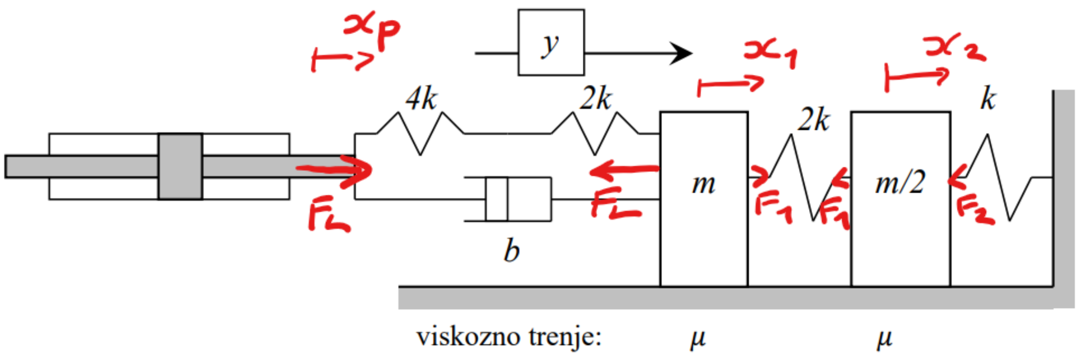
Садржај

1 Project task



Слика 1: Picture of the problem with forces acting

1.1 Model of the system in state space

Differential equations that describe our actuator:

$$A_p P_L = M_t \ddot{x_p} + B_p \dot{x_p} + F_L \tag{1}$$

$$Q_L = A_p \dot{x_p} + C_{tp} P_L + \frac{V_t}{4\beta_e} \dot{P}_L \tag{2}$$

$$Q_L = K_q x_v - K_c P_L \tag{3}$$

$$F_i = K_f x_v \tag{4}$$

$$K_t i = T_L \tag{5}$$

$$u = Ri \tag{6}$$

$$\Theta = \frac{x_v}{r} \tag{7}$$

$$T_L = r F_i \tag{8}$$

Now we are analyzing the mechanical system.

Forces equations:

$$F_L = \frac{4}{3} k(x_1 - x_p) + b(\dot{x_1} - \dot{x_p})$$

$$F_1 = 2k(x_2 - x_1)$$
$$F_2 = kx_2$$
$$F_{tr1} = \mu\dot{x_1}$$
$$F_{tr2} = \mu\dot{x_2}$$

Translation equation for the first body:

$$m\ddot{x_1} = -F_L - F_{tr1} + F_1$$

Translation equation for the second body:

$$\frac{1}{2}m\ddot{x_2} = -F_1 - F_2 - F_{tr2}$$

We adopt state coordinates in the next way: $(z_1, z_2, z_3, z_4, z_5, z_6, z_7) = (x_p, \dot{x_p}, P_L, x_1, \dot{x_1}, x_2, \dot{x_2})$ solving the system of equations above, we obtain that the state coordinates are:

$$\begin{aligned}
\dot{z_1} &= \dot{x_p} = z_2 \\
\dot{z_2} &= -\frac{4k}{3M_t}z_1 - \frac{B_p + b}{M_t}z_2 + \frac{A_p}{M_t}z_3 + \frac{4k}{3M_t}z_4 + \frac{b}{M_t}z_5 \\
\dot{z_3} = \dot{P_L} &= -\frac{4\beta_e}{V_t}\left(K_c + C_{tp}\right)z_3 - \frac{4\beta_e}{V_t}A_p z_2 + \frac{4\beta_e}{V_t}K_q\frac{K_t u}{rRK_f} \\
\dot{z_4} &= \dot{x_1} = z_5 \\
\dot{z_5} = \ddot{x_1} &= \frac{4k}{3m}z_1 + \frac{b}{m}z_2 - \frac{10k}{3m}z_4 - \frac{b+\mu}{m}z_5 + \frac{2k}{m}z_6 \\
\dot{z_6} &= \dot{x_2} = z_7 \\
\dot{z_7} = \ddot{x_2} &= \frac{4k}{m}z_4 - \frac{6k}{m}z_6 - \frac{2\mu}{m}z_7
\end{aligned}$$

1.2 Trapezoidal speed profile

The speed profile consists of three parts: the part when the speed is increasing, the part when the speed is constant, and the part when the speed is decreasing. The traveled distances of these parts are marked in order with L_1, L_2 и L_3, and the total traveled distance with L. Then:

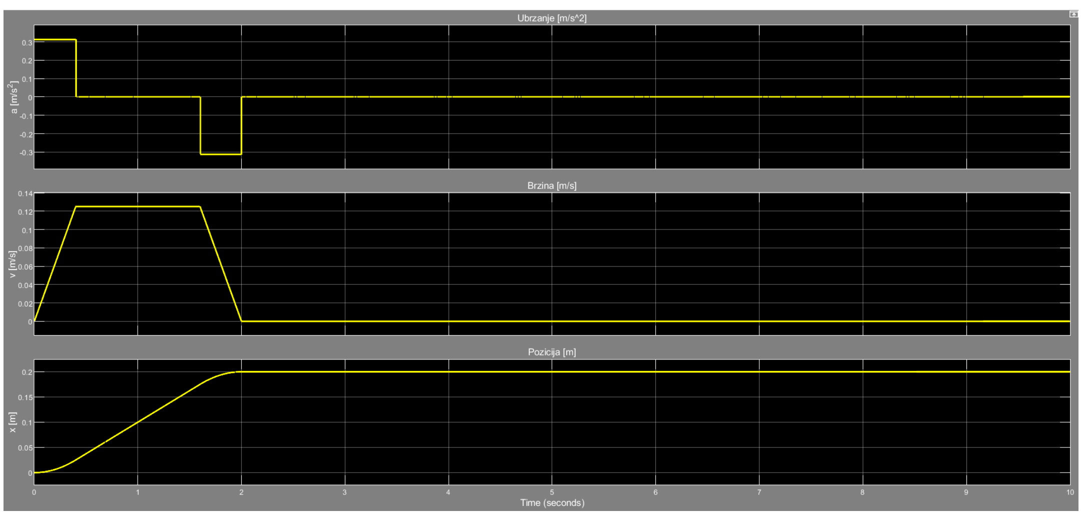
$$L = L_1 + L_2 + L_3$$

$$L = \frac{1}{2}a_{max}\left(0.2T\right)^2 + \left(a_{max}\cdot 0.2T\right)0.6T + \frac{1}{2}a_{max}\left(0.2T\right)^2$$

$$L = 0.16 \cdot a_{max} \cdot T^2$$

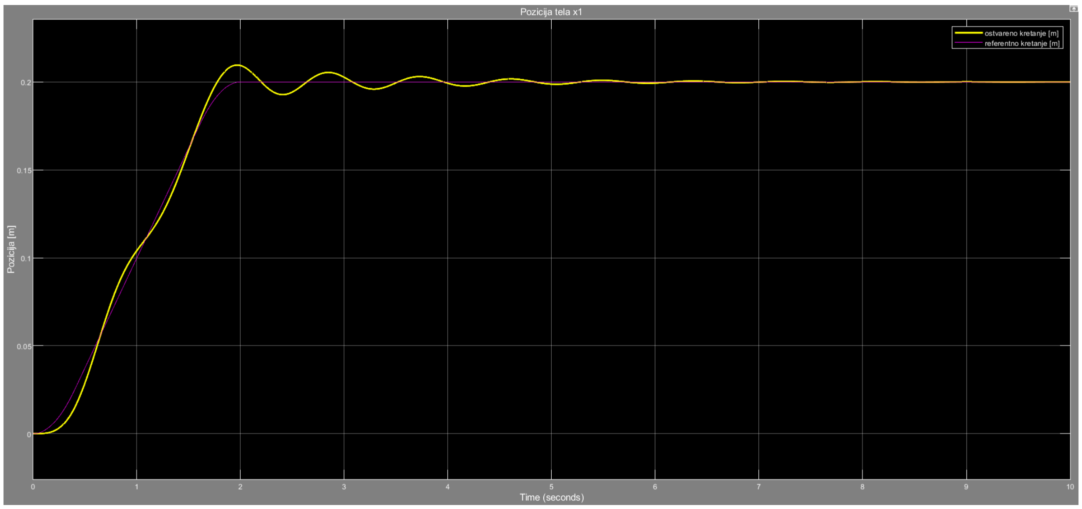
$$\Rightarrow a_{max} = 0.3125\frac{m}{s^2}$$

$$\Rightarrow v_{max} = a_{max} \cdot 0.2T = 0.125\frac{m}{s}$$

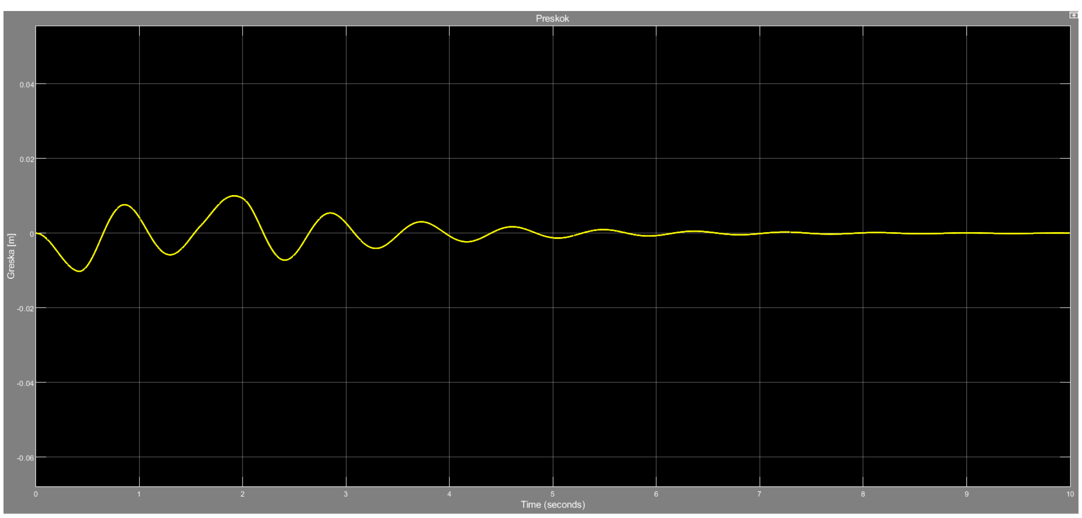


Слика 2: Trapezoidal speed profile

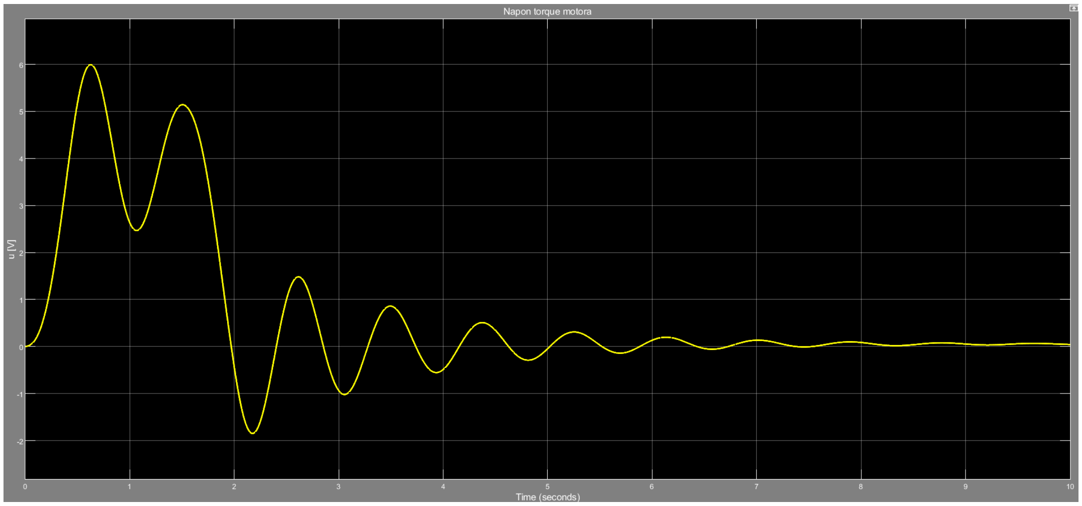
1.3 Graphs from Simulink



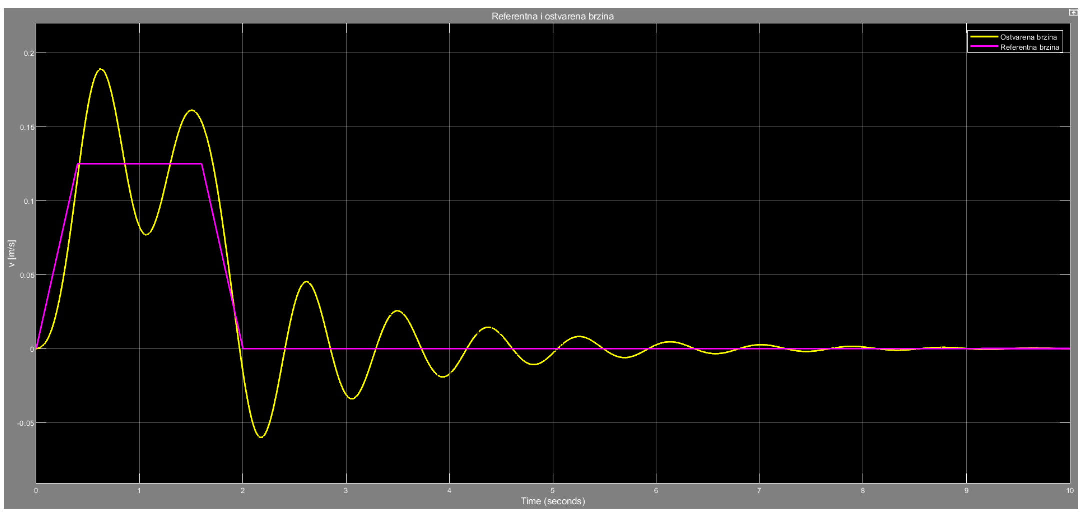
Слика 3: Achieved and reference movement



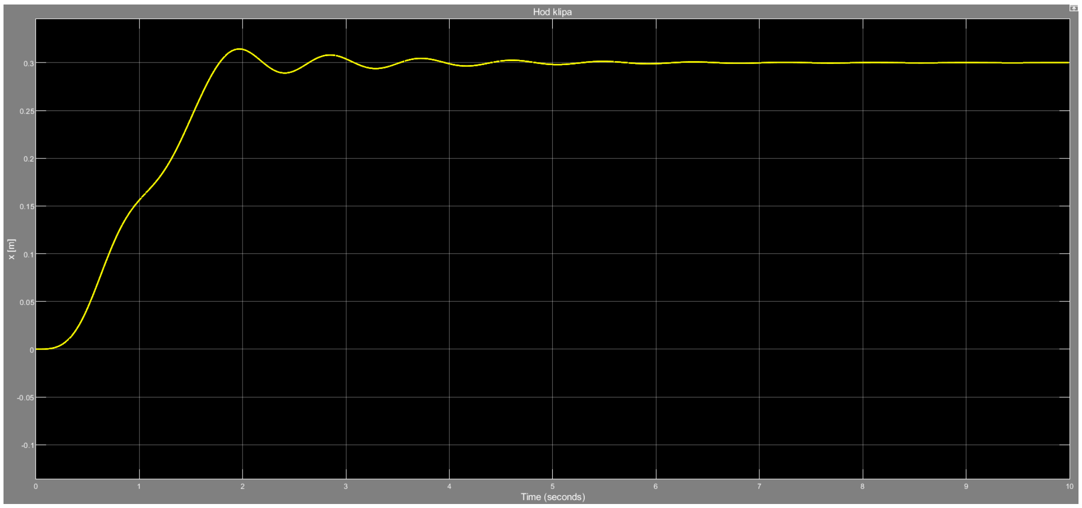
Слика 4: Overshoot



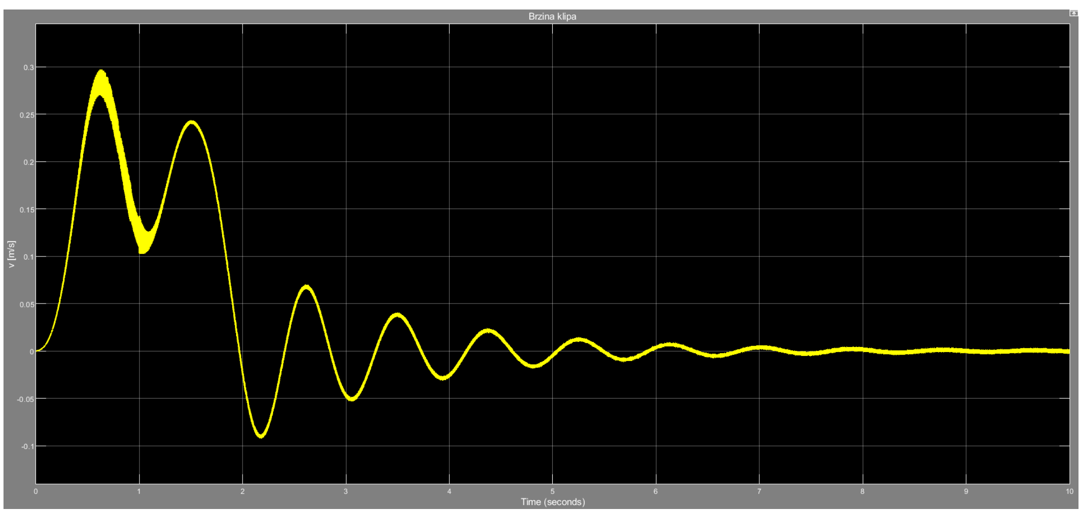
Слика 5: Overall control



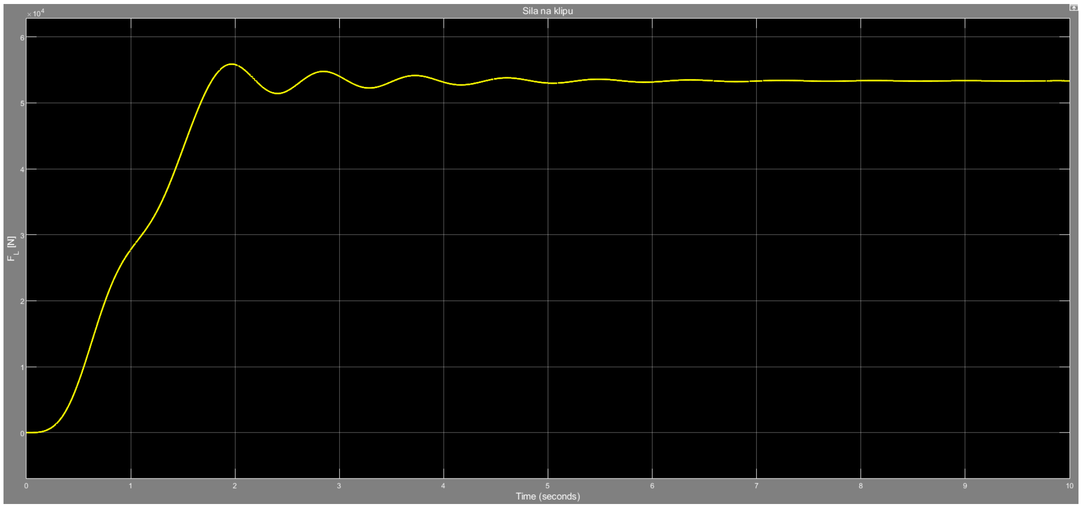
Слика 6: Speeds



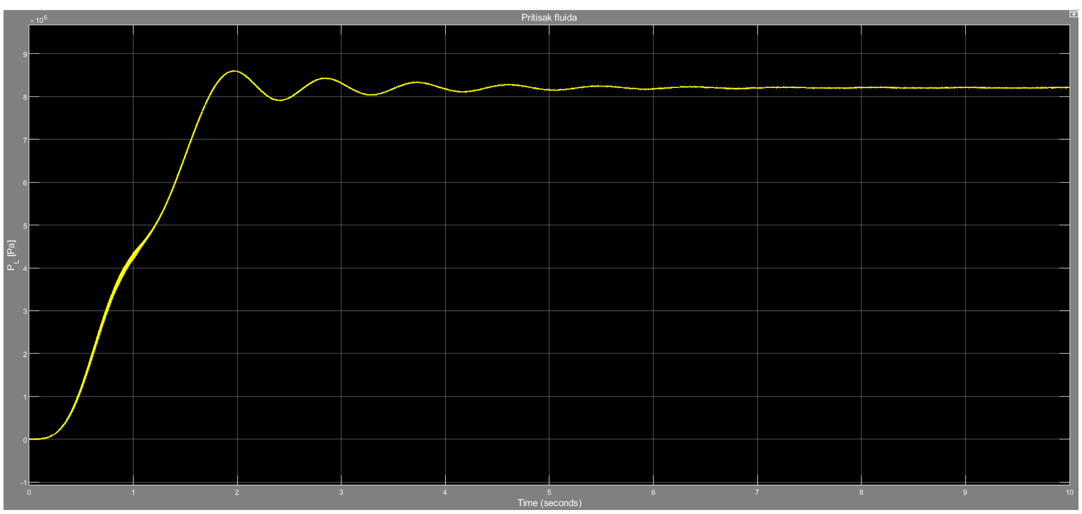
Слика 7: Piston stroke



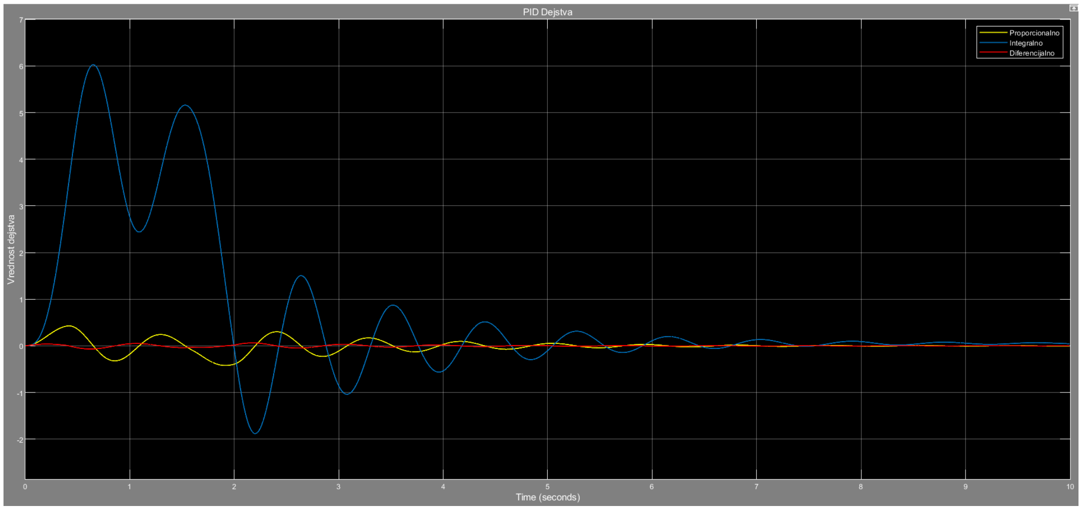
Слика 8: Piston speed



Слика 9: Force on the piston



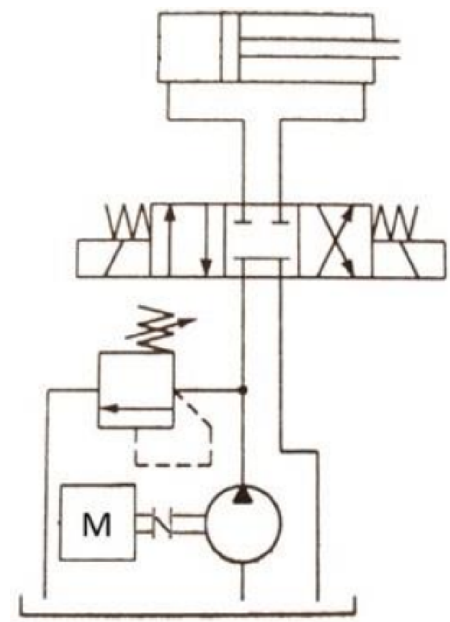
Слика 10: Fluid pressure



Слика 11: PID actions

1.4 Parameters of the hydraulic scheme

The type of hydraulic system is a double-acting cylinder with a passing rod. Read from the simulation, the maximum force acting on the piston is $Fmax = 56kN$, the stroke of the rod $x = 0.315m$, and the maximum speed of the rod $vmax = 0.3m/s$. We design the hydraulic system for an operating pressure of $p = 160bar$.



Слика 12: Hydraulics scheme

To ensure that the rod does not hit the walls at maximum extension, we adopt $l = 0.4m$. Based on Euler's formula for twisting an axially pressed rod, we determine the diameter of the rod:

$$d = \sqrt[4]{64\frac{F_{max}l^2}{\pi^3 E}} = 17.23mm$$

where E is Young's modulus of elasticity and amounts to $2.1 * 10^5 N/mm^2$, and for safety we adopt $d = 20mm$.

The minimum value of the piston diameter is determined from:

$$A_p = \frac{F_{max}}{p\eta_f} = \frac{D^2\pi}{4} = 38.89cm^2$$

where $\eta_f = 0.9$ is the mechanical efficiency coefficient. For D, we get $70.36mm$, and we adopt $D = 75mm$.

We obtain the working volume from:

$$V = \frac{Q}{n\eta_{vol}} = \frac{\frac{D^2\pi}{4}v_{max}}{n\eta_{vol}} = 53cm^3$$

where $\eta_{vol} = 0.75$ is the volumetric efficiency rate, and $n = 2000\frac{obr}{min}$ is the pump rotation speed.

Power of the mechanical aggregate:

$$P = \frac{pQ}{\eta_{vol}\eta_m} = 35.5kW$$

where $\eta_m = 0.8$ is the mechanical efficiency of the motor.