PROJEKTNI ZADATAK IZ DINAMIKE MEHANIČKIH SISTEMA, Elektrotehnički fakultet u Beogradu, školska 2022/23 godina

Cvetković Dositej 2020/0224

-Izvođenja

Θ-otklon kuglice, R-poluprečnik obruča, m-masa kuglice,

Ω - ugaona brzina rotacije

Kinetička energija $T= ½ * m * R * \dot{\Theta}^2 + ½ * m * R^2 * \sin^2\Theta * \Omega^2$

Potencijalna energija $U = m*g*R*(1 - \cos\Theta)$

Lagranžijan $L = T - U$

Jednačina kretanja -> $\partial L/\partial\Theta – d/dt(\partial L/\partial\dot{\Theta})$

Dobijena jednačina kretanja $R*\ddot{\Theta} = \sin\Theta*(R*\Omega^2*\cos\Theta-g)$

Prelazimo na sferni koordinatni sistem (r, ϕ, Θ)

$T = ½*m*(\dot{r}^2 + r^2 * \dot{\Theta}^2 + r^2 * \dot{\phi}^2 * \sin^2\Theta)$

$U = m*g*r*(1 - \cos\Theta)$

$L = T – U$

Funkcije ograničenja $f1(r) = r - R = 0$; $f2(\phi) = \phi - \Omega*t = 0$

Jednačine kretanja:

1) $\partial L/\partial r + \lambda1*\partial f1/\partial r + \lambda2*\partial f2/\partial r – d/dt(\partial L/\partial\dot{r})$
2) $\partial L/\partial\phi + \lambda1*\partial f1/\partial\phi + \lambda2*\partial f2/\partial\phi – d/dt(\partial L/\partial\dot{\phi})$
3) $\partial L/\partial\Theta + \lambda1*\partial f1/\partial\Theta + \lambda2*\partial f2/\partial\Theta – d/dt(\partial L/\partial\dot{\Theta})$

1) $\lambda1 = -m*R*\dot{\Theta}^2 – m * R * \Omega^2 * \sin^2\Theta – m*g*\cos\Theta + m*g$
2) $\lambda2 = 2*m*R^2*\Omega*\sin\Theta*\cos\Theta$
3) $R*\ddot{\Theta} = \sin\Theta*(R*\Omega^2*\cos\Theta-g)$

Sile ograničenja:

$Fcon1 = \lambda1*\partial f1/\partial r = \lambda1 = -m*R*\dot{\Theta}^2 – m*R*\Omega^2*\sin^2\Theta – m*g*\cos\Theta + m*g$

$Fcon2 = \lambda2*\partial f2/\partial\phi = \lambda2 = 2*m*R^2*\Omega*\sin\Theta*\cos\Theta$

Ravnotežna stanja: kada $\dot{\Theta} = 0$ i $\ddot{\Theta} = 0$

$\sin\Theta eq*(R*\Omega^2*\cos\Theta eq-g) = 0$ -> $\sin\Theta eq=0 \ V \ \cos\Theta eq=g/(R*\Omega^2) <= 1 => \Omega >= (g/R)^{1/2}$

$\Theta eq = 0 \ V \ \Theta eq = \pi \ V \ \Theta eq = \arccos(g/(R*\Omega^2))$

I) $\Omega < (g/R)^{1/2}$ -> $\Theta eq = 0$ V $\Theta eq = \pi$

1) $\Theta eq = 0 \quad \Theta = \Theta eq + \delta \quad \dot{\Theta} = \dot{\delta} \quad \dot{\dot{\Theta}} = \dot{\dot{\delta}}$

   $R*\dot{\dot{\delta}} = \sin\delta *(R*\Omega^2*\cos\delta - g) => R*\dot{\dot{\delta}} = \delta*(R*\Omega^2 - g) => \dot{\dot{\delta}} + \delta*(g/R - \Omega^2) = 0$

   $\omega 0^2 = g/R - \Omega^2 > 0 \quad =>$ STABILNA RAVNOTEŽA

2) $\Theta eq = \pi \quad \Theta = \Theta eq + \delta \quad \dot{\Theta} = \dot{\delta} \quad \dot{\dot{\Theta}} = \dot{\dot{\delta}}$

   $R*\dot{\dot{\delta}} = \delta*(R*\Omega^2 + g) => \dot{\dot{\delta}} - \delta*(g/R + \Omega^2) = 0$

   $\omega 0^2 = -g/R - \Omega^2 < 0 \quad =>$ NESTABILNA RAVNOTEŽA

II) $\Omega > (g/R)^{1/2}$ -> $\Theta eq = 0$ V $\Theta eq = \pi$ V $\Theta eq = \arccos(g/(R*\Omega^2))$

1) Sada za $\Theta eq = 0 => \omega 0^2 = g/R - \Omega^2 < 0 =>$ NESTABILNA RAVNOTEŽA
2) Za $\Theta eq = \pi$ isto kao malopre NESTABILNA RAVNOTEŽA
3) Za $\Theta eq = \arccos(g/(R*\Omega^2)) \quad \Theta = \Theta eq + \delta \quad \dot{\Theta} = \dot{\delta} \quad \dot{\dot{\Theta}} = \dot{\dot{\delta}}$

   $R*\dot{\dot{\delta}} = \sin(\Theta eq + \delta) *(R*\Omega^2*\cos(\Theta eq + \delta) - g)$

   $R*\dot{\dot{\delta}} = (\sin\Theta eq*\cos\delta + \sin\delta*\cos\Theta eq) *(R*\Omega^2*(\cos\Theta eq*\cos\delta - \sin\Theta eq*\sin\delta) - g)$

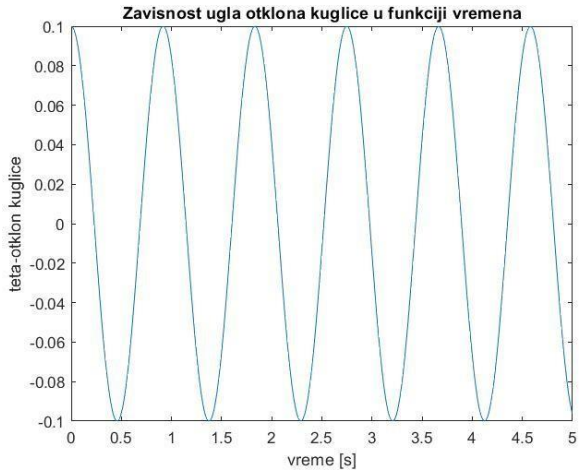
   $\sin^2\Theta eq = 1 - \cos^2\Theta eq = 1 - g^2/(R^2*\Omega^4) => \sin\Theta eq = 1/(R*\Omega^2) * (R^2*\Omega^4 - g^2)^{1/2}$

   $\dot{\dot{\delta}} + \Omega^2 * \sin^2\Theta eq * \delta = 0 \quad => \omega 0^2 = \Omega^2 * \sin^2\Theta eq > 0 =>$ STABILNA RAVNOTEŽA

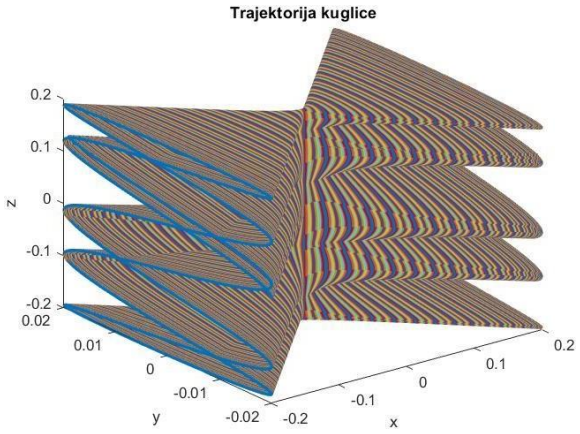
1) Ugaona brzina < (g/R)^(1/2)

   -male oscilacije kuglice oko stabilne ravnoteže teta = 0

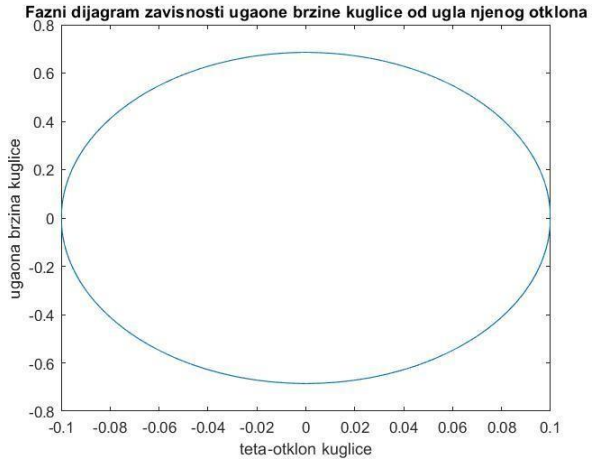
   vidimo sa grafika da su u pitanju oscilacije oko ravnotežnog položaja teta=0



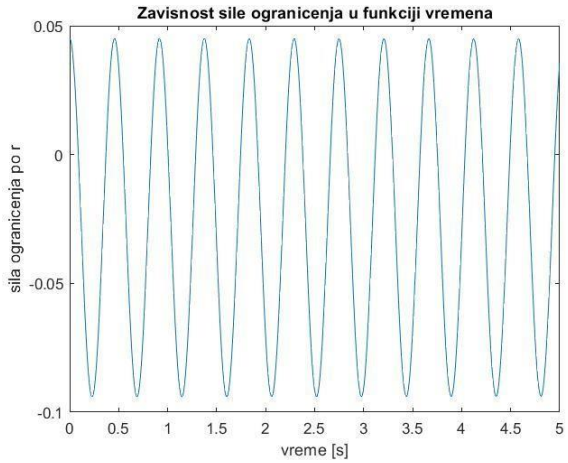
Na trajektorii vidimo da kuglica ponavlja svoju trajektoriju(oscilacije)



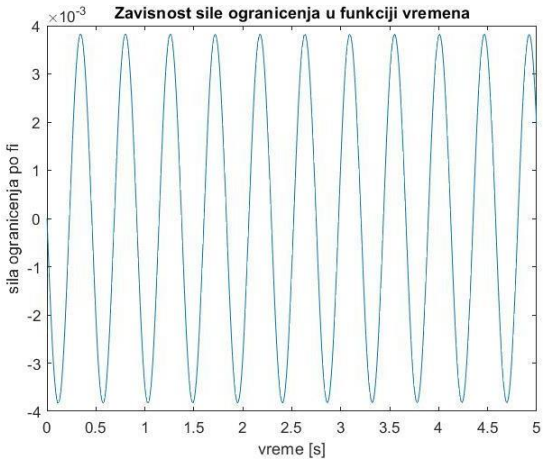
Sa faznog dijagrama vidimo da je amplituda ugaone brzine u ravnotežnom položaju teta=0 i da je ugaona brzina 0 pri amplitudama oscilacija u ovom slucaju +-0.1



Sila ograničenja od f(r) = r-R = 0

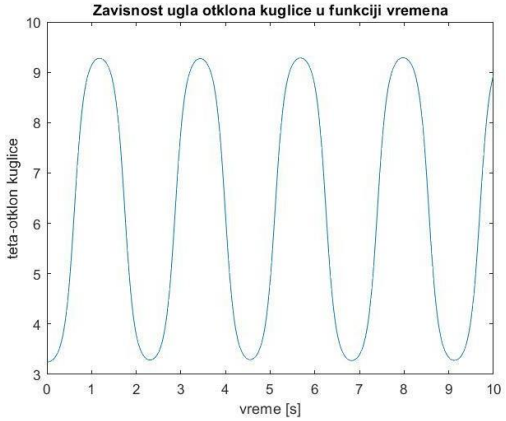


Sila ograničenja od f(ϕ) = ϕ - Ω*t = 0

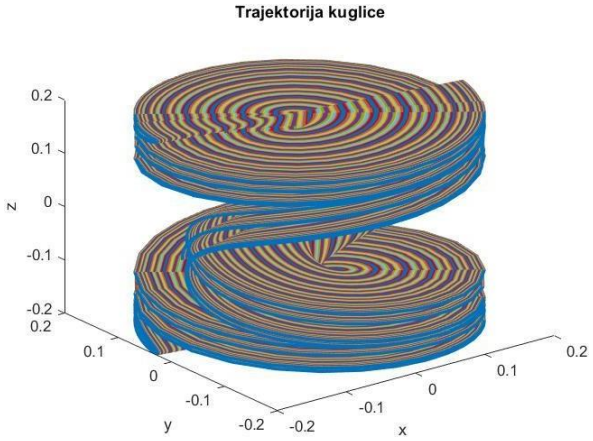


2) Ugaona brzina < (g/R)^(1/2)
-male oscilacije kuglice oko nestabilne ravnoteže teta = pi

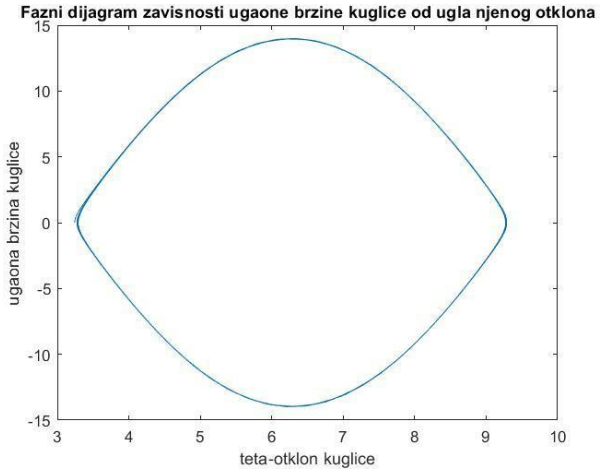
Sa grafika vidimo da kuglica sa gornje pozicije se spušta u smeru suprotnom kazaljke na satu dolazi do ravnotežnog položaja(teta=2pi) i počinje da oscilira



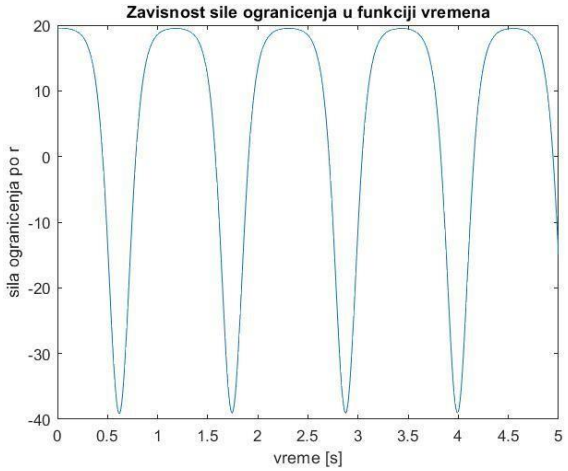
Na trajektoriji takođe vidimo pad i početak oscilacija



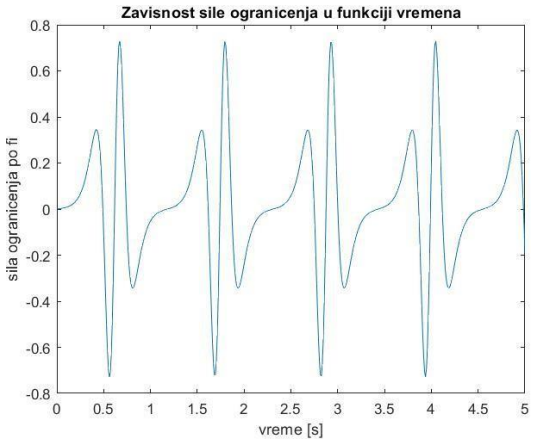
Na faznom diagramu vidimo poremećaj pri početku zbog pada do ravnotežnog položaja pa nastavak očekivanog dijagrama za oscilacije



Sila ograničenja f(r) = r-R = 0

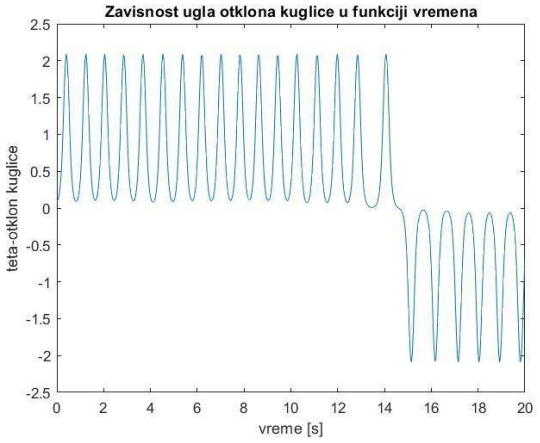


Sila ograničenja f(ϕ) = ϕ - Ω*t = 0

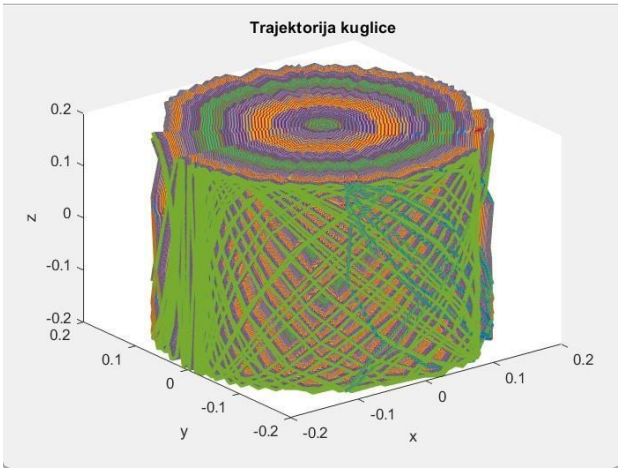


3) Ugaona brzina > (g/R)^(1/2)
-male oscilacije kuglice oko sada nestabilne ravnoteže teta = 0

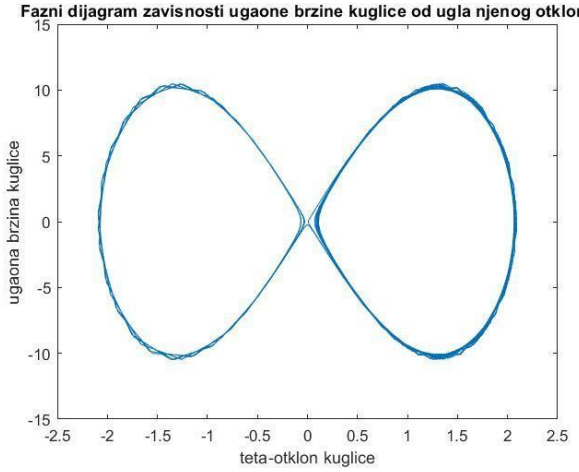
U ovom slučaju na grafiku vidimo da se oscilacije dešavaju do nekog trenutka oko neke tačke pa onda kuglica menja tačku oko koje osciluje



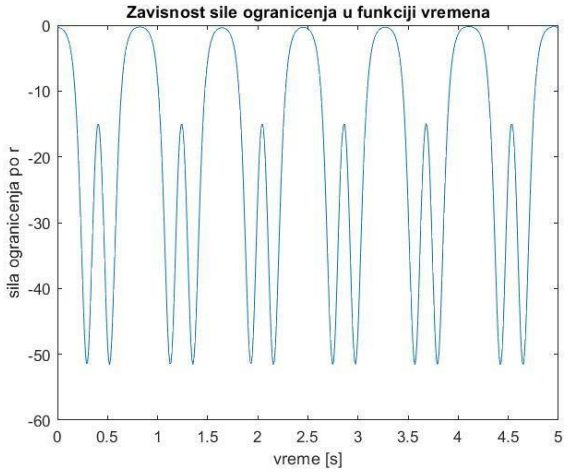
Trajektorija



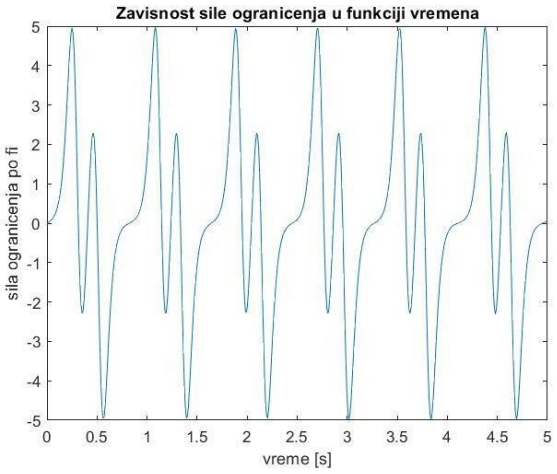
Na faznom diagramu vidimo dva kruga koja predstavljaju oscilacije oko dve različite tačke sa malim poremećajima (razlike u ugaonim brzinama u amplitudama oscilacija)



Sila ograničenja od f(r) = r-R = 0

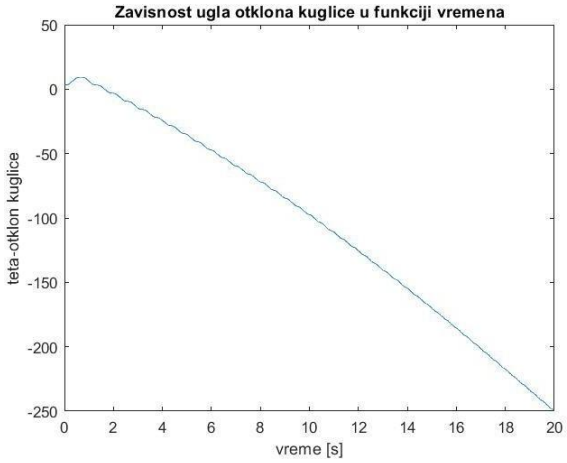


Sila ograničenja od f(ϕ) = ϕ - Ω*t = 0

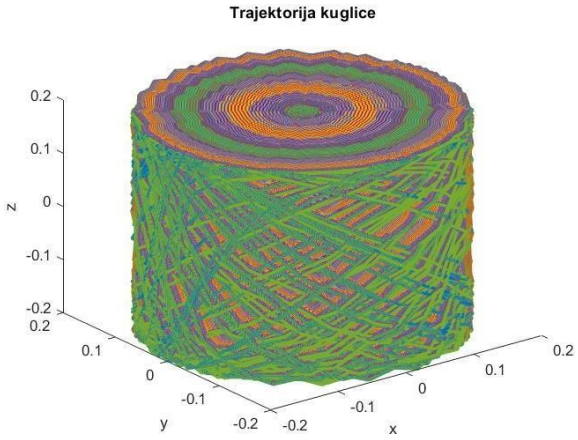


4) Ugaona brzina > (g/R)^(1/2)
-male oscilacije kuglice oko nestabilne ravnoteže teta=pi

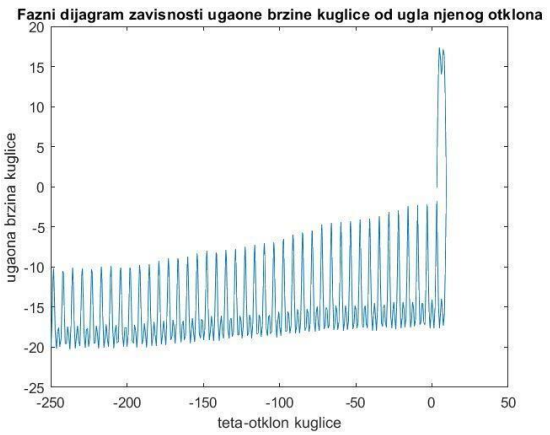
Sada na grafiku vidimo kako ugao na trenutak poraste pa krene konstantno sa poremećajima da opada tj. kuglica se okreće u smeru kazaljke na satu



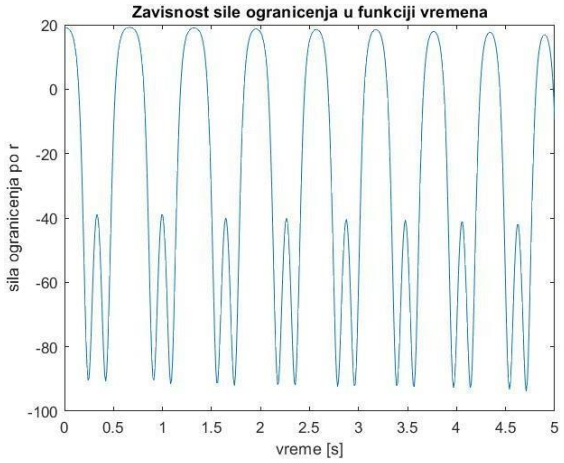
Trajektorija



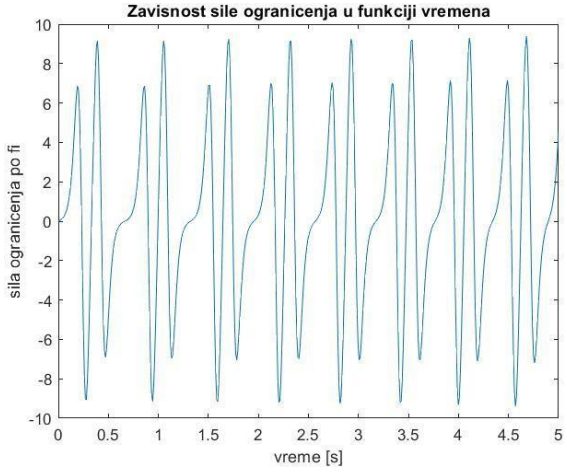
Ugaona brzina



Sila ograničenja od f(r) = r-R = 0

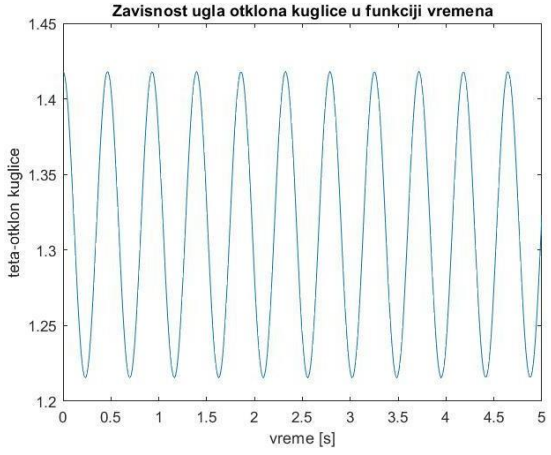


Sila ograničenja od f(ϕ) = ϕ - Ω*t = 0

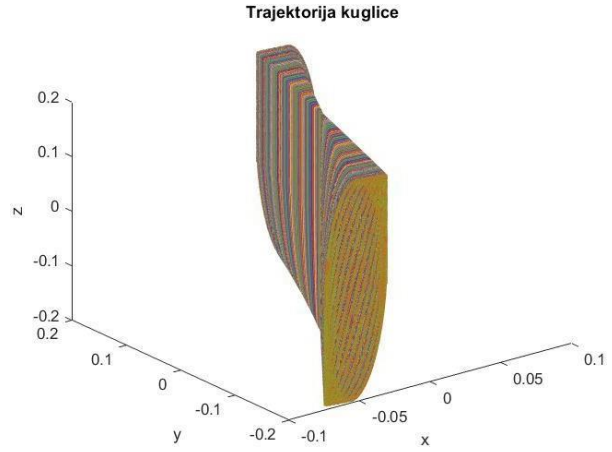


5) Ugaona brzina > (g/R)^(1/2)
-male oscilacije kuglice oko stabilne ravnoteže teta = arccos(g/(w^2*R))

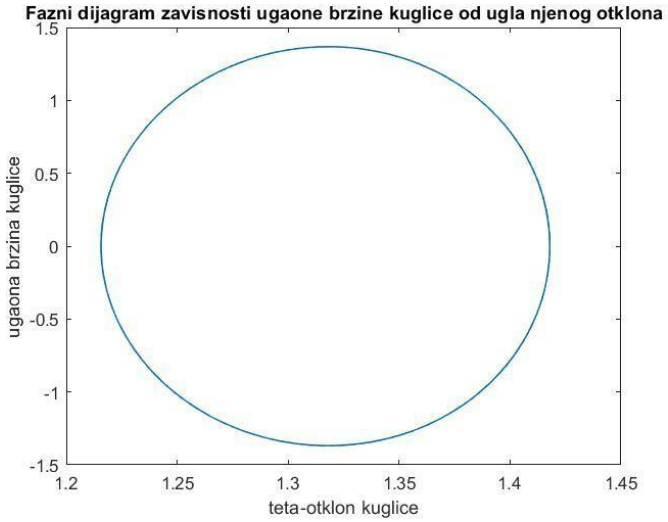
Vidimo oscilacije oko ravnotežnog položaja



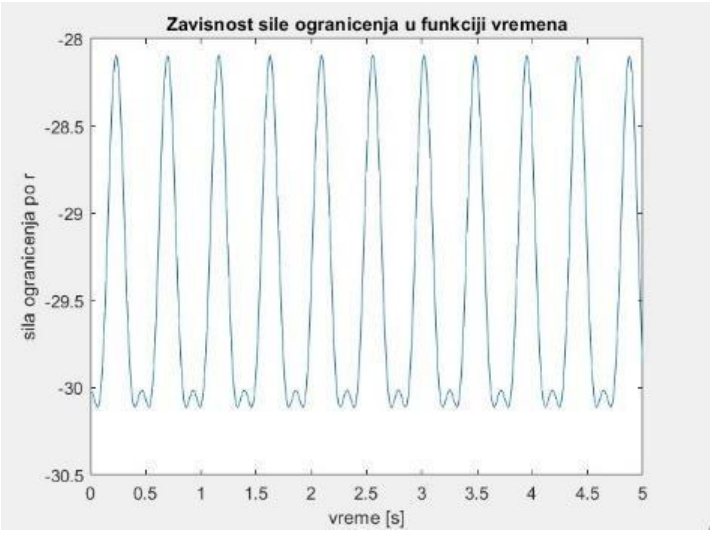
Trajektorija



Fazni diagram kakav očekujemo kod oscilacija



Sila ograničenja od f(r) = r-R = 0



Sila ograničenja od f(ϕ) = ϕ - Ω*t = 0