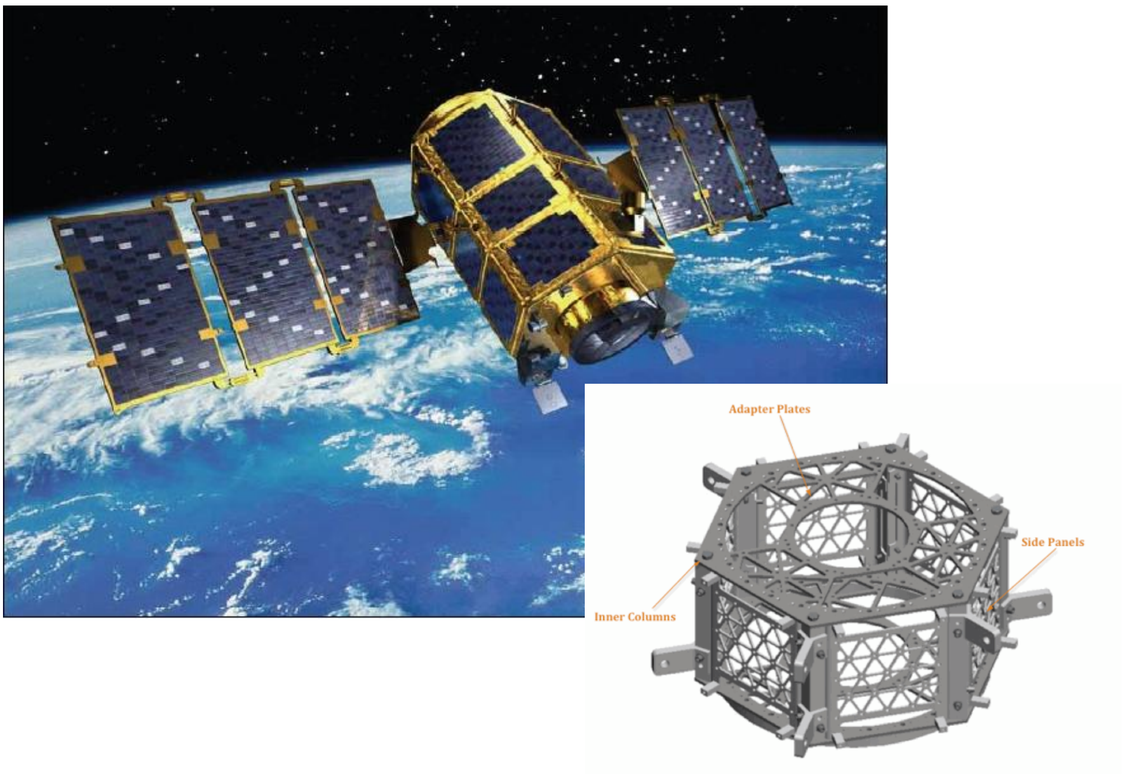
Spacecraft Design

Work Package 4:
A solar array or an RTG attachment



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Version: November 2021

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Project AE2111-I – System design

1 Introduction

Welcome to the second part of the project! This part contains the fourth and fifth work package of the AE2111-I Systems Design project. Whereas the previous work packages were concerned with the overall spacecraft design, and the *conceptual* design of some of its subsystems, work packages 4 and 5 are concerned with several detailed steps necessary to analyse elements of the spacecraft structure. The current work package, number 4, is concerned with the attachment of a solar panel or RTG (Radioisotope Thermoelectric Generator). In work package 5 the focus will be on further details of the fuel tank and aspects of dynamic coupling between various sub-structures of the spacecraft. As for the WP1-3 reader, this document contains the descriptions of project deliverables and hints to help you in your work. Several appendices are also attached, which aim to guide you in the execution of the work for getting to the deliverables.

For project rules, information on the project rules and regulations, you are referred to the general project guide, posted on Brightspace.

In case of questions beyond the knowledge of the teaching assistant, please contact Dr. Roeland de Breuker: R.debreuker@tudelft.nl

2 Activities to be performed

Roughly speaking, the activities that need to be done in WP4 can be divided into two categories. The first class of activities involves a series of steps for the detailed structural design of an attachment lug (**red arrows** in figure 2.1 below). The second class of activities entails an iteration of the design (**orange arrows**), and performing a trade-off based on the numerical outcomes and other qualitative factors you identify to be important.

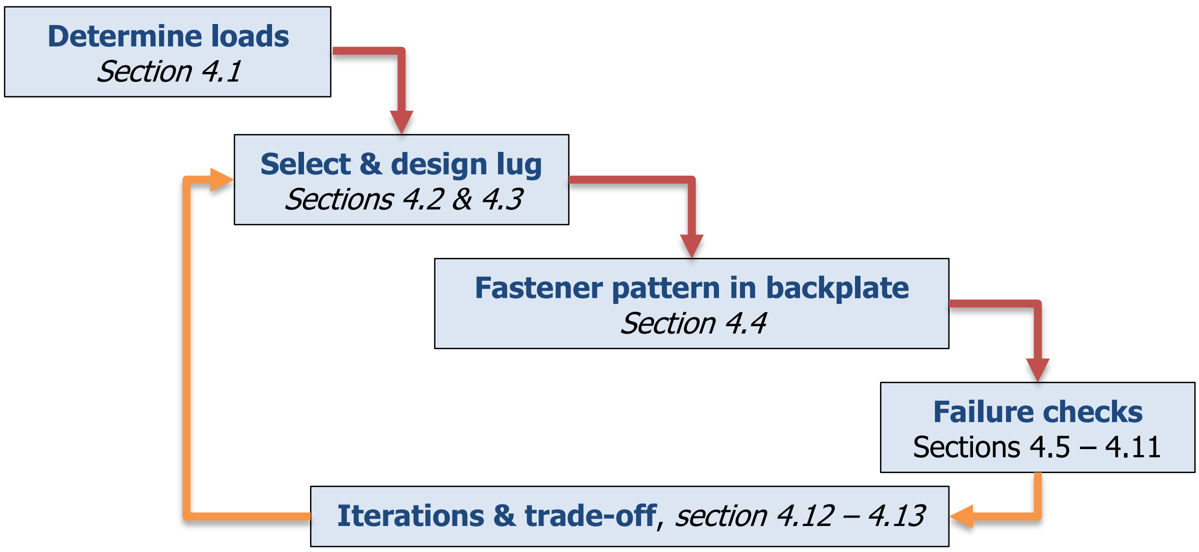

Figure 2.1: Diagram of activities to be performed in this work package

So, to come to a satisfying solution for the design, you will arrive at some form of iteration and a design trade-off. Make sure you anticipate for this during the initial phases of the 'design' and 'failure checks': it is advised to set up calculations in such a way that this iteration is possible in a relatively effortless way.

Trade-offs in the design process you have already done in WP1-WP3; it is self-evident that you are expected to apply the same tools as explained in the appendix of the reader for WP1-3 whenever appropriate. However, the design itself will require several calculations. Although in WP1-3 Excel or Google Spreadsheets may have been the superior choice for calculations, you are thoroughly recommended to use Python (or Matlab) to perform calculations from now on; as time goes on, you will find that Excel or similar programs simply do not offer the same versatility and flexibility that Python does, and you will end up regretting not switching to Python sooner. Even if you are not experienced in programming in Python at all, this project offers an excellent opportunity to brush up your Python skills (and one only learns by doing, after all).

2.1 Use of assumptions

Throughout the course of WP4, you will make assumptions and simplifications at various points. This is encouraged: the model you are making is ultimately a very low-fidelity model. Furthermore, the design calculations should be finished in due time, such that they can be used in an iterative procedure. At the end, as for many engineering exercises, there is simply no time to make a 'perfect' model. Nonetheless, you should be aware of the assumptions and simplifications you are making, and document them diligently. This means, at a minimum, you should describe at least the following:

- The assumption itself.
- The result of the assumption.
- The validity of the assumption.

As an example, unrelated to this project itself, if one were to describe a stress-strain relation for the deformation of a material, then one would need to document 1) that the material elastic deflection is assumed to be proportional to the applied load; 2) the result of this is that the stress-strain relationship is linear; and 3) this assumption is valid because the deflections are small, meaning that plastic deformations are absent. If possible, it is desirable to have at least an order of magnitude estimate of the error caused by the assumption, and if applicable, whether the assumption is conservative or non-conservative (i.e., does it result in an under- or overestimate of the actual result). If it is non-conservative, would it be appropriate to use a safety factor, and if so, what do you deem to be appropriate? Finally, it is preferable to use identifiers when listing assumptions, such that it is convenient to refer to them.

3 Work Package 4 report and deadline

The report for WP4 requires the same format as taught in the course Technical Writing in the first year. Recall, good reporting is very important! The WP reports are graded quite heavily on the reporting aspects (20% of the WP grade), which includes:

- Readability of the text
- Overall grammar and spelling quality

- Quality of figures, equations, and tables
- Logical build-up of content
- Use of references

The report must comply with the page-limits shown in the table below. “Content” is defined as all pages between the introduction and conclusion, including the introduction and conclusion itself. Appendices should *only* contain information that is non-essential to the reader to understand the report; information that is wrongly put in the appendix will result in a lower grade. Technical drawings may be included as A3 pages; this will count as a single page. No other kind of information may be put on A3.

Finally, the names and student numbers of the authors shall be present on the cover page. In addition, the group number and Work Package number must be placed **on the cover in the upper right-hand corner**. **The report shall also include a work division table, indicating which group members have contributed to each one of the tasks/report sections.**

In case Python is used for any kind of calculation, the source code should be included in the report as text, such that it can be checked for plagiarism. The code does not count towards the page limit.

	Content	**Appendices**
WP4	45 pages	15 pages

REPORT DEADLINES

Deadline for the WP4 report is: Monday, 29 November 2021, 12:00hrs-noon, CEST.

Via Ouriginal you must show that the text is really yours. Any form or sort of plagiarism will be reported to the board of examiners.

One last remark: ensure to complete all the required deliverables in your report as a minimum. If it is not in the report, then it has not been done! The deadline is non-negotiable.

4 A solar array or RTG attachment

A proper working and orientation of the solar array or RTG (both hereafter generally called "system") of the space craft is of paramount importance for the success of the mission. This not in the least involves a good attachment of the system to the space craft.

In this work package you are to design these attachment(s), which are placed to the vehicle side wall of the space craft. The attachment will involve some form of *fitting* that connects the system to the vehicle and some *fasteners* (or adhesive which is not considered here). A typical scenario of the attachment configuration is shown in figure 4.1 below, in which a single lug attachment is placed on the vehicle wall.

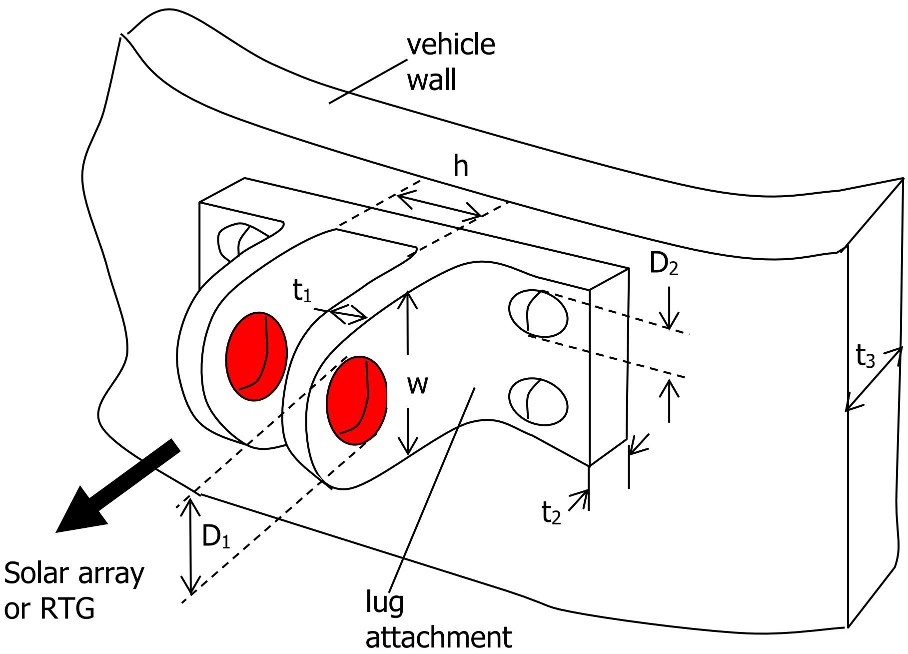

Figure 4.1: Typical attachment configuration using a single lug

DEFINITION OF NOTATION:

Lug: the whole attachment, shown in figure 4.1.
Flanges: are the parts that extrude from the lug, the support for the holes shown in red in figure 4.1. You can have a lug with just one flange, in which case the variable h is not relevant.
Fastener/Pin: how the attachment connects to the rest of the spacecraft (not shown in most figures).

DELIVERABLES

Throughout this work package, you are to determine the detailed design of a lug attachment. Following figure 4.1, this requires you to determine the diameters of the holes, D_1 and D_2, the dimensions of the lug, here taken as w, t_1, t_2 and h (this may vary for your case), and the local thickness of the spacecraft wall t_3.

4.1 **Identify the causes and determine the main loads on the assembly.** With any attachment, the first step is to determine the loads exerted by the system to the wall of the spacecraft. This should include any dynamic loading (although we do neglect vibrational aspects here). These loads should have the form of three forces in the x, y, z directions and, if appropriate, bending moment(s). Please ensure a clear and consistent definition of your coordinate system. An **example** is shown in figure 4.2. Include an FBD with the forces and directions.

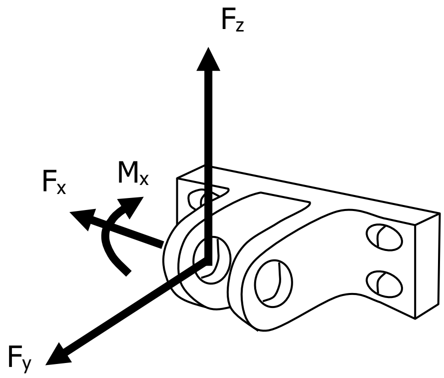

Figure 4.2: Forces acting on a single lug

4.2 **Select an appropriate lug configuration.** A configuration for the lug must be chosen, e.g., a single lug attachment or double lug attachment as seen in figure 4.3. Show how the system is attached to the lug(s) (for example with pins). Be careful if the loading includes some bending moment at the root of the system, to provide an attachment that allows for the creation of at least a force couple to react the moment as shown in figure 4.3 below.

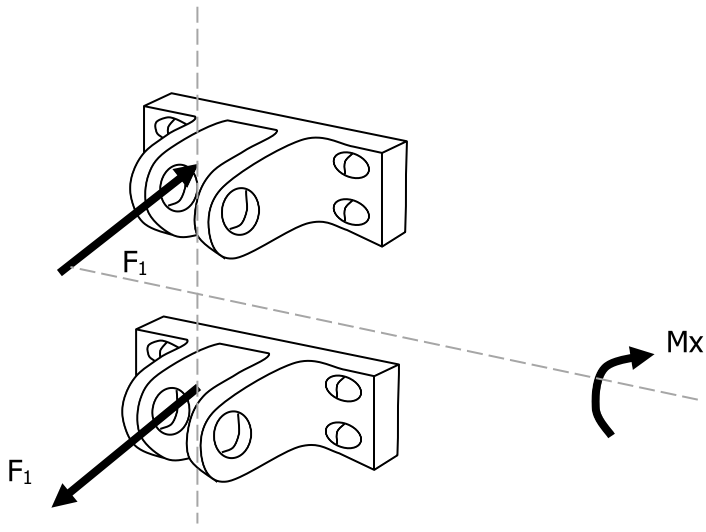

Figure 4.3: Bending moment resolved in a force couple

Also, for the double lug configuration, h (see figure 4.1) is determined by the requirement to provide enough clearance, so the edge of the system fits between the two lugs. Determine h (if you are using a double lug configuration).

4.3 **Design of the lugs.** Determine the P, net force acting on the lug's y-z plane of figure 4.2 and its orientation relative to the y (or z) axis. In general, this would combine, F_1, F_y, F_z. Use the analysis approach in section D1.13 from Bruhn (see Appendix A) to determine D_1, w, t_1 and the material to be used. Use yielding as the failure condition. Note if you use a double lug, P/2 is taken by each of the two lugs. If you use a single lug, the entire P load is taken by the single lug. In addition, check your lug(s) for failure under bending due to the load F_x or F_z. If F_x or $F_z = 0$ in your design, assume F_x or F_z = 10% P. This accounts for misalignments. As for the rest of the design, safety factors must be provided and reasoned for.

4.4 **Select fastener pattern for the back-up plate.** This includes the number of fasteners to be used, and their spacing. See figure 4.4.

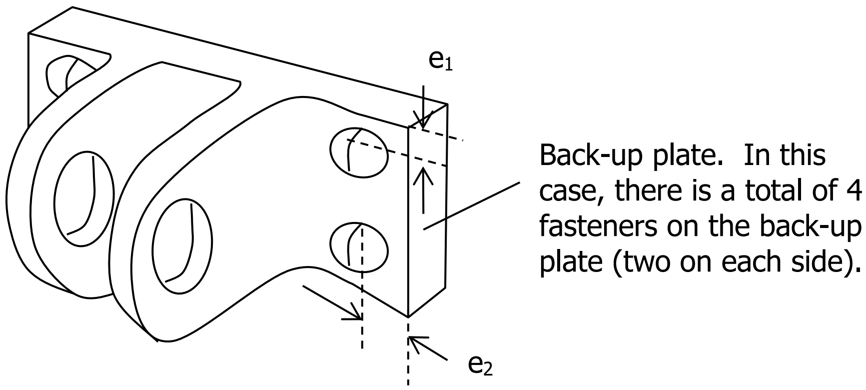

Figure 4.4: Fasteners in back-up plate

Note that if the back-up plate is metal, the fastener spacing, center to center, should be 2-3xD_2. If it is composite it should be 4-5xD_2. Also, the edge distance, (distance between edge of the plate and nearest fastener should be minimum 1.5 D_2 (both metal and composite). There are two edge distances in figure 4.4: e_1 and e_2. The value of w determined in 4.3 and these guidelines for fastener spacing and edge distance should allow you to pick the number of fasteners and D_2. Note that, depending on the results of 4.5 you may have to update w (for example if you need more fasteners in the back-up plate than the w selected in 4.3 allows you to use).

4.5 **Bearing check for fasteners in back-up plate.** Determine the c.g. of the fasteners in the back-up plate. Define an arbitrary coordinate system x-z. Obtain the coordinates x_i, z_i of the center of each fastener. Select a fastener diameter (not necessary at this point if all fasteners have the same diameter and the dependence on D_2 in eq. 4.1 cancels out). Determine x_{cg} and z_{cg} using:

$$x_{cg} = \frac{\sum A_i x_i}{\sum A_i}$$
$$z_{cg} = \frac{\sum A_i z_i}{\sum A_i} \qquad (4.1a,b)$$

where A_i is the area of the ith fastener hole. Analogous to neutral axis calculations, x_i, z_i can be negative if the ith fastener is located at negative x or z location with respect to the coordinate system chosen.

4.6 ***(Bearing check continued).*** Determine the net force F_{cg} in the x-z plane of figure 4.2 as resulting from of F_x and F_z. Resolve this force into two components parallel to the x and z axes: F_{cgx} and F_{cgz} (for the present problem F_{cgx} and F_x must coincide in orientation and magnitude and F_{cgz} and F_z must also coincide). If F_x and F_z do not act through the cg of the fasteners, then there is an additional net moment M_{cgy} about the fastener c.g. (and about the y axis) that these forces create. Determine the value and direction of M_{cgy}. Now determine the in–plane forces on each fastener due to F_{cgx}, F_{cgz}, and M_{cgy} as follows:

$$F_{in-plane-x} = \frac{F_{cgx}}{n_f}$$
$$F_{in-plane-z} = \frac{F_{cgz}}{n_f} \qquad (4.2 - 4.4)$$
$$F_{in-plane-M_y} = \frac{M_y A_i r_i}{\sum A_i r_i^{\,2}}$$

where n_f is the total number of fasteners, A_i is the cross-sectional area of the ith fastener, and r_i is the radial distance of the center of the ith fastener from the fastener c.g. A notional sketch of the situation is shown in figure 4.5. Note that force F_{cg} needs to be translated to the c.g. location.

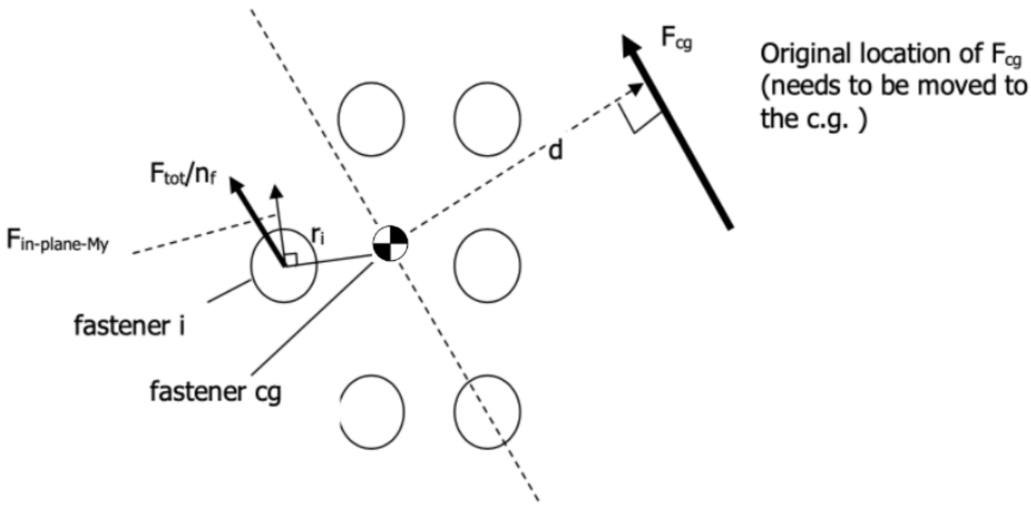

Figure 4.5: Fastener pattern and in-plane fastener load resulting from F_{TOT}

4.7 ***(Bearing check continued).*** Determine the total in-plane force P_i in each fastener as the resultant of the three forces from the previous step. Bearing failure is caused when the total in-plane load at each fastener causes a bearing stress that exceeds the bearing allowable of the material. The bearing stress is the contact stress between fastener and attached parts. The bearing stress is calculated using:

$$\sigma_{br} = \frac{P_i}{D_2 t_2} \tag{4.5}$$

The bearing stress σ_{br} is compared to the material allowable (bearing strength of the material). Bearing allowables are tabulated for different materials. Note that eq. (4.5) should be used to:

a. verify that the current values of D_2 and t_2 are sufficient since there is no failure and will then give an opportunity to reduce weight by reducing t_2 or removing fasteners;
b. show that there is bearing failure and therefore, more fasteners and/or higher t_2 values must be used; if more fasteners are used, the fastener spacing and edge distance requirements must still be satisfied which means the current value of w may need to be updated. Iterations may be required at this step to finalize geometry.

Note that the bearing check should be done for both the lug back-up wall and the spacecraft wall. There is no point in designing a strong lug if the wall of the vehicle to which it will be attached will fail in bearing. For the vehicle wall you need only determine what the thickness would be such that for the material selected for the vehicle wall and the current value of D_2 the wall does not fail in bearing. Also note that this is a local thickness that is not necessary at the rest of the wall where this bearing requirement is not present.

4.8 **Pull-through (or push-through) check.** Pull-through or push-through failure occurs when the out-of-plane fastener load (along the axis of the fastener) is so high that it drives the collar or the fastener head through the part (figure 4.6).

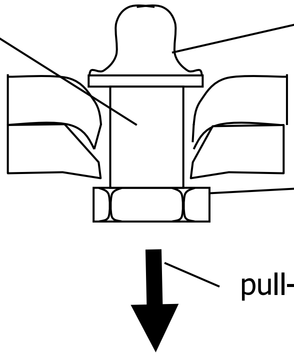

Figure 4.6: Axial load on fastener pulls fastener through the attached parts

Determine the out-of-plane load in each fastener using:

$$F_{pi} = \frac{F_y}{n_f}$$

$$F_{pM_z} = \frac{M_z A_i r_i}{\sum A_i r_i^2} \qquad (4.6\text{-}4.7)$$

These are exactly analogous to equations (4.2)-(4.4) with appropriate changes in variables. Watch out for the sign of F_{pMz}. M_z causes compression in portion of the fastener pattern ("above" the c.g.) and F_{pMz} is thus negative, and tension in portion of the fastener pattern resulting in positive F_{pMz} there. For each fastener, determine now the total out-of-plane force F_{yi} by algebraically adding the forces from eqs. (4.6) and (4.7). (Determine the fastener(s) with the highest out-of-plane load.) Rank the fastener(s) by their out-of-plane load.

4.9 ***(Pull-through failure continued).*** To check for pull-through failure, the shear stress caused by the pull-through load inside each attached part is calculated and compared to the shear yield stress of the material. To calculate the shear stress, determine the normal stress acting on the outer surface of the attached part and assume it is reacted by shear stress on a cylindrical surface inside the part at the <u>inboard</u> edge of the fastener head or nut, whichever is causing the pull-through load on the attached part. Note that in general, the calculation is done for each of the attached parts separately (see figure 4.7). This would mean a check on the lug back-up wall of thickness t_2 and a check on the vehicle wall thickness t_3 and material.

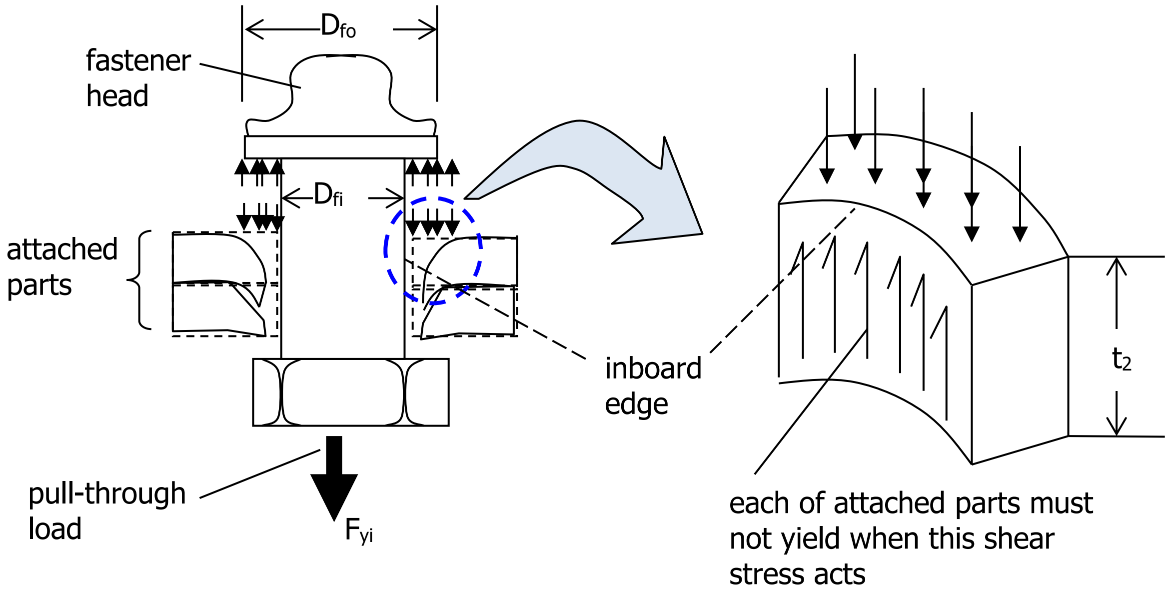

Figure 4.7: Free body diagram of a section of one of the attached parts

It should be noted that if the yield stress in shear for the material(s) selected is not available, it can be calculated using the von Mises stress and the value for tension yield stress. The condition for yielding is:

$$Y = \sqrt{\frac{1}{2}\left[\left(\sigma_x - \sigma_y\right)^2 + \left(\sigma_y - \sigma_z\right)^2 + \left(\sigma_z - \sigma_x\right)^2\right] + 3\tau_{xy}{}^2 + 3\tau_{yz}{}^2 + 3\tau_{xz}{}^2} \tag{4.8}$$

where Y is the tension yield stress.

Calculate the shear stress τ_i inside each attached part and compare to the corresponding yield stress. Note that to do that you will need to know the outer and inner diameters of the fastener head (or collar depending on which part is being driven into the back up wall of the lug or the vehicle wall) D_{fo} and D_{fi}. If there is no failure, consider decreasing t_2 (and t_3) if the bearing requirement in item 4.7 is not violated. If there is failure, increase the thicknesses accordingly.

4.10 **Select fastener type**. Based on the previous calculations and the design guidelines in reference B, select the remaining dimensions and type of fastener head (you can select a standard one). Choose a material for the fastener. This material must be different from the material chosen for the backplate. Include the values of the Young's modulus (E_b), yield and the thermal expansion coefficient (α_b). Based on the selected data, calculate and analyse the force ratio, as defined by the equation:

$$\phi = \frac{\delta_a}{\delta_a + \delta_b}$$

where δ_a is the compliance of the attached parts and δ_b is the compliance of the fastener. The compliance of δ_b can be easily calculated based on the fastener geometry (see equation 7.5.4 reference B). The compliance of the attached parts can be conservatively estimated via:

$$\delta_a = \frac{4\,t}{E_a\,\pi({D_{fo}}^2 - Dfi^2)}$$

where t is the distance of the thickness of the backplate or the vehicle wall, E_a is the young modulus of the backplate or the vehicle wall, and D_{fo} and D_{fi} are the outer and inner diameter of the fastener as defined in Fig 4.7.

4.11 **Thermal stress check.** Temperature differences are known to cause thermal expansion of the material, and this results in stresses when the lug/system is constrained. In the analysis of the thermal stress we consider that the thermal stress is a local issue: the thermal induced loads are constrained to each individual lug (in case you have a configuration like the one in figure 4.3). Based on the thermal control system, designed in WP2 – Part C, calculate the maximum and minimum equilibrium temperatures of the lug and the back-up wall. Obtain the temperature differentials in the lug compared to the assembly reference temperature (e.g. 15° Celsius in an assembly hall in Delft). Using the approach of the ESA handbook (reference B – equation 6.3.21) calculate the thermal induced loads that will appear in each fastener due to the thermal fluctuations. Consider separately the lug back-up plate and the vehicle wall.

Add the thermal induced loads to the resultant of in-plane loads due to mechanical forces at the point of the mission with the maximum and minimum temperatures. Check the new bearing stress and compare it to the material allowable of the back-up plate and vehicle wall. If necessary, evaluate the material used again. Use again eq. (4.5) to verify that there is no failure.

4.12 **Design iterations.** Repeat steps 4.3, 4.7, 4.9 and 4.11 if any portion of your design is failing in the lug, the back-up structure (bearing and pull-through) or the vehicle wall (bearing and pull-through) until no part is failing. Provide a table with the final values of w, D_1, D_2, t_1, t_2 and t_3. Provide a list of margins of safety (Safety factor) for

- lug,
- back-up wall bearing,
- back-up wall bearing including thermal loads
- back-up wall pull-through,
- vehicle wall bearing including thermal loads and
- vehicle wall pull-through.

The Safety factor values in this list should all be positive and as close to zero as possible. If any Safety factor is negative, your design is failing in that failure mode. If it is positive and high, you are carrying too much extra material and you could save weight by removing material and bringing the Safety factor down (still keeping it positive though). Justify the Safety factor differences for the different components. Note that the Safety factor is defined as:

$$MS = \frac{Allowable}{Applied} - 1 \qquad (4.9)$$

where "allowable" is the allowable load or stress and is, usually, the material strength in the corresponding loading case (bearing, shear, etc.). "Applied" is the applied load or stress (make sure you use the same units for allowable and applied in eq. 4.9).

4.13 **Material tradeoff.** Choose now a different material for the attachment and repeat step 4.12. Make changes to the design in order to obtain the lowest Safety factor with this material and to avoid failure if necessary. Compare the results, indicate how the change of material influences the attachment design. Calculate the final weight for both materials and consider which option is better. For making this consideration, a trade-off table is required with a few carefully selected criteria and a scoring system.

4.14 **Design drawing.** Provide a detailed CATIA drawing, with appropriate dimensioning, of the final attachment and its location on the spacecraft.

Appendix A, Lug analysis

Extracted from:
Bruhn, *"Analysis and Design of Flight Vehicle Structures"*, Tri-state offet Co, 1973, section D1.

....
thus allowing the bolt and bushing to move which results in the elongated hole as illustrated in Fig. b. Equating the load P_u to the ultimate bearing strength on the bearing surfaces we can write,

$$P_u = F_{br} D t \quad (4)$$

where F_{br} = allowable bearing stress
D = diameter of bushing
t = plate thickness

It is good practice to require a margin of safety of 50 percent.

Failure by Bearing of Bolt Bushing.

A bushing is pressed into the plate hole and thus it is considered as a tight fit. A fitting bolt is usually considered as removable therefore a certain tolerance between the bolt and bushing inside diameters is necessary in order to insert and remove bolt. If fitting is subjected to reversible loads the small slop in the fitting tends to cause shock on the fitting. Also the fitting may be such that slight rotation takes place on the bolt, which tends to cause wear between bolt and bushing. It is therefore customary to check the bearing pressure between the bolt and the bushing since failure of the bushing could take place in a manner explained in the previous article dealing with bearing of bushing on plate. Then

D1.11 Method 2. Lug Strength Analysis Under Axial Loading.

Due to a comprehensive study and test program by Cozzone, Melcon and Hobbit (Refs. 3 and 4), the procedure as given in Method 1 is somewhat modified. The important difference is that curves derived from test results give the stress concentration factor to use for tension on the net section and the shear out failure as assumed in Method 1 has been replaced by a combined shear-out bearing failure. Fig. D1.11 shows the lug-pin combinations and types of failure as taken from Ref. 3.

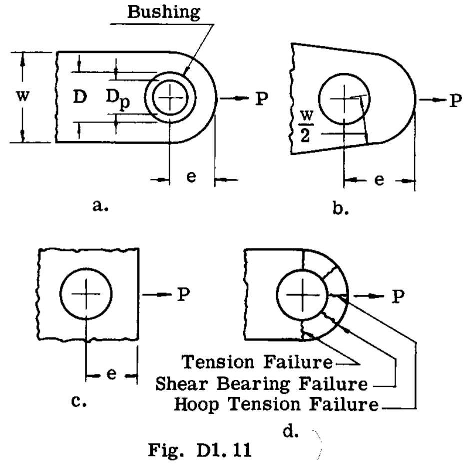

Fig. D1.11

The methods of failure and the methods of lug strength analysis are as follows:

Tension Across the Net Section.

Because of stress concentration, the stress on the net cross-section cannot be taken as uniform. The ultimate allowable tension load P_u for lug equals,

$$P_u = K_t F_{tu} A_t \quad \text{-----------------} \quad (6)$$

where K_t is the stress concentration factor as found from Fig. D1.12 and Table D1.3. F_{tu} = ultimate tensile strength of the material and A_t = net tension area.

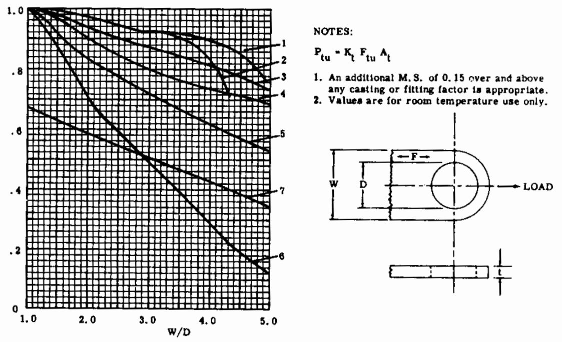

Fig. D1.12 Lug Design Data Tension Efficiency Factors for Axially Loaded Lugs (Ref. 3, 4)

Table D1.3

Curve Nomenclature for Axial Loading for Fig. D1.12

L, LT and ST Indicate Grain in Direction F in Sketch
L - Longitudinal
LT - Long Transverse
ST - Short Transverse (Normal)

MATERIALS
Curve 1 - 2014-T6 and 7075-T6 Die Forging (L)
4130 and 8630 Steel
2014-T6 and 7075-T6 Plate ≤ 0.5 (L, LT)
7075-T6 Bar and Extrusion (L)
2014-T6 Hand Forged Billet ≤ 144 in.^{2} (L)
Curve 2 - 2014-T6 and 7075-T6 Plate > 0.5 in. ≤ 1.0 in. (L, LT)
7075-T6 Extrusion (LT, ST)
2014-T6 Hand Forged Billet > 144 in.^{2} (L)
2014-T6 Hand Forged Billet ≤ 36 in.^{2} (LT)
2014-T6 and 7075-T6 Die Forgings (LT)
Curve 3 - 2024-T4, 2024-T2 Extrusion (L, LT, ST)
Curve 4 - 2014-T6 and 7075-T6 Plate > 1 in. (L, LT)
2024-T4 Bar (L, LT)
2024-T3, 2024-T4 Plate (L, LT)
Curve 5 - 2014-T6 Hand Forged Billet > 36 in.^{2} (LT)
Curve 6 - Aluminum Alloy Plate, Bar, Hand Forged Billet and Die Forging (ST). NOTE: For Die Forgings ST Direction Exists Only at Parting Plane.
7075-T6 Bar (T)
Curve 7 - AZ91C-T6 Mag. Alloy Sand Casting
356-T6 Aluminum Alloy Casting

Shear Out-Bearing Strength.

Failure due to shear out and bearing are closely related and are covered by a single calculation based on empirical curves. The ultimate or failing load in this shear-bearing type of failure is:-

$$P_{bru} = K_{bru} F_{tu} A_{br} \quad \text{-------------} \quad (7)$$

The values of K_{bru}, the shear-bearing efficiency factor, is given by curves in Fig. D1.13.

For shear-bearing yield strength the equation is,

$$P_{bry} = K_{bry} F_{tu} A_{br} \quad \text{-------------} \quad (8)$$

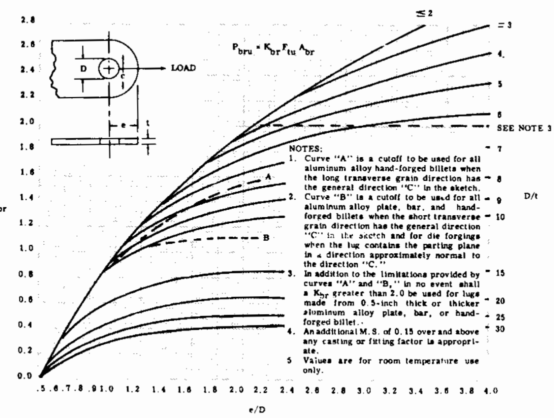

Fig. D1.13 Lug Design Data Shear-Bearing Efficiency Factors for Axially Loaded Lugs (Ref. 3, 4)

Fig. D1.14 gives curves for finding K_{bry}.

Bushing Yield.

Take A_{br} as the smaller of the bearing areas of bushing on pin or bushing on lug. (The latter may be smaller as a result of external chamfer of the bushing, oil grooves, etc.) The allowable yield bearing load on bushing is then,

$$P_{bry} = 1.85 F_{cy} A_{br} \quad \text{-------------} \quad (9)$$

where F_{cy} is compressive yield strength of bushing material.

Bolt or Pin Shear Strength.

The bolt shear strength is calculated in the same manner as given in Method 1.

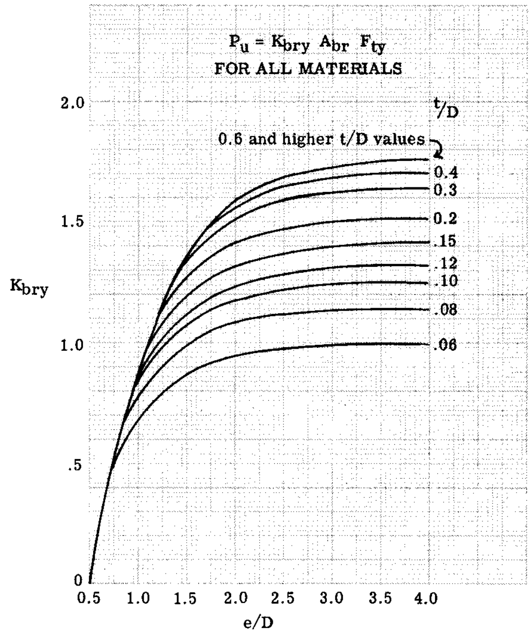

Fig. D1.14 (Ref. 5)
Values of Shear-Bearing Factors of Lugs

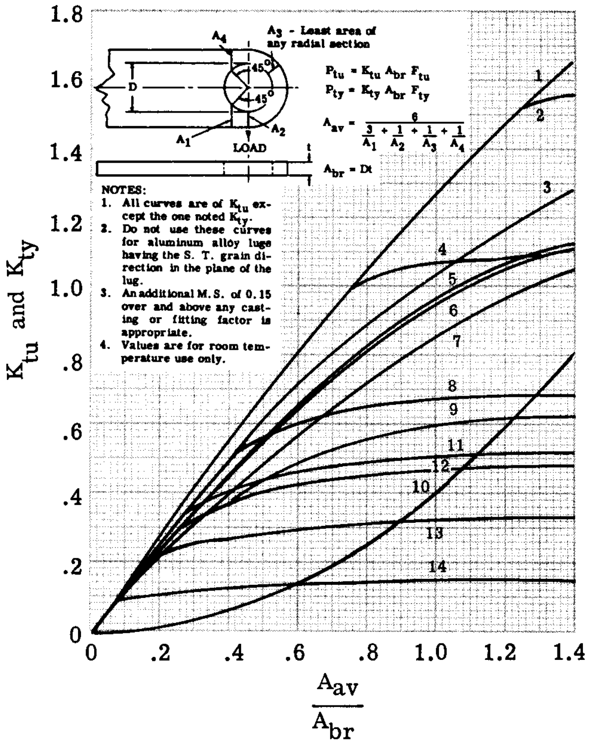

Fig. D1.15 Lug Design Data
Tension Efficiency Factors for Transversely Loaded Lugs (Ref. 3, 4)
(See Table D1.4 for Curve Nomenclature)

Bolt or Pin Bending.

The subject of bolt bending strength is treated in Art. D1.14.

D1.12 Lug Strength Analysis Under Transverse Loading.

Cases arise where the lug of a fitting unit is subjected to only a transverse load. Melcon and Hobbit in (Ref. 4) express the ultimate transverse or failing load by a single equation:-

$$P_{tu} = K_{tu} A_{br} F_{tu} \quad \text{------------ (10)}$$

Similarly the yield strength of lug is,

$$P_{ty} = K_{ty} A_{br} F_{ty} \quad \text{------------ (11)}$$

The efficiency failing and yield coefficients K_{tu} and K_{ty} are given by the curves in Fig. D1.15. The curve nomenclature for the curves in Fig. D1.15 is given in Table D1.4. In using Fig. D1.15, a value called A_{av} is needed, the value of which is shown in the equation shown on Fig. D1.15

D1.13 Lug Strength Analysis Under Oblique Loads.

Fitting lugs are often subjected to oblique loads. Ref. 4 gives the following approach to this loading case.

Resolve the applied load into axial and transverse components. Then use the following interaction equation;-

$$R_a^{1.6} + R_{tr}^{1.6} = 1 \quad \text{------------ (12)}$$

Table D1.4
(To be Used with Fig. D1.15)
Curve Nomenclature for Transverse Loading

Curve 1	4130 and 8630 Steel thru 125 KSI H. T.
Curve 2	4130 and 8630 Steel 150 KSI H. T.
Curve 3	K_{ty} for All Aluminum and Steel Alloys
Curve 4	4130 and 8630 Steel 180 KSI H. T.
Curve 5	356-T6 and AZ91C-T6 Sand Castings
Curve 6	2024-T3 and 2024-T4 Plate ≤ 0.5 in.
Curve 7	220-T4 Sand Casting
Curve 8	2014-T6 and 7075-T6 Plate ≤ 0.5 in.
Curve 9	2024-T3 and 2024-T4 Plate > 0.5 in. also 2024-T4 Bar
Curve 10	Approximate Cantilever Strength for All Aluminum and Steel Alloys. If K_{tu} is Below this Curve a Separate Calculation as a Cantilever Beam is Warranted.
Curve 11	2014-T6 and 7075-T6 Plate > 0.5 in. ≤ 1.0 in. 7075-T6 Extrusions 2014-T6 Hand Forged Billet ≤ 36 in.^{2} 2014-T6 and 7075-T6 Die Forgings
Curve 12	2024-T6 Plate, 2024-T4 & 2024-T42 Extrusions
Curve 13	2014-T6 and 7075-T6 Plate > 1 in.
Curve 14	2014-T6 Hand Forged Billet > 36 in.^{2}

or margin of safety is,

$$M.S. = \frac{1}{(R_a^{1.6} + R_{tr}^{1.6})^{0.625}} - 1 \quad \text{---- (13)}$$

where, R_a = axial component of applied ultimate load divided by the smaller of the values obtained for equations (6) or (7).

R_{tr} = transverse component of applied ultimate load divided by the values of P_{tu} in equation (10).

Appendix B, Reference material: ECSS-E-HB-32-23A standard

5.4.1 Fastener Geometry

For a fastener analysis several dimensions are important. These are depicted in Figure 5-5 and Figure 5-6.

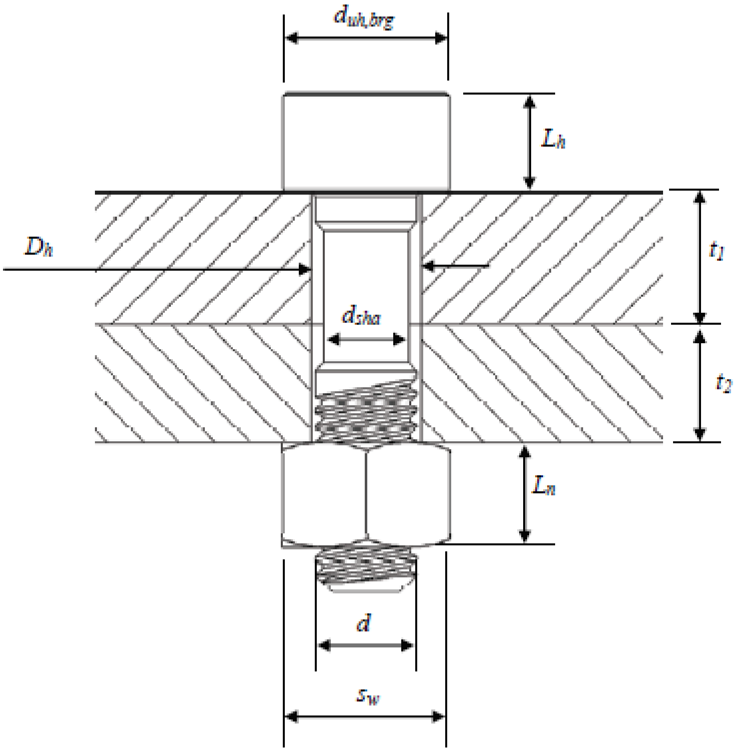

Figure 5-5 – Fastener Dimensions

In the general case of a fastener with varying shank diameters, such as Figure 7-9, the total elongation of the fastener is the sum of the elongations of its constituent segments (i.e. head, nut, and shank). In general, the different parts of the fastener have different local stiffness properties. Thus the total elongation can be written,

$$\Delta L_b = \sum \Delta L_i \qquad [7.5.2]$$

where ΔL_i are the elongations of the constituent segments.

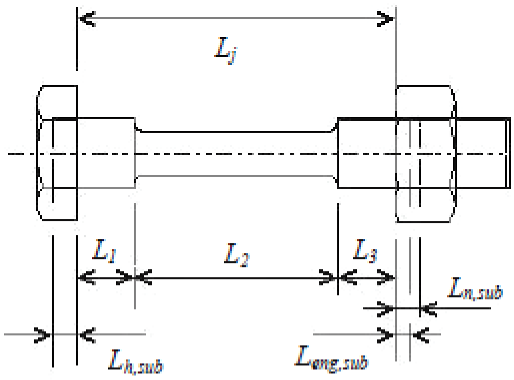

Figure 7-9 – Dimensioning of the Fastener for Compliance Calculations

Applying Hooke's law to each segment,

$$\Delta L_i = \frac{L_i F_b}{E_b A_i} \qquad [7.5.3]$$

where; E_b is the Young's modulus of the fastener material and A_i is the local cross section of the segment.

Combining the above equations, the compliance of the fastener is then,

$$\delta_b = \frac{1}{K_b} = \frac{\Delta L_b}{F_b} = \frac{1}{E_b}\sum \frac{L_i}{A_i} \qquad [7.5.4]$$

Expanding [7.5.4] and introducing substitution lengths for deformations in the head, the fastener's engaged region and the nut's engaged region gives,

$$\delta_b = \frac{1}{E_b}\left[\frac{L_{h,sub}}{A_{nom}} + \frac{L_{eng,sub}}{A_3} + \left(\frac{L_{sha,1}}{A_{sha,1}} + \frac{L_{sha,2}}{A_{sha,2}} + \cdots + \frac{L_{sha,i}}{A_{sha,i}}\right)\right] + \frac{L_{n,sub}}{E_n A_{nom}} \qquad [7.5.5]$$

where; $L_{h,sub}$, $L_{eng,sub}$ and $L_{n,sub}$ are the substitution lengths for deformations within the head, engaged shank and engaged nut or insert (see Table 7-1), A_{nom} is the fastener's nominal cross-sectional area, A_3 is the fasteners minor diameter area (see Figure 5-5), $A_{sha,i}$ is the effective cross-sectional area at the i'th segment of the fastener's shank and $L_{sha,i}$ is the length of the i'th segment of the fastener's shank.

In many cases when using Equation [7.5.5] only one segment of fastener's shank needs to be considered. However, if the fastener has varying shank diameters, the length and effective stiffness area of each segment should be included.

.Table 7-1 shows typical substitution lengths for standardised fasteners, which can be used in Equation .[7.5.5]. The table only includes values for typical hexagon heads, cylindrical heads and nuts. The same analysis method can be applied to other fastener heads and nuts, and the appropriate substitution lengths should be determined by experiment or analysis.

Table 7-1 – Typical Substitution Lengths for Commonly Used Fasteners

(data from Reference .7.3)

Part of Fastener	**Parameter**	**Fastener/Joint Configuration**	**Typical Substitution Length**
Head	$L_{h,sub}$	Hexagon head	0.5 d
		Cylindrical head	0.4 d
Engaged shank	$L_{eng,sub}$	Nut-Tightened	0.4 d
		Threaded hole	0.33 d
Locking device (nut or insert)	$L_{n,sub}$	Any	0.4 d

6.3.5 Forces Induced by Thermal Fluctuation

6.3.5.1 Overview

During thermal fluctuations the preload changes due to differing thermal expansion coefficients of the fastener and flange material. Also, the modulus of elasticity of the materials can be temperature sensitive.

The thermal expansion coefficient relates the elongation $f_{\Delta T}$ of an individual item with the length *L*, to a temperature difference ΔT according to,

$$f_{\Delta T} = \alpha\ L\ \Delta T \qquad [6.3.18]$$

where α is the items's coefficient of thermal expansion and $\Delta T = T_{working} - T_{reference}$.

Assuming clamped parts made from a single material type, the respective elongations of the fastener and clamped parts are,

$$f_{\Delta T,b} = \alpha_b\ L_j\ \Delta T \text{ , and} \qquad [6.3.19]$$

$$f_{\Delta T,c} = \alpha_c\ L_j\ \Delta T \qquad [6.3.20]$$

where L_j is the joint length (equal to the combined thickness of all clamped parts), and α_b and α_c are the coefficients of thermal expansion of the fastener and clamped parts respectively.

Based on the above equations, it can be shown that the thermally induced load (in the fastener) is given by,

$$F_{\Delta T} = (\alpha_c - \alpha_b)\ \Delta T\ E_b\ A_{sm}\ (1 - \Phi) \qquad [6.3.21]$$

where A_{sm} is the stiffness area of the fastener, Φ is the basic (i.e. *n*=1) force ratio of the joint as defined in either Section .7.5 or Section .8 (depending on whether the joint is concentric or eccentric).

The extreme values of the thermally induced force are denoted $F_{\Delta T^+}$ and $F_{\Delta T^-}$, and these are given by the higher and lower results of,

$$(\alpha_c - \alpha_b)\ \Delta T_{\max}\ E_b\ A_{sm}\ (1 - \Phi) \text{, and} \qquad [6.3.22]$$

$$(\alpha_c - \alpha_b)\ \Delta T_{\min}\ E_b\ A_{sm}\ (1 - \Phi) \qquad [6.3.23]$$

where the temperature differentials are given by,

$$\Delta T_{\min} = T_{working,\min} - T_{reference} \qquad [6.3.24]$$

$$\Delta T_{\max} = T_{working,\max} - T_{reference} \qquad [6.3.25]$$