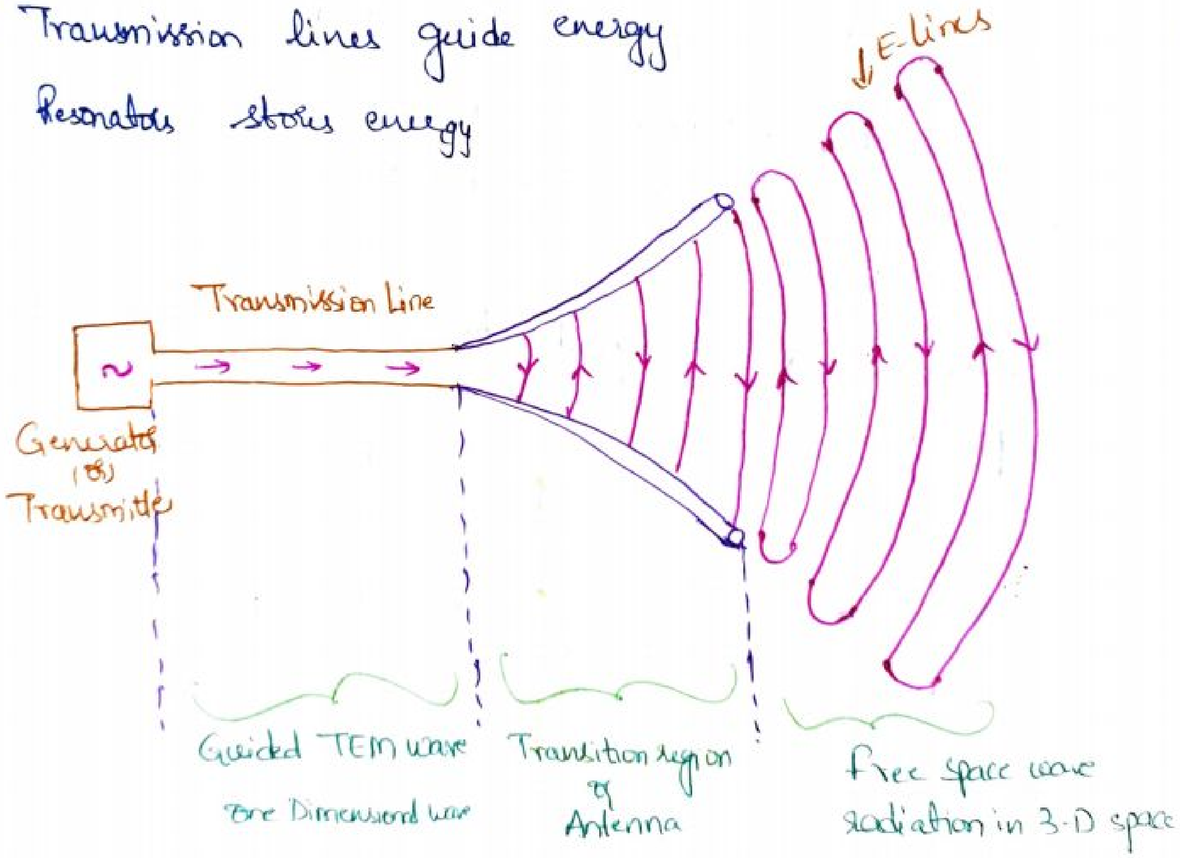
UNIT-I

ANTENNA FUNDAMENTALS :-

A radio antenna may be defined as a structure associated with the region of transition between a guided wave and a free space wave or vice versa.

Plural of radio antenna is antennas
Plural of zoological antenna (insect antenna) is antennae

* Antennas radiate energy
* Transmission lines guide energy
* Resonators stores energy



* The guided wave is a plane wave while the free space wave is a spherically expanding wave.

* Along the uniform part of the line energy is guided as a plane wave with little loss provided the spacing b/w the wire is a small fraction of a wavelength.

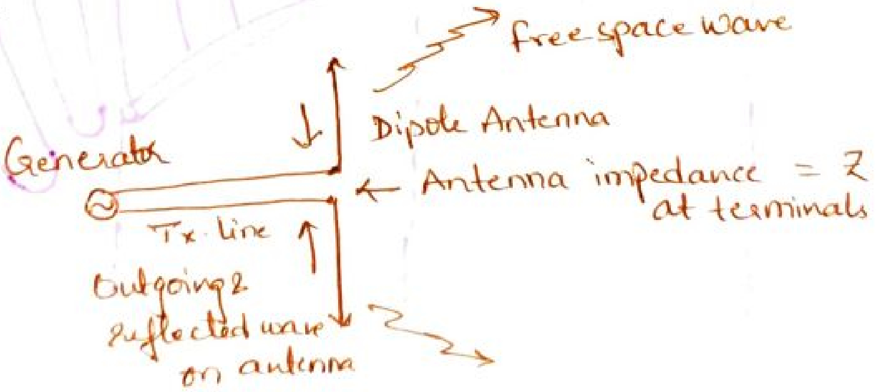
* At the right, as the transmission line separation approaches a wavelength or more, the wave tends to be radiated so that the opened out line acts like an antenna which launches a free space wave.

* The currents on the transmission line flow out on the transmission line and end there, but the fields associated with them keep on going.

* As a receiving device, the definition of an antenna is turned around, An antenna is the region of transition between a free space wave and a guided wave.

* Thus an antenna is a transition device or a transducer between a guided wave and a free space wave or vice versa.

* Antennas are designed to radiate energy as effectively as possible.



The antenna launches a free space wave but appears as a circuit impedance to the transmission line.

* Dipole acts as antenna because it launches a free space wave.

* It may also be regarded as a section of an open ended transmission line.
* Since energy reflected from the ends of the dipole gives rise to a standing wave and energy storage near the antenna. Therefore it exhibits the characteristics of a resonator.

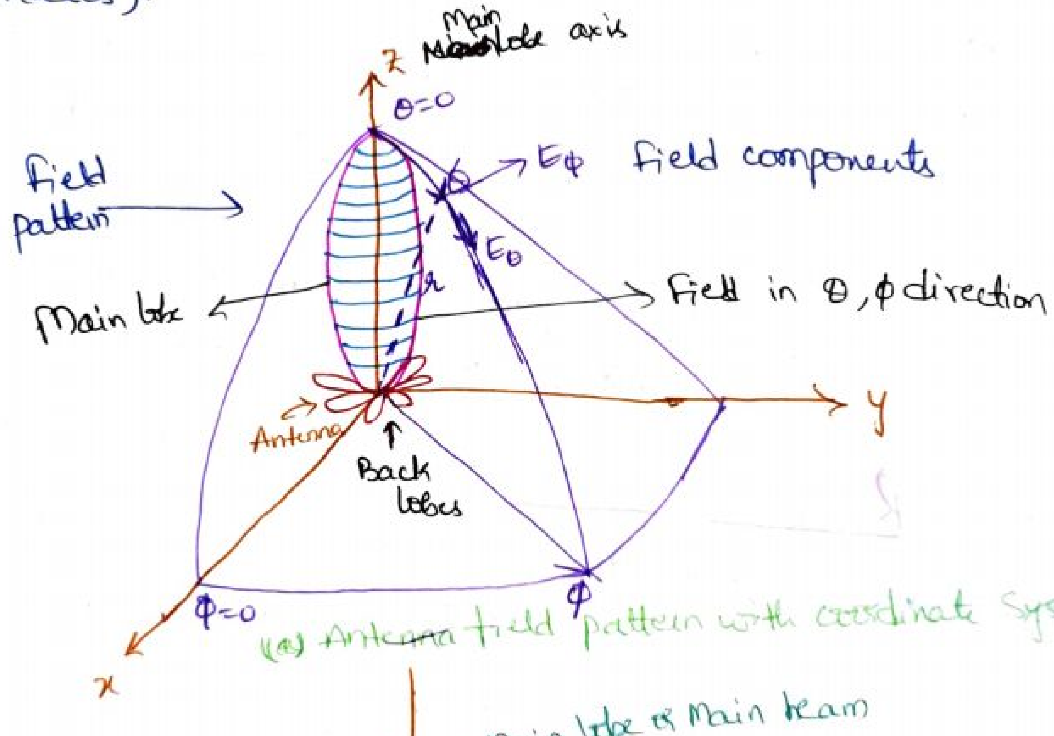
Basic Antenna Parameters :-

* As seen from the transmission line, the antenna appears as a 2-terminal circuit element having an impedance Z with a resistive component called the radiation resistance R_R.
* As seen from the space, the antenna is characterized by its radiation pattern or patterns involving field quantities
* R_R :- **Radiation resistance** is the resistance coupled from the antenna and its environment to the antenna terminals

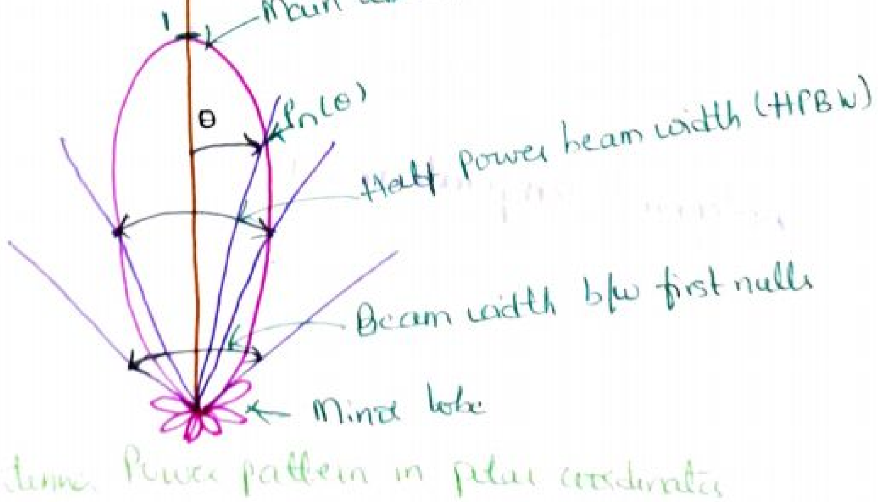
(If the power radiated by the antenna $= I^2R$ $(R = R_R)$. R_R is a ficticious parameter. If R_R is in series with the antenna, then it will absorb the same power as radiated)

* **Antenna Temperature T_A** :-
* For a lossless antenna T_A is not the physical temperature.
* It is the temperature of the distant regions of space (nearer surroundings) coupled to the antenna via its radiation resistance.

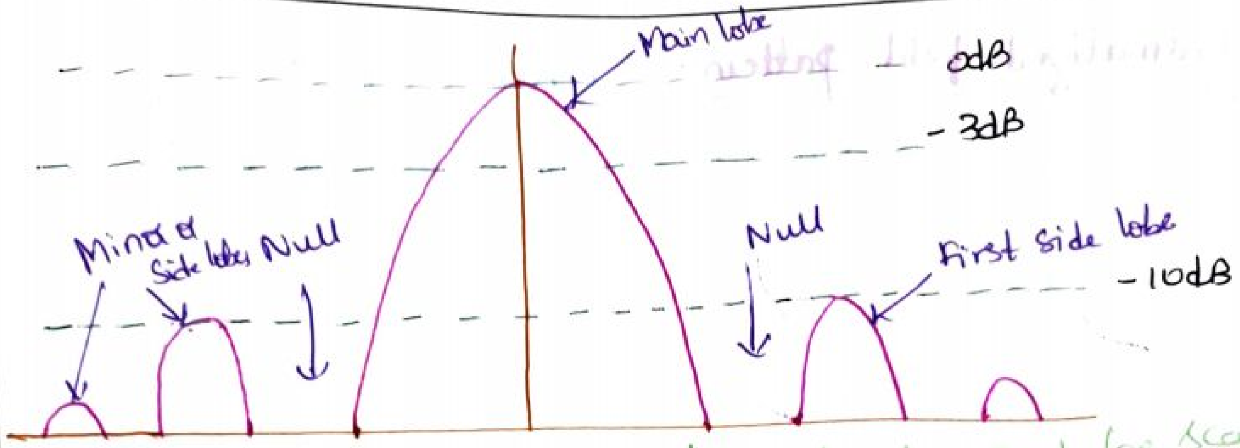
* It is not an inherent property of the antenna.
* It depends on the temperature of the regions the antenna is looking at.
* A receiving antenna may be regarded as a remote sensing temperature measuring device.
* R_r and T_A are single valued quantities
* Radiation pattern (variation of field or power proportional to the field squared) is a function of θ & ϕ (spherical coordinates).



(a) Antenna field pattern with coordinate system



(b) Antenna Power pattern in polar coordinates



(c) Antenna pattern in □lar coordinates and log scale

* In fig (a) 'r' is proportional to field intensity at a certain distance from the antenna
* The main lobe maximum is in the z-direction ($\theta = 0$)
* The minor lobes (side & back are in other directions)
* Between the lobes are nulls in the direction of zero or minimum radiation

To completely specify the radiation pattern with respect to field intensity and polarization requires three patterns

1. The θ-component of the electric field as a function of θ & ϕ $E_\theta(\theta, \phi)$ V/m
2. The ϕ component of the electric field as a function of θ & ϕ $E_\phi(\theta, \phi)$ V/m.
3. The phases of these fields as a function of the angle θ & ϕ or $\delta_\theta(\theta, \phi)$ & $\delta_\phi(\theta, \phi)$ rad or deg.

Normalized field pattern:- Ratio of field component to its maximum value. It is a dimensionless quantity. Its max value is 1.

$$E_\theta(\theta,\phi)_n = \frac{E_\theta(\theta,\phi)}{E_\theta(\theta,\phi)_{max}}$$

* At distances that are large compared to the size of antenna and large compared to the wavelength, the shape of the field pattern is independent of distance.

* Antennas are the basic components of any electric system and are connecting links between the transmitter and the free space and the receiver.
* Irrespective of the type of application of an antenna, all antenna possess certain basic principles / properties like Radiation pattern, Radiation intensity, Polarization Gain, Directivity, Power gain, efficiency, effective Aperture area, Self & Mutual impedance, Radiation resistance, Beam width, Bandwidth etc.

Isotropic Radiators :-

* An isotropic radiator is a ficticious radiator and is defined as a radiator which radiates uniformly in all directions.
* It is also called as isotropic source or omnidirec-tional radiator or unipole
* It is hypothetical lossless radiator or antenna
* It is used as a reference antenna.

Radiation Pattern :-

* In practice the radiated energy from an antenna is not of the same strength in all directions. It is more in one direction and less or zero in other direction
* The energy radiated in a particular direction by an antenna is measured in terms of field strength at a point which is at a particular distance from the antenna
* The field strength is specified at points on the spherical surface of radius r.
* The shape of the radiation pattern does not depend on 'r' provided $r >> \lambda$.
* The direction of field strength (E) for the radiation field is always tangential to the spherical surface of imaginary sphere of radius 'r'

* The shape of the radiation pattern does not depend on 'r' provided $r >> \lambda$.

* The direction of field strength (E) for the radiation field is always tangential to the spherical surface of imaginary sphere of radius 'r'.

For a vertical dipole E-field is in the direction 'θ' & for a horizontal loop, in the direction ϕ.

* Radiation field may have components E_θ & E_ϕ which may or may not be in phase

$$E = \sqrt{E_\theta^2 + E_\phi^2}$$

* The two dimensional patterns obtained from the 3-D field pattern by cutting along a horizontal & vertical planes are respectively known as "Horizontal pattern" & "Vertical pattern"

(Principal plane patterns - XY or YZ or ZX)

* Major lobe is also called as main beam and is defined as the radiation lobe containing the direction of maximum radiation

* Minor lobe is any lobe except major lobe

* Side lobe is a radiation lobe adjacent to the main lobe and occupies the hemisphere in dir. of main lobe.

* The level of minor lobe is expressed as a ratio of the power density in the lobe to that of the major lobe
(−20 dB smaller − not harmful
−30 dB " − difficult to design)

* Patterns may also be expressed in terms of power per unit area at a certain distance from the antenna.

Normalized Power pattern

$$P_n(\theta,\phi) = \frac{S(\theta,\phi)}{S(\theta,\phi)_{max}}$$

$S(\theta,\phi)$ = Poynting vector = $\left[E_\theta^2(\theta,\phi) + E_\phi^2(\theta,\phi)\right] Z_0$ W/m^2

$S(\theta,\phi)_{max}$ = max value of $S(\theta,\phi)$ W/m^2

$Z_0 = 377\,\Omega$.

* To express the minor lobes w.r.t the major lobe

$$dB = 10\log_{10} P_n(\theta,\phi).$$

Beam Area (or Beam Solid Angle):

* Area of sphere $= 4\pi r^2$

* $1\,sr$ = 1 Steradian $= \left(\frac{180}{\pi}\right)^2$ = 1 radian^2 = 3282.8064 sq. degs

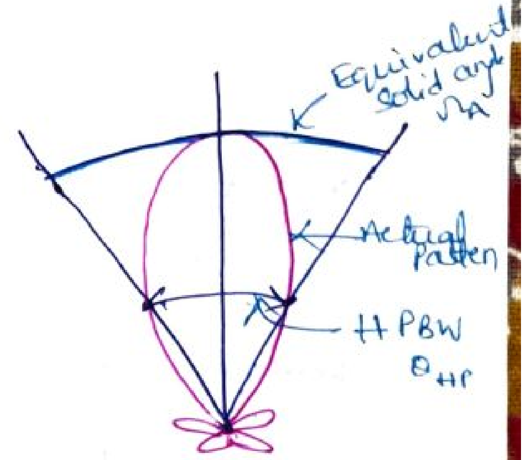
* 4π steradians $= 3282.8064 \times 4\pi \approx 41253$ sq. deg
= Solid angle in sphere.

* $dA = r^2 \sin\theta \, d\theta \, d\phi = r^2 d\Omega$ $\quad d\Omega$ = solid angle subtended by the area of dA.

Beam area or beam solid angle Ω_A for an antenna is given by the integral of the normalized power pattern over a sphere (4π Sr)

$$\Omega_A = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} P_n(\theta,\phi)\, d\Omega \quad Sr$$

$$d\Omega = \sin\theta\, d\theta\, d\phi$$



$$\Omega_A = \theta_{HP}\,\phi_{HP}\ (Sr)$$

θ_{HP} & ϕ_{HP} are half power beam widths in two principal planes, minor lobes being neglected.

Radiation Intensity:- Power radiated from an antenna per unit solid angle is called as Radiation intensity

U w/sr or W/sq.deg.

$$P_n(\theta,\phi) = \frac{U(\theta,\phi)}{U(\theta,\phi)_{max}} = \frac{S(\theta,\phi)}{S(\theta,\phi)_{max}}$$

S (poynting vector) depends on the distance from the antenna (varying inversely as the square of the distance) $1/R^2$ the radiation intensity U is independent of the distance. (assuming in both cases that we are in far field of the antenna)

Beam Efficiency :- The total beam area Ω_A (or beam solid angle) consists of the main beam area (or solid angles Ω_M) plus the minor lobe area (or solid angle Ω_m)

$$\Omega_A = \Omega_M + \Omega_m$$

The ratio of the main beam area to the total beam area is called the beam efficiency

$$\varepsilon_M = \frac{\Omega_M}{\Omega_A} = \text{beam efficiency.}$$

The ratio of the minor lobe area Ω_m to the total beam area is called the stray factor ε_m

$$\therefore \varepsilon_m = \frac{\Omega_m}{\Omega_A} = \text{Stray factor}$$

$$\varepsilon_M + \varepsilon_m = 1.$$

Directivity :- The directivity 'D' of an antenna is given by the ratio of the maximum radiation intensity (power per unit solid angle) $U(\theta,\phi)_{max}$ to the average radiation intensity U_{av} (averaged over a sphere) or at a Certain distance from the antenna the directivity may be expressed as the ratio of the maximum to the average Poynting vector.

$$D = \frac{U(\theta,\phi)_{max}}{U_{av}} = \frac{S(\theta,\phi)_{max}}{S_{av}} \quad \text{(dimensionless)}$$

$$S(\theta,\phi)_{av} = \frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi} S(\theta,\phi)\, d\Omega \quad W/m^2$$

$$D = \frac{1}{\frac{1}{4\pi}\iint \frac{S(\theta,\phi)}{S(\theta,\phi)_{max}}\, d\Omega} = \frac{1}{\frac{1}{4\pi}\iint P_n(\theta,\phi)\, d\Omega} = \frac{4\pi}{\Omega_A}$$

Smaller the beam area, greater the directivity

Directivity & Gain :- The gain of an antenna depends on both directivity & efficiency.

If efficiency is no 100%, gain < directivity.

$$G = KD \text{ (dimensionless)}$$

K = efficiency factor of antenna $(0 \le K \le 1)$ (dimensionless)

This efficiency has to do only with ohmic losses in the antenna. In transmitting, these losses involve power fed to the antenna which is not radiated but heats up the structure.

Directivity & Resolution :- The resolution of the antenna may be defined as equal to half the beam width b/w first nulls (BWFN/2)

Ex. if BWFN of an antenna $= 2^\circ$

its resolution $= 1^\circ$

$$\frac{BWFN}{2} \approx HPBW \qquad \Omega_A = \left(\frac{BWFN}{2}\right)_\theta \left(\frac{BWFN}{2}\right)_\phi$$

1

* If there are 'N' radio transmitters or point sources of radiation distributed uniformly over the sky which an antenna can resolve then

$$N = \frac{4\pi}{\Omega_A} \quad \text{but} \quad D = \frac{4\pi}{\Omega_A} \quad \therefore \quad D = N.$$

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Polarization.

12   J D. Kraus Pg.no 37 (3) (1)

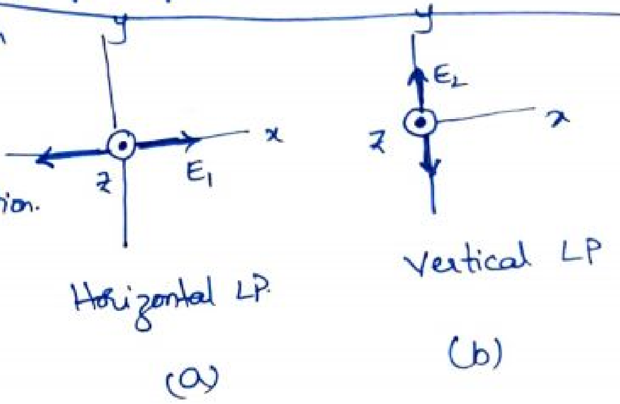
free space wave with time & space variation can be represented In terms:

$e^{-\gamma z}$ i.e $\gamma = \alpha + j\beta$.

Def: The curve traced by the tip of E-field as a function of time at a fixed position in free space

If a plane wave travelling in +ve z-dir with E-field at all times in y-direction as shown in fig (b) then it is called vertical linear polarization.

The wave is said to be linearly polarized.



The E-field can be written as

$$E_y = E_2 \sin(\omega t - \beta z)$$

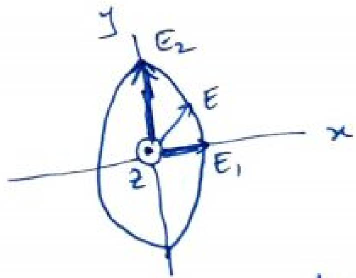
for Horizontal polarization

$$E_x = E_1 \sin(\omega t - \beta z)$$

Def: An EM wave is LPzed at a given pt. in space if E(or H) field vector at that point is always oriented along the same straight line at every instant of time

In general the EM wave in z-direction have both x & y component as shown in fig. (c)

In this case, we call the wave is 'Elliptically polarized'



At a fixed position of the wave 'z', the E-field rotates as a function of time, the tip of the E-vector describes an ellipse called Polarization Ellipse. The ratio of major to minor axes of polarization ellipse is called Axial Ratio (AR).

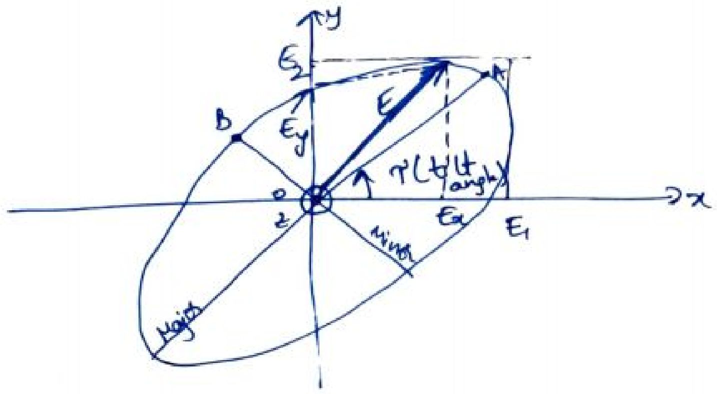
$$AR = \frac{E_2}{E_1}.$$

Linear and Circular Polarization are two extreme cases of Elliptical polarization.

In CP, $E_1 = E_2$ & $AR = 1$

for LP, $E_1 = 0$ & $AR = \infty$

This polarization Ellipse may have any orientation.



$E_x = E_1 \sin(\omega t - \beta z)$ E_1 = amplitude of wave LPzed in x-dir

$E_y = E_2 \sin(\omega t - \beta z + \delta)$ E_2 = " " " " " y-dirc

δ = time phase angle by which E_y leads E_x

$E = \hat{x} E_1 \sin(\omega t - \beta z) + \hat{y} E_2 \sin(\omega t - \beta z + \delta)$

$AR = \frac{OA}{OB} =$ $1 \leq AR \leq \infty$

If $E_1 = 0$, the wave is linearly polarized in y-direction

If $E_2 = 0$, the wave is linearly polarized in x-direction.

If $\delta = 0$ & $E_1 = E_2$ then also the wave is said to be linearly polarized but in a plane at an angle of 45° w.r.t x-axis

If $E_1 = E_2$ & $\delta = \pm 90°$ then the wave circularly polarized

if $\delta = +90°$ the wave is left circularly polarized &
when $\delta = -90°$ the wave is right circularly polarized.

If $t=0,\ z=0$ & $\delta=+90°$

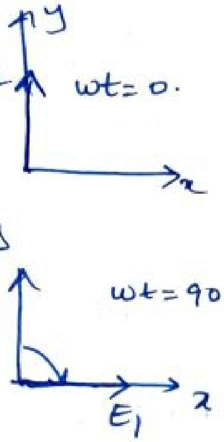
$$E = \hat{x} E_1 \sin(\omega t - \beta z) + \hat{y} E_2 \sin(\omega t - \beta z + \delta)$$

$$E = 0 + \hat{y} E_2.$$

After one quarter cycle
If $\omega t = 90°$.

then

$$E = \hat{x} E_1$$



Def:- The EM wave is CPzed at a given point in space if E(or H) field vector at that pt. traces a circle as a function of time

From Constantine A. Balanis

The necessary & sufficient conditions to accomplish are

(a) The field must have two orthogonal linear components

(b) The two components must have same amplitude

(c) These two " " " a time phase difference of odd multiples of 90°.

The necessary conditions for LP are

1) the field vector posses only one component

2) Two orthogonal linear components that are in time phase of multiples of 180°.

→ The polarization characteristics of an antenna can be represented by its polarization pattern whose definition is "the spatial distribution of the polarizations of a field vector excited (radiated) by an antenna taken over its radiation sphere.

→ At each point on the radiation sphere the polarization is usually resolved into a pair of orthogonal polarizations, the co-polarization and cross polarization.

→ Co-polarization represents the polarization of the antenna that is intended to radiate (receive) while cross polarization represents the polarization orthogonal to a specified polarization which is usually co-polarization.

Importance of Antenna Theorems:

from Jordan & Balmain pg. 345

* In circuit theory, network theorems are used to solve many problems.
* In EM field theory, the solution of antenna problem can be obtained by application Maxwell equations and appropr-iate boundary conditions.

* If the field equations are linear as long as μ, ϵ & σ of the media are truly constant, i.e do not vary; with magnitude of the signal nor with direction, the same network theorems are applied to EMT.

Superposition, Thevinin, Norton, Reciprocity, Max power transfer theory etc.

Application of Network Theorems to Antennas :-

* Antenna theorems relates the properties of transmitting & receiving antennas.
* for a transmitting antenna, the assumption of sinusoidal current distribution yields accurate result but it is not same case as with the receiving ones.
* For receiving antennas, the current distribution varies with direction of arrival of the received field and is not even sinusoidal.
* It is not obvious that the directional and impedance

Properties of an antenna should be identical for transmitting and receiving conditions.

* Hence, for receiving case, the direct computation of its properties is usually relatively complicated.

* Hence, the antenna theorems make it possible to infer the properties of a receiving antenna from its properties as a transmitting antenna and vice versa.

ANTENNA THEOREMS

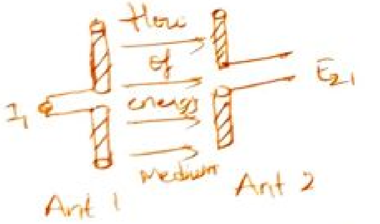
From K.D. Prasad pg. no. 556

Reciprocity Theorem: Statement

If an e.m.f is applied to the terminals of an antenna no.1 and the current measured at the terminals of another antenna no.2, then an equal current both in amplitude and phase will be obtained at the terminals of antenna no.1 if the same e.m.f is applied to the terminals of antenna no.2.

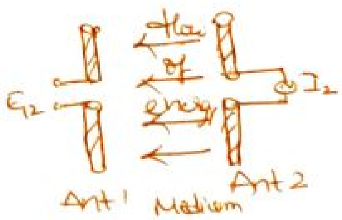
(or)

If a current I_1, at the terminals of Ant.1 induces an emf E_{21} at the open terminals of Ant.2 and a current I_2 at the terminals of Ant.2 induces an emf E_{12} at the open terminals of Ant 1. then $E_{12} = E_{21}$ provided $I_1 = I_2$.



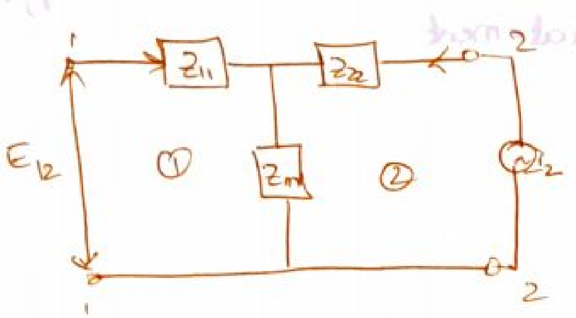
Linear, passive Isotropic medium

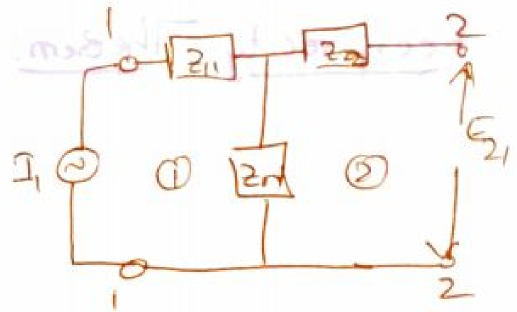
Current I_1 inducing an emf E_{21} in Ant 2



Current I_2 inducing an emf E_{12} in Ant 1.

Proof:- The space between the Ant.1 & Ant.2 are replaced by a network of linear, passive, bilateral impedance





Z_{11}, Z_{22} - Self impedance of Ant. 1 & Ant 2

Z_m - Mutual impedance of two antennas

Applying Kirchoff's mesh law

Loop 2

$$Z_{22} I_2 + Z_m (I_2 - I_1) = 0.$$

$$I_2 (Z_{22} + Z_m) = Z_m I_1$$

$$I_2 = \frac{Z_m I_1}{Z_{22} + Z_m}$$

Illy **Loop 1**

$$Z_{11} I_1 + Z_m (I_1 - I_2) = E_{12}$$

$$(Z_{11} + Z_m) I_1 - Z_m I_2 = E_{12}$$

$$(Z_{11} + Z_m) I_1 - \frac{Z_m^2 I_1}{Z_{22} + Z_m} = E_{12}.$$

$$\Rightarrow (Z_{11} + Z_m)(Z_{22} + Z_m) I_1 - Z_m^2 I_1 = E_{12} (Z_{22} + Z_m)$$

$$\Rightarrow [Z_{11} Z_{22} + (Z_{11} + Z_{22}) Z_m] I_1 = E_{12} (Z_{22} + Z_m)$$

$$I_1 = \frac{E_{12} (Z_{22} + Z_m)}{Z_{11} Z_{22} + Z_m (Z_{11} + Z_{22})}$$

$$I_2 = \frac{Z_m E_{12}}{Z_{11} Z_{22} + Z_m (Z_{11} + Z_{22})} \rightarrow (1)$$

Illy using symmetry nature we can get

$$I_1 = \frac{Z_m E_{21}}{Z_{22} Z_{11} + Z_m (Z_{22} + Z_{11})} \rightarrow (2)$$

from ① & ② it is obvious that

$$\boxed{E_{12} = E_{21} \text{ if } I_1 = I_2}$$

Hence proved.

Application of Reciprocity Theorem :-

This theorem is used to derive two important properties of transmitting and receiving antennas.

(1) Equality of directional patterns
(2) Equality of Effective lengths

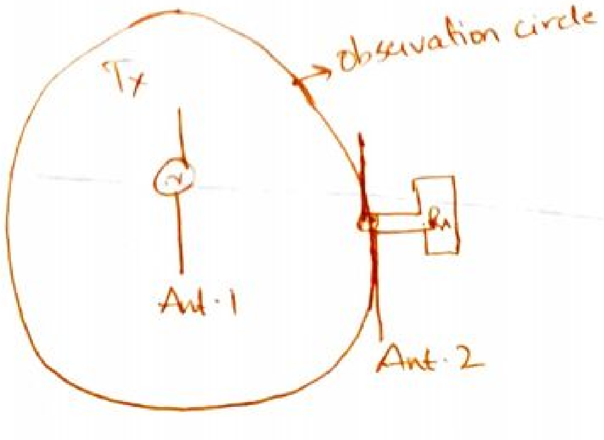
Equality of Directional patterns :-

JB pg no. 349

The directional patterns of transmitting and receiving antennas are identical if all the media are linear, passive, isotropic and reciprocity theorem holds good.

Proof :-

Consider two antennas Ant.1 & Ant.2 (exploring antenna)
for transmission ↳ for reception.



Pattern to be considered may be either field or power pattern

[∵ power pattern $\propto$ (field pattern)^2] $P = \frac{E^2}{\eta}$

Here consider field pattern,

Fix Ant.1 at centre of observation circle and Ant.2 is moved along the surface of the observation circle. This Ant.2 is always kept perpendicular to radius vector of the o/c and parallel to E-vector (for LP).

* If a voltage 'E' is applied at Ant.1 and the resulting current (I) at the terminals of Ant 2 is measured which indicates E-field at Ant.2.

* If the process is reversed i.e the same voltage 'E' is applied to Ant.2 and the resulting current I is measured at Ant.1

* According to reciprocity theorem, for every position of Ant.1, E/I is the same as was in previous case.

* Hence, the radiation pattern of Ant.1 (Tx) observed by moving Ant.2 is identical with the radiation pattern obtained when Ant.2 is txing and Ant.1 is Rxing.

* A variation in the above method is that, instead of reversing the measurement procedure, the resulting current in Ant.2 is measured at a new location by changing the radius of observation circle. Whatever be the position of

Ant. 2 the ratio E/I is always same. Hence the proof of the statement.

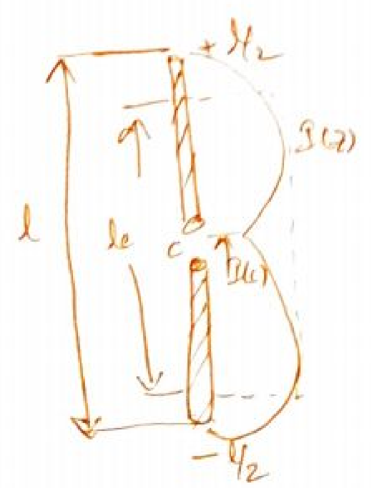
Note:-
If the radiation is elliptically polarized, then it has both E_θ & E_ϕ components which are out of phase. In that case, radiation pattern for each polarization (θ & ϕ pol.) is plotted seperately by keeping receiving (exploring) antenna parallel to polarization

Equality of Effective lengths:-

J&B pg no. 351
K. D. prasad

The value of maximum effective aperture is same for an antenna whether it is transmitting or receiving

For Transmitting antenna Effective length is defined as that length of an equivalent linear antenna that has current $I(c)$ at all points along its length and that radiates the same electric field strength as the actual anteena



(or) It can also be defined as the moment of current distribution of txing antenna divided by i/p current. where, the moment of current distribution is the sum of the moments of its current elements.

Symbolically

$$I(c)\, l_{et} = \int_{-l/2}^{l/2} I(z)\, dz$$

l_{et} → effective length of txing antenna

l → actual length

$$l_{et} = \frac{1}{I(c)} \int_{-l/2}^{l/2} I(z)\,dz.$$

* If an emf 'E' is applied at pt. 'C', then currents I(c) at centre pt. 'C' & I(z) at any point will be produced along the antenna. Then current at centre point 'C' is

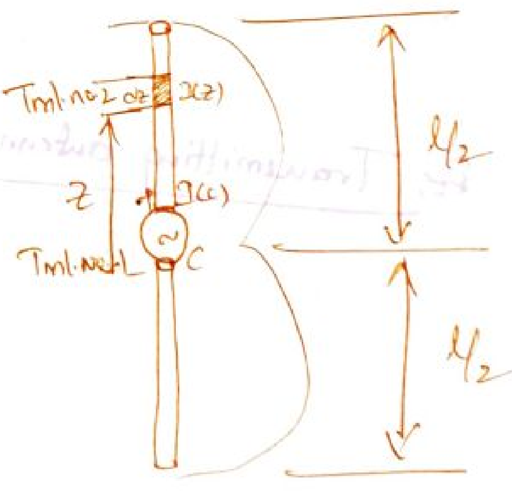
$$\boxed{I(c) = \frac{E}{Z_t}} \quad \text{(a)}$$

$Z_t \rightarrow$ Antenna terminal impedance

* Reciprocity thm, if E_{12} is applied at terminal no. 1 by shorting the tml. no. 2 (at which I_2 is produced) then

$E = E_{12}$

$I(z) = I_2$ — (1)

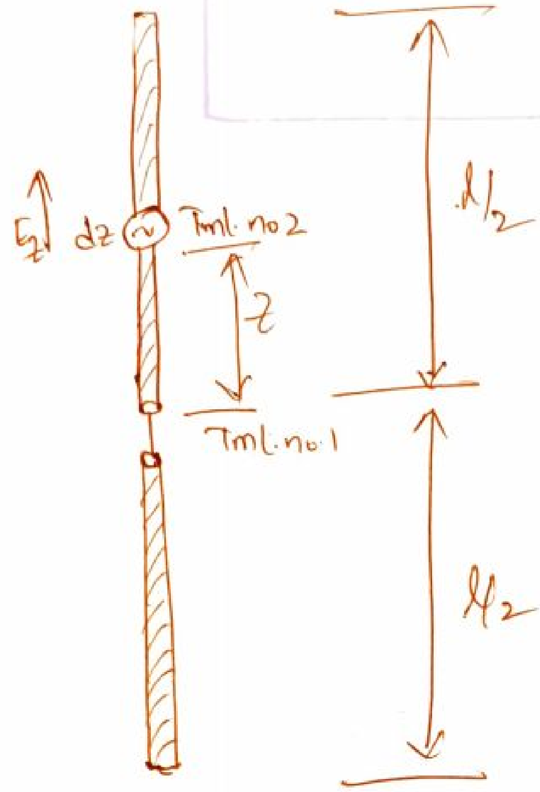


For receiving antenna the effective length is defined as the ratio of open circuit voltage developed at the terminals of antenna and the received field strength

$$l_{er} = \frac{\text{Open Circuit voltage } (V_{oc})}{\text{Electric field Strength } (E)}$$

$$\boxed{V_{oc} = l_{er} E}$$

* Now take the same transmitting antenna for receiving also with tml. no. 1 short circuited.



* Let an EM field of strength E_{zi} is incident on antenna which induces $E_{zi}\,dz$ voltage in the element dz situated (at THL.2) at a distance z from centre.

* Reciprocity thm, the voltage E_{21} is applied at tml. no. 2 by shorting Tml. no 1 (short circuited current dI_{sc} is produced) at which

$$E_{zi}\,dz = E_{21} \quad \longrightarrow (2)$$
$$dI_{sc} = I_1$$

By reciprocity thm

$$\frac{E_{12}}{I_2} = \frac{E_{21}}{\cancel{E_1} I_1}$$

from ①

$$\frac{E}{I(z)} = \frac{E_{zi}\,dz}{dI_{sc}}$$

$$\boxed{dI_{sc} = \frac{E_{zi}\,dz\,I(z)}{E}}$$

Superposition thm, total short circuited current produced at the terminals of receiving antenna is obtained by

$$\int dI_{sc} = \int_{-l/2}^{l/2} \frac{E_{zi}\,dz\,I(z)}{E}$$

and open circuit volta[ge]

$V_{oc} = I_{sc} Z_t$.

$$V_{oc} = Z_t I_{sc} = \frac{Z_t}{E} \int_{-l/2}^{l/2} E_{zi}\, I(z)\, dz$$

For a constant field along entire length of antenna $E_{zi} = E_z$

from eqn ②

$$V_{oc} = \frac{E_z}{I(c)} \int_{-l/2}^{l/2} I(z)\, dz$$

$$\frac{V_{oc}}{E_z} = \frac{\int_{-l/2}^{l/2} I(z)\,dz}{I(c)}$$

$$\boxed{l_{er} = l_{et}}$$

Hence proved.

From Jordan & Balmain
Pg no. 317

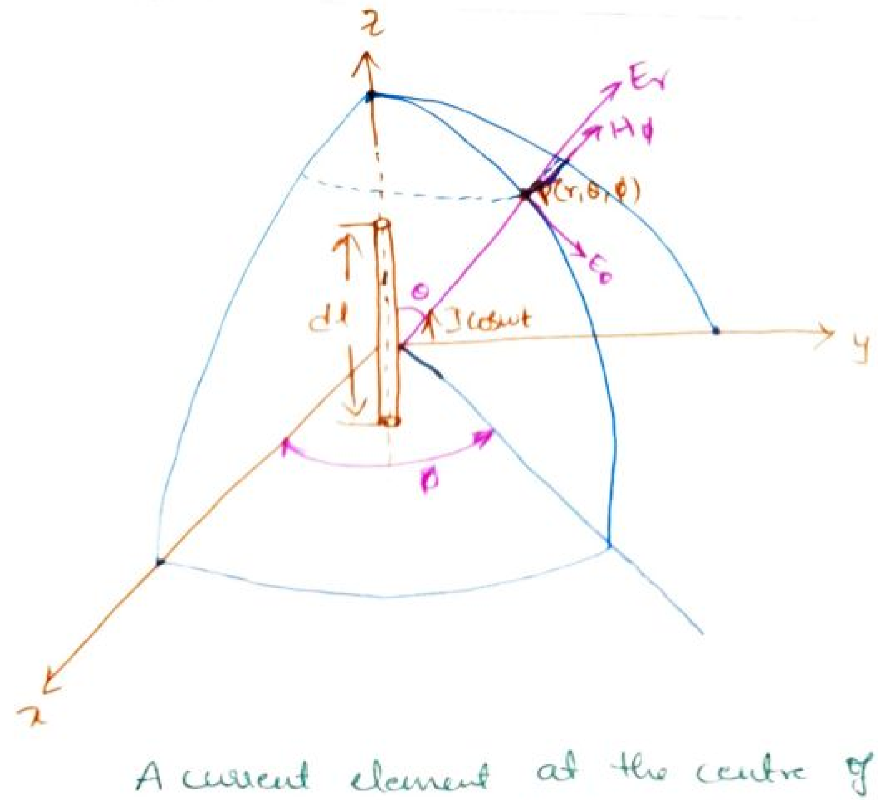
The Alternating Current element (or Oscillating Electric Dipole) :-

→ The application of retarded vector potential occurs in the calculation of EM field of an alternating current element in free space.

→ Consider a current element Idl refers to a current I flowing in a very short length (dl) of thin wire. The wire so short that the current is essentially constant along the length.

→ Such current elements in real do not exist hence an antenna carrying current may be considered to consist of a large no. of such elements joined end-to-end.

* Consider a current element at the centre of a spherical system
* Aim is to calculate EM field at an arbitrary point 'P' due to the current element Idl cos(ωt).



A current element at the centre of the Spherical coordinate system

At first, the magnetic vector potential 'A' has to be calculated at 'P'.

$$A(r) = \frac{\mu}{4\pi} \int \frac{J(t - r/v)}{r} \, dv'$$

The above integration over volume consists of integration over cross sectional area of wire and integration along its length. But

$$\int_A J = I \quad (\because \text{integration of } J \text{ over cross sectional area is } I)$$

and 'I' is assumed to be constant along its length

hence $\int_l I = I\,dl$ ($\because$ integration over the length gives $I\,dl$).

$$\therefore \int \frac{J(t - r/v)}{r} \, dv' = I\,dl.$$

Hence the vector potential 'A' becomes A_z ($\because$ the 'A' has same direction as the current element and is retarded in time by r/v sec.

$$A_z = \frac{\mu}{4\pi} \frac{I\,dl \cos[\omega(t - r/v)]}{r}$$

Hence 'H' can be obtained by using $\mu H = \nabla \times A$.

$\nabla \times A$ in spherical coordinates is

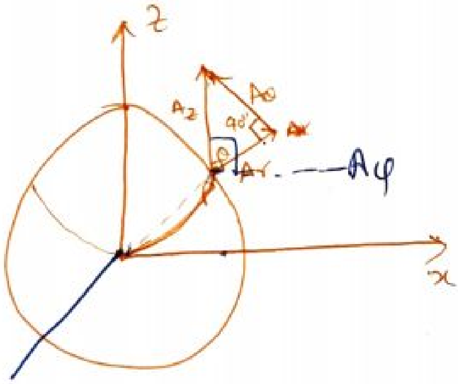
$$= \frac{1}{r^2 \sin\theta} \begin{vmatrix} e_r & r e_\theta & e_\phi r \sin\theta \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_r & r A_\theta & r \sin\theta A_\phi \end{vmatrix}$$

$$(\nabla \times A)_r = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(A_\phi \sin\theta) - \frac{\partial A_\theta}{\partial\phi}\right]$$

$$(\nabla \times A)_\theta = \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\phi} - \frac{\partial}{\partial r}(A_\phi r)\right]$$

$$(\nabla \times A)_\phi = \frac{1}{r}\left[\frac{\partial}{\partial r}(A_\theta r) - \frac{\partial A_r}{\partial\theta}\right]$$

Due to the spherical symmetry, the field is symmetrical w.r.t 'ϕ' so $\frac{\partial}{\partial\phi} = 0$.



$A_r = A_z \cos\theta$

$A_\theta = - A_z \sin\theta$

($\because A_\theta$ is in reverse dir. to $-A_\theta$)

$A_\phi = A_z \cos 90° = 0.$

$(\nabla \times A)_r = 0.$

$(\nabla \times A)_\theta = 0$

$$(\nabla \times A)_\phi = \frac{1}{r}\left[\frac{\partial}{\partial r} A_\theta r - \frac{\partial A_r}{\partial\theta}\right] = \mu H_\phi$$

$$= \frac{1}{r}\left\{\frac{\partial}{\partial r}\left[\frac{\mu I dl \cos\omega(t - r/v)}{4\pi \cancel{r}} \cdot \sin\theta\right] \cdot \cancel{r} - \frac{\partial}{\partial\theta}\left[\frac{\mu I dl \cos\omega(t - r/v)}{4\pi r}\cos\theta\right]\right]$$

$$\cancel{\mu} H_\phi = \frac{1}{r}\left[\left\{-\cancel{\mu}\frac{I dl \sin\theta}{4\pi}\left(-\sin\omega(t - r/v)\right)\left(-\frac{\omega}{v}\right)\right] + \left\{\frac{\cancel{\mu} I dl}{4\pi r}\sin\theta \cos\omega(t - r/v)\right)\right]$$

$$\boxed{H_\phi = \frac{I dl \sin\theta}{4\pi}\left[\frac{-\omega \sin\omega t'}{v r} + \frac{\cos\omega t'}{r^2}\right] \quad \text{where } t' = t - r/c}$$

Now calculate E_θ, E_ϕ, E_r from H-field using

$$\nabla \times H = \epsilon \frac{\partial E}{\partial t}.$$

$$(\nabla \times H)_r = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(H_\phi \sin\theta) - \frac{\partial H_\theta}{\partial\phi}\right] = \epsilon\frac{\partial E_r}{\partial t}$$

$$(\nabla \times H)_\theta = \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial H_r}{\partial\phi} - \frac{\partial}{\partial r}(H_\phi r)\right] = \epsilon\frac{\partial E_\theta}{\partial t}$$

$$(\nabla \times H)_\phi = \frac{1}{r}\left[\frac{\partial}{\partial r}(H_\theta r - \frac{\partial H_r}{\partial\theta}\right] = \epsilon\frac{\partial E_\phi}{\partial t}$$

H_r, H_θ are zero only H_ϕ exists.

$$(\nabla \times H)_r = \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(H_\phi \sin\theta) = \epsilon\frac{\partial E_r}{\partial t}$$

$$(\nabla \times H)_\theta = -\frac{1}{r}\frac{\partial}{\partial r}(H_\phi r) = \epsilon\frac{\partial E_\theta}{\partial t}$$

$(\nabla \times H)_\phi = \quad 0 = \frac{\partial E_\phi}{\partial t}$ i.e E_ϕ is constant or 0.

Let $E_\phi = 0$.

$t' = t - r/v$

$\frac{dt'}{dt} = 1$; $\frac{dt'}{dr} = \frac{-1}{v}$.

$$\epsilon\frac{\partial E_r}{\partial t} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}\left\{\frac{I\,dl\sin^2\theta}{4\pi}\left[-\frac{\omega\sin\omega t'}{vr} + \frac{\cos\omega t'}{r^2}\right]\right\}\right]$$

$$= \frac{1}{r\cancel{\sin\theta}}\frac{I\,dl}{4\pi}\left\{\left\{-\frac{\omega\sin\omega t'}{vr} + \frac{\cos\omega t'}{r^2}\right\}(2\cancel{\sin\theta}\cos\theta)\right\}$$

$$\frac{\partial E_r}{\partial t} = \frac{I\,dl\cos\theta}{2\pi\epsilon}\left[-\frac{\omega\sin\omega t'}{vr^2} + \frac{\cos\omega t'}{r^3}\right]$$

$$E_r = \int (\;)\,dt$$

$$E_r = \frac{Idl\cos\theta}{2\pi\epsilon}\left[\frac{\cancel{\omega}\cos\omega t'}{v r^2 \cancel{\omega}} + \frac{\sin\omega t'}{\omega r^3}\right].$$

$$\boxed{E_r = \frac{Idl\cos\theta}{2\pi\epsilon}\left[\frac{\cos\omega t'}{v r^2} + \frac{\sin\omega t'}{\omega r^3}\right].}$$

Illy

$$\epsilon\frac{\partial E_\theta}{\partial t} = \frac{-1}{r}\frac{\partial}{\partial r}\left[\frac{Idl\sin\theta}{4\pi}\left\{-\frac{\omega\sin\omega t'}{v\cancel{r}} + \frac{\cos\omega t'}{r\cancel{r}}\right\}\cdot\cancel{r}\right]$$

$$= \frac{1}{r}\frac{\partial}{\partial r}\left[\frac{Idl\sin\theta}{4\pi}\left\{\frac{\omega\sin\omega t'}{v} - \frac{\cos\omega t'}{r}\right\}\right].$$

$$= \frac{Idl\sin\theta}{4\pi r}\left[\frac{\omega}{v}\cos\omega t'(-\omega/v) - \left\{\frac{r(-\sin\omega t'(-\omega/v)) - \cos\omega t'}{r^2}\right\}\right.$$

$$\frac{\partial E_\theta}{\partial t} = \frac{Idl\sin\theta}{4\pi\epsilon r}\left[-\frac{\omega^2}{v^2}\cos\omega t' - \frac{\omega}{vr}\sin\omega t' + \frac{\cos\omega t'}{r^2}\right].$$

$$E_\theta = \int(\ )\,dt$$

$$= \frac{Idl\sin\theta}{4\pi\epsilon}\left[-\frac{\omega^2}{v^2}\frac{\sin\omega t'}{\omega r} + \frac{\omega}{vr^2}\frac{\cos\omega t'}{\omega} + \frac{\sin\omega t'}{r^3\omega}\right]$$

$$\boxed{E_\theta = \frac{Idl\sin\theta}{4\pi\epsilon}\left[-\frac{\omega\sin\omega t'}{v^2 r} + \frac{\cos\omega t'}{v r^2} + \frac{\sin\omega t'}{\omega r^3}\right].}$$

where $t' = t - r/v$

Only E_r, E_θ & H_ϕ components exists in the current element or Hertzian dipole.

Each term in the field expressions E_r, E_θ & H_ϕ has a significan

* For eg, Consider H_ϕ expression. It has two terms, one varies inversely as 'r' and other as 'r^2'.

* The term that varies inversely as 'r^2' is called the induction field. It predominates at points close to the current element (r is small)

* when 'r' is large for large distances, the second term becomes negligible (r^2 term). The first term exist and is called radiation & distant field.

* It is important to have the value of 'r' at which amplitudes of both the fields are equal (i.e induction & radiation)

$$\frac{1}{r^2} = \frac{\omega}{r v}$$

$$r = \frac{v}{\omega} = \frac{\lambda}{2\pi} \simeq \frac{\lambda}{6}.$$

* The induction term is $\frac{I dl \sin\theta \cos\omega t'}{4\pi r^2}$.

This is just the magnetic field strength given by Biot-Savart law, except for the fact that t' has replaced t. which accounts for finite time of propagation.

* However, at points close to current element the value r/v becomes small and $t' \simeq t$.

* The first term "the radiation field" is not present for steady currents. It results from the fact of finite time of propagation.

* The radiation term contributes to flow of energy away from the source whereas the induction term contributes to energy storage in the field during one quarter cycle and returned to the circuit during the next quarter cycle.

* E_θ, ~~B_ϕ~~ also has an induction $(\frac{1}{r^2})$ term and radiation $(\frac{1}{r})$ term and E_r has $\frac{1}{r^2}$ term. But both have $\frac{1}{r^3}$ term which is called as the electrostatic field terms due to its similarity with the components of field of an electro static dipole. This term is dominate at points very close to current element (source).

Jordan & Balmain
Pg. no - 321

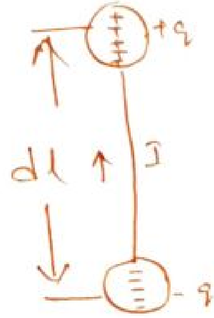
Relation between Current element and a dipole :-

* For an alternating current element

$$\frac{dq}{dt} = I \cos\omega t$$

i.e charge at one end is increasing & at the other end decreasing, by the amount of the current flow.

* In order to obtain a physical approximation of an isolated current element, one could terminate the current element in two small spheres or disks on which the charge accumulates.



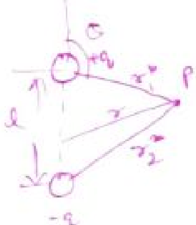
Hertzian dipole

* If the wire is very thin compared to the radius of the spheres, the current in the wire will be uniform. For this the radii of the spheres must be small compared with dl, their distance apart and also $dl << \lambda$.

Then the arrangement is original Hertzian oscillating electric dipole.

$$\frac{dq}{dt} = I\cos\omega t \quad \therefore q = \frac{I\sin\omega t}{\omega}.$$

For an electrostatic dipole, the E-field strength that would result from the seperated charges at the ends of current element is



The proof is given in J & B page no: 37

$$E_\theta = \frac{q\,dl\sin\theta}{4\pi\epsilon r^3} = \frac{I\,dl\sin\theta\,\sin\omega t'}{4\pi\epsilon\omega r^3}$$

$$E_r = \frac{2q\,dl\cos\theta}{4\pi\epsilon r^3} = \frac{I\,dl\cos\theta\,\sin\omega t'}{2\pi\epsilon\omega r^3}$$

$r_1 = r - \frac{l}{2}\cos\theta$

$r_2 = r + \frac{l}{2}\cos\theta$

$V = \frac{1}{4\pi\epsilon}\left[\frac{q}{r_1} - \frac{q}{r_2}\right]$

$E = -\nabla V$

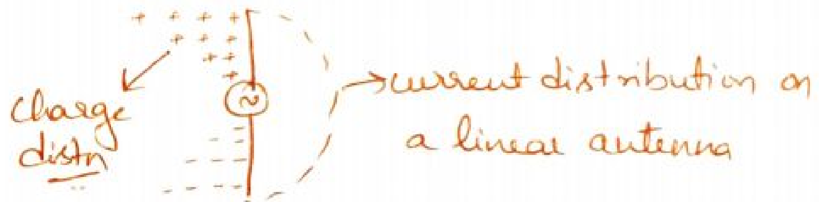
These are the same $1/r^3$ terms that appeared in the solution of EM field due to the current element.

* When a current element is a part of the complete circuit, then there is no accumulation of charge at its end if the current is uniform throughout the circuit. i.e the current from one element flows into the next.

* As there is no accumulated charge in closed circuit the term $1/r^3$ vanishes. Hence only induction & radiation fields are present.

* The +ve charge at the end of one dipole is just cancelled by an equal amount of -ve charge at the opposite end of adjacent dipole.

Chain of Hertzian dipoles



* If the current is not uniform along its length, then the distribution can be represented as chain of current elements of Hertzian dipoles having different amplitudes.
* In this case, the charges in the adjacent dipoles do not cancel out each other and there is an accumulation of charge on the surface of the wire.
* These surface charges are responsible for a relatively strong component of E-field normal to the surface of wire.

Power radiated by a current element :-

* The power flow per unit area at the point 'P' will be given by the Poynting vector at that point.

$$\hat{P}_{(r,\theta,\phi)} = \hat{E} \times \hat{H} = \begin{vmatrix} \hat{r} & \hat{\theta} & \hat{\phi} \\ E_r & E_\theta & E_\phi \\ H_r & H_\theta & H_\phi \end{vmatrix}$$

$$E_\phi = 0 , \quad H_r = H_\theta = 0 .$$

$$\hat{P} = \hat{i}(E_\theta H_\phi) - \hat{j} E_r H_\phi + 0$$

$$P_r = E_\theta H_\phi$$

$$= \left\{ \frac{I dl \sin\theta}{4\pi\epsilon} \left[\frac{-\omega \sin\omega t'}{v^2 r} + \frac{\cos\omega t'}{v r^2} + \frac{\sin\omega t'}{\omega r^3} \right] \right\} \left\{ \frac{I dl \sin\theta}{4\pi} \left[-\frac{\omega \sin\omega t'}{v r} + \frac{\cos\omega t'}{r^2} \right] \right\}$$

$$= \frac{I^2 dl^2 \sin^2\theta}{(4\pi)^2 \epsilon} \left[\underbrace{\frac{\omega^2 \sin^2\omega t'}{v^3 r^2}}_{\frac{1-\cos 2\omega t'}{2}} - \underbrace{\frac{\omega \sin\omega t' \cos\omega t'}{v^2 r^3} - \frac{\omega \sin\omega t' \cos\omega t'}{v^2 r^3}}_{\sin 2\omega t'} + \underbrace{\frac{\cos^2\omega t'}{v r^4} - \frac{\sin^2\omega t'}{v r^4}}_{\cos 2\omega t'} + \underbrace{\frac{\cos\omega t' \sin\omega t'}{\omega r^5}}_{\frac{\sin 2\omega t'}{2}} \right]$$

$$\boxed{P_r = \frac{I^2 dl^2 \sin^2\theta}{(4\pi)^2 \epsilon} \left[\frac{\sin 2\omega t'}{2\omega r^5} + \frac{\cos 2\omega t'}{v r^4} - \frac{\omega \sin 2\omega t'}{v^2 r^3} + \frac{\omega^2 (1 - \cos 2\omega t')}{2 r^2 v^3} \right]}$$

Illy

$$P_\theta = - E_r H_\phi$$

$$= \left\{ (-) \frac{I dl \cos\theta}{2\pi\epsilon} \left[\frac{\cos\omega t'}{v r^2} + \frac{\sin\omega t'}{\omega r^3} \right] \right\} \left\{ \frac{I dl \sin\theta}{4\pi} \left[\frac{-\omega \sin\omega t'}{v r} + \frac{\cos\omega t'}{r^2} \right] \right\}$$

$$= - \frac{2 I^2 dl^2 \sin\theta\cos\theta}{(4\pi)^2 \epsilon} \left[\frac{-\omega \cos\omega t' \sin\omega t'}{v^2 r^3} + \frac{\cos^2\omega t'}{v r^4} - \frac{\sin^2\omega t'}{v r^4} + \frac{\cos\omega t' \sin\omega t'}{\omega r^5} \right]$$

$$= -\frac{I^2 dl^2 \sin 2\theta}{16\pi^2 \epsilon}\left[-\frac{\omega \sin 2\omega t'}{2r^3 v^2} + \frac{\cos 2\omega t'}{r^4 v} + \frac{\sin(2\omega t')}{2\omega r^5}\right]$$

$$\boxed{P_\theta = \frac{I^2 dl^2 \sin 2\theta}{16\pi^2 \epsilon}\left[-\frac{\cos 2\omega t'}{v r^4} - \frac{\sin 2\omega t'}{2\omega r^5} + \frac{\omega \sin 2\omega t'}{2r^3 v^2}\right].}$$

* The average value of $\sin 2\omega t'$ or $\cos 2\omega t'$ over a complete cycle is zero. Hence for any 'r', the $P_{\theta\,avg}$ over a complete cycle is zero.

* P_θ represents only a surging back & forth of power in θ-direction without any net or average flow.

Ily the average value of radial Poynting vector over a cycle will be due to part of final term

$$P_{r(avg)} = \frac{\omega^2 I^2 dl^2 \sin^2\theta}{32\pi^2 r^2 v^3 \epsilon} = \left(\frac{\omega I dl \sin\theta}{4\pi r v}\right)^2 \cdot \frac{1}{2v\epsilon}$$

$\frac{1}{v\epsilon} = ?$ we know $\eta = \sqrt{\mu/\epsilon}$ & $v = \frac{1}{\sqrt{\mu\epsilon}}$ $\Rightarrow$ $\boxed{\frac{\sqrt{\mu\epsilon}}{\epsilon} = \frac{1}{v\epsilon} = \sqrt{\frac{\mu}{\epsilon}} = \eta}$

$$\boxed{P_{r(avg)} = \frac{\eta}{2}\left(\frac{\omega I dl \sin\theta}{4\pi r v}\right)^2 \text{ watts/sq.m}}$$

* The only terms of E & H that contribute to this average power flow are the radiation terms.

* At large distances from the source only these terms have an appreciable value but even for the points close to the current ele where $1/r^2$ & $\frac{1}{r^3}$ terms predominates, the $1/r$ term contribute to an average outward flow of power.

* The amplitudes of radiation fields of an electric current element Idl are

$$E_\theta = \frac{\omega I dl \sin\theta}{4\pi\epsilon r v^2} = \frac{\cancel{\omega}\, \eta I dl \sin\theta}{2\cancel{4}\pi r \left(\frac{\omega}{2\pi}\lambda\right)} \qquad \left(\because \frac{1}{v\epsilon} = \eta\right)$$

$v = f\lambda$

$\omega = 2\pi f$

$v = \frac{\omega}{2\pi}\lambda$

$$E_\theta = \frac{\eta I dl \sin\theta}{2 r\lambda} \rightarrow (1)$$

Illy $H_\phi = \dfrac{\omega I dl \sin\theta}{4\pi v r} = \dfrac{I dl \sin\theta}{2 r\lambda} \rightarrow (2)$

from (1) & (2)

The radiation terms are related by $\boxed{\dfrac{E_\theta}{H_\phi} = \eta.}$

* The total power radiated by the current element can be computed by integrating the radiat Poynting vector over a spherical surface centred at the element.

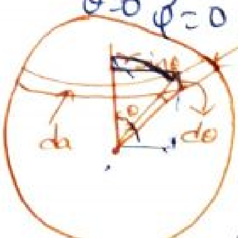
* 'P' is independent of azimuthal angle 'ϕ' so the element of area on the spherical shell will be taken as the strip da where $da = \int (r d\theta)(r\sin\theta\, d\phi)$

r
θ
ϕ

$$A = \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} r^2 \sin\theta\, d\theta\, d\phi$$

$$\boxed{\begin{aligned} dl_1 &= r d\theta \\ dl_2 &= dr \\ dl_3 &= r\sin\theta\, d\phi \end{aligned}}$$

$\int_0^{2\pi} d\phi = 2\pi \quad \therefore da = 2\pi r^2 \sin\theta\, d\theta.$



$ds = da$

$$\text{Power} = \oint_{\text{surface}} P_{r(av)}\, da = \int_0^{\pi} \frac{\eta}{\cancel{2}} \left(\frac{\omega I dl \sin\theta}{4\pi r c}\right)^2 \cancel{2}\pi r^2 \sin\theta\, d\theta$$

dx
dy
dz

$$= \frac{\eta \omega^2 I^2 dl^2}{16\pi c^2} \int_0^{\pi} \sin^3\theta\, d\theta$$

$dx\,dy$
$dy\,dz$
$dx\,dz$

$$= \frac{\eta\omega^2 I^2 dl^2}{\cancel{4}16\pi c^2} \times \frac{\cancel{4}}{3} = \frac{\eta \omega^2 I^2 dl^2}{12\pi c^2}$$

$\sin 3\theta = 3\sin\theta - 4\sin^3\theta.$

$$\int_0^{\pi} \sin^3\theta = \int_0^{\pi} \frac{1}{4}\left[3\sin\theta - \sin 3\theta\right]$$

$$= -\frac{3}{4}\cos\theta + \frac{\cos 3\theta}{12}$$

$$= -\frac{3}{4}\left\{[-1-1] + \frac{1}{12}[-1-1]\right\}$$

$$= +\frac{2\times 3}{4} - \frac{2}{12} =$$

$$= \frac{2(9-1)}{12} = \frac{2\times 8}{12} = 4/3$$

'I' is the peak current $I_{eff} = I/\sqrt{2}$

$$\text{Power} = \frac{\eta \omega^2 I_{eff}^2 \, dl^2}{6\pi v^2}$$

$$= \frac{\cancel{120}^{20}\pi \times 4\pi f^2 I_{eff}^2 \, dl^2}{\cancel{6}\cancel{\pi} \lambda^2}$$

$$= \underbrace{80\pi^2 \left(\frac{dl}{\lambda}\right)^2}\, I_{eff}^2$$

↳ The coefficient of I_{eff}^2 has the dimensions of resistance and is called radiation resistance of current element.

$$\boxed{R_{rad} = 80\pi^2 \left(\frac{dl}{\lambda}\right)^2.}$$

Application to short Antennas :-

So far we derived equations for E_r, E_θ, H_ϕ, P_r, P_θ for oscillating current element which is hypothetical but not for practical antenna.

* The practical 'elementary dipole' is a centre fed antenna having a length that is very short in wavelengths. (usually multiples of λ i.e $\lambda/8$, $\lambda/4$, $\lambda/2$, $3\lambda/4$, λ, $3\lambda/2$, 2λ etc.)

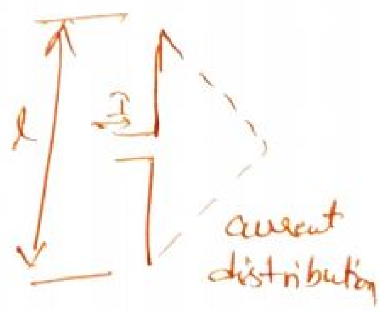
* The current amplitude on such an antenna decreases uniformly from a maximum at the centre to zero at ends.

* For the same current I (at the terminals) the practical dipole of length 'l' will radiate only one quarter as much power as current element of same length, which has the current I throughout its entire length. ($\because$ the field strengths at every point are reduced to one-half & the power density will be reduced to $1/4^{th}$).

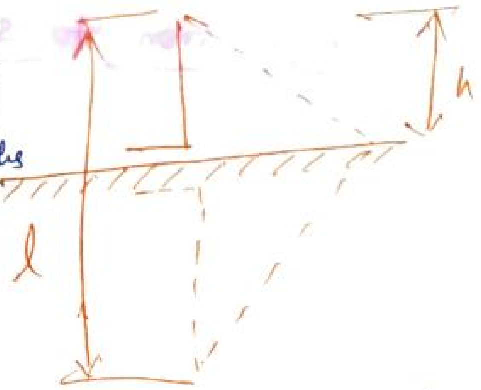
The radiation resistance of short dipole is one quarter that of the current element.

$$R_{rad}\ (\text{Short dipole}) = 20\pi^2 (l/\lambda)^2$$

$$\simeq 200 (l/\lambda)^2\ \Omega.$$



* The monopole of height 'h' or short vertical antenna mounted on a reflecting plane produces the same field strengths above the plane as does the dipole of length $l = 2h$



* It radiates only through the hemispherical surface above the plane, so its radiated power is one half of the corresponding dipole

$$R_{rad} = 10\pi^2 (l/\lambda)^2$$
$$= 40\pi^2 (h/\lambda)^2$$
$$\simeq 400 (h/\lambda)^2.$$

* These formulae hold strictly for very short antennas only, but they are good approximations for dipoles of lengths upto quarter wavelength, and monopoles of heights upto one eigth wavelength.

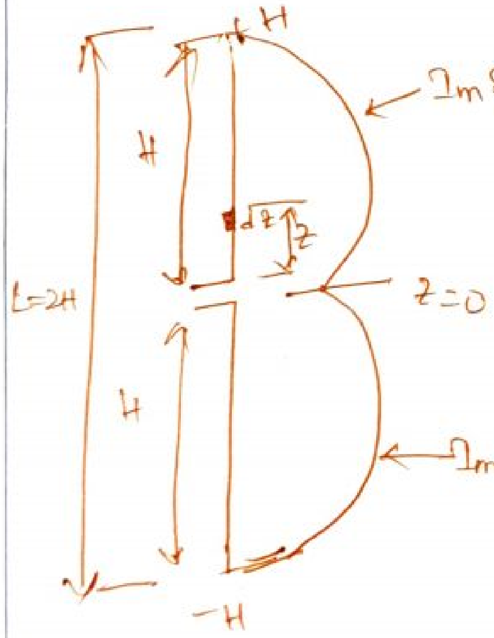
Assumed current distribution:-

* In order to calculate the EM fields of longer antenna it is necessary to know the current distribution along the antennas.
* This information should be obtainable using Maxwell's equations subject to the appropriate boundary conditions along the antenna.

* In the absence of a known antenna current, we can assume a certain distribution and from that to calculate approximate field distributions.

* The accuracy of the fields so calculated will depend upon how good an assumption was made for current distribution.

* For centre fed antenna, a sinusoidal current distribution with current nodes at the ends is suggested.

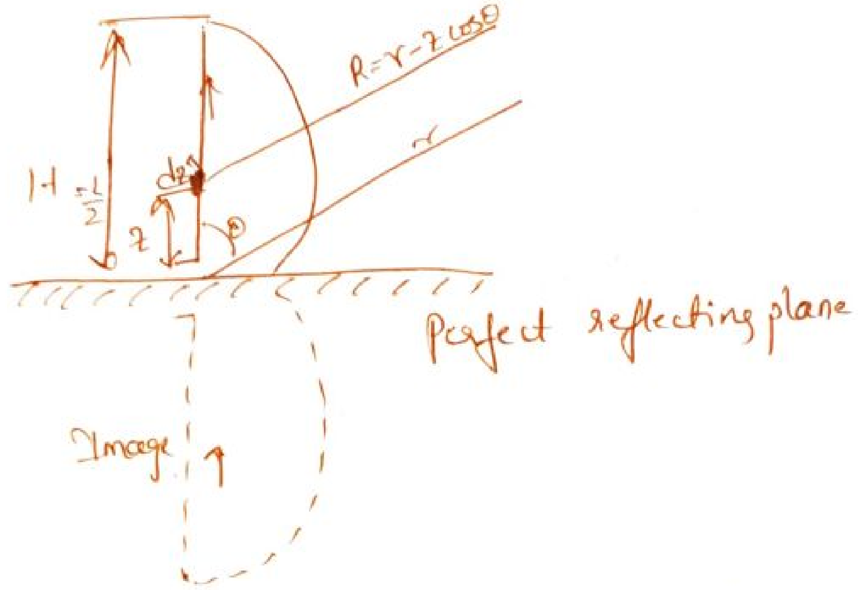


centre fed dipole with sine current distribution.

* A dipole antenna is a straight radiator, usually fed in the centre and producing a maximum of radiation in the plane normal to the axis.

* The vertical antenna of $H = L/2$ length fed against an infinitely large perfectly conducting plane has the same radiation characteristics above the plane as does the dipole antenna of length 'L' in free space. This is because the fields due to a current element $I\,dz$ when

reflected from the plane, appear to originate at an image element located beneath the plane.



Radiation from a quarter wave monopole or Half wave dipole

The current is sinusoidally distributed.

Then
$$\boxed{\begin{aligned} I &= I_m \sin\beta(H-z) \quad z>0 \\ &= I_m \sin\beta(H+z) \quad z<0. \end{aligned}}$$

where I_m is the value of current at maximum.

The expression for Vector potential at a point 'P' due to current element Idz will be

$$dA_z = \frac{\mu I e^{-j\beta R} dz}{4\pi R}.$$

'R' is distance from current element to 'P'.

The total vector potential at 'P' due to all current elements is

$$A_z = \frac{\mu}{4\pi}\int_{-H}^{0} \frac{I_m \sin\beta(H+z)}{R} e^{-j\beta R} dz + \int_{0}^{H} \frac{I_m \sin\beta(H-z) e^{-j\beta R}}{R} dz$$

Because only the radiation fields are required, the inverse distance factor (R in denominator) can be approximated to r

$$R \simeq r.$$

In the numerator 'R' in the phase factor is the difference between R & r.

but for very large values of 'R'

$$R = r - z\cos\theta.$$

$$R - r = -z\cos\theta.$$

$$\therefore A_z = \frac{\mu I_m e^{-j\beta r}}{4\pi r}\left[\int_{-H}^{0} \sin\beta(H+z)e^{j\beta z\cos\theta}\,dz + \int_{0}^{H} \sin\beta(H-z)e^{j\beta z\cos\theta}\,dz\right]$$

For the particular case of $H = \lambda/4$

$$\sin\beta(H+z) = \sin\beta(H-z)$$

$$\sin\frac{2\pi}{\lambda}(\lambda/4 + z) \qquad \sin(\pi/2 - \beta z)$$

$$= \sin(\pi/2 + \beta z) \qquad \cos\beta z$$

$$= \cos\beta z$$

$$A_z = \frac{\mu I_m e^{-j\beta r}}{4\pi r}\int_{0}^{H} \cos\beta z\,(\underbrace{e^{j\beta z\cos\theta} + e^{-j\beta z\cos\theta}}_{2\cos(\beta z\cos\theta)})\,dz$$

because the order of integration is reversed.

$$= \frac{\mu I_m e^{-j\beta r}}{4\pi r}\int_{0}^{H} 2\cos\beta z\cos(\beta z\cos\theta)\,dz.$$

Using $2\cos A\cos B = \cos(A-B) + \cos(A+B)$.

$$A_z = \frac{\mu I_m e^{-j\beta r}}{4\pi r}\int_{0}^{H}\left[\cos(\beta z(1+\cos\theta)) + \cos(\beta z(1-\cos\theta))\right]dz$$

$$= \frac{\mu I_m e^{-j\beta r}}{4\pi r}\left[\frac{\sin(\beta z(1+\cos\theta))}{\beta(1+\cos\theta)} + \frac{\sin(\beta z(1-\cos\theta))}{\beta(1-\cos\theta)}\right]_{0}^{\lambda/4}$$

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$$= \frac{\mu I_m e^{-j\beta r}}{4\pi r\beta}\left[\frac{(1-\cos\theta)\sin[\beta z(1+\cos\theta)] + (1+\cos\theta)\sin[\beta z(1-\cos\theta)]}{1-\cos^2\theta}\right]_0^{\lambda/4}$$

$$= \frac{\mu I_m e^{-j\beta r}}{4\pi\beta r}\left[\frac{(1-\cos\theta)\sin\left(\frac{2\pi}{\lambda}\cdot\frac{\lambda}{4}(1+\cos\theta)\right] + (1+\cos\theta)\sin\left[\frac{\pi}{2}(1-\cos\theta)\right]}{1-\cos^2\theta}\right]$$

$$= \frac{\mu I_m e^{-j\beta r}}{4\pi\beta r}\left[\frac{(1-\cos\theta)\cos(\pi/2\cos\theta) + (1+\cos\theta)\cos(\pi/2\cos\theta)}{\sin^2\theta}\right]$$

$$= \frac{\mu I_m e^{-j\beta r}}{4\pi\beta r}\left[\frac{2\cos(\pi/2\cos\theta)}{\sin^2\theta}\right]$$

$$A_z = \frac{\mu I_m e^{-j\beta r}}{2\pi\beta r}\left[\frac{\cos(\pi/2\cos\theta)}{\sin^2\theta}\right].$$

Find H.

$$\nabla\times A = \mu H.$$

when only the radiation field is considered.

$$\mu H_\phi = (\nabla\times A)_\phi = \frac{1}{r}\left[\frac{\partial}{\partial r}(A_\theta\cdot r)\right].$$

$$A_\theta = -A_z\sin\theta \quad = -\frac{\sin\theta}{r}\frac{\partial(A_z\cdot r)}{\partial r}$$

$$\mu H_\phi = -\frac{1}{r}\left[\frac{\partial}{\partial r}\frac{\mu I_m e^{-j\beta r}}{2\pi\beta r}\frac{\cos(\pi/2\cos\theta)}{\sin^2\theta}\cdot\sin\theta\cdot r\right].$$

$$= \frac{-1}{r}\left[\frac{\mu I_m}{2\pi\beta}\frac{\cos(\pi/2\cos\theta)}{\sin\theta}\right]\frac{\partial}{\partial r}e^{-j\beta r} = \frac{-\mu I_m}{2\pi\beta r}\frac{\cos(\pi/2\cos\theta)}{\sin\theta}\cdot e^{-j\beta r}(-j\beta)$$

$$\boxed{H_\phi = \frac{j I_m e^{-j\beta r}}{2\pi r}\left[\frac{\cos(\pi/2 \cos\theta)}{\sin\theta}\right]}$$

& the electric field strength for the radiation field is.

$$E_\theta = \eta H_\phi \qquad \eta = 120\pi.$$

$$E_\theta = \frac{j 60 I_m e^{-j\beta r}}{r}\left[\frac{\cos(\pi/2 \cos\theta)}{\sin\theta}\right]$$

$$|E_\theta| = \frac{60 I_m}{r} \frac{\cos(\pi/2 \cos\theta)}{\sin\theta}. \quad V/m.$$

E_θ & H_ϕ are in time phase and maximum value of Poynting vector is.

$$P_{max} = |E_\theta||H_\phi|.$$

$$P_{av} = \frac{|E_\theta|}{\sqrt{2}}\frac{|H_\phi|}{\sqrt{2}} = \frac{P_{max}}{2}.$$

$$P_{max} = \eta \frac{I_m^2}{4\pi^2 r^2} \frac{\cos^2(\pi/2 \cos\theta)}{\sin^2\theta} \qquad P_{av} = \frac{\eta I_m^2}{8\pi^2 r^2} \frac{\cos^2(\pi/2 \cos\theta)}{\sin^2\theta} \quad W/m^2$$

Total power radiated from a $\lambda/2$ antenna = Surface integral of Poynting vector over a hemispherical surface of radius r.

$I_{rms} = I_m/\sqrt{2}$.

$$W = \oint P_{av} da = \int_0^{\pi} \frac{30 I_{rms}^2}{\cancel{\pi r^2}} \frac{\cos^2(\pi/2 \cos\theta)}{\sin^{\cancel{2}}\theta} \cdot 2\cancel{\pi r^2} \cancel{\sin\theta}\, d\theta$$

$$= 60 I_{rms}^2 \int_0^{\pi} \frac{1}{2} \left\{ \frac{1 + \cos(\pi \cos\theta)}{\sin\theta} \right\} d\theta .$$

By using numerical integration method or graphical method or by simpson's or trapezoidal rules

$W = 60 I_{rms}^2 (1.219)$.

$W = \underbrace{73.14}_{R_{rad}} I_{rms}^2$

$$\boxed{R_{rad} = 73.14\,\Omega \approx 73\,\Omega.}$$

$\therefore$ Radiation resistance of a centre fed half dipole or simply dipole antenna is approximately 73Ω.

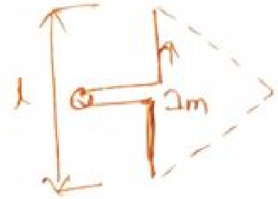
For a quarter wave monopole, the R_{rad} is half the dipole radiation resistance i.e $73/2 \approx 36.5\,\Omega$.

* In practice the radiation resistance is a function of height above the ground and 73Ω will have a tendency to come down and it may range anything between 60Ω to 90Ω.

Effective length or Radiation height (l_e) of linear antennas

(a) Linear current distribution :-

The amplitude of radiative field term in the expression of H_ϕ for an oscillating current in current element is given by.



$$H_\phi = -\frac{I_m dl \sin\theta}{2\lambda r} \sin\omega t'.$$

$$|H_\phi| = \frac{I_m dl}{2\lambda r}$$

This is not true for practical antennas due to non uniform current distribution which is either linear or sinusoidal.

This effect, reduces the amount of power radiated and makes the antenna equivalent to a shorter one with the same current.

The actual length of shorter antenna can be assessed by taking the average value of current over the entire length.

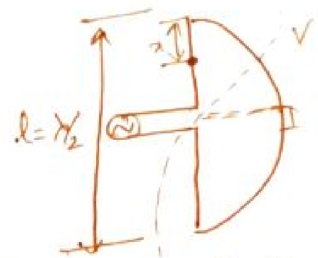
$$|H_\phi| = \frac{I_{av}\, l}{2\lambda r} \qquad I_{av} = \frac{I_m}{2}.$$

$$= \frac{\frac{I_m}{2} \cdot l}{2\lambda r} = \frac{I_m}{2\lambda r} \frac{l}{2}$$

$$|H_\phi| = \frac{I_m}{2\lambda r} l_e \qquad \Rightarrow \boxed{l_e = l/2}$$

(b) For Sinusoidal current distribution :-

The current is maximum at the centre (I_m) and it is zero at its ends.



The current distribution over the entire half of the antenna length is

$$I = I_m \sin \beta x \qquad \beta = \frac{2\pi}{\lambda}.$$

$$= I_m \sin\left(\frac{2\pi}{2l}\right).x$$

$$= I_m \sin\left(\frac{\pi x}{l}\right). \quad \text{where } x \text{ lies between } 0 \text{ to } l/2$$

This is the instantaneous current at point 'x' from the end.

The average value of this current over half the length of the antenna is.

$$I_{av} = \frac{\int_0^{\pi/2} I_m \sin\theta \, d\theta}{\int_0^{\pi/2} d\theta} = \frac{I_m \left[-\cos\theta\right]_0^{\pi/2}}{\pi/2} = \frac{I_m[+1]}{\pi/2}$$

$$I_{av} = \frac{2 I_m}{\pi}.$$

$$|H_\phi| = \frac{l}{2\lambda r} \cdot I_{av} = \frac{2I_m}{\pi} \frac{l}{2\lambda r} = \frac{I_m}{2\lambda r} \cdot l_e$$

$$\boxed{l_e = \frac{2l}{\pi}.}$$

Hence the radiation resistance can be expressed as.

$$R_r = 80\pi^2 \left(\frac{l_e}{\lambda}\right)^2 \quad \text{for oscillating current element}$$

$$= 80\pi^2 \left(\frac{1}{2}\right)^2 \left(\frac{l}{\lambda}\right)^2$$

$$= 20\pi^2 \left(\frac{l}{\lambda}\right)^2 \Omega$$

with Sinusoidal distribution

$$R_r = 80\pi^2 \left(\frac{2l}{\pi}\right)^2 \cdot \left(\frac{1}{\lambda}\right)^2$$

$$= 80\cancel{\pi^2} \cdot \frac{4}{\cancel{\pi^2}} \left(\frac{l}{\lambda}\right)^2$$

$$= 320\, l^2/\lambda^2 \ \Omega.$$

K.D. Prasad

Maximum Effective Aperture of a short dipole.

The maximum effective aperture is defined as

$$(A_e)_{max} = \frac{\text{Maximum received power } (W_{max})}{\text{Power density of incident wave } (P)}$$

$$W_{max} = I_{rms}^2 R_L.$$

$$= \frac{V^2 R_L}{(R_L + R_A)^2 + (X_L + X_A)^2}$$

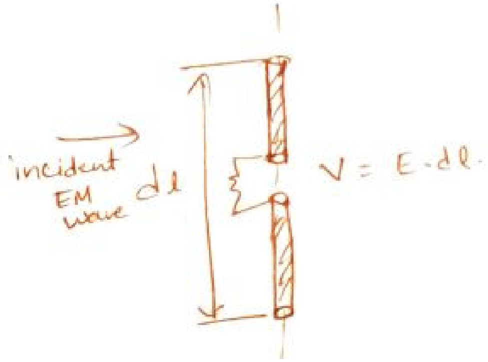
According to maximum power transfer theorem, the max power transferred to the load is when $X_L = -X_A$ & $R_L = R_A$
$= R_r + R_l$

Here we assume antenna losses to be zero.

$$R_L = R_A = R_r .$$

$$W_{max} = \frac{V^2 R_L}{4R_L^2} = \frac{V^2}{4R_L} = \frac{V^2}{4R_r} .$$

$$(A_e)_{max} = \frac{V^2}{4 P R_r}$$



'P' is obtained from poynting vector of incident wave at the short dipole

$P = \frac{E^2}{\eta}$ W/m^2.

η is the intrinsic impedance of free space.

$V = E \cdot dl$. (the induced voltage in short dipole dl)

$$R_r = 80\pi^2 \left(\frac{dl}{\lambda}\right)^2$$

$$(A_e)_{max} = \frac{(E \cdot dl)^2}{4 \times \frac{E^2}{\eta} \times 80\pi^2 \left(\frac{dl}{\lambda}\right)^2}$$

$$= \frac{\eta \lambda^2}{4 \times 80 \times \pi^2} = \frac{\overset{3}{\cancel{120\pi}} \lambda^2}{\cancel{4} \times \cancel{80} \times \pi^{\cancel{2}}} = \frac{3\lambda^2}{8\pi}$$

$(A_e)_{max} =$

The directivity of short dipole can be calculated by using

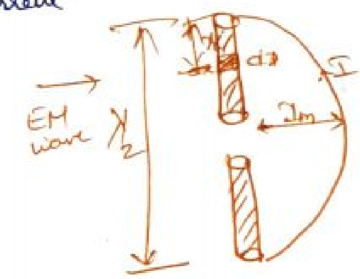
$$D = \frac{4\pi}{\lambda^2} (A_e)_{max}$$

$$D = \frac{4\pi}{\lambda^2} \times$$

$$=$$

Maximum effective aperture of λ/2 half wave dipole.

Let us assume that incident current has sinusoidal distribution and at any point 'x' from the centre the current 'I' is given by



$$I = I_m \cos \frac{2\pi x}{\lambda}$$

x lies between 0 to $\pm \lambda/4$

The above 'I' equation can be written for infinitesimally small current

$$dI = dI_m \cos \frac{2\pi x}{\lambda}.$$

The induced voltage due to dI is dV

$$dV = dV_m \cos \frac{2\pi x}{\lambda}.$$ and due to incident E-field intensity 'E' is

$$dV = E\,dx \cos \frac{2\pi x}{\lambda}. \quad \because dV_m = E\,dx.$$

Total induced voltage 'V' is obtained by 'integrating dv' over entire length '$\lambda/2$'.

$$\int dv = \int_0^{\lambda/2} E \cos\frac{2\pi x}{\lambda}\, dx$$

$$V = 2E \int_0^{\lambda/4} \cos\beta x\, dx.$$

$$= 2E \left.\frac{\sin\beta x}{\beta}\right]_0^{\lambda/4}$$

$$V = \frac{2E\lambda}{2\pi}\left[\sin\frac{2\pi}{\lambda}x\right]_0^{\lambda/4} = \frac{E\lambda}{\pi}\left[1-0\right] = \frac{E\lambda}{\pi}.$$

$$(A_e)_{max} = \frac{V^2}{4PR_r} \quad \& \quad R_r = 73\Omega$$

$$\frac{\left(\frac{E\lambda}{\pi}\right)^2}{4 \times \frac{E^2}{\eta} \times 73} = \frac{120\pi\lambda^2}{4\pi^2 \times 73} = 0.13\lambda^2.$$

To find l_e

$$(A_e)_{max} = \frac{l_e^2 \eta}{4R_r}$$

$$\boxed{(A_e)_{max} = 0.13\lambda^2}$$ → for half wave dipole

& Directivity $= \frac{4\pi}{\lambda^2} \cdot (A_e)_{max}$

$$l_e^2 = 4\left(\frac{R_r l_e}{\eta}\right)$$

$$= \frac{4\pi}{\lambda^2} \cdot \frac{120\pi\lambda^2}{4\pi^2 \times 73} \simeq \frac{120}{73} \simeq$$

$$\frac{l_e^2\eta}{4R_r}$$

$$\frac{\left(\frac{2\lambda}{\pi 2}\right)^2 \cdot 120\pi}{4 \times 73}$$

$$\boxed{D = }$$ → for half wave dipole.

$$\frac{3}{\pi}\lambda^2$$

Power radiated by a vertically earthed antenna

Power due to vertically short dipole, assuming sinusoidal current distribution is

$$P = 80\pi^2\left(\frac{l_e}{\lambda}\right)^2 \cdot I_{rms}^2 .$$

For the case of grounded vertical antenna of effective height (l_e) the apparent height will be ($2l_e$) due to image effect.

$$W = 80\pi^2 \frac{(2l_e)^2}{\lambda^2} \cdot I_{rms}^2$$

$$= 320\pi^2 \left(\frac{l_e}{\lambda}\right)^2 I_{rms}^2 \; w$$ → power radiated through a sphere at the centre of which the antenna of length ($2l_e$) is put.

It radiates through a hemisphere, power radiated by it is half of that given in the above eqn

$$W = 160\pi^2 (l_e/\lambda)^2 I_{rms}^2$$

$$R_r = 160\pi^2 (l_e/\lambda)^2 .$$

* A short vertical grounded antenna is designed to radiate at 1MHz. Cal. R_r, if eff height of antenna is 30m.

$$R_r = 160\pi^2 (l_e/\lambda)^2$$

$l_e = 30m.$ $\quad \lambda = 300\,m. \quad \frac{3\times10^8}{1\times10^6}$

~~Assignment~~

* An antenna has an effective height of 100m & the current at the base is 450 A (rms) at 40kHz. What is the radiated power? If total resistance of the antenna ckt is 1.12 Ω, what is efficiency of the antenna.

$P = R_r I_{rms}^2$

$= 160\pi^2 (l_e/\lambda)^2 \cdot (450)^2$

$f = 40kHz$

$l_e = 100m$

$\lambda = \frac{3\times10^8}{40\times10^3} = \frac{30000}{4} = 7500\ m$

$R_r = 0.2807\Omega$

$\eta = \frac{R_r}{R_r + R_l} = \frac{0.2807}{1.12} \times 100$

Assignment
* Calculate power radiated by $\lambda/16$ dipole in free space if it carries a uniform current of $I = 100\cos\omega t$ A. What is its radiation resistance.

$R_r = 80\pi^2 \times \left(\frac{\lambda}{16\lambda}\right)^2$

$80\pi^2 \times \left(\frac{1}{16}\right)^2$

$P = I_{eff}^2 R_r$ $I_m = 100$

$I_{eff} = \frac{100}{\sqrt{2}}$

$= \frac{(100)^2}{2} \cdot 80\pi^2 (1/16)^2$

$= 15.405\ kW$

Assign
* The radial component of the radiated power density of an antenna is given by $P_{rad} = a_r P_r = a_r \cdot \frac{A_m \sin\theta}{r^2}$ W/m². Calculate total radiated power.

$W_t = \oint_S P_{(avg)}\, ds$

$= \int_0^{2\pi}\int_0^{\pi} \frac{A_m \sin\theta}{r^2} \cdot r^2 \sin\theta\, d\theta\, d\phi = 2\pi \int_0^{\pi} A_m \sin^2\theta\, d\theta$

$= 4\pi A_m \int_0^{\pi/2} \sin^2\theta\, d\theta$

$= 4\pi A_m \cdot \pi/4$

$W_t = 4\pi A_m \cdot \pi/4$
$= \pi^2 A_m$ W

$\cos 2\theta = 1 - 2\sin^2\theta$
$2\cos^2\theta - 1$

$\frac{1 - \cos 2\theta}{2}$

$\frac{1}{2}\frac{\pi}{2} \left(-\frac{\sin 2\theta}{4}\right)_0^{\pi}$

* A loop aerial for use at 500 kHz is of ht 0.5m, width 0.5m & 25 turns, when directed to receive a maximum signal the emf induced in the loop is 150μV. What is the field strength of the signal picked up?

$A = h \times d = 0.25 m^2$.

$N = 25$

$f = 500 kHz$. $\lambda = 600m$.

$V_{rms} = 150 \mu V$.

$$E_{rms} = \frac{V_{rms} \lambda}{2\pi A N}$$

* A loop of 10 turns & area $1m^2$ lies in the plane making an angle 30° with the direction of propagation of an incoming signal of freq 150kHz. What is the field strength of the signal if the emf induced in the loop is 1.5mV.

$V_{rms} = 1.5mV$ $N = 10$, $A = 1m^2$ $\theta = 30°$ $f = 150kHz$ $\lambda = 2000m$.

$$E_{rms} = \frac{V_{rms} \lambda}{2\pi A N \cos\theta}.$$

* Calculate radiation resis of a single turn and an eight turn small circular loop when the radius of the loop is $\lambda/25$ & medium is free space.

$A = \pi a^2 = \pi (\lambda/25)^2 = \frac{\pi \lambda^2}{625}$

$R_r = 31171.2 \left(\frac{A}{\lambda^2}\right)^2 \simeq 0.7868 \Omega$.

for 8 turns $R_r \times N^2 = 50.35\Omega$

* A circular loop antenna with uniform in phase current has a diameter D. Find the (a) far field pattern (b) Radiation resis & gain when $D = \lambda/3$ $a = \lambda/6$.

(b) $\frac{C}{\lambda} = \frac{2\pi a}{\lambda} = \frac{2\pi \times \lambda/6}{\lambda} = \frac{2\pi}{6} = \frac{\pi}{3} = 1.0472$.

For small loop $C/\lambda \le 1/3$

$\frac{C}{\lambda} > 1/3$. Hence we have to consider large loop.

$R_r = 3720 \times (a/\lambda)$

$= 620\,\Omega$.

D gain of antenna for larger loop

$D = 0.682\,(C/\lambda)$

$= 0.682 \times \frac{2\pi a}{\lambda}$

$D = 0.7141$

Calculate for

* $D = 2\lambda$.

$d = \lambda/4$, $3/2\lambda$ $d = 8\lambda$, 0.75λ

(*) The radiating ele is of 10m length & carries a current of 1A. If radiator in $\theta = 30°$ dir in free space at $f = 3MHz$. Estimate the magnitudes of E & H at a point located at 100km from the pt. of origination. (J.D kraus).

Ans. $|E_\theta|$ of radiation field.

$$|E_\theta| = \frac{Idl\sin\theta}{4\pi\epsilon}$$

$$|H_\phi| = \frac{Idl\sin\theta}{4\pi}$$

$\theta = 30°$ $\sin\theta = 0.5$

$\cos\theta = \sqrt{3}/2 = 0.866$.

$Idl = 10$ A-m.

$r = 100km = 10^5 m$. $\epsilon = 8.85 \times 10^{-12}$

$$|E| = \frac{10 \times 0.5}{4\pi \times 8.85 \times 10^{-12}} = 45 \times 10^7.$$

$$|H| = \frac{10 \times 0.5}{4\pi} = \frac{1.25}{\pi}$$

(*) Cal. the distance at which an EM wave will have the same magnitude for induction & radiation fields if its freq is 10MHz.

$$r = \frac{v}{\omega} = \frac{3 \times 10^8}{2\pi \times 10 \times 10^6} = 4.77m$$

(*) If med. of propagation allows the wave to attain only 60% of the velocity of light, at what distance will both induction & radiation fields become equal in magnitude at $f = 3GHz$

$$r = \frac{v}{\omega} = \frac{0.6 \times 3 \times 10^8}{2\pi \times 3 \times 10^9} = \frac{6}{2\pi} = 0.955cm.$$

(*) Cal. the gain of an antenna with $\odot^{lar}$ aperture of diameter 3m at a freq of 5GHZ.

$$G = KD.$$

$$D = \frac{4\pi}{\lambda^2} \cdot A_e$$

$$\cancel{D} = \frac{4\pi}{(3/50)^2} \times \frac{\pi \times 3^2}{4} \qquad \left(\frac{\pi d^2}{4} = A_e\right)$$

$$= \pi^2 \times 2500 = 24649.$$

(*) $\lambda = \frac{3\times10^8}{60\times10^6} = \frac{30^5}{6} $ Calculate length of a $\lambda/2$ dipole antenna meant to have correct half wave length at 60MHZ.

$$\lambda/2 = 2.5m.$$

(*) find out R_r of a $\lambda/16$ wire dipole in free space.

$$R_r = 80\pi^2 (dl/\lambda)^2$$

$$= 80\pi^2 \left(\frac{\lambda}{16\lambda}\right)^2 =$$

$$= \cancel{246.49} \; 3.08\Omega.$$

(*) A thin dipole antenna is $\lambda/15$ long. If its loss resistance is 1.5Ω find R_r & η.

$$R_r = 80\pi^2 \left(\frac{\lambda}{15\lambda}\right)^2 = 3.5\Omega.$$

$$\eta = \frac{3.5}{3.5+1.5} = 0.7.$$

70%.

(*) How much current does an antenna draw when radiating 1000 W & its having a rad resistance of 300Ω.

$$I_{rms}^2 = \frac{W}{R} = \frac{1000}{300} = 3.33 \qquad I_{rms} = \sqrt{3.33} = 1.8A.$$

* Calculate l_e of an $\lambda/2$ antenna, Given $R_r = 73\Omega$, $(A_e)_{max} = 0.13\lambda^2$ $\eta = 120\pi$.

$$l_e = \frac{2\sqrt{(A_e)_{max} R_r}}{\sqrt{Z}}$$

$$\because (A_e)_{max} = \frac{l_e^2 Z}{4R_r}$$

$$= \frac{2\sqrt{0.13\lambda^2 \times 73}}{\sqrt{120 \times 3.14}}$$

$$l_e = 2\lambda\sqrt{\frac{9.49}{376.80}} = 2\lambda(0.1587).$$

$$= 0.3174\lambda.$$

$$A_e = \frac{V^2 R_L}{(R_A+R_L)^2 + (X_A+X_L)^2 P}$$

$$P = \frac{E^2}{Z}$$

$$V^2 = \frac{A_e\left((R_A+R_L)^2 + (X_A+X_L)^2\right)E^2}{2R_L}$$

$$l_e = \frac{V}{E} = \frac{\sqrt{A_e\left((R_A+R_L)^2 + (X_A+X_L)^2\right)}}{\sqrt{2R_L}}$$

$R_A = R_L = R_r + R_l$
$= R_r$

$X_A = -X_L$

$$= \frac{\sqrt{A_{emax}(2R_r)^2}}{\sqrt{2R_L}}$$

* Calculate A_{emax} of an antenna which is operating at a wavelength of 2m & directivity of 100.

$$(A_e)_{max} = \frac{\lambda^2}{4\pi} \times D = \frac{(2)^2}{4\pi} \times 100 = 31.85$$

* Calculate max effective aperture of a pearce antenna which has a directivity of 900.

$$D = \frac{4\pi}{\lambda^2}(A_e)_{max}$$

$$A_{emax} = \frac{900 \times \lambda^2}{4\pi} = 71.619\lambda^2.$$

* Determine the A_{emax} of a beam antenna having HPBW of 30° & 35° in $\perp^r$ planes intersecting in the beam axis. Assume small side lobes.

$$D = \frac{41257}{\theta_E^\circ \times \theta_H^\circ} = \frac{41257}{30^\circ \times 35^\circ} = 39.3^\circ.$$

$$(A_e)_{max} = \frac{D}{4\pi}\lambda^2$$

$$= 3.13\lambda^2$$

Assignment.

41

* An isotropic antenna radiates equally in all directions. The total power delivered to the radiator is 100kW. Cal. the power density at distances of 100m, 1000m, 100,000m.

$P_r = \frac{W_t}{4\pi r^2} \cdot \frac{100 \times 10^3}{4\pi (100)^2} = 0.796178 \ w/m^2$

$P_{r(1000m)} = 0.00796$

$= 7.96 \ mW/m^2$

$P_{r(100km)} = 0.796 \ \mu W/m^2$.

* Calculate the distance at which the ratio of induction and radiation fields will be (a) 1/2 (b) 2 & (c) 10 if the wave freq is 10MHz

* If an ele of 1cm length radiates 1W at 3GHz, estimate effective current carried by the ele.

* Cal. R_r of an element of length L=1m (a) f=3kHz (b) 10MHz (c) 10GHz.

* A 5m long radiating ele carries a current of 2A. It radiates in the $\theta = 45°$ dir in free space at f=10MHz. Estimate the ratio of magnitudes of E & H at a point located at 30km from the pt. of origination. will it be different at 50km.

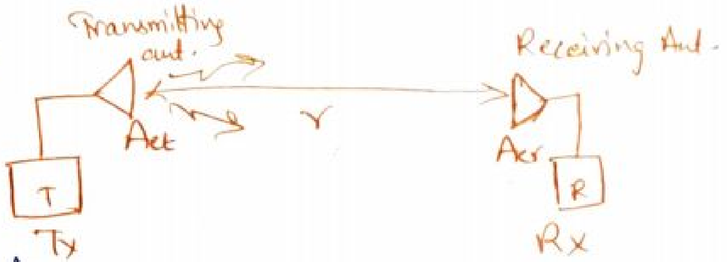
* If the medium of propagation allows the wave to attain only 90% of velocity of light, at what distance will the induction & radiation fields become equal in magnitude at f=5GHz?

Friis Transmission formula 42

This formula gives the power received over a radio communication link

Assume lossless, matched antennas

Let the transmitter feed a power P_t to a transmitting antenna of effective aperture A_{et}.



At a distance 'r' a receiving antenna intercepts some of the power radiated by the transmitting antenna and delivers it to receiver 'R'.

Let the transmitting antenna is isotropic and the power per unit area available at receiver is

$$S_r = \frac{P_t}{4\pi r^2}$$

If the transmitting antenna has gain G_t, the power per unit area (power density) available at receiving antenna is

$$S_r = \frac{P_t G_t}{4\pi r^2}$$

The power collected by receiving antenna having effective aperture A_{er} is

$$P_r = S_r A_{er} = \frac{P_t G_t A_{er}}{4\pi r^2}$$

The gain of transmitting antenna is

$$G_t = \frac{4\pi A_{et}}{\lambda^2} \quad \& \text{ also } G_r = \frac{4\pi A_{er}}{\lambda^2}.$$

$$\boxed{\frac{P_r}{P_t} = \frac{A_{er} A_{et}}{r^2 \lambda^2}}$$ (dimensionless) Friis transmission formula.

$$= \frac{G_t \lambda^2 \cdot G_r \lambda^2}{(4\pi)^2 r^2 \lambda^2}$$

$$\boxed{\frac{P_r}{P_t} = G_t G_r \left(\frac{\lambda}{4\pi r}\right)^2}$$

The term $\frac{\lambda}{4\pi r}$ is called free space loss factor and it takes into account losses due to spherical spreading of energy by the antenna.

Problems. U3

* Show that the directivity of an electric current ele is 3/2.

Directivity is max directive gain

$$G = \frac{\text{Power radiated by current element}}{\text{power radiated by an isotropic antenna}}$$

$$= \frac{P_r}{\frac{W_i}{4\pi r^2}} = \frac{P_r\, 4\pi r^2}{W_i}$$

$4\pi r^2$ is the area of sphere over which isotropic antenna radiates uniformly in all directions. W_i is the input power.

Power radiated by current ele $P_{r(avg)} = \frac{\eta}{2}\left(\frac{\omega I\, dl \sin\theta}{4\pi r c}\right)^2$ w/sqm.

$\frac{2\pi f}{4\pi r c} \cdot \frac{1}{2\lambda r}$ $c = \nu\lambda$

$$= \frac{\eta}{8} \frac{(I\, dl \sin\theta)^2}{\lambda^2 r^2}$$

$[P_{r(avg)}]_{max} \Rightarrow$ when $\sin\theta = 1$.

$$= \frac{\eta}{8} \frac{I_m^2\, dl^2}{\lambda^2 r^2}.$$

Total power radiated

$$P = 80\pi^2 \frac{dl}{\lambda^2} \cdot I_{rms}^2$$

$$= 40\pi^2 \frac{dl^2}{\lambda^2} I_m^2.$$

$$\frac{120\pi}{8 \times 40\pi^2} \times 4\pi.$$

$$G = \frac{\frac{\eta}{8} \frac{I_m^2\, dl^2}{\lambda^2 r^2}}{40\pi^2 \frac{dl^2}{\lambda^2} I_m^2} \cdot 4\pi r^2 = \frac{30\pi \times 4}{8\pi} \quad \frac{120}{80}$$

$$= 1.5$$

* Derive an expression for gain of a half wave antenna.

$$G = \frac{P_r}{P_i} = \frac{P_r}{W_i} \times 4\pi r^2$$

for $\lambda/2$ antenna

$$P_{r(avg)\,max} = \frac{\eta I_m^2}{8\pi^2 r^2}\left(\frac{\cos^2(\frac{\pi}{2}\cos\theta)}{\sin^2\theta}\right) \quad \hookrightarrow 1.$$

$I_{rms} = \frac{I_m}{\sqrt{2}}$

$$= \frac{\eta I_m^2}{8\pi^2 r^2} = \frac{\cancel{120\pi}^{30} \times \cancel{2} \times I_{rms}^2}{\cancel{8}_{2}\pi^2 r^2} = \frac{30 I_{rms}^2}{\pi r^2}$$

$$W_i = 73.14\, I_{rms}^2$$

$$G = \frac{4\cancel{\pi r^2} \cdot \frac{30}{\cancel{\pi r^2}} \cdot \cancel{I_{rms}^2}}{73.14\, \cancel{I_{rms}^2}}$$

$$= \frac{120}{73.14} = 1.641.$$

$$G(dB) = 10\log 1.641 = 2.151\,dB.$$

* A loop operating at 500kHz, having 10 turns, 1 meter square area, it was found that max potential diff. of 2mV was developed across it. The loop is tuned to resonance. Find the field strength if Q of loop is 30.

$$\lambda = \frac{3\times10^8}{500\times10^3} = \frac{3000}{5} = 600\,m.$$

$$|V_{rms}| = \left|\frac{2\pi E_{rms} A N\cos\theta}{\lambda}\right|.$$

$N = 10$

$A = 1\,m^2$

$V_{rms} = 2\times10^{-3}\,V.$

$$E_{rms} = \frac{V_{rms}\cdot\lambda}{2\pi A N}$$

$$= \frac{2\times10^{-3}\times600}{2\pi\times1\times10} = 19.11\,mV/m$$

Antenna Arrays

→ the field strength can be increased in preffered directions by properly exciting group of antennas simultaneously in an arrangement known as "Antenna Arrays".

→ when greater directivity is required than can be obtained by a single antenna, antenna arrays are used.

→ Thus, an antenna array is a system of similar antennas oriented similarly to get greater directivity in a desired direction.

→ Individual antennas of an antenna array system is also termed as "elements".

→ Antenna arrays make use of wave-interference phenomena that occur between the radiations from the different elements of the array.

→ In Antenna arrays an antenna is regarded as point source (or) volume less radiator. In orther words a hypothetical antenna (or) Isotropic (or) omni-directional (or) non-directional antenna which occupies zero volume is considered.

→ An Antenna array is said to be linear, if the individual antennas of the array are equally spaced along a straight line.

→ A uniform Linear array is the one, in which the elements are fed with a current of equal magnitude with uniform progressive phase shift along the Line.

→ Various forms of antenna arrays:

(i) Broad side Array

(ii) End fire Array

(iii) collinear Array

(iv) parasitic Array

Array of two isotropic point sources:

→ In Array theory of antenna's, the superposition (or) addition of fields from the various sources at a great distance will be due to the phases involved, (phase Leading & phase Lagging by other sources.) and hence the following five cases will be dealt with arrays of two isotropic point sources.

i) Two Isotropic point sources of same amplitude and phase.

ii) Two Isotropic point sources of same amplitude but opposite phase.

iii) Two Isotropic point sources of same amplitude and in-phase quadrature.

iv) Two Isotropic point sources of equal amplitude and any phase difference.

(v) Two Isotropic point sources of unequal amplitude and any phase difference.

Advantages of Arrays:

- Higher directivity
- Narrower beam
- Lower sideLobes.
- Electronic steerable beam
- Allow very high resolution for direction finding.
- Combines multiple Antennas
- More flexibility in transmitting / receiving signals - spatial filtering.
- Different information on multiple antennas - increase system throughput (capacity) - spatial multiplexing.

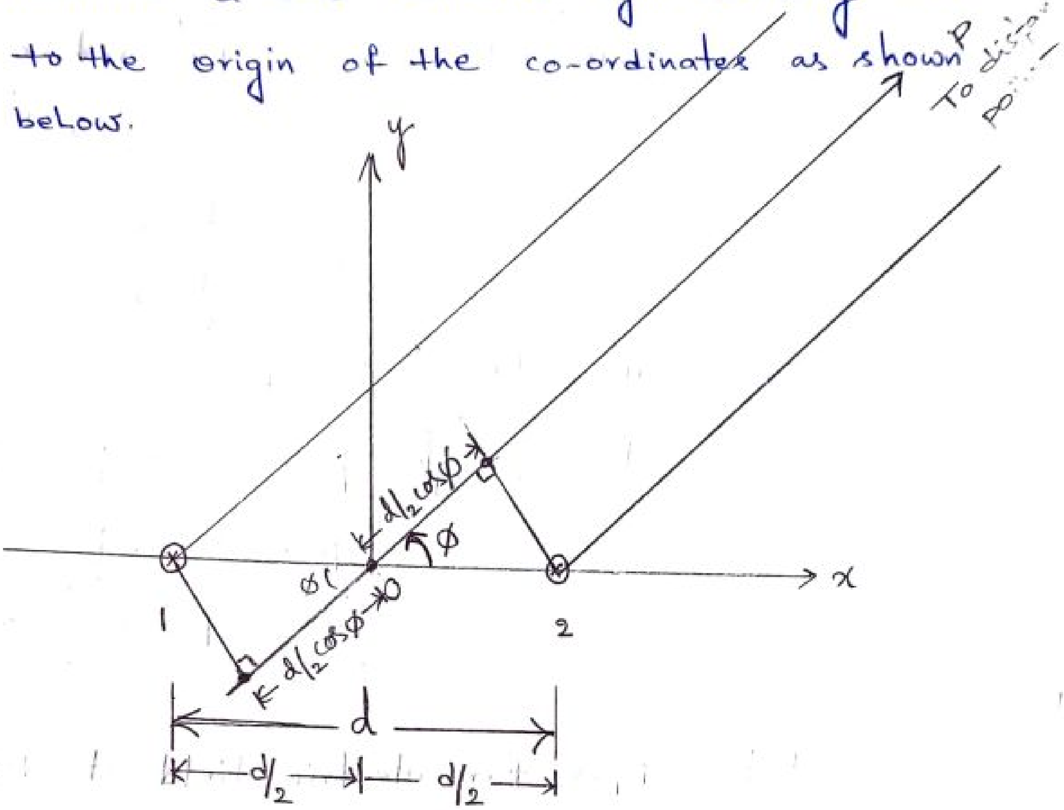
: Design principles of Arrays :

→ In an array of identical elements there are at least five controls that can be used to shape the overall pattern of the antenna, these are :

1. the geometrical configuration of the overall array (Linear, circular, rectangular, spherical e.t.c.
2. the relative displacement between the elements.
3. the excitation Amplitude of the individual elements.
4. the excitation phase of the individual elements.
5. the relative pattern of the individual elements.

1. Two Isotropic point sources of same Amplitude and phase:

→ consider two point sources 1 and 2 be separated by a distance 'd' and Located symmetrically with respect to the origin of the co-ordinates as shown in fig. below.



→ the two isotropic point sources having equal amplitude and osullating in the same phase.

→ the Angle ϕ is measured counter clockwise from the positive x-axis.

→ the origin of the co-ordinates is taken as the reference for phase.

→ To calculate the fields at distant point (say R) from the origin, obviously waves from source 1 reaches the point P at a Later time than the waves from source 2 because the path difference involved between the two waves.

→ thus the fields due to source '1' Lags, while that due to source '2' Leads.

→ the path difference between the two waves is given by

$$\text{path difference} = 1'2' \text{ meters}$$
$$= \frac{d}{2}\cos\phi + \frac{d}{2}\cos\phi$$
$$= d\cos\phi \text{ meters}$$
$$= \frac{d}{\lambda}\cos\phi \text{ wavelengths.}$$

→ from the optics, it is known as

$$\text{phase Angle } \psi = 2\pi \times \text{path difference}$$
$$\psi = 2\pi \times \frac{d}{\lambda}\cos\phi$$
$$\boxed{\psi = \beta d\cos\phi \quad \text{radians.}}$$

→ Let $E_1 \rightarrow$ far electric field at distant point due to source 1.

$E_2 \rightarrow$ far electric field at distant point due to source 2

$E \rightarrow$ Total electric field at distant point.

$\psi \rightarrow \beta d \cos\phi$ radians, phase angle difference between the fields of two sources measured at angle ϕ along radius vector line.

→ then the total field at a Large distance in the direction of ϕ is

$$E = \underbrace{E_1 \cdot e^{-j\psi/2}}_{\text{field component due to source 1}} + \underbrace{E_2 \cdot e^{+j\psi/2}}_{\text{Field component due to source 2}} \longrightarrow (1)$$

In this case it is assumed that amplitudes are same, hence

$$E_1 = E_2 = E_0 \ (\text{say})$$

$$E = E_0 [e^{-j\psi/2} + e^{+j\psi/2}]$$

$$E = 2E_0 \left[\frac{e^{-j\psi/2} + e^{+j\psi/2}}{2} \right]$$

$$E = 2E_0 (\cos \psi/2)$$

$$E = \underbrace{2E_0}_{\text{Amplitude}} \cos \Big(\underbrace{\frac{\beta d \cos\phi}{2}}_{\text{phase.}} \Big)$$

This is the equation of far field pattern of two isotropic point sources of same amplitude and phase.

To normalize the above equation, make its maximum value unity, by putting $2E_0 = 1$, $E_0 = 1/2$, then the pattern is said to be normalized and the above equation becomes

$$E_{nor} = \cos \left(\frac{\beta d \cos\phi}{2} \right)$$

If $d = \lambda/2$

$$E = \cos \left(\frac{2\pi}{\lambda} \cdot \frac{\lambda}{2} \cdot \frac{\cos\phi}{2} \right)$$

$$E = \cos \left(\frac{\pi}{2} \cdot \cos\phi \right)$$

In order to draw the field pattern (E versus ϕ), the directions of maxima, minima and half-power points must be known, which can be calulated with the help of above equation as follows.

Maxima directions:

E is maximum when $\cos(\pi/2 \cos\phi)$ is maximum, and its maximum value is ± 1

$$\cos(\pi/2 \cos\phi) = \pm 1$$

$$\pi/2 \cos\phi_{max} = \pm n\pi, \text{ where } n = 0, 1, 2 \ldots$$

if $n = 0$, $\pi/2 \cos\phi_{max} = 0$

$$\cos\phi_{max} = 0$$

$$\phi_{max} = 90^\circ \text{ and } 270^\circ$$

Minima directions:

E is minimum when $\cos(\pi/2 \cos\phi)$ is minimum, and its minimum value is zero.

$\therefore$ E is minimum when $\cos(\pi/2 \cos\phi) = 0$

$$\pi/2 \cos\phi_{min} = \pm(2n+1)\pi/2$$

where $n = 0, 1, 2, \ldots$

if $n = 0$, $\pi/2 \cos\phi_{min} = \pm\pi/2$

$$\cos\phi_{min} = \pm 1$$

$$\phi_{min} = 0^\circ \text{ and } 180^\circ$$

Half-power point directions:

At half power points power is $1/2$ (or) voltage or current is $1/\sqrt{2}$ times the maximum value of voltage (or) current

$$\cos(\pi/2 \cos\phi) = \pm 1/\sqrt{2}$$

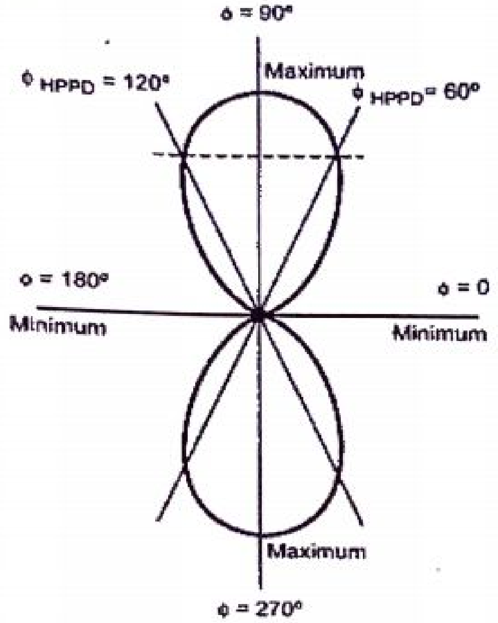
$$\pi/2 \cos\phi_{HP} = \pm(2n+1)\pi/4 \quad \text{where } n = 0, 1, 2 \ldots$$

$$\cos\phi_{HP} = \pm\frac{(2n+1)}{2}$$

if $n=0$, $\cos\phi_{HP} = \pm 1/2$

$\phi_{HP} = 60^\circ, 120^\circ$.

$\therefore$ the field pattern E versus ϕ for the case $d = \frac{\lambda}{2}$ is shown below.



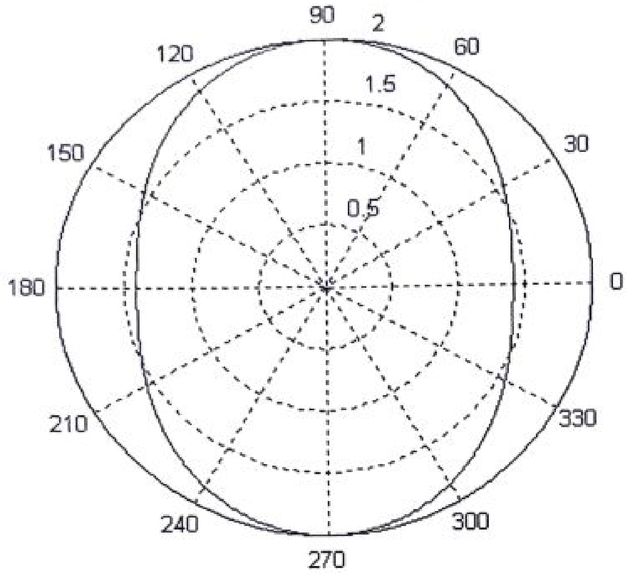
→ the pattern is a bi-directional figure of eight with maxima along the y-axis.

the space pattern is doughnut shape.

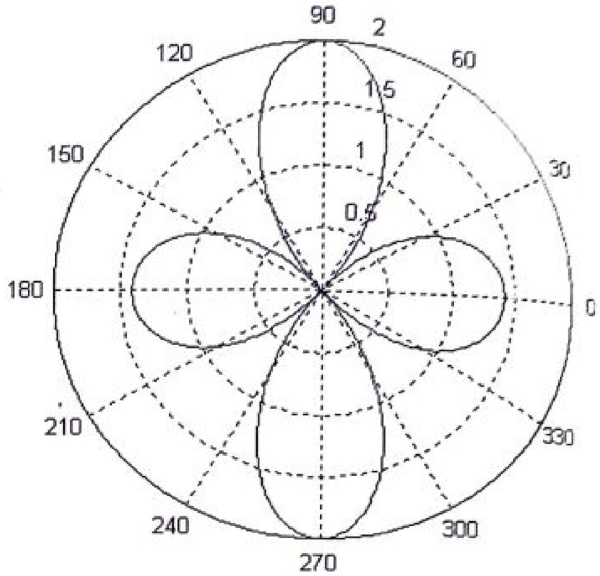
the inphase point sources produce a pattern with the maximum field normal to the line joining the sources.

the two sources in this case may be described as a simple broad side type of array.

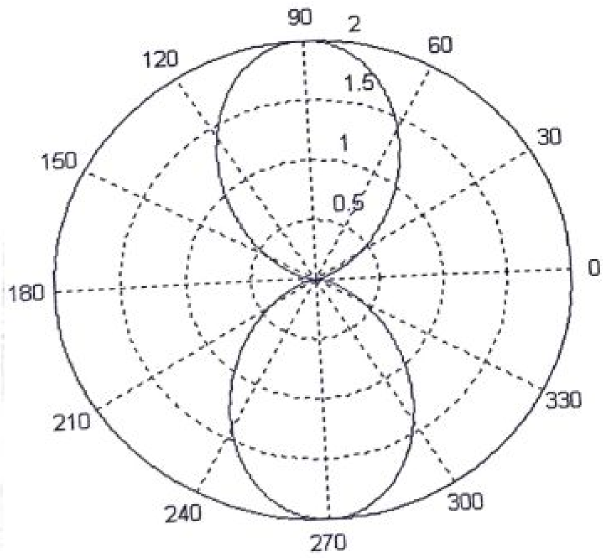
→ the field pattern of E versus ϕ for different values of d/λ, the elements excited in the same phase is shown in fig. below.



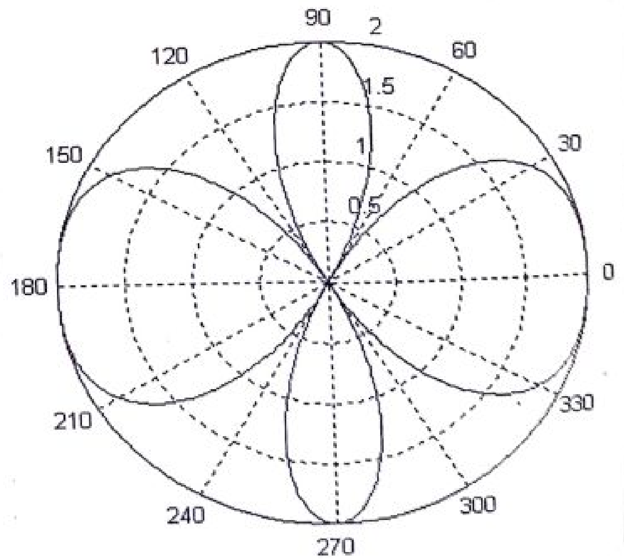
(i) $d = \frac{\lambda}{4}$



(ii) $d = \frac{3\lambda}{4}$

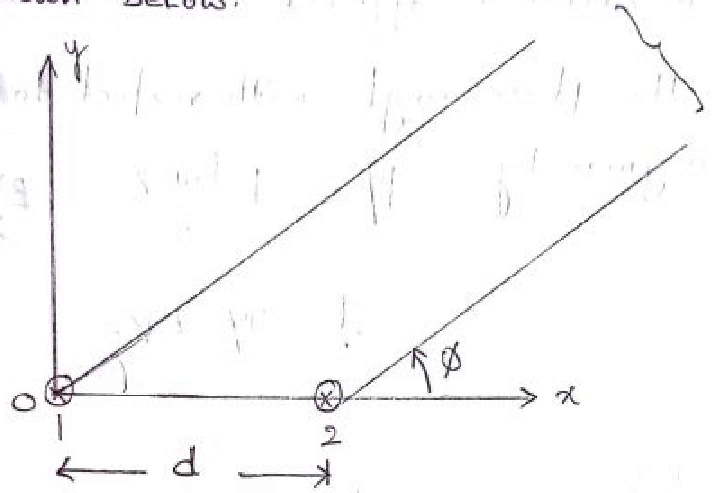


(iii) $d = \frac{\lambda}{2}$



(iv) $d = \lambda$

→ the same pattern can also be obtained by Locating source 1 at the origin of the co-ordinates and source 2 at a distance 'd' along the positive x-axis as shown below.



→ Now taking source 1 as reference, the field from source 2 in the direction ϕ is advanced by $\beta d \cos\phi$.

→ thus, the total electric field E at Large distance is the vector sum of the fields from the two sources as given by

$$E = E_0 + E_0 \cdot e^{j\psi}$$

where $\psi = \beta d \cos\phi$

$$E = E_0 \left[1 + e^{j\psi} \right]$$

$$E = E_0 e^{j\psi/2} \left[e^{-j\psi/2} + e^{+j\psi/2} \right]$$

$$E = E_0 e^{j\psi/2} \cdot 2 \cdot \cos\psi/2$$

$$E = 2E_0 \cdot \cos\psi/2 \cdot \underline{|\psi/2}$$

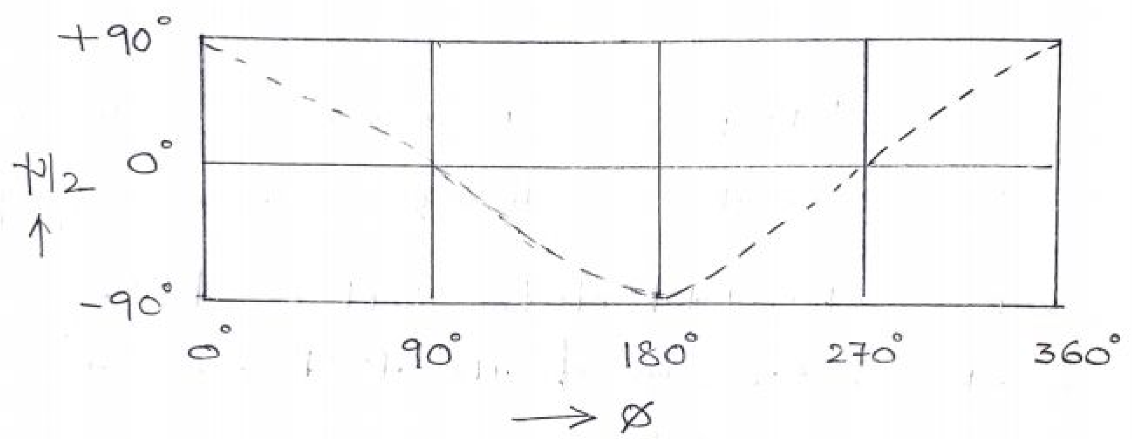
Normalized field $E = \cos\psi/2 \, \underline{|\psi/2}$

→ from above equation, $\cos\psi/2$ the cosine factor gives the amplitude variation of E. and the exponential (or) angle factor gives the phase variation with respect to source 1 as the reference.

→ the phase variation for the case of $\lambda/2$ spacing is shown in fig. below.

→ the phase angle with respect to the phase of source 1 is given by $\psi/2 = \frac{\beta d \cos\phi}{2} = \frac{2\pi}{\lambda} \times \frac{\lambda}{2} \times \frac{\cos\phi}{2}$

$$\frac{\psi}{2} = \pi/2 \cos\phi.$$



→ when the reference is taken the mid way between the sources, there is no phase change around the array. thus an Observer at a fixed distance observes no-phase change when the array is rotated with respect to ϕ around its mid-point.

→ but a phase change is observed if the array is rotated with source 1 as the center of rotation.

2. Two Isotropic point sources of same Amplitude but opposite phase:

→ consider two point sources separated by distance 'd' and supplied with currents of equal in magnitude but opposite in phase.

→ All conditions are exactly same except that the two sources are in opposite phase i,e 180°.

→ then the total field in the direction ϕ at a large distance is given by

$$E = -E_1 \cdot e^{-j\psi/2} + E_2 \cdot e^{+j\psi/2}$$

because phase of source 1 and source 2 at distant point P is $-\psi/2$ and $+\psi/2$, since the reference being at between midway of two sources.

$$E_1 = E_2 = E_0 \text{ (say)}$$

$$E = E_0\left[-e^{-j\psi/2} + e^{+j\psi/2}\right]$$

$$E = 2E_0 \cdot j \sin \psi/2$$

$$E = 2j\, E_0 \sin\left(\frac{\beta d \cos\phi}{2}\right)$$

→ Total field is similar to that of ~~case~~ is eqn but the above eqn involves the sine function instead of cosine and the operator j involved.

the presence of j simply means that opposite phase brings a phase shift of 90° in the total field.

→ the normalized field is given by

$$E = \sin \psi/_2 = \sin\left(\frac{\beta d \cos\phi}{2}\right) \qquad \because 2jE_0 = 1$$

→ If $d = \lambda/_2$

$$E = \sin\left(\frac{2\pi}{\lambda} \cdot \frac{\lambda}{2} \cdot \frac{\cos\phi}{2}\right)$$

$$E = \sin\left(\frac{\pi}{2}\cos\phi\right)$$

→ In order to draw the field pattern E versus ϕ the directions of maxima, minima and half power points must be known, which can be calculated with the help of above eqn as follows.

Maxima directions:

Maximum value of the sine function is ±1

$$\sin\left(\frac{\pi}{2}\cos\phi\right) = \pm 1$$

$$\pi/_2 \cos\phi = \pm(2n+1)\pi/_2$$

where $n = 0, 1, 2, 3, \ldots$

if $n = 0$

$$\pi/_2 \cos\phi_{max} = \pm \pi/_2$$

$$\cos\phi_{max} = \pm 1$$

$$\phi_{max} = 0^\circ \text{ and } 180^\circ$$

Minima directions:

Minimum value of the sine function is zero.

$$\sin(\pi/_2 \cos\phi) = 0$$

$$\pi/_2 \cos\phi = \pm n\pi \quad ; \text{ where } n = 0, 1, 2, 3 \ldots$$

if $n=0$; $\pi/_2 \cos\phi$

$$\cos\phi_{min} = 0$$

$$\phi_{min} = 90^\circ \,\&\, 270$$

Half-power point directions:

$$\sin\left(\pi/_2 \cos\phi\right) = \pm 1/\sqrt{2}$$

$$\pi/_2 \cos\phi = \pm(2n+1)\frac{\pi}{4}$$

if $n=0$; $$\pi/_2 \cos\phi = \pm \pi/4$$

$$\cos\phi_{HP} = \pm 1/_2$$

$$\phi_{HP} = \pm 60^\circ, \pm 120^\circ$$

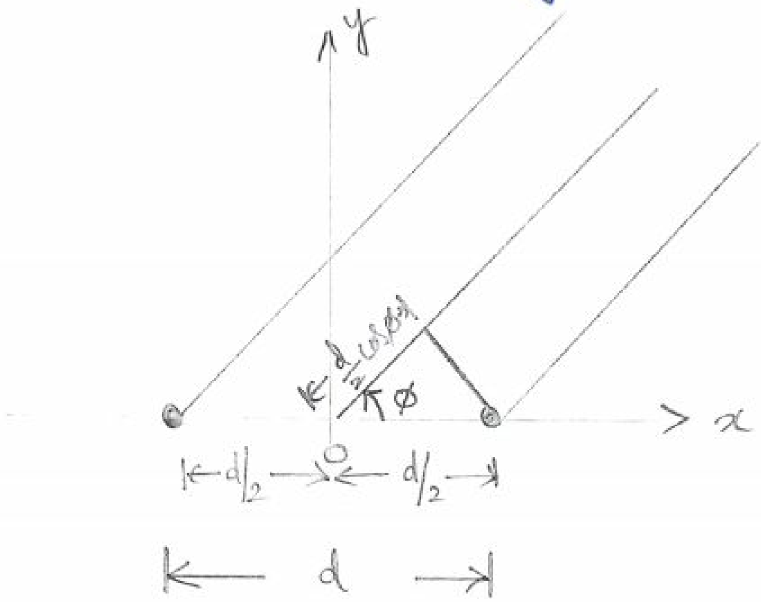
→ the field pattern of two isotropic point sources with spacing $d = \lambda/_2$ and fed with currents in equal magnitude but out of phase by 180°.

"the two sources for this case may be described as a simple type of end-fire array."

→ the pattern is a relatively broad figure of eight with the maximum field in the same direction as the line joining the sources.

Case iii) Two isotropic point sources of the same amplitude and in-phase quadrature:

→ Consider two point sources 1 and 2 be separated by a distance 'd' and Located symmetrically with respect to the origin of the co-ordinates as shown in fig. below.



→ Taking the origin of the co-ordinates as the reference for phase..

→ Let source 1 be retarded by 45° and source 2 advanced by 45°, then the total field in the direction ϕ at a Large distance is given by

$$E = E_0 \cdot e^{-j(\psi/2 + \pi/4)} + E_0 e^{+j(\psi/2+\pi/4)}$$

$$E = 2E_0 \cos\left(\frac{\psi}{2} + \frac{\pi}{4}\right)$$

Let $2E_0 = 1$

then the normalized field is given by

$$E_{nor} = \cos(\psi/2 + \pi/4)$$

Maximum directions:

Maximum value of the cos function is ± 1

$$\cos(\psi/2 + \pi/4) = \pm 1$$

$$\psi = \beta d \cos\phi$$

if $d = \lambda/2$

$$\psi = \pi \cos\phi$$

$$\cos\left(\frac{\pi}{2}\cos\phi + \frac{\pi}{4}\right) = \pm 1$$

$$\pi/2 \cos\phi + \pi/4 = \pm n\pi$$

if $n = 0$, $\quad \pi/2 \cos\phi + \frac{\pi}{4} = 0$

$$\pi/2 \cos\phi = -\pi/4$$

$$\cos\phi = -1/2$$

$$\phi_{max} = 120^\circ \text{ and } 240^\circ$$

Minimum directions:

Minimum value of the function is '0'

$$\cos(\psi/2 + \pi/4) = 0$$

$$\frac{\pi}{4} + \frac{\pi}{2}\cos\phi = \pm(2n+1)\frac{\pi}{2}$$

if $n=0$ $\quad \frac{\pi}{4} + \frac{\pi}{2}\cos\phi = \pm\frac{\pi}{2}$

$$\frac{\pi}{2}\cos\phi = \pi/2 - \pi/4$$

$$\frac{\pi}{2}\cos\phi = \pi/4$$

$$\cos\phi = 1/2$$

$$\phi = 60^\circ$$

→ The field pattern is shown in fig. below. Most of the radiation is in the second and third quadrants.

→ the field in the direction $\phi = 0^\circ$ is the same as in the direction $\phi = 180^\circ$

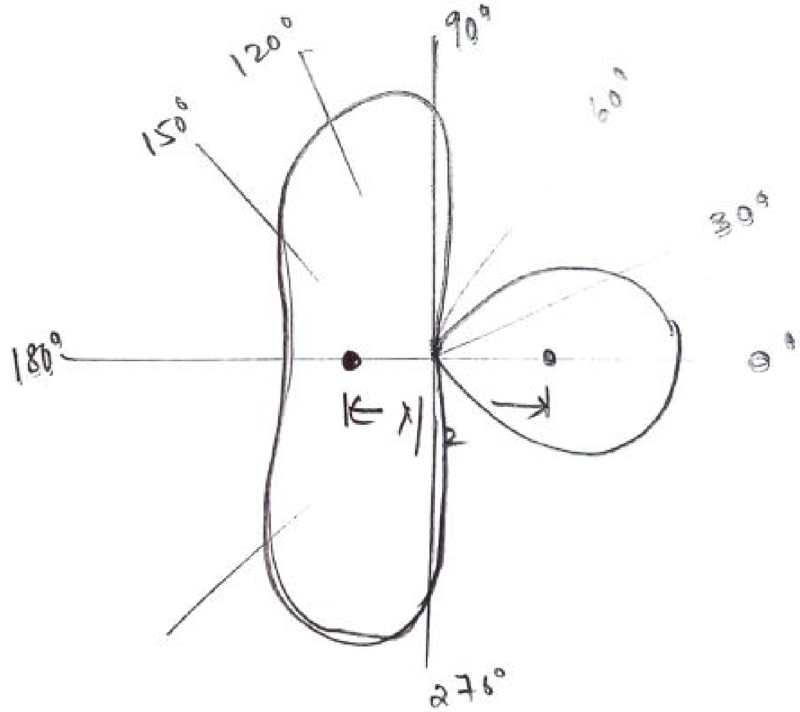
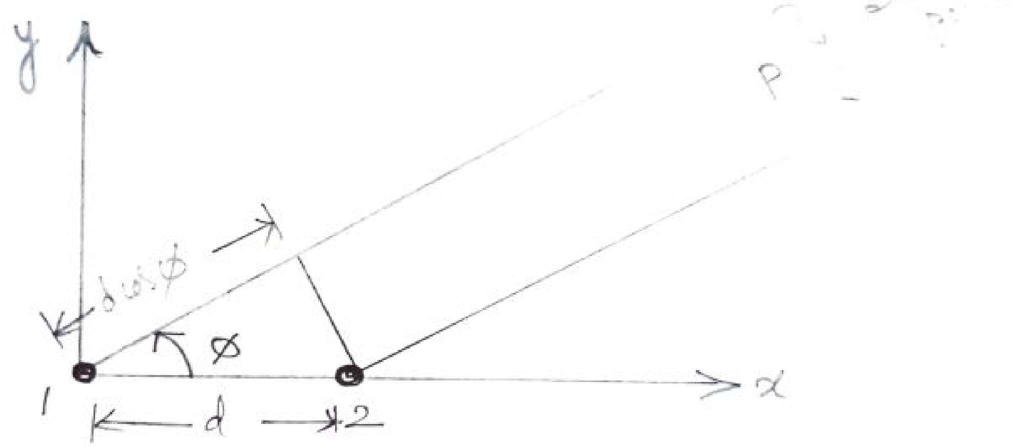


Fig: Relative field pattern of two isotropic point sources of the same amplitude and in phase quadrature for a spacing of $\lambda/2$.

Case iv) Two isotropic point sources of equal Amplitude and any phase difference:

→ Let us consider a general condition in which the amplitudes of two point sources are equal but of any phase difference 'δ'.

→ Let us assume the source 1 is taken as reference for phase and 'd' is the distance between source 1 and 2.



→ The total phase difference 'ψ' between the fields from source 2 and source 1 at a distant point 'p' in the direction ϕ is given by

$$\psi = \beta d \cos\phi \pm \delta \rightarrow (1)$$

→ The positive sign in above eqn (1) indicates that source 2 is advanced in phase by the angle δ. Minus sign would be used to indicate a phase retardation.

→ The total field at a distant point 'p' is given by.

$$E = E_1 \cdot e^{j0} + E_2 \cdot e^{j\psi}$$

$$E = E_0 [1 + e^{j\psi}] \qquad E_1 = E_2 = E_0$$

$$E_0 = E_0 e^{j\psi/2} [e^{-j\psi/2} + e^{+j\psi/2}]$$

$$E = 2E_0 \cos\psi/2 \cdot e^{j\psi/2}$$

$$E_{nor} = \cos\psi/2 \angle\psi/2$$

→ If the center point is reffered as the reference point then phase of the field from source 1 at a distant point is given by $-\frac{\psi}{2}$ and that from source 2 by $+\psi/2$.

then the total field is

$$E = E_1 e^{-j\psi/2} + E_2 e^{+j\psi/2}$$

$$E = E_0 [e^{-j\psi/2} + e^{+j\psi/2}]$$

$$E = 2E_0 \cos\psi/2$$

$$E_{nor} = \cos\psi/2 \longrightarrow ①$$

where $\psi = \beta d \cos\phi + \delta$.

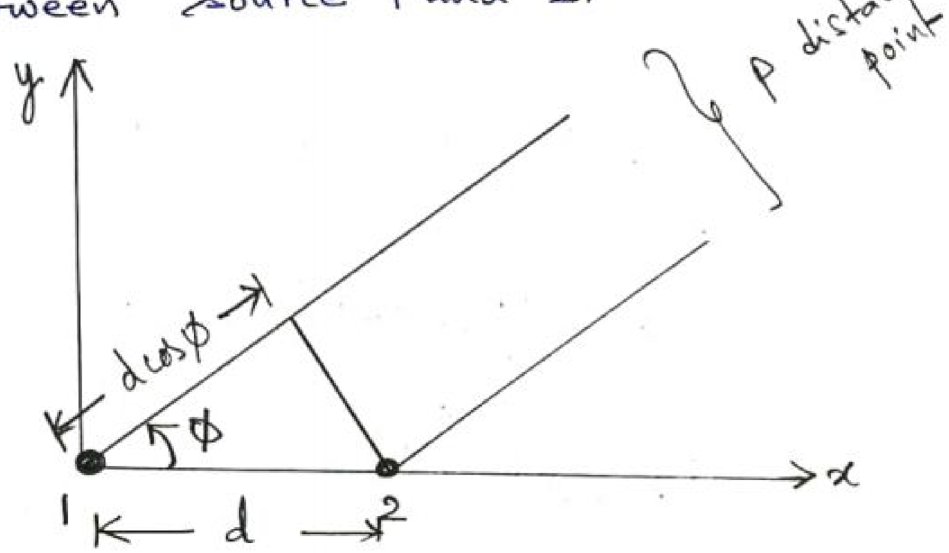
The three cases we have discussed are special cases of eqn ① when $\delta = 0°$, 180° and 90°

Case is 2) & 3) are obtained from above eqn respective

Case 5: Two isotropic point sources of unequal amplitudes and any phase difference:

→ Let us consider two isotropic point sources of unequal amplitudes E_1 and E_2 in which E_1 is greater than E_2 and any phase difference 'δ'.

→ Let us assume that source 1 is taken as reference for phase and 'd' is the distance between source 1 and 2.



→ the total phase difference ψ between the fields from source 2 and source 1 at a distant point 'p' in the direction 'ϕ' is given by

$$\psi = \beta d \cos\phi + \delta$$

the Total field at a distant point is given by

$$E = E_1 \cdot e^{j0} + E_2 \cdot e^{j\psi}$$

$$E = E_1 + E_2 e^{j\psi}$$

$$E = E_1 \left[1 + \frac{E_2}{E_1} \cdot e^{j\psi} \right]$$

Let $\frac{E_2}{E_1} = K$; $E_1 > E_2$ $K < 1$

i,e $0 \leq K \leq 1$.

$$E = E_1 \left[1 + K \cdot e^{j\psi} \right]$$

→ The magnitude and phase angle at point 'p' is given by

$$E = \left| E_1 \left[1 + K e^{j\psi} \right] \right|$$

$$E = \left| E_1 \left[1 + K\cos\psi + jK\sin\psi \right] \right|$$

$$|E| = E_1 \cdot \sqrt{(1 + K\cos\psi)^2 + (K\sin\psi)^2}$$

$$\text{phase Angle} = \tan^{-1}\left[\frac{K\sin\psi}{1 + K\cos\psi} \right]$$

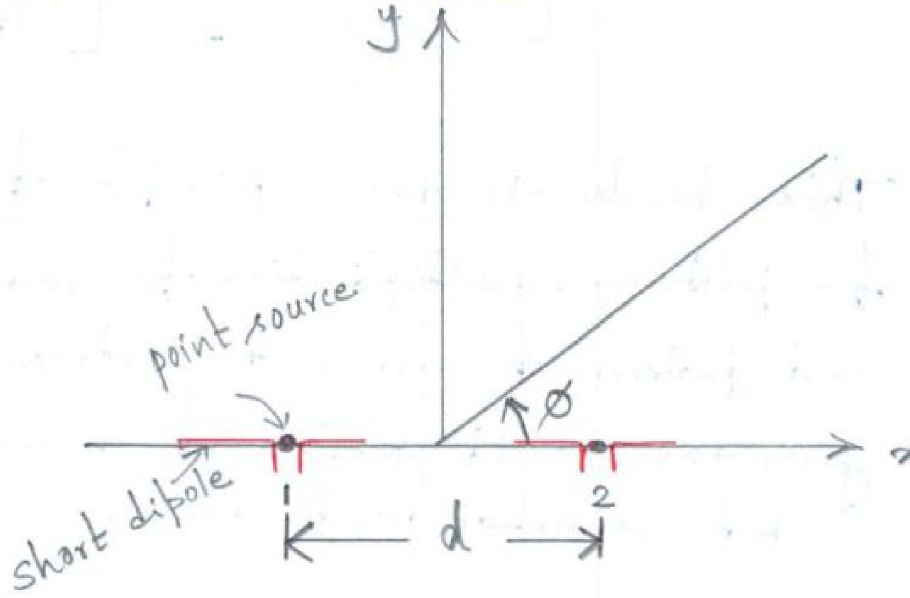
=×=×=

Non-isotropic but similar point sources:

→ Non-isotropic but similar point sources has the meaning that the amplitudes of the individual sources are equal, the source would be non-isotropic, but identical.

→ the pattern's not only must be of the same shape but also must be oriented in the same direction to be called "similar".

→ Let us now consider two short dipoles which are super-imposed over the two isotropic point sources and are separated by a distance 'd' meters.



→ Let the field pattern of each source is given by $E_0 = E_0' \sin\phi \rightarrow ①$

→ The field pattern of Array of two identical isotropic point sources is given by $E = 2E_0 \cos \psi/2 \rightarrow ②$

where $\psi = \beta d \cos\phi + \delta$.

Combining eqn's ① & ②

$$E = 2E_0' \sin\phi \cdot \cos\psi/2$$

$$E_{nor} = \sin\phi \times \cos\psi/2$$

This result is the same as obtained by multiplying the pattern of the individual source [$\sin\phi$] by the pattern of an array of two isotropic point sources [$\cos\psi/2$].

$$E_{norm} = \begin{bmatrix} \text{pattern of} \\ \text{individual} \\ \text{source} \end{bmatrix} \times \begin{bmatrix} \text{pattern of array} \\ \text{of two isotropic} \\ \text{point sources.} \end{bmatrix}$$

This leads to the prinuiple of multiplication of pattern. Multiplication of individual source and pattern of array of isotropic point sources gives the field pattern of non-isotropic but similar point sources..

: Pattern Multiplication :

→ The field pattern of an array of non- isotropic but similar point sources is the product of the pattern of the individual source and the pattern of an array of isotropic point sources having the same Locations, relative amplitudes, and phase as the non-isotropic point sources.

→ This principle may be applied to arrays of any number of sources provided only that they are similar.

→ The individual non-isotropic source (or) antenna may be of finite size, but can be considered as a point source situated at the point in the antenna to which phase is reffered. This point is said to be the " phase center".

→ Let $f(\theta,\phi)$ → field pattern of individual source.

$f_p(\theta,\phi)$ → phase pattern of individual source.

$F(\theta,\phi)$ → Field pattern of array of isotropic sources.

$F_p(\theta,\phi)$ → phase pattern of array of isotropic sources.

Then

The total field pattern of an array of non-isotropic but similar sources is the product of the individual source pattern and the pattern of an array of isotropic point sources each located at the phase center of the individual source and having the same relative amplitude and phase, while the total phase pattern is the sum of the phase pattern's of the individual source and the array of isotropic point sources.

$$E = f(\theta,\phi) \cdot F(\theta,\phi) \underline{|f_p(\theta,\phi) + F_p(\theta,\phi)}$$

→ the principle of multiplication of pattern provides a speedy method for sketching the pattern of complicated arrays just by inspection. and thus the principle proves to be a useful tool in design of antenna arrays.

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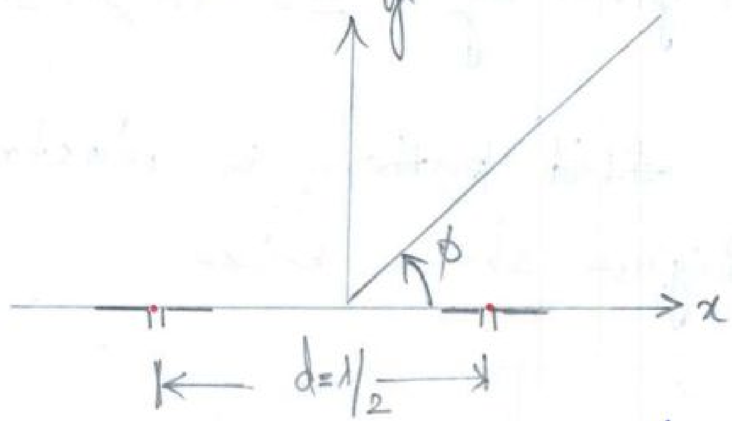
Examples of pattern multiplication..

Ex. 1. → Assume two non-isotropic but identical point sources of the same amplitude & phase spaced $\lambda/2$ apart arrangement is shown in fig. below.



→ Assume the field pattern of each source is given by

$$E_o = E_o' \sin\phi$$

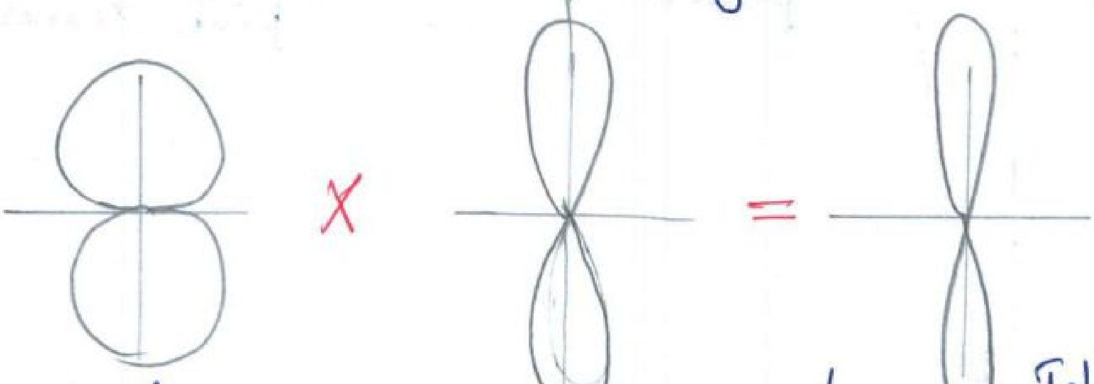
then the total field pattern is given by

$$E = 2E_o' \sin\phi \cdot \cos(\psi/2)$$

$$\psi = \beta d \cos\phi.$$

$$E_{nor} = \sin\phi \cdot \cos\left(\frac{\pi}{2}\cos\phi\right).$$

this pattern is illustrated by the below fig.

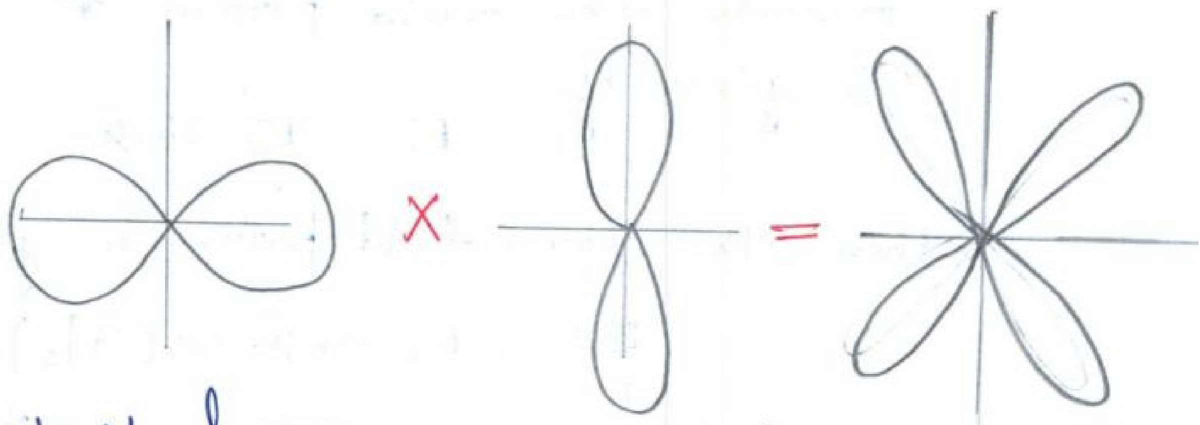


individual source pattern ($\sin\phi$)   Array pattern of two isotropic point   Total Array pattern.

EX:2. If the individual source pattern is given by $E_0 = E_0' \cos\phi$.

→ By using the principle of pattern multiplication, the total normalized field is given by $E = \cos\phi \cdot \cos[\pi/2 \cos\phi]$

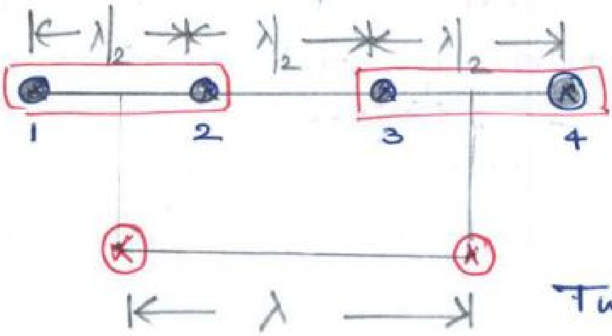
the field pattern is illustrated by the figure shown below.



individual non-isotropic source pattern. × Array pattern of two isotropic point sources = Total Array pattern.

Radiation pattern of 4-isotropic elements fed in phase, spaced $\lambda/2$ apart.

- Let the 4-elements of isotropic radiators are placed at a distance of $\lambda/2$ and fed in phase i,e $\delta = 0$.
- One method to get the radiation pattern of the array is to add the field of individual four elements at a distant point vectorially.

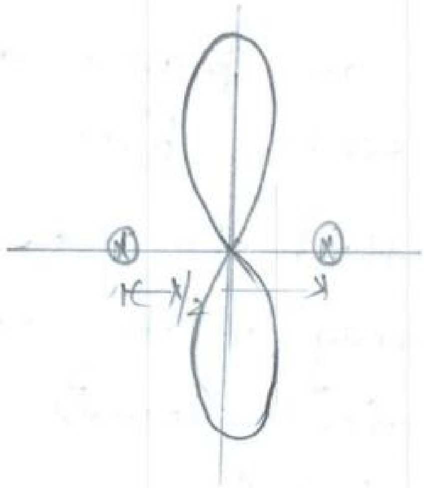


Four isotropic elements spaced $\lambda/2$

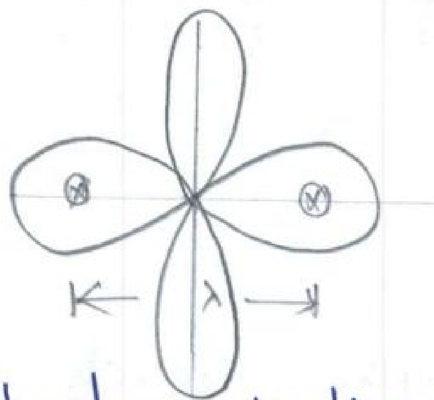
Two unit Array spaced λ apart.

- Now the elements ① & ② are considered as one unit and is considered to be placed between the midway of the elements.
- the elements ③ & ④ are considered another unit. assumed to be placed between the two elements as shown in fig.

→ Two isotropic point sources, spaced $\lambda/2$ apart fed in phase provides a bi-directional pattern [figure of eight] shown in fig. below.



→ the radiation pattern of two isotropic point sources spaced λ apart fed in phase is shown in fig. below.



→ the Resultant radiation pattern of 4-element is obtained by multiplying the radiation pattern of individual unit to the Array of two isotropic point sources spaced λ apart.

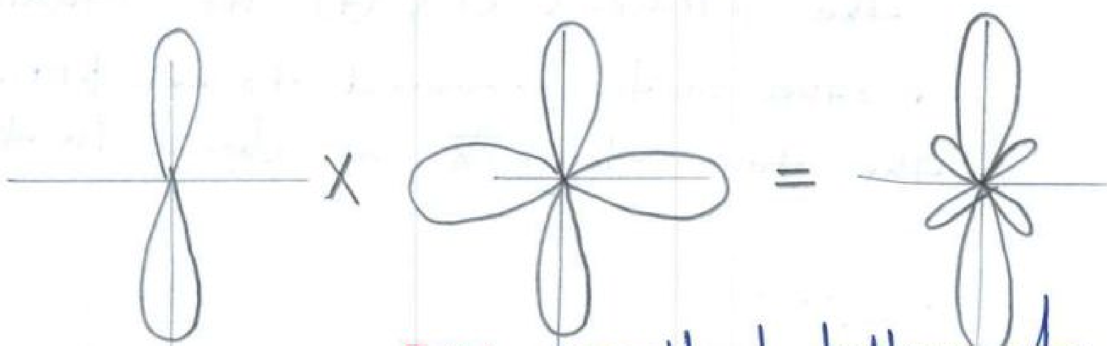
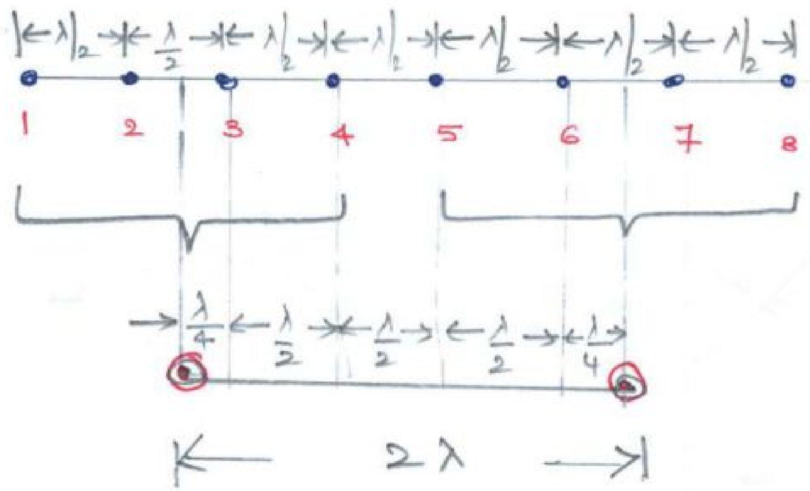


Fig: Resultant pattern of 4-isotropic elements...

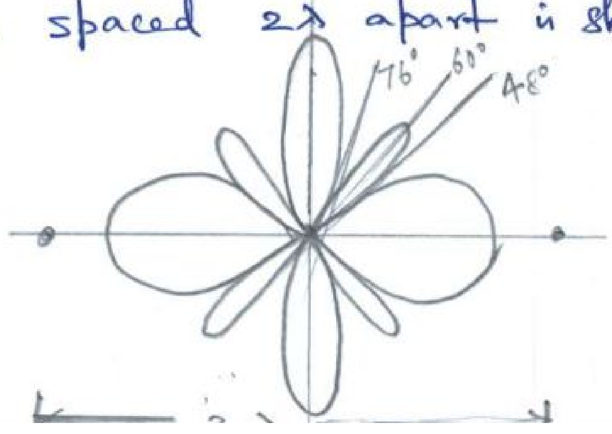
Radiation pattern of 8- isotropic elements fed in-phase, spaced $\lambda/2$ apart:

→ Let the 8-elements of isotropic radiators are placed at a distance of $\lambda/2$ and fed in phase. i,e $\delta = 0$.

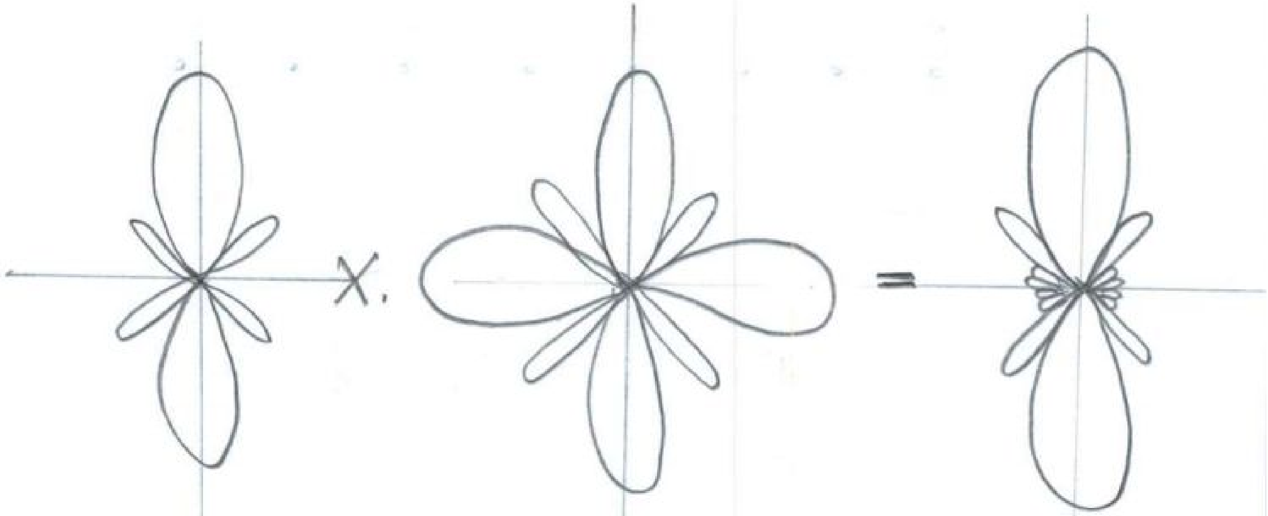


→ In this case 4- isotropic elements are assumed to be one unit, and then find the radiation pattern of two such units spaced at a distance 2λ apart.

→ the radiation pattern of two isotropic radiators spaced 2λ apart is shown in figure below.



→ Thus the radiation pattern of 8- isotropic elements is obtained by multiplying the unit pattern of individual elements and group pattern of two isotropic radiators spaced 2λ is shown in fig. below.



'Unit pattern' due to 4-individual elements.

"Group pattern" due to two isotropic elements spaced 2λ apart

Resultant pattern of 8- isotropic elements.

: Linear Arrays of n-isotropic point sources of equal Amplitude and spacing:

→ Let us consider n-isotropic point sources of equal amplitude and spacing are arranged in a uniform Linear array is shown in fig. below.

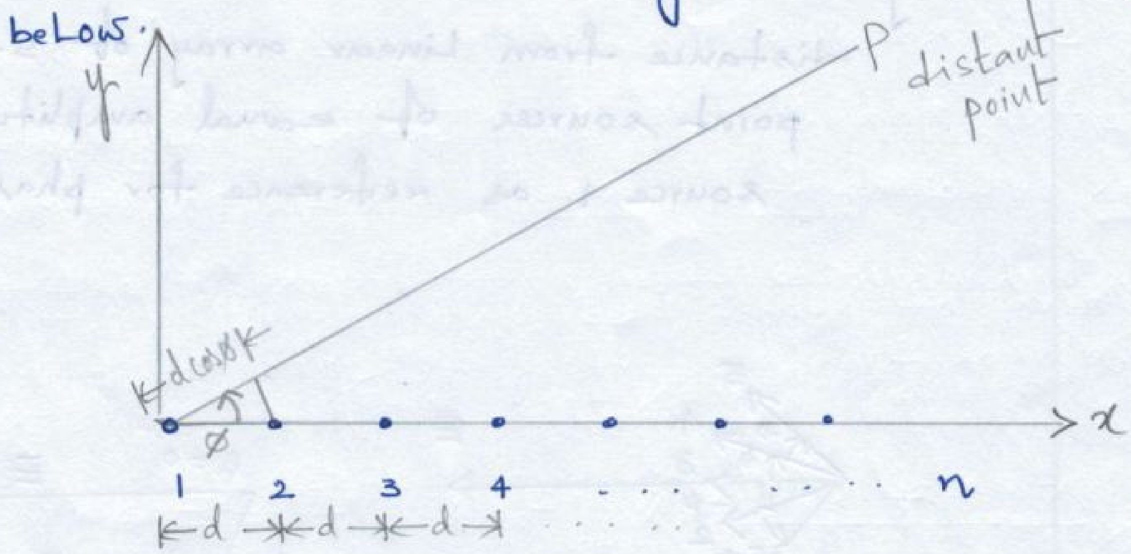


fig: Linear array with n-isotropic point sources with equal amplitude and spacing.

→ The point sources are spaced equally, and are fed with inphase currents of equal amplitudes [Eo].

→ Taking source 1 as the reference for phase, the total field at a distant point 'P' is obtained by adding vectorially fields from individual sources.

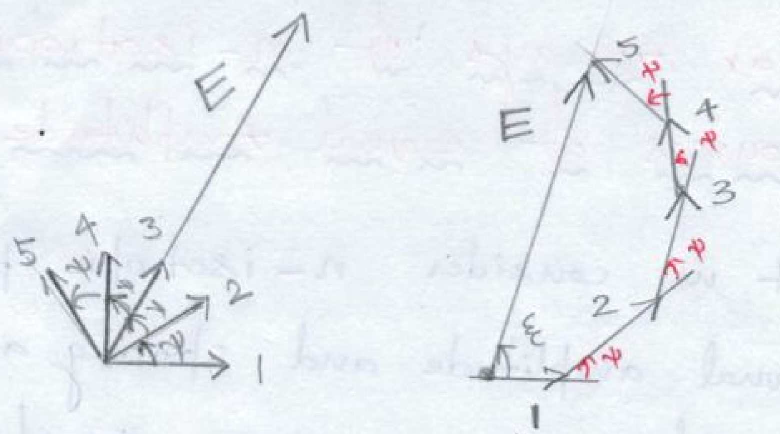


Fig: Vector Addition of fields at a Large distance from Linear array of 5-isotropic point sources of equal amplitude with source 1 as reference for phase.

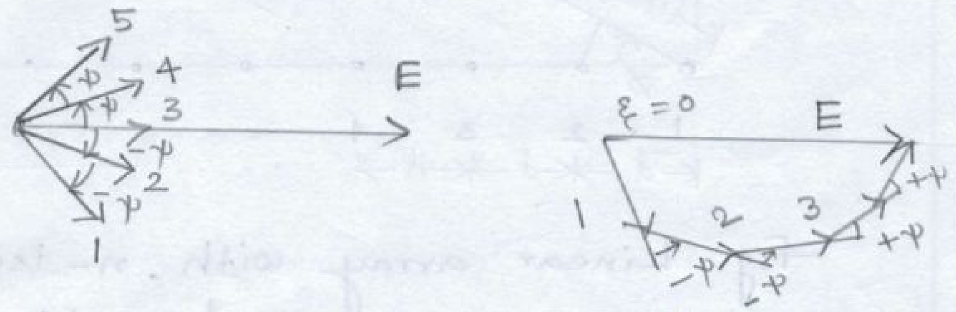


Fig: Same, but mid point of array (source 3) as reference for phase.

$$E_t = E_0 \cdot e^{j0} + E_0 \cdot e^{+j\psi} + E_0 e^{+j2\psi} + \dots + E_0 e^{j(n-1)\psi}$$

$$E_t = E_0 \left[1 + e^{j\psi} + e^{j2\psi} + \dots + e^{j(n-1)\psi} \right] \rightarrow (1)$$

where ψ is the total phase difference of the fields from adjacent sources

$$\psi = \beta d \cos\phi + \delta$$

where $\delta \rightarrow$ phase differe of adjacent sources.

→ By Multiplying eqn (1) by $e^{j\psi}$ on both sides

$$E_t \cdot e^{j\psi} = E_0 \left[e^{j\psi} + e^{j2\psi} + \cdots\cdots + e^{jn\psi} \right] \rightarrow (2)$$

substracting eqn (2) from eqn (1) we get,

$$E_t \left[1 - e^{j\psi} \right] = E_0 \left[1 - e^{jn\psi} \right]$$

$$E_t = E_0 \frac{(1 - e^{jn\psi})}{(1 - e^{j\psi})}$$

$$E_t = E_0 \frac{\left[1 - e^{jn\psi/2} \cdot e^{jn\psi/2} \right]}{\left[1 - e^{j\psi/2} \cdot e^{j\psi/2} \right]}$$

$$E_t = E_0 \cdot \frac{e^{jn\psi/2}}{e^{j\psi/2}} \frac{\left[e^{-jn\psi/2} - e^{jn\psi/2} \right]}{\left[e^{-j\psi/2} - e^{j\psi/2} \right]}$$

$$E_t = E_0 \, e^{j(n-1)\psi/2} \frac{\sin\left(\frac{n\psi}{2}\right)}{\sin(\psi/2)}$$

$$E_t = E_0 \cdot \left[\frac{\sin(n\psi/2)}{\sin(\psi/2)} \right] \angle (n-1)\psi/2 \rightarrow (3)$$

→ this is the equation of field pattern of Linear array of n- isotropic point sources of point source 1 as the reference point for phase.

→ If the phase is reffered to the center point of the array the total field is given by

$$E_t = E_0 \cdot \left[\frac{\sin \frac{n\psi}{2}}{\sin \psi/2} \right]$$

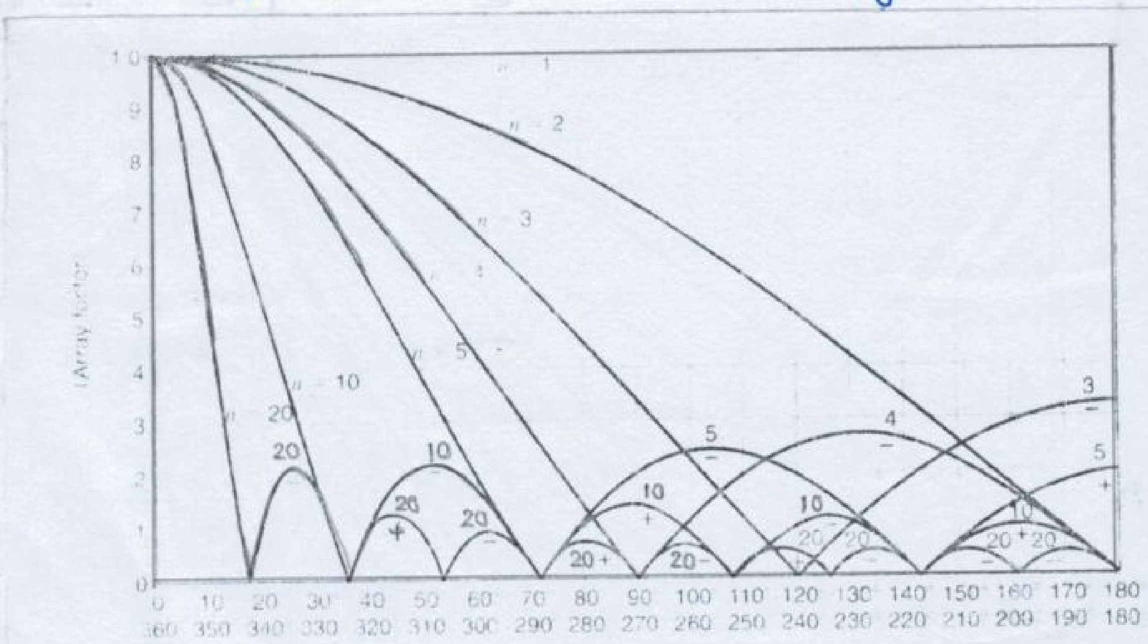
→ As ψ approaches to zero

$$E_{t_{max}} = n E_0$$

→ the normalized value of the field pattern is given by $\dfrac{E_t}{E_{tmax}} = \dfrac{\sin n\psi/2}{n \cdot \sin \psi/2}$ → ④

→ the field given by the above equation will be reffered to as the "Array factor."

→ values of the array factor as obtained from above equation for various number of sources are presented in below figure.



$\therefore$ Array factor $(AF)_n = \dfrac{\sin N\psi/2}{N \sin\psi/2} \rightarrow (1)$

→ For small values of ψ

$$\boxed{(AF)_n \doteq \frac{\sin N\psi/2}{N\,\psi/2}} \rightarrow (2)$$

Null directions:

→ To find the Nulls numerator of eqn (1) i.e array factor is zero.

$$\sin \frac{N\psi}{2} = 0$$

$$\Rightarrow \frac{N\psi}{2} = \pm K\pi$$

where $K = 1, 2, 3, 4 \ldots$

the K value determines the order of the nulls (First, second .. e.t.c)

$$\psi = \pm \frac{2K\pi}{N}$$

$$\beta d \cos\phi + \delta = \pm \frac{2K\pi}{N}$$

$$\beta d \cos\phi = \pm \frac{2K\pi}{N} - \delta$$

$$\cos\phi_n = \frac{1}{\beta d}\left[\pm\frac{2K\pi}{N} - \delta\right]$$

$$\phi_n = \cos^{-1}\left\{\frac{1}{\beta d}\left[\pm\frac{2K\pi}{N} - \delta\right]\right\}$$

$$\boxed{\phi_n = \cos^{-1}\left[\frac{\lambda}{2\pi d}\left[\pm\frac{2K\pi}{N} - \delta\right]\right]}$$

$K = 1, 2, 3, 4 \ldots\ldots$

Maxima directions:-

→ the Maximum values occur when the denominator of array factor is set to Zero...

$$\sin\frac{\psi}{2} = 0$$

$$\Rightarrow \frac{\psi}{2} = \pm K\pi$$

$K = 0, 1, 2, 3 \ldots$

$$\psi = \pm 2K\pi$$

$$\beta d\cos\phi + \delta = \pm 2K\pi$$

$$\beta d\cos\phi = \pm 2K\pi - \delta$$

$$\phi_m = \cos^{-1}\left[\frac{1}{\beta d}\left\{\pm 2K\pi - \delta\right\}\right]$$

$$\boxed{\phi_m = \cos^{-1}\left[\frac{\lambda}{2\pi d}\left[\pm 2K\pi - \delta\right]\right]}$$

$K = 0, 1, 2, 3, \cdots$

→ the Array factor of eqn ① has only one maximum and occur when $K = 0$

$$\phi_m = \cos^{-1}\left[\frac{\lambda}{2\pi d}\left[0 - \delta\right]\right]$$

$$\phi_m = \cos^{-1}\left[-\frac{\lambda\delta}{2\pi d}\right]$$

which is the observation angle that makes $\psi = 0$.

Minor Lobe maxima directions:-

→ For the array factor of eqn ① there are secondary maxima (maxima of minor Lobes) which occurs approximately when the Numerator of eqn ① attains its maximum value.

i,e $$\sin\frac{N\psi}{2} = \pm 1$$

$$\Rightarrow \quad \frac{N\psi}{2} = \pm(2K+1)\frac{\pi}{2}$$

$$\psi = \pm\frac{(2K+1)\pi}{N}$$

$K = 1, 2, 3, 4 \cdots$

$$\left[\beta d\cos\phi + \delta\right] = \pm\frac{(2K+1)\pi}{N}$$

$$\cos\phi = \frac{1}{\beta d}\left[\pm\frac{(2K+1)\pi}{N} - \delta\right]$$

$$\phi_s = \cos^{-1}\left[\frac{1}{\beta d}\left[\pm\frac{(2K+1)\pi}{N} - \delta\right]\right]$$

$$\boxed{\phi_s = \cos^{-1}\left[\frac{\lambda}{2\pi d}\left[\pm\frac{(2K+1)\pi}{N} - \delta\right]\right]}$$

$K = 1, 2, 3, 4 \cdots$

Half power point directions:

→ At half power points the Array factor becomes

$$\frac{\sin N\psi/2}{N\psi/2} = \frac{1}{\sqrt{2}}$$

$\frac{N\psi}{2} = x$ then $\frac{\sin x}{x}$ known as sync function.

$$\frac{\sin x}{x} = \frac{1}{\sqrt{2}} \text{ when } x = \pm 1.391$$

$$\text{ie } \frac{N\psi}{2} = \pm 1.391$$

$$N\psi = \pm 2.782$$

$$\psi = \pm \frac{2.782}{N}$$

$$\beta d \cos\phi + \delta = \pm \frac{2.782}{N}$$

$$\beta d \cos\phi_h = \pm \frac{2.782}{N} - \delta$$

$$\phi_h = \cos^{-1}\left[\frac{1}{\beta d}\left[\pm \frac{2.782}{N} - \delta\right]\right]$$

$$\boxed{\phi_h = \cos^{-1}\left[\frac{\lambda}{2\pi d}\left[\pm \frac{2.782}{N} - \delta\right]\right]}$$

Half power beam width:

→ the Half-power beam width can be found for a symmetrical pattern by

$$HPBW = 2\,|\phi_m - \phi_h|$$

Side Lobe Level ratio:

$$\text{Side Lobe Level ratio} = \frac{\text{First side Lobe level}}{\text{Main Lobe Level}}$$

→ The Maximum of the first minor Lobe of eqn ① occurs approximately when

$$\sin\frac{N\psi}{2} = \pm 1$$

→ $\frac{N\psi}{2} = \pm(2K+1)\frac{\pi}{2}, \quad K = 1, 2, 3 \cdots$

→ For First side Lobe maximum $K=1$

$$\frac{N\psi}{2} = \pm\frac{3\pi}{2}$$

$$(AF)_{SL} = \frac{\sin N\psi/2}{N\psi/2} = \frac{\sin 3\pi/2}{3\pi/2}$$

$$= \frac{1}{3\pi/2} = \frac{2}{3\pi}$$

$$(AF)_{SL} = \frac{2}{3\pi} = 0.212.$$

$$(AF)_{ML} = 1$$

$$\therefore \text{ side Lobe Level ratio} = \frac{(AF)_{SL}}{(AF)_{ML}}$$

$$SLR = \frac{0.212}{1}$$

$$SLR = 0.212$$

$$SLR = 20\log_{10}(0.212)$$

$$\boxed{SLR = -13.46 \text{ dB}}$$

→ Thus the maximum of the first minor Lobe of the array factor of eqn ① is 13.46 dB down from the maximum at the Major Lobe.

Summary :

→ Linear Arrays of n isotropic point sources of equal Amplitude and spacing..

1. Array factor $= (AF)_n = \frac{1}{N}\left[\frac{\sin \frac{N\psi}{2}}{\sin \frac{\psi}{2}}\right]$

2. Null directions:

$$\phi_n = \cos^{-1}\left[\frac{\lambda}{2\pi d}\left[\pm\frac{2K\pi}{N} - \delta\right]\right]$$

$K = 1, 2, 3, 4 \cdots$

3. Maxima directions:

$$\phi_m = \cos^{-1}\left[\frac{\lambda}{2\pi d}\left[\pm 2K\pi - \delta\right]\right]$$

$K = 0, 1, 2, 3 \ldots$

4. MinorLobe maxima directions:

$$\phi_s = \cos^{-1}\left[\frac{\lambda}{2\pi d}\left[\pm\frac{(2K+1)\pi}{N} - \delta\right]\right]$$

$K = 1, 2, 3, 4 \cdots$

5. Half power point direction:

$$\phi_h = \cos^{-1}\left[\frac{\lambda}{2\pi d}\left[\pm\frac{2.782}{N} - \delta\right]\right]$$

6. Half-power beam width:

$$HPBW = 2\left|\phi_m - \phi_h\right|$$

7. side Lobe Level ratio $= -13.46\,dB$

: Broad side Array :

→ Broad side array may be defined as an arrangement in which the principal direction is perpendicular to the array axis and also to the plane containing the array element.

→ In broad side array the maximum radiation of an array directed normal to the axis of the array i,e $\phi = 90^\circ$

→ The First maximum of the array factor Occurs when $\psi = 0$

$$\psi = \beta d \cos\phi + \delta = 0$$

since it is desired to have the first maximum directed towards $\phi = 90^\circ$, then

$$\psi = \beta d \cos\phi + \delta \Big|_{\phi = 90^\circ} = 0$$

$$\Rightarrow \boxed{\delta = 0}$$

Thus to have the maximum of the array factor of a uniform Linear array directed broadside to the axis of the array, it is necessary that all the elements have the same phase excitation.

→ To ensure that there are no prinupal maxima in other directions, which are referred to as grating Lobes, the separation between the elements should not be equal to multiples of a wavelength. i,e $d \neq n\lambda$; $n = 1, 2, 3 \ldots$ when $\delta = 0$.

→ To avoid any grating Lobe, the Largest spacing between the elements should be Less than one wavelength.

$$d_{max} < \lambda$$

Null directions:

$$\phi_n = \cos^{-1}\left[\frac{\lambda}{2\pi d}\left[\pm\frac{2K\pi}{N} - \delta\right]\right]$$

$K = 1, 2, 3, \ldots$

$\delta = 0$

$$\therefore\ \phi_n = \cos^{-1}\left[\frac{\lambda}{\cancel{2}\pi d} \times \pm\frac{\cancel{2}K\pi}{N}\right]$$

$$\phi_n = \cos^{-1}\left[\pm\frac{K\lambda}{Nd}\right]$$

$K = 1, 2, 3 \ldots$

Maxima directions:

$$\phi_m = \cos^{-1}\left[\frac{\lambda}{2\pi d}\left[\pm 2K\pi - \delta\right]\right]$$

$K = 0, 1, 2, 3 \ldots$

$\delta = 0$

$$\phi_m = \cos^{-1}\left[\frac{\lambda}{\cancel{2\pi} d}\left[\pm \cancel{2}K\cancel{\pi}\right]\right]$$

$$\phi_m = \cos^{-1}\left[\pm\frac{K\lambda}{d}\right]$$

for $K=0$

$\phi_m = \frac{\pi}{2}$.

$K = 0, 1, 2, 3 \ldots$

Half power points directions:

$$\phi_h = \cos^{-1}\left[\frac{\lambda}{2\pi d}\left[\pm\frac{2.782}{N} - \delta\right]\right]$$

$\delta = 0$

$$\phi_h = \cos^{-1}\left[\pm\frac{2.782\lambda}{2\pi d N}\right]$$

$$\phi_h = \cos^{-1}\left[\pm\frac{1.391\lambda}{\pi N d}\right]$$

Minor Lobe Maxima directions:

$$\phi_s = \cos^{-1}\left[\frac{\lambda}{2\pi d}\left[\pm\frac{(2K+1)\pi}{N} - \delta\right]\right]$$

$K = 1, 2, 3 \ldots$

$\delta = 0$

$$\phi_s = \cos^{-1}\left[\frac{\lambda}{2\cancel{\pi} d}\left[\pm\frac{(2K+1)\cancel{\pi}}{N}\right]\right]$$

$$\phi_s = \cos^{-1}\left[\pm\frac{\lambda}{2d}\left[\frac{2K+1}{N}\right]\right]$$

$K = 1, 2, 3 \ldots$

First-Null beamwidth (FNBW):

$$\phi_n = \cos^{-1}\left[\pm \frac{K\lambda}{Nd}\right]$$

$K = 1, 2, 3 \ldots$

$K=1$ $$\phi_n = \cos^{-1}\left[\pm \frac{\lambda}{Nd}\right]$$

→ First Null beam width can be found for a symmetrical pattern by

$$FNBW = 2\left|\phi_m - \phi_n\right|$$

$$\phi_m = \cos^{-1}\left[\pm \frac{K\lambda}{d}\right]$$

For $K=0$

$$\phi_m = \pi/2 \text{ (or) } 90^\circ$$

$$\therefore \quad FNBW = 2\left[\frac{\pi}{2} - \cos^{-1}\left[\frac{\lambda}{Nd}\right]\right]$$

Half-power beam width (HPBW):

$$\phi_h = \cos^{-1}\left[\pm \frac{1.391\lambda}{\pi Nd}\right]$$

→ the Half-power beam width can be found for a symmetrical pattern by

$$HPBW = 2\left|\phi_m - \phi_h\right|$$

$$HPBW = 2\left[\frac{\pi}{2} - \cos^{-1}\left[\frac{1.391\lambda}{\pi Nd}\right]\right]$$

First side Lobe beamwidth (FSLBW) :-

$$\phi_s = \cos^{-1}\left[\pm \frac{\lambda}{2d} \left[\frac{2K+1}{N} \right] \right]$$

$K = 1, 2, 3 \cdots$

→ For First side Lobe $K = 1$

$$\phi_s = \cos^{-1}\left[\pm \frac{\lambda}{2d} \left[\frac{3}{N} \right] \right]$$

$$\phi_s = \cos^{-1}\left[\pm \frac{3\lambda}{2dN} \right]$$

→ the First side Lobe beamwidth can be found by

$$FSLBW = 2 | \phi_m - \phi_s |$$

$$FSLBW = 2\left[\frac{\pi}{2} - \cos^{-1}\left[\frac{3\lambda}{2dN} \right] \right]$$

Table : 1 Nulls, Maxima, Half power points and Minor Lobe maxima for Uniform Amplitude broad side array :

Nulls	$\phi_n = \cos^{-1}\left[\pm \frac{K\lambda}{Nd} \right]$ $K = 1, 2, 3 \cdots$
Maxima	$\phi_m = \cos^{-1}\left[\pm \frac{K\lambda}{d} \right]$ $K = 0, 1, 2, 3 \cdots$
Half-power points	$\phi_h = \cos^{-1}\left[\pm \frac{1.391\lambda}{\pi N d} \right]$
Minor Lobe maxima	$\phi_s = \cos^{-1}\left[\pm \frac{\lambda}{2d} \left[\frac{2K+1}{N} \right] \right]$ $K = 1, 2, 3 \cdots$

Table 2: Beam width's for uniform amplitude broadside Arrays:

First-Null Beam width (FNBW)	$FNBW = 2\left[\frac{\pi}{2} - \cos^{-1}\left(\frac{\lambda}{Nd}\right)\right]$
Half-power Beam width (HPBW)	$HPBW = 2\left[\frac{\pi}{2} - \cos^{-1}\left[\frac{1.391\lambda}{\pi N d}\right]\right]$
First side Lobe Beam width (FSLBW)	$FSLBW = 2\left[\frac{\pi}{2} - \cos^{-1}\left[\frac{3\lambda}{2dN}\right]\right]$

Directivity - Broadside Array:

→ For broad-side array $\delta = 0$

$$\psi = \beta d \cos\phi + \delta$$

$$\psi = \beta d \cos\phi$$

→ for broadside radiation, the array factor is given by

$$(AF)_n = \frac{1}{N}\left[\frac{\sin\left[\frac{N\beta d\cos\phi}{2}\right]}{\sin\left[\frac{\beta d\cos\phi}{2}\right]}\right]$$

→ For small spacing between the elements $d << \lambda$

$$(AF)_n \simeq \frac{\sin\left(\frac{N\beta d\cos\phi}{2}\right)}{\left(\frac{N\beta d\cos\phi}{2}\right)}$$

→ The radiation intensity can be written as

$$U(\phi) = \left[(AF)_n\right]^2 = \left[\frac{\sin\left(\frac{N\beta d\cos\phi}{2}\right)}{\left(\frac{N\beta d\cos\phi}{2}\right)}\right]^2$$

→ The Directivity of the Array factor can be evaluated using

$$D = \frac{U_{max}}{U_0} = \frac{U_{max}}{P_{rad}/4\pi}$$

$$D = \frac{4\pi\, U_{max}}{P_{rad}}$$

where $U_0 \rightarrow$ is the Average radiation intensity.

$$U_0 = \frac{P_{rad}}{4\pi}$$

→ $$P_{rad} = \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} U \cdot \sin\theta \, d\theta \, d\phi$$

$$P_{rad} = \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi}\left[\frac{\sin\left(\frac{N\beta d\cos\phi}{2}\right)}{\left(\frac{N\beta d\cos\phi}{2}\right)}\right]^2 \sin\theta\, d\theta\, d\phi$$

$$P_{rad} = 2 \times \int_{\phi=0}^{2\pi}\left[\frac{\sin\left(\frac{N\beta d\cos\phi}{2}\right)}{\left(\frac{N\beta d\cos\phi}{2}\right)}\right]^2 d\phi$$

$$P_{rad} = 2 \times \left(\frac{\pi^2}{N\beta d/2}\right)$$

$$P_{rad} = \frac{4\pi^2}{N\beta d}.$$

$$\rightarrow \quad U_0 = \frac{P_{rad}}{4\pi} = \frac{\cancel{4}\pi^{\cancel{2}}}{N\beta d \times \cancel{4\pi}} = \frac{\pi}{N\beta d}$$

$$\rightarrow \quad U_{max} = 1$$

$$\rightarrow \quad \therefore \quad D = \frac{U_{max}}{U_0}$$

$$D = \frac{1}{\pi/N\beta d} = \frac{N\beta d}{\pi}$$

$$D = N \times \frac{2\cancel{\pi}}{\lambda} \times \frac{d}{\cancel{\pi}}$$

$$\boxed{D = \frac{2Nd}{\lambda} = \frac{2L}{\lambda}}$$

if L is the overall Length of the array

$L = (N-1)d$; for Large array $L >> d$; $L \simeq Nd$

Ordinary End-Fire Array:-

- The end fire array may be defined as the arrangement in which the principal direction of radiation coincides with the direction of the array axis.
- For an array to be end-fire, it may be necessary that it radiates towards only one direction i,e either $\phi = 0^\circ$ or $\phi = 180^\circ$
- To direct the first maximum towards $\phi = 0^\circ$,

$$\psi = \beta d \cos\phi + \delta \Big|_{\phi = 0^\circ} = 0$$

$$\Rightarrow \quad \beta d + \delta = 0$$

$$\delta = -\beta d.$$

- If the first maximum is desired towards $\phi = 180^\circ$, then

$$\psi = \beta d \cos\phi + \delta \Big|_{\phi = 180^\circ} = 0$$

$$\Rightarrow -\beta d + \delta = 0$$

$$\delta = \beta d$$

- Thus end-Fire radiation is accomplished when $\delta = -\beta d$ for $\phi = 0^\circ$ or $\delta = +\beta d$ for $\phi = 180^\circ$

→ If the element separation is $d = \lambda/2$, end-fire radiation exists simultaneously in both directions i,e $\phi = 0°$ and $\phi = 180°$.

→ If the element spacing is a multiple of a waveLength $d = n\lambda$; $n = 1, 2, 3 \ldots$ then in addition to having end-fire radiation in both directions, there also exist maxima in the broad side directions.

→ ∴ To have only one end-fire maximum and to avoid any grating Lobes, the maximum spacing between the elements should be Less than $d_{max} < \frac{\lambda}{2}$.

Null directions:

$$\phi_n = \cos^{-1}\left[\frac{\lambda}{2\pi d}\left[\pm\frac{2K\pi}{N} - \delta\right]\right]$$

$K = 1, 2, 3 \ldots$

$\delta = -\beta d \quad \phi = 0°$

$$\therefore \phi_n = \cos^{-1}\left[\frac{\lambda}{2\pi d}\left[\pm\frac{2K\pi}{N} + \beta d\right]\right]$$

$$= \cos^{-1}\left[\frac{\lambda}{\cancel{2\pi} d}\left[\pm\frac{\cancel{2}K\cancel{\pi}}{N} + \frac{\cancel{2\pi} d}{\lambda}\right]\right]$$

$$= \cos^{-1}\left[\frac{\lambda}{d}\left[\pm\frac{K}{N} + \frac{d}{\lambda}\right]\right]$$

$$\phi_n = \cos^{-1}\left[1 - \frac{K\lambda}{Nd}\right] \; ; \quad K = 1, 2, 3 \ldots$$

Maxima directions:

$$\phi_m = \cos^{-1}\left[\frac{\lambda}{2\pi d}\left[\pm 2K\pi - \delta\right]\right]$$

$K = 0, 1, 2, 3 \ldots$

$\delta = -\beta d$

$$\phi_m = \cos^{-1}\left[\frac{\lambda}{2\pi d}\left[\pm 2K\pi + \beta d\right]\right]$$

$$\phi_m = \cos^{-1}\left[\frac{\lambda}{\cancel{2\pi} d}\left[\pm \cancel{2}K\cancel{\pi} + \frac{\cancel{2\pi} d}{\lambda}\right]\right]$$

$$\phi_m = \cos^{-1}\left[\frac{\lambda}{d}\left[\pm K + \frac{d}{\lambda}\right]\right]$$

$$\phi_m = \cos^{-1}\left[1 - \frac{K\lambda}{d}\right]$$

$K = 0, 1, 2, 3 \ldots$

Half-power point directions:

$$\phi_h = \cos^{-1}\left[\frac{\lambda}{2\pi d}\left[\pm \frac{2.782}{N} - \delta\right]\right]$$

$\delta = -\beta d$

$$\phi_h = \cos^{-1}\left[\frac{\lambda}{2\pi d}\left[\pm \frac{2.782}{N} + \frac{2\pi d}{\lambda}\right]\right]$$

$$\phi_h = \cos^{-1}\left[1 - \frac{1.391\lambda}{\pi d N}\right]$$

Minor Lobe maxima directions:

$$\phi_s = \cos^{-1}\left[\frac{\lambda}{2\pi d}\left[\pm \frac{(2K+1)\pi}{N} - \delta\right]\right]$$

$\delta = -\beta d$

$$\phi_s = \cos^{-1}\left[\frac{\lambda}{2\cancel{\pi} d}\left[\pm \frac{(2K+1)\cancel{\pi}}{N} + \frac{2\cancel{\pi}}{\lambda} d\right]\right]$$

$$\phi_s = \cos^{-1}\left[1 - \frac{(2K+1)\lambda}{2Nd}\right] \quad K = 1, 2, 3, \ldots$$

First - Null beamwidth (FNBW):

$$\phi_n = \cos^{-1}\left[1 - \frac{K\lambda}{Nd}\right], K = 1, 2, 3, \cdots$$

K = 1

$$\phi_n = \cos^{-1}\left[1 - \frac{\lambda}{Nd}\right]$$

→ First null beamwidth can be found for a symmetrical pattern by

$$FNBW = 2|\phi_m - \phi_n|$$

$$\phi_m = \cos^{-1}\left[1 - \frac{K\lambda}{d}\right] \quad K = 0, 1, 2, 3 \cdots$$

for K = 0

$$\phi_m = 0^\circ$$

$$FNBW = 2\left|0 - \cos^{-1}\left[1 - \frac{\lambda}{Nd}\right]\right|$$

$$FNBW = 2\cos^{-1}\left[1 - \frac{\lambda}{Nd}\right]$$

Half-power beam width (HPBW):

$$\phi_n = \cos^{-1}\left[1 - \frac{1.391\lambda}{\pi d N}\right]$$

→ the Half-power beam width can be found for a symmetrical pattern by

$$HPBW = 2|\phi_m - \phi_n|$$

$$HPBW = 2\cos^{-1}\left[1 - \frac{1.391\lambda}{\pi d N}\right]$$

First side Lobe beamwidth (FSLBW):

$$\phi_s = \cos^{-1}\left[1 - \frac{(2K+1)\lambda}{2Nd}\right]$$

$K = 1, 2, 3, \ldots$

→ For First side Lobe $K=1$

$$\phi_s = \cos^{-1}\left[1 - \frac{3\lambda}{2Nd}\right]$$

→ The First side Lobe beamwidth can be found by

$$FSLBW = 2\left|\phi_m - \phi_s\right|$$

$$FSLBW = 2\cos^{-1}\left[1 - \frac{3\lambda}{2Nd}\right]$$

Table 1: Nulls, Maxima, Half-power points, and minor Lobe maxima for Uniform Amplitude Ordinary End-fire Arrays..

Nulls	$\phi_n = \cos^{-1}\left[1 - \frac{K\lambda}{Nd}\right]$ $K = 1, 2, 3, \ldots$
Maxima	$\phi_m = \cos^{-1}\left[1 - \frac{K\lambda}{d}\right]$ $K = 0, 1, 2, 3 \ldots$
Half-power points	$\phi_h = \cos^{-1}\left[1 - \frac{1.391\lambda}{\pi Nd}\right]$
Minor-Lobe maxima	$\phi_s = \cos^{-1}\left[1 - \frac{(2K+1)\lambda}{2Nd}\right]$ $K = 1, 2, 3, \ldots$

Table 2: Beam width's for Uniform Amplitude Ordinary End-fire Arrays:

First-Null Beam Width (FNBW)	$FNBW = 2\cos^{-1}\left[1-\frac{\lambda}{Nd}\right]$
Half-power beam width (HPBW)	$HPBW = 2\cos^{-1}\left[1-\frac{1.391\lambda}{\pi d N}\right]$
First side Lobe beam width (FSLBW)	$FSLBW = 2\cos^{-1}\left[1-\frac{3\lambda}{2Nd}\right]$

Directivity - End Fire Array:

→ For End Fire Array $\delta = -\beta d$ $\phi = 0°$

$$\psi = \beta d \cos\phi + \delta$$

$$\psi = \beta d \cos\phi - \beta d$$

$$\psi = \beta d(\cos\phi - 1)$$

→ For ordinary end-fire radiation, the array factor is given by

$$(AF)_n = \frac{\sin\left(\frac{N\beta d(\cos\phi-1)}{2}\right)}{\left(\frac{N\beta d(\cos\phi-1)}{2}\right)}$$

→ The radiation intensity can be written as

$$U(\phi) = [(AF)_n]^2 = \left[\frac{\sin\left(\frac{N\beta d(\cos\phi-1)}{2}\right)}{\left(\frac{N\beta d(\cos\phi-1)}{2}\right)}\right]^2$$

→ the Directivity of the Array factor can be evaluated using

$$D = \frac{U_{max}}{U_0} = \frac{U_{max}}{P_{rad}/4\pi}$$

$$D = \frac{4\pi\, U_{max}}{P_{rad}}$$

where U_0 → is the average radiation intensity

$$U_0 = \frac{P_{rad}}{4\pi}$$

$$P_{rad} = \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} U \sin\theta \, d\theta \, d\phi$$

$$P_{rad} = \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} \left[\frac{\sin\left(\frac{N\beta d(\cos\phi-1)}{2}\right)}{\left(\frac{N\beta d(\cos\phi-1)}{2}\right)}\right]^2 \sin\theta \, d\theta \, d\phi$$

$$P_{rad} = 2 \times \int_{\phi=0}^{2\pi} \left[\frac{\sin\left(\frac{N\beta d(\cos\phi - 1)}{2}\right)}{\left(\frac{N\beta d(\cos\phi-1)}{2}\right)} \right]^2 d\phi$$

$$P_{rad} = 2 \times \left[\frac{\pi^2}{N\beta d}\right]$$

$$U_0 = \frac{P_{rad}}{4\pi} = \frac{2\pi^2}{4\pi \times N\beta d}$$

$$U_0 = \frac{\pi}{2N\beta d}$$

$$U_{max} = 1$$

$$\therefore D = \frac{U_{max}}{U_0}$$

$$D = \frac{1}{\pi/2N\beta d} = \frac{2N\beta d}{\pi}$$

$$D = 2N \times \frac{2\cancel{\pi}}{\lambda} \times \frac{d}{\cancel{\pi}}$$

$$\boxed{D = 4\frac{Nd}{\lambda} = 4\frac{L}{\lambda}}$$

If L is the overall Length of the array

$L = (N-1)d$; for Large array $L >> d$

$L \simeq Nd$

: Hansen – Woodyard End – Fire Array :

→ The conditions for an ordinary end-fire array was concluded that the maximum radiation can be directed along the axis of the uniform array by allowing the progressive phase shift 'δ' between the elements to be equal to $\delta = -\beta d$ for $\phi = 0°$, and $\delta = +\beta d$ for $\phi = 180°$.

→ To enhance the directivity of an end-fire array without destroying any of the other characteristics, Hansen and Woodyard proposed that the required phase shift between closely spaced elements of a very long array should be

$$\delta = -\beta d - \frac{\pi}{n} \quad \text{for maximum in } \phi = 0° \qquad \rightarrow (1)$$

$$\boxed{\delta = +\beta d + \frac{\pi}{n} \quad \text{for maximum in } \phi = 180°} \qquad \rightarrow (2)$$

→ These requirements are known as the Hansen – Woodyard conditions for end-fire radiation. they Lead to a Larger directivity than the conditions given by ordinary end-fire array...

→ To realize the increase in directivity as a result of the Hansen-Woodyard conditions, it is necessary that, in addition to the above conditions $|\psi|$ assumes the values of

→ For maximum radiation along $\phi = 0°$

$$|\psi| = |\beta d \cos\phi + \delta|_{\phi=0°} = \frac{\pi}{N} \quad \text{and}$$

$$|\psi| = |\beta d \cos\phi + \delta|_{\phi=180°} \simeq \pi \quad \rightarrow (3)$$

→ For maximum radiation along $\phi = 180°$

$$|\psi| = |\beta d \cos\phi + \delta|_{\phi=180°} = \frac{\pi}{N} \quad \text{and}$$

$$|\psi| = |\beta d \cos\phi + \delta|_{\phi=0°} \simeq \pi \quad \rightarrow (4)$$

→ The condition of $|\psi| = \frac{\pi}{N}$ in eqns (3) & (4) is realized by the use of eqns ① & ② respectively.

→ The condition of $|\psi| \simeq \pi$ is satisfied by the use of eqns ① & ②, and choosing for each a spacing of

$$d = \left[\frac{N-1}{N}\right]\frac{\lambda}{4}$$

If the number of elements is Large, then the above eqn can be approximated by $d \simeq \frac{\lambda}{4}$

→ Thus for a Large uniform array, the Hansen-Woodyard condition can only yield an improved directivity provided the spacing between the elements is approximately $\frac{\lambda}{4}$.

Null directions:

$$\phi_n = \cos^{-1}\left[\frac{\lambda}{2\pi d}\left[\pm\frac{2K\pi}{N} - \delta\right]\right]$$

$K = 1, 2, 3, \ldots$

$$\delta = -\beta d - \frac{\pi}{N}$$

$$\phi_n = \cos^{-1}\left[\frac{\lambda}{2\pi d}\left[\pm\frac{2K\pi}{N} + \beta d + \frac{\pi}{N}\right]\right]$$

$$\phi_n = \cos^{-1}\left[\frac{\lambda}{2\pi d}\left[\pm\frac{2K\pi}{N} + \frac{2\pi}{\lambda}d + \frac{\pi}{N}\right]\right]$$

$$\phi_n = \cos^{-1}\left[\pm\frac{K\lambda}{Nd} + 1 + \frac{\lambda}{2Nd}\right]$$

$$\phi_n = \cos^{-1}\left[1 + \frac{\lambda}{2Nd} \pm \frac{K\lambda}{Nd}\right]$$

$$\phi_n = \cos^{-1}\left[1 + \frac{\lambda}{2Nd}\left[1 - \frac{K\cancel{\lambda}}{\cancel{Nd}} \times \frac{2\cancel{Nd}}{\cancel{\lambda}}\right]\right]$$

$$\boxed{\phi_n = \cos^{-1}\left[1 + \frac{\lambda}{2Nd}\left[1 - 2K\right]\right]}$$

$K = 1, 2, 3 \ldots$

Maxima directions:

$$\phi_m = \cos^{-1}\left[\frac{\lambda}{2\pi d}\left[\pm 2K\pi - \delta\right]\right]$$

$K = 0, 1, 2, 3, \cdots$

$$\delta = -\beta d - \frac{\pi}{N}$$

$$\phi_m = \cos^{-1}\left[\frac{\lambda}{2\pi d}\left[\pm 2K\pi + \beta d + \frac{\pi}{N}\right]\right]$$

$$\phi_m = \cos^{-1}\left[\frac{\lambda}{2\pi d}\left[\pm 2K\pi + \frac{2\pi}{\lambda}.d + \frac{\pi}{N}\right]\right]$$

$$\phi_m = \cos^{-1}\left[\pm\frac{K\lambda}{d} + 1 + \frac{\lambda}{2Nd}\right]$$

$$\phi_m = \cos^{-1}\left[1 + \frac{\lambda}{2Nd}\left[1 - \frac{K\lambda}{d} \times \frac{2Nd}{\lambda}\right]\right]$$

$$\boxed{\phi_m = \cos^{-1}\left[1 + \frac{\lambda}{2Nd}\left[1 - 2KN\right]\right]}$$

$K = 1, 2, 3 \cdots$

Minor Lobe Maxima directions:-

$$\phi_s = \cos^{-1}\left[\frac{\lambda}{2\pi d}\left[\pm\frac{(2K+1)\pi}{N} - \delta\right]\right]$$

$K = 1, 2, 3 \cdots$

$$\delta = -\beta d - \frac{\pi}{N}$$

$$\phi_s = \cos^{-1}\left[\frac{\lambda}{2\pi d}\left[\pm\frac{(2K+1)\pi}{N} + \beta d + \frac{\pi}{N}\right]\right]$$

$$\phi_s = \cos^{-1}\left[\frac{\lambda}{2\pi d}\left[\pm\frac{(2K+1)\pi}{N} + \frac{2\pi d}{\lambda} + \frac{\pi}{N}\right]\right]$$

$$\phi_s = \cos^{-1}\left[\pm \frac{(2K+1)\cancel{\pi}}{N} \times \frac{\lambda}{2\cancel{\pi} d} + 1 + \frac{\cancel{\pi}}{N} \times \frac{\lambda}{2\cancel{\pi} d} \right]$$

$$\phi_s = \cos^{-1}\left[1 + \frac{\lambda}{2Nd} \pm \frac{(2K+1)\lambda}{2Nd} \right]$$

$$\phi_s = \cos^{-1}\left[1 + \cancel{\frac{\lambda}{2Nd}} - \frac{2K\lambda}{2Nd} - \cancel{\frac{\lambda}{2Nd}} \right]$$

$$\boxed{\phi_s = \cos^{-1}\left[1 - \frac{K\lambda}{Nd} \right]}$$

$K = 1, 2, 3, \dots$

Half-power point directions :

$$E_t = E_0 \frac{\sin N\psi/2}{\sin \psi/2}$$

$$\psi = \beta d \cos\phi + \delta$$

$\delta = -\beta d - \pi/N$ to get maximum in $\phi = 0°$

$$\psi = \beta d \cos\phi - \beta d - \frac{\pi}{N}$$

At $\phi = 0°$

$$\psi = \beta d - \beta d - \frac{\pi}{N}$$

$$\psi = -\pi/N$$

$$\therefore E_{t\max} = E_0 \left[\frac{\sin\left(-\frac{\cancel{N} \times \pi}{2\cancel{N}}\right)}{\sin\left(-\frac{\pi}{2N}\right)} \right]$$

$$\rightarrow \quad E_{tmax} = E_0 \frac{\sin \pi/2}{\sin(\pi/2N)}$$

$$E_{tmax} = E_0 \times \frac{1}{\sin(\pi/2N)}$$

$$\rightarrow \quad E_{nor} = \frac{E_t}{E_{tmax}} = \frac{\cancel{E_0} \dfrac{\sin(N\psi/2)}{\sin(\psi/2)}}{\cancel{E_0} \cdot \dfrac{1}{\sin(\pi/2N)}}$$

$$E_{nor} = \sin\left(\frac{\pi}{2N}\right) \cdot \frac{\sin \frac{N\psi}{2}}{\sin \psi/2}$$

$$\rightarrow \quad (AF)_n = \sin\left(\frac{\pi}{2N}\right) \cdot \frac{\sin N\psi/2}{\sin \psi/2}$$

As the Array is Large $\psi \to 0$

$$(AF)_n \simeq \left(\frac{\pi}{2N}\right) \cdot \frac{\sin N\psi/2}{\psi/2}$$

$$(AF)_n \simeq \frac{\pi}{2} \cdot \frac{\sin N\psi/2}{N\psi/2}$$

→ At half power points the Array factor becomes

$$\frac{\pi}{2} \cdot \frac{\sin N\psi/2}{N\psi/2} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \frac{\sin N\psi/2}{N\psi/2} = \frac{1}{\sqrt{2}} \times \frac{2}{\pi} = 0.45031$$

$$\frac{\sin N\psi/2}{N\psi/2} = 0.45031$$

$$\frac{\sin x}{x} = 0.45031 \quad \text{when} \quad x \simeq 2$$

$$\text{i,e} \quad \frac{N\psi}{2} = \pm 2$$

$$\psi = \pm \frac{4}{N}$$

$$\beta d \cos\phi_n + \delta = \pm \frac{4}{N}$$

$$\beta d \cos\phi_n - \beta d - \frac{\pi}{N} = \pm \frac{4}{N}$$

$$\beta d \cos\phi_n = \pm \frac{4}{N} + \beta d + \frac{\pi}{N}$$

$$\cos\phi_n = \pm \frac{4}{N\beta d} + \cancel{\frac{\beta d}{\beta d}} + \frac{\pi}{N\beta d}$$

$$\cos\phi_n = \pm \frac{4}{N\beta d} + 1 + \frac{\pi}{N\beta d}$$

$$\phi_n = \cos^{-1}\left[1 + \frac{\cancel{\pi}}{N \times \frac{2\cancel{\pi}}{\lambda} \times d} \pm \frac{4}{N \times \frac{2\pi}{\lambda} \times d}\right]$$

$$\phi_n = \cos^{-1}\left[1 + \frac{\lambda}{2Nd} \pm \frac{2\lambda}{\pi N d}\right]$$

$$\phi_n = \cos^{-1}\left[1 + \frac{\lambda}{Nd}\left[\frac{1}{2} - \frac{2}{\pi}\right]\right]$$

$$\boxed{\phi_n = \cos^{-1}\left[1 - 0.1398 \frac{\lambda}{Nd}\right]}$$

First-Null beamwidth (FNBW):

$$\phi_n = \cos^{-1}\left[1 + \frac{\lambda}{2Nd}[1-2K]\right]$$

$K = 1, 2, 3, \ldots$

$$K = 1 \quad \phi_n = \cos^{-1}\left[1 - \frac{\lambda}{2Nd}\right]$$

→ First null beamwidth can be found for a symmetrical pattern by

$$FNBW = 2|\phi_m - \phi_n|$$

$$\phi_m = 0^\circ$$

$$FNBW = 2\cos^{-1}\left[1 - \frac{\lambda}{2Nd}\right]$$

Half-power beamwidth (HPBW):

$$\phi_n = \cos^{-1}\left[1 - 0.1398\frac{\lambda}{Nd}\right]$$

→ The Half-power beam width can be found for a symmetrical pattern by

$$HPBW = 2|\phi_m - \phi_n|$$

$$HPBW = 2\cos^{-1}\left[1 - 0.1398\frac{\lambda}{Nd}\right]$$

First side Lobe beamwidth (FSLBW):

$$\phi_s = \cos^{-1}\left[1 - \frac{K\lambda}{Nd}\right]$$

$K = 1, 2, 3, \ldots$

→ For First side Lobe, $K=1$

$$\phi_s = \cos^{-1}\left[1 - \frac{\lambda}{Nd}\right]$$

→ The first side Lobe beamwidth can be found by $FSLBW = 2\left|\phi_m - \phi_s\right|$

$$FSLBW = 2\cos^{-1}\left[1 - \frac{\lambda}{Nd}\right]$$

Table 1: Nulls, Maxima, Half-power points, and minor Lobe maxima for Uniform Amplitude Hansen-Woodyard End-Fire Arrays:

Nulls	$\phi_n = \cos^{-1}\left[1 + \frac{\lambda}{2Nd}[1-2K]\right]$ $K = 1, 2, 3, \ldots$
Maxima	$\phi_m = \cos^{-1}\left[1 + \frac{\lambda}{2Nd}[1-2KN]\right]$ $K = 1, 2, 3 \ldots$
Half-power points	$\phi_h = \cos^{-1}\left[1 - 0.1398\frac{\lambda}{Nd}\right]$
minor Lobe maxima	$\phi_s = \cos^{-1}\left[1 - \frac{K\lambda}{Nd}\right]$ $K = 1, 2, 3, \ldots$

Table 2 : Beamwidths for Uniform Amplitude Hansen – Woodyard End-Fire Arrays..

First – Null Beam width (FNBW)	$FNBW = 2\cos^{-1}\left[1-\frac{\lambda}{2Nd}\right]$
Half- power beam width (HPBW)	$HPBW = 2\cos^{-1}\left[1-\frac{0.1398\lambda}{Nd}\right]$
First side Lobe beam width (FSLBW)	$FSLBW = 2\cos^{-1}\left[1-\frac{\lambda}{Nd}\right]$

Directivity – Hansen - Woodyard End-Fire Array:

→ For Hansen – Woodyard End-fire Array

$$\delta = -\beta d - \frac{\pi}{N} \; ; \; \phi = 0^{\circ}$$

$$\psi = \beta d \cos\phi + \delta$$

$$\psi = \beta d \cos\phi - \beta d - \frac{\pi}{N}$$

$$\psi = \beta d(\cos\phi - 1) - \frac{\pi}{N}$$

→ For Hansen – Woodyard End – Fire Array radiation, the array factor is given by

$$(AF)_n = \frac{\pi}{2} \cdot \frac{\sin N\psi/2}{N\psi/2}$$

→ The radiation intensity can be written as

$$U(\phi) = [(AF)_n]^2 = \left[\frac{\pi}{2} \cdot \frac{\sin N\psi/2}{N\psi/2}\right]^2$$

→ the Directivity of the Array factor can be evaluated using

$$D = \frac{U_{max}}{U_0} = \frac{U_{max}}{P_{rad}/4\pi}$$

$$D = \frac{4\pi U_{max}}{P_{rad}}$$

where $U_0 \rightarrow$ is the average radiation intensity.

$$U_0 = \frac{P_{rad}}{4\pi}$$

$$P_{rad} = \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} U \cdot \sin\theta \, d\theta \, d\phi$$

$$P_{rad} = \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \left[\frac{\pi}{2} \frac{\sin N\psi/2}{N\psi/2}\right]^2 \cdot \sin\theta \, d\theta \, d\phi$$

$$P_{rad} = 2 \times \frac{\pi^2}{4} \int_{\phi=0}^{2\pi} \frac{\sin\left(\left(N\beta d(\cos\phi - 1) - \pi\right)/2\right)}{\left[N\beta d(\cos\phi - 1) - \pi\right]/2} d\phi$$

$$P_{rad} = 0.554 \times \left[\frac{2\pi^2}{N\beta d}\right]$$

$$U_0 = \frac{P_{rad}}{4\pi}$$

$$U_0 = 0.554 \left[\frac{\pi}{2N\beta d}\right]$$

$$U_0 = \frac{0.871}{N\beta d} = \frac{1.742}{2N\beta d}$$

$$U_{max} = 1$$

$$\therefore D = \frac{U_{max}}{U_0}$$

$$D = \frac{1}{0.871/N\beta d}$$

$$D = \frac{N\beta d}{0.871} = \frac{N \times \frac{2\pi}{\lambda} \times d}{0.871}$$

$$D = 1.805\left[4N\frac{d}{\lambda}\right]$$

For the Large array

$$\boxed{D \simeq 1.805\left[4\left(\frac{L}{\lambda}\right)\right]}$$

Table:

Directivities for broad side and End-Fire Arrays:

Broad-side [$d_{max} < \lambda$]	$D = 2N\left(\frac{d}{\lambda}\right)$ $L >>> d$ $D = 2\left(\frac{L}{\lambda}\right)$
End-Fire (Ordinary) [$d_{max} < \lambda/2$]	$D = 4N\left(\frac{d}{\lambda}\right)$ $L >> d$ $D = 4\left(\frac{L}{\lambda}\right)$
End-Fire (Hansen — Woodyard) [$d \simeq \lambda/4$]	$D = 1.805\left[4N\left(\frac{d}{\lambda}\right)\right]$ $L >> d$ $D = 1.805\left[4\left[\frac{L}{\lambda}\right]\right]$

Example: Given a Linear broadside uniform array of 4 isotropic elements with a separation of $\frac{\lambda}{2}$ between the elements. Draw its radiation pattern.

Sol$\underline{n}$:

→ Null directions:

$$\phi_n = \cos^{-1}\left(\pm \frac{K\lambda}{Nd}\right)$$

given $d = \lambda/2$ and $N = 4$

$$\phi_n = \cos^{-1}\left[\pm \frac{K\cancel{\lambda}}{4 \times \cancel{\lambda}/\cancel{2}}\right]$$

$$\phi_n = \cos^{-1}\left[\pm \frac{K}{2}\right]$$

if $K = 1$ $\quad \phi_n = \cos^{-1}[\pm 1/2]$

$$\phi_n = \pm 60^\circ, \pm 120^\circ$$

→ Maxmima directions:

$$\phi_m = \cos^{-1}\left[\pm \frac{K\lambda}{d}\right]$$

if $K = 0$ $\quad \phi_m = 90^\circ, 270^\circ$

→ Half-power point directions:

$$\phi_h = \cos^{-1}\left[\pm \frac{1.391\lambda}{\pi N d}\right]$$

$$\phi_h = \cos^{-1}\left[\pm \frac{1.391}{2\pi}\right]$$

$$\phi_h = \pm 77.209, \pm 102.79.$$

Minor Lobe maxima directions:

$$\phi_s = \cos^{-1}\left[\pm\frac{\lambda}{2d}\left[\frac{2K+1}{N}\right]\right]$$

$K=1$,

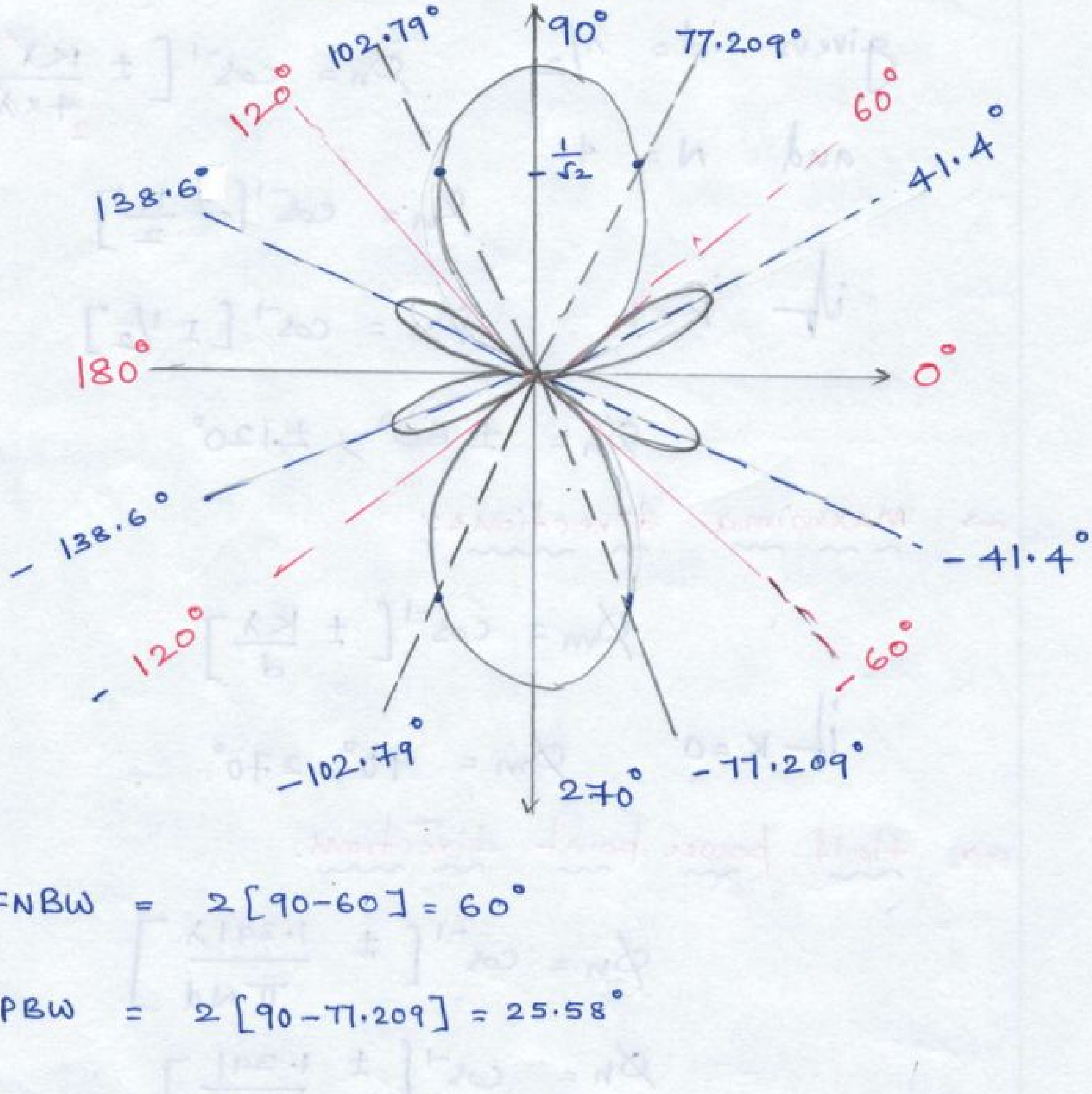
$d=\lambda/2$

$N=4$

$$\phi_s = \cos^{-1}\left[\pm\frac{\cancel{\lambda}}{\cancel{2}\times\cancel{\lambda}/\cancel{2}}\left[\frac{3}{4}\right]\right]$$

$$\phi_s = \cos^{-1}\left[\pm 3/4\right]$$

$$\phi_s = \pm 41.41^\circ, \ \pm 138.6^\circ$$



$$FNBW = 2[90-60] = 60^\circ$$

$$HPBW = 2[90-77.209] = 25.58^\circ$$

Example: Given a linear Ordinary end-fire uniform array of 4 isotropic elements with a separation of $\lambda/2$ between the elements. Draw its radiation pattern.

Sol^n:

Null directions:

$$\phi_n = \cos^{-1}\left[1 - \frac{K\lambda}{Nd}\right]$$

$N = 4$

$d = \lambda/2$

$$\phi_n = \cos^{-1}\left[1 - \frac{K\lambda}{4 \cdot \lambda/2}\right]$$

$$\phi_n = \cos^{-1}\left(1 - \frac{K}{2}\right]$$

$K = 1 \qquad \phi_n = \cos^{-1}[1/2]$

$$\phi_n = \pm 60^\circ$$

$K = 2 \qquad \phi_n = \cos^{-1}(0)$

$$\phi_n = \pm 90^\circ$$

$K = 3 \qquad \phi_n = \cos^{-1}[-1/2]$

$$\phi_n = \pm 120^\circ$$

Maxima directions:

$$\phi_m = \cos^{-1}\left[1 - \frac{K\lambda}{d}\right] = \cos^{-1}[1 - 2K]$$

$K = 0 \qquad \phi_m = \cos^{-1}(1) \qquad K = 1 \quad \phi_m = \cos^{-1}[-1]$

$\phi_m = 0^\circ \qquad \phi_m = 180^\circ$

Half-power point directions:

$$\phi_h = \cos^{-1}\left[1 - \frac{1.391\lambda}{\pi N d}\right]$$

$N = 4$

$d = \lambda/2$

$$\phi_h = \cos^{-1}\left[1 - \frac{1.391\cancel{\lambda}}{\pi \times \cancel{4}_{2} \times \cancel{\lambda}/\cancel{2}}\right]$$

$$\phi_h = \cos^{-1}\left[1 - \frac{1.391}{2\pi}\right]$$

$$\phi_h = \pm 38.86^\circ$$

Minor Lobe maxima directions:

$$\phi_s = \cos^{-1}\left[1 - \frac{(2K+1)\lambda}{2Nd}\right]$$

$N = 4$

$d = \lambda/2$

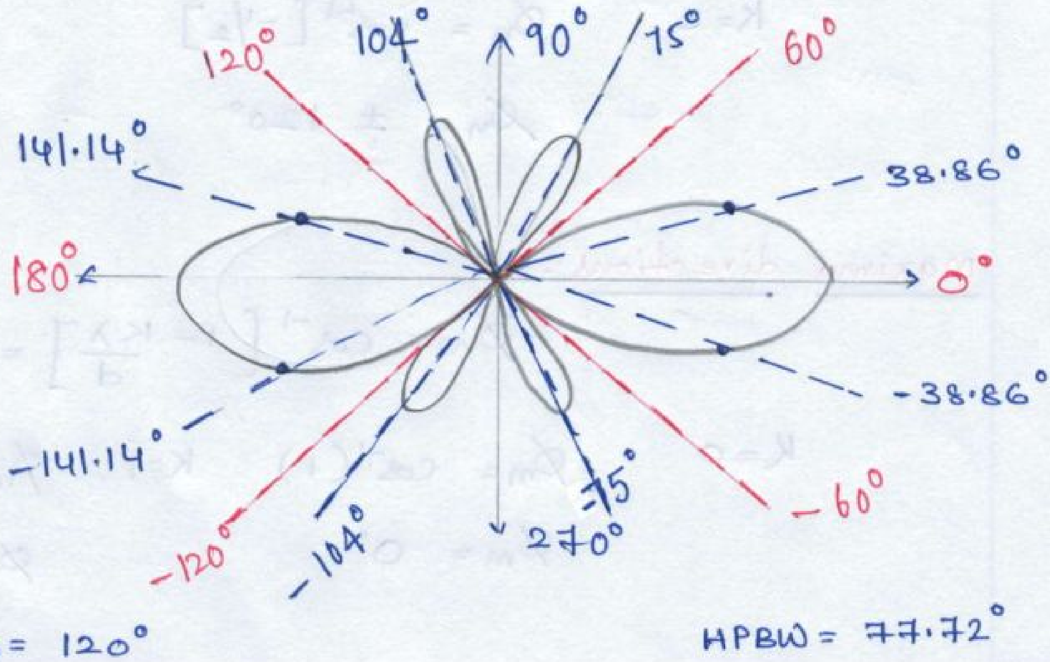
$$\phi_s = \cos^{-1}\left[1 - \frac{(2K+1)\cancel{\lambda}}{\cancel{2} \times 4 \times \cancel{\lambda}/\cancel{2}}\right]$$

$$\phi_s = \cos^{-1}\left[1 - \frac{(2K+1)}{4}\right]$$

if $K=1$ $\quad \phi_s = \cos^{-1}\left[1 - 3/4\right]$

$$\phi_s = \cos^{-1}\left[1/4\right] = \pm 75.52^\circ$$

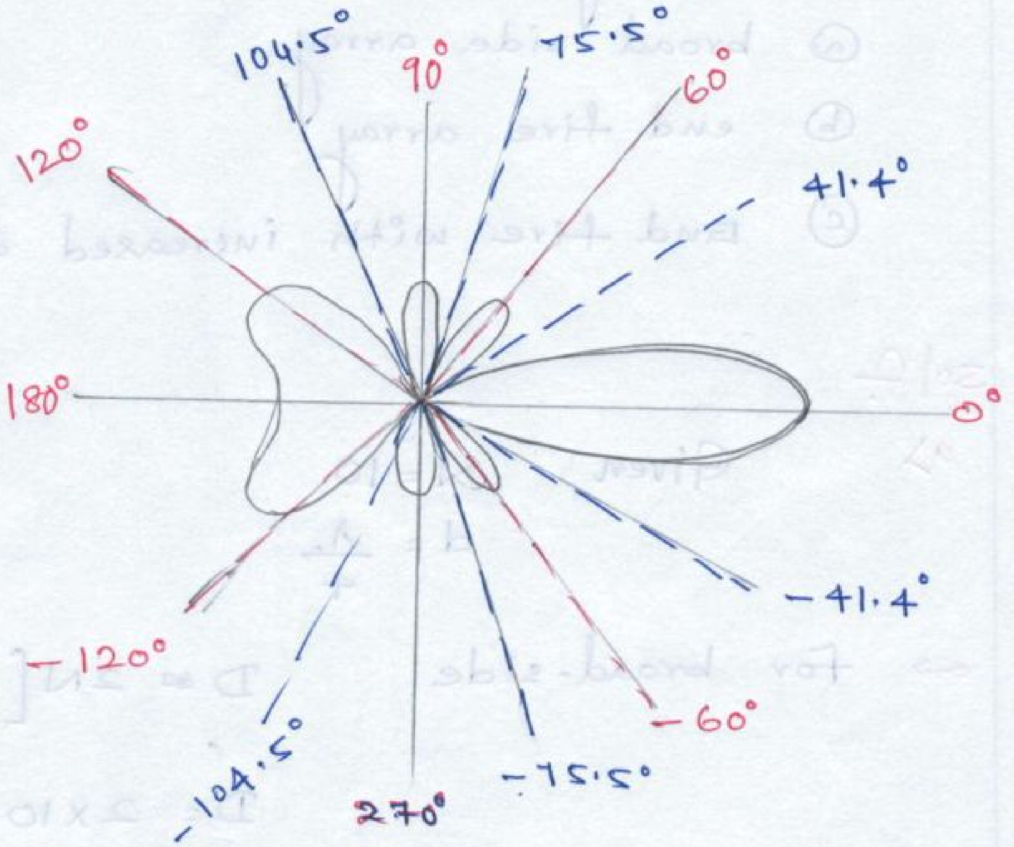
if $K=2$ $\quad \phi_s = \cos^{-1}(1 - 5/4) = \cos^{-1}(-1/4) = \pm 104.47^\circ$



FNBW = 120° $\qquad$ HPBW = 77.72°

Example: Given a Linear Hansen-Woodyard end-fire uniform array of 4-isotropic elements with a separation of $\lambda/2$ between the elements. Draw its radiation pattern..

soln:



$\phi_{max} = 0°$

$\phi_{nulls} = \pm 41.4°, \pm 75.5°, \pm 104.5°$

$\phi_{s} = \pm 60°, \pm 90°, \pm 120°, 180°$

Example:

Given a Linear, uniform array of 10 (N=10) elements with a separation of $\frac{\lambda}{4}$ ($d=\frac{\lambda}{4}$) between the elements, Find the directivity of the array factor. For

(a) broad side array

(b) end fire array

(c) End fire with increased directivity.

Sol^n:

a) Given $N=10$

$d=\frac{\lambda}{4}$

→ For broad-side $D = 2N\left[\frac{d}{\lambda}\right]$

$D = 2 \times 10 \times \frac{\lambda/4}{\lambda}$

$D = \frac{20}{4} = 5$

$D = 10\log 5 = 6.99\,dB$

→ For End-Fire $D = 4N\left(\frac{d}{\lambda}\right)$

$= 4 \times 10 \times \frac{\lambda/4}{\lambda}$

$= 10$

$D = 10\log 10 = 10\,dB$

→ For End Fire with increased directivity $D = 1.805\left[4N\left(\frac{d}{\lambda}\right)\right]$

$= 1.805 \times 4 \times 10 \times \frac{\lambda/4}{\lambda}$

$= 18.05 = 12.56\,dB.$

Binomial Arrays:

→ If the radiating sources must have current amplitudes proportional to the co-efficients of the binomial series, then the secondary (or) side Lobes in the linear broad side array are to be eliminated totally, this was proposed by John stone in 1929.

→ The Arrays are arranged in such a way that radiating sources in the centre of broad side array radiates more strongly than the radiating sources at the edges, then the side Lobes are eliminated entirely, if the following two conditions are satisfied.

i) Spacing between the two consecutive radiating sources does not exceed $\lambda/2$, and

ii) the current amplitudes in radiating sources are proportional to the co-efficients of the successive terms of the binomial series.

→ Excitation co-efficients of a binomial array can be determined by using the binomial expansion, as

$$(1+x)^{n-1} = 1 + (n-1)x + \frac{(n-1)(n-2)}{2!}x^2 + \cdots$$

→ the positive co-efficients of the series expansion for different values of 'n' are:

$n=1$						1					
$n=2$					1		1				
$n=3$				1		2		1			
$n=4$			1		3		3		1		
$n=5$		1		4		6		4		1	
$n=6$	1		5		10		10		5		1

→ The above represents pascal's triangle.

→ if the values of the 'n' are used to represent the number of elements of the array, then the co-efficients of the expansion represent the relative amplitudes of the elements.

since the co-efficients are determined from a binomial series expansion, the array is known as a binomial array...

→ Binomial arrays do not exhibit any minor lobes provided the spacing between the elements is equal or less than one-half of a wavelength. $d \leq \frac{\lambda}{2}$

→ Minor Lobes elimination in binomial array takes place at the cost of decrease in directivity in comparison with the same array of equal amplitude sources.

→ i,e, Half-power beam width of binomial array is more than that of uniform array for the same Length.

Thus in uniform array secondary Lobes appear, but prinicipal Lobe is sharp and narrow. where as in binomial array width of beam widens but without secondary Lobes.

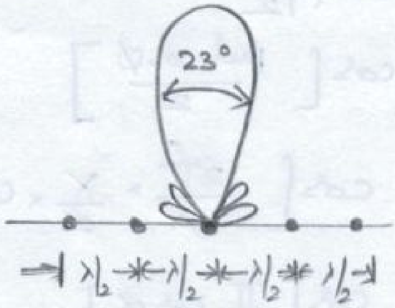


Fig: Uniform Array

$n = 5$, $d = \frac{\lambda}{2}$

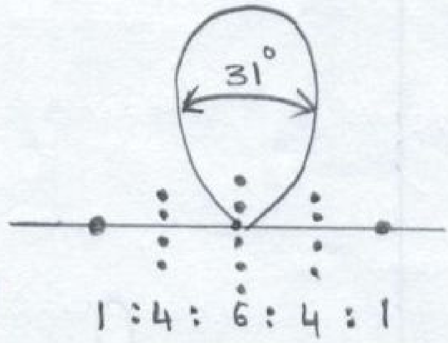


fig: Binomial Array

$n = 5$, Amplitude ratio : 1:4:6:4:1

→ if $d = \frac{\lambda}{2}$, For binomial arrays

$$HPBW \simeq \frac{1.06}{\sqrt{N-1}} = \frac{1.06}{\sqrt{2L/\lambda}} = \frac{0.75}{\sqrt{L/\lambda}}$$

$$L = (N-1)\frac{\lambda}{2}$$

→ Directivity $D \simeq 1.77\sqrt{N}$

$$D \simeq 1.77\sqrt{1 + 2L/\lambda}$$

→ In the field derivation of binomial arrays the principle of multiplication pattern is used...

Consider the far field pattern of two point sources of same amplitude and phase is given by

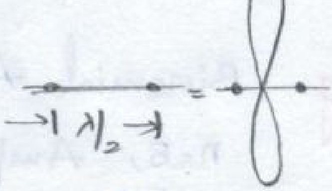
$$E = 2E_0 \cdot \cos \psi/2 \qquad \psi = \beta d \cos\phi$$

$$E_{nor} = \cos \psi/2$$

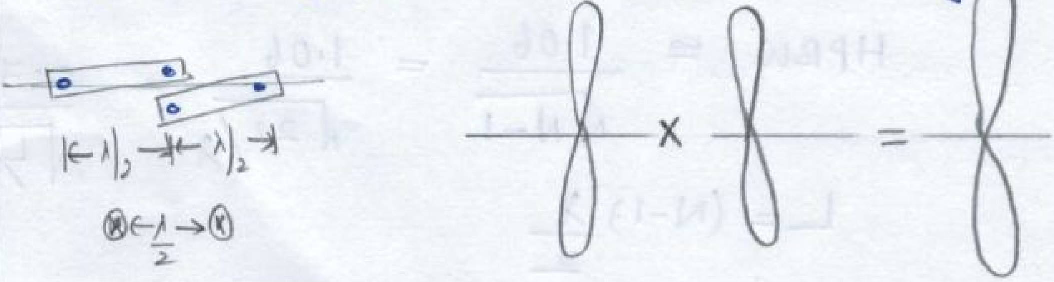
$$= \cos\left[\frac{\beta d \cos\phi}{2}\right] \qquad d = \lambda/2$$

$$= \cos\left[\frac{\cancel{2\pi}}{\cancel{\lambda}} \times \frac{\cancel{\lambda}}{\cancel{2}} \times \frac{\cos\phi}{2}\right]$$

$$E_{nor} = \cos\left[\frac{\pi}{2}\cos\phi\right]$$



if another identical array of two point source is superimposed then the far field pattern by pattern multiplication is given by



$$E_{nor} = \cos^{n}\left(\frac{\pi}{2}\cos\phi\right)$$

→ ∴ it is possible to have a pattern of any desired directivity without any minor Lobes provided, the current amplitudes corresponds to the co-efficients of the binomial series, the field pattern of 'n' sources is given by

$$E_{nor} = \cos^{(N-1)}\left[\pi/2 \cos\phi\right]$$

Advantages:

→ No side Lobes

→ Larger Half power beamwidth & wide beam

Disadvantages:

→ Half power beam width increases and hence the directivity decreases.

→ To design a Large array, Larger amplitude ratio of sources is required.

→ the Array factor for the binomial array is represented by

$$(AF)_{2M}(\text{even}) = \sum_{n=1}^{M} a_n \cos\left[(2n-1)\frac{\pi d}{\lambda}\cos\theta\right]$$

$$(AF)_{2M+1}(\text{odd}) = \sum_{n=1}^{M+1} a_n \cos\left[2(n-1)\frac{\pi d}{\lambda}\cos\theta\right]$$

Example: For a 10-element binomial array with a spacing of $\frac{\lambda}{2}$ between the elements, determine the half-power beam width and maximum directivity.

Sol$\underline{n}$:

$$HPBW = \frac{1.06}{\sqrt{N-1}} = \frac{1.06}{\sqrt{9}}$$

$$= \frac{1.06}{3} = 0.353 \text{ radians}$$

$$HPBW = 20.23^\circ$$

$$\text{Directivity } D = 1.77\sqrt{N}$$

$$= 1.77\sqrt{10}$$

$$= 5.59723$$

$$D = 10\log(5.597)$$

$$D = 7.47\,dB.$$

: Helical Antenna :

→ Helical antenna is simple and practical configuration of an electromagnetic radiator, consists of a conducting wire wound in the form of a screw thread forming a helix, and is used in conjunction with a flat metal plate called a ground plate.

→ the typical diameter of the ground plane should be at least $\frac{3\lambda}{4}$.

→ Helical antenna is a broad band VHF and UHF antenna to provide circularly polarized waves, which are used in extra terrestrial communications in which satellite relays e.t.c.

→ the helix is generally fed by a coaxial cable.

→ the one end of the helix is connected to the centre of the coaxial cable and the outer conductor the cable is connected to the ground plane.

→ the geometrical configuration of a helix consists:

$D \rightarrow$ diameter of helix

$C \rightarrow$ circumference of helix $= \pi D$

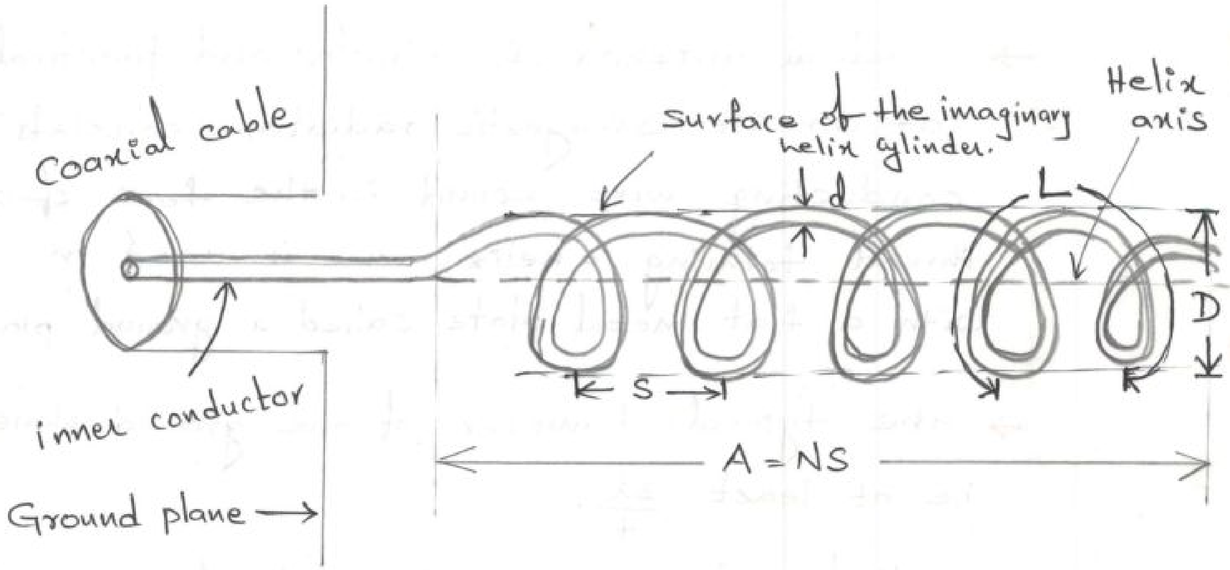
$S \rightarrow$ spacing between turns (center to center)

$A \rightarrow$ Axial Length $= NS$

$d \rightarrow$ diameter of helix conductor

$L \rightarrow$ Length of one turn

$\alpha \rightarrow$ pitch angle $= \tan^{-1}\left(\frac{S}{\pi D}\right)$



→ the helix is a basic 3-dimensional geometric form, combines the geometric forms of straight line, a circle and a cylinder.

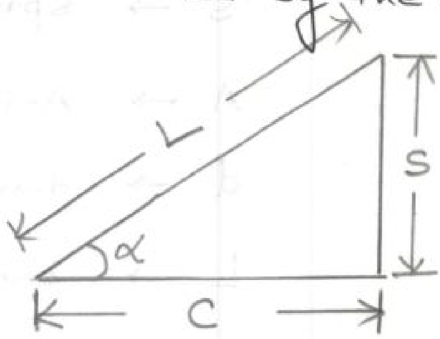
→ the diameter 'D' and circumference 'C' refers to the imaginary cylinder whose surface passes through the centerline ~~line~~ of the helix conductor.

→ for N turn helix, the total Length of the antenna is equal to NS.

→ If one turn of helix is unrolled on a plane surface, the circumference C, spacing S, turn length L, and pitch angle α are related by the triangle shown below.

$$L = \sqrt{S^2 + C^2}$$

$$L = \sqrt{S^2 + (\pi D)^2}$$



pitch angle:

the pitch angle is the angle between a line tangent to the helix wire and the plane normal to the helix axis.

from the above triangle

$$\tan\alpha = \frac{S}{C} = \frac{S}{\pi D}$$

$$\alpha = \tan^{-1}\left(\frac{S}{\pi D}\right)$$

→ The mode of radiation depends on the diameter of helix 'D' and turn spacing 'S'.

→ The helical antenna may radiate in many modes but principle modes of radiations are two.

i) Normal (or) broad side (or) perpendicular mode of radiation.

ii) Axial (or) End-fire (or) beam mode of radiation.

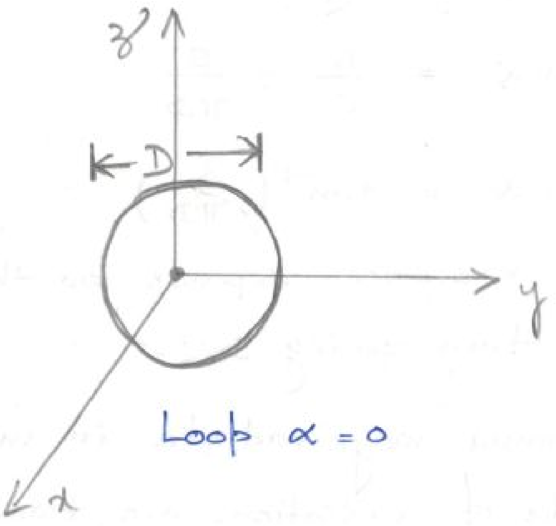
Normal Mode of radiation:

→ In the normal mode of operation, the field radiated by the antenna is maximum in a plane normal to the helix axis and minimum along its axis.

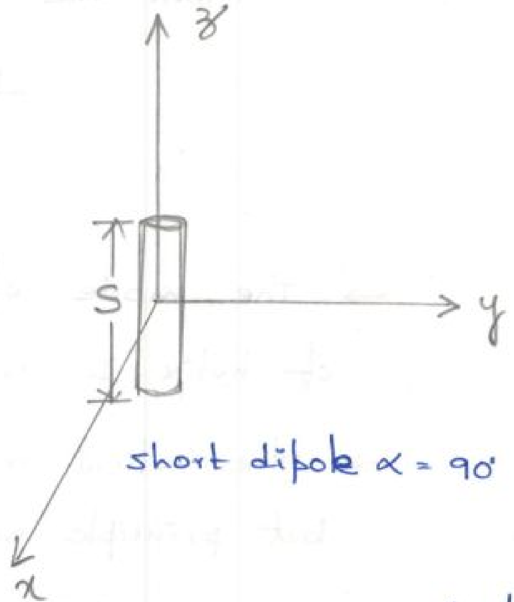
→ this mode of radiation is obtained if the dimensions of the helix is small compared with wave Length. i,e $NL \ll \lambda$

→ the geometry of the helix reduces to a Loop of diameter D when the pitch angle approaches to zero. and a linear wire of Length S when it approaches to 90°.

→ Since the Limiting geometries of the helix are a Loop and a dipole, ∴ the far field radiated by a small helix in the normal mode can be described in terms of E_θ and E_ϕ components of the dipole and Loop respectively.

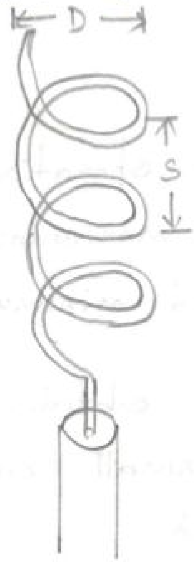


Loop $\alpha = 0$

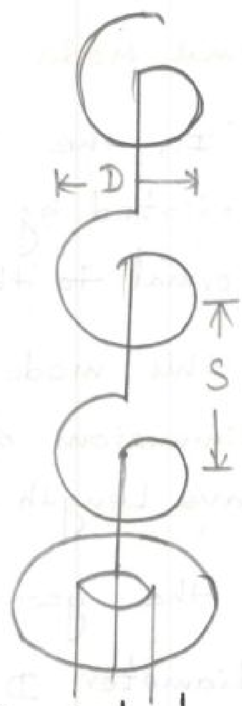


short dipole $\alpha = 90^\circ$

→ ∴ In the normal mode, the helix consists of 'N' small Loops and 'N' short dipoles connected together in series as shown in figure.



Normal mode



equivalent.

→ since in the normal mode the helix dimensions are small, the current through out its Length can be assumed to be constant and its relative far field pattern to be independent of the number of Loops and short dipoles.

→ the radiation pattern's of these two radiator's (Loop and short dipole) are same, However the polarizations are at right angle and the phase angles at any point in space are at 90° apart.

→ Hence the resultant field is either circularly polarized (or) elliptically polarized depending upon the amplitudes of the two components, which intern depends on the pitch angle α.

→ if α is small Loop type of radiation pre-dominates, if α becomes very Large the dipole polarization pre- dominates.

→ The far - field radiated by a short dipole of Length s and constant current I is given by

$$E_\theta = \frac{60\pi\,[I]\sin\theta}{r} \times \frac{s}{\lambda} \longrightarrow ①$$

$[I] \rightarrow$ retarded current

$r \rightarrow$ the distance

$s \rightarrow$ Length of dipole.

→ The far field of the small Loop is given by

$$E_\phi = \frac{120\pi^2\,[I]\sin\theta}{r} \times \frac{A}{\lambda^2} \longrightarrow ②$$

$A \rightarrow$ Area of the Loop $= \frac{\pi D^2}{4}$.

→ Axial ratio:

the ratio of the magnitudes of the E_θ and E_ϕ components is defined as the "Axial ratio" (AR) and is given by

$$AR = \frac{|E_\theta|}{|E_\phi|}$$

$$AR = \frac{\frac{\cancel{60\pi}\,[\cancel{I}]\,\cancel{\sin\theta}}{\cancel{r}} \times \frac{S}{\cancel{\lambda}}}{\frac{2\,\cancel{120}\,\pi^2[\cancel{I}]\,\cancel{\sin\theta}}{\cancel{r}} \times \frac{A}{\lambda^2}}$$

$$AR = \frac{S}{\frac{2\pi A}{\lambda}} = \frac{S\lambda}{2\pi A}$$

$$AR = \frac{S\lambda}{2\pi \cdot \frac{\pi D^2}{4}} = \frac{2S\lambda}{\pi^2 D^2}$$ → Axial ratio of elliptical polarization.

→ By varying D and/or S the axial ratio attains the values of $0 \leq AR \leq \infty$

→ when $AR = 0$ is a special case occurs when $E_\theta = 0$ then elliptical polarization becomes Linear horizontal polarization (the helix is a Loop.)

→ when $AR = \infty$, $E_\phi = 0$ and the radiated wave is linearly polarized with vertical polarization. (the helix is a vertical dipole.)

→ when $AR = 1$, the elliptical polarization becomes circular polarization.

thus for circular polarization

$$AR = 1 \rightarrow \left|\frac{E_\theta}{E_\phi}\right| = 1 \quad \text{(or)} \quad |E_\theta| = |E_\phi|$$

$$2S\lambda = (\pi D)^2$$

$$S = \frac{\pi^2 D^2}{2\lambda}$$

$$S = \frac{C^2}{2\lambda}$$

$$C = \sqrt{2S\lambda}$$

→ pitch angle $\alpha = \tan^{-1}\left(\frac{S}{\pi D}\right)$

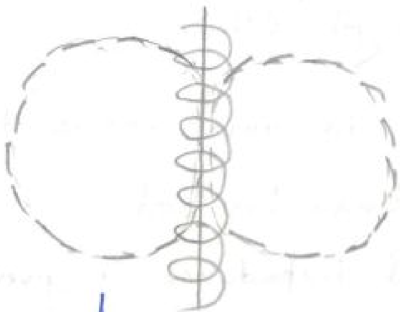
$$\alpha = \tan^{-1}\left(\frac{\pi^2 D^2}{2\lambda \cdot \pi D}\right)$$

$$\alpha = \tan^{-1}\left(\frac{\pi D}{2\lambda}\right)$$

$$\boxed{\alpha = \tan^{-1}\left(\frac{C}{2\lambda}\right)}$$

This is the condition for pitch angle to get circular polarization.

→ the radiation pattern in normal mode, for circular polarization is shown in fig. below.



→ This mode of operation is very narrow in band width and its radiation efficiency is very small. practically this mode of operation is limited and it is hardly used.

: Axial (or) beam mode of operation :

→ In axial mode of operation, the radiation is maximum in the end-fire direction i,e along the helix axis and the polarization is circular (or) nearly circular.

→ This mode occurs when the helix circumference, and spacing (S) are appreciable of the order of wave Length. $\frac{3}{4}\lambda < C < \frac{4}{3}\lambda$

→ This mode produces a broad and fairly directional beam in the axial direction with minor Lobes at oblique angles.

→ The helix is operated in conjunction with ground plane and is fed by a coaxial cable.

→ the pitch angle α varies from 12° to 18° and about 14° is optimum pitch angle.

→ the antenna gain and beamwidth depends upon the helix Length (NS).

→ In general in axial mode the terminal impedance of helical antenna lies between $100\,\Omega$ to $200\,\Omega$.

the terminal impedance is given by $R = \frac{140C}{\lambda}\,\Omega$

→ The beam pattern has axial symmetry i,e it is same in any plane containing the axis.

→ The beam width between half power points is given by

$$HPBW = \frac{52}{C}\sqrt{\frac{\lambda^3}{NS}} \text{ degrees.}$$

λ → free space wave Length

C → circumference. S → spacing.

N → Number of turns

→ Beam width between first nulls is given by

$$BWFN = \frac{115}{C}\sqrt{\frac{\lambda^3}{NS}}$$

→ The maximum directive gain in axial mode is given by $D = \dfrac{15\,NSC^2}{\lambda^3}$.

→ Axial ratio $AR = 1 + \frac{1}{2N}$

→ The normalized far-field pattern is given by

$$E = \sin\left(\frac{\pi}{2N}\right) \cdot \cos\theta \cdot \frac{\sin\left(\frac{N\phi}{2}\right)}{\sin(\phi/2)}$$

→ Much of the theory and art have been developed by J. D. Kraus and his associates.

→ This is the more practical mode of operation which is generated with great ease.

: Applications:

→ Helical antenna is used to receive (or) transmit the VHF signals through ionosphere.

→ Helical antenna is frequently used for satellite and space probe communications.

→ The wide band width, simplicity, highest directivity and circular polarization of the helical beam antenna made indispensible for space communication applications.

→ Helical antenna has been used transmitting telemetry data from moon to earth.

→ Helical antenna finds many applications in radio Astronomy.

→ the dimensions of the helix in axial mode are not critical and there by resulting in greater band width.

problems:

1. Design an end fire right hand circularly polarized helix having a half power beam width of $45°$, $\alpha = 13°$ and a circumference of 60 cm at a frequency of 500 MHz. determine a) turns needed b) Directivity c) Axial ratio d) Lower & upper cut off frequencies over which the required parameters (e) i/p impedance at f_0, f_1 and f_2.

Sol$\underline{n}$. given $\alpha = 13°$, $C = 60\text{cm}$, $f = 500\text{MHz}$

$$\alpha = \tan^{-1}\left(\frac{S}{C}\right)$$

$$\frac{S}{C} = \tan 13°$$

$$S = 60 \tan 13° = 13.852\text{cm}.$$

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{500\times10^6}$$

$$\lambda = 60\text{cm}$$

a) $$HPBW = \frac{52\,\lambda^{3/2}}{C\sqrt{NS}}$$

$$45° = \frac{52\times(60)^{3/2}}{60\sqrt{N\times13.852}}$$

$$N \simeq 5.7839$$

$$N \simeq 6$$

b) $$D = 15N\frac{C^2 S}{\lambda^3}$$

$$D = \frac{15\times6\times60^2\times13.852}{[60]^3}$$

$$D = 20.778 \simeq 13.176\,\text{dB}$$

c) Axial ratio $$AR = \frac{2N+1}{2N}$$

$$= \frac{13}{12} = 1.0833$$

d) $$\frac{3}{4} < \frac{C}{\lambda} < \frac{4}{3}$$

$$C = \frac{3\lambda_1}{4} \Rightarrow \lambda_1 = 80\text{cm}$$

$$C = \frac{4\lambda_2}{3} \Rightarrow \lambda_2 = 45\text{cm}$$

$f_1 = 375\text{MHz}$, $f_2 = 666.67\text{MHz}$

e) i/p impedance $R = 140(C/\lambda)$

$R = 140$ at $C = \lambda$

$R = 105$ at $C/\lambda = 3/4$;

Q: Design a 10-turn helix to operate in the axial mode. For an optimum design, determine

a) Circumference (in λ) ; pitch angle (in degrees) and separation between turns (in λ).

b) Axial ratio.

Sol$\underline{n}$:

→ For an optimum design

$$C \simeq \lambda \quad \alpha \simeq 13^\circ$$

$$S = C \tan\alpha$$

$$S = C \tan 13^\circ$$

$$S = \lambda \tan 13^\circ$$

$$S = 0.231\,\lambda$$

→ the Length of a single turn is

$$L = \sqrt{S^2 + C^2} = \lambda\sqrt{(0.231)^2 + (1)^2}$$

$$L = 1.0263\,\lambda$$

→ $$\text{HPBW} = \frac{52\,\lambda^{3/2}}{C\sqrt{NS}}$$

$$= \frac{52}{\sqrt{10(0.231)}} = 34.2135^\circ$$

→ Directivity $$D = \frac{15NC^2S}{\lambda^3}$$

$$D = \frac{15(10)(1)^2(0.231)\cdot\cancel{\lambda^3}}{\cancel{\lambda^3}}$$

$$D = 34.65$$

$$D = 15.397\ dB$$

→ Axial ratio

$$AR = \frac{2N+1}{2N}$$

$$= \frac{20+1}{20} = \frac{21}{20}$$

$$AR = 1.05$$

$$AR = 0.21\ dB.$$

= x =

2. Design a helical antenna with a directivity of 15dB that is operating in the axial mode and whose polarization is nearly circular. the spacing between the turns is $\lambda/10$. Determine

(a) N (b) AR.

Soln:

given $D = 15dB$.

$$10 \log D = 15$$

$$D = 10^{1.5} = 31.62.$$

$$S = \frac{\lambda}{10} \qquad C \simeq \lambda.$$

$$D = \frac{15 N C^2 S}{\lambda^3}$$

$$D = 15N \cdot \frac{\cancel{\lambda^2}}{\cancel{\lambda^3}} \cdot \frac{\cancel{\lambda}}{10} = \frac{15N}{10}$$

$$31.62 = \frac{15N}{10}$$

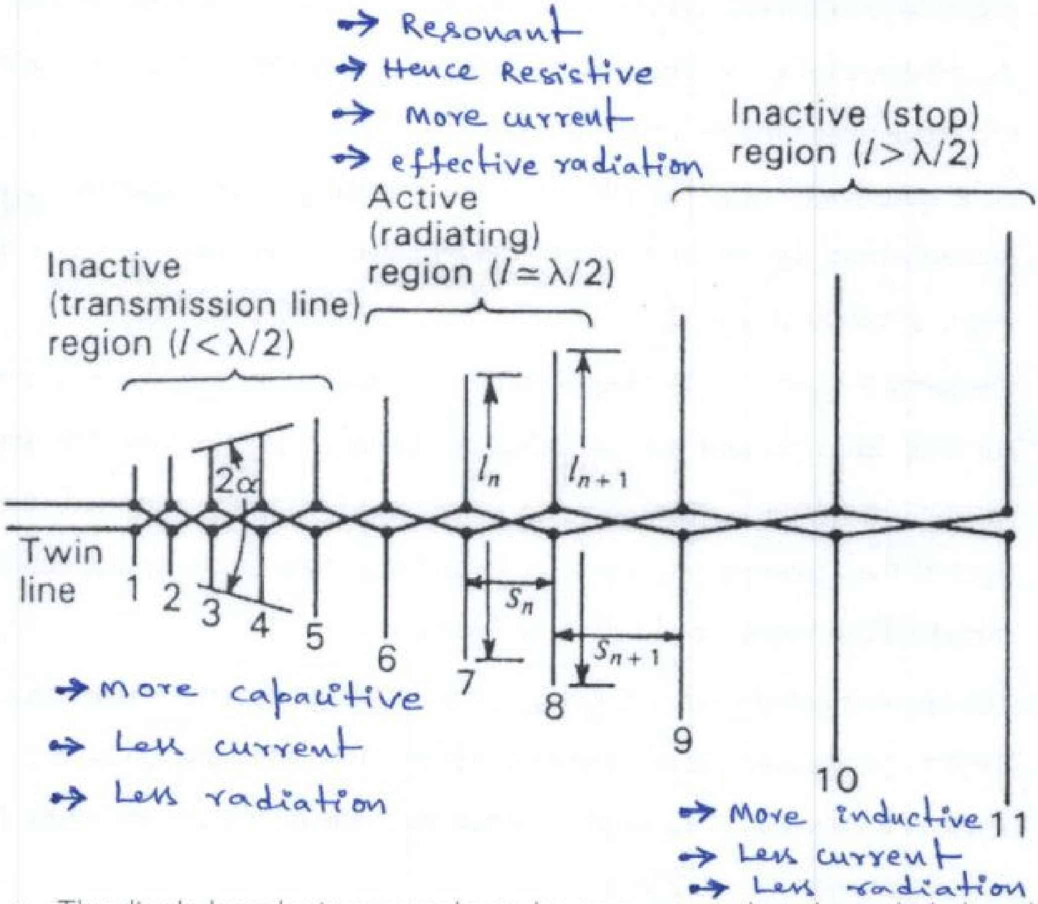
$$\Rightarrow \quad N = \frac{31.62 \times 10}{15} \simeq 21.$$

Axial ratio $\quad AR = \frac{2N+1}{2N} = \frac{2 \times (21) + 1}{2 \times (21)}$

$$AR = \frac{43}{42} = 1.02 = 0.2 \text{ dB}.$$

Log Periodic Antenna

→ LPDA is an antenna having structural geometry such that its input impedance and radiation characteristics repeat periodically as the logarithm of frequency.



- The dipole lengths increase along the antenna so that the included angle α is a constant, and the lengths 'l' and spacings 's' of adjacent elements are scaled so that

$$\frac{l_{n+1}}{l_n} = \frac{s_{n+1}}{s_n} = K$$

Where 'K' is a constant.

Basic Principle of Operation:

- The role of a specific dipole element is linked to the operating frequency: if its length, L, is around half of the wavelength, it is an active dipole and within the active region; Otherwise it is in an inactive region.
- A gradually expanding periodic structure array radiates most effectively when the array elements are near resonance so that with the change in frequency the active region moves along the array.
- At a wavelength near the middle of the operating range, radiation occurs primarily from the central region of the antenna. The elements in this active region are about $\lambda/2$ long.
- Elements $\geq \frac{\lambda}{2}$ are in the neighborhood of 1λ long and carry only small currents (they present a large inductive reactance to the line). The small currents in elements mean that the antenna is effectively truncated at the right of the active region. Any small fields from these elements also tend to cancel in both forward and backward directions.
- The elements at the left ($\leq \frac{\lambda}{2}$long) present a large capacitive reactance to the line. Hence, currents in these elements are small and radiation is small.
- Thus, at a wavelength 'λ', radiation occurs from the middle portion where the dipole elements are $\sim\lambda/2$ long.
- When the wavelength is increased the radiation zone moves to the right (larger element side) and when the wavelength is decreased it moves to the left (shorter element side) with maximum radiation toward the apex or feed point of the array.
- At any given frequency only a fraction of the antenna is used (where the dipoles are about $\lambda/2$ long). At the short-wavelength limit of the bandwidth only 15 percent of the length may be used, while at the long-wavelength limit a larger fraction is used but still less than 50 percent.

From the geometry

$$\tan\alpha = \frac{(l_{n+1} - l_n)/2}{s}$$

but $\frac{l_{n+1}}{l_n} = K$

$\Rightarrow l_n = \frac{l_{n+1}}{K}$

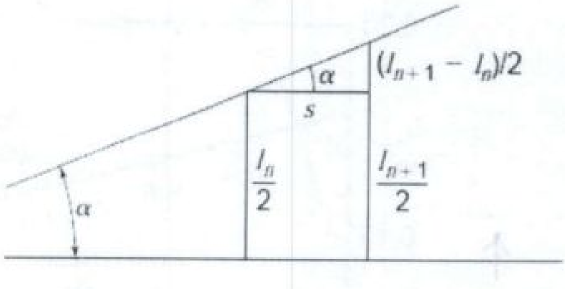


Fig: Log-periodic array geometry for determing the relation parameters.

$$\therefore \tan\alpha = \frac{\left(l_{n+1} - \frac{l_{n+1}}{K}\right)}{2s}$$

$$\tan\alpha = \frac{\left(1 - \frac{1}{K}\right) l_{n+1}}{2s}$$

→ Taking $l_{n+1} = \lambda/2$ (when active), we have

$$\tan\alpha = \frac{\left(1 - \frac{1}{K}\right)}{4s/\lambda} = \frac{\left(1 - \frac{1}{K}\right)}{4 s_\lambda}$$

where α = apex angle

K = scale factor

s_λ = spacing in wavelengths shortward of $\lambda/2$ element

→ specifying any two of the three parameters α, K and S_λ determines the third.

→ the relationship of the 3 parameters is displayed in fig. below.

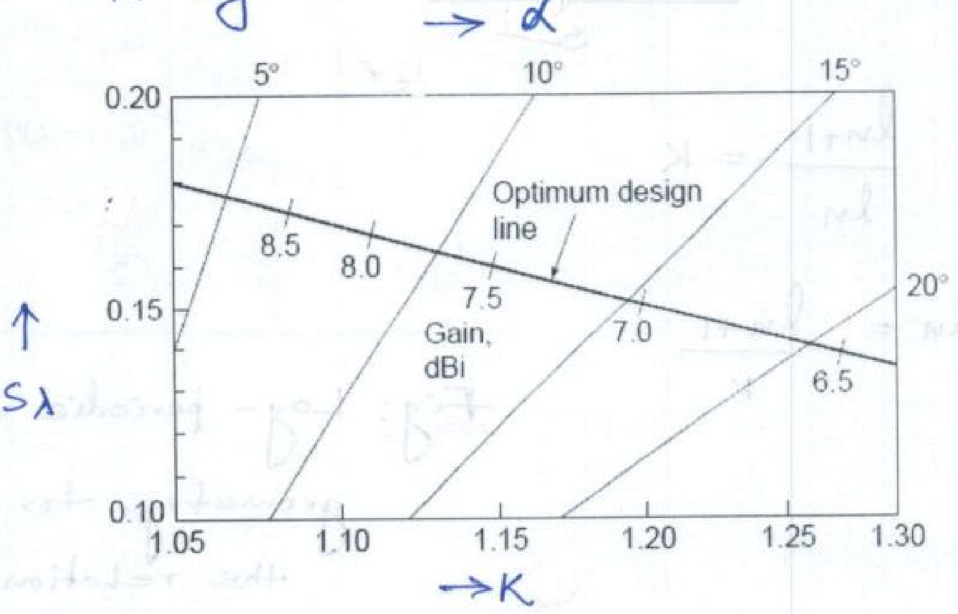


Fig: Relation of log-periodic array parameters of apex angle α, scale factor k and spacing $s\lambda$

→ the Length 'l' (and spacing 's') for any element n+1 is K^n greater than for element 1.

$$\text{i,e} \quad \frac{l_{n+1}}{l_1} = K^n = F$$

where F = Frequency ratio (or) bandwidth.

→ Thus if K = 1.19 and n = 4

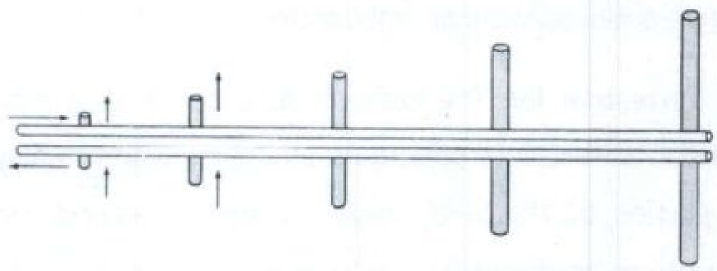
$$F = K^n \; ; \; F = 1.19^4 = 2$$

and element 5 (= n+1) is twice the Length l_1 of element 1. Thus with 5 elements and K = 1.19 the frequency ratio is 2 to 1.

Feeding Types

Straight Connection

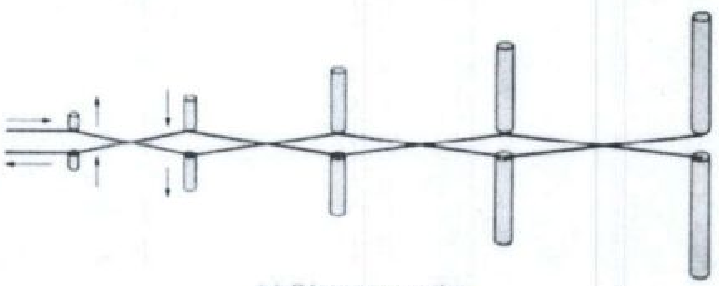
- Currents are in same phase relationship as the terminal phases
- If elements are closely spaced, the phase progression of the currents is to the right.
- End-fire beam in the direction of the longer elements and results in interference effects



(b) Straight connection

Crisscross Connection

- Criss cross – 180^0 phase is added to the terminal of each element.
- Phase diff between closely spaced elements is 180^0, causes very little radiation and results in less interference
- The longer and lager spaced elements radiate
- Radiation is in the direction of the shorter elements



(c) Crisscross connection

Applications:

Log periodic antenna is used in a number of applications where a wide bandwidth is required along with directivity and a modest level of gain.

- **HF communications:** Log periodic antenna arrays are often used for diplomatic traffic on the HF bands. A single log periodic antenna will give access to a sufficient number of frequencies over the HF bands to enable communications to be made despite the variations in the ionosphere changing optimum working frequencies.
- **UHF Terrestrial TV**: The log periodic antenna is sometimes used for UHF terrestrial television reception. As television channels may be located over a wide portion of the UHF spectrum, the log periodic enables a sufficient bandwidth to be covered.
- **EMC measurements:** EMC is a key issue for all electronic products. Testing requires frequency scans to be undertaken over wide bands of frequencies. Log periodic antenna is able to provide a flat response over a wide band of frequencies is needed. The log periodic is able to offer the performance required and is widely used in this form of application.

Q: Design a Log-periodic antenna dipole array with 7 dBi gain and 4 to 1 bandwidth. specify the number of elements.

choose $\alpha = 15^\circ$, scaling factor $K = 1.2$

Soln:

given bandwidth $BW = F = 4$;

$K = 1.2$

$$\frac{l_{n+1}}{l_1} = K^n = F$$

$$(1.2)^n = 4$$

$$\log(1.2)^n = \log(4)$$

$$n = \frac{\log(4)}{\log(1.2)} = 7.6$$

$$\boxed{n \simeq 8}$$

→ Taking $n=8$, $n+1=9$, adding 2 more elements for a conservative design brings the total to 11.

$$\tan\alpha = \frac{1-1/K}{4 S_n\lambda}$$

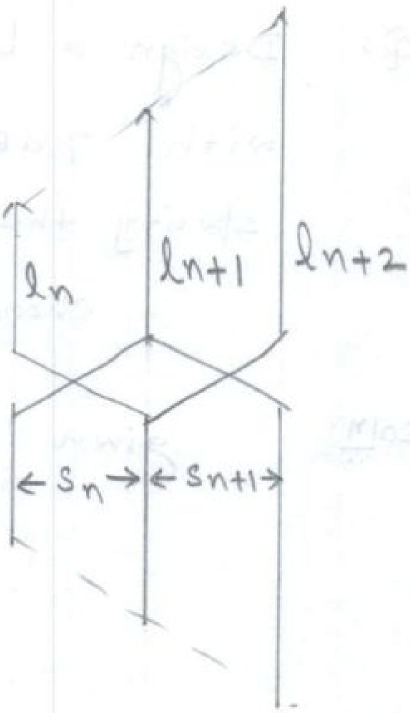
$$\tan 15^\circ = \frac{1-1/1.2}{4 S_n\lambda}$$

$$\rightarrow \quad S_n\lambda = \frac{1-1/1.2}{4\tan 15^\circ} = 0.155$$

$$S_n\lambda = 0.155$$

$$\boxed{S_8\lambda = 0.155} \qquad S_8 = 0.155\lambda$$

$$\frac{l_{n+1}}{l_n} = \frac{s_{n+1}}{s_n} = K$$



$$\frac{S_9}{S_8} = K$$

$$S_9 = K \times S_8$$

$$S_9 = 1.2 \times 0.155\lambda$$

$$S_9 = 0.186\lambda$$

$$S_8 = 0.155\lambda$$

$$S_7 = \frac{S_8}{K} = \frac{0.155\lambda}{1.2} = 0.129\lambda$$

$$S_6 = \frac{S_7}{K} = \frac{0.129\lambda}{1.2} = 0.107\lambda$$

$$S_5 = \frac{S_6}{K} = \frac{0.107\lambda}{1.2} = 0.089\lambda$$

$$S_4 = \frac{S_5}{K} = \frac{0.089\lambda}{1.2} = 0.074\lambda$$

$$S_3 = \frac{S_4}{K} = \frac{0.074\lambda}{1.2} = 0.062\lambda$$

$$S_2 = \frac{S_3}{K} = \frac{0.062\lambda}{1.2} = 0.051\lambda$$

$$S_1 = \frac{S_2}{K} = \frac{0.051\lambda}{1.2} = 0.0425\lambda$$

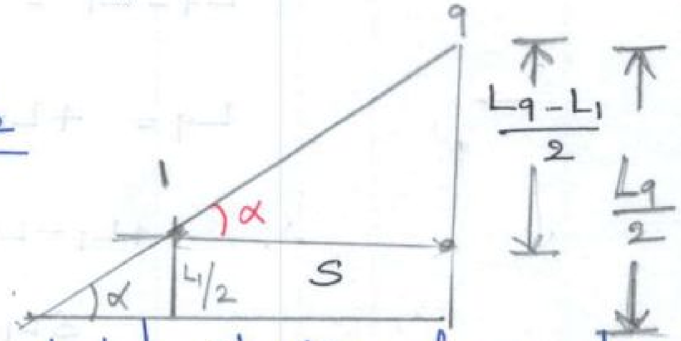
$$\frac{l_{n+1}}{l_1} = K^n = F$$

if $n = 8$ $$\frac{L_9}{L_1} = 4$$

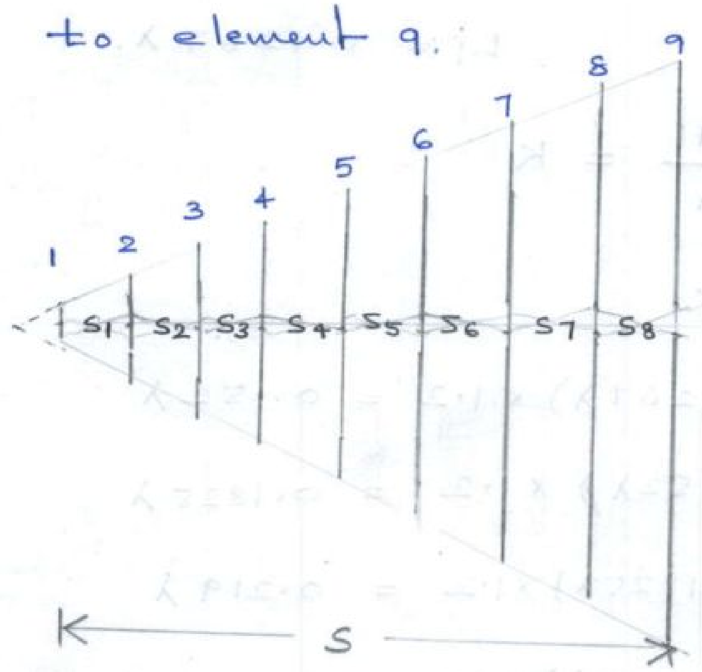
$$L_9 = 4L_1.$$

$$\tan\alpha = \frac{(L_{n+1} - L_n)/2}{S_n}$$

$$\tan\alpha = \frac{(L_9 - L_1)/2}{S}$$



where S is the total spacing from element 1 to element 9.



$$S = S_1 + S_2 + S_3 + S_4 + S_5 + S_6 + S_7 + S_8$$

$$S = [0.0425 + 0.051 + 0.062 + 0.074 + 0.089 + 0.107 + 0.129 + 0.155]\lambda$$

$$\boxed{S = 0.7095\lambda}$$

$$\tan\alpha = \frac{L_9 - L_1}{2S}$$

$$\tan 15^\circ = \frac{L_9 - L_1}{2\times 0.7095\lambda}$$

$$L_9 - L_1 = 1.419 \tan 15^\circ \cdot \lambda$$

$$L_9 - L_1 = 0.3802\lambda$$

$$L_9 = 4L_1$$

$$\therefore \quad 4L_1 - L_1 = 0.3802\lambda$$

$$3L_1 = 0.3802\lambda$$

$$L_1 = 0.1267\lambda.$$

$$\frac{L_{n+1}}{L_n} = K$$

$$\frac{L_2}{L_1} = K$$

$$L_2 = (0.1267\lambda)\times 1.2 = 0.152\lambda$$

$$L_3 = (0.152\lambda)\times 1.2 = 0.1825\lambda$$

$$L_4 = (0.1825\lambda)\times 1.2 = 0.219\lambda$$

$$L_5 = (0.219\lambda)\times 1.2 = 0.2628\lambda$$

$$L_9 = 0.5449\lambda$$

$$L_6 = (0.2628\lambda)\times 1.2 = 0.3153\lambda$$

$$L_7 = (0.3153\lambda)\times 1.2 = 0.3784\lambda$$

$$L_8 = (0.3784\lambda)\times 1.2 = 0.4541\lambda$$

Q: Design an optimum Log-periodic antenna to operate at frequencies from 50 MHz to 250 MHz with 11 elements..

choose $\alpha = 15^\circ$ and $K = 1.195$.

Soln

→ $f_1 = 50\,MHz$

$$\lambda_1 = \frac{c}{f_1} = \frac{3\times10^8}{50\times10^6} = 6\,mt.$$

$$\frac{\lambda_1}{2} = 3\,mt$$

$f_2 = 250\,MHz$

$$\lambda_2 = \frac{c}{f_2} = \frac{3\times10^8}{250\times10^6} = 1.2\,mt$$

$$\frac{\lambda_2}{2} = 0.6\,mt$$

→ $$F = \frac{f_2}{f_1} = \frac{250}{50} = 5$$

$$K^n = F$$

$$(1.195)^n = 5$$

$$\log(1.195)^n = \log 5$$

$$n = \frac{\log(5)}{\log(1.195)}$$

$$n = 9.034$$

$$\boxed{n \simeq 9}$$

→ the higher cutoff frequency occurs when the shortest element is nearly $\lambda/2$.

$$\therefore \; n=1 \Rightarrow l_1 = \frac{\lambda_2}{2} = 0.6\,\text{mt}$$

→ $$\frac{l_{n+1}}{l_1} = F$$

$$n=9 \Rightarrow \frac{l_{10}}{l_1} = 5$$

$$l_{10} = 5 \times l_1 = 5 \times 0.6 = 3\,\text{mt}$$

→ $$\frac{l_{n+1}}{l_n} = K$$

$$l_2 = K \times l_1$$

$$l_2 = 1.195 \times 0.6 = 0.717\,\text{mt}$$

$$l_3 = 1.195 \times 0.717 = 0.8568\,\text{mt}$$

$$l_4 = 1.195 \times 0.8568 = 1.0238\,\text{mt}$$

$$l_5 = 1.195 \times 1.0238 = 1.2235\,\text{mt}$$

$$l_6 = 1.2235 \times 1.195 = 1.4621\,\text{mt}$$

$$l_7 = 1.195 \times 1.4621 = 1.7472\,\text{mt}$$

$$l_8 = 1.195 \times 1.7472 = 2.0879\,\text{mt}$$

$$l_9 = 1.195 \times 2.0879 = 2.495\,\text{mt}$$

$$l_{10} = 1.195 \times 2.495 = 2.981\,\text{mt}$$ $l_{11} = 3.563\,\text{mt}$

7.

$$\tan\alpha = \frac{1-1/K}{4\, S_n \lambda}$$

$$S_n\lambda = \frac{1-1/K}{4\tan\alpha}$$

$$S_9\lambda = \frac{1-1/1.195}{4\tan 15^\circ}$$

$$S_9\lambda = 0.152$$

$l_{n+1} = \frac{\lambda}{2}$

$l_{10} = \lambda/2$

$\lambda = 2\times l_{10}.$

$$S_9 = 0.152\times(2\times l_{10})$$

$$S_9 = 0.152\times 2\times 2.981$$

$$S_9 = 0.9062 \text{ mt}$$

$$\frac{S_{n+1}}{S_n} = K$$

$$S_n = \frac{S_{n+1}}{K}$$

$$S_8 = \frac{S_9}{1.195} = \frac{0.9062}{1.195} = 0.7583 \text{ mt}$$

$$S_7 = \frac{0.7583}{1.195} = 0.6345 \text{ mt}$$

$$S_6 = \frac{0.6345}{1.195} = 0.531 \text{ mt}$$

$$S_5 = \frac{0.531}{1.195} = 0.4443 \text{ mt}$$

$$S_4 = \frac{0.4443}{1.195} = 0.3718 \text{ mt}$$

$$\rightarrow \quad S_3 = \frac{0.3718}{1.195} = 0.3111 \text{ mt}$$

$$S_2 = \frac{0.3111}{1.195} = 0.2604 \text{ mt}$$

$$S_1 = \frac{0.2604}{1.195} = 0.217 \text{ mt.}$$

= x =.