

# Introduction to reinforcement learning

October 5, 2019

# Introduction

- ▶ We will study an important AI paradigm : **Reinforcement learning (RL)**

# Applications of RL

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- ▶ RL has many applications and is quite a hot topic.
- ▶ Especially **Deep Reinforcement Learning** has received a lot of attention recently.

# Applications of Deep Reinforcement Learning I

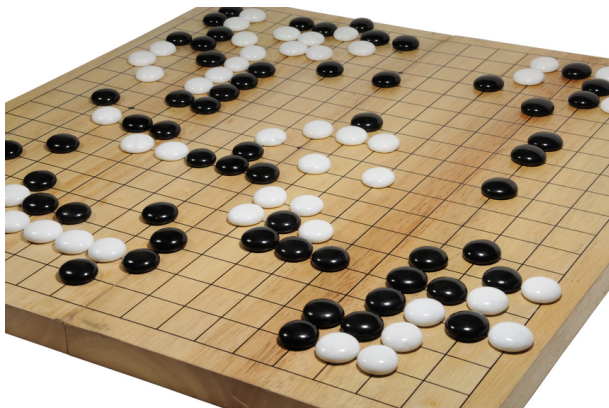
## ► Atari games



Figure: Atari game

# Applications of Deep Reinforcement Learning II

## ► AlphaGo



**Figure:** Go game, beaten by AlphaGo in 2017 [Silver et al., 2016]

## Applications of Reinforcement Learning III

- ▶ Reinforcement Learning is also begin used in the community of **Computational neuroscience**.

# References

- ▶ General Reinforcement Learning : [Andrew and Sutton, 1998]



# Overview

## The framework

Supervised learning

Reinforcement learning

## Dynamic programming

Value Iteration

Policy Iteration

## Model free Reinforcement learning

Temporal Difference learning

Additional considerations

# Supervised learning and Correction

- ▶ In **supervised learning**, the supervisor indicates the **expected answer** the agent should answer.
- ▶ With our mnist digit classification example, the action of the agent is the **prediction of the class**.

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- ▶ In **supervised learning**, the supervisor indicates the **expected answer** the agent should answer.
- ▶ The feedback does not depend on the action performed by the agent (for instance the prediction from the agent)
- ▶ We say that the agent receives an **instructive feedback**

## Supervised learning Correction

- ▶ In **supervised learning**, the supervisor indicates the **expected answer** the agent should answer.
- ▶ The agent must then **correct its model** based on this answer.

## Cost sensitive learning

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- ▶ The agent receives an **evaluative feedback**. The feedback depends on the action performed by the agent.
- ▶ **Examples :**
  - ▶ AI playing a game and receiving "victory" or "defeat" as a feedback.
  - ▶ Child playing with an animal.

# Reinforcement learning

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- ▶ In reinforcement learning, the feedback is a **real number**
- ▶ **Example** : amount of coins won after a poker turn.

# Reinforcement learning

- ▶ First, the agent does **not** know if a reward is good or bad per se.
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# Reinforcement learning

- ▶ First, the agent does not know if a reward is good or bad per se.
- ▶ A reward of  $-10$  good be good or bad depending on the other rewards that are possible to obtain.
- ▶ The objective of the agent will be to optimize the **agregation of rewards**

# Reinforcement learning

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The agent performs **actions**  $a$  and receives rewards  $r$ .

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The agent performs **actions**  $a$  and receives rewards  $r$ .
- ▶ **Examples :**
  - ▶ world =  $\mathbb{R}^2$
  - ▶ state = position
  - ▶ actions = moving somewhere
  - ▶ reward = amount of food found

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- ▶ We will consider **discrete spaces** :
  - ▶ the time will be discrete
  - ▶ the number of possible states will be **finite**
  - ▶ the number of possible actions will be **finite**



## Question

- ▶ We will consider **discrete spaces** :
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  - ▶ the number of possible actions will be **finite**
- ▶ Are these hypothesis valid in the case of AlphaGo ?

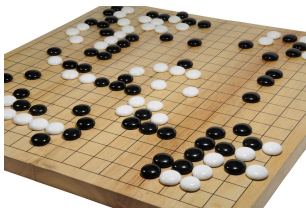
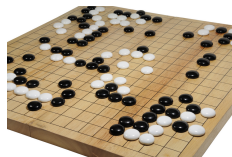


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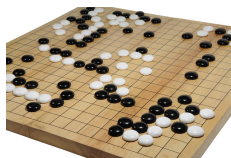


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- ▶ Yes.

## Question

- ▶ Are these hypothesis valid in the case of AlphaGo ?



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- ▶ Yes.
- ▶ However, please note that this is not always the case. Sometimes the possible actions are continuous, the available positions are continuous, etc.

## Let us continue with the formalization

- ▶ we will write :
  - ▶  $s_t$  : state at time  $t$
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- ▶ how is the action chosen ?

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  - ▶  $a_t$  : action performed at time  $t$
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- ▶ the actions are chosen according to a **policy**  $\pi$

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- ▶ It can be **deterministic** : the action chosen is chosen with probability 1
- ▶ Or **stochastic** : the action performed in a given state is drawn from a **distribution**

## Two levels of randomness

- ▶ The policy can be deterministic or stochastic.
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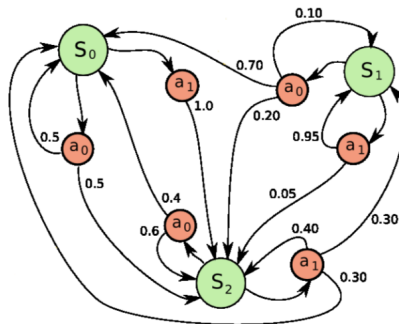


Figure: A stochastic policy with a stochastic transition function.

## Exercise 1: Computing a probability.

- What is the probability of staying in state  $S_0$  when performing an action from  $S_0$  ? and from  $S_1$  and  $S_2$  ?

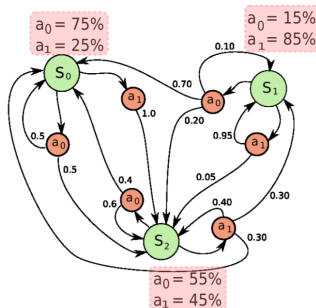


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- ▶ However, what exactly does the agent maximise ?
- ▶ There are several ways to agregate the rewards.

## Agregation of rewards

- If the horizon is finite, we can take the sum

$$V^{\pi}(s_0) = r_0 + \dots + r_N \quad (1)$$

## Agregation of rewards

- ▶ If the horizon is finite, we can take the sum

$$V^{\pi}(s_0) = r_0 + \dots + r_N \quad (2)$$

- ▶ We could also average a window. For instance a window of size 3 :

$$V^{\pi}(s_0) = \frac{r_0 + r_1 + r_2}{3} \quad (3)$$



## Agregation of rewards : discount factor

- ▶ the **discount factor**  $\gamma \in [0, 1]$  allows you to weight the rewards  $r_k$

$$V^\pi(s_0) = \sum_{t=t_0}^{+\infty} \gamma^{t-t_0} r_t \quad (4)$$

## More considerations

- ▶ The Markov hypothesis
- ▶ Exploitation exploration compromise

# Art

"RL is a science, but dealing with the exploration-exploitation compromise is an art" (Sutton)

## Dynamic programming

- ▶ Today we will study a simple case of Reinforcement learning
- ▶ In that case, the result of our actions is deterministic.

# World

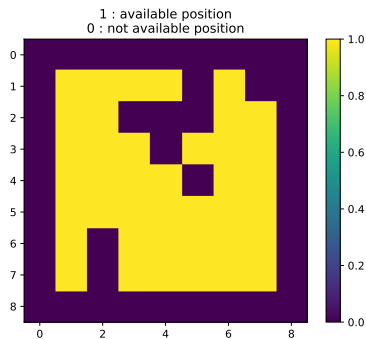


Figure: 2 dimensional world.

# Reward

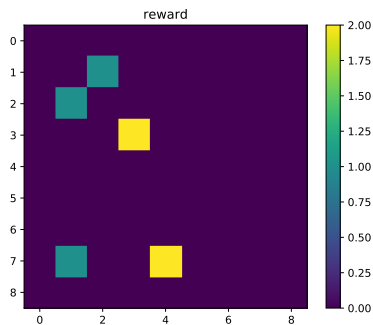


Figure: Reward function.

## 2D world

- ▶ Our agent can move in the 4 directions, one step at a time.
- ▶ We will progressively build an agent that learns to evaluate the states and then learns how to go to the best state.

## Value function

- ▶ For each state (=position in the 2D world), we want to compute the **value function**.



$$V(s_0) = r_0 + \gamma r_1 + \gamma^2 r_2 \dots \quad (5)$$



## States and rewards.

### Exercise 2: Bellman equation.

- ▶ For each state (=position in the 2D world), we want to compute the **value function**.



$$V(s_0) = r_0 + \gamma r_1 + \gamma^2 r_2 \dots \quad (6)$$

- ▶ Can you express  $V(s_0)$  as a function of  $V(s_1)$  ?

## Bellman equation

- ▶ This equation is the Bellman equation.

# Value Iteration

- ▶ First, the initial Value function for all the states is 0.

## Value Iteration

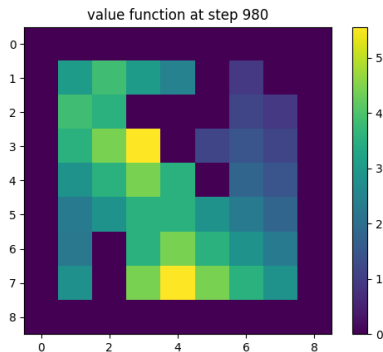
- ▶ First, the initial Value function for all the states is 0.
- ▶ Then we propagate the information about the rewards between the states, in order to **update the value function**
- ▶ We can find an optimal policy in the following way :

$$\forall s \in V(s_t) \leftarrow \max_{a_t} (r_{s_t} + \gamma V(s_{t+1})) \quad (7)$$

( $s_{t+1}$  depends on  $a_t$ ).

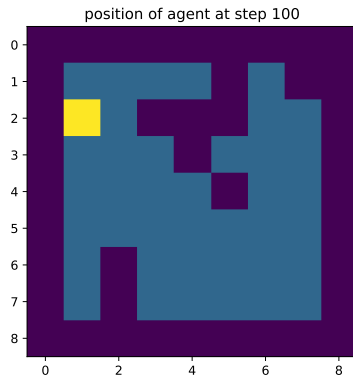
## Value iteration

- After learning, we will obtain a value function



## Introduction to reinforcement learning

- └ Dynamic programming
  - └ Value Iteration



# World

## Exercise 3: Creating the environment.

- ▶ **cd ./reinforcement\_learning**
- ▶ use the file **create\_world.py** in order to generate your own environment.
- ▶ You can use the one that is already there if you prefer.
- ▶ We store the data about the world in **.npz** files.

## Random policy

### Exercise 4: Moving agent

- ▶ In **value\_iteration.py**, modify the function **move\_agent** so that the agent is randomly moved.



## Bellman update

### Exercise 4: Update value

- ▶ In **value\_iteration.py**, modify the function **update\_value\_function** in order to modify the value function according to the Bellman equation.

# Optimal value

## Exercise 4: Update value

- ▶ Finally, make the algorithm run in order to **converge to the optimal value function**.

## Optimal policy

### Exercise 5 : Choosing a policy

- ▶ Please use the file **value\_iteration\_policy** in order to design an optimal policy for our agent.

## Optimal policy

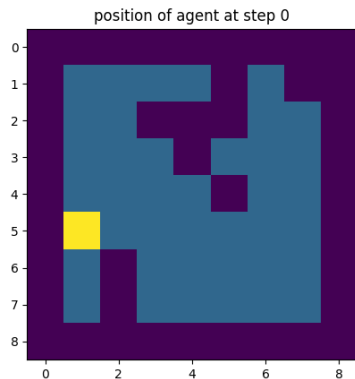


Figure: After learning, the agent can go to the reward.

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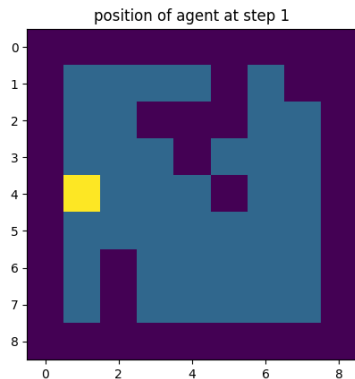


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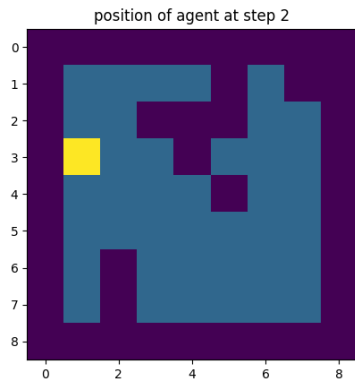


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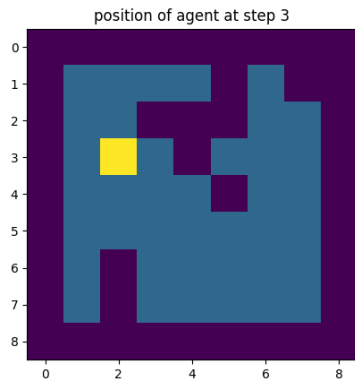


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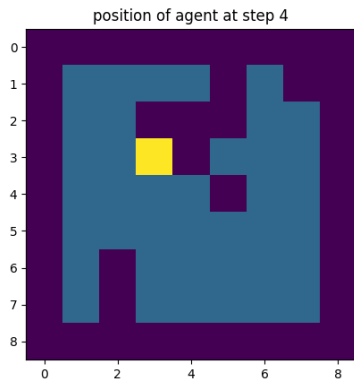


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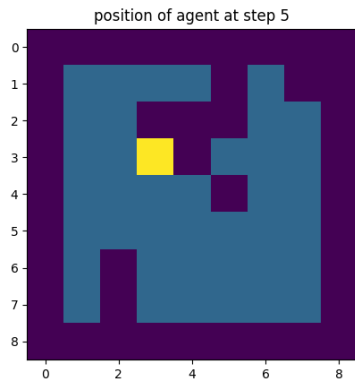


Figure: After learning, the agent can go to the reward.

## Remark

- ▶ Before going closer to RL, let us do another example of **dynamic programming**.

## Policy Iteration

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  - ▶ **Policy improvement**

## Policy Iteration

### Exercise 6 : Implementing the algorithm

- ▶ Please use the file **policy\_iteration.py** in order to perform the algorithm.

# Policy Iteration

## Exercise 6 : Implementing the algorithm with randomness

- ▶ Please use the file **policy\_iteration.py** in order to perform the algorithm.
- ▶ Add randomness to the actions of the agent to **guarantee exploration**.

## Multiple paradigms

- ▶ Reinforcement learning has many variants.
- ▶ In the ones we studied, a model of the effect of our actions were known.
- ▶ This is not always the case.



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$$V(S_t) \leftarrow V(S_t) + \alpha[R_{t+1} + \gamma V(S_{t+1}) - V(S_t)] \quad (8)$$

## Monte Carlo methods

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- ▶ For instance in episodic games, we can do statistics on the values of the states.

## Actor critic methods

- ▶ Sometimes you can use **two** policies

## Actor critic methods

- ▶ Sometimes you can use **two** policies
  - ▶ the **behavior policy** provides actions and guarantees exploration
  - ▶ the **target policy** is the optimal policy learned in parallel by the agent, that would be used in exploitation mode.

## Bias variance compromise

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  - ▶ more complex : less bias, more variance
  - ▶ less complex : more bias, less variance

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- ▶ We studied **finite** (and thus discrete situations).
- ▶ However, RL can also be applied to continuous state / discrete action spaces (DQN) [Mnih et al., 2013]
- ▶ And even to continous state / continous action spaces (DDPG) [Bengio, 2009] .

## References I



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