Formulaire

NICOLAS LE HIR
nicolaslehir@gmail.com

1 DÉCOMPOSITION BINAIRE

Result: Entier n en binaire
$L \leftarrow$ liste vide [];
$r \leftarrow 0$;
while $n > 0$ **do**
  $r \leftarrow n\%2$;
  $l \leftarrow l + [r]$;
  $n \leftarrow (n - r)/2$;
end
$L \leftarrow reversed(L)$;
return L

Algorithm 1: Binary decomposition of integer n

2 DISTANCES

Distances in two dimensions

Two points M_1 and M_2 in the 2D space with coordinates (x_1, y_1), and (x_2, y_2), respectively.

L2

$$d(M_1, M_2) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \tag{1}$$

L1

$$d(M_1, M_2) = |x_1 - x_2| + |y_1 - y_2| \tag{2}$$

L∞

$$d(M_1, M_2) = \max(|x_1 - x_2|, |y_1 - y_2|) \tag{3}$$

weighted L1

$$d(M_1, M_2) = \alpha_1 |x_1 - x_2| + \alpha_2 |y_1 - y_2| \tag{4}$$

Distances in three dimensions

Two points M_1 and M_2 in the 3D space with coordinates (x_1, y_1, z_1), and (x_2, y_2, z_2), respectively.

L2

$$d(M_1, M_2) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2} \tag{5}$$

L1

$$d(M_1, M_2) = |x_1 - x_2| + |y_1 - y_2| + |z_1 - z_2| \tag{6}$$

$L\infty$

$$d(M_1, M_2) = \max(|x_1 - x_2|, |y_1 - y_2|, |z_1 - z_2|) \tag{7}$$

weighted L1

$$d(M_1, M_2) = \alpha_1|x_1 - x_2| + \alpha_2|y_1 - y_2| + \alpha_3|z_1 - z_2| \tag{8}$$

3 LIKELIHOOD

— Observations : $(x_1, \ldots, x_n)$
— Model : p (for instance a normal law)
— Parameters : θ (for instance (μ, σ), the mean and the standard deviation of the normal law)

$$L(\theta) = p(x_1, \ldots, x_n|\theta) \tag{9}$$

3.1 Exercise 5

Here p **is a parameter** (that of a Bernoulli law), not to be mixed with the letter p that stands for a **model** of a probability distribution.

$$L(p) = p \times (1 - p) \tag{10}$$

$$L(p) = p - p^2 \tag{11}$$

$$L'(p) = 1 - 2p \tag{12}$$

$L'(p) = 0$ if and only if $p = \frac{1}{2}$
Studying the variations of L as a funciton of p, we see that the maximum is at $p = \frac{1}{2}$.

4 DERIVATIVE

$f : x \to f(x)$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \tag{13}$$

$g : x \to 3x$
$\forall \in \mathbb{R}, x, g'(x) = 3$
$h : x \to x^2$
$h' = ?$

5 EXPECTED VALUE

X constant random variable : $X = \alpha$

$$\sum_{i=1}^{n} p_i x_i = \sum_{i=1}^{n} p_i \alpha = \alpha \sum_{i=1}^{n} p_i \tag{14}$$

6 K-MEANS

— Datapoints $(x_1, \dots, x_n)$
— Centroids $(c_1, \dots, c_n)$ (one centroid per point, however the number of different centroids is smaller than the number of datapoints)

The inertia I is given by :

$$I = \sum_{i=1}^{n} d(x_i, c_i)^2 \tag{15}$$

7 ENTROPY

Entropy of certain distribution.

$$H = 0 \tag{16}$$

Entropy of uniform distribution with n values :

$$\begin{aligned} H &= -\sum_{i=1}^{n} \frac{1}{n} \log \frac{1}{n} \\ &= -n \times \frac{1}{n} \times \log \frac{1}{n} \\ &= \log n \end{aligned} \tag{17}$$

8 COMPLEXITY

Let n be the size of the problem.
Polynomial complexity :

$$a_k n^k + a_{k-1} n^{k-1} + \dots + n \tag{18}$$

Exponential complexity :

$$k^n \tag{19}$$

avec $k > 1$

9 EXERCISE 2 (TIDE LEVEL)

Hypothèse :
L : tide level in meters
t : time in hours
A : amplitude
ϕ : phase
f : frequence (Hz = s^{-1})jn
$\lambda = \frac{1}{f}$: periode en secondes
$\omega = 2\pi f$: pulsation (radian par seconde)

$$L = A\sin(\omega t + \phi) + c \tag{20}$$

error E :

$$\sum_{\text{samples}} (\text{prediction} - \text{truth})^2 \tag{21}$$