Main Questions Part A

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a) $\vec{OC} = \vec{OA} + \vec{AC} = \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}$

b) $\theta = \cos^{-1}\left(\frac{|AC|^2 - |OA|^2 - |OC|^2}{2 \times |OA| \times |OC|}\right)$

$= \cos^{-1}\left(\frac{30 - 14 - 14}{2 \times 14 \times 14}\right)$

$= \cos^{-1}\left(\frac{2}{392}\right) \approx \boxed{89.7^\circ}$

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a) $v = \begin{pmatrix} 0 \\ 125 \end{pmatrix}$

$w = \begin{pmatrix} 0 \\ -40 \end{pmatrix}$

$\Rightarrow v_{final} = v + w = \begin{pmatrix} 0 \\ 85 \end{pmatrix}$

$\rightarrow$ 85 km/h to the North

b) $w = \begin{pmatrix} x \\ x \end{pmatrix}$ where $|w| = 80$

$\Rightarrow \sqrt{2x^2} = 80$

$x = 40\sqrt{2}$

$\Rightarrow v = \begin{pmatrix} 0 \\ 120 \end{pmatrix} + \begin{pmatrix} 40\sqrt{2} \\ 40\sqrt{2} \end{pmatrix}$

$= \begin{pmatrix} 40\sqrt{2} \\ 120 + 40\sqrt{2} \end{pmatrix}$, $|w| \approx 190.2$ km/h

~~m~~ at a ~~bearing~~ angle of ~ 017.3° from North

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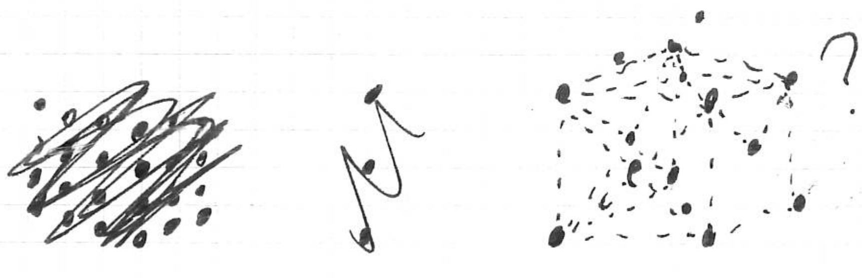
a)



I don't understand how to draw this structure

b) Where does the origin start from? Middle of the cube? I don't think there is an atom at that position?

If it starts at a corner of the cube:

1) ~~m~~ $r_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$$n_2 = \begin{pmatrix} 0 \\ d/2 \\ d/2 \end{pmatrix}$$

$$n_3 = \begin{pmatrix} d/2 \\ 0 \\ d/2 \end{pmatrix}$$

$$n_4 = \begin{pmatrix} d/2 \\ d/2 \\ 0 \end{pmatrix}$$

c) $$n_1 = \begin{pmatrix} -d/4 \\ -d/4 \\ -d/4 \end{pmatrix}$$

$$n_2 = \begin{pmatrix} -d/4 \\ d/4 \\ d/4 \end{pmatrix}$$

$$n_3 = \begin{pmatrix} d/4 \\ -d/4 \\ d/4 \end{pmatrix}$$

$$n_4 = \begin{pmatrix} d/4 \\ d/4 \\ -d/4 \end{pmatrix}$$

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$$\cancel{P = \begin{pmatrix} 0 \\ 30 \end{pmatrix} + t}$$

$$B = \begin{pmatrix} 10 \\ 10 \end{pmatrix} + t\begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

$$P = \begin{pmatrix} 0 \\ 30 \end{pmatrix} + (t-10)\begin{pmatrix} 5 \\ 2 \end{pmatrix}$$

$$F = \begin{pmatrix} 50 \\ 0 \end{pmatrix} + (t-20)\begin{pmatrix} 5 \\ 3 \end{pmatrix}$$

Bugsy:

$$\cancel{B = \begin{pmatrix} 10 \\ 10 \end{pmatrix} + \begin{pmatrix} 3t \\ 4t \end{pmatrix} = F = \begin{pmatrix} 0 \\ 50 \end{pmatrix}}$$

$$B = \begin{pmatrix} 10+3t \\ 10+4t \end{pmatrix} = F = \begin{pmatrix} 5t-50 \\ 3t-60 \end{pmatrix}$$

$\rightarrow 10+3t = 5t-50$

$2t = 60$

$t = 30$ ✓

$\rightarrow 10+4t = 3t-60$

$t = -70$ ✗ $\rightarrow$ Does not catch Bugsy

Peter:

$$P = \begin{pmatrix} 5t-50 \\ 2t+10 \end{pmatrix} = \begin{pmatrix} 5t-50 \\ 3t-60 \end{pmatrix} = F$$

$\rightarrow 5t-50 = 5t-50$ ✓

$\rightarrow 2t+10 = 3t-60$

$t = 70s$

$\rightarrow$ Fox catches Peter.

→ a) Yes

b) Peter

c) $70s - 20s = \boxed{50s}$

d) Point of capture $= \begin{pmatrix} 5\times70-50 \\ 2\times70+10 \end{pmatrix} = \begin{pmatrix} 300 \\ 150 \end{pmatrix}$

$\to \vec{FP_c} = \begin{pmatrix} 250 \\ 150 \end{pmatrix} \to |FP_c| \approx \underline{291.5m}$ (fox)

$\vec{PP_c} = \begin{pmatrix} 300 \\ 120 \end{pmatrix} \to |PP_c| \approx \underline{323.1m}$ (peter)

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a) $d = b + \frac{1}{2}(c-b) = \frac{1}{2}b + \frac{1}{2}c = \frac{1}{2}(b+c)$

b) $\vec{Ad} = \vec{AO} + d$

$= -a + d$

$\to p = \lambda(d-a) = \boxed{\lambda\left(\frac{1}{2}(b+c) - a\right)} + a$

c) $p_b = \lambda\left(\frac{1}{2}(a+c) - b\right) + b$

$p_c = \lambda\left(\frac{1}{2}(a+b) - c\right) + b$

d) $\lambda = \frac{1}{2}$?

$\rightarrow P_a \; \frac{1}{4}(b+c) + \frac{1}{2}a + a = \frac{1}{4}b + \frac{1}{4}c + \frac{3}{2}a$

$P_b = \frac{1}{4}a + \frac{1}{4}c + \frac{3}{2}b$

$P_c = \frac{1}{4}a + \frac{1}{4}b + \frac{3}{2}c$

$\rightarrow$ ~~4EIC:~~ $\cancel{\frac{1}{4}c = \frac{1}{4}c = \frac{1}{4}a + \frac{3}{2}b = \frac{1}{4}b + \frac{3}{2}a}$

$\rightarrow \cancel{\frac{1}{4}c = \frac{5}{4}a = \frac{5}{4}b}$

$P_a = P_b = P_c$

$\rightarrow \lambda\left(\frac{1}{2}(b+c) - a\right) + a = \lambda\left(\frac{1}{2}(a+c) - b\right) + b = \lambda\left(\frac{1}{2}(a+b) - c\right) + c$

$\rightarrow \cancel{\frac{\lambda}{2}b + \frac{\lambda}{2}c = (\lambda+1)a}$

$\frac{\lambda}{2}b + \frac{\lambda}{2}c - \lambda a + a$

$= \frac{\lambda}{2}a + \frac{\lambda}{2}c - \lambda b + b$

$= \frac{\lambda}{2}a + \frac{\lambda}{2}b - \lambda c + c$

$\rightarrow \frac{\lambda}{2}b - \frac{\lambda}{2}a - \lambda a + a + \lambda b - b = 0$

and: $\frac{\lambda}{2}b - \frac{\lambda}{2}c - \lambda b + b + \lambda c - c = 0$

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$\Rightarrow \frac{\lambda}{2}b - \frac{\lambda}{2}a - \lambda a + a + \lambda b - b$

$= \frac{\lambda}{2}b - \frac{\lambda}{2}c - \lambda b + b + \lambda c - c$

$\Rightarrow \left(\frac{3}{2}\lambda b + \cancel{b}\right) + \left(-\frac{3}{2}\lambda a + a\right)$

$= \left(-\frac{\lambda}{2}b + \cancel{b}\right) + \left(-\frac{\lambda}{2}c - c\right)$

$2\lambda b - \frac{3}{2}\lambda a + a + \frac{\lambda}{2}c + c = 0$ X

$\boxed{???}$

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a) ~~$r = a + \lambda(b-a)$~~

$r = a + \lambda(b-a)$

b) $r = \begin{pmatrix} 1+2\lambda \\ 1+0 \\ 0+4\lambda \end{pmatrix} = \begin{pmatrix} 1+2\lambda \\ 1 \\ 4\lambda \end{pmatrix}$

i) No
ii) Yes
iii) No
iv) No

$\Rightarrow$ Eliminate ones where $y \neq 1 \Rightarrow$ Not e, f

ii) $1+2\lambda = 5 \Rightarrow \lambda = 2$, $4\lambda = 4\times 2 = 8$ ✓

iii) $1+2\lambda = 1 \Rightarrow \lambda = 0$, $4\lambda = 4\times 0 = 0 \neq -4 \Rightarrow$ No X

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a) ① $r = \begin{pmatrix}1\\1\\1\end{pmatrix} + \lambda\begin{pmatrix}3\\1\\4\end{pmatrix}$

② $r = \begin{pmatrix}3\\1\\6\end{pmatrix} + \mu\begin{pmatrix}-1\\-1\\1\end{pmatrix}$

b) ① $\dfrac{x-1}{3} = \dfrac{y-1}{1} = \dfrac{z-1}{4}$

② $\dfrac{x-3}{-1} = \dfrac{y-1}{-1} = \dfrac{z-6}{1}$

c) $\Rightarrow \begin{pmatrix}1+3\lambda\\1+\lambda\\1+4\lambda\end{pmatrix} = \begin{pmatrix}3-\mu\\1-\mu\\6+\mu\end{pmatrix}$

$\Rightarrow 1+3\lambda = 3-\mu \Rightarrow 3\lambda+\mu = 2$

$1+\lambda = 1-\mu \Rightarrow \lambda = -\mu$

$\Rightarrow$ $\cancel{2\lambda = 2\mu = 2 \Rightarrow \lambda = \mu = \tfrac{1}{2}}$ $\quad 3\lambda + \mu = 2\lambda = 2$

$\lambda = 1, \mu = -1$

Check:

$1+4(1) = 5, \; 6-1 = 5$ ✓

$\Rightarrow p = \begin{pmatrix}1+3\\1+1\\1+4\end{pmatrix} = \boxed{\begin{pmatrix}4\\2\\5\end{pmatrix}}$

OR: $\cancel{① \dfrac{x-1}{3} = \dfrac{x-3}{-1} \Rightarrow x-1 = 3x-9, \; 2x = 8 \Rightarrow x = 4}$

$\cancel{② \dfrac{y-1}{1} = \dfrac{y-1}{-1} \Rightarrow 1-y = y-1 \Rightarrow 2}$

$\cancel{③ \dfrac{z-1}{4} = \dfrac{z-6}{1}, \; z-1 = 4z-24, \; 3z =}$

OR

① $x = 3y - 3 + 1 = 3y - 2$

② $x = y - 1 + 3 = y + 2$

$\rightarrow \quad 3y - 2 = y + 2$

$2y = 4$

$\boxed{y = 2}$

$\rightarrow \quad x = y + 2 = \boxed{4}$

$z = \underset{-1}{\underline{y}} - 1 + 6$

$= \underset{-1}{\underline{2}} - 1 + 6 = -1 + 6 = \boxed{5}$

$\rightarrow \quad \boxed{p = \begin{pmatrix} 4 \\ 2 \\ 5 \end{pmatrix}}$

→ Both methods give thesame answer.