Discrete Mathematics Proofs

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dg681

1 ~~A~~ Let n be a non-prime integer, $n > 2$ 1

Let $n = 8$

$2 \times 8 + 13 = 29$

29 is prime, statement is disproven by counter example.

② $x^2 + y = 13 \Rightarrow (y \neq 4 \Rightarrow x \neq 3)$ ②

Establish contrapositive:

$x = 3 \Rightarrow y = 4$

Assume $x = 3$,

$3^2 + y = 13$

$\underline{y = 4}$

So statement is proven

③ $(n^2 = 2i) \Leftrightarrow (n = 2j)$

Assume $n = 2j$

$n^2 = (2j)^2 = 4j = 2[2j]$, proven

Assume $n^2 = 2i$

Contrapositive:

Assume n odd

$\Rightarrow (2i+1)^2 = 4i^2 + 4i + 1$

$= \underline{2[2i+2] + 1}$

4. Assume:

$$x + z = y - z$$

$$2z = y - x$$

$$z = \frac{y-x}{2}$$

Hence

$$x + \frac{y-x}{2} = \frac{2x - x + y}{2} = \frac{y+x}{2}$$

$$= \frac{2y}{2} - \frac{(y-x)}{2} = y - \left(\frac{y-x}{2}\right) = \underline{y - z}$$

Hence statement is proven

5. Assume true

$$\Rightarrow x + z = y - z$$

$$\Rightarrow 2z = y - x$$

$$z = \frac{y-x}{2}, \text{ where } z \text{ is an integer}$$

Prove by contradiction:

Suppose $y = 2$, $x = 1$

$$\rightarrow z = \frac{2-1}{2} = \frac{1}{2}$$

Which is not an integer, so original assumption is false & statement is disproven

Let a be arbitrary rational number, so $a = \frac{m}{n}$ $(m, n \in \mathbb{Z}$
Let b be arbitrary rational number so $b = \frac{x}{y}$ $(x, y \in \mathbb{Z})$

$$a + b = \frac{m}{n} + \frac{x}{y} = \frac{my + xn}{ny}$$, is a rational number so

statement is proven.

7

Assume $x \neq 2$, x is a real number

$x = \frac{2y}{y+1}$ for a unique real number y

$(y+1)x = 2y$

$(2y - yx) = x$

$y(2-x) = x$

$y = \frac{x}{2-x}$

...fore for all real numbers z, if $x = \frac{2z}{z+1}$ then $z = y = \frac{2 \cdot x}{2-x}$

Assume $\frac{2z}{z+1} = x$

$$\frac{x}{2-x} = \frac{\frac{2z}{z+1}}{2 - \frac{2z}{z+1}} = \frac{2z}{(z+1)\left(2 - \frac{2z}{z+1}\right)} = \frac{2z}{2(z+1) - 2z} = \frac{2z}{2} = \boxed{z}$$

→ Statement is proven.

8 Let m be an arbitrary integer
Let n be an arbitrary integer

Establish contrapositive:
- If neither m nor n is even, then $m \cdot n$ is not even

Assume m is odd - $m = 2i + 1$
Assume n is odd - $n = 2j + 1$

$$m \cdot n = (2i+1)(2j+1) = 4ij + 2i + 2j + 1$$

$$= 2[2ij + i + j] + 1$$, hence $m \cdot n$ is odd

Statement is proven.

1
a) $n = \emptyset$
b) $d = \mathbb{Z}$

2 Let k be positive integer, m, n be integers

$\cancel{\Rightarrow 2.\ (k \cdot m) \mid (k \cdot n) =}$

$\Rightarrow$ Assume $(k \cdot m) \mid (k \cdot n)$

$$\rightarrow s(k \cdot m) = (k \cdot n)$$
$$s\,km = kn$$
$$sm = n$$
$$\rightarrow \boxed{m \mid n}$$

$\Leftarrow$: Assume $m \mid n$

$$\rightarrow sm = n$$
$$kms = kn$$
$$\rightarrow km \mid kn$$
$$\rightarrow \boxed{(k \cdot m) \mid (k \cdot n)}$$

3. ~~Let n be an arbitrary~~

Let $n = 0$

$2^0 = 1$

$2 | 1$ is false, so statement is disproven.

4. Let n be an arbitrary integer, m & n be integers

① Assume $l | m$ so $k_1 l = m$
② Assume $m | n$ so $k_2 m = n$

$\Rightarrow k_1 k_2 l = n$ so $\boxed{l | n}$

5. $k = 5$, $m = 5$, $n = 5$

$5 | 5$, $5 | 5$, but $(5 \times 5) | 5$ is not true

6.

a) $d | m$ so $k_1 d = m$
$d | n$ so $k_2 d = n$

$m + n = k_1 d + k_2 d$

$= (k_1 + k_2) d$ so $d | (m+n)$

b) $d | m$ so $k_1 d = m$

$\underbrace{k_1}\, k d = k m$ so $d | (k \cdot m)$

c) $d|m$ so $\cancel{d/k}$ $k_1 d = m$
$d|n$ so $k_2 d = n$

$km = k_1 k d$, is an integer
$ln = k_2 l d$, is an integer)

$\Rightarrow k \cdot m + l \cdot n = d(k_1 k + k_2 l)$

so $\underline{\underline{d | (k \cdot m + l \cdot n)}}$

7 Let n be an arbitrary integer

$\Rightarrow$: Assume $30|n$ so $n = 30k$

$n = 2 \times 15k$ so $2|n$
$n = 3 \times 10k$ so $3|n$
$n = 5 \times 6k$ so $5|n$

$\Leftarrow$: Assume $2|n$, $3|n$, $5|n$

① $2|n$ so $n = 2k_1$
② $3|n$ so $n = 3k_2$
③ $5|n$ so $n = 5k_3$

$15n = 30k_1$
$10n = 30k_2$
$6n = 30k_3$

$15n - 10n - 6n = 30(k_1 - k_2 - k_3)$
$-n = 30(k_1 - k_2 - k_3)$
$n = 30(k_2 + k_3 - k_1)$ so $\boxed{30|n}$

8) Let m, n be arbitrary integers

① $m|n$ so ~~$m=kn$~~ $k_1 m = n$
② $n|m$ so ~~$n=m$~~ $k_2 n = m$

$k_1(k_2 n) = n$?
so $k_1 k_2 = 1$? unsure

~~$\Rightarrow k_2(k_1 m) = m$~~

9) $m=5, n=5, k=25$

$25 | 5\times5$ ✓

$25 | 5$ X
$25 | 5$ X
→ Disproven by counterexample

10) a) 1 is a natural number

$0 \leq 1 \leq 1$ so $P(1)$

so, $P^{\#}(1) \Rightarrow P(1)$

b) $P(m)$ ~~is~~ $=$ m is even

$m = 2$

$P(2)$ ~~even~~ is true
But $P^{\#}(1)$ includes $P(1)$ which is false

e) ~~a~~ $P^{\#}(0)$

$\Rightarrow$: Assume $P^{\#}(0)$

Sine $0 \leq 0 \leq 0$, $P(0)$ holds

$\Leftarrow$: Assume $P(0)$

Assume ~~a~~ $0 \leq n \leq 0$, so $n = 0$ so $P^{\#}(0)$ holds

b) ?
~~c~~) ?