

Digital Electronics Paper

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dg68!

1.

①

a)

ab	\bar{a}	$\overline{a \cdot b}$	\bar{b}	$\overline{b \cdot a}$	π
00	1	1	1	1	0
01	1	0	0	1	1
10	0	1	1	0	1
11	0	1	0	1	0

b)

ab	\bar{a}	$\overline{a + b}$	\bar{b}	$\overline{b + a}$	π
00	1	0	1	0	1
01	1	0	0	1	0
10	0	1	1	0	0
11	0	0	0	0	1

2.

a) $d \cdot b \cdot c + a \cdot b \cdot \bar{c}$

$$= d \cdot b \cdot (c + \bar{c})$$

$$= d \cdot b \cdot 1$$

$$= \underline{\underline{d \cdot b}}$$

b) $a \cdot (\bar{a} + b)$

$$= a \cdot \bar{a} + a \cdot b$$

$$= 0 + a \cdot b$$

$$= d \cdot b$$

~~c) $a \cdot b + \bar{a} \cdot c$~~

~~$$= (a \cdot \bar{a} + a \cdot b) + (a \cdot \bar{a} + a \cdot c)$$~~

~~$$=$$~~

~~$$(a + c) \cdot (\bar{a} + b)$$~~

~~$$= a \cdot \bar{a} + a \cdot b + \bar{a} \cdot c + c \cdot b$$~~

~~$$= 0 + c \cdot b + a \cdot b + \bar{a} \cdot c$$~~

→ Consensus:

$$= d \cdot b + \bar{a} \cdot c$$

$$d) (a+c). (a+d). (b+e). (b+d)$$

②

$$= (a.a + a.d + c.a + c.d). (b.b + b.d + e.b + e.d)$$

~~$$= (a + a.d + a.c). (b + b.d + b.c)$$~~

~~$$= (a.d + a.c). (b.d + b.c)$$~~

~~$$= a.d.d.b + a.d.b.c + a.c.b.c + a.c.c.b$$~~

~~$$= a.b + a.c$$~~

$$= (a + a.d + a.c + c.d). (b + b.d + b.c + e.d)$$

$$= [a.(1+d+c) + c.d]. [b.(1+d+c) + e.d]$$

$$= (a.1 + c.d). (b.1 + e.d)$$

$$= (a + c.d). (b + e.d)$$

~~$$= a.b + a.c.d + b.c.d + e.c.d.d$$~~

$$= a.b + e.d + a.c.d + b.c.d$$

Consensus:

$$= \underline{a.b + e.d}$$

(3)

$$3 \quad z = b + \overline{a \cdot a + a \cdot b \cdot c}$$

~~$$= b + \overline{a \cdot a}$$~~

$$= b + \overline{a} + a \cdot b \cdot c$$

$$= b + (a \cdot (\bar{a} + \bar{b} + \bar{c}))$$

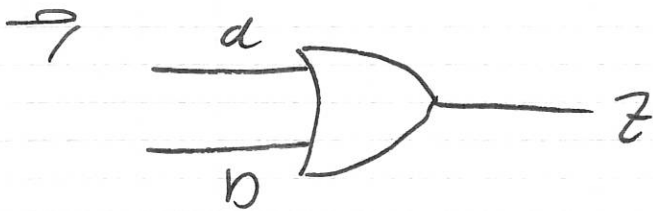
$$= b + a \cdot \bar{a} + a \cdot \bar{b} + a \cdot \bar{c}$$

$$= b + 0 + a \cdot \bar{b} + a \cdot \bar{c}$$

$$= b + a \cdot \bar{b} + a \cdot \bar{c}$$

$$= b + a + a \cdot \bar{c}$$

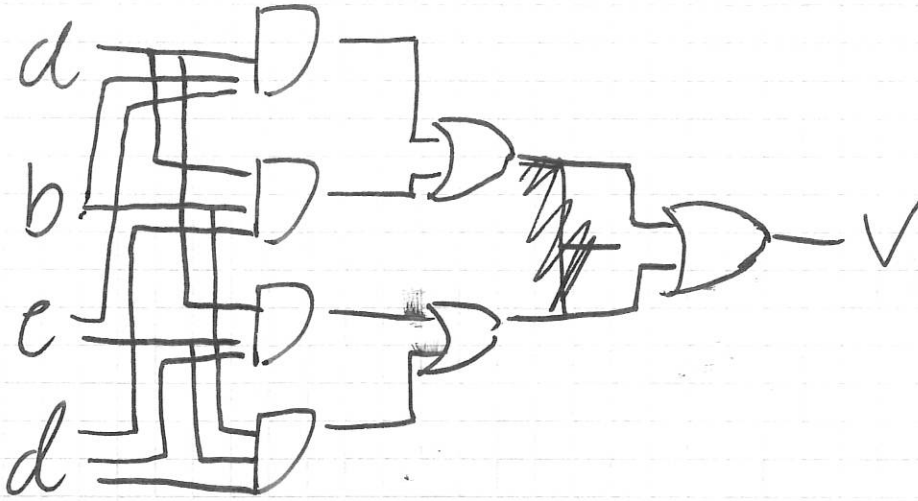
$$= a + b$$



(4)

4

$$V = a.b.c + a.b.d + a.c.d + b.c.d + \cancel{a.c.d}$$

5

a) Currently in SOP form ~~and~~ $\overline{a.b} = \overline{a} + \overline{b}$

$$\rightarrow V = \overline{(\overline{a} + \overline{b} + \overline{c})} + \overline{(\overline{a} + \overline{b} + \overline{d})} + \overline{(\overline{a} + \overline{c} + \overline{d})} + \overline{(\overline{b} + \overline{c} + \overline{d})}$$

$$= \overline{a.b.c} + \overline{a.b.d} + \overline{a.c.d} + \overline{b.c.d}$$

$$= \overline{a.b.c.d} + \overline{a.c.d.b.c.d}$$

$$= \overline{c.d} + \overline{a.d}$$

$$= \overline{a.b.c.d} = \cancel{a.b} \cdot \overline{c.d}$$

b) Let ∇ in SOP

(5)

$cd \backslash b$	00	01	11	10
00	0	0	0	1
01	0	0	1	0
11	1	1	1	1
10	0	0	1	0

$$\nabla = \bar{a} \cdot \bar{b} + \bar{c} \cdot \bar{d} + \bar{a} \cdot \bar{c} + \bar{a} \cdot \bar{d} + \bar{b} + \bar{d} + \bar{b} \cdot \bar{c}$$

$$\Rightarrow \nabla = \overline{a+b} + \overline{c+d} + \overline{a+c} + \overline{a+d} + \overline{b+d} + \overline{b+c}$$

$$\nabla = \overline{(a+b) \cdot (c+d) \cdot (a+c) \cdot (a+d) \cdot (b+d) \cdot (b+c)}$$

$$\Rightarrow \nabla = (a+b) \cdot (c+d) \cdot (a+c) \cdot (a+d) \cdot (b+d) \cdot (b+c)$$

Into circuit:

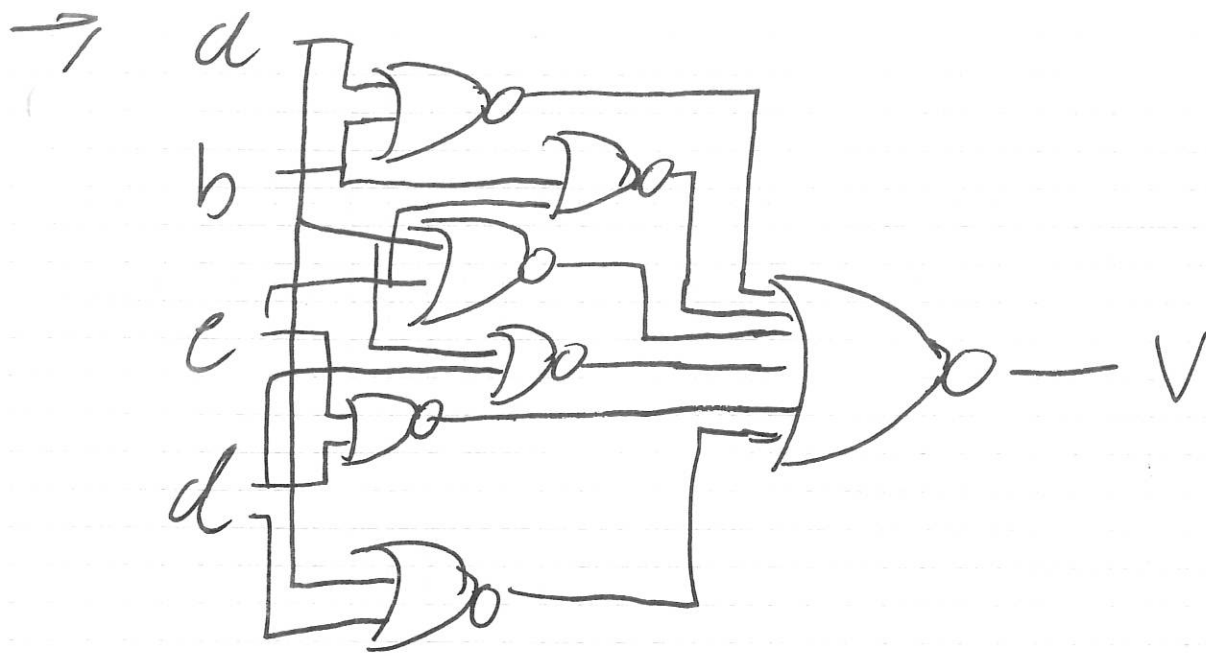
$$(a+b) \cdot (c+d) = \overline{\overline{a+b} + \overline{c+d}}$$

$$\Rightarrow \nabla = \overline{a+b + c+d} \cdot \overline{a+c + a+d} \cdot \overline{b+d + b+c}$$

$$= \overline{a+b + c+d + a+c + a+d} \cdot \overline{b+d + b+c}$$

$$= \overline{a+b + c+d + a+c + a+d + b+d + b+c}$$

(6)

b

$\begin{matrix} ab \\ \backslash \\ ed \end{matrix}$	00	01	11	10
00	1	1	0	1
01	1	1	0	1
11	0	0	0	0
10	1	1	0	X

$$f = \bar{a} \cdot \bar{c} + \bar{b} \cdot \bar{c} + \bar{a} \cdot \bar{d}$$

$$\bar{f} = a \cdot b + c \cdot d$$

7

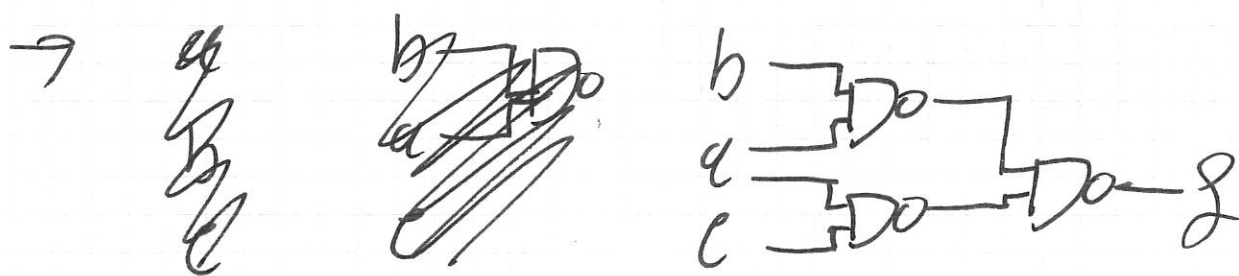
⑦

$\begin{matrix} ab \\ e \end{matrix}$	00	01	11	10
0	0	0	1	0
1	0	0	1	1

$$f = \underline{a \cdot b} + a \cdot c \quad (= a \cdot (b + c))$$

$$\bar{f} = \bar{a} + \bar{c} \cdot \bar{b}$$

$$a) f = \cancel{a \cdot b} \cdot \cancel{c \cdot a} \quad \overline{\overline{a \cdot b} \cdot \overline{a \cdot c}}$$



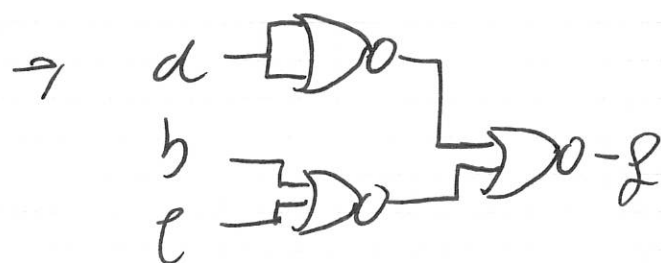
$$b) \bar{f} = \bar{a} + (\bar{c} \cdot \bar{b})$$

$$\bar{f} = \bar{a} + \overline{c + b}$$

$$\bar{f} = \overline{(a) \cdot (c + b)}$$

$$f = (a) \cdot (c + b)$$

$$f = \overline{\bar{a} \cdot \overline{c + b}}$$



8 Q-M method

0101, 0100, 0111, 1000, 1010, 1011, 1100,
1101, 1110, 1111

	1	2	3	4
→ 1000	✓	10-0	✓	1--0
		1-00	✓	
0101	✓	01-0	✓	-1-1
0110	✓	-101	✓	-1-1
1010	✓	011-	✓	-11-
1100	✓	-110	✓	1-1-
		101-	✓	11--
0111	✓	1-10	✓	
1011	✓	1100	✓	
1101	✓	11-0	✓	
1110	✓	-111	✓	
1111	✓	111-	✓	

(9)

	5	6	7	8
8, 10, 12, 14 11, 13, 15 1--0	—	1	—	(X)
5, 7, 13, 15 -1-1	(X)	—	X	—
6, 7, 14, 15 -1-1	—	(X)	X	—
0, 11, 14, 15 -1-1	—	—	—	—
12, 13, 14, 15 1--	—	—	—	—

→ ~~g~~ $g = a \cdot \bar{a} + b \cdot d + b \cdot c$

Q

Jan, Feb, Mar, April, Sep, Oct, Nov, Dec

→ 0001, 0010, 0011, 0100, ~~0101, 0110, 0111, 1000, 1001, 1010, 1011, 1100~~

$\begin{matrix} A_3 \\ A_2 \\ A_1 \\ A_0 \end{matrix}$	00	01	11	10
00	(X)	1	1	0
01	1	0	(X)	1
11	1	0	X	1
10	1	0	X	1

→ $m = \bar{A}_3 \cdot \bar{A}_2 + A_2 \cdot \bar{A}_1 \cdot \bar{A}_0 + A_3 \cdot A_1 + A_3 \cdot \bar{A}_1 \cdot A_0$

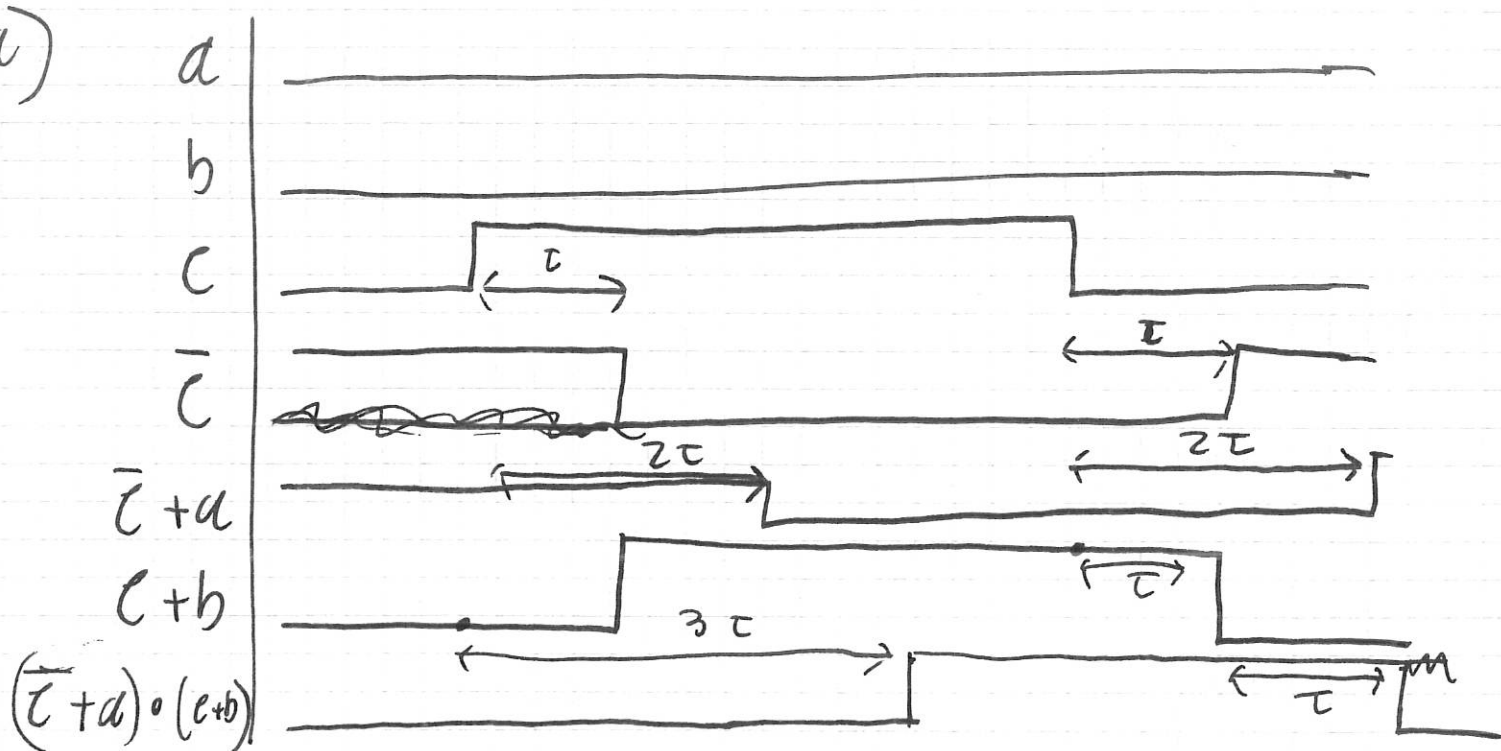
b) 0000, 0001, 0010, 0011, 1000, 1001, 1010, 1011 10

$A_2 \backslash A_1 A_0$	00	01	11	10
00	1	0	1	1
01	1	0	1	1
11	1	0	1	1
10	1	0	1	1

$$\Rightarrow m = \bar{A}_2$$

10

a)



b) There may be a static 0 hazard because $(\bar{c}+a) \cdot (c+b)$ may not change to 1 if c changes before 3τ , even though the output should've changed.

$$b) z = (\bar{c} + a) \cdot (c + b)$$

~~7/11~~

$$z = \overline{c \cdot \bar{a}} + \overline{\bar{c} \cdot \bar{b}}$$

$$\bar{z} = \underline{(c \cdot \bar{a}) + (\bar{c} \cdot \bar{b})}$$

c)

$c \backslash ab$	00	01	11	10
0	0	1	0	1
1	1	1	0	0

Protection

Answer: An or gate should be added to "connect" the essential prime implicants, so

$$\bar{z} = c \cdot \bar{a} + \bar{c} \cdot \bar{b} + \boxed{\bar{a} \cdot \bar{b}}$$

$$\overline{\bar{a} \cdot \bar{b}} \rightarrow (a + b)$$

→ z should become:

