Main Questions Part C

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$$\theta = \cos^{-1}\left(\frac{\begin{pmatrix}?\\1\\1\end{pmatrix}\cdot\begin{pmatrix}3\\-1\\-5\end{pmatrix}}{\left|\begin{pmatrix}?\\1\\1\end{pmatrix}\right| \times \left|\begin{pmatrix}3\\-1\\-5\end{pmatrix}\right|}\right)$$

$$= \underline{90^\circ}$$

Direction cosines? meaning?

Vector perp:

$$\begin{pmatrix}?\\1\\1\end{pmatrix} \wedge \begin{pmatrix}3\\-1\\-5\end{pmatrix} = \begin{vmatrix} i & j & k \\ ? & 1 & 1 \\ 3 & -1 & -5 \end{vmatrix} = \boxed{\begin{pmatrix}-4\\13\\-5\end{pmatrix}}$$

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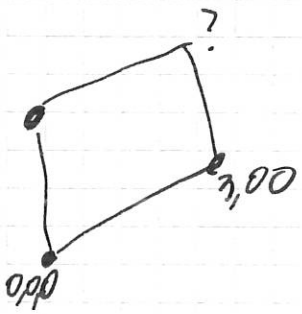
a) $3 \times 2 = \underline{6}$



b) $V = b \times h$

$b = 6$

h = component of $\vec{OC}$ in direction perp to OA & OB

$$\rightarrow h = \begin{pmatrix}1\\1\\5\end{pmatrix} \cdot \left(\begin{pmatrix}?\\1\\1\end{pmatrix} \wedge \begin{pmatrix}3\\-1\\-5\end{pmatrix}\right) = \left|\begin{pmatrix}1\\1\\5\end{pmatrix} \cdot \begin{pmatrix}-4\\13\\-5\end{pmatrix}\right| = 16$$

$$\rightarrow V = 6 \times 16 = \boxed{96 \text{ units}^3}$$

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$$(b-a) \wedge (c-a)$$

$$= b \wedge c + b \wedge -a + -a \wedge c + -a \wedge -a$$

$$= b \wedge c + a \wedge c + c \wedge a + 0$$

$$= \underline{a \wedge b + b \wedge c + c \wedge a}$$

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$$l_1 = \lambda \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$l_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + m \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

Find perp to both:

$$n = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \wedge \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 1 & 2 & 3 \end{vmatrix} = \underline{\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}}, \quad \hat{n} = \frac{1}{\sqrt{6}} \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$

Find distance:

$$\overrightarrow{\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Project onto $\hat{n}$ $\rightarrow$ $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \cdot \frac{1}{\sqrt{6}} \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{6}} = \boxed{\frac{\sqrt{6}}{6}} = d$

b

Vectors forming plane:

① $A = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$

② $B = \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix}$

$$A \wedge B = n = \begin{vmatrix} i & j & k \\ 0 & 1 & 1 \\ -1 & 2 & 0 \end{vmatrix} = \begin{pmatrix} -2 \\ -1 \\ 1 \end{pmatrix}$$

→ Line:

$$L_1 = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + m \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + r \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix}$$

→ ① $2 - 2\lambda = 1 - r$

② $1 - \lambda = 1 + m - 2r$

③ $\lambda = 1 + m$

→ $-2\lambda + 0m + r = -1$

$-\lambda - m + 2r = 0$

$\lambda - m + 0r = 1$

Solver:

$\lambda = \frac{1}{2}, m = -\frac{1}{2}, r = 0$

→ Length = [illegible]

$= \frac{1}{2}$ units

$\text{Point} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + -\frac{1}{2} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + 0$

$= \begin{pmatrix} 1 \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}$

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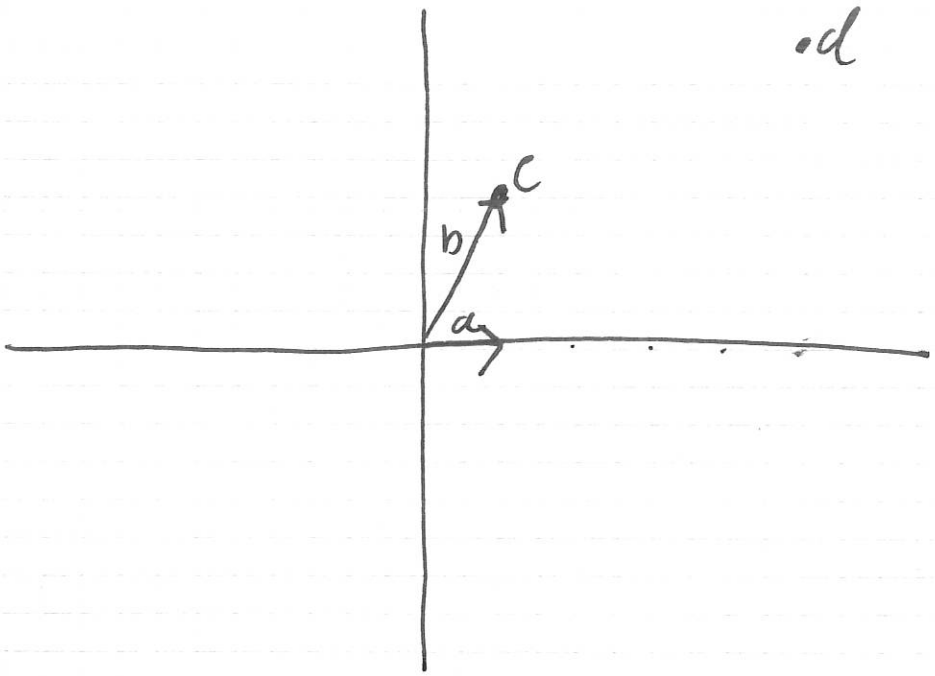
a)



b) $c = 0a + 1b$

$d = 3a + 2b$

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a) 3 vectors form a basis if none are parallel?

Unsure how to show

b) Unsure how to do with S.T.D