Main Questions Part B

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a) $A \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0 \Rightarrow 3x + 2y + z = 0$

$B \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0 \Rightarrow x + y = 0$

$\Rightarrow x = -y$

$\Rightarrow 3x - 2x + z = 0$

$x + z = 0$

$\Rightarrow x = -y = -z$

$\Rightarrow$ E.g, $n = \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$

$\Rightarrow \boxed{\hat{n} = \begin{pmatrix} \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{3}} \end{pmatrix} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}}$

b) $\left(r - \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}\right) \cdot \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$

c) Line from origin to plane $\rightarrow$ $r = \lambda \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} =$

~~r = ...~~ ~~r = ...~~

$$\rightarrow \begin{pmatrix} \lambda - 1 \\ -\lambda - 1 \\ -\lambda - 1 \end{pmatrix} \cdot \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix} = 0$$

$$\frac{\lambda - 1}{\sqrt{3}} + \frac{\lambda + 1}{\sqrt{3}} + \frac{\lambda + 1}{\sqrt{3}} = 0$$

$$3\lambda + 1 = 0$$

$$\lambda = -\frac{1}{3}$$

$$\rightarrow \text{Distance} = \left| -\frac{1}{3} \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix} \right|$$

$$= \frac{\sqrt{3}}{3}$$

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a) $\cancel{\frac{t}{|\hat{n}|}}$

$$\frac{|\hat{n} \cdot p - d|}{|\hat{n}|} \; ? \quad \Rightarrow n \quad \boxed{|\hat{n} \cdot p - d|}$$

b) $n = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$, $d = 6$

$$\rightarrow S: \; \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} \Big/ \sqrt{3^2 + (-1)^2 + 2^2} - 6 = \boxed{\cancel{\frac{5\sqrt{14}}{7}}}$$

$$\left|\frac{-42+5\sqrt{14}}{7}\right| \approx \boxed{3.33}$$

T: $$\left|\frac{\begin{pmatrix}3\\-1\\2\end{pmatrix}\cdot\begin{pmatrix}1\\0\\-1\end{pmatrix}}{\sqrt{14}}\right| - 6$$

$$= \left|\frac{-84+\sqrt{14}}{14}\right| \approx \underline{5.73}$$

c) Yes, because the ~~expression is negative for bo~~ formula gives a negative value for both.

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a) $(r-a)\cdot b = 0$

$\rightarrow$ b is normal

$\rightarrow$ $\cancel{\frac{b}{|b|}}$ $= \frac{b}{|b|}$

b) $\cancel{\frac{|N\cdot(c-a)|}{|\hat{n}|}}$ $\left|\frac{b}{|b|}\cdot(c-a)\right|$

c) $\boxed{\frac{b}{|b|}\cdot(c-a)}$

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a) Both planes go through $\begin{pmatrix}0\\0\\0\end{pmatrix}$

$\rightarrow$ $\cancel{\Pi_1 : r\cdot\begin{pmatrix}1\\2\\3\end{pmatrix} = 0}$

$\cancel{\Pi_2 : r\cdot\begin{pmatrix}3\\2\\1\end{pmatrix} = 0}$

and:

$x + 2y + 3z = 3x + 2y + z$

$2x - 2z = 0$

$\boxed{x = z}$ for all points r that lie on both planes

b) Find another point:

Let $x = z = 1$

$\Rightarrow 1 + 2y + 3 = 0$

$2y = -4$

$\boxed{y = -2}$

$\Rightarrow$ Equation of line:

$$r = \lambda \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$

$\Rightarrow$ Angle between y-axis

$$\theta = \cos^{-1}\left(\frac{\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}}{\left|\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}\right| \times \left|\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}\right|} \right)$$

$$= \cos^{-1}\left(\frac{-2}{\sqrt{6}} \right) = \boxed{\cos^{-1}\left(-\sqrt{2/3} \right)}$$

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Cube at e.g

$$\begin{pmatrix}0\\0\\0\end{pmatrix}, \begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\1\\0\end{pmatrix}, \begin{pmatrix}1\\1\\0\end{pmatrix}, \begin{pmatrix}0\\0\\1\end{pmatrix}, \begin{pmatrix}1\\0\\1\end{pmatrix}, \begin{pmatrix}0\\1\\1\end{pmatrix}, \begin{pmatrix}1\\1\\1\end{pmatrix}$$

→ Line 1:

$$= \begin{pmatrix}1\\1\\1\end{pmatrix} = l_1$$

Line 2:

$$= \begin{pmatrix}1\\1\\-1\end{pmatrix} = l_2$$

$$\to \theta = \cos^{-1}\left(\frac{l_1 \cdot l_2}{|l_1| \times |l_2|}\right) = 70.5^\circ$$

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a) $\cancel{AB \cdot BC = 0}$

$\cancel{BD \cdot AC = 0}$

$\cancel{AB = AC + CB}$

$\cancel{CD = CA + AD}$

$CD = (d - c)$

$AB = (b - a)$

$OA = a,\ OB = b,\ OC = c,\ OD = d$

$\to$ ① $(d-a)\cdot(c-b) = 0$

② $(d-b)\cdot(c-a) = 0$

$\to (d-a)\cdot(c-b) - (d-b)\cdot(c-a) = 0$

$\to d\cdot c - d\cdot b - a\cdot c + a\cdot b$

$- (d\cdot c - d\cdot a - b\cdot c + a\cdot b) = 0$

$$\Rightarrow -(d\cdot b) - (a\cdot c) + (d\cdot a) + (b\cdot c) = 0$$

$$\Leftrightarrow (d-c)\cdot(a-b) = 0 \quad \underline{\text{QED}}$$

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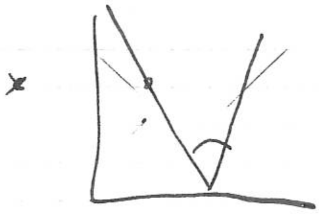

a) Sphere

b) Plane

c) Cone

c) ???