Main Questions Part 1e

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1

a) $\tanh(in)$

$= \dfrac{e^{in} - e^{-in}}{e^{in} + e^{-in}}$

$= \dfrac{e^{ix} - e^{-ix}}{\cos x} \cdot \dfrac{\cos x}{e^{ix} - e^{-ix}}$

$= \dfrac{2i \sin x}{2 \cos x} = \underline{i \tan x}$

b) $\operatorname{sech}(ix)$

$= \dfrac{2}{2(e^{ix} - e^{-ix})}$

$= \dfrac{2}{2 \cos x} = \sec x$

c) $\frac{1}{4}(e^{2x} + 2 + e^{-2x}) - \frac{1}{4}(e^{2x} - 2 + e^{-2x})$

$$= 0 + \frac{1}{2} - -\frac{1}{2} + 0$$

$$= \underline{1}$$

d) $1 - \tanh^2 x$

$$= 1 - \left(\frac{e^{2x} - 2 + e^{-2x}}{e^{2x} + 2 + e^{-2x}} \right)$$

$$= \frac{e^{2x} + 2 + e^{-2x} - e^{2x} - e^{-2x} + 2}{e^{2x} + 2 + e^{-2x}}$$

$$= \frac{4}{e^{2x} + 2 + e^{-2x}} = \frac{2^2}{(e^x + e^{-x})^2} = \underline{\operatorname{sech}^2 x}$$

2

$$\cosh(x+y) = \frac{e^x e^y + e^{-x} e^{-y}}{2}$$

$$= \frac{e^x e^y + e^{-x} e^{-y} - e^x e^{-y} - e^{-x} e^y}{2}$$

$$= (e^x + e^{-x})(e^y + e^{-y})$$

$$= \frac{2e^{x}e^{y} + 2e^{-x}e^{-y} + e^{-x}e^{y} + e^{x}e^{-y} - e^{-x}e^{y} - e^{x}e^{-y}}{4}$$

$$= \frac{(e^{x}+e^{-x})(e^{y}+e^{-y}) + (e^{x}-e^{-x})(e^{y}-e^{-y})}{4}$$

$$= \cosh x \cosh y + \sinh y \sinh x$$

b) $\sinh(x+y)$

$$= \frac{e^{x}e^{y} - e^{-x}e^{-y}}{2}$$

$$= \frac{e^{x}e^{y} - e^{-x}e^{-y} - e^{-y}e^{x} + e^{-x}e^{y} + e^{-y}e^{x} - e^{-x}e^{y}}{4}$$

$$= (e^{x}-e^{-x})(e^{y}+e^{-y}) + (e^{x}+e^{-x})(e^{y}-e^{-y})$$

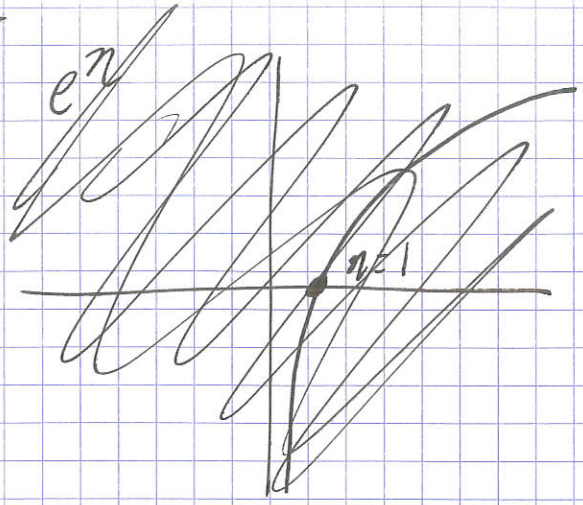
$$= \sinh x \cosh y + \cosh x \sinh y$$

c) $\tanh(x+y) = \dfrac{e^{x}e^{y} - e^{-x}e^{-y}}{e^{x}e^{y} + e^{-x}e^{-y}}$

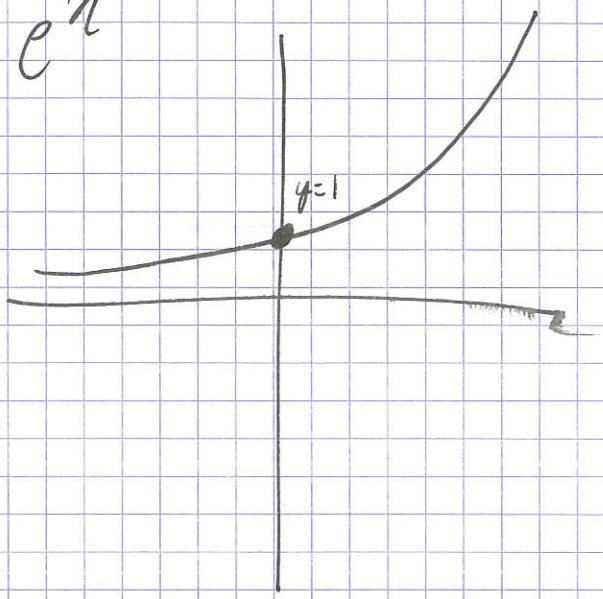
???

3.

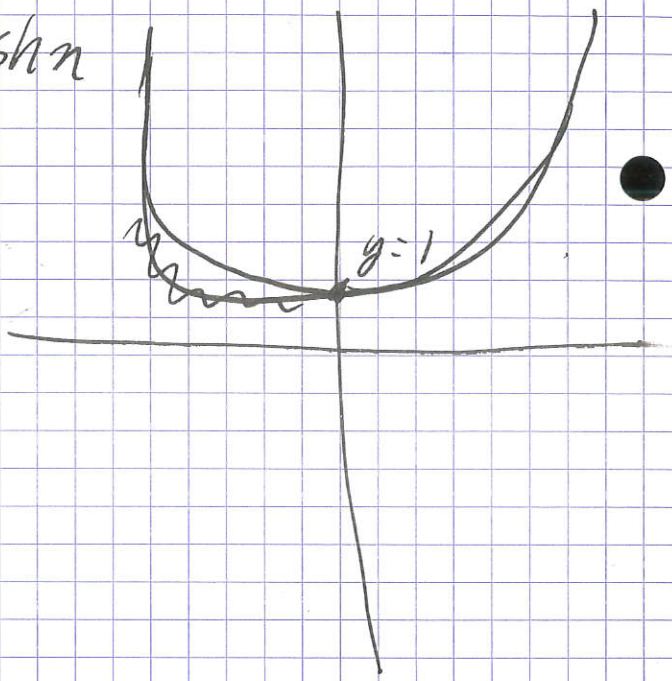
a) e^x



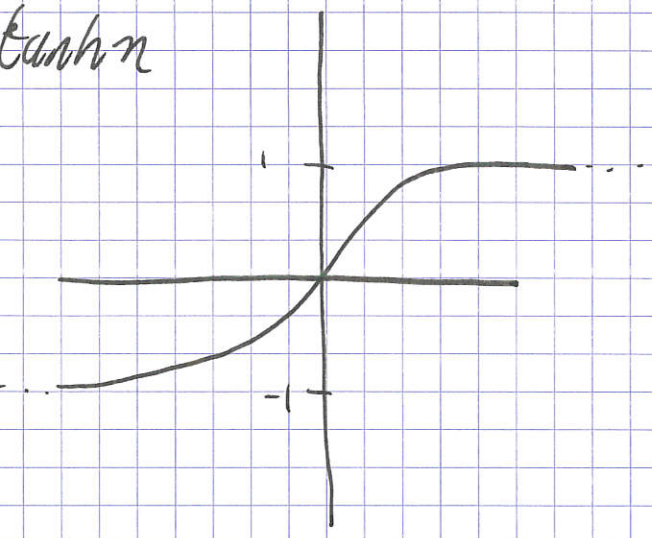
e^x



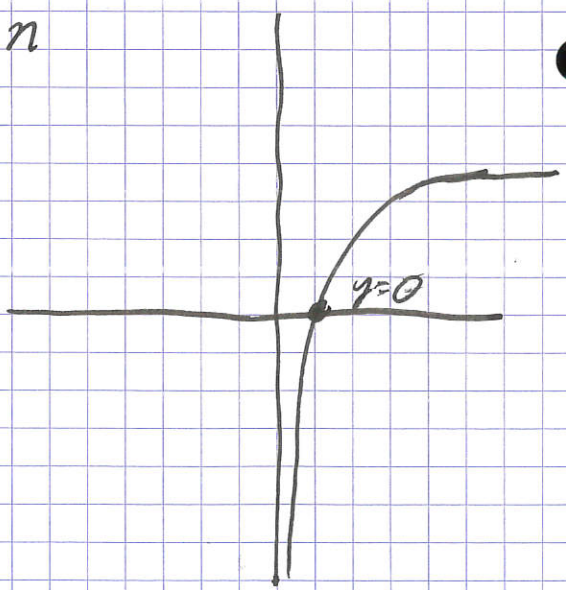
$\cosh x$



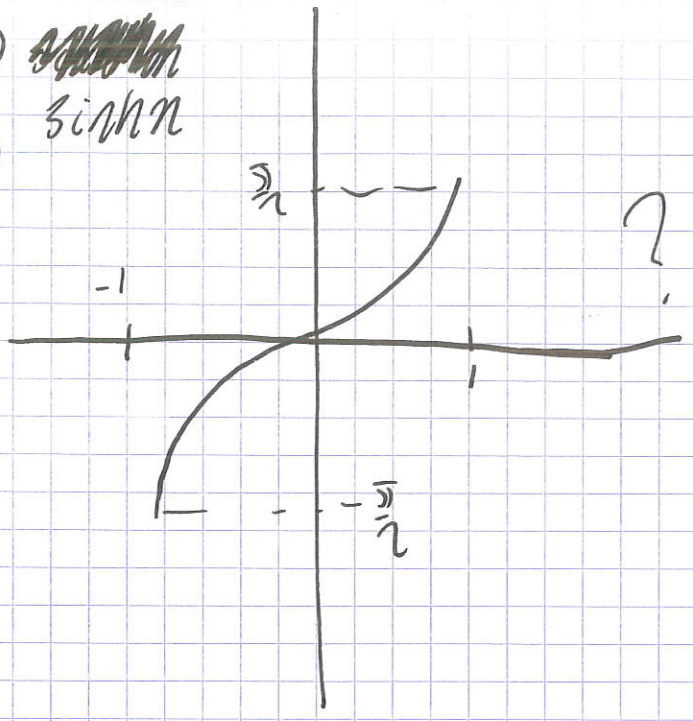
$\tanh x$



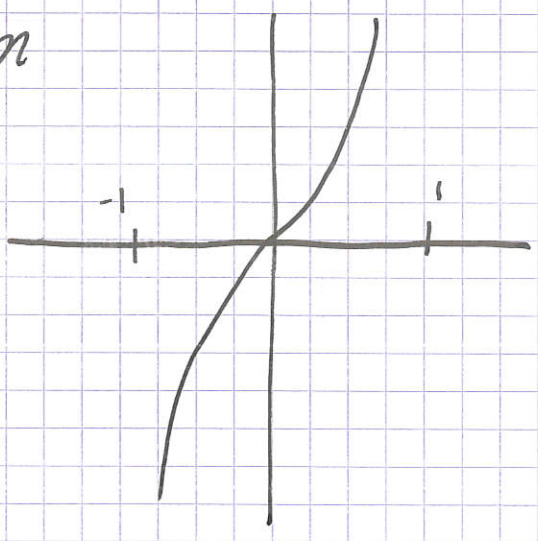
$\log x$



e) $\sinh n$

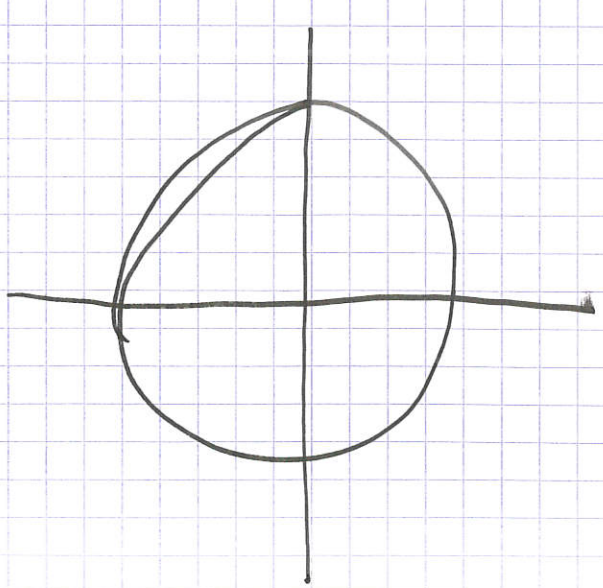


g) $\tanh^{-1} n$

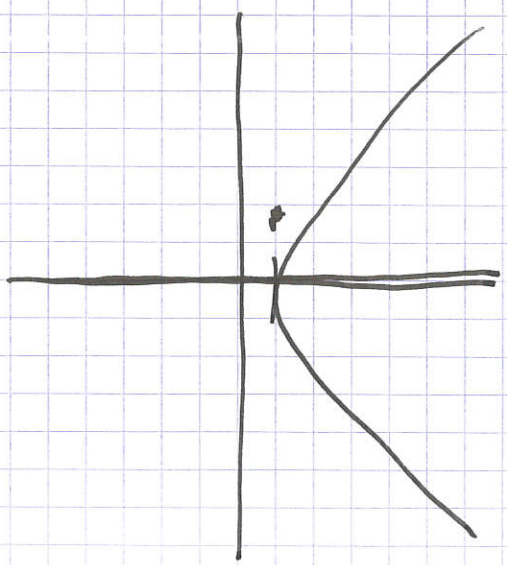


4

a)



b)



5

a) $y = \sinh x = \frac{e^x - e^{-x}}{2}$

$2y = e^x - e^{-x}$

$2ye^x = e^{2x} - 1$

$e^{2x} - 2ye^x - 1 = 0$

$e^x = \frac{2y \pm \sqrt{4y+4}}{2}$

$= \frac{2y \pm 2\sqrt{y+1}}{2}$

$= y \pm \sqrt{y+1}$

$\ln x = \ln(y \pm \sqrt{y+1})$

$\rightarrow \sinh^{-1} x = \ln(x + \sqrt{x+1})$

b) $y = \cosh^{-1} x = \frac{e^x + e^{-x}}{2}$

$2y = e^x + e^{-x}$

$2ye^x = e^{2x} + 1$

$e^{2x} - 2ye^x + 1 = 0$

$e^x = \frac{2y \pm \sqrt{4y-4}}{2}$

$\rightarrow e^x = y \pm \sqrt{y-1}$

$x = \ln y + \sqrt{y-1}$

$\rightarrow \cosh^{-1} x = \ln(x + \sqrt{x-1})$

c) ~~tanh⁻¹x~~

$$y = \tanh x = \frac{e^x - e^{-x}}{e^x + e^x}$$

$$ye^x + ye^{-x} = e^x - e^{-x}$$

$$e^x(y-1) + e^{-x}(y+1) = 0$$

$$e^{2x}(y-1) + (y+1) = 0$$

$$e^{2x} = \frac{-1-y}{y-1}$$

$$2x = \ln\left(\frac{y+1}{1-y}\right)$$

$$x = \frac{\ln\left(\frac{y+1}{1-y}\right)}{2}$$

$$\rightarrow \tanh^{-1} x = \frac{1}{2}\ln\left(\frac{y+1}{1-y}\right)$$