

Graphs

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Basics

Definitions

Graph	A collection of nodes and edges
Node (or vertex)	A point on a graph
Edge (or arc)	A line segment connecting two nodes
Weighted graph (or network)	A graph with weights associated with its edges
Directed graph (or digraph)	A graph where the edges only go one way
Subgraph	A subset of the nodes and vertices from the original graph
Degree (or valency, or order)	The number of edges connected to a given vertex
Walk	A route through a graph, connecting vertices along edges
Path	A walk in which no <i>vertex</i> is visited more than once
Trail	A walk in which no <i>edge</i> is visited more than once
Eulerian circuit	A trail which starts where it ends and traverses every edge
Cycle	A walk which ends where it started, and no other vertex is visited more than once
Hamiltonian cycle	A cycle that includes every vertex
Connected (vertices)	There is a path between the vertices
Connected (graph)	All the vertices in the graph are connected
Loop	An edge that starts and finishes at the same vertex
Simple graph	A graph where there are no loops and there is at most one edge connecting any pair of vertices
Tree	A connected graph with no cycles
Spanning tree (of graph \mathbf{G})	A subgraph of \mathbf{G} which contains all vertices and is a tree
Complete graph	A graph in which every vertex is directly connected by a single edge to all other vertices
Isomorphic graphs	Graphs which represent the same information but are drawn differently

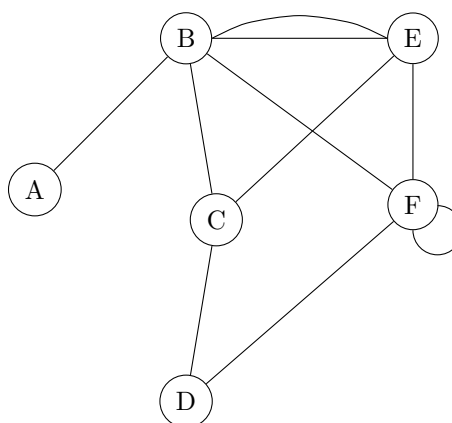
Matrices

A graph can be represented by an adjacency matrix, and a weighted graph can be represented by a distance matrix.

Adjacency matrices

Each entry in an adjacency matrix describes the number of edges joining the corresponding nodes.

	A	B	C	D	E	F
A	0	1	0	0	0	0
B	1	0	1	0	2	1
C	0	1	0	1	1	0
D	0	0	1	0	0	1
E	0	2	1	0	0	1
F	0	1	0	1	1	2



Distance matrices

Each entry in a distance matrix describes the weight of the edge joining the corresponding nodes, if any.

	A	B	C	D	E
A	–	17	18	–	–
B	17	–	15	19	23
C	18	15	–	20	–
D	–	19	20	–	16
E	–	23	–	16	–

