Stats

Contents

Probability Distributions

Binomial

The binomial distribution is used to model a situation with a fixed number of independent trials each with a constant probability of success.

You can model X as a binomial distribution if:

- There a fixed number of trials, n

- Each trial must succeed or fail

- There is a fixed probability of success, p

- Each trial is independent

If $X \sim \mathrm{B}(n, p)$, then
$$\boxed{\text{Formula Book} \quad P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}} \quad (0 \leq x \leq n)$$

Normal

The normal distribution $X \sim \mathrm{N}(\mu, \sigma^2)$ is symmetrical, meaning the mean and median are equal.

When doing questions that involve the normal distribution, sketching the bell curve on the right is always a good idea.

The standard normal distribution $Z \sim \mathrm{N}(0, 1^2)$ is very useful, since it allows you to find values for μ and σ when they're unknown. The normal random variable X can be coded using
$$\boxed{\text{Remember} \quad Z = \frac{X - \mu}{\sigma}}$$
and you can use this equation to find the unknown parameters of X.

The 'Normal CD' function on your calculator will calculate the area between an upper and lower bound on the bell curve. The 'Inverse Normal' function will find a value for which the area to the *left* of that value is the area you specify.
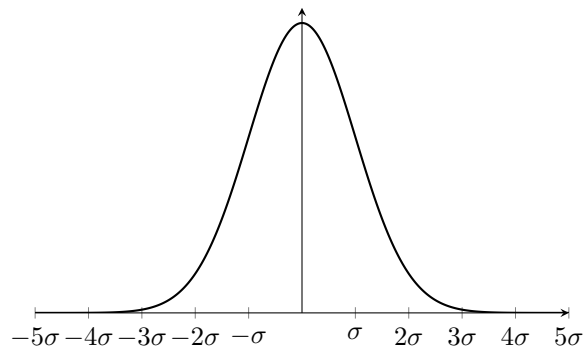
Hypothesis Testing

Every hypothesis test has two hypotheses:

H_0 : The null hypothesis - this is what you assume to be true by default

H_1 : The alternative hypothesis

The hypotheses are written in different forms depending on whether the test is one- or two-tailed.

<table>
<tr><td>One-tailed:</td><td>Two-tailed:</td></tr>
<tr><td>$H_0 : p = k$</td><td>$H_0 : p = k$</td></tr>
<tr><td>$H_1 : p \lessgtr k$</td><td>$H_1 : p \neq k$</td></tr>
</table>

If the question says that someone measured and got a value, then you plug that value into a probability calculation with the parameters from the null hypothesis, with the inequality sign in the same direction as the alternative hypothesis. If the probability of the event happening when assuming H_0 is less than the level of significance, then we reject H_0 and accept the alternative hypothesis.

A critical value is the smallest or largest value (depending on the direction of the inequality) obtained by a random variable such that H_0 would be rejected. Finding a critical value is often best done with the tables in the back of the book.
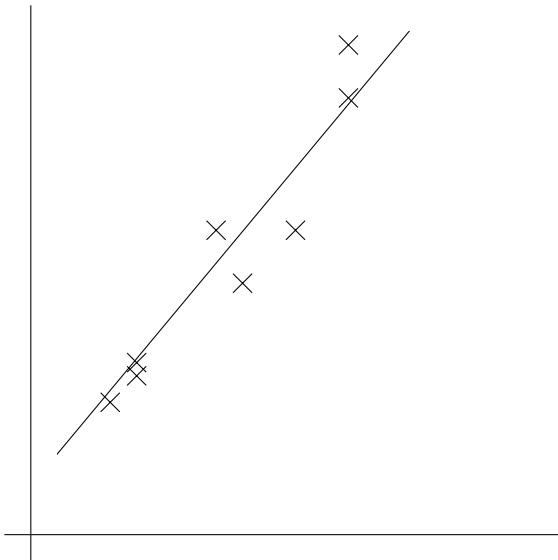
Correlation

Figure 1: A strong positive correlation

The **product moment correlation coefficient** or "Pearson's correlation coefficient" is a measure of correlation between two variables. It is often called r and is measured in the range $[-1, 1]$. $r = \pm 1$ means the data perfectly follows a positive or negative correlation respectively.

Hypothesis testing

Use $H_0 : \rho = 0$ and $H_1 : \rho \lessgtr 0$ or $H_1 : \rho \neq 0$. Get r from your calculator (use section 6 and option 4 regression calc) and ρ from the table in the formula book by finding the sample size and significance level. Remember to halve the significance level if the test is two-tailed.

Conditional probability

$P(A)$

$P(A \cup B)$

$P(A \cap B)$

$P(A \cap B')$

$P(A' \cup B)$

$P\big((A \cap B') \cup (B \cap A')\big)$
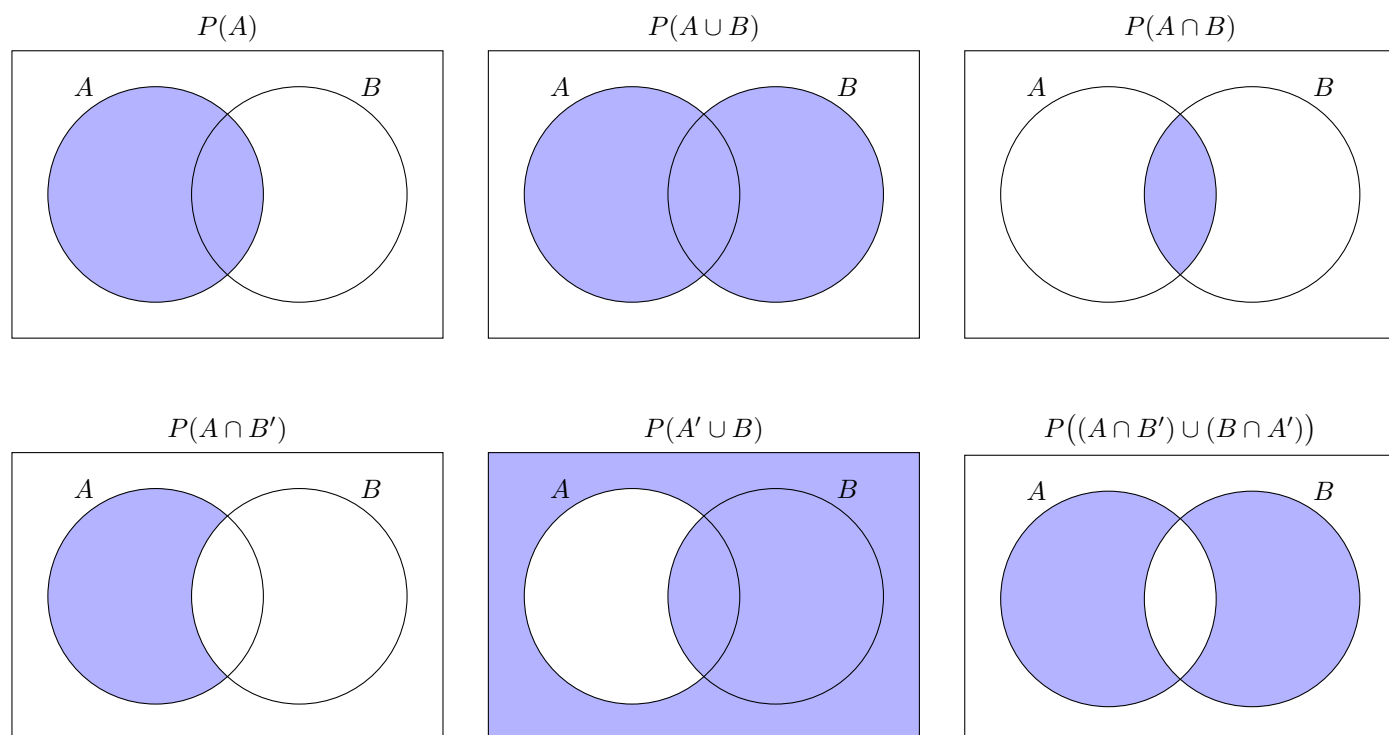
Remember

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

For <u>independent</u> events A and B, we know that $P(A \cap B) = P(A) \times P(B)$. This means that

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A) \times P(B)}{P(B)} = P(A)$$