Pure

Contents

Binomial Expansion

Definitions

The factorial $n! \equiv n \times (n-1) \times (n-2) \times \cdots \times 3 \times 2 \times 1.$

The falling factorial $n^{\underline{k}} \equiv n \times (n-1) \times (n-2) \times \cdots \times (n-(k-2)) \times (n-(k-1))$. It has k terms.

$$\boxed{0! = n^{\underline{0}} = 1}$$

The choose function ${}^nC_r \equiv \boxed{\binom{n}{r} \equiv \frac{n!}{r!(n-r)!}}$

Expansions

For a natural number n, the expansion of $(a+b)^n$ is

$$a^n + na^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots \binom{n}{r}a^{n-r}b^r + \cdots b^n$$

In general, $\boxed{(a+b)^n = \sum_{r=0}^{n} \binom{n}{r} a^{n-r} b^r}$ $(n \in \mathbb{N})$

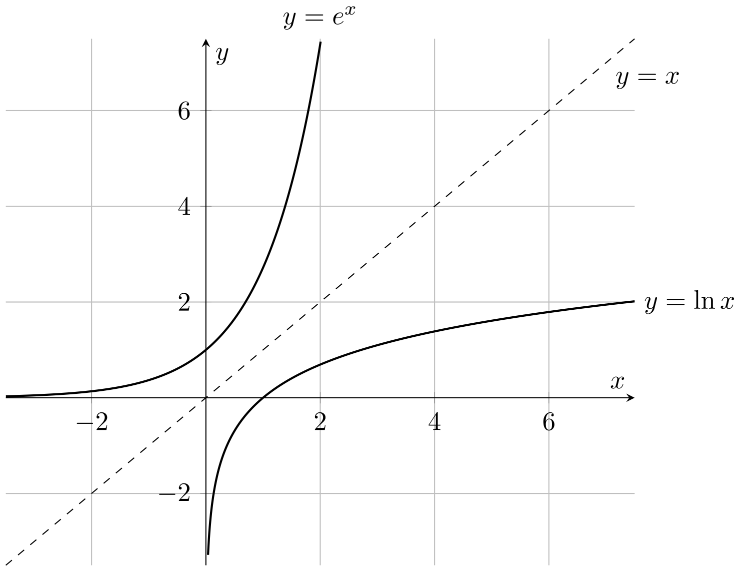
That's true if n is a natural number, but there is a version that works for all real numbers. For an expression $(a+bx)^n$, it should first be normalised to $a^n(1+\frac{b}{a}x)^n$. Let $y = \frac{b}{a}x$. Then the expansion of $(1+y)^n$ is given by

$$1 + ny + \frac{n(n-1)}{2!}y^2 + \frac{n(n-1)(n-2)}{3!}y^3 + \cdots + \frac{n(n-1)(n-2)\cdots(n-(r-1))}{r!}y^r + \cdots$$

In general, $\boxed{(a+bx)^n = a^n \sum_{r=0}^{\infty} \frac{n^{\underline{r}}}{r!} \left(\frac{b}{a}x\right)^r}$ $(n \in \mathbb{R})$

Exponentials and Logarithms

Exponentials



$$\frac{\mathrm{d}}{\mathrm{d}x} e^x = e^x$$

$$\frac{\mathrm{d}}{\mathrm{d}x} e^{kx} = k e^{kx}$$

$$\frac{\mathrm{d}}{\mathrm{d}x} e^{f(x)} = f'(x) e^{f(x)}$$

$\ln x$ is the inverse of e^x, meaning its graph is reflected in the line $y = x$.

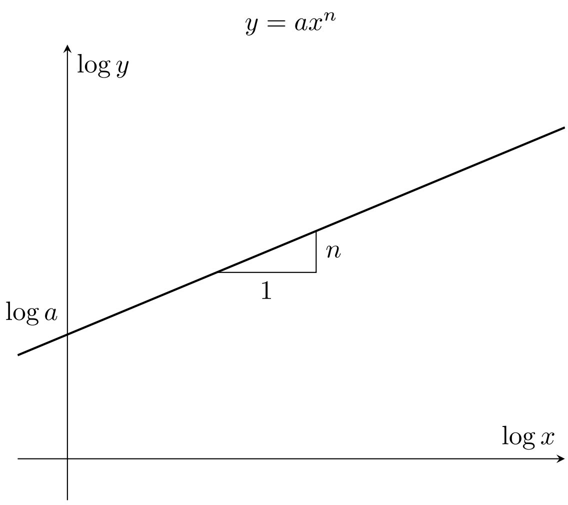
Log Laws

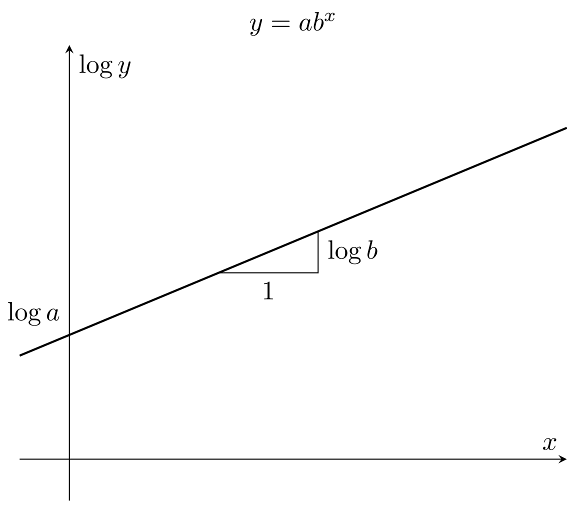
$$\log_a b \equiv \frac{\ln b}{\ln a}$$

$$\log xy \equiv \log x + \log y \qquad \log \frac{x}{y} \equiv \log x - \log y \qquad \log x^y \equiv y \log x$$

$$\log_a a \equiv 1 \qquad \log 1 \equiv 0 \qquad \log \frac{1}{x} \equiv -\log x$$

Log Plots





Sequences and Series

Arithmetic

a is the first term. d is the common difference. Counting starts from $n = 1$.

The nth term of the sequence is given by:

$$\boxed{u_n = a + (n-1)d}$$

The sum of the first n terms (inclusive) of the series is given by:

$$\boxed{S_n = \frac{n}{2}(2a + (n-1)d)}$$

Or $\boxed{S_n = \frac{n}{2}(a + l)}$ where l is the last term.

Geometric

a is the first term. r is the common ratio. Counting starts from $n = 1$.

The nth term of the sequence is given by:

$$\boxed{u_n = ar^{n-1}}$$

The sum of the first n terms (inclusive) of the series is given by:

$$\boxed{S_n = \frac{a(1-r^n)}{1-r} = \frac{a(r^n-1)}{r-1}} \ (r \neq 1)$$

The sum to infinity of a geometric series is **only valid** when $|r| < 1$ and is given by:

$$\boxed{S_\infty = \frac{a}{1-r}} \ (|r| < 1)$$