

Maths Crib Sheets

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Binomial Expansion

Definitions

The factorial $n! \equiv n \times (n-1) \times (n-2) \times \cdots \times 3 \times 2 \times 1$.

The falling factorial $n^{\underline{k}} \equiv n \times (n-1) \times (n-2) \times \cdots \times (n-(k-2)) \times (n-(k-1))$. It has k terms.

$$0! = n^{\underline{0}} = 1$$

$$\text{The choose function } {}^nC_r \equiv \binom{n}{r} \equiv \frac{n!}{r!(n-r)!}$$

Expansions

For a natural number n , the expansion of $(a+b)^n$ is

$$a^n + na^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + b^n$$

$$\text{In general, } (a+b)^n = \sum_{r=0}^n \binom{n}{r} a^{n-r} b^r \quad (n \in \mathbb{N})$$

That's true if n is a natural number, but there is a version that works for all real numbers. For an expression $(a+bx)^n$, it should first be normalised to $a^n(1+\frac{b}{a}x)^n$. Let $y = \frac{b}{a}x$. Then the expansion of $(1+y)^n$ is given by

$$1 + ny + \frac{n(n-1)}{2!}y^2 + \frac{n(n-1)(n-2)}{3!}y^3 + \cdots + \frac{n(n-1)(n-2)\cdots(n-(r-1))}{r!}y^r + \cdots$$

$$\text{In general, } (a+bx)^n = a^n \sum_{r=0}^{\infty} \frac{n^{\underline{r}}}{r!} \left(\frac{b}{a}x\right)^r \quad (n \in \mathbb{R})$$

Calculus

Elementary Derivatives

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

f	C	x^n	$\sin x$	$\cos x$	a^x	$\ln x$
f'	0	nx^{n-1}	$\cos x$	$-\sin x$	$a^x \ln a$	$\frac{1}{x}$

Composition Laws

Let f and g be differentiable functions over x .

The $'$ mark denotes the derivative with respect to x , so $f' = \frac{df}{dx}$ and $g' = \frac{dg}{dx}$.

The \circ symbol denotes function composition, so $(f \circ g)(x) = f(g(x))$.

$$(f \pm g)' = f' \pm g'$$

$$(fg)' = fg' + f'g$$

$$(f \circ g)' = (f' \circ g)g'$$

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

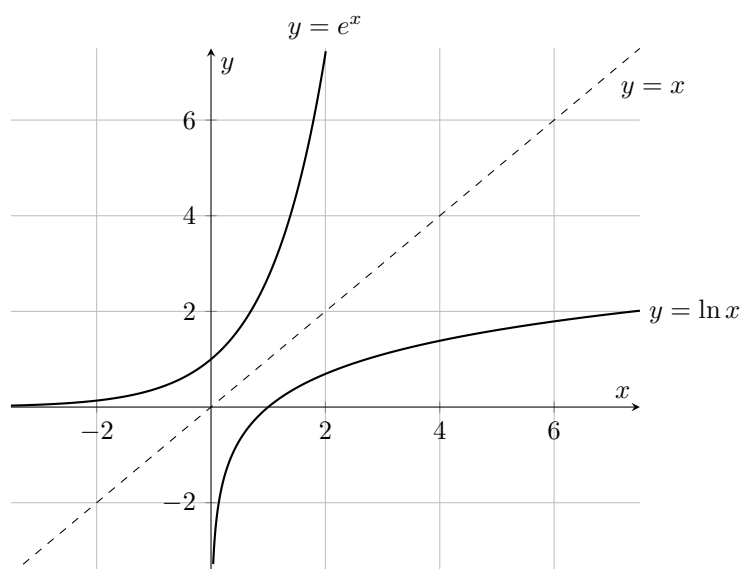
Integral Tricks

For integrals of the form on the left, consider the function on the right.

$\int k f' f^n dx$	f^{n+1}
$\int k \frac{f'}{f} dx$	$\ln f $

Exponentials and Logarithms

Exponentials



$$\frac{d}{dx}e^x = e^x$$

$$\frac{d}{dx}e^{kx} = ke^{kx}$$

$$\frac{d}{dx}e^{f(x)} = f'(x)e^{f(x)}$$

$\ln x$ is the inverse of e^x , meaning its graph is reflected in the line $y = x$.

Log Laws

$$\log_a b \equiv \frac{\ln b}{\ln a}$$

$$\log xy \equiv \log x + \log y$$

$$\log \frac{x}{y} \equiv \log x - \log y$$

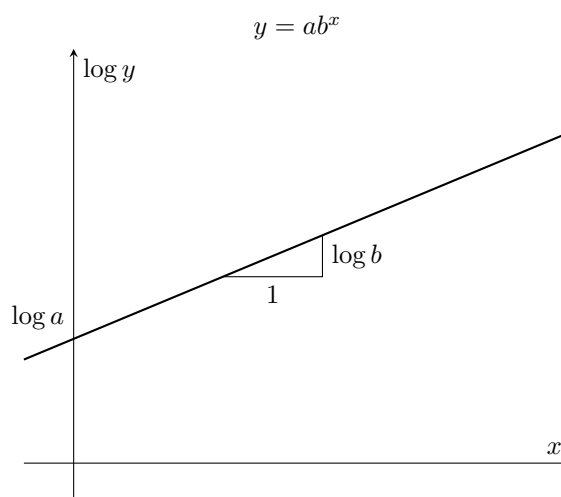
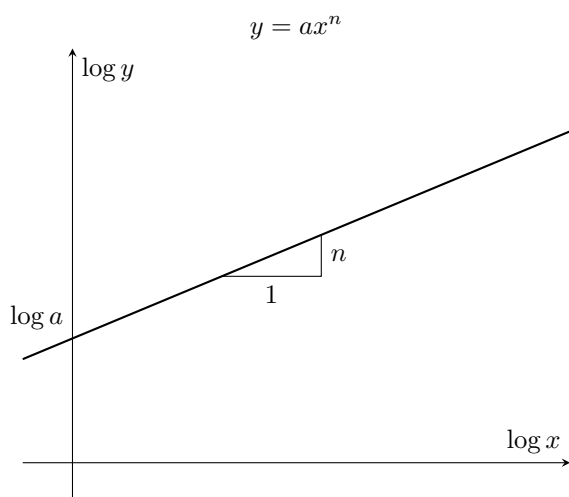
$$\log x^y \equiv y \log x$$

$$\log_a a \equiv 1$$

$$\log 1 \equiv 0$$

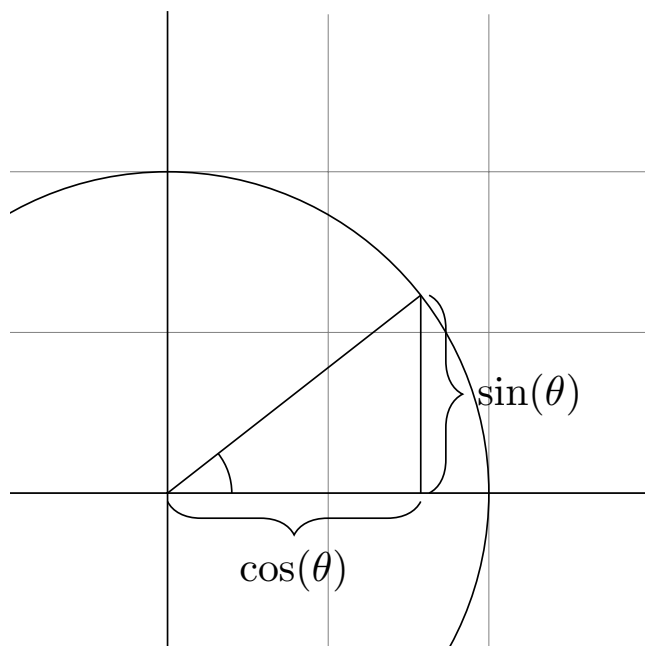
$$\log \frac{1}{x} \equiv -\log x$$

Log Plots



Trigonometry

Definitions



$$\tan \theta \equiv \frac{\sin \theta}{\cos \theta}$$

$$\sec \theta \equiv \frac{1}{\cos \theta}$$

$$\operatorname{cosec} \theta \equiv \frac{1}{\sin \theta}$$

$$\cot \theta \equiv \frac{1}{\tan \theta} \equiv \frac{\cos \theta}{\sin \theta}$$

Identities

$$\sin^2 \theta + \cos^2 \theta \equiv 1$$

$$1 + \tan^2 \theta \equiv \sec^2 \theta$$

$$1 + \cot^2 \theta \equiv \operatorname{cosec}^2 \theta$$

$$\sin(\alpha + \beta) \equiv \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) \equiv \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) \equiv \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos(\alpha - \beta) \equiv \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\tan(\alpha + \beta) \equiv \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\tan(\alpha - \beta) \equiv \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

$$\sin 2\theta \equiv 2 \sin \theta \cos \theta$$

$$\cos 2\theta \equiv \cos^2 \theta - \sin^2 \theta$$

$$\tan 2\theta \equiv \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

$$\equiv 2 \cos^2 \theta - 1$$

$$\equiv 1 - 2 \sin^2 \theta$$

Calculus

$$\frac{d}{dx} \sin x = \cos x$$

$$\int \sin x \, dx = -\cos x + C$$

$$\frac{d}{dx} \cos x = -\sin x$$

$$\int \cos x \, dx = \sin x + C$$

$$\frac{d}{dx} \tan x = \sec^2 x$$

$$\int \tan x \, dx = \ln |\sec x| + C$$

$$\frac{d}{dx} \sec x = \sec x \tan x$$

$$\int \sec x \, dx = \ln |\tan x + \sec x| + C$$

$$\frac{d}{dx} \operatorname{cosec} x = -\operatorname{cosec} x \cot x$$

$$\int \operatorname{cosec} x \, dx = -\ln |\cot x + \operatorname{cosec} x| + C$$

$$\frac{d}{dx} \cot x = -\operatorname{cosec}^2 x$$

$$\int \cot x \, dx = \ln |\sin x| + C$$

$$\frac{d}{dx} \arcsin x = \frac{1}{\sqrt{1-x^2}}$$

$$\int \arcsin x \, dx = x \arcsin x + \sqrt{1-x^2} + C$$

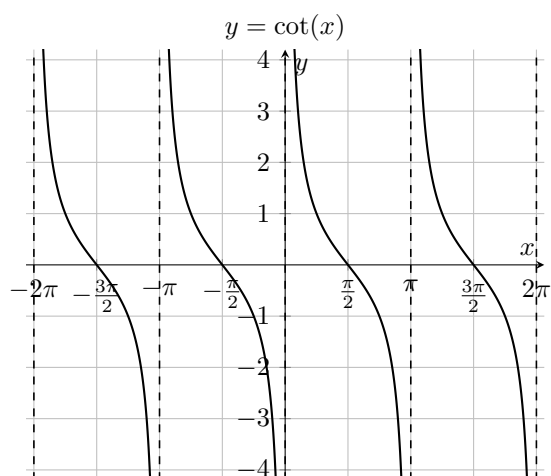
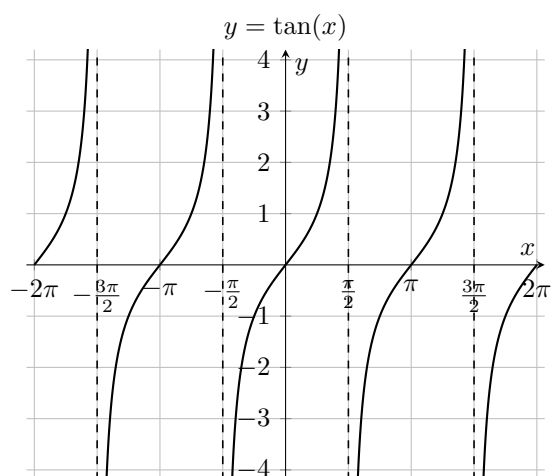
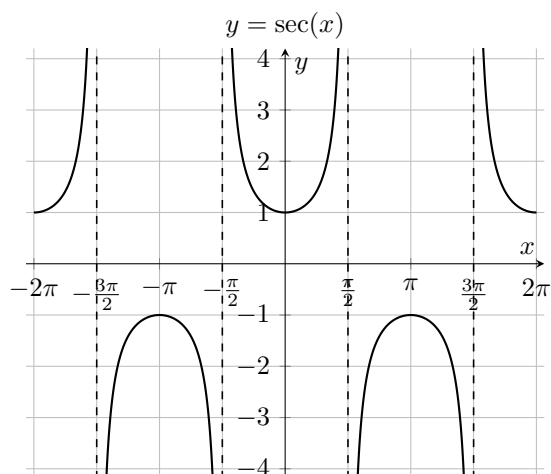
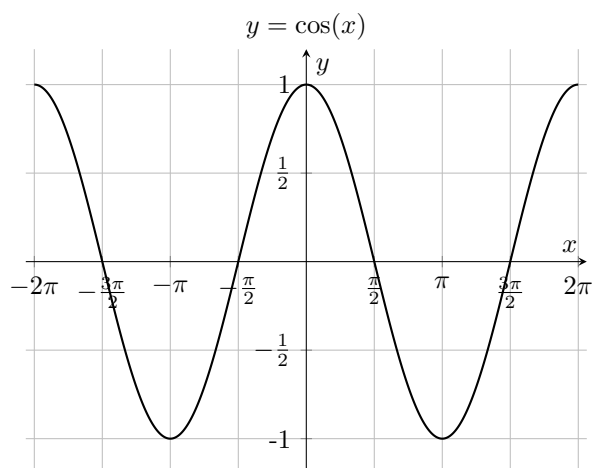
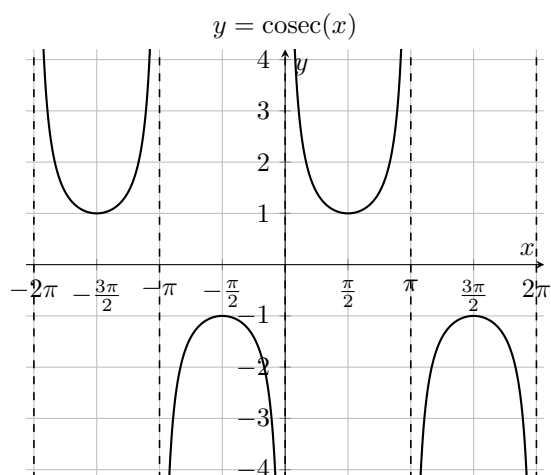
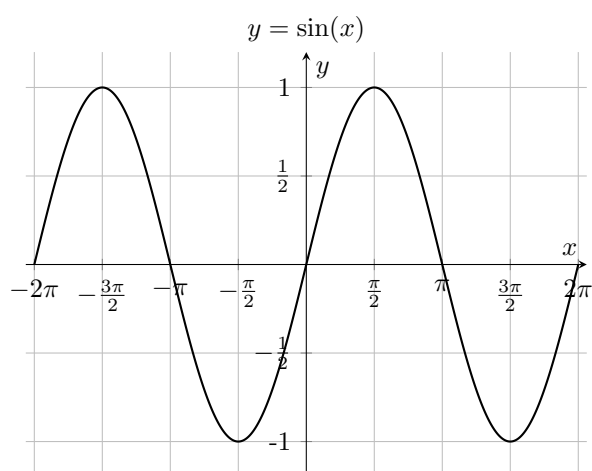
$$\frac{d}{dx} \arccos x = -\frac{1}{\sqrt{1-x^2}}$$

$$\int \arccos x \, dx = x \arccos x - \sqrt{1-x^2} + C$$

$$\frac{d}{dx} \arctan x = \frac{1}{x^2+1}$$

$$\int \arctan x \, dx = x \arctan x - \frac{\ln(x^2+1)}{2} + C$$

Graphs



Problem Solving Matters

General Tips

- Be lazy; only do necessary work
- Avoid long and/or complicated calculations
- Draw diagrams and make them big
- In diagrams, label things and add lines
- Look for similar shapes (often triangles)

Tips For Sketching Graphs

- Look for symmetries
- Think about periodicity
- Look for turning points (0 derivative)
- Look for asymptotes
- Try values of x like 0, 1, -1, etc.
- If there's a trig function involved, try multiples of π
- See what happens when x tends to 0 or $\pm\infty$

Things To Remember

- $\log_a b \times \log_b a = 1$
- $\log_{a^c} b^c = \log_a b$
- When graphing $y^2 = f(x)$, draw the positive branch and don't forget to reflect it in the x axis as well
- $\log x$ is negative when $0 < x < 1$