Complex Numbers

Contents

Basics

The imaginary number i is defined to satisfy

$$\boxed{i^2 \equiv -1}$$

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A complex number $a + bi$ is the sum of a real number and an imaginary number (which is a real multiple of i).

Complex numbers are added element-by-element, and multiplied by expanding brackets à la foil.

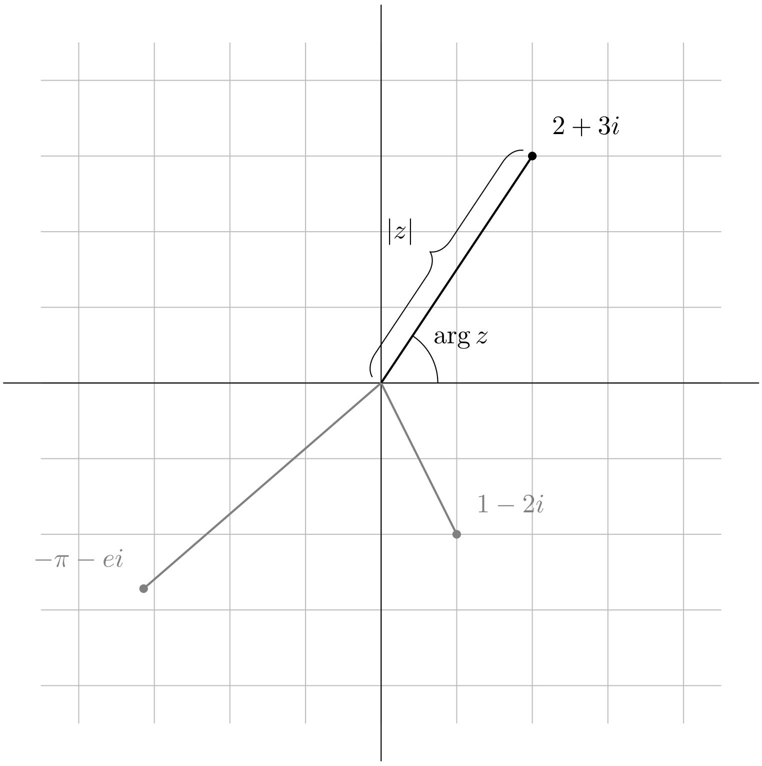
The *complex conjugate* for a complex number $z = a + bi$ is $z^* = a - bi$.

For a polynomial $f(x)$ with real coefficients, complex roots must occur in conjugate pairs.

Argand Diagrams

The Diagram

An Argand diagram is a way of representing complex numbers on a 2D plane. The horizontal axis is the real numbers, and the vertical axis is the imaginary numbers.



The modulus $|z|$ of a complex number z is the distance from the point to the origin. The argument $\arg z$ is the angle between the vector line and the positive real axis.

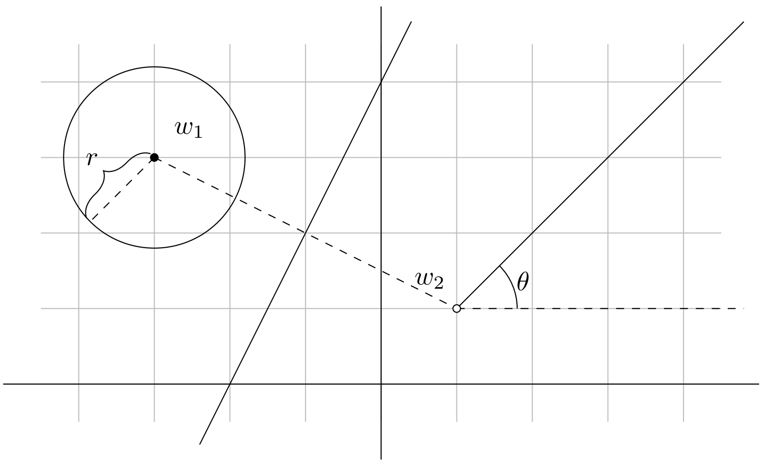
For a modulus $|z| = r$ and an argument $\arg z = \theta$, the modulus-argument form of z is $\boxed{r(\cos\theta + i\sin\theta)}$.

Multiplication is unaffected by the modulus, so $|zw| = |z||w|$ and $\left|\frac{z}{w}\right| = \frac{|z|}{|w|}$.

Multiplication is additive over arg, so $\arg(zw) = \arg z + \arg w$ and $\arg\left(\frac{z}{w}\right) = \arg z - \arg w$.

$|z - w|$ is the distance between z and w.

Loci



For a complex number $w_1 = x + yi$, the locus of points $|z - w_1| = r \Leftrightarrow |z - (x + yi)| = r$ is a circle of radius r around the point w.

$|z - w_1| = |z - w_2|$ is the perpendicular bisector of the line joining w_1 and w_2.

$\arg(z - w_2) = \theta$ is a half-line from, but not including, the point w making an angle θ from the real axis.