

# Pure

## Contents

Binomial Expansion . . . . .	2
Definitions . . . . .	2
Expansions . . . . .	2
Exponentials and Logarithms . . . . .	3
Exponentials . . . . .	3
Log Laws . . . . .	3
Log Plots . . . . .	3
Sequences and Series . . . . .	4
Arithmetic . . . . .	4
Geometric . . . . .	4

# Binomial Expansion

## Definitions

The factorial  $n! \equiv n \times (n-1) \times (n-2) \times \cdots \times 3 \times 2 \times 1$ .

The falling factorial  $n^{\underline{k}} \equiv n \times (n-1) \times (n-2) \times \cdots \times (n-(k-2)) \times (n-(k-1))$ . It has  $k$  terms.

$$0! = n^{\underline{0}} = 1$$

Formula Book

The choose function

$${}^nC_r \equiv \binom{n}{r} \equiv \frac{n!}{r!(n-r)!}$$

## Expansions

For a natural number  $n$ , the expansion of  $(a+b)^n$  is

Formula Book

$$a^n + na^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots \binom{n}{r}a^{n-r}b^r + \cdots b^n$$

In general,  $(a+b)^n = \sum_{r=0}^n \binom{n}{r} a^{n-r} b^r \quad (n \in \mathbb{N})$

That's true if  $n$  is a natural number, but there is a version that works for all real numbers. For an expression  $(a+bx)^n$ , it should first be normalised to  $a^n(1+\frac{b}{a}x)^n$ . Let  $y = \frac{b}{a}x$ . Then the expansion of  $(1+y)^n$  is given by

Formula Book

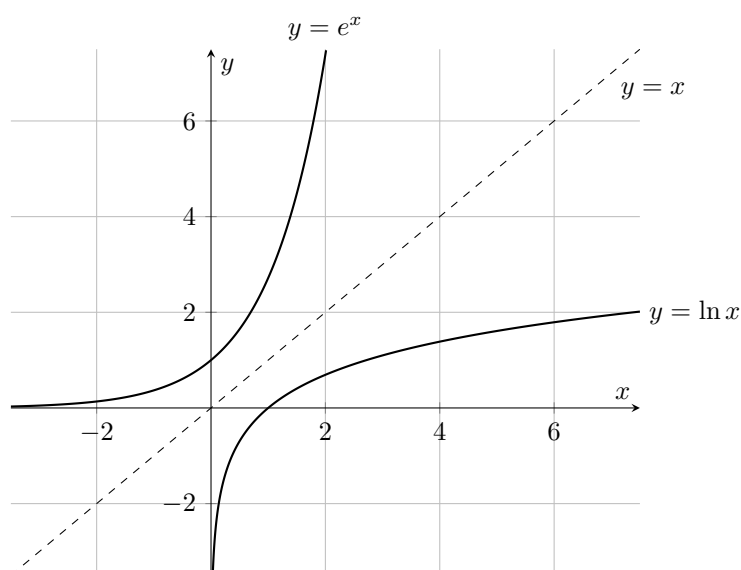
$$(1+y)^n \equiv 1 + ny + \frac{n(n-1)}{2!}y^2 + \frac{n(n-1)(n-2)}{3!}y^3 + \cdots + \frac{n(n-1)(n-2)\cdots(n-(r-1))}{r!}y^r + \cdots$$

Remember

In general,  $(a+bx)^n = a^n \sum_{r=0}^{\infty} \frac{n^{\underline{r}}}{r!} \left(\frac{b}{a}x\right)^r \quad (n \in \mathbb{R})$

# Exponentials and Logarithms

## Exponentials



### Remember

$$\frac{d}{dx} e^x = e^x$$

$$\frac{d}{dx} e^{kx} = k e^{kx}$$

$$\frac{d}{dx} e^{f(x)} = f'(x) e^{f(x)}$$

$\ln x$  is the inverse of  $e^x$ , meaning its graph is reflected in the line  $y = x$ .

## Log Laws

### Formula Book

$$\log_a b \equiv \frac{\ln b}{\ln a}$$

### Remember

$$\log xy \equiv \log x + \log y$$

$$\log \frac{x}{y} \equiv \log x - \log y$$

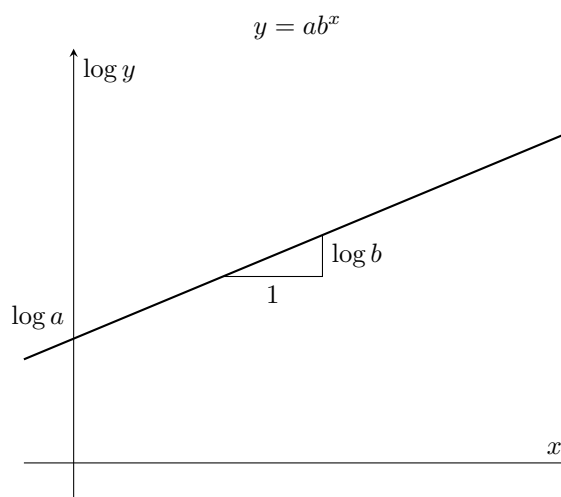
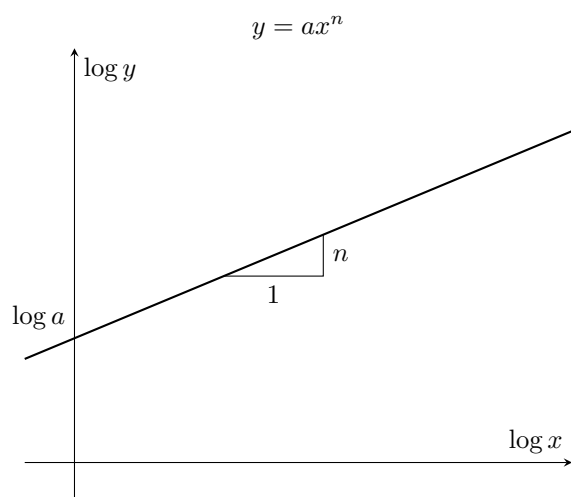
$$\log x^y \equiv y \log x$$

$$\log_a a \equiv 1$$

$$\log 1 \equiv 0$$

$$\log \frac{1}{x} \equiv -\log x$$

## Log Plots



# Sequences and Series

## Arithmetic

$a$  is the first term.  $d$  is the common difference. Counting starts from  $n = 1$ .

The  $n$ th term of the sequence is given by:

Remember

$$u_n = a + (n - 1)d$$

The sum of the first  $n$  terms (inclusive) of the series is given by:

Formula Book

$$S_n = \frac{n}{2}(2a + (n - 1)d)$$

Or 

Formula Book

$$S_n = \frac{n}{2}(a + l)$$

 where  $l$  is the last term.

## Geometric

$a$  is the first term.  $r$  is the common ratio. Counting starts from  $n = 1$ .

The  $n$ th term of the sequence is given by:

Remember

$$u_n = ar^{n-1}$$

The sum of the first  $n$  terms (inclusive) of the series is given by:

Formula Book

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1} \quad (r \neq 1)$$

The sum to infinity of a geometric series is **only valid** when  $|r| < 1$  and is given by:

Formula Book

$$S_\infty = \frac{a}{1 - r} \quad (|r| < 1)$$