

# Maths Crib Sheets

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# Binomial Expansion

## Definitions

The factorial  $n! \equiv n \times (n-1) \times (n-2) \times \cdots \times 3 \times 2 \times 1$ .

The falling factorial  $n^{\underline{k}} \equiv n \times (n-1) \times (n-2) \times \cdots \times (n-(k-2)) \times (n-(k-1))$ . It has  $k$  terms.

$$\boxed{0! = n^{\underline{0}} = 1}$$

$$\text{The choose function } {}^nC_r \equiv \boxed{\binom{n}{r} \equiv \frac{n!}{r!(n-r)!}}$$

## Expansions

For a natural number  $n$ , the expansion of  $(a+b)^n$  is

$$a^n + na^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + b^n$$

$$\text{In general, } \boxed{(a+b)^n = \sum_{r=0}^n \binom{n}{r} a^{n-r} b^r \quad (n \in \mathbb{N})}$$

That's true if  $n$  is a natural number, but there is a version that works for all real numbers. For an expression  $(a+bx)^n$ , it should first be normalised to  $a^n(1+\frac{b}{a}x)^n$ . Let  $y = \frac{b}{a}x$ . Then the expansion of  $(1+y)^n$  is given by

$$1 + ny + \frac{n(n-1)}{2!}y^2 + \frac{n(n-1)(n-2)}{3!}y^3 + \cdots + \frac{n(n-1)(n-2)\cdots(n-(r-1))}{r!}y^r + \cdots$$

$$\text{In general, } \boxed{(a+bx)^n = a^n \sum_{r=0}^{\infty} \frac{n^{\underline{r}}}{r!} \left(\frac{b}{a}x\right)^r \quad (n \in \mathbb{R})}$$

# Calculus

## Elementary Derivatives

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$f$	$C$	$x^n$	$\sin x$	$\cos x$	$a^x$	$\ln x$
$f'$	0	$nx^{n-1}$	$\cos x$	$-\sin x$	$a^x \ln a$	$\frac{1}{x}$

## Composition Laws

Let  $f$  and  $g$  be differentiable functions over  $x$ .

The  $'$  mark denotes the derivative with respect to  $x$ , so  $f' = \frac{df}{dx}$  and  $g' = \frac{dg}{dx}$ .

The  $\circ$  symbol denotes function composition, so  $(f \circ g)(x) = f(g(x))$ .

$$(f \pm g)' = f' \pm g'$$

$$(fg)' = fg' + f'g$$

$$(f \circ g)' = (f' \circ g)g'$$

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

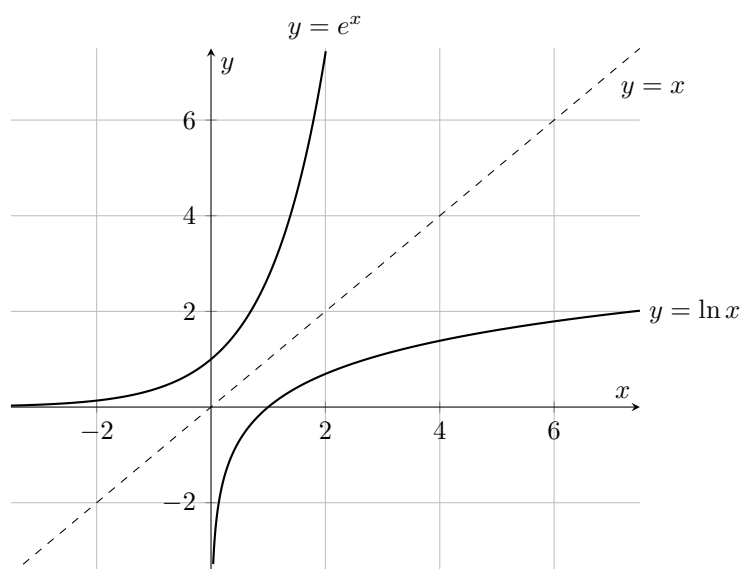
## Integral Tricks

For integrals of the form on the left, consider the function on the right.

$\int k f' f^n dx$	$f^{n+1}$
$\int k \frac{f'}{f} dx$	$\ln  f $

# Exponentials and Logarithms

## Exponentials



$$\frac{d}{dx}e^x = e^x$$

$$\frac{d}{dx}e^{kx} = ke^{kx}$$

$$\frac{d}{dx}e^{f(x)} = f'(x)e^{f(x)}$$

$\ln x$  is the inverse of  $e^x$ , meaning its graph is reflected in the line  $y = x$ .

## Log Laws

$$\log_a b \equiv \frac{\ln b}{\ln a}$$

$$\log xy \equiv \log x + \log y$$

$$\log \frac{x}{y} \equiv \log x - \log y$$

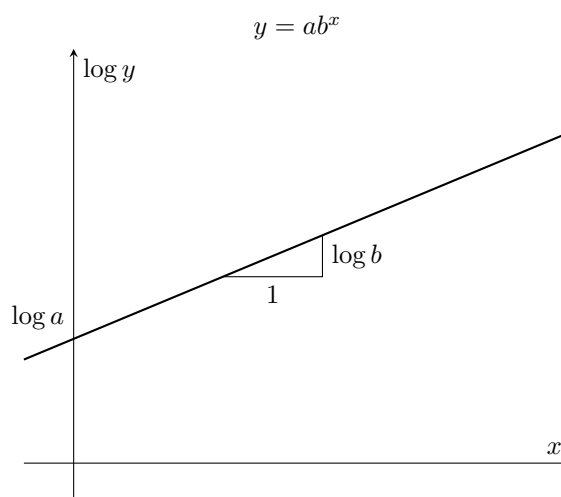
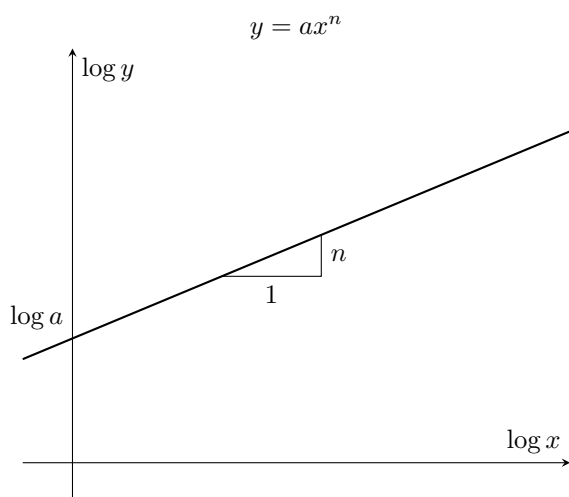
$$\log x^y \equiv y \log x$$

$$\log_a a \equiv 1$$

$$\log 1 \equiv 0$$

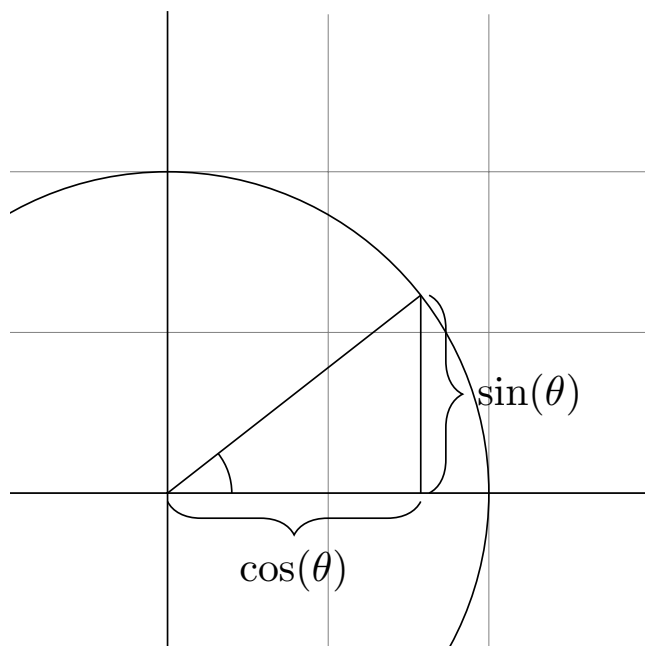
$$\log \frac{1}{x} \equiv -\log x$$

## Log Plots



# Trigonometry

## Definitions



$$\tan \theta \equiv \frac{\sin \theta}{\cos \theta}$$

$$\sec \theta \equiv \frac{1}{\cos \theta}$$

$$\operatorname{cosec} \theta \equiv \frac{1}{\sin \theta}$$

$$\cot \theta \equiv \frac{1}{\tan \theta} \equiv \frac{\cos \theta}{\sin \theta}$$

## Identities

$$\sin^2 \theta + \cos^2 \theta \equiv 1$$

$$1 + \tan^2 \theta \equiv \sec^2 \theta$$

$$1 + \cot^2 \theta \equiv \operatorname{cosec}^2 \theta$$

$$\sin(\alpha + \beta) \equiv \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) \equiv \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) \equiv \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos(\alpha - \beta) \equiv \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\tan(\alpha + \beta) \equiv \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\tan(\alpha - \beta) \equiv \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

$$\sin 2\theta \equiv 2 \sin \theta \cos \theta$$

$$\cos 2\theta \equiv \cos^2 \theta - \sin^2 \theta$$

$$\tan 2\theta \equiv \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

$$\equiv 2 \cos^2 \theta - 1$$

$$\equiv 1 - 2 \sin^2 \theta$$

## Calculus

$$\frac{d}{dx} \sin x = \cos x$$

$$\int \sin x \, dx = -\cos x + C$$

$$\frac{d}{dx} \cos x = -\sin x$$

$$\int \cos x \, dx = \sin x + C$$

$$\frac{d}{dx} \tan x = \sec^2 x$$

$$\int \tan x \, dx = \ln |\sec x| + C$$

$$\frac{d}{dx} \sec x = \sec x \tan x$$

$$\int \sec x \, dx = \ln |\tan x + \sec x| + C$$

$$\frac{d}{dx} \operatorname{cosec} x = -\operatorname{cosec} x \cot x$$

$$\int \operatorname{cosec} x \, dx = -\ln |\cot x + \operatorname{cosec} x| + C$$

$$\frac{d}{dx} \cot x = -\operatorname{cosec}^2 x$$

$$\int \cot x \, dx = \ln |\sin x| + C$$

$$\frac{d}{dx} \arcsin x = \frac{1}{\sqrt{1-x^2}}$$

$$\int \arcsin x \, dx = x \arcsin x + \sqrt{1-x^2} + C$$

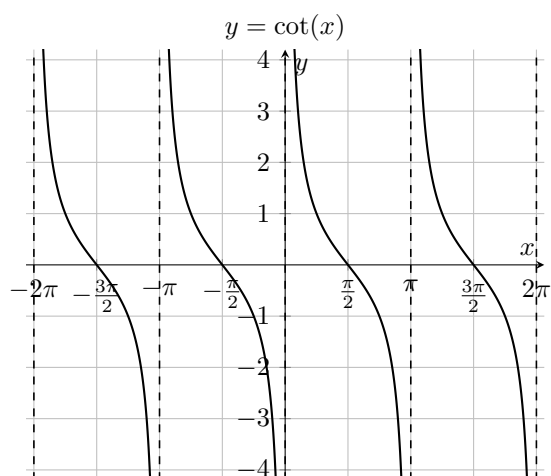
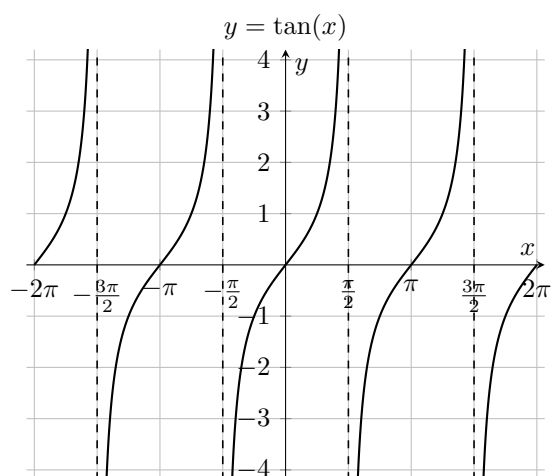
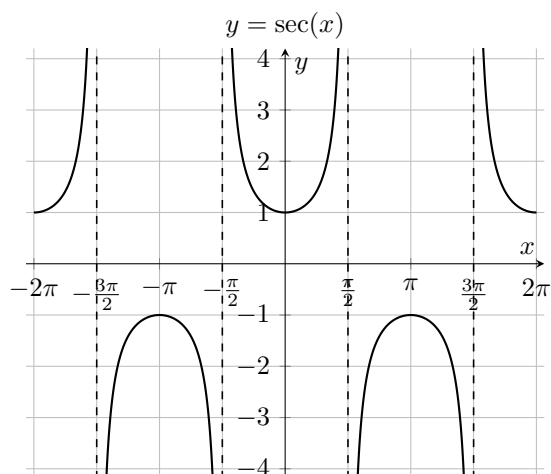
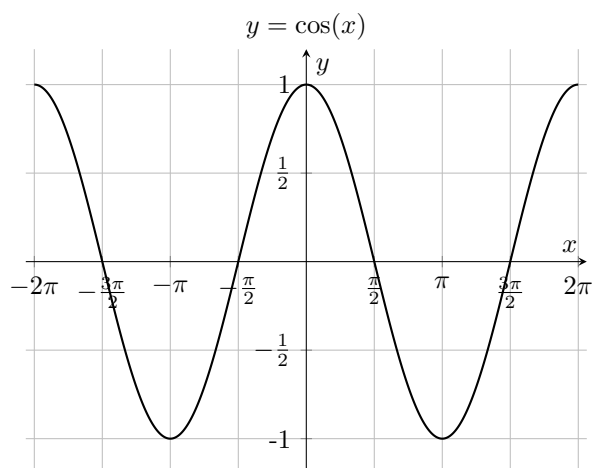
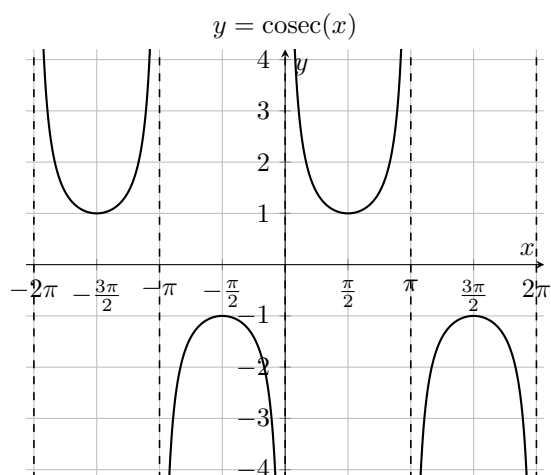
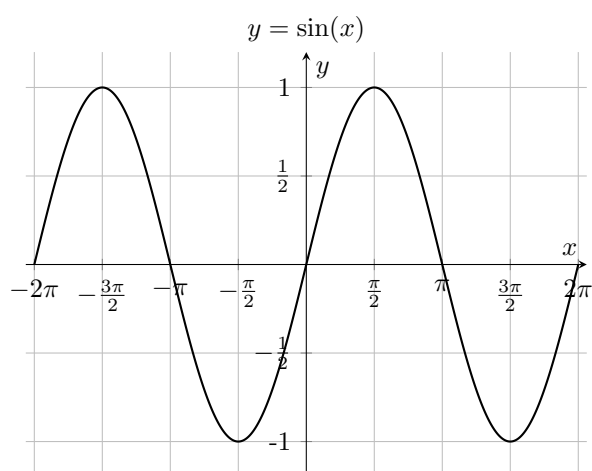
$$\frac{d}{dx} \arccos x = -\frac{1}{\sqrt{1-x^2}}$$

$$\int \arccos x \, dx = x \arccos x - \sqrt{1-x^2} + C$$

$$\frac{d}{dx} \arctan x = \frac{1}{x^2+1}$$

$$\int \arctan x \, dx = x \arctan x - \frac{\ln(x^2+1)}{2} + C$$

# Graphs



# Problem Solving Matters

## General Tips

- Be lazy; only do necessary work
- Write in sentences to explain (especially in proofs)
- Avoid long and/or complicated calculations
- Draw diagrams and make them big
- In diagrams, label things and add lines
- Look for similar shapes (often triangles)

## Tips For Sketching Graphs

- Look for symmetries
- Think about periodicity
- Look for turning points (0 derivative)
- Look for asymptotes
- Try values of  $x$  like 0, 1, -1, etc.
- If there's a trig function involved, try multiples of  $\pi$
- See what happens when  $x$  tends to 0 or  $\pm\infty$

## Things To Remember

- $\log_a b \times \log_b a = 1$
- $\log_{a^c} b^c = \log_a b$
- When graphing  $y^2 = f(x)$ , draw the positive branch of  $y = \sqrt{f(x)}$  and reflect it in the  $x$  axis
- $\log x$  is negative when  $0 < x < 1$