Trigonometry

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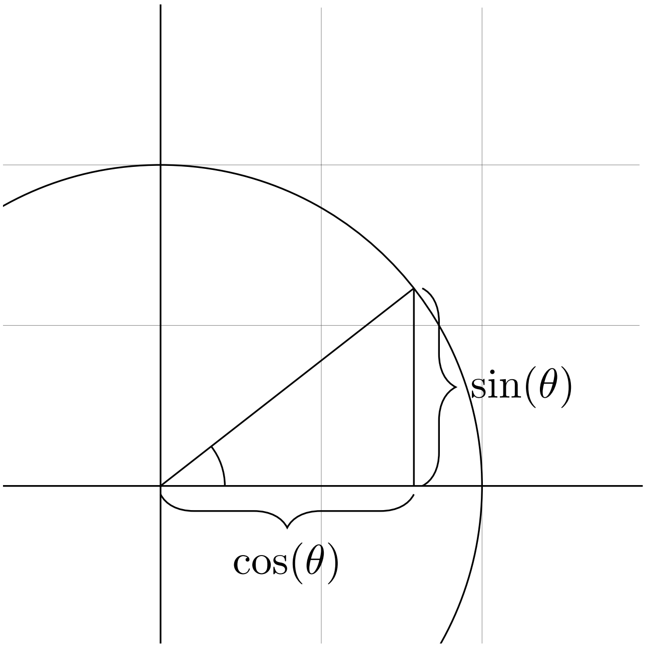
Basics

Definitions



Remember

$$\tan\theta \equiv \frac{\sin\theta}{\cos\theta}$$

$$\sec\theta \equiv \frac{1}{\cos\theta}$$

$$\operatorname{cosec}\theta \equiv \frac{1}{\sin\theta}$$

$$\cot\theta \equiv \frac{1}{\tan\theta} \equiv \frac{\cos\theta}{\sin\theta}$$

Identities

Remember

$$\sin^2\theta + \cos^2\theta \equiv 1 \qquad 1 + \tan^2\theta \equiv \sec^2\theta \qquad 1 + \cot^2\theta \equiv \operatorname{cosec}^2\theta$$

Formula Book

$$\sin(\alpha \pm \beta) \equiv \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\cos(\alpha \pm \beta) \equiv \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\tan(\alpha \pm \beta) \equiv \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$

Remember

$$\sin 2\theta \equiv 2\sin\theta\cos\theta$$

$$\tan 2\theta \equiv \frac{2\tan\theta}{1 - \tan^2\theta}$$

$$\begin{aligned}\cos 2\theta &\equiv \cos^2\theta - \sin^2\theta\\ &\equiv 2\cos^2\theta - 1\\ &\equiv 1 - 2\sin^2\theta\end{aligned}$$

Calculus

Remember

$$\frac{\mathrm{d}}{\mathrm{d}x}\sin x = \cos x \qquad \int \sin x \ \mathrm{d}x = -\cos x + C$$
$$\frac{\mathrm{d}}{\mathrm{d}x}\cos x = -\sin x \qquad \int \cos x \ \mathrm{d}x = \sin x + C$$

$$\int \arcsin x \ \mathrm{d}x = x \arcsin x + \sqrt{1-x^2} + C$$
$$\int \arccos x \ \mathrm{d}x = x \arccos x - \sqrt{1-x^2} + C$$
$$\int \arctan x \ \mathrm{d}x = x \arctan x - \frac{\ln(x^2+1)}{2} + C$$

Formula Book

$$\frac{\mathrm{d}}{\mathrm{d}x}\tan x = \sec^2 x \qquad \int \tan x \ \mathrm{d}x = \ln|\sec x| + C \qquad \frac{\mathrm{d}}{\mathrm{d}x}\arcsin x = \frac{1}{\sqrt{1-x^2}}$$
$$\frac{\mathrm{d}}{\mathrm{d}x}\sec x = \sec x \tan x \qquad \int \sec x \ \mathrm{d}x = \ln|\tan x + \sec x| + C \qquad \frac{\mathrm{d}}{\mathrm{d}x}\arccos x = -\frac{1}{\sqrt{1-x^2}}$$
$$\frac{\mathrm{d}}{\mathrm{d}x}\operatorname{cosec} x = -\operatorname{cosec} x \cot x \qquad \int \operatorname{cosec} x \ \mathrm{d}x = -\ln|\cot x + \operatorname{cosec} x| + C \qquad \frac{\mathrm{d}}{\mathrm{d}x}\arctan x = \frac{1}{x^2+1}$$
$$\frac{\mathrm{d}}{\mathrm{d}x}\cot x = -\operatorname{cosec}^2 x \qquad \int \cot x \ \mathrm{d}x = \ln|\sin x| + C$$

Simplifying combinations of trig functions

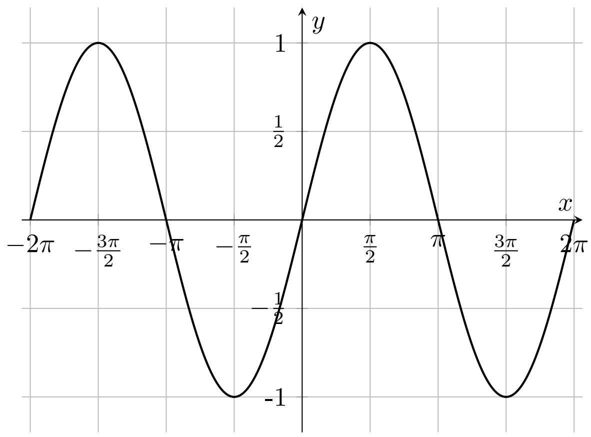
Remember

$$a\sin x \pm b\cos x \equiv R\sin(x \pm \alpha)$$
$$a\cos x \pm b\sin x \equiv R\cos(x \mp \alpha)$$
$$R\cos\alpha = a \text{ and } R\sin\alpha = b \implies \tan\alpha = \frac{b}{a}$$
$$R = \sqrt{a^2+b^2}$$

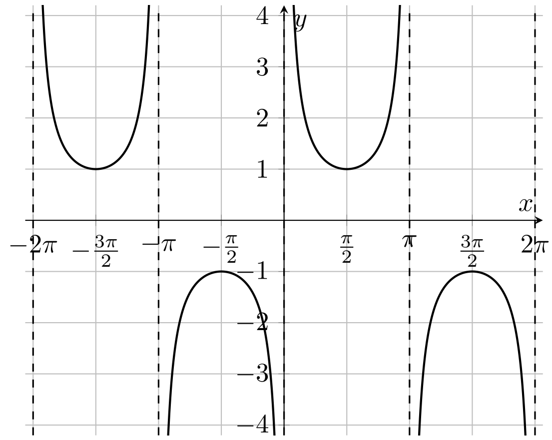
Simplify the expression to use the R form with whichever trig function comes first in the pair. Then $\tan\alpha \equiv \frac{b}{a}$.

Graphs

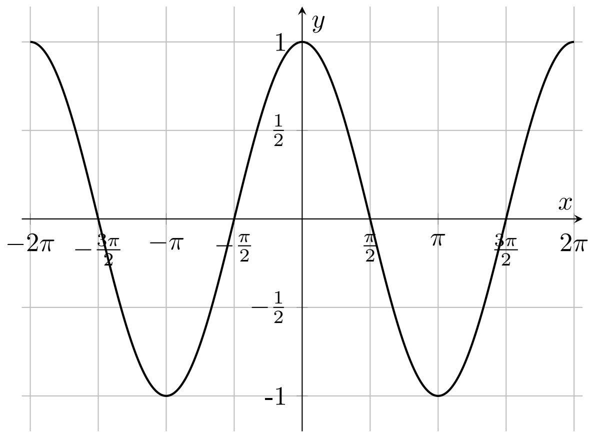
$y = \sin(x)$



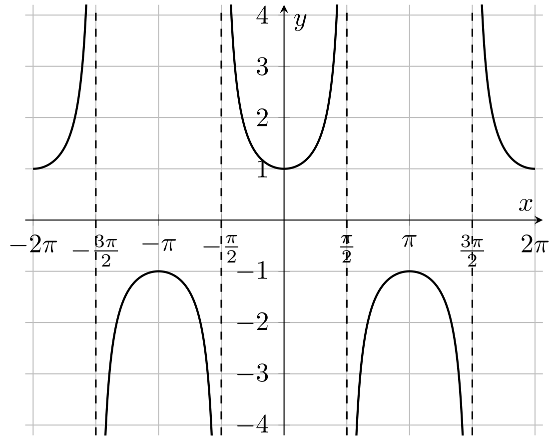
$y = \mathrm{cosec}(x)$



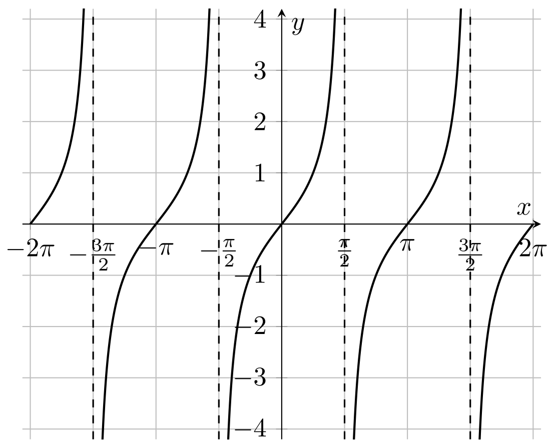
$y = \cos(x)$



$y = \sec(x)$



$y = \tan(x)$



$y = \cot(x)$