<u>Further Stats 1</u>

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Probability Distributions

<u>Poisson</u>

The Poisson distribution is used to model a situation where an event occurs at a fixed rate.

You can model X as a Poisson distribution if:

- The events must occur independently
- They must occur singly in space or time
- The events must occur at a constant average rate

If
$$X \sim \text{Po}(\lambda)$$
, then
$$P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!} \qquad (x \ge 0)$$

Geometric

The Geometric distribution is used to model a situation where you try an event several times until a success occurs, and you want to know how many tries it will take.

You can model X as a Geometric distribution if:

- Each attempt is independent
- Each attempt has the same probability

If
$$X \sim \text{Geo}(p)$$
, then $P(X = x) = p(1-p)^{x-1}$ $(x > 0)$

Remember
$$P(X \le x) = 1 - (1 - p)^x \qquad P(X \ge x) = (1 - p)^{x - 1}$$

$$P(X > x) = (1 - p)^x \qquad P(X < x) = 1 - (1 - p)^{x - 1}$$

Errors

The first step of an errors problem is always to find the critical region. The critical region is the region where H_0 would be rejected.

	Truth		
Conc.		H_0	H_1
	H_0	True negative	False negative (Type II error)
	H_1	False positive (Type I error)	True positive (Power)

The $Type\ I\ error$ or size is the probability of $being\ in$ the critical region with the original parameter.

The **Type II error** is the probability of *not being in* the critical region with a *new* parameter.

The **Power** (true positive) is the probability of being in the critical region with a new parameter. The power function is a function to find the power given a specific new parameter.

Probability Generating Functions

The **probability generating function (PGF)** of a probability distribution X is some function $G_X(t)$ of a dummy variable t such that:

Remember

$$G_X(t) = E(t^X) = \sum_x P(X = x)t^x$$

$$G_X(1) = 1$$

$$Y = aX + b \implies G_Y(t) = t^b G_X(t^a)$$

Formula Book

$$E(X) = G'_X(1)$$

$$Var(X) = G''_X(1) + G'_X(1) - (G'_X(1))^2$$

$$Z = X + Y \implies G_Z(t) = G_X(t) \times G_Y(t)$$