# Binomial Expansion

## *Definitions*

The factorial $n! \equiv n \times (n-1) \times (n-2) \times \cdots \times 3 \times 2 \times 1$.

The falling factorial $n^{\underline{k}} \equiv n \times (n-1) \times (n-2) \times \cdots \times (n-(k-2)) \times (n-(k-1))$. It has $k$ terms.

$$\boxed{0! = n^{\underline{0}} = 1}$$

The choose function ${}^nC_r \equiv \boxed{\binom{n}{r} \equiv \frac{n!}{r!(n-r)!}}$

## *Expansions*

For a natural number $n$, the expansion of $(a+b)^n$ is

$$a^n + na^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots \binom{n}{r}a^{n-r}b^r + \cdots b^n$$

In general, $\boxed{(a+b)^n = \sum_{r=0}^{n} \binom{n}{r} a^{n-r}b^r \ (n \in \mathbb{N})}$

That's true if $n$ is a natural number, but there is a version that works for all real numbers. For an expression $(a+bx)^n$, it should first be normalised to $a^n(1+\frac{b}{a}x)^n$. Let $y = \frac{b}{a}x$. Then the expansion of $(1+y)^n$ is given by

$$1 + ny + \frac{n(n-1)}{2!}y^2 + \frac{n(n-1)(n-2)}{3!}y^3 + \cdots + \frac{n(n-1)(n-2)\cdots(n-(r-1))}{r!}y^r + \cdots$$

In general, $\boxed{(a+bx)^n = a^n \sum_{r=0}^{\infty} \frac{n^{\underline{r}}}{r!} \left(\frac{b}{a}x\right)^r \ \ (n \in \mathbb{R})}$

# Calculus

## *Elementary Derivatives*

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

| $f$ | $C$ | $x^n$ | $\sin x$ | $\cos x$ | $a^x$ | $\ln x$ |
|---|---|---|---|---|---|---|
| $f'$ | $0$ | $nx^{n-1}$ | $\cos x$ | $-\sin x$ | $a^x \ln a$ | $\frac{1}{x}$ |

## *Composition Laws*

Let $f$ and $g$ be differentiable functions over $x$.

The $'$ mark denotes the derivative with respect to $x$, so $f' = \dfrac{df}{dx}$ and $g' = \dfrac{dg}{dx}$.

The $\circ$ symbol denotes function composition, so $(f \circ g)(x) = f(g(x))$.

$$(f \pm g)' = f' \pm g'$$

$$(fg)' = fg' + f'g$$

$$(f \circ g)' = (f' \circ g)g'$$

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

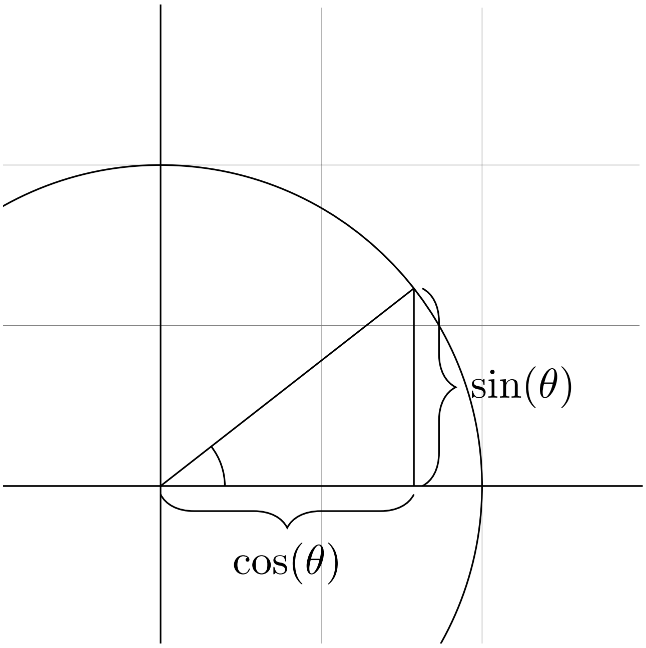
## *Integral Tricks*

For integrals of the form on the left, consider the function on the right.

| | |
|---|---|
| $\int kf'f^n \ dx$ | $f^{n+1}$ |
| $\int k\frac{f'}{f} \ dx$ | $\ln \lvert f \rvert$ |

# Trigonometry

## *Definitions*




$$\tan\theta \equiv \frac{\sin\theta}{\cos\theta}$$

$$\sec\theta \equiv \frac{1}{\cos\theta}$$

$$\operatorname{cosec}\theta \equiv \frac{1}{\sin\theta}$$

$$\cot\theta \equiv \frac{1}{\tan\theta} \equiv \frac{\cos\theta}{\sin\theta}$$

## *Identities*

$$\sin^2\theta + \cos^2\theta \equiv 1 \qquad 1 + \tan^2\theta \equiv \sec^2\theta \qquad 1 + \cot^2\theta \equiv \operatorname{cosec}^2\theta$$

$$\sin(\alpha+\beta) \equiv \sin\alpha\cos\beta + \cos\alpha\sin\beta \qquad \sin(\alpha-\beta) \equiv \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\cos(\alpha+\beta) \equiv \cos\alpha\cos\beta - \sin\alpha\sin\beta \qquad \cos(\alpha-\beta) \equiv \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\tan(\alpha+\beta) \equiv \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} \qquad \tan(\alpha-\beta) \equiv \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

$$\sin 2\theta \equiv 2\sin\theta\cos\theta$$

$$\tan 2\theta \equiv \frac{2\tan\theta}{1 - \tan^2\theta}$$

$$\begin{aligned}\cos 2\theta &\equiv \cos^2\theta - \sin^2\theta \\ &\equiv 2\cos^2\theta - 1 \\ &\equiv 1 - 2\sin^2\theta\end{aligned}$$

# *Calculus*

$$\frac{d}{dx}\sin x = \cos x \qquad \int \sin x \ dx = -\cos x + C$$

$$\frac{d}{dx}\cos x = -\sin x \qquad \int \cos x \ dx = \sin x + C$$

$$\frac{d}{dx}\tan x = \sec^2 x \qquad \int \tan x \ dx = \ln|\sec x| + C$$

$$\frac{d}{dx}\sec x = \sec x \tan x \qquad \int \sec x \ dx = \ln|\tan x + \sec x| + C$$

$$\frac{d}{dx}\operatorname{cosec} x = -\operatorname{cosec} x \cot x \qquad \int \operatorname{cosec} x \ dx = -\ln|\cot x + \operatorname{cosec} x| + C$$

$$\frac{d}{dx}\cot x = -\operatorname{cosec}^2 x \qquad \int \cot x \ dx = \ln|\sin x| + C$$

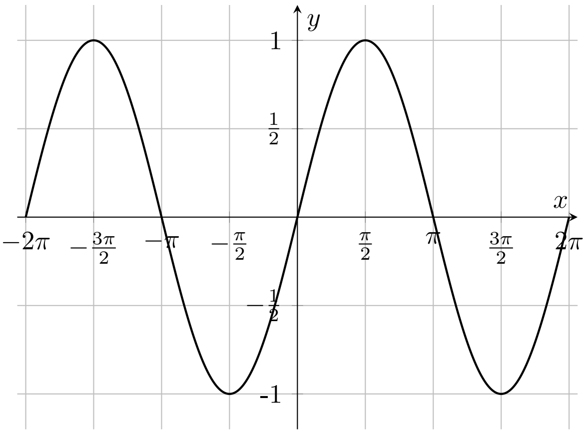
$$\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}} \qquad \int \arcsin x \ dx = x\arcsin x + \sqrt{1-x^2} + C$$

$$\frac{d}{dx}\arccos x = -\frac{1}{\sqrt{1-x^2}} \qquad \int \arccos x \ dx = x\arccos x - \sqrt{1-x^2} + C$$

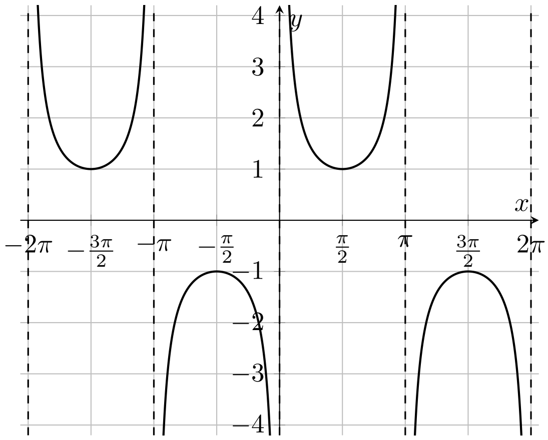
$$\frac{d}{dx}\arctan x = \frac{1}{x^2+1} \qquad \int \arctan x \ dx = x\arctan x - \frac{\ln(x^2+1)}{2} + C$$

# *Graphs*

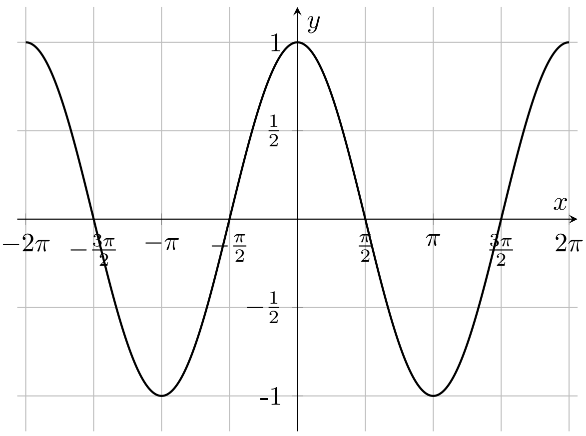
$y = \sin(x)$



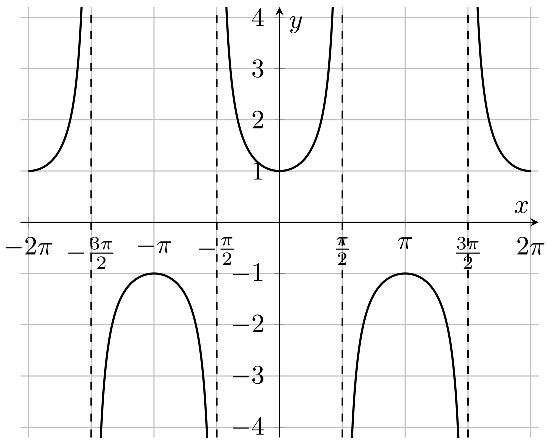

$y = \text{cosec}(x)$



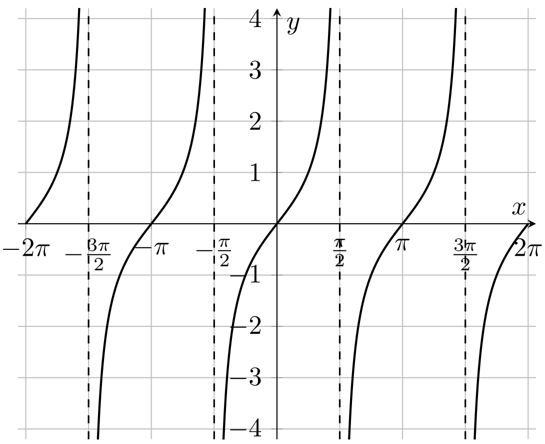

$y = \cos(x)$




$y = \sec(x)$




$y = \tan(x)$




$y = \cot(x)$