Further Stats 1

Contents

Probability Distributions

Poisson

The Poisson distribution is used to model a situation where an event occurs at a fixed rate.

You can model X as a Poisson distribution if:

- The events must occur independently
- They must occur singly in space or time
- The events must occur at a constant average rate

Formula Book

If $X \sim \text{Po}(\lambda)$, then $\boxed{P(X = x) = \dfrac{e^{-\lambda}\lambda^x}{x!}}$ $(x \geq 0)$

Geometric

The Geometric distribution is used to model a situation where you try an event several times until a success occurs, and you want to know how many tries it will take.

You can model X as a Geometric distribution if:

- Each attempt is independent
- Each attempt has the same probability

Formula Book

If $X \sim \text{Geo}(p)$, then $\boxed{P(X = x) = p(1-p)^{x-1}}$ $(x > 0)$

Remember

$P(X \leq x) = 1 - (1-p)^x$ $\quad$ $P(X \geq x) = (1-p)^{x-1}$

$P(X > x) = (1-p)^x$ $\quad$ $P(X < x) = 1 - (1-p)^{x-1}$

Errors

The first step of an errors problem is always to find the critical region. The critical region is the region where H_0 would be *rejected*.

	Truth	
Conc.	H_0	H_1
H_0	True negative	False negative (Type II error)
H_1	False positive (Type I error)	True positive (Power)

The **Type I error** or **size** is the probability of *being in* the critical region with the *original* parameter.

The **Type II error** is the probability of *not being in* the critical region with a *new* parameter.

The **Power** (true positive) is the probability of *being in* the critical region with a *new* parameter. The power function is a function to find the power given a specific new parameter.

Probability Generating Functions

The **probability generating function (PGF)** of a probability distribution X is some function $G_X(t)$ of a dummy variable t such that:

Remember

$$G_X(t) = E\left(t^X\right) = \sum_x P(X = x)t^x$$

$$G_X(1) = 1$$

$$Y = aX + b \implies G_Y(t) = t^b G_X\left(t^a\right)$$

$$P(X = n) = \frac{G_X^{(n)}(0)}{n!}, \ n \in \mathbb{N}$$

Formula Book

$$\mathrm{E}(X) = G'_X(1)$$

$$\mathrm{Var}(X) = G''_X(1) + G'_X(1) - (G'_X(1))^2$$

$$Z = X + Y \implies G_Z(t) = G_X(t) \times G_Y(t)$$