Misc

Contents

Problem Solving Matters

General Tips

- Be lazy; only do necessary work
- Write in sentences to explain (especially in proofs)
- Avoid long and/or complicated calculations
- Draw diagrams and make them big
- In diagrams, label things and add lines
- Look for similar shapes (often triangles)

Tips For Sketching Graphs

- Look for symmetries
- Think about periodicity
- Look for turning points (0 derivative)
- Look for asymptotes
- Try values of x like 0, 1, -1, etc.
- If there's a trig function involved, try multiples of π
- See what happens when x tends to 0 or $\pm\infty$

Things To Remember

- $\log_a b \times \log_b a = 1$
- $\log_{a^c} b^c = \log_a b$
- When graphing $y^2 = f(x)$, draw the positive branch of $y = \sqrt{f(x)}$ *and* reflect it in the x axis
- $\log x$ is negative when $0 < x < 1$

STEP Tips

General

- Take a step back and try to avoid getting tunnel vision for a specific technique
- Be very careful with the stem; it will be used for the rest of the question
- Explore the stem to get everything out of it that you can
- Check every line of algebra when you write it
- Don't take shortcuts unless you can justify ***why*** they're allowed
- Look for similar shapes (often triangles)
- If you get stuck on a 'show that' question, then just go onto the next part and use the previous result
- Try completing the square

Calculus

- If you can get an integral I in two forms, try adding them
- If given a substitution and asked to find a similar one for a slightly different function, find what made the first substitution work. What cancelled?
- To differentiate an equation of the form $y = f(x)^{g(x)}$, take logs and differentiate $\ln y = g(x) \ln\big(f(x)\big)$ implicitly
- In an ODE, always try to separate the variables. An awkward fraction is better than some terrible integral

Trig

- If you've got multiple trig terms on the top of a fraction and just one on the bottom, try using tan
- If you have a sum of products of n trig terms like $\cos^2 x + \cos x \sin x$ $(n = 2)$, then you can divide by $\cos^n x$ (in this example, you get $1 + \tan x$)
- If you're struggling to simplify a fraction with $\sin x$ and $\cos x$ mixed together on top and bottom, multiply top and bottom by $\frac{1}{\cos x}$

Proof

- A powerful form of proof is to try some simple cases, make a conjecture, and prove it to be true by induction
- If you're trying to prove that $a > b$, it's probably easier to prove that $a - b > 0$
- Use the format of the answer to inform your solution: if it wants something with x^{-1}, try using x^{-1}

Specifics

- Be careful when cancelling fractions with a factorial on the bottom $\left(\frac{x^2}{x!} = \frac{x}{(x-1)!} \neq \frac{1}{(x-2)!}\right)$
- $a^2 - b^2 = (a+b)(a-b)$
- $a^3 - b^3 = (a-b)(a^2+ab+b^2)$
- If you need all the things except one of them, then get all of them and then remove one (perhaps by dividing it out) (example: $C_p(x) = \frac{x^p-1}{x-1}$)

Stats

- PDF $= \frac{d}{dx}$CDF (alt. $P(X=x) = \frac{d}{dx}P(X \leq x)$)
- $E(X) = \sum_{\min(X)}^{\max(X)} xP(X=x)$ (for discrete X) or $\int_{-\infty}^{\infty} xP(X=x)dx$ (for continuous X)