# Stats

## Contents

# Probability Distributions

## *Binomial*

The binomial distribution is used to model a situation with a fixed number of independent trials each with a constant probability of success.

You can model $X$ as a binomial distribution if:

- There a fixed number of trials, $n$
- Each trial must succeed or fail
- There is a fixed probability of success, $p$
- Each trial is independent

If $X \sim \mathrm{B}(n, p)$, then

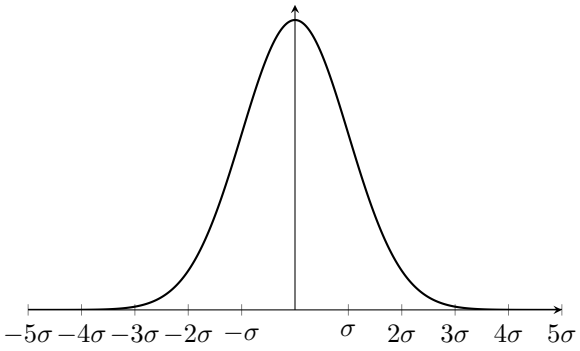
**Formula Book**

$$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x} \quad (0 \leq x \leq n)$$

## *Normal*

The normal distribution $X \sim \mathrm{N}(\mu, \sigma^2)$ is symmetrical, meaning the mean and median are equal.

When doing questions that involve the normal distribution, sketching the bell curve on the right is always a good idea.




The standard normal distribution $Z \sim \mathrm{N}(0, 1^2)$ is very useful, since it allows you to find values for $\mu$ and $\sigma$ when they're unknown. The normal random variable $X$ can be coded using

**Remember**

$$Z = \frac{X - \mu}{\sigma}$$

and you can use this equation to find the unknown parameters of $X$.

The 'Normal CD' function on your calculator will calculate the area between an upper and lower bound on the bell curve. The 'Inverse Normal' function will find a value for which the area to the *left* of that value is the area you specify.

# Hypothesis Testing

Every hypothesis test has two hypotheses:

$H_0$ : The null hypothesis - this is what you assume to be true by default
$H_1$ : The alternative hypothesis

The hypotheses are written in different forms depending on whether the test is one- or two-tailed.

| One-tailed: | Two-tailed: |
|---|---|
| $H_0 : p = k$ | $H_0 : p = k$ |
| $H_1 : p \lessgtr k$ | $H_1 : p \neq k$ |

If the question says that someone measured and got a value, then you plug that value into a probability calculation with the parameters from the null hypothesis, with the inequality sign in the same direction as the alternative hypothesis. If the probability of the event happening when assuming $H_0$ is less than the level of significance, then we reject $H_0$ and accept the alternative hypothesis.

A critical value is the smallest or largest value (depending on the direction of the inequality) obtained by a random variable such that $H_0$ would be rejected. Finding a critical value is often best done with the tables in the back of the book.

# Correlation

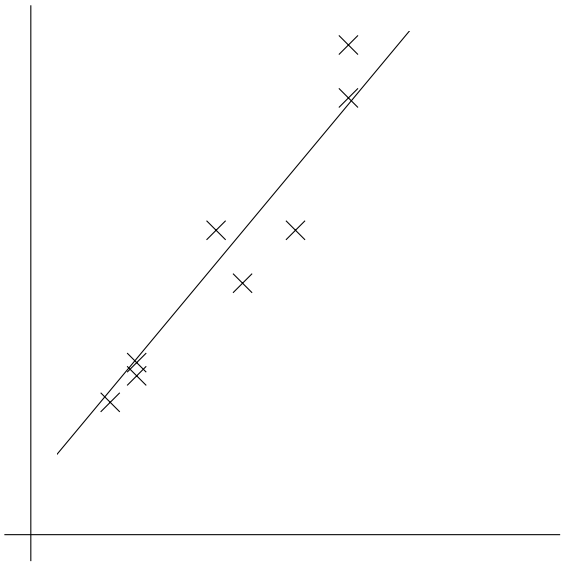


Figure 1: A strong positive correlation

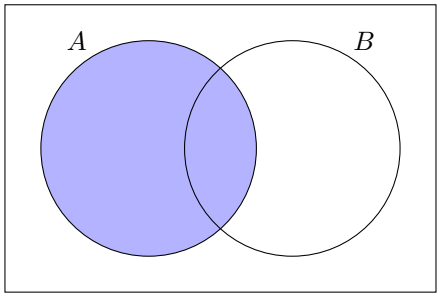
The **product moment correlation coefficient** or "Pearson's correlation coefficient" is a measure of correlation between two variables. It is often called $r$ and is measured in the range $[-1, 1]$. $r = \pm 1$ means the data perfectly follows a positive or negative correlation respectively.

## *Hypothesis testing*

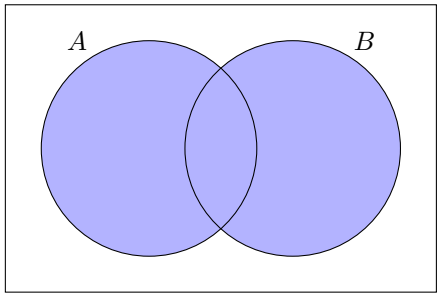
Use $H_0 : \rho = 0$ and $H_1 : \rho \lessgtr 0$ or $H_1 : \rho \neq 0$. Get $r$ from your calculator (use section 6 and option 4 regression calc) and $\rho$ from the table in the formula book by finding the sample size and significance level. Remember to halve the significance level if the test is two-tailed.

# Conditional probability

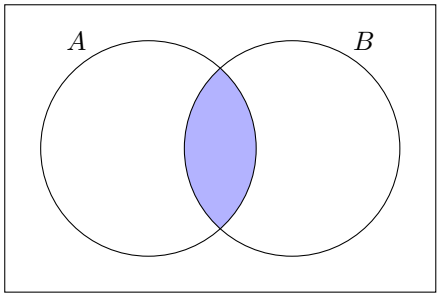
$P(A)$



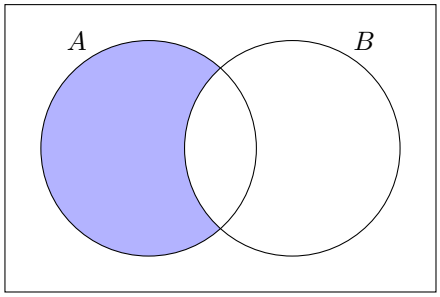

$P(A \cup B)$



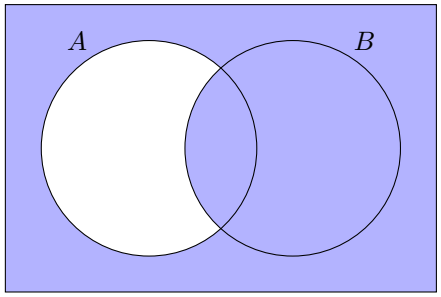

$P(A \cap B)$



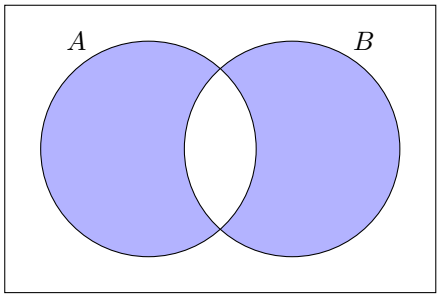

$P(A \cap B')$




$P(A' \cup B)$




$P\big((A \cap B') \cup (B \cap A')\big)$




**Remember**

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

For independent events $A$ and $B$, we know that $P(A \cap B) = P(A) \times P(B)$. This means that

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A) \times \cancel{P(B)}}{\cancel{P(B)}} = P(A)$$