

# Calculus

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# Basics

## Elementary Derivatives

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

|      |     |            |          |           |             |               |
|------|-----|------------|----------|-----------|-------------|---------------|
| $f$  | $C$ | $x^n$      | $\sin x$ | $\cos x$  | $a^x$       | $\ln x$       |
| $f'$ | 0   | $nx^{n-1}$ | $\cos x$ | $-\sin x$ | $a^x \ln a$ | $\frac{1}{x}$ |

## Composition Laws

Let  $f$  and  $g$  be differentiable functions over  $x$ .

The  $'$  mark denotes the derivative with respect to  $x$ , so  $f' = \frac{df}{dx}$  and  $g' = \frac{dg}{dx}$ .

The  $\circ$  symbol denotes function composition, so  $(f \circ g)(x) = f(g(x))$ .

$$(f \pm g)' = f' \pm g'$$

$$(fg)' = fg' + f'g$$

$$(f \circ g)' = (f' \circ g)g'$$

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

# Integration

If it's a pattern that you know, then just use the pattern. Here are some examples:

Set patterns

|                          |           |
|--------------------------|-----------|
| $\int k f' f^n dx$       | $f^{n+1}$ |
| $\int k \frac{f'}{f} dx$ | $\ln  f $ |

Formula book

Some integrals are given in the formula book. These include many common trig-based integrals, as well as some fractions. Check the formula book before spending ages on a tricky integral.

Trig

Many integrals benefit from transforming their insides using trig identities. Integrating a trig function to a power is very hard. Try to apply identities to reduce it.

Partial fractions

Large algebraic fractions are very awkward. Simplify them into partial fractions. These should then become natural logs and/or and inverse trig functions when integrated.

Substitution

Integrals can often be simplified by using an appropriate substitution. Always remember to find  $dx$  in terms of  $du$  and exchange the bounds of the integral.

By parts

Integrating by parts is the most complicated method, and should be avoided wherever possible. Use the following formula:

$$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

Since the second integral involves  $\frac{du}{dx}$ , we should choose  $u$  to be the function in the original integral that reduces the fastest. To find the function that reduces fastest, follow the acronym **LATE** - **L**ogs, **A**lgebra, **T**rig, **E**xponentials. Pick the function that comes first in the acronym.