lintrans

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1 Analysis

One of the topics in the A Level Further Maths course is linear transformations, as represented by matrices. This is a topic all about how vectors move and get transformed in the plane. It's a topic that lends itself exceedingly well to visualization, but students often find it hard to visualize this themselves, and there is a considerable lack of good tools to provide visual intuition on the subject. There is the YouTube series *Essence of Linear Algebra* by 3blue1brown[7], which is excellent, but I couldn't find any good interactive visualizations.

My solution is to develop a desktop application that will allow the user to define 2×2 matrices and view these matrices and compositions thereof as linear transformations of a 2D plane. This will give students a way to get to grips with linear transformations in a more hands-on way, and will give teachers the ability to easily and visually show concepts like the determinant and invariant lines.

1.1 Computational Approach

This solution is particularly well suited to a computational approach since it is entirely focussed on visualizing transformations, which require complex mathematics to properly display. It will also have lots of settings to allow the user to configure aspects of the visualization. As previously mentioned, visualizing transformations in one's own head is difficult, so a piece of software to do it would be very valuable to teachers and learners, but current solutions are considerably lacking.

My solution will make use of abstraction by allowing the user to define a set of matrices which they can use in expressions. This allows them to use a matrix multiple times and they don't have to keep track of any of the numbers. All the actual processing and mathematics happens behind the scenes and the user never has to worry about it - they just compose their defined matrices into transformations. This abstraction allows the user to focus on exploring the transformations themselves without having to do any actual computations. This will make learning the subject much easier, as they will able to gain a visual intuition for linear transformations without worrying about computation until after they've built up that intuition.

I will also employ decomposition and modularization by breaking the project down into many smaller parts, such as one module to keep track of defined matrices, one module to validate and parse matrix expressions, one module for the main GUI, as well as sub-modules for the widgets and dialog boxes, etc. This decomposition allows for simpler project design, easier code maintenance (since module coupling is kept to a minimum, so bugs are isolated in their modules), inheritance of classes to reduce code repetition, and unit testing to inform development. I also intend this unit testing to be automated using GitHub Actions.

Selection will also be used widely in the application. The GUI will provide many settings for visualization, and these settings will need to be checked when rendering the transformation. For example, the user will have the option to render the determinant, so I will need to check this setting on every render cycle and only render the determinant parallelogram if the user has enabled that option. The app will have many options for visualization, which will be useful in learning, but if all these options were being rendered at the same time, then there would be too much information for the user to properly process, so I will let the user configure these display options to their liking and only render the things they want to be rendered.

Validation will also be prevalent because the matrix expressions will need to follow a strict format, which will be validated. The buttons to render and animate the matrix will only be clickable when the given expression is valid, so I will need to check this and update the buttons every time the text in the text box is changed. I will also need to parse matrix expressions so that I can evaluate them properly. All this validation ensures that crashes due to malformed input are practically impossible, and makes the user's life easier since they don't need to worry about if their input is in the right format - the app will tell them.

I will also make use of iteration, primarily in animation. I will have to re-calculate positions and

values to render everything for every frame of the animation and this will likely be done with a simple `for` loop. A `for` loop will allow me to just loop over every frame and use the counter variable as a way to measure how far through the animation we are on each frame. This is preferable to a `while` loop, since that would require me to keep track of which frame we're on with a separate variable.

Finally, the core of the application is visualization, so that will definitely be used a lot. I will have to calculate positions of points and lines based on given matrices, and when animating, I will also have to calculate these matrices based on the current frame. Then I will have to use the rendering capabilities of the GUI framework that I choose to render these calculated points and lines onto a widget, which will form the viewport of the main GUI. I may also have to convert between coordinate systems. I will have the origin in the middle with positive x going to the right and positive y going up, but I may need to convert that to standard computer graphics coordinates with the origin in the top left, positive x going to the right, and positive y going down. This visualization of linear transformations is the core component of the app and is the primary feature, so it is incredibly important.

1.2 Stakeholders

Stakeholders for my app include A Level Further Maths students and teachers, who learn and teach linear transformations respectively. They will be able to provide useful input as to what they would like to see in the app, and they can provide feedback on what they like and what I can add or improve. I already know from experience that linear transformations are tricky to visualize and a computer-based visualization would be useful. My stakeholders agreed with this. Many teachers said that a desktop app that could render and animate linear transformations would be useful in a classroom environment and students said that it would be helpful to have something that they could play around with at home and use to get to grips with matrices and linear transformations.

Some teachers also suggested that it would be useful to have an option to save and load sets of matrices. This would allow them to have a single save file containing some matrices, and then just load this file to use for demonstrations in the classroom. This would probably be quite easy to implement. I could just wrap all the relevant information into one object and use Python's `pickle` module to save the binary data to a file, and then load this data back into the app in a similar way.

My stakeholders agreed that being able to see incremental animation - where, for example, we apply matrix **A** to the current scene, pause, and then apply matrix **B** - would be beneficial. This would be a good demonstration of matrix multiplication being non-commutative. **AB** is not always equal to **BA**. Being able to see this in terms of animating linear transformations would be good for learning.

They also agreed that a tutorial on using the software would be useful, so I plan to implement this through an online written tutorial hosted with GitHub Pages, and perhaps a video tutorial as well. This would make the app much easier to use for people who have never seen it before. It wouldn't be a lesson on the maths itself, just a guide on how to use the software.

1.3 Research on existing solutions

There are actually quite a few web apps designed to help visualize 2D linear transformations but many of them are hard to use and lacking many features.

1.3.1 MIT 'Matrix Vector' Mathlet

Arguably the best app that I found was an MIT 'Mathlet' - a simple web app designed to help visualize a maths concept. This one is called 'Matrix Vector'[8] and allows the user to drag an input vector around the plane and see the corresponding output vector, transformed by a matrix that the user can define, although this definition is finicky since it involves sliders rather than keyboard input.

This app fails in two crucial ways in my opinion. It doesn't show the basis vectors or let the user drag them around, and the user can only define and therefore visualize a single matrix at once. This second problem was common among every solution I found, so I won't mention it again, but it is a big issue in my opinion and my app will allow for multiple matrices. I like the idea of having a draggable input vector and rendering its output, so I will probably have this feature in my app, but I also want the ability to define multiple matrices and be able to drag the basis vectors to visually define a matrix. Being able to drag the basis vectors will help build intuition, so I think this would greatly benefit the app.

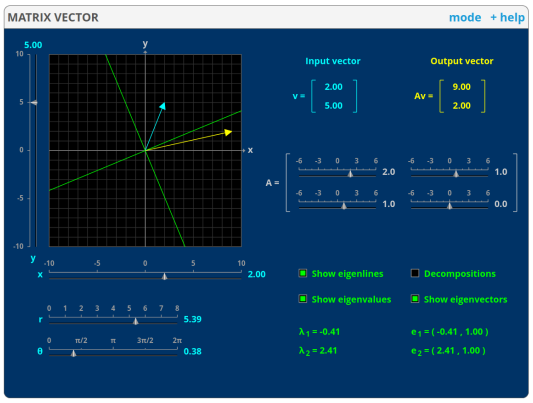


Figure 1.1: The MIT 'Matrix Vector' Mathlet

However, in the comments on this Mathlet, a user called 'David S. Bruce' suggested that the Mathlet should display the basis vectors, to which a user called 'hrm' (who I assume to be the 'H. Miller' to whom the copyright of the whole website is accredited) replied saying that this Mathlet is primarily focussed on eigenvectors, that it is perhaps badly named, and that displaying the basis vectors 'would make a good focus for a second Mathlet about 2×2 matrices'. This Mathlet does not exist. But I do like the idea of showing the eigenvectors and eigenlines, so I will definitely have that in my app. Showing the invariant lines or lack thereof will help with learning, since these are often hard to visualize.

1.3.2 Linear Transformation Visualizer

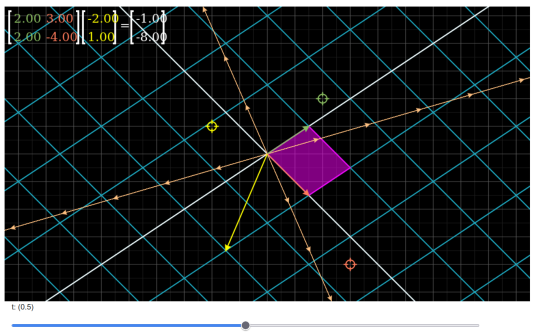


Figure 1.2: 'Linear Transformation Visualizer' halfway through an animation

Another web app that I found was one simply called 'Linear Transformation Visualizer' by Shad Sharma[22]. This one was similarly inspired by 3blue1brown's YouTube series. This app has the ability to render input and output vectors and eigenlines, but it can also render the determinant parallelogram; it allows the user to drag the basis vectors; and it has the option to snap vectors to the background grid, which is quite useful. It also implements a simple form of animation where the tips of the vectors move in straight lines from where they start to where they end, and the animation is controlled by dragging a slider labelled t. This isn't particularly intuitive.

I really like the vectors snapping to the grid, the input and output vectors, and rendering the determinant. This app also renders positive and negative determinants in different colours, which is really nice - I intend to use that idea in my own app, since it helps create understanding about negative determinants in terms of orientation changes. However, I think that the animation system here is flawed and not very easy to use. My animation will likely be a button, which just triggers an animation, rather than a slider. I also don't like the way vector dragging is handled. If you click anywhere on the grid, then the closest vector target (the final position of the target's associated vector) snaps to that location. I think it would be more intuitive to have to drag the vector from its current location to where you want it. This was also a problem with the MIT Mathlet.

1.3.3 Desmos app

One of the solutions I found was a Desmos app[6], which was quite hard to use and arguably overcomplicated. Desmos is not designed for this kind of thing - it's designed to graph pure mathematical functions - and it shows here. However, this app brings some really interesting ideas to the table, mainly functions. This app allows you to define custom functions and view them before and after the transformation. This is achieved by treating the functions parametrically as the set of points $(t, f(t))$ and then transforming each coordinate by the given matrix to get a new coordinate.

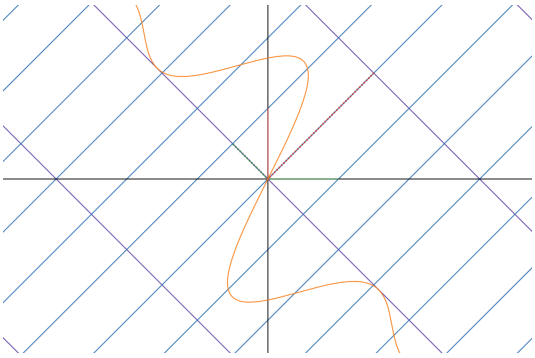


Figure 1.3: The Desmos app halfway through an animation, rendering $f(x) = \frac{\sin^2 x}{x}$ in orange

Desmos does this for every point and then renders the resulting transformed function parametrically. This is a really interesting technique and idea, but I'm not going to use it in my app. I don't think arbitrary functions fit with the linearity of the whole app, and I don't think it's necessary. It's just overcomplicating things, and rendering it on a widget would be tricky, because I'd have to render every point myself, possibly using something like OpenGL. It's just not worth implementing.

Additionally, this Desmos app makes things quite hard to see. It's hard to tell where any of the vectors are - they just get lost in the sea of grid lines. This image also hides some of the extra information. For instance, this image doesn't show the original function $f(x) = \frac{\sin^2 x}{x}$, only the transformed version. This app easily gets quite cluttered. I will give my vectors arrowheads to make them easily identifiable amongst the grid lines.

1.3.4 Visualizing Linear Transformations

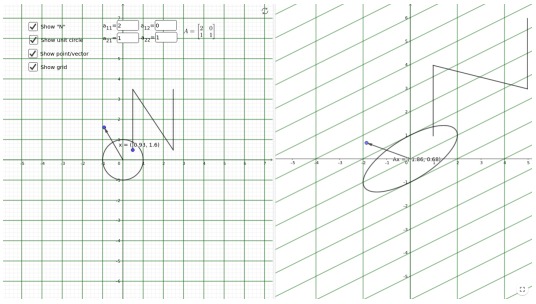


Figure 1.4: The GeoGebra applet rendering its default matrix

The last solution that I want to talk about is a GeoGebra applet simply titled 'Visualizing Linear Transformations'[10]. This applet has input and output vectors, original and transformed grid lines, a unit circle, and the letter N. It allows the user to define a matrix as 4 numbers and view the aforementioned N (which the user can translate to anywhere on the grid), the unit circle, the input/output vectors, and the grid lines. It also has the input vector snapping to integer coordinates, but that's a standard part of GeoGebra.

I've already talked about most of these features but the thing I wanted to talk about here is the N. I don't particularly want the letter N to be a prominent part of my own app, but I really like the idea of being able to define a custom polygon and see how that polygon gets transformed by a given transformation. I think that would really help with building intuition and it shouldn't be too hard to implement.

1.4 Essential features

The primary aim of this application is to visualize linear transformations, so this will obviously be the centre of the app and an essential feature. I will have a widget which can render a background grid and a second version of the grid, transformed according to a user-defined matrix expression. This is necessary because it is the entire purpose of the app. It's designed to visualize linear transformations and would be completely useless without this visual component. I will give the user the ability to render a custom matrix expression containing matrices they have previously defined, as well as reset the canvas to the default identity matrix transformation. This will obviously require an input box to enter the expression, a render button, a reset button, and various dialog boxes to define matrices in different ways. I want the user to be able to define a matrix as a set of 4 numbers, and by dragging the basis vectors i and j. These dialogs will allow the user to define new matrices to be used in expressions, and having multiple ways to do it will make it easier, and will aid learning.

Another essential feature is animation. I want the user to be able to smoothly animate between matrices. I see two options for how this could work. If **C** is the matrix for the currently displayed transformation, and **T** is the matrix for the target transformation, then we could either animate from **C** to **T** or we could animate from **C** to **TC**. I would probably call these transitional and applicative animation respectively. Perhaps I'll give the user the option to choose which animation method they want to use. I might even have an option for sequential animation, where the user can define a sequence of matrices, perhaps separated with commas or semicolons, and the app will animate through the sequence, applying one at a time. Sequential animation would be nice, but is not crucial.

Either way, animation is used in most of the alternative solutions that I found, and it's a great way to build intuition, by allowing students to watch the transformation happen in real time. Compared to simply rendering the transformations, animating them would profoundly benefit learning, and since that's the main aim of the project, I think animation is a necessary part of the app.

Something that I thought was a big problem in every alternative solution I found was the fact that the user could only visualize a single matrix at once. I see this as a fatal flaw and I will allow the user to define 25 different matrices (all capital letters except **I** for the identity matrix) and use all of them in expressions. This will allow teachers to define multiple matrices and then just change the expression to demonstrate different concepts rather than redefine a new transformation every time. It will also make things easier for students as it will allow them to visualize compositions of different matrix transformations without having to do any computations themselves.

Additionally, being able to show information on the currently displayed matrix is an essential tool for learning. Rendering things like the determinant parallelogram and the invariant lines of the transformation will greatly assist with learning and building understanding, so I think that having the option to render these attributes of the currently displayed transformation is necessary for success.

1.5 Limitations

The main limitation in this app is likely to be drawing grid lines. Most transformations will be fine but in some cases, the app will be required to draw potentially thousands of grid lines on the canvas and this will probably cause noticeable lag, especially in the animations. I will have to artificially limit the number of grid lines that can be drawn on the screen. This won't look fantastic, because it means that the grid lines will only extend a certain distance from the origin, but it's an inherent limitation of computers. Perhaps if I was using a faster, compiled language like C++ rather than Python, this processing would happen faster and I could render more grid lines, but it's impossible to render all the grid lines and any implementation of this idea must limit them for performance.

An interesting limitation is that I don't think I'll implement panning. I suspect that I'll have to convert between coordinate systems and having the origin in the centre of the canvas will probably make the code much simpler. Also, linear transformations always leave the origin fixed, so always having it in the centre of the canvas seems thematically appropriate. Panning is certainly an option - the Desmos solution in §1.3.3 and GeoGebra solution in §1.3.4 both allow panning as a default part

of Desmos and GeoGebra respectively, for example - but I don't think I'll implement it myself. I just don't think it's worth it.

I'm also not going to do any work with 3D linear transformations. 3D transformations are often harder to visualize and thus it would make sense to target them in an app like this, designed to help with learning and intuition, but 3D transformations are also harder to code. I would have to use a full graphics package rather than a simple widget, and I think it would be too much work for this project and I wouldn't be able to do it in the time frame. It's definitely a good idea, but I'm currently incapable of creating an app like that.

There are other limitations inherent to matrices. For instance, it's impossible to take an inverse of a singular matrix. There's nothing I can do about that without rewriting most of mathematics. Matrices can also only represent linear transformations. There's definitely a market for an app that could render any arbitrary transformation from $\mathbb{R}^2 \to \mathbb{R}^2$ - I know I'd want an app like that - but matrices can only represent linear transformations, so those are the only kind of transformations that I'll be looking at with this project.

1.6 Hardware and software requirements

1.6.1 Hardware

Hardware requirements for the project are the same between the release and development environments and they're quite simple. I expect the app to require a processor with at least 1 GHz clock speed, `$BINARY_SIZE` free disk space, and about 1 GB of available RAM. The processor and RAM requirements are needed by the Python runtime and mainly by Qt5 - the GUI library I'll be using. The `$BINARY_SIZE` disk space is just for the executable binary that I'll compile for the public release. The code itself is less than 1 MB, but the compiled binary has to package all the dependencies and the entire CPython runtime to allow it to run on systems that don't have that, so the file size is much bigger.

I will also require that the user has a monitor that is at least 1920×1080 pixels in resolution. This isn't necessarily required, because the app will likely run in a smaller window, but a HD monitor is highly recommended. This allows the user to go fullscreen if they want to, and it gives them enough resolution to easily see everything in the app. A large, wall-mounted screen is also highly recommended for use in the classroom, although this is common among schools.

I will also require a keyboard with all standard Latin alphabet characters. This is because the matrices are defined as uppercase Latin letters. Any UK or US keyboard will suffice for this. The app will also require a mouse with at least one button. I don't intend to have right click do anything, so only the primary mouse button is required, although getting a single button mouse to actually work on modern computers is probably quite a challenge. A separate mouse is not strictly required - a laptop trackpad is equally sufficient.

1.6.2 Software

Software requirements differ slightly between release and development, although everything that the release environment requires is also required by the development environment. I will require a modern operating system - namely Windows 10 or later, macOS 10.9 'Mavericks'[1] or later, or any modern Linux distro[2]. Basically, it just requires an operating system that is compatible with Python 3.10 and Qt5, since I'll be using these in the project. Of course, Qt5 will need to be installed on the user's computer, although it's standard pretty much everywhere these days.

Python 3.10 won't actually be required for the end user, because I will be compiling the app into a

[1] Python 3.10 won't compile on any earlier versions of macOS[16]

[2] Specifying a Linux version is practically impossible. Python 3.10 isn't available in many package repositories, but will compile on any modern distro. Qt5 is available in many package repositories and can be compiled on any `x86` or `x86_64` generic Linux machine with `gcc` version 5 or later[17]

stand-alone binary executable for release, and this binary will contain the required Python runtime and dependencies. However, if the user wishes to download and run the source code themself, then they will need Python 3.10 and the package dependencies: `numpy`, `nptyping`, and `pyqt5`. These can be automatically installed with the command `python -m pip install -r requirements.txt` from the root of the repository. `numpy` is a maths library that allows for fast matrix maths; `nptyping` is used by `mypy` for type-checking and isn't actually a runtime dependency but the imports in the `typing` module fail if it's not installed at runtime; and `pyqt5` is a library that just allows interop between Python and Qt5, which is originally a C++ library.

In the development environment, I use PyCharm for actually writing my code, and I use a virtual environment to isolate my project dependencies. There are also some development dependencies listed in the file `dev_requirements.txt`. They are: `mypy`, `pyqt5-stubs`, `flake8`, `pycodestyle`, `pydocstyle`, and `pytest`. `mypy` is a static type checker[3]; `pyqt5-stubs` is a collection of type annotations for the PyQt5 API for `mypy` to use; `flake8`, `pycodestyle`, and `pydocstyle` are all linters; and `pytest` is a unit testing framework. I use these libraries to make sure my code is good quality and actually working properly during development.

1.7 Success criteria

The main aim of the app is to help teach students about linear transformations. As such, the primary measure of success will be letting teachers get to grips with the app and then asking if they would use it in the classroom or recommend it to students to use at home.

Additionally, the app must fulfil some basic requirements:

1. It must allow the user to define multiple matrices in at least two different ways (numerically and visually)
2. It must be able to validate arbitrary matrix expressions
3. It must be able to render any valid matrix expression
4. It must be able to animate any valid matrix expression
5. It must be able to apply a matrix expression to the current scene and animate this (animate from **C** to **TC**, and perhaps do sequential animation)
6. It must be able to display information about the currently rendered transformation (determinant, eigenlines, etc.)
7. It must be able to save and load sessions (defined matrices, display settings, etc.)
8. It must allow the user to define and transform arbitrary polygons

Defining multiple matrices is a feature that I thought was lacking from every other solution I researched, and I think it would make the app much easier to use, so I think it's necessary for success. Validating matrix expressions is necessary because if the user tries to render an expression that doesn't make sense, has an undefined matrix, or contains the inverse of a singular matrix, then we have to disallow that or else the app will crash.

Visualizing matrix expressions as linear transformations is the core part of the app, so basic rendering of them is definitely a requirement for success. Animating these expressions is also a pretty crucial part of the app, so I would consider this necessary for success. Displaying the information of a matrix transformation is also very useful for building understanding, so I would consider this needed to succeed.

Saving and loading isn't strictly necessary for success, but it is a standard part of many apps, so will likely be expected by users, and it will benefit the app by allowing teachers to plan lessons in advance and save the matrices they've defined for that lesson to be loaded later.

[3]Python has weak, dynamic typing with optional type annotations but `mypy` enforces these static type annotations

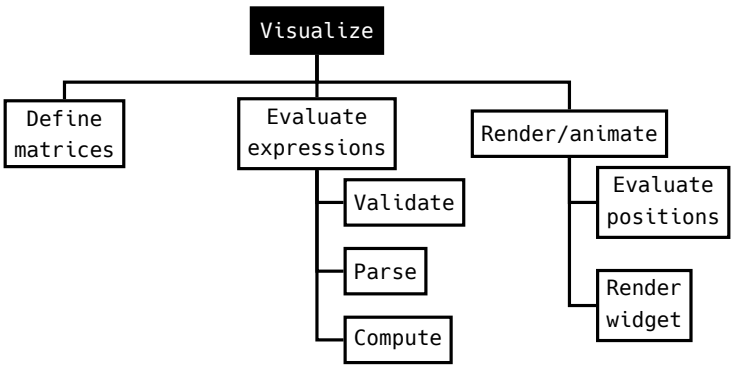
Transforming polygons is the lowest priority item on this list and will likely be implemented last, but it would definitely benefit learning. I wouldn't consider it necessary for success, but it would be very good to include, and it's certainly a feature that I want to have.

If the majority of teachers would use and/or recommend the app and it meets all of these points, then I will consider the app as a whole to be a success.

2 Design

2.1 Problem decomposition

I have decomposed the problem of visualization as follows:



Defining matrices is key to visualization because we need to have matrices to actually visualize. This is a key part of the app, and the user will be able to define multiple separate matrices numerically and visually using the GUI.

Evaluating expressions is another key part of the app and can be further broken down into validating, parsing, and computing the value. Validating an expression simply consists of checking that it adheres to a set of syntax rules for matrix expressions, and that it only contains matrices which have already been defined. Parsing consists of breaking an expression down into tokens, which are then much easier to evaluate. Computing the expression with these tokens is then just a series of simple operations, which will produce a final matrix at the end.

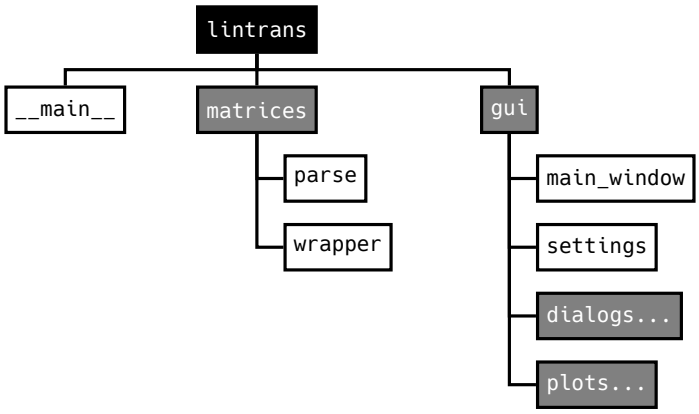
Rendering and animating will likely be the largest part in reality, but I've only decomposed it into simple blocks here. Evaluating positions involves evaluating the matrix expression that the user has input and using the columns of the resultant matrix to find the new positions of the basis vectors, and then extrapolating this for the rest of the plane. Rendering onto the widget is likely to be quite complicated and framework-dependent, so I've abstracted away the details for brevity here. Rendering will involve using the previously calculated values to render grid lines and vectors. Animating will probably be a `for` loop which just renders slightly different matrices onto the widget and sleeps momentarily between frames.

I have deliberately broken this problem down into parts that can be easily translated into modules in my eventual coded solution. This is simply to ease the design and development process, since now I already know my basic project structure. This problem could've been broken down into the parts that the user will directly interact with, but that would be less useful to me when actually starting development, since I would then have to decompose the problem differently to write the actual code.

2.2 Structure of the solution

2.2.1 The main project

I have decomposed my solution like so:



The `lintrans` node is simply the root of the whole project. `__main__` is the Python way to make the project executable as `python -m lintrans` on the command line. For release, I will package it into a standalone binary executable.

`matrices` is the package that will allow the user to define, validate, parse, evaluate, and use matrices. The `parse` module will contain functions to validate matrix expressions - likely using regular expressions - and functions to parse matrix expressions. It will not know which matrices are defined, so validation will be naïve and evaluation will be elsewhere. The `wrapper` module will contain a `MatrixWrapper` class, which will hold a dictionary of matrix names and values. It is this class which will have aware validation - making sure that all matrices are actually defined - as well the ability to evaluate matrix expressions, in addition to its basic behaviour of setting and getting matrices. This `matrices` package will also have a `create_rotation_matrix` function that will generate a rotation matrix from an angle using the formula $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$. It will be in the `wrapper` module since it's related to defining and manipulating matrices, but it will be exported and accessible as `lintrans.matrices.create_rotation_matrix`.

`gui` is the package that will contain all the frontend code for everything GUI-related. `main_window` is the module that will contain a `LintransMainWindow` class, which will act as the main window of the application and have an instance of `MatrixWrapper` to keep track of which matrices are defined and allow for evaluation of matrix expressions. It will also have methods for rendering and animating matrix expressions, which will be connected to buttons in the GUI. This module will also contain a simple `main()` function to instantiate and launch the application GUI.

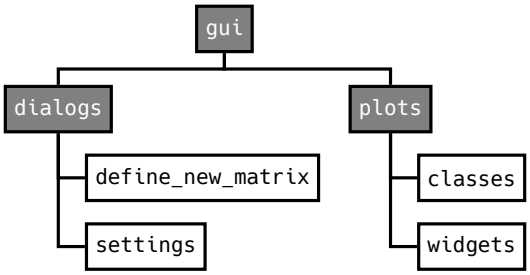
The `settings` module will contain a `DisplaySettings` dataclass[4] that will represent the settings for visualizing transformations. The `LintransMainWindow` class will have an instance of this class and check against it when rendering things. The user will be able to open a dialog to change these display settings, which will update the main window's instance of this class.

The `settings` module will also have a `GlobalSettings` class, which will represent the global settings for the application, such as the logging level, where to save the logs, whether to ask the user if they want to be prompted with a tutorial whenever they open the app, etc. This class will have defaults for everything, but the constructor will try to read these settings from a config file if possible. This allows for persistent settings between sessions. This config file will be `~/.config/lintrans.conf` on Unix-like systems, including macOS, and `C:\Users\%USER%\AppData\Roaming\lintrans\config.txt` on Windows. This difference is to remain consistent with operating system conventions[5].

[4] This is the Python equivalent of a `struct` or `record` in other languages

[5] And also to avoid confusing Windows users with a `.conf` file

2.2.2 The `gui` subpackages



The `dialogs` subpackage will contain modules with different dialog classes. It will have a `define_new_matrices` module, which will have a `DefineDialog` abstract superclass. It will also contain classes that inherit from this superclass and provide dialogs for defining new matrices visually, numerically, and as an expression in terms of other matrices. Additionally, this subpackage will contain a `settings` module, which will provide a `SettingsDialog` superclass and a `DisplaySettingsDialog` class, which will allow the user to configure the aforementioned display settings. It will also have a `GlobalSettingsDialog` class, which will similarly allow the user to configure the app's global settings through a dialog.

The `plots` subpackage will have a `classes` module and a `widgets` module. The `classes` module will have the abstract superclasses `BackgroundPlot` and `VectorGridPlot`. The former will provide helped methods to convert between coordinate systems and draw the background grid, while the latter will provide helper methods to draw transformations and their components. It will have `point_i` and `point_j` attributes and will provide methods to draw the transformed version of the grid, the vectors and their arrowheads, the eigenlines of the transformation, etc. These methods can then be called from the Qt5 `paintEvent` handler which will be declared abstract and must therefore be implemented by all subclasses.

The `plots` subpackage will also contain a `widgets` module, which will have the classes `VisualizeTransformationWidget` and `DefineVisuallyWidget`, both of which will inherit from `VectorGridPlot`. They will both implement their own `paintEvent` handler to actually draw the respective widgets, and `DefineVisuallyWidget` will also implement handlers for mouse events, allowing the user to drag around the basis vectors.

It's also worth noting here that I don't currently know how I'm going to implement the transformation of arbitrary polygons. It will likely consist of an attribute in `VisualizeTransformationWidget` which is a list of points, and these points can be dragged around with mouse event handlers and then the transformed versions can be rendered, but I'm not yet sure about how I'm going to implement it.

2.3 Algorithm design

This section will be completed later.

2.4 Usability features

My main concern in terms of usability is colour. In the 3blue1brown videos on linear algebra, red and green are used for the basis vectors, but these colours are often hard to distinguish in most common forms of colour blindness. The most common form is deuteranopia[25], which makes red and green look incredibly similar. I will use blue and red for my basis vectors. These colours are easy to distinguish for people with deuteranopia and protanopia - the two most common forms of colour blindness. Tritanopia makes it harder to distinguish blue and yellow, but my colour scheme is still be accessible for people with tritanopia, as red and blue are very distinct in this form of colour blindness.

I will probably use green for the eigenvectors and eigenlines, which will be hard to distinguish from the red basis vector for people with red-green colour blindness, but I think that the basis vectors and

eigenvectors/eigenlines will look physically different enough from each other that the colour shouldn't be too much of a problem. Additionally, I will use a tool called Color Oracle[11] to make sure that my app is accessible to people with different forms of colour blindness[6].

Another solution would be to have one default colour scheme, and allow the user to change the colour scheme to something more accessible for colour blind people, but I don't see the point in this. I think it's easier for colour blind people to just have the main colour scheme be accessible, and it's not really an inconvenience to non-colour blind people, so I think this is the best option.

The layout of my app will be self-consistent and follow standard conventions. I will have a menu bar at the top of the main window for actions like saving and loading, as well as accessing the tutorial (which will also be accessible by pressing `F1` at any point) and documentation. The dialogs will always have the confirm button in the bottom right and the cancel button just to the left of that. They will also have the matrix name drop-down on the left. This consistency will make the app easier to learn and understand.

I will also have hotkeys for everything that can have hotkeys - buttons, checkboxes, etc. This makes my life easier, since I'm used to having hotkeys for everything, and thus makes the app faster to test because I don't need to click everything. This also makes things easier for other people like me, who prefer to stay at the keyboard and not use the mouse. Obviously a mouse will be required for things like dragging basis vectors and polygon vertices, but hotkeys will be available wherever possible to help people who don't like using the mouse or find it difficult.

2.5 Variables and validation

This project won't actually have many variables. The main ones will be instance attributes on the `LintransMainWindow` class. It will have a `MatrixWrapper` instance, a `DisplaySettings` instance, and a `GlobalSettings` instance. These will handle the matrices and various settings respectively. Having these as instance attributes allows them to be referenced from any method in the class, and Qt5 uses lots of slots (basically callback methods) and handlers, so it's good to be able to access the attributes I need right there rather than having to pass them around from method to method.

The `MatrixWrapper` class will have a dictionary of names and matrices. The names will be single letters[7] and the matrices will be of type `MatrixType`. This will be a custom type alias representing a 2×2 `numpy` array of floats. When setting the values for these matrices, I will have to manually check the types. This is because Python has weak typing, and if we got, say, an integer in place of a matrix, then operations would fail when trying to evaluate a matrix expression, and the program would crash. To prevent this, we have to validate the type of every matrix when it's set. I have chosen to use a dictionary here because it makes accessing a matrix by its name easier. We don't have to check against a list of letters and another list of matrices, we just index into the dictionary.

The settings dataclasses will have instance attributes for each setting. Most of these will be booleans, since they will be simple binary options like *Show determinant*, which will be represented with checkboxes in the GUI. The `DisplaySettings` dataclass will also have an attribute of type `int` representing the time in milliseconds to pause during animations.

The `DefineDialog` superclass have a `MatrixWrapper` instance attribute, which will be a parameter in the constructor. When `LintransMainWindow` spawns a definition dialog (which subclasses `DefineDialog`), it will pass in a copy of its own `MatrixWrapper` and connect the `accepted` signal for the dialog. The slot (method) that this signal is connected to will get called when the dialog is closed with the *Confirm* button[8]. This allows the dialog to mutate its own `MatrixWrapper` object and then the main window can copy that mutated version back into its own instance attribute when the user confirms the change. This reduces coupling and makes everything easier to reason about and debug, as well as reducing

[6] I actually had to clone a fork of this project[1] to get it working on Ubuntu 20.04 and adapt it slightly to create a working `jar` file

[7] I would make these `char` but Python only has a `str` type for strings

[8] Actually when the dialog calls `.accept()`. The `Confirm` button is actually connected to a method which first takes the info and updates the instance `MatrixWrapper`, and then calls `.accept()`

the number of bugs, since the classes will be independent of each other. In another language, I could pass a pointer to the wrapper and let the dialog mutate it directly, but this is potentially dangerous, and Python doesn't have pointers anyway.

Validation will also play a very big role in the application. The user will be able to enter matrix expressions and these must be validated. I will define a BNF schema and either write my own RegEx or use that BNF to programmatically generate a RegEx. Every matrix expression input will be checked against it. This is to ensure that the matrix wrapper can actually evaluate the expression. If we didn't validate the expression, then the parsing would fail and the program could crash. I've chosen to use a RegEx here rather than any other option because it's the simplest. Creating a RegEx can be difficult, especially for complicated patterns, but it's then easier to use it. Also, Python can compile a RegEx pattern, which makes it much faster to match against, so I will compile the pattern at initialization time and just compare expressions against that pre-compiled pattern, since we know it won't change at runtime.

Additionally, the buttons to render and animate the current matrix expression will only be enabled when the expression is valid. Textboxes in Qt5 emit a `textChanged` signal, which can be connected to a slot. This is just a method that gets called whenever the text in the textbox is changed, so I can use this method to validate the input and update the buttons accordingly. An empty string will count as invalid, so the buttons will be disabled when the box is empty.

I will also apply this matrix expression validation to the textbox in the dialog which allows the user to define a matrix as an expression involving other matrices, and I will validate the input in the numeric definition dialog to make sure that all the inputs are floats. Again, this is to prevent crashes, since a matrix with non-number values in it will likely crash the program.

2.6 Iterative test data

In unit testing, I will test the validation, parsing, and generation of rotation matrices from an angle. I will also unit test the utility functions for the GUI, like `is_valid_float`.

For the validation of matrix expressions, I will have data like the following:

Valid	Invalid
`"A"`	`""`
`"AB"`	`"A^"`
`"-3.4A"`	`"rot()"`
`"A^2"`	`"A^{2"`
`"A^T"`	`"^12"`
`"A^{-1}"`	`"A^3.2"`
`"rot(45)"`	`"A^B"`
`"3A^{12}"`	`".A"`
`"2B^2+A^TC^{-1}"`	`"--A"`
`"3.5A^45.6rot(19.2^T-B^-14.1C^5"`	`"A--B"`

This list is not exhaustive, mostly to save space and time, but the full unit testing code is included in appendix B.

The invalid expressions presented here have been chosen to be almost valid, but not quite. They are edge cases. I will also test blatantly invalid expressions like `"This is a matrix expression"` to make sure the validation works.

Here's an example of some test data for parsing:

Input	Expected
`"A"`	`[[("", "A", "")]]`
`"AB"`	`[[("", "A", ""), ("", "B", "")]]`
`"2A+B^2"`	`[[("2", "A", ""), ("", "B", "2")]]`
`"3A^T2.4B^{-1}-C"`	`[[("3", "A", "T"), ("2.4", "B", "-1")], [("-1", "C", "")]]`

The parsing output is pretty verbose and this table doesn't have enough space for most of the more complicated inputs, so here's a monster one:

```
"2.14A^{3} 4.5rot(14.5)^-1 + 8.5B^T 5.97C^14 - 3.14D^{-1} 6.7E^T"
```

which should parse to give:

```
[[("2.14", "A", "3"), ("4.5", "rot(14.5)", "-1")], [("8.5", "B", "T"), ("5.97", "C", "14")],
                        [("-3.14", "D", "-1"), ("6.7", "E", "T")]]
```

Any invalid expression will also raise a parse error, so I will check every invalid input previously mentioned and make sure it raises the appropriate error.

Again, this section is brief to save space and time. All unit tests are included in appendix B.

2.7 Post-development test data

This section will be completed later.

2.8 Issues with testing

Since `lintrans` is a graphical application about visualizing things, it will be mainly GUI focussed. Unfortunately, unit testing GUIs is a lot harder than unit testing library or API code. I don't think there's any way to easily and reliably unit test a graphical interface, so my unit tests will only cover the backend code for handling matrices. Testing the GUI will be entirely manual; mostly defining matrices, thinking about what I expect them to look like, and then making sure they look like that. I don't see a way around this limitation. I will make my backend unit tests very thorough, but testing the GUI can only be done manually.

3 Development

Please note, throughout this section, every code snippet will have two comments at the top. The first is the git commit hash that the snippet was taken from[9]. The second comment is the file name. The line numbers of the snippet reflect the line numbers of the file from where the snippet was taken. After a certain point, I introduced copyright comments at the top of every file. These are always omitted here.

3.1 Matrices backend

3.1.1 `MatrixWrapper` class

The first real part of development was creating the `MatrixWrapper` class. It needs a simple instance dictionary to be created in the constructor, and it needs a way of accessing the matrices. I decided to use Python's `__getitem__()` and `__setitem__()` special methods[15] to allow indexing into a `MatrixWrapper` object like `wrapper['M']`. This simplifies using the class.

```python
# 29ec1fedbf307e3b7ca731c4a381535fec899b0b
# src/lintrans/matrices/wrapper.py

"""A module containing a simple MatrixWrapper class to wrap matrices and context."""

import numpy as np

from lintrans.typing import MatrixType


class MatrixWrapper:
    """A simple wrapper class to hold all possible matrices and allow access to them."""

    def __init__(self):
        """Initialise a MatrixWrapper object with a matrices dict."""
        self._matrices: dict[str, MatrixType | None] = {
            'A': None, 'B': None, 'C': None, 'D': None,
            'E': None, 'F': None, 'G': None, 'H': None,
            'I': np.eye(2),  # I is always defined as the identity matrix
            'J': None, 'K': None, 'L': None, 'M': None,
            'N': None, 'O': None, 'P': None, 'Q': None,
            'R': None, 'S': None, 'T': None, 'U': None,
            'V': None, 'W': None, 'X': None, 'Y': None,
            'Z': None
        }

    def __getitem__(self, name: str) -> MatrixType | None:
        """Get the matrix with `name` from the dictionary.

        Raises:
            KeyError:
                If there is no matrix with the given name
        """
        return self._matrices[name]

    def __setitem__(self, name: str, new_matrix: MatrixType) -> None:
        """Set the value of matrix `name` with the new_matrix.

        Raises:
            ValueError:
                If `name` isn't a valid matrix name
        """
        name = name.upper()

        if name == 'I' or name not in self._matrices:
            raise NameError('Matrix name must be a capital letter and cannot be "I"')
```

[9] A history of all commits can be found in the GitHub repository[2]

```
44
45              self._matrices[name] = new_matrix
```

This code is very simple. The constructor (`__init__()`) creates a dictionary of matrices which all start out as having no value, except the identity matrix **I**. The `__getitem__()` and `__setitem__()` methods allow the user to easily get and set matrices just like a dictionary, and `__setitem__()` will raise an error if the name is invalid. This is a very early prototype, so it doesn't validate the type of whatever the user is trying to assign it to yet. This validation will come later.

I could make this class subclass `dict`, since it's basically just a dictionary at this point, but I want to extend it with much more functionality later, so I chose to handle the dictionary stuff myself.

I then had to write unit tests for this class, and I chose to do all my unit tests using a framework called `pytest`.

```
# 29ec1fedbf307e3b7ca731c4a381535fec899b0b
# tests/test_matrix_wrapper.py

"""Test the MatrixWrapper class."""

import numpy as np
import pytest
from lintrans.matrices import MatrixWrapper

valid_matrix_names = 'ABCDEFGHJKLMNOPQRSTUVWXYZ'
test_matrix = np.array([[1, 2], [4, 3]])


@pytest.fixture
def wrapper() -> MatrixWrapper:
    """Return a new MatrixWrapper object."""
    return MatrixWrapper()


def test_get_matrix(wrapper) -> None:
    """Test MatrixWrapper.__getitem__()."""
    for name in valid_matrix_names:
        assert wrapper[name] is None

    assert (wrapper['I'] == np.array([[1, 0], [0, 1]])).all()


def test_get_name_error(wrapper) -> None:
    """Test that MatrixWrapper.__getitem__() raises a KeyError if called with an invalid name."""
    with pytest.raises(KeyError):
        _ = wrapper['bad name']
        _ = wrapper['123456']
        _ = wrapper['Th15 Is an 1nV@l1D n@m3']
        _ = wrapper['abc']


def test_set_matrix(wrapper) -> None:
    """Test MatrixWrapper.__setitem__()."""
    for name in valid_matrix_names:
        wrapper[name] = test_matrix
        assert (wrapper[name] == test_matrix).all()


def test_set_identity_error(wrapper) -> None:
    """Test that MatrixWrapper.__setitem__() raises a NameError when trying to assign to I."""
    with pytest.raises(NameError):
        wrapper['I'] = test_matrix


def test_set_name_error(wrapper) -> None:
    """Test that MatrixWrapper.__setitem__() raises a NameError when trying to assign to an invalid name."""
    with pytest.raises(NameError):
        wrapper['bad name'] = test_matrix
        wrapper['123456'] = test_matrix
```

```
52          wrapper['Th15 Is an 1nV@l1D n@m3'] = test_matrix
53          wrapper['abc'] = test_matrix
```

These tests are quite simple and just ensure that the expected behaviour works the way it should, and that the correct errors are raised when they should be. It verifies that matrices can be assigned, that every valid name works, and that the identity matrix **I** cannot be assigned to.

The function decorated with `@pytest.fixture` allows functions to use a parameter called `wrapper` and `pytest` will automatically call this function and pass it as that parameter. It just saves on code repetition.

3.1.2 Rudimentary parsing and evaluating

This first thing I did here was improve the `__setitem__()` and `__getitem__()` methods to validate input and easily get transposes and simple rotation matrices.

```
# f89fc9fd8d5917d07557fc50df3331123b55ad6b
# src/lintrans/matrices/wrapper.py
```

```
60      def __setitem__(self, name: str, new_matrix: MatrixType) -> None:
61          """Set the value of matrix `name` with the new_matrix.
62
63          :param str name: The name of the matrix to set the value of
64          :param MatrixType new_matrix: The value of the new matrix
65          :rtype: None
66
67          :raises NameError: If the name isn't a valid matrix name or is 'I'
68          """
69          if name not in self._matrices.keys():
70              raise NameError('Matrix name must be a single capital letter')
71
72          if name == 'I':
73              raise NameError('Matrix name cannot be "I"')
74
75          # All matrices must have float entries
76          a = float(new_matrix[0][0])
77          b = float(new_matrix[0][1])
78          c = float(new_matrix[1][0])
79          d = float(new_matrix[1][1])
80
81          self._matrices[name] = np.array([[a, b], [c, d]])
```

In this method, I'm now casting all the values to floats. This is very simple validation, since this cast will raise `ValueError` if it fails to cast the value to a float. I should've declared `:raises ValueError:` in the docstring, but this was an oversight at the time.

```
# f89fc9fd8d5917d07557fc50df3331123b55ad6b
# src/lintrans/matrices/wrapper.py
```

```
27      def __getitem__(self, name: str) -> Optional[MatrixType]:
28          """Get the matrix with the given name.
29
30          If it is a simple name, it will just be fetched from the dictionary.
31          If the name is followed with a 't', then we will return the transpose of the named matrix.
32          If the name is 'rot()', with a given angle in degrees, then we return a new rotation matrix with that angle.
33
34          :param str name: The name of the matrix to get
35          :returns: The value of the matrix (may be none)
36          :rtype: Optional[MatrixType]
37
38          :raises NameError: If there is no matrix with the given name
39          """
40          # Return a new rotation matrix
```

```
41          match = re.match(r'rot\((\d+)\)', name)
42          if match is not None:
43              return create_rotation_matrix(float(match.group(1)))
44
45          # Return the transpose of this matrix
46          match = re.match(r'([A-Z])t', name)
47          if match is not None:
48              matrix = self[match.group(1)]
49
50              if matrix is not None:
51                  return matrix.T
52              else:
53                  return None
54
55          if name not in self._matrices:
56              raise NameError(f'Unrecognised matrix name "{name}"')
57
58          return self._matrices[name]
```

This `__getitem__()` method now allows for easily accessing transposes and rotation matrices by checking input with regular expressions. This makes getting matrices easier and thus makes evaluating full expressions simpler.

The `create_rotation_matrix()` method is also defined in this file and just uses the $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ formula from before:

```
# f89fc9fd8d5917d07557fc50df3331123b55ad6b
# src/lintrans/matrices/wrapper.py

158 def create_rotation_matrix(angle: float) -> MatrixType:
159     """Create a matrix representing a rotation by the given number of degrees anticlockwise.
160
161     :param float angle: The number of degrees to rotate by
162     :returns MatrixType: The resultant rotation matrix
163     """
164     rad = np.deg2rad(angle)
165     return np.array([
166         [np.cos(rad), -1 * np.sin(rad)],
167         [np.sin(rad), np.cos(rad)]
168     ])
```

At this stage, I also implemented a simple parser and evaluator using regular expressions. It's not great and it's not very flexible, but it can evaluate simple expressions.

```
# f89fc9fd8d5917d07557fc50df3331123b55ad6b
# src/lintrans/matrices/wrapper.py

83      def parse_expression(self, expression: str) -> MatrixType:
84          """Parse a given expression and return the matrix for that expression.
85
86          Expressions are written with standard LaTeX notation for exponents. All whitespace is ignored.
87
88          Here is documentation on syntax:
89              A single matrix is written as 'A'.
90              Matrix A multiplied by matrix B is written as 'AB'
91              Matrix A plus matrix B is written as 'A+B'
92              Matrix A minus matrix B is written as 'A-B'
93              Matrix A squared is written as 'A^2'
94              Matrix A to the power of 10 is written as 'A^10' or 'A^{10}'
95              The inverse of matrix A is written as 'A^-1' or 'A^{-1}'
96              The transpose of matrix A is written as 'A^T' or 'At'
97
98          :param str expression: The expression to be parsed
99          :returns MatrixType: The matrix result of the expression
100
101         :raises ValueError: If the expression is invalid, such as an empty string
102         """
```

```
103             if expression == '':
104                 raise ValueError('The expression cannot be an empty string')
105 
106             match = re.search(r'[^-+A-Z^{}rot()\d.]', expression)
107             if match is not None:
108                 raise ValueError(f'Invalid character "{match.group(0)}"')
109 
110             # Remove all whitespace in the expression
111             expression = re.sub(r'\s', '', expression)
112 
113             # Wrap all exponents and transposition powers with {}
114             expression = re.sub(r'(?<=\^)(-?\d+|T)(?=[^}]|$)', r'{\g<0>}', expression)
115 
116             # Replace all subtractions with additions, multiplied by -1
117             expression = re.sub(r'(?<=.)-(?=[A-Z])', '+-1', expression)
118 
119             # Replace a possible leading minus sign with -1
120             expression = re.sub(r'^-(?=[A-Z])', '-1', expression)
121 
122             # Change all transposition exponents into lowercase
123             expression = expression.replace('^{T}', 't')
124 
125             # Split the expression into groups to be multiplied, and then we add those groups at the end
126             # We also have to filter out the empty strings to reduce errors
127             multiplication_groups = [x for x in expression.split('+') if x != '']
128 
129             # Start with the 0 matrix and add each group on
130             matrix_sum: MatrixType = np.array([[0., 0.], [0., 0.]])
131 
132             for group in multiplication_groups:
133                 # Generate a list of tuples, each representing a matrix
134                 # These tuples are (the multiplier, the matrix (with optional
135                 # 't' at the end to indicate a transpose), the exponent)
136                 string_matrices: list[tuple[str, str, str]]
137 
138                 # The generate tuple is (multiplier, matrix, full exponent, stripped exponent)
139                 # The full exponent contains ^{}, so we ignore it
140                 # The multiplier and exponent might be '', so we have to set them to '1'
141                 string_matrices = [(t[0] if t[0] != '' else '1', t[1], t[3] if t[3] != '' else '1')
142                                    for t in re.findall(r'(-?\d*\.?\d*)([A-Z]t?|rot\(\d+\))(\^{(-?\d+|T)})?', group)]
143 
144                 # This list is a list of tuple, where each tuple is (a float multiplier,
145                 # the matrix (gotten from the wrapper's __getitem__()), the integer power)
146                 matrices: list[tuple[float, MatrixType, int]]
147                 matrices = [(float(t[0]), self[t[1]], int(t[2])) for t in string_matrices]
148 
149                 # Process the matrices and make actual MatrixType objects
150                 processed_matrices: list[MatrixType] = [t[0] * np.linalg.matrix_power(t[1], t[2]) for t in matrices]
151 
152                 # Add this matrix product to the sum total
153                 matrix_sum += reduce(lambda m, n: m @ n, processed_matrices)
154 
155             return matrix_sum
```

I think the comments in the code speak for themselves, but we basically split the expression up into groups to be added, and then for each group, we multiply every matrix in that group to get its value, and then add all these values together at the end.

This code is objectively bad. At the time of writing, it's now quite old, so I can say that. This code has no real error handling, and line 127 introduces the glaring error that `'A++B'` is now a valid expression because we disregard empty strings. Not to mention the fact that the method is called `parse_expression()` but actually evaluates an expression. All these issues will be fixed in the future, but this was the first implementation of matrix evaluation, and it does the job decently well.

I then implemented several tests for this parsing.

```
# 60e0c713b244e097bab8ee0f71142b709fde1a8b
# tests/test_matrix_wrapper_parse_expression.py
```

```python
"""Test the MatrixWrapper parse_expression() method."""

import numpy as np
from numpy import linalg as la
import pytest
from lintrans.matrices import MatrixWrapper


@pytest.fixture
def wrapper() -> MatrixWrapper:
    """Return a new MatrixWrapper object with some preset values."""
    wrapper = MatrixWrapper()

    root_two_over_two = np.sqrt(2) / 2

    wrapper['A'] = np.array([[1, 2], [3, 4]])
    wrapper['B'] = np.array([[6, 4], [12, 9]])
    wrapper['C'] = np.array([[-1, -3], [4, -12]])
    wrapper['D'] = np.array([[13.2, 9.4], [-3.4, -1.8]])
    wrapper['E'] = np.array([
        [root_two_over_two, -1 * root_two_over_two],
        [root_two_over_two, root_two_over_two]
    ])
    wrapper['F'] = np.array([[-1, 0], [0, 1]])
    wrapper['G'] = np.array([[np.pi, np.e], [1729, 743.631]])

    return wrapper


def test_simple_matrix_addition(wrapper: MatrixWrapper) -> None:
    """Test simple addition and subtraction of two matrices."""

    # NOTE: We assert that all of these values are not None just to stop mypy complaining
    # These values will never actually be None because they're set in the wrapper() fixture
    # There's probably a better way  do this, because this method is a bit of a bdoge, but this works for now
    assert wrapper['A'] is not None and wrapper['B'] is not None and wrapper['C'] is not None and \
        wrapper['D'] is not None and wrapper['E'] is not None and wrapper['F'] is not None and \
        wrapper['G'] is not None

    assert (wrapper.parse_expression('A+B') == wrapper['A'] + wrapper['B']).all()
    assert (wrapper.parse_expression('E+F') == wrapper['E'] + wrapper['F']).all()
    assert (wrapper.parse_expression('G+D') == wrapper['G'] + wrapper['D']).all()
    assert (wrapper.parse_expression('C+C') == wrapper['C'] + wrapper['C']).all()
    assert (wrapper.parse_expression('D+A') == wrapper['D'] + wrapper['A']).all()
    assert (wrapper.parse_expression('B+C') == wrapper['B'] + wrapper['C']).all()


def test_simple_two_matrix_multiplication(wrapper: MatrixWrapper) -> None:
    """Test simple multiplication of two matrices."""
    assert wrapper['A'] is not None and wrapper['B'] is not None and wrapper['C'] is not None and \
           wrapper['D'] is not None and wrapper['E'] is not None and wrapper['F'] is not None and \
           wrapper['G'] is not None

    assert (wrapper.parse_expression('AB') == wrapper['A'] @ wrapper['B']).all()
    assert (wrapper.parse_expression('BA') == wrapper['B'] @ wrapper['A']).all()
    assert (wrapper.parse_expression('AC') == wrapper['A'] @ wrapper['C']).all()
    assert (wrapper.parse_expression('DA') == wrapper['D'] @ wrapper['A']).all()
    assert (wrapper.parse_expression('ED') == wrapper['E'] @ wrapper['D']).all()
    assert (wrapper.parse_expression('FD') == wrapper['F'] @ wrapper['D']).all()
    assert (wrapper.parse_expression('GA') == wrapper['G'] @ wrapper['A']).all()
    assert (wrapper.parse_expression('CF') == wrapper['C'] @ wrapper['F']).all()
    assert (wrapper.parse_expression('AG') == wrapper['A'] @ wrapper['G']).all()


def test_identity_multiplication(wrapper: MatrixWrapper) -> None:
    """Test that multiplying by the identity doesn't change the value of a matrix."""
    assert wrapper['A'] is not None and wrapper['B'] is not None and wrapper['C'] is not None and \
           wrapper['D'] is not None and wrapper['E'] is not None and wrapper['F'] is not None and \
           wrapper['G'] is not None

    assert (wrapper.parse_expression('I') == wrapper['I']).all()
    assert (wrapper.parse_expression('AI') == wrapper['A']).all()
    assert (wrapper.parse_expression('IA') == wrapper['A']).all()
```

```
74          assert (wrapper.parse_expression('GI') == wrapper['G']).all()
75          assert (wrapper.parse_expression('IG') == wrapper['G']).all()
76
77          assert (wrapper.parse_expression('EID') == wrapper['E'] @ wrapper['D']).all()
78          assert (wrapper.parse_expression('IED') == wrapper['E'] @ wrapper['D']).all()
79          assert (wrapper.parse_expression('EDI') == wrapper['E'] @ wrapper['D']).all()
80          assert (wrapper.parse_expression('IEIDI') == wrapper['E'] @ wrapper['D']).all()
81          assert (wrapper.parse_expression('EI^3D') == wrapper['E'] @ wrapper['D']).all()
82
83
84      def test_simple_three_matrix_multiplication(wrapper: MatrixWrapper) -> None:
85          """Test simple multiplication of two matrices."""
86          assert wrapper['A'] is not None and wrapper['B'] is not None and wrapper['C'] is not None and \
87                 wrapper['D'] is not None and wrapper['E'] is not None and wrapper['F'] is not None and \
88                 wrapper['G'] is not None
89
90          assert (wrapper.parse_expression('ABC') == wrapper['A'] @ wrapper['B'] @ wrapper['C']).all()
91          assert (wrapper.parse_expression('ACB') == wrapper['A'] @ wrapper['C'] @ wrapper['B']).all()
92          assert (wrapper.parse_expression('BAC') == wrapper['B'] @ wrapper['A'] @ wrapper['C']).all()
93          assert (wrapper.parse_expression('EFG') == wrapper['E'] @ wrapper['F'] @ wrapper['G']).all()
94          assert (wrapper.parse_expression('DAC') == wrapper['D'] @ wrapper['A'] @ wrapper['C']).all()
95          assert (wrapper.parse_expression('GAE') == wrapper['G'] @ wrapper['A'] @ wrapper['E']).all()
96          assert (wrapper.parse_expression('FAG') == wrapper['F'] @ wrapper['A'] @ wrapper['G']).all()
97          assert (wrapper.parse_expression('GAF') == wrapper['G'] @ wrapper['A'] @ wrapper['F']).all()
98
99
100     def test_matrix_inverses(wrapper: MatrixWrapper) -> None:
101         """Test the inverses of single matrices."""
102         assert wrapper['A'] is not None and wrapper['B'] is not None and wrapper['C'] is not None and \
103                wrapper['D'] is not None and wrapper['E'] is not None and wrapper['F'] is not None and \
104                wrapper['G'] is not None
105
106         assert (wrapper.parse_expression('A^{-1}') == la.inv(wrapper['A'])).all()
107         assert (wrapper.parse_expression('B^{-1}') == la.inv(wrapper['B'])).all()
108         assert (wrapper.parse_expression('C^{-1}') == la.inv(wrapper['C'])).all()
109         assert (wrapper.parse_expression('D^{-1}') == la.inv(wrapper['D'])).all()
110         assert (wrapper.parse_expression('E^{-1}') == la.inv(wrapper['E'])).all()
111         assert (wrapper.parse_expression('F^{-1}') == la.inv(wrapper['F'])).all()
112         assert (wrapper.parse_expression('G^{-1}') == la.inv(wrapper['G'])).all()
113
114         assert (wrapper.parse_expression('A^-1') == la.inv(wrapper['A'])).all()
115         assert (wrapper.parse_expression('B^-1') == la.inv(wrapper['B'])).all()
116         assert (wrapper.parse_expression('C^-1') == la.inv(wrapper['C'])).all()
117         assert (wrapper.parse_expression('D^-1') == la.inv(wrapper['D'])).all()
118         assert (wrapper.parse_expression('E^-1') == la.inv(wrapper['E'])).all()
119         assert (wrapper.parse_expression('F^-1') == la.inv(wrapper['F'])).all()
120         assert (wrapper.parse_expression('G^-1') == la.inv(wrapper['G'])).all()
121
122
123     def test_matrix_powers(wrapper: MatrixWrapper) -> None:
124         """Test that matrices can be raised to integer powers."""
125         assert wrapper['A'] is not None and wrapper['B'] is not None and wrapper['C'] is not None and \
126                wrapper['D'] is not None and wrapper['E'] is not None and wrapper['F'] is not None and \
127                wrapper['G'] is not None
128
129         assert (wrapper.parse_expression('A^2') == la.matrix_power(wrapper['A'], 2)).all()
130         assert (wrapper.parse_expression('B^4') == la.matrix_power(wrapper['B'], 4)).all()
131         assert (wrapper.parse_expression('C^{12}') == la.matrix_power(wrapper['C'], 12)).all()
132         assert (wrapper.parse_expression('D^12') == la.matrix_power(wrapper['D'], 12)).all()
133         assert (wrapper.parse_expression('E^8') == la.matrix_power(wrapper['E'], 8)).all()
134         assert (wrapper.parse_expression('F^{-6}') == la.matrix_power(wrapper['F'], -6)).all()
135         assert (wrapper.parse_expression('G^-2') == la.matrix_power(wrapper['G'], -2)).all()
```

These test lots of simple expressions, but don't test any more complicated expressions, nor do they test any validation, mostly because validation doesn't really exist at this point. `'A++B'` is still a valid expression and is equivalent to `'A+B'`.

3.1.3 Simple matrix expression validation

My next major step was to implement proper parsing, but I procrastinated for a while and first implemented proper validation.

```python
# 39b918651f60bc72bc19d2018075b24a6fc3af17
# src/lintrans/_parse/matrices.py

def compile_valid_expression_pattern() -> Pattern[str]:
    """Compile the single regular expression that will match a valid matrix expression."""
    digit_no_zero = '[123456789]'
    digits = '\\d+'
    integer_no_zero = '-?' + digit_no_zero + '(' + digits + ')?'
    real_number = f'({integer_no_zero}(\\.{digits})?|-?0?\\.{digits})'

    index_content = f'({integer_no_zero}|T)'
    index = f'(\\^\\{{{index_content}\\}}|\\^{index_content}|t)'
    matrix_identifier = f'([A-Z]|rot\\({real_number}\\))'
    matrix = '(' + real_number + '?' + matrix_identifier + index + '?)'
    expression = f'{matrix}+((\\+|-){matrix}+)*'

    return re.compile(expression)


# This is an expensive pattern to compile, so we compile it when this module is initialized
valid_expression_pattern = compile_valid_expression_pattern()


def validate_matrix_expression(expression: str) -> bool:
    """Validate the given matrix expression.

    This function simply checks the expression against a BNF schema. It is not
    aware of which matrices are actually defined in a wrapper. For an aware
    version of this function, use the MatrixWrapper().is_valid_expression() method.

    Here is the schema for a valid expression given in a version of BNF:

        expression        ::=  matrices { ( "+" | "-" ) matrices };
        matrices          ::=  matrix { matrix };
        matrix            ::=  [ real_number ] matrix_identifier [ index ];
        matrix_identifier ::=  "A" .. "Z" | "rot(" real_number ")";
        index             ::=  "^{" index_content "}" | "^" index_content | "t";
        index_content     ::=  integer_not_zero | "T";

        digit_no_zero     ::=  "1" | "2" | "3" | "4" | "5" | "6" | "7" | "8" | "9";
        digit             ::=  "0" | digit_no_zero;
        digits            ::=  digit | digits digit;
        integer_not_zero  ::=  [ "-" ] digit_no_zero [ digits ];
        real_number       ::=  ( integer_not_zero [ "." digits ] | [ "-" ] [ "0" ] "." digits );

    :param str expression: The expression to be validated
    :returns bool: Whether the expression is valid according to the schema
    """
    match = valid_expression_pattern.match(expression)
    return expression == match.group(0) if match is not None else False
```

Here, I'm using a BNF schema to programmatically generate a regular expression. I use a function to generate this pattern and assign it to a variable when the module is initialized. This is because the pattern compilation is expensive and it's more efficient to compile the pattern once and then just use it in the `validate_matrix_expression()` function.

I also created a method `is_valid_expression()` in `MatrixWrapper`, which just validates a given expression. It uses the aforementioned `validate_matrix_expression()` and also checks that every matrix referenced in the expression is defined in the wrapper.

```python
# 39b918651f60bc72bc19d2018075b24a6fc3af17
# src/lintrans/matrices/wrapper.py
```

```
99          def is_valid_expression(self, expression: str) -> bool:
100             """Check if the given expression is valid, using the context of the wrapper.
101
102             This method calls _parse.validate_matrix_expression(), but also ensures
103             that all the matrices in the expression are defined in the wrapper.
104
105             :param str expression: The expression to validate
106             :returns bool: Whether the expression is valid according the schema
107             """
108             # Get rid of the transposes to check all capital letters
109             expression = re.sub(r'\^T', 't', expression)
110             expression = re.sub(r'\^{T}', 't', expression)
111
112             # Make sure all the referenced matrices are defined
113             for matrix in {x for x in expression if re.match('[A-Z]', x)}:
114                 if self[matrix] is None:
115                     return False
116
117             return _parse.validate_matrix_expression(expression)
```

I then implemented some simple tests to make sure the function works with valid and invalid expressions.

```
# a0fb029f7da995803c24ee36e7e8078e5621f676
# tests/_parse/test_parse_and_validate_expression.py

1   """Test the _parse.matrices module validation and parsing."""
2
3   import pytest
4   from lintrans._parse import validate_matrix_expression
5
6   valid_inputs: list[str] = [
7       'A', 'AB', '3A', '1.2A', '-3.4A', 'A^2', 'A^-1', 'A^{-1}',
8       'A^12', 'A^T', 'A^{5}', 'A^{T}', '4.3A^7', '9.2A^{18}',
9
10      'rot(45)', 'rot(12.5)', '3rot(90)',
11      'rot(135)^3', 'rot(51)^T', 'rot(-34)^-1',
12
13      'A+B', 'A+2B', '4.3A+9B', 'A^2+B^T', '3A^7+0.8B^{16}',
14      'A-B', '3A-4B', '3.2A^3-16.79B^T', '4.752A^{17}-3.32B^{36}',
15      'A--1B', '-A', '--1A'
16
17      '3A4B', 'A^TB', 'A^{T}B', '4A^6B^3',
18      '2A^{3}4B^5', '4rot(90)^3', 'rot(45)rot(13)',
19      'Arot(90)', 'AB^2', 'A^2B^2', '8.36A^T3.4B^12',
20
21      '3.5A^{4}5.6rot(19.2)^T-B^{-1}4.1C^5',
22  ]
23
24  invalid_inputs: list[str] = [
25      '', 'rot()', 'A^', 'A^1.2', 'A^{3.4}', '1,2A', 'ro(12)', '5', '12^2',
26      '^T', '^{12}', 'A^{13', 'A^3}', 'A^A', '^2', 'A--B', '--A'
27
28      'This is 100% a valid matrix expression, I swear'
29  ]
30
31
32  @pytest.mark.parametrize('inputs,output', [(valid_inputs, True), (invalid_inputs, False)])
33  def test_validate_matrix_expression(inputs: list[str], output: bool) -> None:
34      """Test the validate_matrix_expression() function."""
35      for inp in inputs:
36          assert validate_matrix_expression(inp) == output
```

Here, we test some valid data, some definitely invalid data, and some edge cases. At this stage, `'A--1B'` was considered a valid expression. This was a quirk of the validator at the time, but I fixed it later. This should obviously be an invalid expression, especially since `'A--B'` is considered invalid, but `'A--1B'` is valid.

The `@pytest.mark.parametrize` decorator on line 32 means that `pytest` will run one test for valid inputs, and then another test for invalid inputs, and these will count as different tests. This makes it easier to see which tests failed and then debug the app.

3.1.4 Parsing matrix expressions

Parsing is quite an interesting problem and something I didn't feel able to tackle head-on, so I wrote the unit tests first. I had a basic idea of what I wanted the parser to return, but no real idea of how to implement that. My unit tests looked like this:

```
# e9f7a81892278fe70684562052f330fb3a02bf9b
# tests/_parse/test_parse_and_validate_expression.py

40  expressions_and_parsed_expressions: list[tuple[str, MatrixParseList]] = [
41      # Simple expressions
42      ('A', [[('', 'A', '')]]),
43      ('A^2', [[('', 'A', '2')]]),
44      ('A^{2}', [[('', 'A', '2')]]),
45      ('3A', [[('3', 'A', '')]]),
46      ('1.4A^3', [[('1.4', 'A', '3')]]),
47
48      # Multiplications
49      ('4A^{3} 6B^2', [[('4', 'A', '3'), ('6', 'B', '2')]]),
50      ('4.2A^{T} 6.1B^-1', [[('4.2', 'A', 'T'), ('6.1', 'B', '-1')]]),
51      ('-1.2A^2 rot(45)^2', [[('-1.2', 'A', '2'), ('', 'rot(45)', '2')]]),
52      ('3.2A^T 4.5B^{5} 9.6rot(121.3)', [[('3.2', 'A', 'T'), ('4.5', 'B', '5'), ('9.6', 'rot(121.3)', '')]]),
53      ('-1.18A^{-2} 0.1B^{2} 9rot(34.6)^-1', [[('-1.18', 'A', '-2'), ('0.1', 'B', '2'), ('9', 'rot(34.6)', '-1')]]),
54
55      # Additions
56      ('A + B', [[('', 'A', '')], [('', 'B', '')]]),
57      ('A + B - C', [[('', 'A', '')], [('', 'B', '')], [('-1', 'C', '')]]),
58      ('2A^3 + 8B^T - 3C^-1', [[('2', 'A', '3')], [('8', 'B', 'T')], [('-3', 'C', '-1')]]),
59
60      # Additions with multiplication
61      ('2.14A^{3} 4.5rot(14.5)^-1 + 8B^T - 3C^-1', [[('2.14', 'A', '3'), ('4.5', 'rot(14.5)', '-1')],
62                                                    [('8', 'B', 'T')], [('-3', 'C', '-1')]]),
63      ('2.14A^{3} 4.5rot(14.5)^-1 + 8.5B^T 5.97C^4 - 3.14D^{-1} 6.7E^T',
64       [[('2.14', 'A', '3'), ('4.5', 'rot(14.5)', '-1')], [('8.5', 'B', 'T'), ('5.97', 'C', '4')],
65        [('-3.14', 'D', '-1'), ('6.7', 'E', 'T')]]),
66  ]
67
68
69  @pytest.mark.skip(reason='parse_matrix_expression() not implemented')
70  def test_parse_matrix_expression() -> None:
71      """Test the parse_matrix_expression() function."""
72      for expression, parsed_expression in expressions_and_parsed_expressions:
73          # Test it with and without whitespace
74          assert parse_matrix_expression(expression) == parsed_expression
75          assert parse_matrix_expression(expression.replace(' ', '')) == parsed_expression
```

I just had example inputs and what I expected as output. I also wanted the parser to ignore whitespace. The decorator on line 69 just skips the test because the parser wasn't implemented yet.

When implementing the parser, I first had to tighten up validation to remove anomalies like `'A--1B'` being valid. I did this by factoring out the optional minus signs from being part of a number, to being optionally in front of a number. This eliminated this kind of repetition and made `'A--1B'` invalid, as it should be.

```
# fd80d8d3b0e975e92dcc7c10f1f0f1276879f408
# src/lintrans/_parse/matrices.py

32  def compile_valid_expression_pattern() -> Pattern[str]:
33      """Compile the single regular expression that will match a valid matrix expression."""
34      digit_no_zero = '[123456789]'
35      digits = '\\d+'
```

```
36          integer_no_zero = digit_no_zero + '(' + digits + ')?'
37          real_number = f'({integer_no_zero}(\\.{digits})?|0?\\.{digits})'
38
39          index_content = f'(-?{integer_no_zero}|T)'
40          index = f'(\\^\\{{{index_content}\\}}|\\^{index_content}|t)'
41          matrix_identifier = f'([A-Z]|rot\\(-?{real_number}\\))'
42          matrix = '(' + real_number + '?' + matrix_identifier + index + '?)'
43          expression = f'-?{matrix}+((\\+|-){matrix}+)*'
44
45          return re.compile(expression)
```

The code can be a bit hard to read with all the RegEx stuff, but the BNF illustrates these changes nicely.

Compare the old version:

```
# 39b918651f60bc72bc19d2018075b24a6fc3af17
# src/lintrans/_parse/matrices.py

38  expression        ::=  matrices { ( "+" | "-" ) matrices };
39  matrices          ::=  matrix { matrix };
40  matrix            ::=  [ real_number ] matrix_identifier [ index ];
41  matrix_identifier ::=  "A" .. "Z" | "rot(" real_number ")";
42  index             ::=  "^{" index_content "}" | "^" index_content | "t";
43  index_content     ::=  integer_not_zero | "T";
44
45  digit_no_zero     ::=  "1" | "2" | "3" | "4" | "5" | "6" | "7" | "8" | "9";
46  digit             ::=  "0" | digit_no_zero;
47  digits            ::=  digit | digits digit;
48  integer_not_zero  ::=  [ "-" ] digit_no_zero [ digits ];
49  real_number       ::=  ( integer_not_zero [ "." digits ] | [ "-" ] [ "0" ] "." digits );
```

to the new version:

```
# fd80d8d3b0e975e92dcc7c10f1f0f1276879f408
# src/lintrans/_parse/matrices.py

61  expression        ::=  [ "-" ] matrices { ( "+" | "-" ) matrices };
62  matrices          ::=  matrix { matrix };
63  matrix            ::=  [ real_number ] matrix_identifier [ index ];
64  matrix_identifier ::=  "A" .. "Z" | "rot(" [ "-" ] real_number ")";
65  index             ::=  "^{" index_content "}" | "^" index_content | "t";
66  index_content     ::=  [ "-" ] integer_not_zero | "T";
67
68  digit_no_zero     ::=  "1" | "2" | "3" | "4" | "5" | "6" | "7" | "8" | "9";
69  digit             ::=  "0" | digit_no_zero;
70  digits            ::=  digit | digits digit;
71  integer_not_zero  ::=  digit_no_zero [ digits ];
72  real_number       ::=  ( integer_not_zero [ "." digits ] | [ "0" ] "." digits );
```

Then once I'd fixed the validation, I could implement the parser itself.

```
# fd80d8d3b0e975e92dcc7c10f1f0f1276879f408
# src/lintrans/_parse/matrices.py

86  def parse_matrix_expression(expression: str) -> MatrixParseList:
87      """Parse the matrix expression and return a list of results.
88
89      The return value is a list of results. This results list contains lists of tuples.
90      The top list is the expressions that should be added together, and each sublist
91      is expressions that should be multiplied together. These expressions to be
92      multiplied are tuples, where each tuple is (multiplier, matrix identifier, index).
93      The multiplier can be any real number, the matrix identifier is either a named
94      matrix or a new rotation matrix declared with 'rot()', and the index is an
95      integer or 'T' for transpose.
96
```

```
97          :param str expression: The expression to be parsed
98          :returns MatrixParseTuple: A list of results
99          """
100         # Remove all whitespace
101         expression = re.sub(r'\s', '', expression)
102
103         # Check if it's valid
104         if not validate_matrix_expression(expression):
105             raise MatrixParseError('Invalid expression')
106
107         # Wrap all exponents and transposition powers with {}
108         expression = re.sub(r'(?<=\^)(-?\d+|T)(?=[^}]|$)', r'{\g<0>}', expression)
109
110         # Remove any standalone minuses
111         expression = re.sub(r'-(?=[A-Z])', '-1', expression)
112
113         # Replace subtractions with additions
114         expression = re.sub(r'-(?=\d+\.?\d*([A-Z]|rot))', '+-', expression)
115
116         # Get rid of a potential leading + introduced by the last step
117         expression = re.sub(r'^\+', '', expression)
118
119         return [
120             [
121                 # The tuple returned by re.findall is (multiplier, matrix identifier, full index, stripped index),
122                 # so we have to remove the full index, which contains the {}
123                 (t[0], t[1], t[3])
124                 for t in re.findall(r'(-?\d+\.?\d*)?([A-Z]|rot\(-?\d+\.?\d*\))(\^{(-?\d+|T)})?', group)
125             ]
126             # We just split the expression by '+' to have separate groups
127             for group in expression.split('+')
128         ]
```

It works similarly to the old `MatrixWrapper.parse_expression()` method in §3.1.2 but with a powerful list comprehension at the end. It splits the expression up into groups and then uses some RegEx magic to find all the matrices in these groups as a tuple.

This method passes all the unit tests, as expected.

My next step was then to rewrite the evaluation to use this new parser, like so (method name and docstring removed):

```
# a453774bcdf824676461f9b9b441d7b94969ea55
# src/lintrans/matrices/wrapper.py
```

```
168         if not self.is_valid_expression(expression):
169             raise ValueError('The expression is invalid')
170
171         parsed_result = _parse.parse_matrix_expression(expression)
172         final_groups: list[list[MatrixType]] = []
173
174         for group in parsed_result:
175             f_group: list[MatrixType] = []
176
177             for matrix in group:
178                 if matrix[2] == 'T':
179                     m = self[matrix[1]]
180                     assert m is not None
181                     matrix_value = m.T
182                 else:
183                     matrix_value = np.linalg.matrix_power(self[matrix[1]],
184                                                           1 if (index := matrix[2]) == '' else int(index))
185
186                 matrix_value *= 1 if (multiplier := matrix[0]) == '' else float(multiplier)
187                 f_group.append(matrix_value)
188
189             final_groups.append(f_group)
190
191         return reduce(add, [reduce(matmul, group) for group in final_groups])
```

Here, we go through the list of tuples and evaluate the matrix represented by each tuple, putting this together in a list as we go. Then at the end, we simply reduce the sublists and then reduce these new matrices using a list comprehension in the `reduce()` call using `add` and `matmul` from the `operator` library. It's written in a functional programming style, and it passes all the previous tests.

3.2 Initial GUI

3.2.1 First basic GUI

The discrepancy in all the GUI code between `snake_case` and `camelCase` is because Qt5 was originally a C++ framework that was adapted into PyQt5 for Python. All the Qt API is in `camelCase`, but my Python code is in `snake_case`.

```
# 93ce763f7b993439fc0da89fad39456d8cc4b52c
# src/lintrans/gui/main_window.py

"""The module to provide the main window as a QMainWindow object."""

import sys

from PyQt5 import QtCore, QtGui, QtWidgets
from PyQt5.QtWidgets import QApplication, QHBoxLayout, QMainWindow, QVBoxLayout

from lintrans.matrices import MatrixWrapper


class LintransMainWindow(QMainWindow):
    """The class for the main window in the lintrans GUI."""

    def __init__(self):
        """Create the main window object, creating every widget in it."""
        super().__init__()

        self.matrix_wrapper = MatrixWrapper()

        self.setWindowTitle('Linear Transformations')
        self.setMinimumWidth(750)

        # === Create widgets

        # Left layout: the plot and input box

        # NOTE: This QGraphicsView is only temporary
        self.plot = QtWidgets.QGraphicsView(self)

        self.text_input_expression = QtWidgets.QLineEdit(self)
        self.text_input_expression.setPlaceholderText('Input matrix expression...')
        self.text_input_expression.textChanged.connect(self.update_render_buttons)

        # Right layout: all the buttons

        # Misc buttons

        self.button_create_polygon = QtWidgets.QPushButton(self)
        self.button_create_polygon.setText('Create polygon')
        # TODO: Implement create_polygon()
        # self.button_create_polygon.clicked.connect(self.create_polygon)
        self.button_create_polygon.setToolTip('Define a new polygon to view the transformation of')

        self.button_change_display_settings = QtWidgets.QPushButton(self)
        self.button_change_display_settings.setText('Change\ndisplay settings')
        # TODO: Implement change_display_settings()
        # self.button_change_display_settings.clicked.connect(self.change_display_settings)
        self.button_change_display_settings.setToolTip('Change which things are rendered on the plot')

        # Define new matrix buttons

```

```
52          self.label_define_new_matrix = QtWidgets.QLabel(self)
53          self.label_define_new_matrix.setText('Define a\nnew matrix')
54          self.label_define_new_matrix.setAlignment(QtCore.Qt.AlignCenter)
55
56          # TODO: Implement defining a new matrix visually, numerically, as a rotation, and as an expression
57
58          self.button_define_visually = QtWidgets.QPushButton(self)
59          self.button_define_visually.setText('Visually')
60          self.button_define_visually.setToolTip('Drag the basis vectors')
61
62          self.button_define_numerically = QtWidgets.QPushButton(self)
63          self.button_define_numerically.setText('Numerically')
64          self.button_define_numerically.setToolTip('Define a matrix just with numbers')
65
66          self.button_define_as_rotation = QtWidgets.QPushButton(self)
67          self.button_define_as_rotation.setText('As a rotation')
68          self.button_define_as_rotation.setToolTip('Define an angle to rotate by')
69
70          self.button_define_as_expression = QtWidgets.QPushButton(self)
71          self.button_define_as_expression.setText('As an expression')
72          self.button_define_as_expression.setToolTip('Define a matrix in terms of other matrices')
73
74          # Render buttons
75
76          self.button_render = QtWidgets.QPushButton(self)
77          self.button_render.setText('Render')
78          self.button_render.setEnabled(False)
79          self.button_render.clicked.connect(self.render_expression)
80          self.button_render.setToolTip('Render the expression<br><b>(Ctrl + Enter)</b>')
81
82          self.button_render_shortcut = QtWidgets.QShortcut(QtGui.QKeySequence('Ctrl+Return'), self)
83          self.button_render_shortcut.activated.connect(self.button_render.click)
84
85          self.button_animate = QtWidgets.QPushButton(self)
86          self.button_animate.setText('Animate')
87          self.button_animate.setEnabled(False)
88          self.button_animate.clicked.connect(self.animate_expression)
89          self.button_animate.setToolTip('Animate the expression<br><b>(Ctrl + Shift + Enter)</b>')
90
91          self.button_animate_shortcut = QtWidgets.QShortcut(QtGui.QKeySequence('Ctrl+Shift+Return'), self)
92          self.button_animate_shortcut.activated.connect(self.button_animate.click)
93
94          # === Arrange widgets
95
96          self.setContentsMargins(10, 10, 10, 10)
97
98          self.vlay_left = QVBoxLayout()
99          self.vlay_left.addWidget(self.plot)
100         self.vlay_left.addWidget(self.text_input_expression)
101
102         self.vlay_misc_buttons = QVBoxLayout()
103         self.vlay_misc_buttons.setSpacing(20)
104         self.vlay_misc_buttons.addWidget(self.button_create_polygon)
105         self.vlay_misc_buttons.addWidget(self.button_change_display_settings)
106
107         self.vlay_define_new_matrix = QVBoxLayout()
108         self.vlay_define_new_matrix.setSpacing(20)
109         self.vlay_define_new_matrix.addWidget(self.label_define_new_matrix)
110         self.vlay_define_new_matrix.addWidget(self.button_define_visually)
111         self.vlay_define_new_matrix.addWidget(self.button_define_numerically)
112         self.vlay_define_new_matrix.addWidget(self.button_define_as_rotation)
113         self.vlay_define_new_matrix.addWidget(self.button_define_as_expression)
114
115         self.vlay_render = QVBoxLayout()
116         self.vlay_render.setSpacing(20)
117         self.vlay_render.addWidget(self.button_animate)
118         self.vlay_render.addWidget(self.button_render)
119
120         self.vlay_right = QVBoxLayout()
121         self.vlay_right.setSpacing(50)
122         self.vlay_right.addLayout(self.vlay_misc_buttons)
123         self.vlay_right.addLayout(self.vlay_define_new_matrix)
124         self.vlay_right.addLayout(self.vlay_render)
```

```
125
126             self.hlay_all = QHBoxLayout()
127             self.hlay_all.setSpacing(15)
128             self.hlay_all.addLayout(self.vlay_left)
129             self.hlay_all.addLayout(self.vlay_right)
130
131             self.central_widget = QtWidgets.QWidget()
132             self.central_widget.setLayout(self.hlay_all)
133             self.setCentralWidget(self.central_widget)
134
135         def update_render_buttons(self) -> None:
136             """Enable or disable the render and animate buttons according to the validity of the matrix expression."""
137             valid = self.matrix_wrapper.is_valid_expression(self.text_input_expression.text())
138             self.button_render.setEnabled(valid)
139             self.button_animate.setEnabled(valid)
140
141         def render_expression(self) -> None:
142             """Render the expression in the input box, and then clear the box."""
143             # TODO: Render the expression
144             self.text_input_expression.setText('')
145
146         def animate_expression(self) -> None:
147             """Animate the expression in the input box, and then clear the box."""
148             # TODO: Animate the expression
149             self.text_input_expression.setText('')
150
151
152     def main() -> None:
153         """Run the GUI."""
154         app = QApplication(sys.argv)
155         window = LintransMainWindow()
156         window.show()
157         sys.exit(app.exec_())
158
159
160     if __name__ == '__main__':
161         main()
```

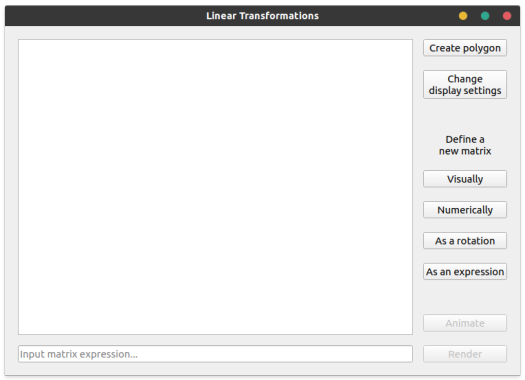


Figure 3.1: The first version of the GUI

A lot of the methods here don't have implementations yet, but they will. This version is just a very early prototype to get a rough draft of the GUI.

I create the widgets and layouts in the constructor as well as configuring all of them. The most important non-constructor method is `update_render_buttons()`. It gets called whenever the text in `text_input_expression` is changed. This happens because we connect it to the `textChanged` signal on line 32.

The big white box here will eventually be replaced with an actual viewport. This is just a prototype.

3.2.2 Numerical definition dialog

My next major addition was a dialog that would allow the user to define a matrix numerically.

```
# cedbd3ed126a1183f197c27adf6dabb4e5d301c7
# src/lintrans/gui/dialogs/define_new_matrix.py

1   """The module to provide dialogs for defining new matrices."""
2
3   from numpy import array
4   from PyQt5 import QtGui, QtWidgets
5   from PyQt5.QtWidgets import QDialog, QGridLayout, QHBoxLayout, QVBoxLayout
```

```
6
7   from lintrans.matrices import MatrixWrapper
8
9   ALPHABET_NO_I = 'ABCDEFGHJKLMNOPQRSTUVWXYZ'
10
11
12  def is_float(string: str) -> bool:
13      """Check if a string is a float."""
14      try:
15          float(string)
16          return True
17      except ValueError:
18          return False
19
20
21  class DefineNumericallyDialog(QDialog):
22      """The dialog class that allows the user to define a new matrix numerically."""
23
24      def __init__(self, matrix_wrapper: MatrixWrapper, *args, **kwargs):
25          """Create the dialog, but don't run it yet.
26
27          :param matrix_wrapper: The MatrixWrapper that this dialog will mutate
28          :type matrix_wrapper: MatrixWrapper
29          """
30          super().__init__(*args, **kwargs)
31
32          self.matrix_wrapper = matrix_wrapper
33          self.setWindowTitle('Define a matrix')
34
35          # === Create the widgets
36
37          self.button_confirm = QtWidgets.QPushButton(self)
38          self.button_confirm.setText('Confirm')
39          self.button_confirm.setEnabled(False)
40          self.button_confirm.clicked.connect(self.confirm_matrix)
41          self.button_confirm.setToolTip('Confirm this as the new matrix<br><b>(Ctrl + Enter)</b>')
42
43          QtWidgets.QShortcut(QtGui.QKeySequence('Ctrl+Return'), self).activated.connect(self.button_confirm.click)
44
45          self.button_cancel = QtWidgets.QPushButton(self)
46          self.button_cancel.setText('Cancel')
47          self.button_cancel.clicked.connect(self.close)
48          self.button_cancel.setToolTip('Cancel this definition<br><b>(Ctrl + Q)</b>')
49
50          QtWidgets.QShortcut(QtGui.QKeySequence('Ctrl+Q'), self).activated.connect(self.button_cancel.click)
51
52          self.element_tl = QtWidgets.QLineEdit(self)
53          self.element_tl.textChanged.connect(self.update_confirm_button)
54
55          self.element_tr = QtWidgets.QLineEdit(self)
56          self.element_tr.textChanged.connect(self.update_confirm_button)
57
58          self.element_bl = QtWidgets.QLineEdit(self)
59          self.element_bl.textChanged.connect(self.update_confirm_button)
60
61          self.element_br = QtWidgets.QLineEdit(self)
62          self.element_br.textChanged.connect(self.update_confirm_button)
63
64          self.matrix_elements = (self.element_tl, self.element_tr, self.element_bl, self.element_br)
65
66          self.letter_combo_box = QtWidgets.QComboBox(self)
67
68          # Everything except I, because that's the identity
69          for letter in ALPHABET_NO_I:
70              self.letter_combo_box.addItem(letter)
71
72          self.letter_combo_box.activated.connect(self.load_matrix)
73
74          # === Arrange the widgets
75
76          self.setContentsMargins(10, 10, 10, 10)
77
78          self.grid_matrix = QGridLayout()
```

```
79              self.grid_matrix.setSpacing(20)
80              self.grid_matrix.addWidget(self.element_tl, 0, 0)
81              self.grid_matrix.addWidget(self.element_tr, 0, 1)
82              self.grid_matrix.addWidget(self.element_bl, 1, 0)
83              self.grid_matrix.addWidget(self.element_br, 1, 1)
84
85              self.hlay_buttons = QHBoxLayout()
86              self.hlay_buttons.setSpacing(20)
87              self.hlay_buttons.addWidget(self.button_cancel)
88              self.hlay_buttons.addWidget(self.button_confirm)
89
90              self.vlay_right = QVBoxLayout()
91              self.vlay_right.setSpacing(20)
92              self.vlay_right.addLayout(self.grid_matrix)
93              self.vlay_right.addLayout(self.hlay_buttons)
94
95              self.hlay_all = QHBoxLayout()
96              self.hlay_all.setSpacing(20)
97              self.hlay_all.addWidget(self.letter_combo_box)
98              self.hlay_all.addLayout(self.vlay_right)
99
100             self.setLayout(self.hlay_all)
101
102             # Finally, we load the default matrix A into the boxes
103             self.load_matrix(0)
104
105         def update_confirm_button(self) -> None:
106             """Enable the confirm button if there are numbers in every box."""
107             for elem in self.matrix_elements:
108                 if elem.text() == '' or not is_float(elem.text()):
109                     # If they're not all numbers, then we can't confirm it
110                     self.button_confirm.setEnabled(False)
111                     return
112
113             # If we didn't find anything invalid
114             self.button_confirm.setEnabled(True)
115
116         def load_matrix(self, index: int) -> None:
117             """If the selected matrix is defined, load it into the boxes."""
118             matrix = self.matrix_wrapper[ALPHABET_NO_I[index]]
119
120             if matrix is None:
121                 for elem in self.matrix_elements:
122                     elem.setText('')
123
124             else:
125                 self.element_tl.setText(str(matrix[0][0]))
126                 self.element_tr.setText(str(matrix[0][1]))
127                 self.element_bl.setText(str(matrix[1][0]))
128                 self.element_br.setText(str(matrix[1][1]))
129
130             self.update_confirm_button()
131
132         def confirm_matrix(self) -> None:
133             """Confirm the inputted matrix and assign it to the name."""
134             letter = self.letter_combo_box.currentText()
135             matrix = array([
136                 [float(self.element_tl.text()), float(self.element_tr.text())],
137                 [float(self.element_bl.text()), float(self.element_br.text())]
138             ])
139
140             self.matrix_wrapper[letter] = matrix
141             self.close()
```

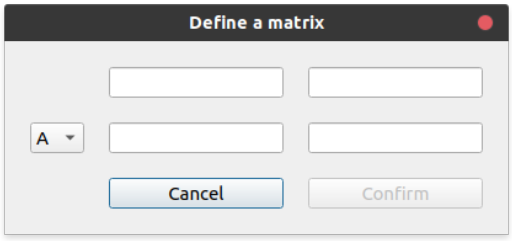


Figure 3.2: The first version of the numerical definition dialog

When I add more definition dialogs, I will factor out a superclass, but this is just a prototype to make sure it all works as intended.

Hopefully the methods are relatively self explanatory, but they're just utility methods to update the GUI when things are changed. We connect the `QLineEdit` widgets to the `update_confirm_button()` slot to make sure the confirm button is always up to date.

The `confirm_matrix()` method just updates the instance's matrix wrapper with the new matrix. We pass a reference to the `LintransMainWindow` instance's matrix wrapper when we open the dialog, so we're just updating the referenced object directly.

In the `LintransMainWindow` class, we're just connecting a lambda slot to the button so that it opens the dialog, as seen here:

```
# cedbd3ed126a1183f197c27adf6dabb4e5d301c7
# src/lintrans/gui/main_window.py

66  self.button_define_numerically.clicked.connect(
67      lambda: DefineNumericallyDialog(self.matrix_wrapper, self).exec()
68  )
```

3.2.3 More definition dialogs

I then factored out the constructor into a `DefineDialog` superclass so that I could easily create other definition dialogs.

```
# 5d04fb7233a03d0cd8fa0768f6387c6678da9df3
# src/lintrans/gui/dialogs/define_new_matrix.py

22  class DefineDialog(QDialog):
23      """A superclass for definitions dialogs."""
24
25      def __init__(self, matrix_wrapper: MatrixWrapper, *args, **kwargs):
26          """Create the dialog, but don't run it yet.
27
28          :param matrix_wrapper: The MatrixWrapper that this dialog will mutate
29          :type matrix_wrapper: MatrixWrapper
30          """
31          super().__init__(*args, **kwargs)
32
33          self.matrix_wrapper = matrix_wrapper
34          self.setWindowTitle('Define a matrix')
35
36          # === Create the widgets
37
38          self.button_confirm = QtWidgets.QPushButton(self)
39          self.button_confirm.setText('Confirm')
40          self.button_confirm.setEnabled(False)
41          self.button_confirm.clicked.connect(self.confirm_matrix)
42          self.button_confirm.setToolTip('Confirm this as the new matrix<br><b>(Ctrl + Enter)</b>')
43          QShortcut(QKeySequence('Ctrl+Return'), self).activated.connect(self.button_confirm.click)
44
45          self.button_cancel = QtWidgets.QPushButton(self)
46          self.button_cancel.setText('Cancel')
47          self.button_cancel.clicked.connect(self.close)
48          self.button_cancel.setToolTip('Cancel this definition<br><b>(Ctrl + Q)</b>')
49          QShortcut(QKeySequence('Ctrl+Q'), self).activated.connect(self.button_cancel.click)
50
51          self.label_equals = QtWidgets.QLabel()
```

```
52          self.label_equals.setText('=')
53
54          self.letter_combo_box = QtWidgets.QComboBox(self)
55
56          # Everything except I, because that's the identity
57          for letter in ALPHABET_NO_I:
58              self.letter_combo_box.addItem(letter)
59
60          self.letter_combo_box.activated.connect(self.load_matrix)
```

This superclass just has a constructor that subclasses can use. When I added the `DefineAsARotationDialog` class, I also moved the cancel and confirm buttons into the constructor and added abstract methods that all dialog subclasses must implement.

```
# 0d534c35c6a4451e317d41a0d2b3ecb17827b45f
# src/lintrans/gui/dialogs/define_new_matrix.py

61          # === Arrange the widgets
62
63          self.setContentsMargins(10, 10, 10, 10)
64
65          self.horizontal_spacer = QSpacerItem(50, 5, hPolicy=QSizePolicy.Expanding, vPolicy=QSizePolicy.Minimum)
66
67          self.hlay_buttons = QHBoxLayout()
68          self.hlay_buttons.setSpacing(20)
69          self.hlay_buttons.addItem(self.horizontal_spacer)
70          self.hlay_buttons.addWidget(self.button_cancel)
71          self.hlay_buttons.addWidget(self.button_confirm)
72
73      @property
74      def selected_letter(self) -> str:
75          """The letter currently selected in the combo box."""
76          return self.letter_combo_box.currentText()
77
78      @abc.abstractmethod
79      def update_confirm_button(self) -> None:
80          """Enable the confirm button if it should be enabled."""
81          ...
82
83      @abc.abstractmethod
84      def confirm_matrix(self) -> None:
85          """Confirm the inputted matrix and assign it.
86
87          This should mutate self.matrix_wrapper and then call self.accept().
88          """
89          ...
```

I then added the class for the rotation definition dialog.

```
# 0d534c35c6a4451e317d41a0d2b3ecb17827b45f
# src/lintrans/gui/dialogs/define_new_matrix.py

182 class DefineAsARotationDialog(DefineDialog):
183     """The dialog that allows the user to define a new matrix as a rotation."""
184
185     def __init__(self, matrix_wrapper: MatrixWrapper, *args, **kwargs):
186         """Create the dialog, but don't run it yet."""
187         super().__init__(matrix_wrapper, *args, **kwargs)
188
189         # === Create the widgets
190
191         self.label_equals.setText('= rot(')
192
193         self.text_angle = QtWidgets.QLineEdit(self)
194         self.text_angle.setPlaceholderText('angle')
195         self.text_angle.textChanged.connect(self.update_confirm_button)
196
197         self.label_close_paren = QtWidgets.QLabel(self)
```

```
198             self.label_close_paren.setText(')')
199
200             self.checkbox_radians = QtWidgets.QCheckBox(self)
201             self.checkbox_radians.setText('Radians')
202
203             # === Arrange the widgets
204
205             self.hlay_checkbox_and_buttons = QHBoxLayout()
206             self.hlay_checkbox_and_buttons.setSpacing(20)
207             self.hlay_checkbox_and_buttons.addWidget(self.checkbox_radians)
208             self.hlay_checkbox_and_buttons.addItem(self.horizontal_spacer)
209             self.hlay_checkbox_and_buttons.addLayout(self.hlay_buttons)
210
211             self.hlay_definition = QHBoxLayout()
212             self.hlay_definition.addWidget(self.letter_combo_box)
213             self.hlay_definition.addWidget(self.label_equals)
214             self.hlay_definition.addWidget(self.text_angle)
215             self.hlay_definition.addWidget(self.label_close_paren)
216
217             self.vlay_all = QVBoxLayout()
218             self.vlay_all.setSpacing(20)
219             self.vlay_all.addLayout(self.hlay_definition)
220             self.vlay_all.addLayout(self.hlay_checkbox_and_buttons)
221
222             self.setLayout(self.vlay_all)
223
224         def update_confirm_button(self) -> None:
225             """Enable the confirm button if there is a valid float in the angle box."""
226             self.button_confirm.setEnabled(is_float(self.text_angle.text()))
227
228         def confirm_matrix(self) -> None:
229             """Confirm the inputted matrix and assign it."""
230             self.matrix_wrapper[self.selected_letter] = create_rotation_matrix(
231                 float(self.text_angle.text()),
232                 degrees=not self.checkbox_radians.isChecked()
233             )
234             self.accept()
```

This dialog class just overrides the abstract methods of the superclass with its own implementations. This will be the pattern that all of the definition dialogs will follow.

It has a checkbox for radians, since this is supported in `create_rotation_matrix()`, but the textbox only supports numbers, so the user would have to calculate some multiple of π and paste in several decimal places. I expect people to only use degrees, because these are easier to use.

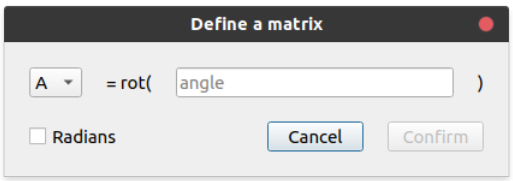


Figure 3.3: The first version of the rotation definition dialog

Additionally, I created a helper method in `LintransMainWindow`. Rather than connecting the `clicked` signal of the buttons to lambdas that instantiate an instance of the `DefineDialog` subclass and call `.exec()` on it, I now connect the `clicked` signal of the buttons to lambdas that call `self.dialog_define_matrix()` with the specific subclass.

```
# 6269e04d453df7be2d2f9c7ee176e83406ccc139
# src/lintrans/gui/main_window.py

170         def dialog_define_matrix(self, dialog_class: Type[DefineDialog]) -> None:
171             """Open a generic definition dialog to define a new matrix.
172
173             The class for the desired dialog is passed as an argument. We create an
174             instance of this class and the dialog is opened asynchronously and modally
175             (meaning it blocks interaction with the main window) with the proper method
176             connected to the ``dialog.finished`` slot.
177
178             .. note::
```

```
179             ``dialog_class`` must subclass :class:`lintrans.gui.dialogs.define_new_matrix.DefineDialog`.
180
181         :param dialog_class: The dialog class to instantiate
182         :type dialog_class: Type[lintrans.gui.dialogs.define_new_matrix.DefineDialog]
183         """
184         # We create a dialog with a deepcopy of the current matrix_wrapper
185         # This avoids the dialog mutating this one
186         dialog = dialog_class(deepcopy(self.matrix_wrapper), self)
187
188         # .open() is asynchronous and doesn't spawn a new event loop, but the dialog is still modal (blocking)
189         dialog.open()
190
191         # So we have to use the finished slot to call a method when the user accepts the dialog
192         # If the user rejects the dialog, this matrix_wrapper will be the same as the current one, because we copied
            ↪ it
193         # So we don't care, we just assign the wrapper anyway
194         dialog.finished.connect(lambda: self._assign_matrix_wrapper(dialog.matrix_wrapper))
195
196     def _assign_matrix_wrapper(self, matrix_wrapper: MatrixWrapper) -> None:
197         """Assign a new value to self.matrix_wrapper.
198
199         This is a little utility function that only exists because a lambda
200         callback can't directly assign a value to a class attribute.
201
202         :param matrix_wrapper: The new value of the matrix wrapper to assign
203         :type matrix_wrapper: MatrixWrapper
204         """
205         self.matrix_wrapper = matrix_wrapper
```

I also then implemented a simple `DefineAsAnExpressionDialog`, which evaluates a given expression in the current `MatrixWrapper` context and assigns the result to the given matrix name.

```
# d5f930e15c3c8798d4990486532da46e926a6cb9
# src/lintrans/gui/dialogs/define_new_matrix.py

241 class DefineAsAnExpressionDialog(DefineDialog):
242     """The dialog that allows the user to define a matrix as an expression."""
243
244     def __init__(self, matrix_wrapper: MatrixWrapper, *args, **kwargs):
245         """Create the dialog, but don't run it yet."""
246         super().__init__(matrix_wrapper, *args, **kwargs)
247
248         self.setMinimumWidth(450)
249
250         # === Create the widgets
251
252         self.text_box_expression = QtWidgets.QLineEdit(self)
253         self.text_box_expression.setPlaceholderText('Enter matrix expression...')
254         self.text_box_expression.textChanged.connect(self.update_confirm_button)
255
256         # === Arrange the widgets
257
258         self.hlay_definition.addWidget(self.text_box_expression)
259
260         self.vlay_all = QVBoxLayout()
261         self.vlay_all.setSpacing(20)
262         self.vlay_all.addLayout(self.hlay_definition)
263         self.vlay_all.addLayout(self.hlay_buttons)
264
265         self.setLayout(self.vlay_all)
266
267     def update_confirm_button(self) -> None:
268         """Enable the confirm button if the expression is valid."""
269         self.button_confirm.setEnabled(
270             self.matrix_wrapper.is_valid_expression(self.text_box_expression.text())
271         )
272
273     def confirm_matrix(self) -> None:
274         """Evaluate the matrix expression and assign its value to the chosen matrix."""
275         self.matrix_wrapper[self.selected_letter] = \
```

```
276                 self.matrix_wrapper.evaluate_expression(self.text_box_expression.text())
277             self.accept()
```

My next dialog that I wanted to implement was a visual definition dialog, which would allow the user to drag around the basis vectors to define a transformation. However, I would first need to create the `lintrans.gui.plots` package to allow for actually visualizing matrices and transformations.

3.3 Visualizing matrices

3.3.1 Asking strangers on the internet for help

After creating most of the GUI skeleton, I wanted to build the viewport. Unfortunately, I had no idea what I was doing.

While looking through the PyQt5 docs, I found a pretty comprehensive explanation of the Qt5 'Graphics View Framework'[14], which seemed pretty good, but not really what I was looking for. I wanted a way to easily draw lots of straight, parallel lines. This framework seemed more focussed on manipulating objects on a canvas, almost like sprites. I knew of a different Python library called `matplotlib`, which has various backends available. I learned that it could be embedded in a standard PyQt5 GUI, so I started doing some research.

I didn't get very far with `matplotlib`. I hadn't used it much before and it's designed for visualizing data. It can draw manually defined straight lines on a canvas, but that's not what it's designed for and it's not very good at it. Thankfully, my horrific `matplotlib` code has been lost to time. I used the `Qt5Agg` backend from `matplotlib` to create a custom PyQt5 widget for the GUI and I could graph randomly generated data with it after following a tutorial[13].

I realised that I wasn't going to get very far with `matplotlib`, but I didn't know what else to do. I couldn't find any relevant examples on the internet, so I decided to post a question on a forum myself. I'd had experience with StackOverflow and its unfriendly community before, so I decided to ask the `r/learnpython` subreddit[3].

I only got one response, but it was incredibly helpful. The person told me that if I couldn't find an easy way to do what I wanted, I could write a custom PyQt5 widget. I knew this was possible with a class that just inherited from `QWidget`, but had no idea how to actually make something useful. Thankfully, this person provided a link to a GitLab repository of theirs, where they had multiple examples of custom widgets with PyQt5[4].

When looking through this repo, I found out how to draw on a widget like a simple canvas. All I have to do is override the `paintEvent()` method and use a `QPainter` object to draw on the widget. I used this knowledge to start creating the actual viewport for the GUI, starting with the background axes.

3.3.2 Creating the plots package

Initially, the `lintrans.gui.plots` package just has some classes for widgets. `TransformationPlotWidget` acts as a base class and then `ViewTransformationWidget` acts as a wrapper. I will expand this class in the future.

```
# 4af63072b383dc9cef9adbb8900323aa007e7f26
# src/lintrans/gui/plots/plot_widget.py

"""This module provides the basic classes for plotting transformations."""

from __future__ import annotations

from PyQt5.QtCore import Qt
```

```python
from PyQt5.QtGui import QColor, QPainter, QPaintEvent, QPen
from PyQt5.QtWidgets import QWidget


class TransformationPlotWidget(QWidget):
    """An abstract superclass for plot widgets.

    This class provides a background (untransformed) plane, and all the backend
    details for a Qt application, but does not provide useful functionality. To
    be useful, this class must be subclassed and behaviour must be implemented
    by the subclass.

    .. warning:: This class should never be directly instantiated, only subclassed.

    .. note::
       I would make this class have ``metaclass=abc.ABCMeta``, but I can't because it subclasses ``QWidget``,
       and a every superclass of a class must have the same metaclass, and ``QWidget`` is not an abstract class.
    """

    def __init__(self, *args, **kwargs):
        """Create the widget, passing ``*args`` and ``**kwargs`` to the superclass constructor (``QWidget``)."""
        super().__init__(*args, **kwargs)

        self.setAutoFillBackground(True)

        # Set the background to white
        palette = self.palette()
        palette.setColor(self.backgroundRole(), Qt.white)
        self.setPalette(palette)

        # Set the gird colour to grey and the axes colour to black
        self.grid_colour = QColor(128, 128, 128)
        self.axes_colour = QColor(0, 0, 0)

        self.grid_spacing: int = 50
        self.line_width: float = 0.4

    @property
    def w(self) -> int:
        """Return the width of the widget."""
        return self.size().width()

    @property
    def h(self) -> int:
        """Return the height of the widget."""
        return self.size().height()

    def paintEvent(self, e: QPaintEvent):
        """Handle a ``QPaintEvent`` by drawing the widget."""
        qp = QPainter()
        qp.begin(self)
        self.draw_widget(qp)
        qp.end()

    def draw_widget(self, qp: QPainter):
        """Draw the grid and axes in the widget."""
        qp.setRenderHint(QPainter.Antialiasing)
        qp.setBrush(Qt.NoBrush)

        # Draw the grid
        qp.setPen(QPen(self.grid_colour, self.line_width))

        # We draw the background grid, centered in the middle
        # We deliberately exclude the axes - these are drawn separately
        for x in range(self.w // 2 + self.grid_spacing, self.w, self.grid_spacing):
            qp.drawLine(x, 0, x, self.h)
            qp.drawLine(self.w - x, 0, self.w - x, self.h)

        for y in range(self.h // 2 + self.grid_spacing, self.h, self.grid_spacing):
            qp.drawLine(0, y, self.w, y)
            qp.drawLine(0, self.h - y, self.w, self.h - y)

        # Now draw the axes
```

```
79          qp.setPen(QPen(self.axes_colour, self.line_width))
80          qp.drawLine(self.w // 2, 0, self.w // 2, self.h)
81          qp.drawLine(0, self.h // 2, self.w, self.h // 2)
82
83
84  class ViewTransformationWidget(TransformationPlotWidget):
85      """This class is used to visualise matrices as transformations."""
86
87      def __init__(self, *args, **kwargs):
88          """Create the widget, passing ``*args`` and ``**kwargs`` to the superclass constructor."""
89          super().__init__(*args, **kwargs)
```

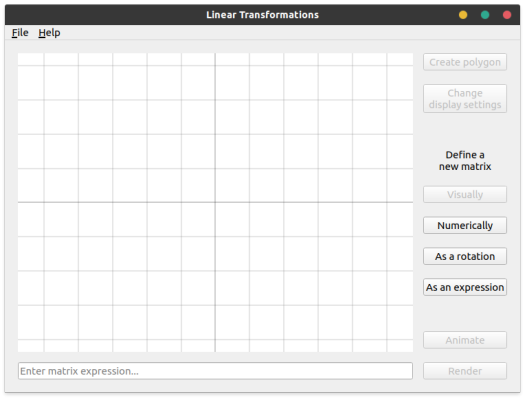


Figure 3.4: The GUI with background axes

The meat of this class is the `draw_widget()` method. Right now, this method only draws the background axes. My next step is to implement basis vector attributes and draw them in `draw_widget()`. After changing the the `plot` attribute in `LintransMainWindow` to an instance of `ViewTransformationWidget`, the plot was visible in the GUI.

I then refactored the code slightly to rename `draw_widget()` to `draw_background()` and then call it from the `paintEvent()` method in `ViewTransformationWidget`.

3.3.3 Implementing basis vectors

My first step in implementing basis vectors was to add some utility methods to convert between coordinate systems. The matrices are using Cartesian coordinates with $(0, 0)$ in the middle, positive x going to the right, and positive y going up. However, Qt5 is using standard computer graphics coordinates, with $(0, 0)$ in the top left, positive x going to the right, and positive y going down. I needed a way to convert Cartesian 'grid' coordinates to Qt5 'canvas' coordinates, so I wrote some little utility methods.

```
# 1fa7e1c61d61cb6aeff773b9698541f82fee39ea
# src/lintrans/gui/plots/plot_widget.py

45      @property
46      def origin(self) -> tuple[int, int]:
47          """Return the canvas coords of the origin."""
48          return self.width() // 2, self.height() // 2
49
50      def trans_x(self, x: float) -> int:
51          """Transform an x coordinate from grid coords to canvas coords."""
52          return int(self.origin[0] + x * self.grid_spacing)
53
54      def trans_y(self, y: float) -> int:
55          """Transform a y coordinate from grid coords to canvas coords."""
56          return int(self.origin[1] - y * self.grid_spacing)
57
58      def trans_coords(self, x: float, y: float) -> tuple[int, int]:
59          """Transform a coordinate in grid coords to canvas coords."""
60          return self.trans_x(x), self.trans_y(y)
```

Once I had a way to convert coordinates, I could add the basis vectors themselves. I did this by creating attributes for the points in the constructor and creating a `transform_by_matrix()` method to change these point attributes accordingly.

```
# 37e7c208a33d7cbbc8e0bb6c94cd889e2918c605
# src/lintrans/gui/plots/plot_widget.py
```

```
92  class ViewTransformationWidget(TransformationPlotWidget):
93      """This class is used to visualise matrices as transformations."""
94
95      def __init__(self, *args, **kwargs):
96          """Create the widget, passing ``*args`` and ``**kwargs`` to the superclass constructor."""
97          super().__init__(*args, **kwargs)
98
99          self.point_i: tuple[float, float] = (1., 0.)
100         self.point_j: tuple[float, float] = (0., 1.)
101
102         self.colour_i = QColor(37, 244, 15)
103         self.colour_j = QColor(8, 8, 216)
104
105         self.width_vector_line = 1
106         self.width_transformed_grid = 0.6
107
108     def transform_by_matrix(self, matrix: MatrixType) -> None:
109         """Transform the plane by the given matrix."""
110         self.point_i = (matrix[0][0], matrix[1][0])
111         self.point_j = (matrix[0][1], matrix[1][1])
112         self.update()
```

I also created a `draw_transformed_grid()` method which gets called in `paintEvent()`.

```
# 37e7c208a33d7cbbc8e0bb6c94cd889e2918c605
# src/lintrans/gui/plots/plot_widget.py

122     def draw_transformed_grid(self, painter: QPainter) -> None:
123         """Draw the transformed version of the grid, given by the unit vectors."""
124         # Draw the unit vectors
125         painter.setPen(QPen(self.colour_i, self.width_vector_line))
126         painter.drawLine(*self.origin, *self.trans_coords(*self.point_i))
127         painter.setPen(QPen(self.colour_j, self.width_vector_line))
128         painter.drawLine(*self.origin, *self.trans_coords(*self.point_j))
```

I then changed the `render_expression()` method in `LintransMainWindow` to call this new `transform_by_matrix()` method.

```
# 37e7c208a33d7cbbc8e0bb6c94cd889e2918c605
# src/lintrans/gui/main_window.py

229     def render_expression(self) -> None:
230         """Render the expression in the input box, and then clear the box."""
231         self.plot.transform_by_matrix(
232             self.matrix_wrapper.evaluate_expression(
233                 self.lineedit_expression_box.text()
234             )
235         )
```

Testing this new code shows that it works well.

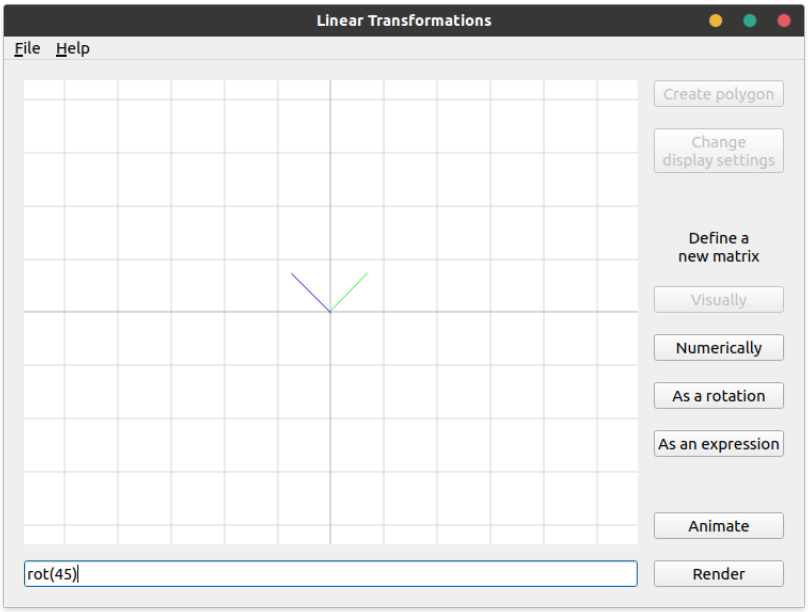


Figure 3.5: Basis vectors drawn for a 45° rotation

3.3.4 Drawing the transformed grid

After drawing the basis vectors, I wanted to draw the transformed version of the grid. I first created a `grid_corner()` utility method to return the grid coordinates of the top right corner of the canvas. This allows me to find the bounding box in which to draw the grid lines.

```
# 2ade98ac28d1c3f6691e4afa819142a3ab8e9fd9
# src/lintrans/gui/plots/plot_widget.py

64      def grid_corner(self) -> tuple[float, float]:
65          """Return the grid coords of the top right corner."""
66          return self.width() / (2 * self.grid_spacing), self.height() / (2 * self.grid_spacing)
```

I then created a `draw_parallel_lines()` method that would fill the bounding box with a set of lines parallel to a given vector with spacing defined by the intersection with a given point.

```
# 2ade98ac28d1c3f6691e4afa819142a3ab8e9fd9
# src/lintrans/gui/plots/plot_widget.py

126     def draw_parallel_lines(self, painter: QPainter, vector: tuple[float, float], point: tuple[float, float]) ->
        ↪ None:
127         """Draw a set of grid lines parallel to ``vector`` intersecting ``point``."""
128         max_x, max_y = self.grid_corner()
129         vector_x, vector_y = vector
130         point_x, point_y = point
131
132         if vector_x == 0:
133             painter.drawLine(self.trans_x(0), 0, self.trans_x(0), self.height())
134
135             for i in range(int(max_x / point_x)):
136                 painter.drawLine(
137                     self.trans_x((i + 1) * point_x),
138                     0,
139                     self.trans_x((i + 1) * point_x),
140                     self.height()
141                 )
142                 painter.drawLine(
143                     self.trans_x(-1 * (i + 1) * point_x),
```

```
                    0,
                    self.trans_x(-1 * (i + 1) * point_x),
                    self.height()
                )

        elif vector_y == 0:
            painter.drawLine(0, self.trans_y(0), self.width(), self.trans_y(0))

            for i in range(int(max_y / point_y)):
                painter.drawLine(
                    0,
                    self.trans_y((i + 1) * point_y),
                    self.width(),
                    self.trans_y((i + 1) * point_y)
                )
                painter.drawLine(
                    0,
                    self.trans_y(-1 * (i + 1) * point_y),
                    self.width(),
                    self.trans_y(-1 * (i + 1) * point_y)
                )
```

I then called this method from `draw_transformed_grid()`.

```
# 2ade98ac28d1c3f6691e4afa819142a3ab8e9fd9
# src/lintrans/gui/plots/plot_widget.py

    def draw_transformed_grid(self, painter: QPainter) -> None:
        """Draw the transformed version of the grid, given by the unit vectors."""
        # Draw the unit vectors
        painter.setPen(QPen(self.colour_i, self.width_vector_line))
        painter.drawLine(*self.origin, *self.trans_coords(*self.point_i))
        painter.setPen(QPen(self.colour_j, self.width_vector_line))
        painter.drawLine(*self.origin, *self.trans_coords(*self.point_j))

        # Draw all the parallel lines
        painter.setPen(QPen(self.colour_i, self.width_transformed_grid))
        self.draw_parallel_lines(painter, self.point_i, self.point_j)
        painter.setPen(QPen(self.colour_j, self.width_transformed_grid))
        self.draw_parallel_lines(painter, self.point_j, self.point_i)
```

This worked quite well when the matrix involved no rotation, as seen on the right, but this didn't work with rotation. When trying `'rot(45)'` for example, it looked the same as in Figure 3.5.

Also, the vectors aren't particularly clear. They'd be much better with arrowheads on their tips, but this is just a prototype. The arrowheads will come later.

My next step was to make the transformed grid lines work with rotations.

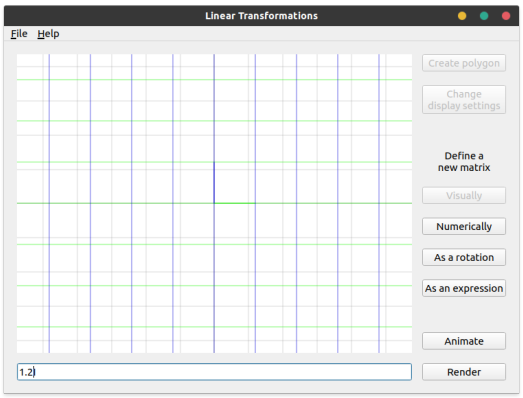


Figure 3.6: Parallel lines being drawn for matrix $1.2\mathbf{I}$

```
# 7dfe1e24729562501e2fd88a839dca6b653a3375
# src/lintrans/gui/plots/plot_widget.py

def draw_parallel_lines(self, painter: QPainter, vector: tuple[float, float], point: tuple[float, float]) -> None:
    """Draw a set of grid lines parallel to ``vector`` intersecting ``point``."""
    max_x, max_y = self.grid_corner()
    vector_x, vector_y = vector
    point_x, point_y = point
```

```
131
132         print(max_x, max_y, vector_x, vector_y, point_x, point_y)
133
134         # We want to use y = mx + c but m = y / x and if either of those are 0, then this
135         # equation is harder to work with, so we deal with these edge cases first
136         if abs(vector_x) < 1e-12 and abs(vector_y) < 1e-12:
137             # If both components of the vector are practically 0, then we can't render any grid lines
138             return
139
140         elif abs(vector_x) < 1e-12:
141             painter.drawLine(self.trans_x(0), 0, self.trans_x(0), self.height())
142
143             for i in range(abs(int(max_x / point_x))):
144                 painter.drawLine(
145                     self.trans_x((i + 1) * point_x),
146                     0,
147                     self.trans_x((i + 1) * point_x),
148                     self.height()
149                 )
150                 painter.drawLine(
151                     self.trans_x(-1 * (i + 1) * point_x),
152                     0,
153                     self.trans_x(-1 * (i + 1) * point_x),
154                     self.height()
155                 )
156
157         elif abs(vector_y) < 1e-12:
158             painter.drawLine(0, self.trans_y(0), self.width(), self.trans_y(0))
159
160             for i in range(abs(int(max_y / point_y))):
161                 painter.drawLine(
162                     0,
163                     self.trans_y((i + 1) * point_y),
164                     self.width(),
165                     self.trans_y((i + 1) * point_y)
166                 )
167                 painter.drawLine(
168                     0,
169                     self.trans_y(-1 * (i + 1) * point_y),
170                     self.width(),
171                     self.trans_y(-1 * (i + 1) * point_y)
172                 )
173
174         else:  # If the line is not horizontal or vertical, then we can use y = mx + c
175             m = vector_y / vector_x
176             c = point_y - m * point_x
177
178             # For c = 0
179             painter.drawLine(
180                 *self.trans_coords(
181                     -1 * max_x,
182                     m * -1 * max_x
183                 ),
184                 *self.trans_coords(
185                     max_x,
186                     m * max_x
187                 )
188             )
189
190             # Count up how many multiples of c we can have without wasting time rendering lines off screen
191             multiples_of_c: int = 0
192             ii: int = 1
193             while True:
194                 y1 = m * max_x + ii * c
195                 y2 = -1 * m * max_x + ii * c
196
197                 if y1 < max_y or y2 < max_y:
198                     multiples_of_c += 1
199                     ii += 1
200
201                 else:
202                     break
203
```

```
204             # Once we know how many lines we can draw, we just draw them all
205             for i in range(1, multiples_of_c + 1):
206                 painter.drawLine(
207                     *self.trans_coords(
208                         -1 * max_x,
209                         m * -1 * max_x + i * c
210                     ),
211                     *self.trans_coords(
212                         max_x,
213                         m * max_x + i * c
214                     )
215                 )
216                 painter.drawLine(
217                     *self.trans_coords(
218                         -1 * max_x,
219                         m * -1 * max_x - i * c
220                     ),
221                     *self.trans_coords(
222                         max_x,
223                         m * max_x - i * c
224                     )
225                 )
```

This code checks if x or y is zero[10] and if they're not, then we have to use the standard straight line equation $y = mx + c$ to create parallel lines. We find our value of m and then iterate through all the values of c that keep the line within the bounding box.

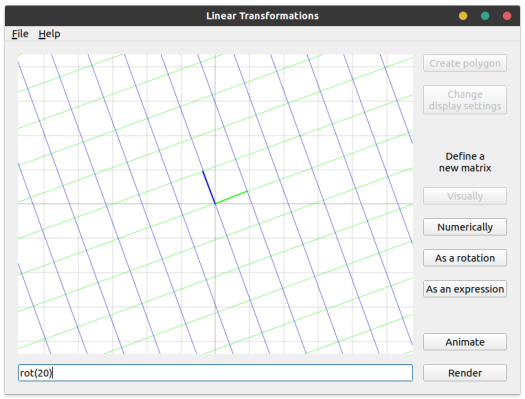


Figure 3.7: An example of a 20° rotation

There are some serious logical errors in this code. It works fine for things like `'3rot(45)'` or `'0.5rot(20)'`, but something like `'rot(115)'` will leave the program hanging indefinitely.

In fact, this code only works for rotations between 0° and 90°, and will hang forever when given a matrix like $\left(\begin{smallmatrix} 12 & 4 \\ -2 & 3 \end{smallmatrix}\right)$, because it's just not very good.

I will fix these issues in the future, but it works somewhat decently, so I decided to do animation next, because that sounded more fun.

3.3.5 Implementing animation

Now that I had a very crude renderer, I could create a method to animate a matrix. Eventually I want to be able to apply a given matrix to the currently rendered scene and animate between them. However, I wanted to start simple by animating from the identity to the given matrix.

```
# 829a130af5aee9819bf0269c03ecfb20bec1a108
# src/lintrans/gui/main_window.py

238     def animate_expression(self) -> None:
239         """Animate the expression in the input box, and then clear the box."""
240         self.button_render.setEnabled(False)
241         self.button_animate.setEnabled(False)
242
243         matrix = self.matrix_wrapper.evaluate_expression(self.lineedit_expression_box.text())
244         matrix_move = matrix - self.matrix_wrapper['I']
245         steps: int = 100
246
247         for i in range(0, steps + 1):
```

[10] We actually check if they're less than 10^{-12} to allow for floating point errors

```
248             self.plot.visualize_matrix_transformation(
249                 self.matrix_wrapper['I'] + (i / steps) * matrix_move
250             )
251
252             self.update()
253             self.repaint()
254
255             time.sleep(0.01)
256
257         self.button_render.setEnabled(False)
258         self.button_animate.setEnabled(False)
```

This code creates the `matrix_move` variable and adds scaled versions of it to the identity matrix and renders that each frame. It's simple, but it works well for this simple use case. Unfortunately, it's very hard to show off an animation in a PDF, since all these images are static. The git commit hashes are included in the code snippets if you want to clone the repo[2], checkout this commit, and run it yourself if you want.

3.3.6 Preserving determinants

Ignoring the obvious flaw with not being able to render transformations with a more than 90° rotation, the animations don't respect determinants. When rotating 90°, the determinant changes during the animation, even though we're going from a determinant 1 matrix (the identity) to another determinant 1 matrix. This is because we're just moving each vector to its new position in a straight line. I want to animate in a way that smoothly transitions the determinant.

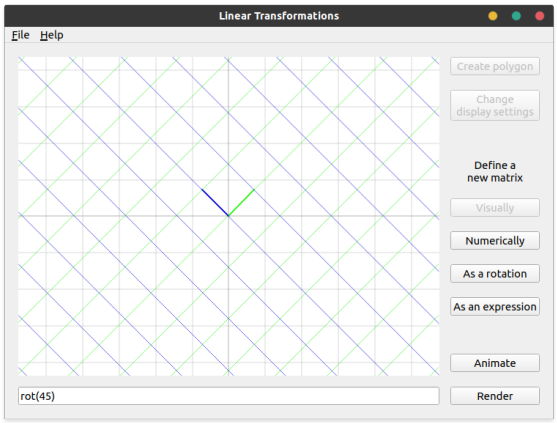


Figure 3.8: What we would expect halfway through a 90° rotation

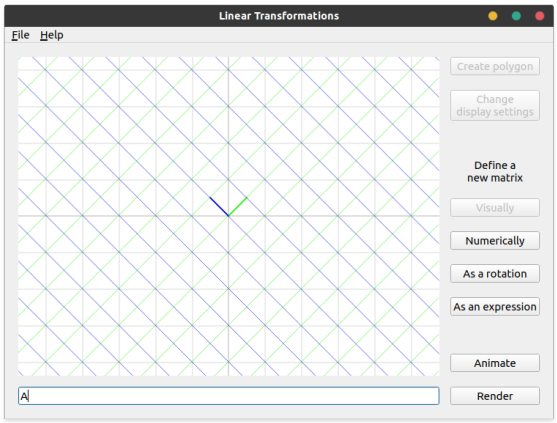


Figure 3.9: What we actually get halfway through a 90° rotation

In order to smoothly animate the determinant, I had to do some maths. I first defined the matrix $\mathbf{A}$ to be equivalent to the `matrix_move` variable from before - the target matrix minus the identity, scaled by the proportion. I then wanted to normalize $\mathbf{A}$ so that it had a determinant of 1 so that I could scale it up with the `proportion` variable through the animation.

I think I first tried just multiplying $\mathbf{A}$ by $\frac{1}{\det(\mathbf{A})}$ but that didn't work, so I googled it. I found a post[12] on ResearchGate about the topic, and thanks to a very helpful comment from Jeffrey L Stuart, I learned that for a 2×2 matrix $\mathbf{A}$ and a scalar c, $\det(c\mathbf{A}) = c^2 \det(\mathbf{A})$.

I wanted a c such that $\det(c\mathbf{A}) = 1$. Therefore $c = \frac{1}{\sqrt{|\det(\mathbf{A})|}}$. I then defined matrix $\mathbf{B}$ to be $c\mathbf{A}$.

Then I wanted to scale this normalized matrix $\mathbf{B}$ to have the same determinant as the target matrix $\mathbf{T}$ using some scalar d. We know that $\det(d\mathbf{B}) = d^2 \det(\mathbf{B}) = \det(\mathbf{T})$. We can just rearrange to find d

and get $d = \sqrt{\left|\frac{\det(\mathbf{T})}{\det(\mathbf{B})}\right|}$. But $\mathbf{B}$ is defined so that $\det(\mathbf{B}) = 1$, so we can get $d = \sqrt{|\det(\mathbf{T})|}$.

However, we want to scale this over time with our `proportion` variable p, so our final scalar $s = 1 + p\left(\sqrt{|\det(\mathbf{T})|} - 1\right)$. We define a matrix $\mathbf{C} = s\mathbf{B}$ and render $\mathbf{C}$ each frame. When in code form, this is the following:

```
# 6ff49450d8438ea2b2e7d2a97125dc518e648bc5
# src/lintrans/gui/main_window.py
```

```
245             # Get the target matrix and it's determinant
246             matrix_target = self.matrix_wrapper.evaluate_expression(self.lineedit_expression_box.text())
247             det_target = linalg.det(matrix_target)
248 
249             identity = self.matrix_wrapper['I']
250             steps: int = 100
251 
252             for i in range(0, steps + 1):
253                 # This proportion is how far we are through the loop
254                 proportion = i / steps
255 
256                 # matrix_a is the identity plus some part of the target, scaled by the proportion
257                 # If we just used matrix_a, then things would animate, but the determinants would be weird
258                 matrix_a = identity + proportion * (matrix_target - identity)
259 
260                 # So to fix the determinant problem, we get the determinant of matrix_a and use it to normalise
261                 det_a = linalg.det(matrix_a)
262 
263                 # For a 2x2 matrix A and a scalar c, we know that det(cA) = c^2 det(A)
264                 # We want B = cA such that det(B) = 1, so then we can scale it with the animation
265                 # So we get c^2 det(A) = 1 => c = sqrt(1 / abs(det(A)))
266                 # Then we scale A down to get a determinant of 1, and call that matrix_b
267                 if det_a == 0:
268                     c = 0
269                 else:
270                     c = np.sqrt(1 / abs(det_a))
271 
272                 matrix_b = c * matrix_a
273 
274                 # matrix_c is the final matrix that we transform by
275                 # It's B, but we scale it up over time to have the target determinant
276 
277                 # We want some C = dB such that det(C) is some target determinant T
278                 # det(dB) = d^2 det(B) = T => d = sqrt(abs(T / det(B))
279                 # But we defined B to have det 1, so we can ignore it there
280 
281                 # We're also subtracting 1 and multiplying by the proportion and then adding one
282                 # This just scales the determinant along with the animation
283                 scalar = 1 + proportion * (np.sqrt(abs(det_target)) - 1)
284 
285                 matrix_c = scalar * matrix_b
286 
287                 self.plot.visualize_matrix_transformation(matrix_c)
288 
289                 self.repaint()
290                 time.sleep(0.01)
```

Unfortunately, the system I use to render matrices is still quite bad at its job. This makes it hard to test properly. But, transformations like `'2rot(90)'` work exactly as expected, which is very good.

3.4 Improving the GUI

3.4.1 Fixing rendering

Now that I had the basics of matrix visualization sorted, I wanted to make the GUI and UX better. My first step was overhauling the rendering code to make it actually work with rotations of more than 90°.

I narrowed down the issue with PyCharm's debugger and found that the loop in `VectorGridPlot.draw_parallel_lines()` was looping forever if it tried to doing anything outside of the top right quadrant. To fix this, I decided to instead delegate this task of drawing a set of oblique lines to a separate method, and work on that instead.

```
# cf05e09e5ebb6ea7a96db8660d0d8de6b946490a
# src/lintrans/gui/plots/classes.py
```

```
203             else:  # If the line is not horizontal or vertical, then we can use y = mx + c
204                 m = vector_y / vector_x
205                 c = point_y - m * point_x
206 
207                 # For c = 0
208                 painter.drawLine(
209                     *self.trans_coords(
210                         -1 * max_x,
211                         m * -1 * max_x
212                     ),
213                     *self.trans_coords(
214                         max_x,
215                         m * max_x
216                     )
217                 )
218 
219                 # We keep looping and increasing the multiple of c until we stop drawing lines on the canvas
220                 multiple_of_c = 1
221                 while self.draw_pair_of_oblique_lines(painter, m, multiple_of_c * c):
222                     multiple_of_c += 1
```

This separation of functionality made designing and debugging this part of the solution much easier. The `draw_pair_of_oblique_lines()` method looked like this:

```
# cf05e09e5ebb6ea7a96db8660d0d8de6b946490a
# src/lintrans/gui/plots/classes.py
```

```
224     def draw_pair_of_oblique_lines(self, painter: QPainter, m: float, c: float) -> bool:
225         """Draw a pair of oblique lines, using the equation y = mx + c.
226 
227         This method just calls :meth:`draw_oblique_line` with ``c`` and ``-c``,
228         and returns True if either call returned True.
229 
230         :param QPainter painter: The ``QPainter`` object to use for drawing the vectors and grid lines
231         :param float m: The gradient of the lines to draw
232         :param float c: The y-intercept of the lines to draw. We use the positive and negative versions
233         :returns bool: Whether we were able to draw any lines on the canvas
234         """
235         return any([
236             self.draw_oblique_line(painter, m, c),
237             self.draw_oblique_line(painter, m, -c)
238         ])
239 
240     def draw_oblique_line(self, painter: QPainter, m: float, c: float) -> bool:
241         """Draw an oblique line, using the equation y = mx + c.
242 
243         We only draw the part of the line that fits within the canvas, returning True if
244         we were able to draw a line within the boundaries, and False if we couldn't draw a line
245 
246         :param QPainter painter: The ``QPainter`` object to use for drawing the vectors and grid lines
```

```
247         :param float m: The gradient of the line to draw
248         :param float c: The y-intercept of the line to draw
249         :returns bool: Whether we were able to draw a line on the canvas
250         """
251         max_x, max_y = self.grid_corner()
252 
253         # These variable names are shortened for convenience
254         # myi is max_y_intersection, mmyi is minus_max_y_intersection, etc.
255         myi = (max_y - c) / m
256         mmyi = (-max_y - c) / m
257         mxi = max_x * m + c
258         mmxi = -max_x * m + c
259 
260         # The inner list here is a list of coords, or None
261         # If an intersection fits within the bounds, then we keep its coord,
262         # else it is None, and then gets discarded from the points list
263         # By the end, points is a list of two coords, or an empty list
264         points: list[tuple[float, float]] = [
265             x for x in [
266                 (myi, max_y) if -max_x < myi < max_x else None,
267                 (mmyi, -max_y) if -max_x < mmyi < max_x else None,
268                 (max_x, mxi) if -max_y < mxi < max_y else None,
269                 (-max_x, mmxi) if -max_y < mmxi < max_y else None
270             ] if x is not None
271         ]
272 
273         # If no intersections fit on the canvas
274         if len(points) < 2:
275             return False
276 
277         # If we can, then draw the line
278         else:
279             painter.drawLine(
280                 *self.trans_coords(*points[0]),
281                 *self.trans_coords(*points[1])
282             )
283             return True
```

To illustrate what this code is doing, I'll use a diagram.

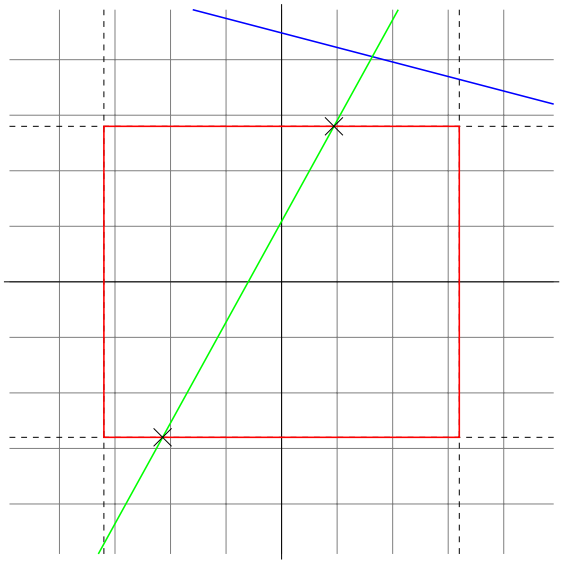


Figure 3.10: Two example lines and the viewport box

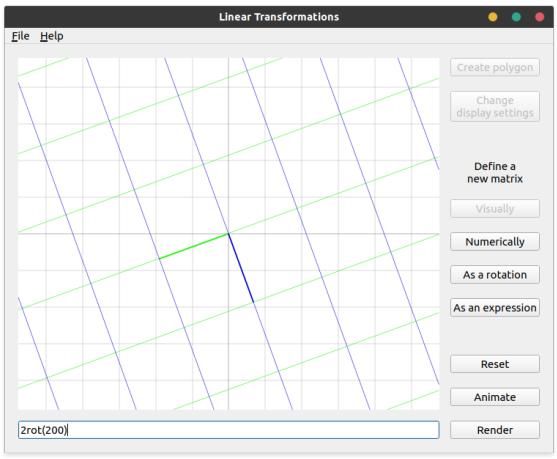


Figure 3.11: A demonstration of the new oblique lines system.

The red box represents the viewport of the GUI. The dashed lines represent the extensions of the red box. For a given line we want to draw, we first want to find where it intersects these orthogonal lines. Any oblique line will intersect each of these lines exactly once. This is what the `myi`, `mmyi`, `mxi`, and

`mmxi` variables represent. The value of `myi` is the x value where the line intersects the maximum y line, for example.

In the case of the blue line, all 4 intersection points are outside the bounds of the box, whereas the green line intersects with the box, as shown with the crosses. We use a list comprehension over a list of ternaries to get the `points` list. This list contains 0 or 2 coordinates, and we may or may not draw a line accordingly.

That's how the `draw_oblique_line()` method works, and the `draw_pair_of_oblique_lines()` method just calls it with positive and negative values of c.

3.4.2 Adding vector arrowheads

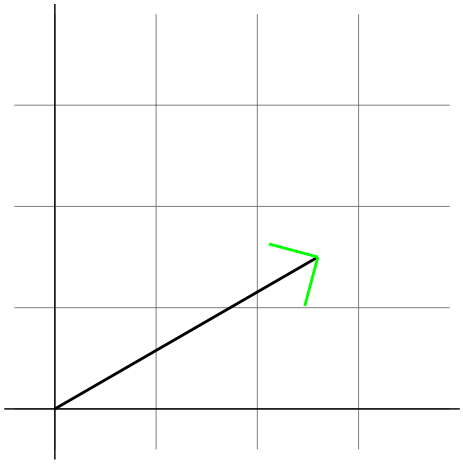


Figure 3.12: An example of a vector with the arrowheads highlighted in green

Now that I had a good renderer, I wanted to add arrowheads to the vectors to make them easier to see. They were already thicker than the gridlines, but adding arrowheads like in the 3blue1brown series would make them much easier to see. Unfortunately, I couldn't work out how to do this.

I wanted a function that would take a coordinate, treat it as a unit vector, and draw lines at 45° angles at the tip. This wasn't how I was conceptualising the problem at the time and because of that, I couldn't work out how to solve this problem. I could create this 45° lines in the top right quadrant, but none of my possible solutions worked for any arbitrary point.

So I started googling and found a very nice algorithm on `csharphelper.com`[23][11], which I adapted for Python.

```
# 5373b1ad8040f6726147cccea523c0570251cf67
# src/lintrans/gui/plots/widgets.py
```

```
52      def draw_arrowhead_away_from_origin(self, painter: QPainter, point: tuple[float, float]) -> None:
53          """Draw an arrowhead at ``point``, pointing away from the origin.
54
55          :param QPainter painter: The ``QPainter`` object to use to draw the arrowheads with
56          :param point: The point to draw the arrowhead at, given in grid coords
57          :type point: tuple[float, float]
58          """
59          # This algorithm was adapted from a C# algorithm found at
60          # http://csharphelper.com/blog/2014/12/draw-lines-with-arrowheads-in-c/
61
62          # Get the x and y coords of the point, and then normalize them
63          # We have to normalize them, or else the size of the arrowhead will
64          # scale with the distance of the point from the origin
65          x, y = point
66          nx = x / np.sqrt(x * x + y * y)
67          ny = y / np.sqrt(x * x + y * y)
68
69          # We choose a length and do some magic to find the steps in the x and y directions
70          length = 0.15
71          dx = length * (-nx - ny)
72          dy = length * (nx - ny)
73
74          # Then we just plot those lines
75          painter.drawLine(*self.trans_coords(x, y), *self.trans_coords(x + dx, y + dy))
76          painter.drawLine(*self.trans_coords(x, y), *self.trans_coords(x - dy, y + dx))
```

[11] This website is currently being rewritten and this arrowheads tutorial is listed as 'not rebuilt' on `http://csharphelper.com`

```
77
78          def draw_vector_arrowheads(self, painter: QPainter) -> None:
79              """Draw arrowheads at the tips of the basis vectors.
80
81              :param QPainter painter: The ``QPainter`` object to use to draw the arrowheads with
82              """
83              painter.setPen(QPen(self.colour_i, self.width_vector_line))
84              self.draw_arrowhead_away_from_origin(painter, self.point_i)
85              painter.setPen(QPen(self.colour_j, self.width_vector_line))
86              self.draw_arrowhead_away_from_origin(painter, self.point_j)
```

As the comments suggest, we get the x and y components of the normalised vector, and then do some magic with a chosen length and get some distance values, and then draw those lines. I don't really understand how this code works, but I'm happy that it does. All we have to do is call `draw_vector_arrowheads()` from `paintEvent()`.

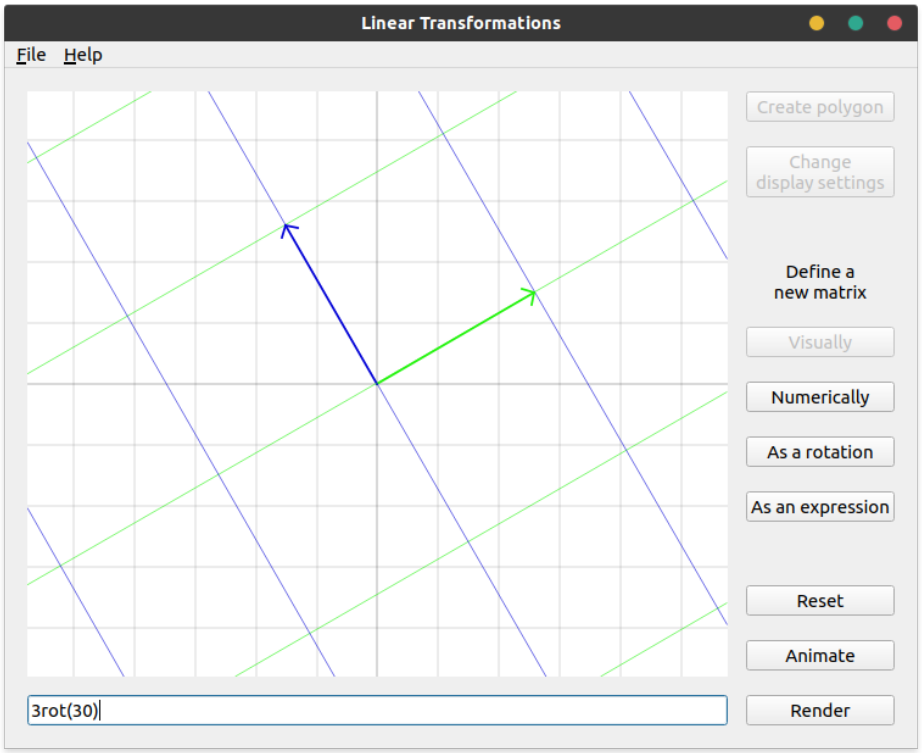


Figure 3.13: An example of the i and j vectors with arrowheads

3.4.3 Implementing zoom

The next thing I wanted to do was add the ability to zoom in and out of the viewport, and I wanted a button to reset the zoom level as well. I added a `default_grid_spacing` class attribute in `BackgroundPlot` and used that as the `grid_spacing` instance attribute in `__init__()`.

```
# d944e86e1d0fdc2c4be4d63479bc6bc3a31568ef
# src/lintrans/gui/plots/classes.py

27          default_grid_spacing: int = 50
28
29          def __init__(self, *args, **kwargs):
30              """Create the widget and setup backend stuff for rendering.
31
32              .. note:: ``*args`` and ``**kwargs`` are passed the superclass constructor (``QWidget``).
33              """
```

```
34          super().__init__(*args, **kwargs)
35
36          self.setAutoFillBackground(True)
37
38          # Set the background to white
39          palette = self.palette()
40          palette.setColor(self.backgroundRole(), Qt.white)
41          self.setPalette(palette)
42
43          # Set the gird colour to grey and the axes colour to black
44          self.colour_background_grid = QColor(128, 128, 128)
45          self.colour_background_axes = QColor(0, 0, 0)
46
47          self.grid_spacing = BackgroundPlot.default_grid_spacing
```

The reset button in `LintransMainWindow` simply sets `plot.grid_spacing` to the default.

To actually allow for zooming, I had to implement the `wheelEvent()` method in `BackgroundPlot` to listen for mouse wheel events. After reading through the docs for the `QWheelEvent` class[18], I learned how to handle this event.

```
# d944e86e1d0fdc2c4be4d63479bc6bc3a31568ef
# src/lintrans/gui/plots/classes.py

119     def wheelEvent(self, event: QWheelEvent) -> None:
120         """Handle a ``QWheelEvent`` by zooming in or our of the grid."""
121         # angleDelta() returns a number of units equal to 8 times the number of degrees rotated
122         degrees = event.angleDelta() / 8
123
124         if degrees is not None:
125             self.grid_spacing = max(1, self.grid_spacing + degrees.y())
126
127         event.accept()
128         self.update()
```

All we do is get the amount that the user scrolled and add that to the current spacing, taking the max with 1, which acts as a minimum grid spacing. We need to use `degrees.y()` on line 125 because Qt5 allows for mice that can scroll in the x and y directions, and we only want the y component. Line 127 marks the event as accepted so that the parent widget doesn't try to act on it.

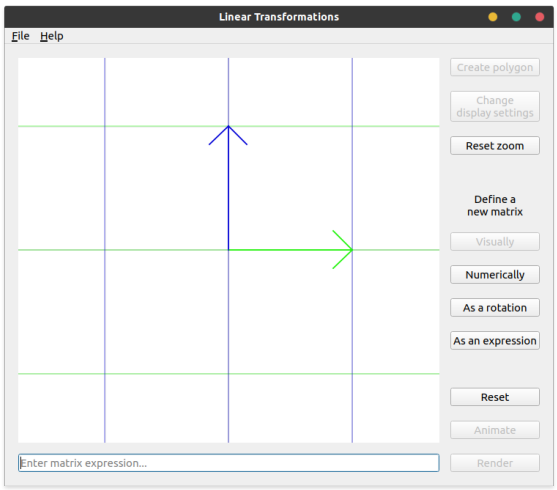


Figure 3.14: The GUI zoomed in a bit

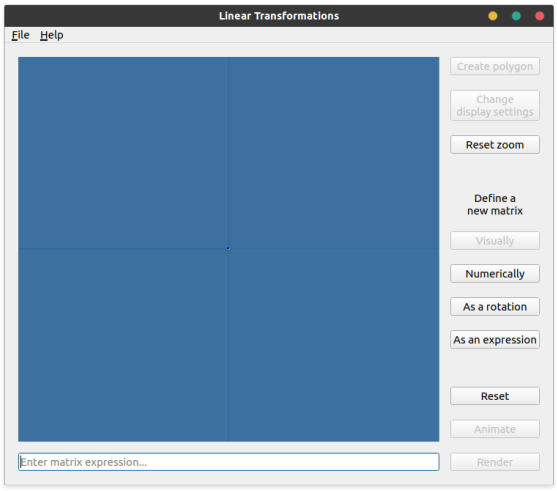


Figure 3.15: The GUI zoomed out as far as possible

There are two things I don't like here. Firstly, the minimum grid spacing is too small. The user can zoom out too far. Secondly, the arrowheads are too big in figure 3.14.

The first problem is minor and won't be fixed for quite a while, but I fixed the second problem quite quickly.

We want the arrowhead length to not just be 0.15, but to scale with the zoom level (the ratio between default grid spacing and current spacing).

This creates a slight issue when zoomed out all the way, because the arrowheads are then far larger than the vectors themselves, so we take the minimum of the scaled length and the vector length.

I factored out the default arrowhead length into the `arrowhead_length` instance attribute and initialize it in `__init__()`.

```
# 3d19a003368ae992ebb60049685bb04fde0836b5
# src/lintrans/gui/plots/widgets.py

68          vector_length = np.sqrt(x * x + y * y)
69          nx = x / vector_length
70          ny = y / vector_length
71
72          # We choose a length and find the steps in the x and y directions
73          length = min(
74              self.arrowhead_length * self.default_grid_spacing / self.grid_spacing,
75              vector_length
76          )
```

This code results in arrowheads that stay the same length unless the user is zoomed out basically as far as possible.

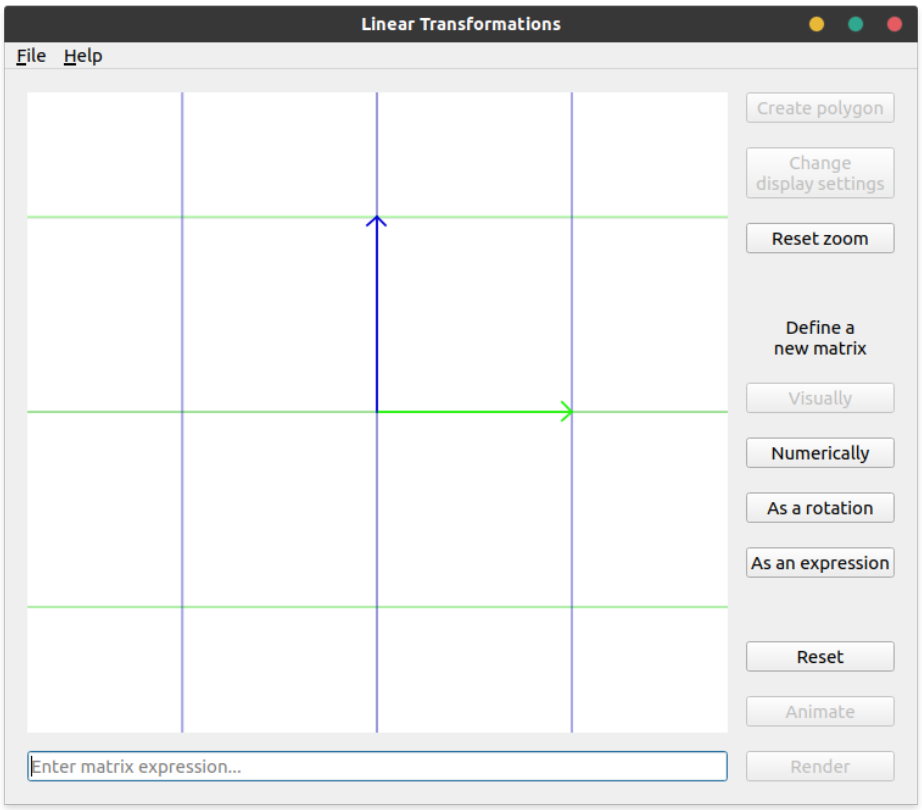


Figure 3.16: The arrowheads adjusted for zoom level

3.4.4 Animation blocks zooming

The biggest problem with this new zoom feature is that when animating between matrices, the user is unable to zoom. This is because when `LintransMainWindow.animate_expression()` is called, it uses Python's standard library `time.sleep()` function to delay each frame, which prevents Qt from handling user interaction while we're animating. This was a problem.

I did some googling and found a helpful post on StackOverflow[9] that gave me a nice solution. The user `ekhumoro` used the functions `QApplication.processEvents()` and `QThread.msleep()` to solve the problem, and I used these functions in my own app, with much success.

After reading 'The Event System' in the Qt5 documentation[24], I learned that Qt5 uses an event loop, a lot like JavaScript. This means that events are scheduled to be executed on the next pass of the event loop. I also read the documentation for the `repaint()` and `update()` methods on the `QWidget` class[20, 21] and decided that it would be better to just queue a repaint by calling `update()` on the plot rather than immediately repaint with `repaint()`, and then call `QApplication.processEvents()` to process the pending events on the main thread. This is a nicer way of repainting, which reduces potential flickering issues, and using `QThread.msleep()` allows for asynchronous processing and therefore non-blocking animation.

3.4.5 Rank 1 transformations

The rank of a matrix is the dimension of its column space. This is the dimension of the span of its columns, which is to say the dimension of the output space. The rank of a matrix must be less than or equal to the dimension of the matrix, so we only need to worry about ranks 0, 1, and 2. There is only one rank 0 matrix, which is the **0** matrix itself. I've already covered this case by just not drawing any transformed grid lines.

Rank 2 matrices encompass most 2D matrices, and I've already covered this case in §3.3.4 and §3.4.1. A rank 1 matrix collapses all of 2D space onto a single line, so for this type of matrix, we should just draw this line.

This code is in `VectorGridPlot.draw_parallel_lines()`. We assemble the matrix $\begin{pmatrix} \texttt{vector\_x} & \texttt{point\_x} \\ \texttt{vector\_y} & \texttt{point\_y} \end{pmatrix}$ (which is actually the matrix used to create the transformation we're trying to render lines for) and use this matrix to check determinant and rank.

```
# 677b38c87bb6722b16aaf35058cf3cef66e43c21
# src/lintrans/gui/plots/classes.py

        # If the determinant is 0
        if abs(vector_x * point_y - vector_y * point_x) < 1e-12:
            rank = np.linalg.matrix_rank(
                np.array([
                    [vector_x, point_x],
                    [vector_y, point_y]
                ])
            )

            # If the matrix is rank 1, then we can draw the column space line
            if rank == 1:
                self.draw_oblique_line(painter, vector_y / vector_x, 0)

            # If the rank is 0, then we don't draw any lines
            else:
                return
```

Additionally, there was a bug with animating these determinant 0 matrices, since we try to scale the determinant through the animation, as documented in §3.3.6, but when the determinant is 0, this causes

issues. To fix this, we just check the `det_target` variable in `LintransMainWindow.animate_expression` and if it's 0, we use the non-scaled version of the matrix.

```
# b889b686d997c2b64124bee786bccba3fc4f6b08
# src/lintrans/gui/main_window.py

307             # If we're animating towards a det 0 matrix, then we don't want to scale the
308             # determinant with the animation, because this makes the process not work
309             # I'm doing this here rather than wrapping the whole animation logic in an
310             # if block mainly because this looks nicer than an extra level of indentation
311             # The extra processing cost is negligible thanks to NumPy's optimizations
312             if det_target == 0:
313                 matrix_c = matrix_a
314             else:
315                 matrix_c = scalar * matrix_b
```

3.4.6 Matrices that are too big

One of my friends was playing around with the prototype and she discovered a bug. When trying to render really big matrices, we can get errors like '`OverflowError: argument 3 overflowed: value must be in the range -2147483648 to 2147483647`' because PyQt5 is a wrapper over Qt5, which is a C++ library that uses the C++ `int` type for the `painter.drawLine()` call. This type is a 32-bit integer. Python can store integers of arbitrary precision, but when PyQt5 calls the underlying C++ library code, this gets cast to a C++ `int` and we can get an `OverflowError`.

This isn't a problem with the gridlines, because we only draw them inside the viewport, as discussed in §3.4.1, and these calculations all happen in Python, so integer precision is not a concern. However, when drawing the basis vectors, we just draw them directly, so we'll have to check that they're within the limit.

I'd previously created a `LintransMainWindow.show_error_message()` method for telling the user when they try to take the inverse of a singular matrix[12].

```
# 0f699dd95b6431e95b2311dcb03e7af49c19613f
# src/lintrans/gui/main_window.py

378     def show_error_message(self, title: str, text: str, info: str | None = None) -> None:
379         """Show an error message in a dialog box.
380
381         :param str title: The window title of the dialog box
382         :param str text: The simple error message
383         :param info: The more informative error message
384         :type info: Optional[str]
385         """
386         dialog = QMessageBox(self)
387         dialog.setIcon(QMessageBox.Critical)
388         dialog.setWindowTitle(title)
389         dialog.setText(text)
390
391         if info is not None:
392             dialog.setInformativeText(info)
393
394         dialog.open()
395
396         dialog.finished.connect(self.update_render_buttons)
```

I then created the `is_matrix_too_big()` method to just check that the elements of the matrix are within the desired bounds. If it returns `True` when we try to render or animate, then we call `show_error_message()`.

```
# 4682a7b225747cfd77aca0fe3abcdd1397b7c5dd
# src/lintrans/gui/main_window.py
```

[12] This commit didn't get a standalone section in this write-up because it was so small

```
407         def is_matrix_too_big(self, matrix: MatrixType) -> bool:
408             """Check if the given matrix will actually fit onto the canvas.
409 
410             Convert the elements of the matrix to canvas coords and make sure they fit within Qt's 32-bit integer limit.
411 
412             :param MatrixType matrix: The matrix to check
413             :returns bool: Whether the matrix fits on the canvas
414             """
415             coords: list[tuple[int, int]] = [self.plot.trans_coords(*vector) for vector in matrix.T]
416 
417             for x, y in coords:
418                 if not (-2147483648 <= x <= 2147483647 and -2147483648 <= y <= 2147483647):
419                     return True
420 
421             return False
```

3.4.7 Creating the `DefineVisuallyDialog`

Next, I wanted to allow the user to define a matrix visually by dragging the basis vectors. To do this, I obviously needed a new `DefineDialog` subclass for it.

```
# 16ca0229aab73b3f4a8fe752dee3608f3ed6ead5
# src/lintrans/gui/dialogs/define_new_matrix.py

135 class DefineVisuallyDialog(DefineDialog):
136     """The dialog class that allows the user to define a matrix visually."""
137 
138     def __init__(self, matrix_wrapper: MatrixWrapper, *args, **kwargs):
139         """Create the widgets and layout of the dialog.
140 
141         :param MatrixWrapper matrix_wrapper: The MatrixWrapper that this dialog will mutate
142         """
143         super().__init__(matrix_wrapper, *args, **kwargs)
144 
145         self.setMinimumSize(500, 450)
146 
147         # === Create the widgets
148 
149         self.combobox_letter.activated.connect(self.show_matrix)
150 
151         self.plot = DefineVisuallyWidget(self)
152 
153         # === Arrange the widgets
154 
155         self.hlay_definition.addWidget(self.plot)
156         self.hlay_definition.setStretchFactor(self.plot, 1)
157 
158         self.vlay_all = QVBoxLayout()
159         self.vlay_all.setSpacing(20)
160         self.vlay_all.addLayout(self.hlay_definition)
161         self.vlay_all.addLayout(self.hlay_buttons)
162 
163         self.setLayout(self.vlay_all)
164 
165         # We load the default matrix A into the plot
166         self.show_matrix(0)
167 
168         # We also enable the confirm button, because any visually defined matrix is valid
169         self.button_confirm.setEnabled(True)
170 
171     def update_confirm_button(self) -> None:
172         """Enable the confirm button.
173 
174         .. note::
175            The confirm button is always enabled in this dialog and this method is never actually used,
176            so it's got an empty body. It's only here because we need to implement the abstract method.
177         """
178 
```

```
179         def show_matrix(self, index: int) -> None:
180             """Show the selected matrix on the plot. If the matrix is None, show the identity."""
181             matrix = self.matrix_wrapper[ALPHABET_NO_I[index]]
182 
183             if matrix is None:
184                 matrix = self.matrix_wrapper['I']
185 
186             self.plot.visualize_matrix_transformation(matrix)
187             self.plot.update()
188 
189         def confirm_matrix(self) -> None:
```

This `DefineVisuallyDialog` class just implements the normal methods needed for a `DefineDialog` and has a `plot` attribute to handle drawing graphics and handling mouse movement. After creating the `DefineVisuallyWidget` as a skeleton and doing some more research in the Qt5 docs[19], I renamed the `trans_coords()` methods to `canvas_coords()` to make the intent more clear, and created a `grid_coords()` method.

```
# 417aea6555029b049c470faff18df29f064f6101
# src/lintrans/gui/plots/classes.py

85          def grid_coords(self, x: int, y: int) -> tuple[float, float]:
86              """Convert a coordinate from canvas coords to grid coords.
87 
88              :param int x: The x component of the canvas coordinate
89              :param int y: The y component of the canvas coordinate
90              :returns: The resultant grid coordinates
91              :rtype: tuple[float, float]
92              """
93              # We get the maximum grid coords and convert them into canvas coords
94              return (x - self.canvas_origin[0]) / self.grid_spacing, (-y + self.canvas_origin[1]) / self.grid_spacing
```

I then needed to implement the methods to handle mouse movement in the `DefineVisuallyWidget` class. Thankfully, Ross Wilson, the person who helped me learn about the `QWidget.paintEvent()` method in §3.3.1, also wrote an example of draggable points[5]. In my post, I had explained that I needed draggable points on my canvas, and Ross was helpful enough to create an example in their own time. I probably could've worked it out myself eventually, but this example allowed me to learn a lot quicker.

```
# 417aea6555029b049c470faff18df29f064f6101
# src/lintrans/gui/plots/widgets.py

56  class DefineVisuallyWidget(VisualizeTransformationWidget):
57      """This class is the widget that allows the user to visually define a matrix.
58 
59      This is just the widget itself. If you want the dialog, use
60      :class:`lintrans.gui.dialogs.define_new_matrix.DefineVisuallyDialog`.
61      """
62 
63      def __init__(self, *args, **kwargs):
64          """Create the widget and enable mouse tracking. ``*args`` and ``**kwargs`` are passed to ``super()``."""
65          super().__init__(*args, **kwargs)
66 
67          # self.setMouseTracking(True)
68          self.dragged_point: tuple[float, float] | None = None
69 
70          # This is the distance that the cursor needs to be from the point to drag it
71          self.epsilon: int = 5
72 
73      def mousePressEvent(self, event: QMouseEvent) -> None:
74          """Handle a QMouseEvent when the user pressed a button."""
75          mx = event.x()
76          my = event.y()
77          button = event.button()
78 
79          if button != Qt.LeftButton:
```

```
80              event.ignore()
81              return
82
83          for point in (self.point_i, self.point_j):
84              px, py = self.canvas_coords(*point)
85              if abs(px - mx) <= self.epsilon and abs(py - my) <= self.epsilon:
86                  self.dragged_point = point[0], point[1]
87
88          event.accept()
89
90      def mouseReleaseEvent(self, event: QMouseEvent) -> None:
91          """Handle a QMouseEvent when the user release a button."""
92          if event.button() == Qt.LeftButton:
93              self.dragged_point = None
94              event.accept()
95          else:
96              event.ignore()
97
98      def mouseMoveEvent(self, event: QMouseEvent) -> None:
99          """Handle the mouse moving on the canvas."""
100         mx = event.x()
101         my = event.y()
102
103         if self.dragged_point is not None:
104             x, y = self.grid_coords(mx, my)
105
106             if self.dragged_point == self.point_i:
107                 self.point_i = x, y
108
109             elif self.dragged_point == self.point_j:
110                 self.point_j = x, y
111
112             self.dragged_point = x, y
113
114             self.update()
115
116             print(self.dragged_point)
117             print(self.point_i, self.point_j)
118
119             event.accept()
120
121         event.ignore()
```

This snippet has the line '`self.setMouseTracking(True)`' commented out. This line was in the example, but it turns out that I don't want it. Mouse tracking means that a widget will receive a `QMouseEvent` every time the mouse moves. But if it's disabled (the default), then the widget will only receive a `QMouseEvent` for mouse movement when a button is held down at the same time.

I've also left in some print statements on lines 116 and 117. These small oversights are there because I just forgot to remove them before I committed these changes. They were removed 3 commits later.

3.4.8 Fixing a division by zero bug

When drawing the rank line for a determinant 0, rank 1 matrix, we can encounter a division by zero error. I'm sure this originally manifested in a crash with a `ZeroDivisionError` at runtime, but now I can only get a `RuntimeWarning` when running the old code from commit `16ca0229aab73b3f4a8fe752dee3608f3ed6ead5`. Whether it crashes or just warns the user, there is a division by zero bug when trying to render $\begin{pmatrix} k & 0 \\ 0 & 0 \end{pmatrix}$ or $\begin{pmatrix} 0 & 0 \\ 0 & k \end{pmatrix}$. To fix this, I just handled those cases separately in `VectorGridPlot.draw_parallel_lines(⌟)`.

```
# 40bee6461d477a5c767ed132359cd511c0051e3b
# src/lintrans/gui/plots/classes.py
```

```
196             # If the matrix is rank 1, then we can draw the column space line
197             if rank == 1:
198                 if abs(vector_x) < 1e-12:
199                     painter.drawLine(self.width() // 2, 0, self.width() // 2, self.height())
200                 elif abs(vector_y) < 1e-12:
201                     painter.drawLine(0, self.height() // 2, self.width(), self.height() // 2)
202                 else:
203                     self.draw_oblique_line(painter, vector_y / vector_x, 0)
204 
205             # If the rank is 0, then we don't draw any lines
206             else:
207                 return
```

3.4.9 Implementing transitional animation

Currently, all animation animates from $\mathbf{I}$ to the target matrix $\mathbf{T}$. This means it resets the plot at the start. I eventually want an applicative animation system, where the matrix in the box is applied to the current scene. But I also want an option for a transitional animation, where the program animates from the start matrix $\mathbf{S}$ to the target matrix $\mathbf{T}$, and this seems easier to implement, so I'll do it first.

In `LintransMainWindow`, I created a new method called `animate_between_matrices()` and I call it from `animate_expression()`. The maths for smoothening determinants in §3.3.6 assumed the starting matrix had a determinant of 1, but when using transitional animation, this may not always be true.

If we let $\mathbf{S}$ be the starting matrix, and $\mathbf{A}$ be the matrix from the first stage of calculation as specified in §3.3.6, then we want a c such that $\det(c\mathbf{A}) = \det(\mathbf{S})$, so we get $c = \sqrt{\left|\dfrac{\det(\mathbf{S})}{\det(\mathbf{A})}\right|}$ by the identity $\det(c\mathbf{A}) = c^2 \det(\mathbf{A})$.

Following the same logic as in §3.3.6, we can let $\mathbf{B} = c\mathbf{A}$ and then scale it by d to get the same determinant as the target matrix $\mathbf{T}$ and find that $d = \sqrt{\left|\dfrac{\det(\mathbf{T})}{\det(\mathbf{B})}\right|}$. Unlike previously, $\det(\mathbf{B})$ could be any scalar, so we can't simplify our expression for d.

We then scale this with our `proportion` variable p to get a scalar $s = 1 + p\left(\sqrt{\left|\dfrac{\det(\mathbf{T})}{\det(\mathbf{B})}\right|} - 1\right)$ and render $\mathbf{C} = s\mathbf{B}$ on each frame.

In code, that looks like this:

```
# 4017b84fbce67d8e041bc9ce84cefcb0b6e65e1f
# src/lintrans/gui/main_window.py

275     def animate_expression(self) -> None:
276         """Animate from the current matrix to the matrix in the expression box."""
277         self.button_render.setEnabled(False)
278         self.button_animate.setEnabled(False)
279 
280         # Get the target matrix and it's determinant
281         try:
282             matrix_target = self.matrix_wrapper.evaluate_expression(self.lineedit_expression_box.text())
283 
284         except linalg.LinAlgError:
285             self.show_error_message('Singular matrix', 'Cannot take inverse of singular matrix')
286             return
287 
288         matrix_start: MatrixType = np.array([
289             [self.plot.point_i[0], self.plot.point_j[0]],
290             [self.plot.point_i[1], self.plot.point_j[1]]
291         ])
292 
293         self.animate_between_matrices(matrix_start, matrix_target)
```

```
294
295             self.button_render.setEnabled(True)
296             self.button_animate.setEnabled(True)
297
298         def animate_between_matrices(self, matrix_start: MatrixType, matrix_target: MatrixType, steps: int = 100) ->
        ↪  None:
299             """Animate from the start matrix to the target matrix."""
300             det_target = linalg.det(matrix_target)
301             det_start = linalg.det(matrix_start)
302
303             for i in range(0, steps + 1):
304                 # This proportion is how far we are through the loop
305                 proportion = i / steps
306
307                 # matrix_a is the start matrix plus some part of the target, scaled by the proportion
308                 # If we just used matrix_a, then things would animate, but the determinants would be weird
309                 matrix_a = matrix_start + proportion * (matrix_target - matrix_start)
310
311                 # So to fix the determinant problem, we get the determinant of matrix_a and use it to normalise
312                 det_a = linalg.det(matrix_a)
313
314                 # For a 2x2 matrix A and a scalar c, we know that det(cA) = c^2 det(A)
315                 # We want B = cA such that det(B) = det(S), where S is the start matrix,
316                 # so then we can scale it with the animation, so we get
317                 # det(cA) = c^2 det(A) = det(S) => c = sqrt(abs(det(S) / det(A)))
318                 # Then we scale A to get the determinant we want, and call that matrix_b
319                 if det_a == 0:
320                     c = 0
321                 else:
322                     c = np.sqrt(abs(det_start / det_a))
323
324                 matrix_b = c * matrix_a
325                 det_b = linalg.det(matrix_b)
326
327                 # matrix_c is the final matrix that we then render for this frame
328                 # It's B, but we scale it over time to have the target determinant
329
330                 # We want some C = dB such that det(C) is some target determinant T
331                 # det(dB) = d^2 det(B) = T => d = sqrt(abs(T / det(B))
332
333                 # We're also subtracting 1 and multiplying by the proportion and then adding one
334                 # This just scales the determinant along with the animation
335                 scalar = 1 + proportion * (np.sqrt(abs(det_target / det_b)) - 1)
336
337                 # If we're animating towards a det 0 matrix, then we don't want to scale the
338                 # determinant with the animation, because this makes the process not work
339                 # I'm doing this here rather than wrapping the whole animation logic in an
340                 # if block mainly because this looks nicer than an extra level of indentation
341                 # The extra processing cost is negligible thanks to NumPy's optimizations
342                 if det_target == 0:
343                     matrix_c = matrix_a
344                 else:
345                     matrix_c = scalar * matrix_b
346
347                 if self.is_matrix_too_big(matrix_c):
348                     self.show_error_message('Matrix too big', "This matrix doesn't fit on the canvas")
349                     return
350
351                 self.plot.visualize_matrix_transformation(matrix_c)
352
353                 # We schedule the plot to be updated, tell the event loop to
354                 # process events, and asynchronously sleep for 10ms
355                 # This allows for other events to be processed while animating, like zooming in and out
356                 self.plot.update()
```

This change results in an animation system that will transition from the current matrix to whatever the user types into the input box.

3.4.10 Allowing for sequential animation with commas

Applicative animation has two main forms. There's the version where a standard matrix expression gets applied to the current scene, and the kind where the user defines a sequence of matrices and we animate through the sequence, applying one at a time. Both of these are referenced in success criterion 5.

I want the user to be able to decide if they want applicative animation or transitional animation, so I'll need to create some form of display settings. However, transitional animation doesn't make much sense for sequential animation[13], so I can implement this now.

Applicative animation is just animating from the matrix **C** representing the current scene to the composition **TC** with the target matrix **T**.

We use **TC** instead of **CT** because matrix multiplication can be thought of as applying successive transformations from right to left. **TC** is the same as starting with the identity **I**, applying **C** (to get to the current scene), and then applying **T**.

Doing this in code is very simple. We just split the expression on commas, and then apply each sub-expression to the current scene one by one, pausing on each comma.

```
# 60584d2559cacbf23479a1bebbb986a800a32331
# src/lintrans/gui/main_window.py
```

```
284         def animate_expression(self) -> None:
285             """Animate from the current matrix to the matrix in the expression box."""
286             self.button_render.setEnabled(False)
287             self.button_animate.setEnabled(False)
288 
289             matrix_start: MatrixType = np.array([
290                 [self.plot.point_i[0], self.plot.point_j[0]],
291                 [self.plot.point_i[1], self.plot.point_j[1]]
292             ])
293 
294             text = self.lineedit_expression_box.text()
295 
296             # If there's commas in the expression, then we want to animate each part at a time
297             if ',' in text:
298                 current_matrix = matrix_start
299 
300                 # For each expression in the list, right multiply it by the current matrix,
301                 # and animate from the current matrix to that new matrix
302                 for expr in text.split(',')[::-1]:
303                     new_matrix = self.matrix_wrapper.evaluate_expression(expr) @ current_matrix
304 
305                     self.animate_between_matrices(current_matrix, new_matrix)
306                     current_matrix = new_matrix
307 
308                     # Here we just redraw and allow for other events to be handled while we pause
309                     self.plot.update()
310                     QApplication.processEvents()
311                     QThread.msleep(500)
312 
313             # If there's no commas, then just animate directly from the start to the target
314             else:
315                 # Get the target matrix and it's determinant
316                 try:
317                     matrix_target = self.matrix_wrapper.evaluate_expression(text)
318 
319                 except linalg.LinAlgError:
320                     self.show_error_message('Singular matrix', 'Cannot take inverse of singular matrix')
321                     return
322 
323                 self.animate_between_matrices(matrix_start, matrix_target)
```

[13] I have since changed my thoughts on this, and I allowed sequential transitional animation much later, in commit `41907b81661f3878e435b794d9d719491ef14237`

```
324
325             self.update_render_buttons()
```

We're deliberately not checking if the sub-expressions are valid here. We would normally validate the expression in `LintransMainWindow.update_render_buttons()` and only allow the user to render or animate an expression if it's valid. Now we have to check all the sub-expressions if the expression contains commas. Additionally, we can only animate these expressions with commas in them, so rendering should be disabled when the expression contains commas.

Compare the old code to the new code:

```
# 4017b84fbce67d8e041bc9ce84cefcb0b6e65e1f
# src/lintrans/gui/main_window.py

243         def update_render_buttons(self) -> None:
244             """Enable or disable the render and animate buttons according to whether the matrix expression is valid."""
245             valid = self.matrix_wrapper.is_valid_expression(self.lineedit_expression_box.text())
246             self.button_render.setEnabled(valid)
247             self.button_animate.setEnabled(valid)
```

```
# 60584d2559cacbf23479a1bebbb986a800a32331
# src/lintrans/gui/main_window.py

243         def update_render_buttons(self) -> None:
244             """Enable or disable the render and animate buttons according to whether the matrix expression is valid."""
245             text = self.lineedit_expression_box.text()
246
247             if ',' in text:
248                 self.button_render.setEnabled(False)
249
250                 valid = all(self.matrix_wrapper.is_valid_expression(x) for x in text.split(','))
251                 self.button_animate.setEnabled(valid)
252
253             else:
254                 valid = self.matrix_wrapper.is_valid_expression(text)
255                 self.button_render.setEnabled(valid)
256                 self.button_animate.setEnabled(valid)
```

3.5 Adding display settings

3.5.1 Creating the dataclass

The first step of adding display settings is creating a dataclass to hold all of the settings. This dataclass will hold attributes to manage how a matrix transformation is displayed. Things like whether to show eigenlines or the determinant parallelogram. It will also hold information for animation. We can factor out the code used to smoothen the determinant, as written in §3.3.6, and make it dependant on a `bool` attribute of the `DisplaySettings` dataclass.

This is a standard class rather than some form of singleton to allow different plots to have different display settings. For example, the user might want different settings for the main view and the visual definition dialog. Allowing each instance of a subclass of `VectorGridPlot` to have its own `DisplaySettings` attribute allows for separate settings for separate plots.

However, this class initially just contained attributes relevant to animation, so it was only an attribute on `LintransMainWindow`.

```
# 2041c7a24d963d8d142d6f0f20ec3828ba8257c6
# src/lintrans/gui/settings.py

1   """This module contains the :class:`DisplaySettings` class, which holds configuration for display."""
2
```

```python
from dataclasses import dataclass


@dataclass
class DisplaySettings:
    """This class simply holds some attributes to configure display."""

    animate_determinant: bool = True
    """This controls whether we want the determinant to change smoothly during the animation."""

    applicative_animation: bool = True
    """There are two types of simple animation, transitional and applicative.

    Let ``C`` be the matrix representing the currently displayed transformation, and let ``T`` be the target matrix.
    Transitional animation means that we animate directly from ``C`` from ``T``,
    and applicative animation means that we animate from ``C`` to ``TC``, so we apply ``T`` to ``C``.
    """

    animation_pause_length: int = 400
    """This is the number of milliseconds that we wait between animations when using comma syntax."""
```

Once I had the dataclass, I just had to add '`from .settings import DisplaySettings`' to the top of the file, and '`self.display_settings = DisplaySettings()`' to the constructor of `LintransMainWindow`. I could then use the attributes of this dataclass in `animate_expression()`.

```python
# 2041c7a24d963d8d142d6f0f20ec3828ba8257c6
# src/lintrans/gui/main_window.py

    def animate_expression(self) -> None:
        """Animate from the current matrix to the matrix in the expression box."""
        self.button_render.setEnabled(False)
        self.button_animate.setEnabled(False)

        matrix_start: MatrixType = np.array([
            [self.plot.point_i[0], self.plot.point_j[0]],
            [self.plot.point_i[1], self.plot.point_j[1]]
        ])

        text = self.lineedit_expression_box.text()

        # If there's commas in the expression, then we want to animate each part at a time
        if ',' in text:
            current_matrix = matrix_start

            # For each expression in the list, right multiply it by the current matrix,
            # and animate from the current matrix to that new matrix
            for expr in text.split(',')[::-1]:
                new_matrix = self.matrix_wrapper.evaluate_expression(expr) @ current_matrix

                self.animate_between_matrices(current_matrix, new_matrix)
                current_matrix = new_matrix

                # Here we just redraw and allow for other events to be handled while we pause
                self.plot.update()
                QApplication.processEvents()
                QThread.msleep(self.display_settings.animation_pause_length)

        # If there's no commas, then just animate directly from the start to the target
        else:
            # Get the target matrix and it's determinant
            try:
                matrix_target = self.matrix_wrapper.evaluate_expression(text)

            except linalg.LinAlgError:
                self.show_error_message('Singular matrix', 'Cannot take inverse of singular matrix')
                return

            # The concept of applicative animation is explained in /gui/settings.py
            if self.display_settings.applicative_animation:
                matrix_target = matrix_target @ matrix_start
```

```
328
329             self.animate_between_matrices(matrix_start, matrix_target)
330
331         self.update_render_buttons()
```

I also wrapped the main logic of `animate_between_matrices()` in an **`if`** block to check if the user wants the determinant to be smoothed.

```
# 03e154e1326dc256ffc1a539e97d8ef5ec89f6fd
# src/lintrans/gui/main_window.py

333     def animate_between_matrices(self, matrix_start: MatrixType, matrix_target: MatrixType, steps: int = 100) ->
        ↪ None:
334         """Animate from the start matrix to the target matrix."""
335         det_target = linalg.det(matrix_target)
336         det_start = linalg.det(matrix_start)
337
338         for i in range(0, steps + 1):
339             # This proportion is how far we are through the loop
340             proportion = i / steps
341
342             # matrix_a is the start matrix plus some part of the target, scaled by the proportion
343             # If we just used matrix_a, then things would animate, but the determinants would be weird
344             matrix_a = matrix_start + proportion * (matrix_target - matrix_start)
345
346             if self.display_settings.animate_determinant and det_target != 0:
347                 # To fix the determinant problem, we get the determinant of matrix_a and use it to normalise
348                 det_a = linalg.det(matrix_a)
349
350                 # For a 2x2 matrix A and a scalar c, we know that det(cA) = c^2 det(A)
351                 # We want B = cA such that det(B) = det(S), where S is the start matrix,
352                 # so then we can scale it with the animation, so we get
353                 # det(cA) = c^2 det(A) = det(S) => c = sqrt(abs(det(S) / det(A)))
354                 # Then we scale A to get the determinant we want, and call that matrix_b
355                 if det_a == 0:
356                     c = 0
357                 else:
358                     c = np.sqrt(abs(det_start / det_a))
359
360                 matrix_b = c * matrix_a
361                 det_b = linalg.det(matrix_b)
362
363                 # matrix_to_render is the final matrix that we then render for this frame
364                 # It's B, but we scale it over time to have the target determinant
365
366                 # We want some C = dB such that det(C) is some target determinant T
367                 # det(dB) = d^2 det(B) = T => d = sqrt(abs(T / det(B))
368
369                 # We're also subtracting 1 and multiplying by the proportion and then adding one
370                 # This just scales the determinant along with the animation
371                 scalar = 1 + proportion * (np.sqrt(abs(det_target / det_b)) - 1)
372                 matrix_to_render = scalar * matrix_b
373
374             else:
375                 matrix_to_render = matrix_a
376
377             if self.is_matrix_too_big(matrix_to_render):
378                 self.show_error_message('Matrix too big', "This matrix doesn't fit on the canvas")
379                 return
380
381             self.plot.visualize_matrix_transformation(matrix_to_render)
382
383             # We schedule the plot to be updated, tell the event loop to
384             # process events, and asynchronously sleep for 10ms
385             # This allows for other events to be processed while animating, like zooming in and out
386             self.plot.update()
387             QApplication.processEvents()
388             QThread.msleep(1000 // steps)
```

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A Project code

A.1 `__init__.py`

```
# lintrans - The linear transformation visualizer
# Copyright (C) 2021-2022 D. Dyson (DoctorDalek1963)

# This program is licensed under GNU GPLv3, available here:
# <https://www.gnu.org/licenses/gpl-3.0.html>

"""This is the top-level ``lintrans`` package, which contains all the subpackages of the project."""

from . import gui, matrices, typing_

__version__ = '0.3.0-alpha'

__all__ = ['gui', 'matrices', 'typing_', '__version__']
```

A.2 `__main__.py`

```
#!/usr/bin/env python

# lintrans - The linear transformation visualizer
# Copyright (C) 2021-2022 D. Dyson (DoctorDalek1963)

# This program is licensed under GNU GPLv3, available here:
# <https://www.gnu.org/licenses/gpl-3.0.html>

"""This module provides a :func:`main` function to interpret command line arguments and run the program."""

import sys
from argparse import ArgumentParser
from textwrap import dedent
from typing import List

from lintrans import __version__, gui


def main(args: List[str]) -> None:
    """Interpret program-specific command line arguments and run the main window in most cases.

    If the user supplies --help or --version, then we simply respond to that and then return.
    If they don't supply either of these, then we run :func:`lintrans.gui.main_window.main`.

    :param List[str] args: The full argument list (including program name)
    """
    parser = ArgumentParser(add_help=False)

    parser.add_argument(
        '-h',
        '--help',
        default=False,
        action='store_true'
    )

    parser.add_argument(
        '-V',
        '--version',
        default=False,
        action='store_true'
    )

    parsed_args, unparsed_args = parser.parse_known_args()

    if parsed_args.help:
        print(dedent('''
        Usage: lintrans [option]

```

```
49              Options:
50                  -h, --help       Display this help text and exit
51                  -V, --version    Display the version information and exit
52
53              Any other options will get passed to the QApplication constructor.
54              If you don't know what that means, then don't provide any arguments and just the run the program.'''[1:]))
55              return
56
57          if parsed_args.version:
58              print(dedent(f'''
59              lintrans (version {__version__})
60              The linear transformation visualizer
61
62              Copyright (C) 2021-2022 D. Dyson (DoctorDalek1963)
63
64              This program is licensed under GNU GPLv3, available here:
65              <https://www.gnu.org/licenses/gpl-3.0.html>'''[1:]))
66              return
67
68          for arg in unparsed_args:
69              print(f'Passing "{arg}" to QApplication. See --help for recognised args')
70
71          gui.main(args[:1] + unparsed_args)
72
73
74      if __name__ == '__main__':
75          main(sys.argv)
```

A.3 gui/main_window.py

```
1       # lintrans - The linear transformation visualizer
2       # Copyright (C) 2021-2022 D. Dyson (DoctorDalek1963)
3
4       # This program is licensed under GNU GPLv3, available here:
5       # <https://www.gnu.org/licenses/gpl-3.0.html>
6
7       """This module provides the :class:`LintransMainWindow` class, which provides the main window for the GUI."""
8
9       from __future__ import annotations
10
11      import re
12      import sys
13      import webbrowser
14      from copy import deepcopy
15      from typing import List, Tuple, Type
16
17      import numpy as np
18      from numpy import linalg
19      from numpy.linalg import LinAlgError
20      from PyQt5 import QtWidgets
21      from PyQt5.QtCore import pyqtSlot, QCoreApplication, QThread
22      from PyQt5.QtGui import QCloseEvent, QKeySequence
23      from PyQt5.QtWidgets import (QApplication, QHBoxLayout, QMainWindow, QMessageBox,
24                                   QShortcut, QSizePolicy, QSpacerItem, QStyleFactory, QVBoxLayout)
25
26      import lintrans
27      from lintrans.matrices import MatrixWrapper
28      from lintrans.matrices.parse import validate_matrix_expression
29      from lintrans.matrices.utility import polar_coords, rotate_coord
30      from lintrans.typing_ import MatrixType, VectorType
31      from .dialogs import (AboutDialog, DefineAsAnExpressionDialog, DefineDialog,
32                            DefineNumericallyDialog, DefineVisuallyDialog, InfoPanelDialog)
33      from .dialogs.settings import DisplaySettingsDialog
34      from .plots import VisualizeTransformationWidget
35      from .settings import DisplaySettings
36      from .validate import MatrixExpressionValidator
37
38
39      class LintransMainWindow(QMainWindow):
40          """This class provides a main window for the GUI using the Qt framework.
```

```
41
42          This class should not be used directly, instead call :func:`lintrans.gui.main_window.main` to create the GUI.
43          """
44
45          def __init__(self):
46              """Create the main window object, and create and arrange every widget in it.
47
48              This doesn't show the window, it just constructs it.
49              Use :func:`lintrans.gui.main_window.main` to show the GUI.
50              """
51              super().__init__()
52
53              self.matrix_wrapper = MatrixWrapper()
54
55              self.setWindowTitle('lintrans')
56              self.setMinimumSize(1000, 750)
57
58              self.animating: bool = False
59              self.animating_sequence: bool = False
60
61              # === Create menubar
62
63              self.menubar = QtWidgets.QMenuBar(self)
64
65              self.menu_file = QtWidgets.QMenu(self.menubar)
66              self.menu_file.setTitle('&File')
67
68              self.menu_help = QtWidgets.QMenu(self.menubar)
69              self.menu_help.setTitle('&Help')
70
71              self.action_new = QtWidgets.QAction(self)
72              self.action_new.setText('&New')
73              self.action_new.setShortcut('Ctrl+N')
74              self.action_new.triggered.connect(lambda: print('new'))
75
76              self.action_open = QtWidgets.QAction(self)
77              self.action_open.setText('&Open')
78              self.action_open.setShortcut('Ctrl+O')
79              self.action_open.triggered.connect(lambda: print('open'))
80
81              self.action_save = QtWidgets.QAction(self)
82              self.action_save.setText('&Save')
83              self.action_save.setShortcut('Ctrl+S')
84              self.action_save.triggered.connect(lambda: print('save'))
85
86              self.action_save_as = QtWidgets.QAction(self)
87              self.action_save_as.setText('Save as...')
88              self.action_save_as.triggered.connect(lambda: print('save as'))
89
90              self.action_tutorial = QtWidgets.QAction(self)
91              self.action_tutorial.setText('&Tutorial')
92              self.action_tutorial.setShortcut('F1')
93              self.action_tutorial.triggered.connect(lambda: print('tutorial'))
94
95              self.action_docs = QtWidgets.QAction(self)
96              self.action_docs.setText('&Docs')
97
98              # If this is an old release, use the docs for this release. Else, use the latest docs
99              # We use the latest because most use cases for non-stable releases will be in development and testing
100             docs_link = 'https://lintrans.readthedocs.io/en/'
101
102             if re.match(r'^\d+\.\d+\.\d+$', lintrans.__version__):
103                 docs_link += 'v' + lintrans.__version__
104             else:
105                 docs_link += 'latest'
106
107             self.action_docs.triggered.connect(
108                 lambda: webbrowser.open_new_tab(docs_link)
109             )
110
111             self.action_about = QtWidgets.QAction(self)
112             self.action_about.setText('&About')
113             self.action_about.triggered.connect(lambda: AboutDialog(self).open())
```

```
114
115             # TODO: Implement these actions and enable them
116             self.action_new.setEnabled(False)
117             self.action_open.setEnabled(False)
118             self.action_save.setEnabled(False)
119             self.action_save_as.setEnabled(False)
120             self.action_tutorial.setEnabled(False)
121
122             self.menu_file.addAction(self.action_new)
123             self.menu_file.addAction(self.action_open)
124             self.menu_file.addSeparator()
125             self.menu_file.addAction(self.action_save)
126             self.menu_file.addAction(self.action_save_as)
127
128             self.menu_help.addAction(self.action_tutorial)
129             self.menu_help.addAction(self.action_docs)
130             self.menu_help.addSeparator()
131             self.menu_help.addAction(self.action_about)
132
133             self.menubar.addAction(self.menu_file.menuAction())
134             self.menubar.addAction(self.menu_help.menuAction())
135
136             self.setMenuBar(self.menubar)
137
138             # === Create widgets
139
140             # Left layout: the plot and input box
141
142             self.plot = VisualizeTransformationWidget(self, display_settings=DisplaySettings())
143
144             self.lineedit_expression_box = QtWidgets.QLineEdit(self)
145             self.lineedit_expression_box.setPlaceholderText('Enter matrix expression...')
146             self.lineedit_expression_box.setValidator(MatrixExpressionValidator(self))
147             self.lineedit_expression_box.textChanged.connect(self.update_render_buttons)
148
149             # Right layout: all the buttons
150
151             # Misc buttons
152
153             self.button_create_polygon = QtWidgets.QPushButton(self)
154             self.button_create_polygon.setText('Create polygon')
155             # self.button_create_polygon.clicked.connect(self.create_polygon)
156             self.button_create_polygon.setToolTip('Define a new polygon to view the transformation of')
157
158             # TODO: Implement this and enable button
159             self.button_create_polygon.setEnabled(False)
160
161             self.button_change_display_settings = QtWidgets.QPushButton(self)
162             self.button_change_display_settings.setText('Change\ndisplay settings')
163             self.button_change_display_settings.clicked.connect(self.dialog_change_display_settings)
164             self.button_change_display_settings.setToolTip(
165                 "Change which things are rendered and how they're rendered<br><b>(Ctrl + D)</b>"
166             )
167             QShortcut(QKeySequence('Ctrl+D'), self).activated.connect(self.button_change_display_settings.click)
168
169             self.button_reset_zoom = QtWidgets.QPushButton(self)
170             self.button_reset_zoom.setText('Reset zoom')
171             self.button_reset_zoom.clicked.connect(self.reset_zoom)
172             self.button_reset_zoom.setToolTip('Reset the zoom level back to normal<br><b>(Ctrl + Shift + R)</b>')
173             QShortcut(QKeySequence('Ctrl+Shift+R'), self).activated.connect(self.button_reset_zoom.click)
174
175             # Define new matrix buttons and their groupbox
176
177             self.button_define_visually = QtWidgets.QPushButton(self)
178             self.button_define_visually.setText('Visually')
179             self.button_define_visually.setToolTip('Drag the basis vectors<br><b>(Alt + 1)</b>')
180             self.button_define_visually.clicked.connect(lambda: self.dialog_define_matrix(DefineVisuallyDialog))
181             QShortcut(QKeySequence('Alt+1'), self).activated.connect(self.button_define_visually.click)
182
183             self.button_define_numerically = QtWidgets.QPushButton(self)
184             self.button_define_numerically.setText('Numerically')
185             self.button_define_numerically.setToolTip('Define a matrix just with numbers<br><b>(Alt + 2)</b>')
186             self.button_define_numerically.clicked.connect(lambda: self.dialog_define_matrix(DefineNumericallyDialog))
```

```
187             QShortcut(QKeySequence('Alt+2'), self).activated.connect(self.button_define_numerically.click)
188
189             self.button_define_as_expression = QtWidgets.QPushButton(self)
190             self.button_define_as_expression.setText('As an expression')
191             self.button_define_as_expression.setToolTip('Define a matrix in terms of other matrices<br><b>(Alt +
                ↪  3)</b>')
192             self.button_define_as_expression.clicked.connect(lambda:
                ↪  self.dialog_define_matrix(DefineAsAnExpressionDialog))
193             QShortcut(QKeySequence('Alt+3'), self).activated.connect(self.button_define_as_expression.click)
194
195             self.vlay_define_new_matrix = QVBoxLayout()
196             self.vlay_define_new_matrix.setSpacing(20)
197             self.vlay_define_new_matrix.addWidget(self.button_define_visually)
198             self.vlay_define_new_matrix.addWidget(self.button_define_numerically)
199             self.vlay_define_new_matrix.addWidget(self.button_define_as_expression)
200
201             self.groupbox_define_new_matrix = QtWidgets.QGroupBox('Define a new matrix', self)
202             self.groupbox_define_new_matrix.setLayout(self.vlay_define_new_matrix)
203
204             # Info panel button
205
206             self.button_info_panel = QtWidgets.QPushButton(self)
207             self.button_info_panel.setText('Show defined matrices')
208             self.button_info_panel.clicked.connect(
209                 # We have to use a lambda instead of 'InfoPanelDialog(self.matrix_wrapper, self).open' here
210                 # because that would create an unnamed instance of InfoPanelDialog when LintransMainWindow is
211                 # constructed, but we need to create a new instance every time to keep self.matrix_wrapper up to date
212                 lambda: InfoPanelDialog(self.matrix_wrapper, self).open()
213             )
214             self.button_info_panel.setToolTip(
215                 'Open an info panel with all matrices that have been defined in this session<br><b>(Ctrl + M)</b>'
216             )
217             QShortcut(QKeySequence('Ctrl+M'), self).activated.connect(self.button_info_panel.click)
218
219             # Render buttons
220
221             self.button_reset = QtWidgets.QPushButton(self)
222             self.button_reset.setText('Reset')
223             self.button_reset.clicked.connect(self.reset_transformation)
224             self.button_reset.setToolTip('Reset the visualized transformation back to the identity<br><b>(Ctrl +
                ↪  R)</b>')
225             QShortcut(QKeySequence('Ctrl+R'), self).activated.connect(self.button_reset.click)
226
227             self.button_render = QtWidgets.QPushButton(self)
228             self.button_render.setText('Render')
229             self.button_render.setEnabled(False)
230             self.button_render.clicked.connect(self.render_expression)
231             self.button_render.setToolTip('Render the expression<br><b>(Ctrl + Enter)</b>')
232             QShortcut(QKeySequence('Ctrl+Return'), self).activated.connect(self.button_render.click)
233
234             self.button_animate = QtWidgets.QPushButton(self)
235             self.button_animate.setText('Animate')
236             self.button_animate.setEnabled(False)
237             self.button_animate.clicked.connect(self.animate_expression)
238             self.button_animate.setToolTip('Animate the expression<br><b>(Ctrl + Shift + Enter)</b>')
239             QShortcut(QKeySequence('Ctrl+Shift+Return'), self).activated.connect(self.button_animate.click)
240
241             # === Arrange widgets
242
243             self.vlay_left = QVBoxLayout()
244             self.vlay_left.addWidget(self.plot)
245             self.vlay_left.addWidget(self.lineedit_expression_box)
246
247             self.vlay_misc_buttons = QVBoxLayout()
248             self.vlay_misc_buttons.setSpacing(20)
249             self.vlay_misc_buttons.addWidget(self.button_create_polygon)
250             self.vlay_misc_buttons.addWidget(self.button_change_display_settings)
251             self.vlay_misc_buttons.addWidget(self.button_reset_zoom)
252
253             self.vlay_info_buttons = QVBoxLayout()
254             self.vlay_info_buttons.setSpacing(20)
255             self.vlay_info_buttons.addWidget(self.button_info_panel)
256
```

```
257             self.vlay_render = QVBoxLayout()
258             self.vlay_render.setSpacing(20)
259             self.vlay_render.addWidget(self.button_reset)
260             self.vlay_render.addWidget(self.button_animate)
261             self.vlay_render.addWidget(self.button_render)
262 
263             self.vlay_right = QVBoxLayout()
264             self.vlay_right.setSpacing(50)
265             self.vlay_right.addLayout(self.vlay_misc_buttons)
266             self.vlay_right.addItem(QSpacerItem(100, 2, hPolicy=QSizePolicy.Minimum, vPolicy=QSizePolicy.Expanding))
267             self.vlay_right.addWidget(self.groupbox_define_new_matrix)
268             self.vlay_right.addItem(QSpacerItem(100, 2, hPolicy=QSizePolicy.Minimum, vPolicy=QSizePolicy.Expanding))
269             self.vlay_right.addLayout(self.vlay_info_buttons)
270             self.vlay_right.addItem(QSpacerItem(100, 2, hPolicy=QSizePolicy.Minimum, vPolicy=QSizePolicy.Expanding))
271             self.vlay_right.addLayout(self.vlay_render)
272 
273             self.hlay_all = QHBoxLayout()
274             self.hlay_all.setSpacing(15)
275             self.hlay_all.addLayout(self.vlay_left)
276             self.hlay_all.addLayout(self.vlay_right)
277 
278             self.central_widget = QtWidgets.QWidget()
279             self.central_widget.setLayout(self.hlay_all)
280             self.central_widget.setContentsMargins(10, 10, 10, 10)
281 
282             self.setCentralWidget(self.central_widget)
283 
284         def closeEvent(self, event: QCloseEvent) -> None:
285             """Handle a :class:`QCloseEvent` by cancelling animation first."""
286             self.animating = False
287             event.accept()
288 
289         def update_render_buttons(self) -> None:
290             """Enable or disable the render and animate buttons according to whether the matrix expression is valid."""
291             text = self.lineedit_expression_box.text()
292 
293             # Let's say that the user defines a non-singular matrix A, then defines B as A^-1
294             # If they then redefine A and make it singular, then we get a LinAlgError when
295             # trying to evaluate an expression with B in it
296             # To fix this, we just do naive validation rather than aware validation
297             if ',' in text:
298                 self.button_render.setEnabled(False)
299 
300                 try:
301                     valid = all(self.matrix_wrapper.is_valid_expression(x) for x in text.split(','))
302                 except LinAlgError:
303                     valid = all(validate_matrix_expression(x) for x in text.split(','))
304 
305                 self.button_animate.setEnabled(valid)
306 
307             else:
308                 try:
309                     valid = self.matrix_wrapper.is_valid_expression(text)
310                 except LinAlgError:
311                     valid = validate_matrix_expression(text)
312 
313                 self.button_render.setEnabled(valid)
314                 self.button_animate.setEnabled(valid)
315 
316         @pyqtSlot()
317         def reset_zoom(self) -> None:
318             """Reset the zoom level back to normal."""
319             self.plot.grid_spacing = self.plot.default_grid_spacing
320             self.plot.update()
321 
322         @pyqtSlot()
323         def reset_transformation(self) -> None:
324             """Reset the visualized transformation back to the identity."""
325             self.plot.visualize_matrix_transformation(self.matrix_wrapper['I'])
326             self.animating = False
327             self.animating_sequence = False
328             self.plot.update()
329 
```

```
330         @pyqtSlot()
331         def render_expression(self) -> None:
332             """Render the transformation given by the expression in the input box."""
333             try:
334                 matrix = self.matrix_wrapper.evaluate_expression(self.lineedit_expression_box.text())
335 
336             except LinAlgError:
337                 self.show_error_message('Singular matrix', 'Cannot take inverse of singular matrix')
338                 return
339 
340             if self.is_matrix_too_big(matrix):
341                 self.show_error_message('Matrix too big', "This matrix doesn't fit on the canvas")
342                 return
343 
344             self.plot.visualize_matrix_transformation(matrix)
345             self.plot.update()
346 
347         @pyqtSlot()
348         def animate_expression(self) -> None:
349             """Animate from the current matrix to the matrix in the expression box."""
350             self.button_render.setEnabled(False)
351             self.button_animate.setEnabled(False)
352 
353             matrix_start: MatrixType = np.array([
354                 [self.plot.point_i[0], self.plot.point_j[0]],
355                 [self.plot.point_i[1], self.plot.point_j[1]]
356             ])
357 
358             text = self.lineedit_expression_box.text()
359 
360             # If there's commas in the expression, then we want to animate each part at a time
361             if ',' in text:
362                 current_matrix = matrix_start
363                 self.animating_sequence = True
364 
365                 # For each expression in the list, right multiply it by the current matrix,
366                 # and animate from the current matrix to that new matrix
367                 for expr in text.split(',')[::-1]:
368                     try:
369                         new_matrix = self.matrix_wrapper.evaluate_expression(expr)
370 
371                         if self.plot.display_settings.applicative_animation:
372                             new_matrix = new_matrix @ current_matrix
373                     except LinAlgError:
374                         self.show_error_message('Singular matrix', 'Cannot take inverse of singular matrix')
375                         return
376 
377                     if not self.animating_sequence:
378                         break
379 
380                     self.animate_between_matrices(current_matrix, new_matrix)
381                     current_matrix = new_matrix
382 
383                     # Here we just redraw and allow for other events to be handled while we pause
384                     self.plot.update()
385                     QApplication.processEvents()
386                     QThread.msleep(self.plot.display_settings.animation_pause_length)
387 
388                 self.animating_sequence = False
389 
390             # If there's no commas, then just animate directly from the start to the target
391             else:
392                 # Get the target matrix and it's determinant
393                 try:
394                     matrix_target = self.matrix_wrapper.evaluate_expression(text)
395 
396                 except LinAlgError:
397                     self.show_error_message('Singular matrix', 'Cannot take inverse of singular matrix')
398                     return
399 
400                 # The concept of applicative animation is explained in /gui/settings.py
401                 if self.plot.display_settings.applicative_animation:
402                     matrix_target = matrix_target @ matrix_start
```

```
403
404             # If we want a transitional animation and we're animating the same matrix, then restart the animation
405             # We use this check rather than equality because of small floating point errors
406             elif (abs(matrix_start - matrix_target) < 1e-12).all():
407                 matrix_start = self.matrix_wrapper['I']
408
409                 # We pause here for 200 ms to make the animation look a bit nicer
410                 self.plot.visualize_matrix_transformation(matrix_start)
411                 self.plot.update()
412                 QApplication.processEvents()
413                 QThread.msleep(200)
414
415             self.animate_between_matrices(matrix_start, matrix_target)
416
417         self.update_render_buttons()
418
419     def _get_animation_frame(self, start: MatrixType, target: MatrixType, proportion: float) -> MatrixType:
420         """Get the matrix to render for this frame of the animation.
421
422         This method will smoothen the determinant if that setting in enabled and if the determinant is positive.
423         It also animates rotation-like matrices using a logarithmic spiral to rotate around and scale continuously.
424         Essentially, it just makes things look good when animating.
425
426         :param MatrixType start: The starting matrix
427         :param MatrixType start: The target matrix
428         :param float proportion: How far we are through the loop
429         """
430         det_target = linalg.det(target)
431         det_start = linalg.det(start)
432
433         # This is the matrix that we're applying to get from start to target
434         # We want to check if it's rotation-like
435         if linalg.det(start) == 0:
436             matrix_application = None
437         else:
438             matrix_application = target @ linalg.inv(start)
439
440         # For a matrix to represent a rotation, it must have a positive determinant,
441         # its vectors must be perpendicular, and its vectors must be the same length
442         # The checks for 'abs(value) < 1e-10' are to account for floating point error
443         if matrix_application is not None \
444                 and self.plot.display_settings.smoothen_determinant \
445                 and linalg.det(matrix_application) > 0 \
446                 and abs(np.dot(matrix_application.T[0], matrix_application.T[1])) < 1e-10 \
447                 and abs(np.hypot(*matrix_application.T[0]) - np.hypot(*matrix_application.T[1])) < 1e-10:
448             rotation_vector: VectorType = matrix_application.T[0]  # Take the i column
449             radius, angle = polar_coords(*rotation_vector)
450
451             # We want the angle to be in [-pi, pi), so we have to subtract 2pi from it if it's too big
452             if angle > np.pi:
453                 angle -= 2 * np.pi
454
455             i: VectorType = start.T[0]
456             j: VectorType = start.T[1]
457
458             # Scale the coords with a list comprehension
459             # It's a bit janky, but rotate_coords() will always return a 2-tuple,
460             # so new_i and new_j will always be lists of length 2
461             scale = (radius - 1) * proportion + 1
462             new_i = [scale * c for c in rotate_coord(i[0], i[1], angle * proportion)]
463             new_j = [scale * c for c in rotate_coord(j[0], j[1], angle * proportion)]
464
465             return np.array(
466                 [
467                     [new_i[0], new_j[0]],
468                     [new_i[1], new_j[1]]
469                 ]
470             )
471
472         # matrix_a is the start matrix plus some part of the target, scaled by the proportion
473         # If we just used matrix_a, then things would animate, but the determinants would be weird
474         matrix_a = start + proportion * (target - start)
475
```

```
476             if not self.plot.display_settings.smoothen_determinant or det_start * det_target <= 0:
477                 return matrix_a
478 
479             # To fix the determinant problem, we get the determinant of matrix_a and use it to normalize
480             det_a = linalg.det(matrix_a)
481 
482             # For a 2x2 matrix A and a scalar c, we know that det(cA) = c^2 det(A)
483             # We want B = cA such that det(B) = det(S), where S is the start matrix,
484             # so then we can scale it with the animation, so we get
485             # det(cA) = c^2 det(A) = det(S) => c = sqrt(abs(det(S) / det(A)))
486             # Then we scale A to get the determinant we want, and call that matrix_b
487             if det_a == 0:
488                 c = 0
489             else:
490                 c = np.sqrt(abs(det_start / det_a))
491 
492             matrix_b = c * matrix_a
493             det_b = linalg.det(matrix_b)
494 
495             # We want to return B, but we have to scale it over time to have the target determinant
496 
497             # We want some C = dB such that det(C) is some target determinant T
498             # det(dB) = d^2 det(B) = T => d = sqrt(abs(T / det(B))
499 
500             # We're also subtracting 1 and multiplying by the proportion and then adding one
501             # This just scales the determinant along with the animation
502 
503             # That is all of course, if we can do that
504             # We'll crash if we try to do this with det(B) == 0
505             if det_b == 0:
506                 return matrix_a
507 
508             scalar = 1 + proportion * (np.sqrt(abs(det_target / det_b)) - 1)
509             return scalar * matrix_b
510 
511         def animate_between_matrices(self, matrix_start: MatrixType, matrix_target: MatrixType) -> None:
512             """Animate from the start matrix to the target matrix."""
513             self.animating = True
514 
515             # Making steps depend on animation_time ensures a smooth animation without
516             # massive overheads for small animation times
517             steps = self.plot.display_settings.animation_time // 10
518 
519             for i in range(0, steps + 1):
520                 if not self.animating:
521                     break
522 
523                 matrix_to_render = self._get_animation_frame(matrix_start, matrix_target, i / steps)
524 
525                 if self.is_matrix_too_big(matrix_to_render):
526                     self.show_error_message('Matrix too big', "This matrix doesn't fit on the canvas")
527                     self.animating = False
528                     return
529 
530                 self.plot.visualize_matrix_transformation(matrix_to_render)
531 
532                 # We schedule the plot to be updated, tell the event loop to
533                 # process events, and asynchronously sleep for 10ms
534                 # This allows for other events to be processed while animating, like zooming in and out
535                 self.plot.update()
536                 QApplication.processEvents()
537                 QThread.msleep(self.plot.display_settings.animation_time // steps)
538 
539             self.animating = False
540 
541         @pyqtSlot(DefineDialog)
542         def dialog_define_matrix(self, dialog_class: Type[DefineDialog]) -> None:
543             """Open a generic definition dialog to define a new matrix.
544 
545             The class for the desired dialog is passed as an argument. We create an
546             instance of this class and the dialog is opened asynchronously and modally
547             (meaning it blocks interaction with the main window) with the proper method
548             connected to the :meth:`QDialog.accepted` signal.
```

```
549
550             .. note:: ``dialog_class`` must subclass :class:`lintrans.gui.dialogs.define_new_matrix.DefineDialog`.
551
552             :param dialog_class: The dialog class to instantiate
553             :type dialog_class: Type[lintrans.gui.dialogs.define_new_matrix.DefineDialog]
554             """
555             # We create a dialog with a deepcopy of the current matrix_wrapper
556             # This avoids the dialog mutating this one
557             dialog: DefineDialog
558
559             if dialog_class == DefineVisuallyDialog:
560                 dialog = DefineVisuallyDialog(
561                     self,
562                     matrix_wrapper=deepcopy(self.matrix_wrapper),
563                     display_settings=self.plot.display_settings
564                 )
565             else:
566                 dialog = dialog_class(self, matrix_wrapper=deepcopy(self.matrix_wrapper))
567
568             # .open() is asynchronous and doesn't spawn a new event loop, but the dialog is still modal (blocking)
569             dialog.open()
570
571             # So we have to use the accepted signal to call a method when the user accepts the dialog
572             dialog.accepted.connect(self.assign_matrix_wrapper)
573
574         @pyqtSlot()
575         def assign_matrix_wrapper(self) -> None:
576             """Assign a new value to ``self.matrix_wrapper`` and give the expression box focus."""
577             self.matrix_wrapper = self.sender().matrix_wrapper
578             self.lineedit_expression_box.setFocus()
579             self.update_render_buttons()
580
581         @pyqtSlot()
582         def dialog_change_display_settings(self) -> None:
583             """Open the dialog to change the display settings."""
584             dialog = DisplaySettingsDialog(self, display_settings=self.plot.display_settings)
585             dialog.open()
586             dialog.accepted.connect(lambda: self.assign_display_settings(dialog.display_settings))
587
588         @pyqtSlot(DisplaySettings)
589         def assign_display_settings(self, display_settings: DisplaySettings) -> None:
590             """Assign a new value to ``self.plot.display_settings`` and give the expression box focus."""
591             self.plot.display_settings = display_settings
592             self.plot.update()
593             self.lineedit_expression_box.setFocus()
594             self.update_render_buttons()
595
596         def show_error_message(self, title: str, text: str, info: str | None = None) -> None:
597             """Show an error message in a dialog box.
598
599             :param str title: The window title of the dialog box
600             :param str text: The simple error message
601             :param info: The more informative error message
602             :type info: Optional[str]
603             """
604             dialog = QMessageBox(self)
605             dialog.setIcon(QMessageBox.Critical)
606             dialog.setWindowTitle(title)
607             dialog.setText(text)
608
609             if info is not None:
610                 dialog.setInformativeText(info)
611
612             dialog.open()
613
614             # This is `finished` rather than `accepted` because we want to update the buttons no matter what
615             dialog.finished.connect(self.update_render_buttons)
616
617         def is_matrix_too_big(self, matrix: MatrixType) -> bool:
618             """Check if the given matrix will actually fit onto the canvas.
619
620             Convert the elements of the matrix to canvas coords and make sure they fit within Qt's 32-bit integer limit.
621
```

```
622             :param MatrixType matrix: The matrix to check
623             :returns bool: Whether the matrix is too big to fit on the canvas
624             """
625             coords: List[Tuple[int, int]] = [self.plot.canvas_coords(*vector) for vector in matrix.T]
626 
627             for x, y in coords:
628                 if not (-2147483648 <= x <= 2147483647 and -2147483648 <= y <= 2147483647):
629                     return True
630 
631             return False
632 
633 
634     def qapp() -> QCoreApplication:
635         """Return the equivalent of the global :class:`qApp` pointer.
636 
637         :raises RuntimeError: If :meth:`QCoreApplication.instance` returns ``None``
638         """
639         instance = QCoreApplication.instance()
640 
641         if instance is None:
642             raise RuntimeError('qApp undefined')
643 
644         return instance
645 
646 
647     def main(args: List[str]) -> None:
648         """Run the GUI by creating and showing an instance of :class:`LintransMainWindow`.
649 
650         :param List[str] args: The args to pass to :class:`QApplication`
651         """
652         app = QApplication(args)
653         app.setApplicationName('lintrans')
654         app.setApplicationVersion(lintrans.__version__)
655 
656         qapp().setStyle(QStyleFactory.create('fusion'))
657 
658         window = LintransMainWindow()
659         window.show()
660 
661         sys.exit(app.exec_())
```

A.4 gui/settings.py

```
1   # lintrans - The linear transformation visualizer
2   # Copyright (C) 2021-2022 D. Dyson (DoctorDalek1963)
3 
4   # This program is licensed under GNU GPLv3, available here:
5   # <https://www.gnu.org/licenses/gpl-3.0.html>
6 
7   """This module contains the :class:`DisplaySettings` class, which holds configuration for display."""
8 
9   from __future__ import annotations
10 
11  from dataclasses import dataclass
12 
13 
14  @dataclass
15  class DisplaySettings:
16      """This class simply holds some attributes to configure display."""
17 
18      # === Basic stuff
19 
20      draw_background_grid: bool = True
21      """This controls whether we want to draw the background grid.
22 
23      The background axes will always be drawn. This makes it easy to identify the center of the space.
24      """
25 
26      draw_transformed_grid: bool = True
27      """This controls whether we want to draw the transformed grid. Vectors are handled separately."""
```

```
28
29          draw_basis_vectors: bool = True
30          """This controls whether we want to draw the transformed basis vectors."""
31
32          # === Animations
33
34          smoothen_determinant: bool = True
35          """This controls whether we want the determinant to change smoothly during the animation.
36
37          .. note::
38             Even if this is True, it will be ignored if we're animating from a positive det matrix to
39             a negative det matrix, or vice versa, because if we try to smoothly animate that determinant,
40             things blow up and the app often crashes.
41          """
42
43          applicative_animation: bool = True
44          """There are two types of simple animation, transitional and applicative.
45
46          Let ``C`` be the matrix representing the currently displayed transformation, and let ``T`` be the target matrix.
47          Transitional animation means that we animate directly from ``C`` from ``T``,
48          and applicative animation means that we animate from ``C`` to ``TC``, so we apply ``T`` to ``C``.
49          """
50
51          animation_time: int = 1200
52          """This is the number of milliseconds that an animation takes."""
53
54          animation_pause_length: int = 400
55          """This is the number of milliseconds that we wait between animations when using comma syntax."""
56
57          # === Matrix info
58
59          draw_determinant_parallelogram: bool = False
60          """This controls whether or not we should shade the parallelogram representing the determinant of the matrix."""
61
62          show_determinant_value: bool = True
63          """This controls whether we should write the text value of the determinant inside the parallelogram.
64
65          The text only gets draw if :attr:`draw_determinant_parallelogram` is also True.
66          """
67
68          draw_eigenvectors: bool = False
69          """This controls whether we should draw the eigenvectors of the transformation."""
70
71          draw_eigenlines: bool = False
72          """This controls whether we should draw the eigenlines of the transformation."""
```

A.5 gui/__init__.py

```
1   # lintrans - The linear transformation visualizer
2   # Copyright (C) 2021-2022 D. Dyson (DoctorDalek1963)
3
4   # This program is licensed under GNU GPLv3, available here:
5   # <https://www.gnu.org/licenses/gpl-3.0.html>
6
7   """This package supplies the main GUI and associated dialogs for visualization."""
8
9   from . import dialogs, plots, settings, validate
10  from .main_window import main
11
12  __all__ = ['dialogs', 'main', 'plots', 'settings', 'validate']
```

A.6 gui/validate.py

```
1   # lintrans - The linear transformation visualizer
2   # Copyright (C) 2021-2022 D. Dyson (DoctorDalek1963)
3
4   # This program is licensed under GNU GPLv3, available here:
5   # <https://www.gnu.org/licenses/gpl-3.0.html>
```

```

    """This simple module provides a :class:`MatrixExpressionValidator` class to validate matrix expression input."""

    from __future__ import annotations

    import re
    from typing import Tuple

    from PyQt5.QtGui import QValidator

    from lintrans.matrices import parse


    class MatrixExpressionValidator(QValidator):
        """This class validates matrix expressions in a Qt input box."""

        def validate(self, text: str, pos: int) -> Tuple[QValidator.State, str, int]:
            """Validate the given text according to the rules defined in the :mod:`lintrans.matrices` module."""
            clean_text = re.sub(parse.NAIVE_CHARACTER_CLASS[:-1] + ',]', '', text)

            if clean_text == '':
                if parse.validate_matrix_expression(clean_text):
                    return QValidator.Acceptable, text, pos
                else:
                    return QValidator.Intermediate, text, pos

            return QValidator.Invalid, text, pos
```

A.7 gui/dialogs/misc.py

```

    #  lintrans - The linear transformation visualizer
    #  Copyright (C) 2022 D. Dyson (DoctorDalek1963)
    #
    #  This program is licensed under GNU GPLv3, available here:
    #  <https://www.gnu.org/licenses/gpl-3.0.html>

    """This module provides miscellaneous dialog classes like :class:`AboutDialog`."""

    from __future__ import annotations

    import platform
    from typing import Union

    from PyQt5.QtCore import PYQT_VERSION_STR, QT_VERSION_STR, Qt
    from PyQt5.QtWidgets import QDialog, QGridLayout, QLabel, QVBoxLayout, QWidget

    import lintrans
    from lintrans.matrices.utility import round_float
    from lintrans.matrices import MatrixWrapper
    from lintrans.typing_ import is_matrix_type, MatrixType


    class FixedSizeDialog(QDialog):
        """A simple superclass to create modal dialog boxes with fixed size.

        We override the :meth:`open` method to set the fixed size as soon as the dialog is opened modally.
        """

        def open(self) -> None:
            """Override :meth:`QDialog.open` to set the dialog to a fixed size."""
            super().open()
            self.setFixedSize(self.size())


    class AboutDialog(FixedSizeDialog):
        """A simple dialog class to display information about the app to the user.

        It only has an :meth:`__init__` method because it only has label widgets, so no other methods are necessary
    ↪   here.
        """
```

```
40
41          def __init__(self, *args, **kwargs):
42              """Create an :class:`AboutDialog` object with all the label widgets."""
43              super().__init__(*args, **kwargs)
44
45              self.setWindowTitle('About lintrans')
46
47              # === Create the widgets
48
49              label_title = QLabel(self)
50              label_title.setText(f'lintrans (version {lintrans.__version__})')
51              label_title.setAlignment(Qt.AlignCenter)
52
53              font_title = label_title.font()
54              font_title.setPointSize(font_title.pointSize() * 2)
55              label_title.setFont(font_title)
56
57              label_version_info = QLabel(self)
58              label_version_info.setText(
59                  f'With Python version {platform.python_version()}\n'
60                  f'Qt version {QT_VERSION_STR} and PyQt5 version {PYQT_VERSION_STR}\n'
61                  f'Running on {platform.platform()}'
62              )
63              label_version_info.setAlignment(Qt.AlignCenter)
64
65              label_info = QLabel(self)
66              label_info.setText(
67                  'lintrans is a program designed to help visualise<br>'
68                  '2D linear transformations represented with matrices.<br><br>'
69                  "It's designed for teachers and students and any feedback<br>"
70                  'is greatly appreciated at <a href="https://github.com/DoctorDalek1963/lintrans" '
71                  'style="color: black;">my GitHub page</a><br>or via email '
72                  '(<a href="mailto:dyson.dyson@icloud.com" style="color: black;">dyson.dyson@icloud.com</a>).'
73              )
74              label_info.setAlignment(Qt.AlignCenter)
75              label_info.setTextFormat(Qt.RichText)
76              label_info.setOpenExternalLinks(True)
77
78              label_copyright = QLabel(self)
79              label_copyright.setText(
80                  'This program is free software.<br>Copyright 2021-2022 D. Dyson (DoctorDalek1963).<br>'
81                  'This program is licensed under GPLv3, which can be found '
82                  '<a href="https://www.gnu.org/licenses/gpl-3.0.html" style="color: black;">here</a>.'
83              )
84              label_copyright.setAlignment(Qt.AlignCenter)
85              label_copyright.setTextFormat(Qt.RichText)
86              label_copyright.setOpenExternalLinks(True)
87
88              # === Arrange the widgets
89
90              self.setContentsMargins(10, 10, 10, 10)
91
92              vlay = QVBoxLayout()
93              vlay.setSpacing(20)
94              vlay.addWidget(label_title)
95              vlay.addWidget(label_version_info)
96              vlay.addWidget(label_info)
97              vlay.addWidget(label_copyright)
98
99              self.setLayout(vlay)
100
101
102     class InfoPanelDialog(FixedSizeDialog):
103         """A simple dialog class to display an info panel that shows all currently defined matrices."""
104
105         def __init__(self, matrix_wrapper: MatrixWrapper, *args, **kwargs):
106             """Create the dialog box with all the widgets needed to show the information."""
107             super().__init__(*args, **kwargs)
108             self.wrapper = matrix_wrapper
109
110             self.setWindowTitle('Defined matrices')
111
112             grid_layout = QGridLayout()
```

```
113             grid_layout.setSpacing(20)
114
115             bold_font = self.font()
116             bold_font.setBold(True)
117
118             name_value_pair: tuple[str, Union[MatrixType, str]]
119
120             # Each defined matrix will get a widget group. Each group will be a label for the name,
121             # a label for '=', and a container widget to either show the matrix numerically, or to
122             # show the expression that it's defined as
123             for i, name_value_pair in enumerate(self.wrapper.get_defined_matrices()):
124                 name, value = name_value_pair
125
126                 # Create all the widgets first
127                 label_name = QLabel(self)
128                 label_name.setText(name)
129                 label_name.setFont(bold_font)
130
131                 label_equals = QLabel(self)
132                 label_equals.setText('=')
133
134                 widget_matrix = self._get_matrix_widget(value)
135
136                 # We want columns of at most 6 widget groups
137                 # This column variable manages which column of defined matrices we're on
138                 # It's multiplied by 3 because all the widgets are in a single grid layout
139                 # I could factor out each triplet of widgets for a defined matrix into a container widget,
140                 # but I prefer to keep the widget count lower to reduce any possible lag
141                 column = 3 * (i // 6)
142
143                 grid_layout.addWidget(
144                     label_name,
145                     i - 2 * column,
146                     column,
147                     Qt.AlignCenter
148                 )
149                 grid_layout.addWidget(
150                     label_equals,
151                     i - 2 * column,
152                     column + 1,
153                     Qt.AlignCenter
154                 )
155                 grid_layout.addWidget(
156                     widget_matrix,
157                     i - 2 * column,
158                     column + 2,
159                     Qt.AlignCenter
160                 )
161
162             self.setContentsMargins(10, 10, 10, 10)
163             self.setLayout(grid_layout)
164
165         def _get_matrix_widget(self, matrix: Union[MatrixType, str]) -> QWidget:
166             """Return a :class:`QWidget` containing the value of the matrix.
167
168             If the matrix is defined as an expression, it will be a simple :class:`QLabel`.
169             If the matrix is defined as a matrix, it will be a :class:`QWidget` container
170             with multiple :class:`QLabel` objects in it.
171             """
172             if isinstance(matrix, str):
173                 label = QLabel(self)
174                 label.setText(matrix)
175                 return label
176
177             elif is_matrix_type(matrix):
178                 # tl = top left, br = bottom right, etc.
179                 label_tl = QLabel(self)
180                 label_tl.setText(round_float(matrix[0][0]))
181
182                 label_tr = QLabel(self)
183                 label_tr.setText(round_float(matrix[0][1]))
184
185                 label_bl = QLabel(self)
```

```
186                 label_bl.setText(round_float(matrix[1][0]))
187
188                 label_br = QLabel(self)
189                 label_br.setText(round_float(matrix[1][1]))
190
191                 # The parens need to be bigger than the numbers, but increasing the font size also
192                 # makes the font thicker, so we have to reduce the font weight by the same factor
193                 font_parens = self.font()
194                 font_parens.setPointSize(int(font_parens.pointSize() * 2.5))
195                 font_parens.setWeight(int(font_parens.weight() / 2.5))
196
197                 label_paren_left = QLabel(self)
198                 label_paren_left.setText('(')
199                 label_paren_left.setFont(font_parens)
200
201                 label_paren_right = QLabel(self)
202                 label_paren_right.setText(')')
203                 label_paren_right.setFont(font_parens)
204
205                 container = QWidget(self)
206                 grid_layout = QGridLayout()
207
208                 grid_layout.addWidget(label_paren_left, 0, 0, -1, 1)
209                 grid_layout.addWidget(label_tl, 0, 1)
210                 grid_layout.addWidget(label_tr, 0, 2)
211                 grid_layout.addWidget(label_bl, 1, 1)
212                 grid_layout.addWidget(label_br, 1, 2)
213                 grid_layout.addWidget(label_paren_right, 0, 3, -1, 1)
214
215                 container.setLayout(grid_layout)
216
217                 return container
218
219             raise ValueError('Matrix was not MatrixType or str')
```

A.8 gui/dialogs/settings.py

```
1   # lintrans - The linear transformation visualizer
2   # Copyright (C) 2021-2022 D. Dyson (DoctorDalek1963)
3
4   # This program is licensed under GNU GPLv3, available here:
5   # <https://www.gnu.org/licenses/gpl-3.0.html>
6
7   """This module provides dialogs to edit settings within the app."""
8
9   from __future__ import annotations
10
11  import abc
12  from typing import Dict
13
14  from PyQt5 import QtWidgets
15  from PyQt5.QtGui import QIntValidator, QKeyEvent, QKeySequence
16  from PyQt5.QtWidgets import QCheckBox, QGroupBox, QHBoxLayout, QShortcut, QSizePolicy, QSpacerItem, QVBoxLayout
17
18  from lintrans.gui.dialogs.misc import FixedSizeDialog
19  from lintrans.gui.settings import DisplaySettings
20
21
22  class SettingsDialog(FixedSizeDialog):
23      """An abstract superclass for other simple dialogs."""
24
25      def __init__(self, *args, **kwargs):
26          """Create the widgets and layout of the dialog, passing ``*args`` and ``**kwargs`` to super."""
27          super().__init__(*args, **kwargs)
28
29          # === Create the widgets
30
31          self.button_confirm = QtWidgets.QPushButton(self)
32          self.button_confirm.setText('Confirm')
33          self.button_confirm.clicked.connect(self.confirm_settings)
```

```
34            self.button_confirm.setToolTip('Confirm these new settings<br><b>(Ctrl + Enter)</b>')
35            QShortcut(QKeySequence('Ctrl+Return'), self).activated.connect(self.button_confirm.click)
36
37            self.button_cancel = QtWidgets.QPushButton(self)
38            self.button_cancel.setText('Cancel')
39            self.button_cancel.clicked.connect(self.reject)
40            self.button_cancel.setToolTip('Revert these settings<br><b>(Escape)</b>')
41
42            # === Arrange the widgets
43
44            self.setContentsMargins(10, 10, 10, 10)
45
46            self.hlay_buttons = QHBoxLayout()
47            self.hlay_buttons.setSpacing(20)
48            self.hlay_buttons.addItem(QSpacerItem(50, 5, hPolicy=QSizePolicy.Expanding, vPolicy=QSizePolicy.Minimum))
49            self.hlay_buttons.addWidget(self.button_cancel)
50            self.hlay_buttons.addWidget(self.button_confirm)
51
52            self.vlay_options = QVBoxLayout()
53            self.vlay_options.setSpacing(20)
54
55            self.vlay_all = QVBoxLayout()
56            self.vlay_all.setSpacing(20)
57            self.vlay_all.addLayout(self.vlay_options)
58            self.vlay_all.addLayout(self.hlay_buttons)
59
60            self.setLayout(self.vlay_all)
61
62        @abc.abstractmethod
63        def load_settings(self) -> None:
64            """Load the current settings into the widgets."""
65
66        @abc.abstractmethod
67        def confirm_settings(self) -> None:
68            """Confirm the settings chosen in the dialog."""
69
70
71    class DisplaySettingsDialog(SettingsDialog):
72        """The dialog to allow the user to edit the display settings."""
73
74        def __init__(self, *args, display_settings: DisplaySettings, **kwargs):
75            """Create the widgets and layout of the dialog.
76
77            :param DisplaySettings display_settings: The :class:`lintrans.gui.settings.DisplaySettings` object to mutate
78            """
79            super().__init__(*args, **kwargs)
80
81            self.display_settings = display_settings
82            self.setWindowTitle('Change display settings')
83
84            self.dict_checkboxes: Dict[str, QCheckBox] = dict()
85
86            # === Create the widgets
87
88            # Basic stuff
89
90            self.checkbox_draw_background_grid = QCheckBox(self)
91            self.checkbox_draw_background_grid.setText('Draw &background grid')
92            self.checkbox_draw_background_grid.setToolTip(
93                'Draw the background grid (axes are always drawn)'
94            )
95            self.dict_checkboxes['b'] = self.checkbox_draw_background_grid
96
97            self.checkbox_draw_transformed_grid = QCheckBox(self)
98            self.checkbox_draw_transformed_grid.setText('Draw t&ransformed grid')
99            self.checkbox_draw_transformed_grid.setToolTip(
100               'Draw the transformed grid (vectors are handled separately)'
101           )
102           self.dict_checkboxes['r'] = self.checkbox_draw_transformed_grid
103
104           self.checkbox_draw_basis_vectors = QCheckBox(self)
105           self.checkbox_draw_basis_vectors.setText('Draw basis &vectors')
106           self.checkbox_draw_basis_vectors.setToolTip(
```

```
107                 'Draw the transformed basis vectors'
108             )
109             self.dict_checkboxes['v'] = self.checkbox_draw_basis_vectors
110 
111             # Animations
112 
113             self.checkbox_smoothen_determinant = QCheckBox(self)
114             self.checkbox_smoothen_determinant.setText('&Smoothen determinant')
115             self.checkbox_smoothen_determinant.setToolTip(
116                 'Smoothly animate the determinant transition during animation (if possible)'
117             )
118             self.dict_checkboxes['s'] = self.checkbox_smoothen_determinant
119 
120             self.checkbox_applicative_animation = QCheckBox(self)
121             self.checkbox_applicative_animation.setText('&Applicative animation')
122             self.checkbox_applicative_animation.setToolTip(
123                 'Animate the new transformation applied to the current one,\n'
124                 'rather than just that transformation on its own'
125             )
126             self.dict_checkboxes['a'] = self.checkbox_applicative_animation
127 
128             self.label_animation_time = QtWidgets.QLabel(self)
129             self.label_animation_time.setText('Total animation length (ms)')
130             self.label_animation_time.setToolTip(
131                 'How long it takes for an animation to complete'
132             )
133 
134             self.lineedit_animation_time = QtWidgets.QLineEdit(self)
135             self.lineedit_animation_time.setValidator(QIntValidator(1, 9999, self))
136 
137             self.label_animation_pause_length = QtWidgets.QLabel(self)
138             self.label_animation_pause_length.setText('Animation pause length (ms)')
139             self.label_animation_pause_length.setToolTip(
140                 'How many milliseconds to pause for in comma-separated animations'
141             )
142 
143             self.lineedit_animation_pause_length = QtWidgets.QLineEdit(self)
144             self.lineedit_animation_pause_length.setValidator(QIntValidator(1, 999, self))
145 
146             # Matrix info
147 
148             self.checkbox_draw_determinant_parallelogram = QCheckBox(self)
149             self.checkbox_draw_determinant_parallelogram.setText('Draw &determinant parallelogram')
150             self.checkbox_draw_determinant_parallelogram.setToolTip(
151                 'Shade the parallelogram representing the determinant of the matrix'
152             )
153             self.checkbox_draw_determinant_parallelogram.clicked.connect(self.update_gui)
154             self.dict_checkboxes['d'] = self.checkbox_draw_determinant_parallelogram
155 
156             self.checkbox_show_determinant_value = QCheckBox(self)
157             self.checkbox_show_determinant_value.setText('Show de&terminant value')
158             self.checkbox_show_determinant_value.setToolTip(
159                 'Show the value of the determinant inside the parallelogram'
160             )
161             self.dict_checkboxes['t'] = self.checkbox_show_determinant_value
162 
163             self.checkbox_draw_eigenvectors = QCheckBox(self)
164             self.checkbox_draw_eigenvectors.setText('Draw &eigenvectors')
165             self.checkbox_draw_eigenvectors.setToolTip('Draw the eigenvectors of the transformations')
166             self.dict_checkboxes['e'] = self.checkbox_draw_eigenvectors
167 
168             self.checkbox_draw_eigenlines = QCheckBox(self)
169             self.checkbox_draw_eigenlines.setText('Draw eigen&lines')
170             self.checkbox_draw_eigenlines.setToolTip('Draw the eigenlines (invariant lines) of the transformations')
171             self.dict_checkboxes['l'] = self.checkbox_draw_eigenlines
172 
173             # === Arrange the widgets in QGroupBoxes
174 
175             # Basic stuff
176 
177             self.vlay_groupbox_basic_stuff = QVBoxLayout()
178             self.vlay_groupbox_basic_stuff.setSpacing(20)
179             self.vlay_groupbox_basic_stuff.addWidget(self.checkbox_draw_background_grid)
```

```
180             self.vlay_groupbox_basic_stuff.addWidget(self.checkbox_draw_transformed_grid)
181             self.vlay_groupbox_basic_stuff.addWidget(self.checkbox_draw_basis_vectors)
182
183             self.groupbox_basic_stuff = QGroupBox('Basic stuff', self)
184             self.groupbox_basic_stuff.setLayout(self.vlay_groupbox_basic_stuff)
185
186             # Animations
187
188             self.hlay_animation_time = QHBoxLayout()
189             self.hlay_animation_time.addWidget(self.label_animation_time)
190             self.hlay_animation_time.addWidget(self.lineedit_animation_time)
191
192             self.hlay_animation_pause_length = QHBoxLayout()
193             self.hlay_animation_pause_length.addWidget(self.label_animation_pause_length)
194             self.hlay_animation_pause_length.addWidget(self.lineedit_animation_pause_length)
195
196             self.vlay_groupbox_animations = QVBoxLayout()
197             self.vlay_groupbox_animations.setSpacing(20)
198             self.vlay_groupbox_animations.addWidget(self.checkbox_smoothen_determinant)
199             self.vlay_groupbox_animations.addWidget(self.checkbox_applicative_animation)
200             self.vlay_groupbox_animations.addLayout(self.hlay_animation_time)
201             self.vlay_groupbox_animations.addLayout(self.hlay_animation_pause_length)
202
203             self.groupbox_animations = QGroupBox('Animations', self)
204             self.groupbox_animations.setLayout(self.vlay_groupbox_animations)
205
206             # Matrix info
207
208             self.vlay_groupbox_matrix_info = QVBoxLayout()
209             self.vlay_groupbox_matrix_info.setSpacing(20)
210             self.vlay_groupbox_matrix_info.addWidget(self.checkbox_draw_determinant_parallelogram)
211             self.vlay_groupbox_matrix_info.addWidget(self.checkbox_show_determinant_value)
212             self.vlay_groupbox_matrix_info.addWidget(self.checkbox_draw_eigenvectors)
213             self.vlay_groupbox_matrix_info.addWidget(self.checkbox_draw_eigenlines)
214
215             self.groupbox_matrix_info = QGroupBox('Matrix info', self)
216             self.groupbox_matrix_info.setLayout(self.vlay_groupbox_matrix_info)
217
218             # Now arrange the groupboxes
219             self.vlay_options.addWidget(self.groupbox_basic_stuff)
220             self.vlay_options.addWidget(self.groupbox_animations)
221             self.vlay_options.addWidget(self.groupbox_matrix_info)
222
223             # Finally, we load the current settings and update the GUI
224             self.load_settings()
225             self.update_gui()
226
227         def load_settings(self) -> None:
228             """Load the current display settings into the widgets."""
229             # Basic stuff
230             self.checkbox_draw_background_grid.setChecked(self.display_settings.draw_background_grid)
231             self.checkbox_draw_transformed_grid.setChecked(self.display_settings.draw_transformed_grid)
232             self.checkbox_draw_basis_vectors.setChecked(self.display_settings.draw_basis_vectors)
233
234             # Animations
235             self.checkbox_smoothen_determinant.setChecked(self.display_settings.smoothen_determinant)
236             self.checkbox_applicative_animation.setChecked(self.display_settings.applicative_animation)
237             self.lineedit_animation_time.setText(str(self.display_settings.animation_time))
238             self.lineedit_animation_pause_length.setText(str(self.display_settings.animation_pause_length))
239
240             # Matrix info
241             self.checkbox_draw_determinant_parallelogram.setChecked(
                ↪  self.display_settings.draw_determinant_parallelogram)
242             self.checkbox_show_determinant_value.setChecked(self.display_settings.show_determinant_value)
243             self.checkbox_draw_eigenvectors.setChecked(self.display_settings.draw_eigenvectors)
244             self.checkbox_draw_eigenlines.setChecked(self.display_settings.draw_eigenlines)
245
246         def confirm_settings(self) -> None:
247             """Build a :class:`lintrans.gui.settings.DisplaySettings` object and assign it."""
248             # Basic stuff
249             self.display_settings.draw_background_grid = self.checkbox_draw_background_grid.isChecked()
250             self.display_settings.draw_transformed_grid = self.checkbox_draw_transformed_grid.isChecked()
251             self.display_settings.draw_basis_vectors = self.checkbox_draw_basis_vectors.isChecked()
```

```
252
253             # Animations
254             self.display_settings.smoothen_determinant = self.checkbox_smoothen_determinant.isChecked()
255             self.display_settings.applicative_animation = self.checkbox_applicative_animation.isChecked()
256             self.display_settings.animation_time = int(self.lineedit_animation_time.text())
257             self.display_settings.animation_pause_length = int(self.lineedit_animation_pause_length.text())
258
259             # Matrix info
260             self.display_settings.draw_determinant_parallelogram =
                ↪  self.checkbox_draw_determinant_parallelogram.isChecked()
261             self.display_settings.show_determinant_value = self.checkbox_show_determinant_value.isChecked()
262             self.display_settings.draw_eigenvectors = self.checkbox_draw_eigenvectors.isChecked()
263             self.display_settings.draw_eigenlines = self.checkbox_draw_eigenlines.isChecked()
264
265             self.accept()
266
267         def update_gui(self) -> None:
268             """Update the GUI according to other widgets in the GUI.
269
270             For example, this method updates which checkboxes are enabled based on the values of other checkboxes.
271             """
272             self.checkbox_show_determinant_value.setEnabled(self.checkbox_draw_determinant_parallelogram.isChecked())
273
274         def keyPressEvent(self, event: QKeyEvent) -> None:
275             """Handle a :class:`QKeyEvent` by manually activating toggling checkboxes.
276
277             Qt handles these shortcuts automatically and allows the user to do ``Alt + Key``
278             to activate a simple shortcut defined with ``&``. However, I like to be able to
279             just hit ``Key`` and have the shortcut activate.
280             """
281             letter = event.text().lower()
282             key = event.key()
283
284             if letter in self.dict_checkboxes:
285                 self.dict_checkboxes[letter].animateClick()
286
287             # Return or keypad enter
288             elif key == 0x01000004 or key == 0x01000005:
289                 self.button_confirm.click()
290
291             # Escape
292             elif key == 0x01000000:
293                 self.button_cancel.click()
294
295             else:
296                 event.ignore()
```

A.9 gui/dialogs/__init__.py

```
1   # lintrans - The linear transformation visualizer
2   # Copyright (C) 2021-2022 D. Dyson (DoctorDalek1963)
3
4   # This program is licensed under GNU GPLv3, available here:
5   # <https://www.gnu.org/licenses/gpl-3.0.html>
6
7   """This package provides separate dialogs for the main GUI.
8
9   These dialogs are for defining new matrices in different ways and editing settings.
10  """
11
12  from .define_new_matrix import DefineAsAnExpressionDialog, DefineDialog, DefineNumericallyDialog,
    ↪  DefineVisuallyDialog
13  from .misc import AboutDialog, InfoPanelDialog
14  from .settings import DisplaySettingsDialog
15
16  __all__ = ['AboutDialog', 'DefineAsAnExpressionDialog', 'DefineDialog', 'DefineNumericallyDialog',
17             'DefineVisuallyDialog', 'DisplaySettingsDialog', 'InfoPanelDialog']
```

A.10 gui/dialogs/define_new_matrix.py

```
# lintrans - The linear transformation visualizer
# Copyright (C) 2021-2022 D. Dyson (DoctorDalek1963)

# This program is licensed under GNU GPLv3, available here:
# <https://www.gnu.org/licenses/gpl-3.0.html>

"""This module provides an abstract :class:`DefineDialog` class and subclasses, allowing definition of new
↪  matrices."""

from __future__ import annotations

import abc

from numpy import array
from PyQt5 import QtWidgets
from PyQt5.QtCore import pyqtSlot
from PyQt5.QtGui import QDoubleValidator, QKeySequence
from PyQt5.QtWidgets import QGridLayout, QHBoxLayout, QShortcut, QSizePolicy, QSpacerItem, QVBoxLayout

from lintrans.gui.dialogs.misc import FixedSizeDialog
from lintrans.gui.plots import DefineVisuallyWidget
from lintrans.gui.settings import DisplaySettings
from lintrans.gui.validate import MatrixExpressionValidator
from lintrans.matrices import MatrixWrapper
from lintrans.matrices.utility import is_valid_float, round_float
from lintrans.typing_ import MatrixType

ALPHABET_NO_I = 'ABCDEFGHJKLMNOPQRSTUVWXYZ'


class DefineDialog(FixedSizeDialog):
    """An abstract superclass for definitions dialogs.

    .. warning:: This class should never be directly instantiated, only subclassed.

    .. note::
       I would make this class have ``metaclass=abc.ABCMeta``, but I can't because it subclasses :class:`QDialog`,
       and every superclass of a class must have the same metaclass, and :class:`QDialog` is not an abstract class.
    """

    def __init__(self, *args, matrix_wrapper: MatrixWrapper, **kwargs):
        """Create the widgets and layout of the dialog.

        .. note:: ``*args`` and ``**kwargs`` are passed to the super constructor (:class:`QDialog`).

        :param MatrixWrapper matrix_wrapper: The MatrixWrapper that this dialog will mutate
        """
        super().__init__(*args, **kwargs)

        self.matrix_wrapper = matrix_wrapper
        self.setWindowTitle('Define a matrix')

        # === Create the widgets

        self.button_confirm = QtWidgets.QPushButton(self)
        self.button_confirm.setText('Confirm')
        self.button_confirm.setEnabled(False)
        self.button_confirm.clicked.connect(self.confirm_matrix)
        self.button_confirm.setToolTip('Confirm this as the new matrix<br><b>(Ctrl + Enter)</b>')
        QShortcut(QKeySequence('Ctrl+Return'), self).activated.connect(self.button_confirm.click)

        self.button_cancel = QtWidgets.QPushButton(self)
        self.button_cancel.setText('Cancel')
        self.button_cancel.clicked.connect(self.reject)
        self.button_cancel.setToolTip('Cancel this definition<br><b>(Escape)</b>')

        self.label_equals = QtWidgets.QLabel()
        self.label_equals.setText('=')

        self.combobox_letter = QtWidgets.QComboBox(self)
```

```
70
71              for letter in ALPHABET_NO_I:
72                  self.combobox_letter.addItem(letter)
73
74              self.combobox_letter.activated.connect(self.load_matrix)
75
76              # === Arrange the widgets
77
78              self.setContentsMargins(10, 10, 10, 10)
79
80              self.hlay_buttons = QHBoxLayout()
81              self.hlay_buttons.setSpacing(20)
82              self.hlay_buttons.addItem(QSpacerItem(50, 5, hPolicy=QSizePolicy.Expanding, vPolicy=QSizePolicy.Minimum))
83              self.hlay_buttons.addWidget(self.button_cancel)
84              self.hlay_buttons.addWidget(self.button_confirm)
85
86              self.hlay_definition = QHBoxLayout()
87              self.hlay_definition.setSpacing(20)
88              self.hlay_definition.addWidget(self.combobox_letter)
89              self.hlay_definition.addWidget(self.label_equals)
90
91              self.vlay_all = QVBoxLayout()
92              self.vlay_all.setSpacing(20)
93
94              self.setLayout(self.vlay_all)
95
96          @property
97          def selected_letter(self) -> str:
98              """Return the letter currently selected in the combo box."""
99              return str(self.combobox_letter.currentText())
100
101         @abc.abstractmethod
102         @pyqtSlot()
103         def update_confirm_button(self) -> None:
104             """Enable the confirm button if it should be enabled, else, disable it."""
105
106         @pyqtSlot(int)
107         def load_matrix(self, index: int) -> None:
108             """Load the selected matrix into the dialog.
109
110             This method is optionally able to be overridden. If it is not overridden,
111             then no matrix is loaded when selecting a name.
112
113             We have this method in the superclass so that we can define it as the slot
114             for the :meth:`QComboBox.activated` signal in this constructor, rather than
115             having to define that in the constructor of every subclass.
116             """
117
118         @abc.abstractmethod
119         @pyqtSlot()
120         def confirm_matrix(self) -> None:
121             """Confirm the inputted matrix and assign it.
122
123             .. note:: When subclassing, this method should mutate ``self.matrix_wrapper`` and then call
    ↪  ``self.accept()``.
124             """
125
126
127     class DefineVisuallyDialog(DefineDialog):
128         """The dialog class that allows the user to define a matrix visually."""
129
130         def __init__(self, *args, matrix_wrapper: MatrixWrapper, display_settings: DisplaySettings, **kwargs):
131             """Create the widgets and layout of the dialog.
132
133             :param MatrixWrapper matrix_wrapper: The MatrixWrapper that this dialog will mutate
134             """
135             super().__init__(*args, matrix_wrapper=matrix_wrapper, **kwargs)
136
137             self.setMinimumSize(700, 550)
138
139             # === Create the widgets
140
141             self.plot = DefineVisuallyWidget(self, display_settings=display_settings)
```

```
142
143             # === Arrange the widgets
144
145             self.hlay_definition.addWidget(self.plot)
146             self.hlay_definition.setStretchFactor(self.plot, 1)
147
148             self.vlay_all.addLayout(self.hlay_definition)
149             self.vlay_all.addLayout(self.hlay_buttons)
150
151             # We load the default matrix A into the plot
152             self.load_matrix(0)
153
154             # We also enable the confirm button, because any visually defined matrix is valid
155             self.button_confirm.setEnabled(True)
156
157         @pyqtSlot()
158         def update_confirm_button(self) -> None:
159             """Enable the confirm button.
160
161             .. note::
162                The confirm button is always enabled in this dialog and this method is never actually used,
163                so it's got an empty body. It's only here because we need to implement the abstract method.
164             """
165
166         @pyqtSlot(int)
167         def load_matrix(self, index: int) -> None:
168             """Show the selected matrix on the plot. If the matrix is None, show the identity."""
169             matrix = self.matrix_wrapper[self.selected_letter]
170
171             if matrix is None:
172                 matrix = self.matrix_wrapper['I']
173
174             self.plot.visualize_matrix_transformation(matrix)
175             self.plot.update()
176
177         @pyqtSlot()
178         def confirm_matrix(self) -> None:
179             """Confirm the matrix that's been defined visually."""
180             matrix: MatrixType = array([
181                 [self.plot.point_i[0], self.plot.point_j[0]],
182                 [self.plot.point_i[1], self.plot.point_j[1]]
183             ])
184
185             self.matrix_wrapper[self.selected_letter] = matrix
186             self.accept()
187
188
189     class DefineNumericallyDialog(DefineDialog):
190         """The dialog class that allows the user to define a new matrix numerically."""
191
192         def __init__(self, *args, matrix_wrapper: MatrixWrapper, **kwargs):
193             """Create the widgets and layout of the dialog.
194
195             :param MatrixWrapper matrix_wrapper: The MatrixWrapper that this dialog will mutate
196             """
197             super().__init__(*args, matrix_wrapper=matrix_wrapper, **kwargs)
198
199             # === Create the widgets
200
201             # tl = top left, br = bottom right, etc.
202             self.element_tl = QtWidgets.QLineEdit(self)
203             self.element_tl.textChanged.connect(self.update_confirm_button)
204             self.element_tl.setValidator(QDoubleValidator())
205
206             self.element_tr = QtWidgets.QLineEdit(self)
207             self.element_tr.textChanged.connect(self.update_confirm_button)
208             self.element_tr.setValidator(QDoubleValidator())
209
210             self.element_bl = QtWidgets.QLineEdit(self)
211             self.element_bl.textChanged.connect(self.update_confirm_button)
212             self.element_bl.setValidator(QDoubleValidator())
213
214             self.element_br = QtWidgets.QLineEdit(self)
```

```
215             self.element_br.textChanged.connect(self.update_confirm_button)
216             self.element_br.setValidator(QDoubleValidator())
217
218             self.matrix_elements = (self.element_tl, self.element_tr, self.element_bl, self.element_br)
219
220             # === Arrange the widgets
221
222             self.grid_matrix = QGridLayout()
223             self.grid_matrix.setSpacing(20)
224             self.grid_matrix.addWidget(self.element_tl, 0, 0)
225             self.grid_matrix.addWidget(self.element_tr, 0, 1)
226             self.grid_matrix.addWidget(self.element_bl, 1, 0)
227             self.grid_matrix.addWidget(self.element_br, 1, 1)
228
229             self.hlay_definition.addLayout(self.grid_matrix)
230
231             self.vlay_all.addLayout(self.hlay_definition)
232             self.vlay_all.addLayout(self.hlay_buttons)
233
234             # We load the default matrix A into the boxes
235             self.load_matrix(0)
236
237             self.element_tl.setFocus()
238
239         @pyqtSlot()
240         def update_confirm_button(self) -> None:
241             """Enable the confirm button if there are valid floats in every box."""
242             for elem in self.matrix_elements:
243                 if not is_valid_float(elem.text()):
244                     # If they're not all numbers, then we can't confirm it
245                     self.button_confirm.setEnabled(False)
246                     return
247
248             # If we didn't find anything invalid
249             self.button_confirm.setEnabled(True)
250
251         @pyqtSlot(int)
252         def load_matrix(self, index: int) -> None:
253             """If the selected matrix is defined, load its values into the boxes."""
254             matrix = self.matrix_wrapper[self.selected_letter]
255
256             if matrix is None:
257                 for elem in self.matrix_elements:
258                     elem.setText('')
259
260             else:
261                 self.element_tl.setText(round_float(matrix[0][0]))
262                 self.element_tr.setText(round_float(matrix[0][1]))
263                 self.element_bl.setText(round_float(matrix[1][0]))
264                 self.element_br.setText(round_float(matrix[1][1]))
265
266             self.update_confirm_button()
267
268         @pyqtSlot()
269         def confirm_matrix(self) -> None:
270             """Confirm the matrix in the boxes and assign it to the name in the combo box."""
271             matrix: MatrixType = array([
272                 [float(self.element_tl.text()), float(self.element_tr.text())],
273                 [float(self.element_bl.text()), float(self.element_br.text())]
274             ])
275
276             self.matrix_wrapper[self.selected_letter] = matrix
277             self.accept()
278
279
280     class DefineAsAnExpressionDialog(DefineDialog):
281         """The dialog class that allows the user to define a matrix as an expression of other matrices."""
282
283         def __init__(self, *args, matrix_wrapper: MatrixWrapper, **kwargs):
284             """Create the widgets and layout of the dialog.
285
286             :param MatrixWrapper matrix_wrapper: The MatrixWrapper that this dialog will mutate
287             """
```

```
288             super().__init__(*args, matrix_wrapper=matrix_wrapper, **kwargs)
289
290             self.setMinimumWidth(450)
291
292             # === Create the widgets
293
294             self.lineedit_expression_box = QtWidgets.QLineEdit(self)
295             self.lineedit_expression_box.setPlaceholderText('Enter matrix expression...')
296             self.lineedit_expression_box.textChanged.connect(self.update_confirm_button)
297             self.lineedit_expression_box.setValidator(MatrixExpressionValidator())
298
299             # === Arrange the widgets
300
301             self.hlay_definition.addWidget(self.lineedit_expression_box)
302
303             self.vlay_all.addLayout(self.hlay_definition)
304             self.vlay_all.addLayout(self.hlay_buttons)
305
306             # Load the matrix if it's defined as an expression
307             self.load_matrix(0)
308
309             self.lineedit_expression_box.setFocus()
310
311         @pyqtSlot()
312         def update_confirm_button(self) -> None:
313             """Enable the confirm button if the matrix expression is valid in the wrapper."""
314             text = self.lineedit_expression_box.text()
315             valid_expression = self.matrix_wrapper.is_valid_expression(text)
316
317             self.button_confirm.setEnabled(valid_expression and self.selected_letter not in text)
318
319         @pyqtSlot(int)
320         def load_matrix(self, index: int) -> None:
321             """If the selected matrix is defined an expression, load that expression into the box."""
322             if (expr := self.matrix_wrapper.get_expression(self.selected_letter)) is not None:
323                 self.lineedit_expression_box.setText(expr)
324             else:
325                 self.lineedit_expression_box.setText('')
326
327         @pyqtSlot()
328         def confirm_matrix(self) -> None:
329             """Evaluate the matrix expression and assign its value to the name in the combo box."""
330             self.matrix_wrapper[self.selected_letter] = self.lineedit_expression_box.text()
331             self.accept()
```

A.11 gui/plots/__init__.py

```
1   # lintrans - The linear transformation visualizer
2   # Copyright (C) 2021-2022 D. Dyson (DoctorDalek1963)
3
4   # This program is licensed under GNU GPLv3, available here:
5   # <https://www.gnu.org/licenses/gpl-3.0.html>
6
7   """This package provides widgets for the visualization plot in the main window and the visual definition dialog."""
8
9   from .classes import BackgroundPlot, VectorGridPlot
10  from .widgets import DefineVisuallyWidget, VisualizeTransformationWidget
11
12  __all__ = ['BackgroundPlot', 'DefineVisuallyWidget', 'VectorGridPlot', 'VisualizeTransformationWidget']
```

A.12 gui/plots/widgets.py

```
1   # lintrans - The linear transformation visualizer
2   # Copyright (C) 2021-2022 D. Dyson (DoctorDalek1963)
3
4   # This program is licensed under GNU GPLv3, available here:
5   # <https://www.gnu.org/licenses/gpl-3.0.html>
6
```

```python
"""This module provides the actual widgets that can be used to visualize transformations in the GUI."""

from __future__ import annotations

from math import ceil, dist, floor
from typing import List, Tuple

from PyQt5.QtCore import Qt
from PyQt5.QtGui import QMouseEvent, QPainter, QPaintEvent

from lintrans.typing_ import MatrixType
from lintrans.gui.settings import DisplaySettings
from .classes import VectorGridPlot


class VisualizeTransformationWidget(VectorGridPlot):
    """This class is the widget that is used in the main window to visualize transformations.

    It handles all the rendering itself, and the only method that the user needs to
    worry about is :meth:`visualize_matrix_transformation`, which allows you to visualize
    the given matrix transformation.
    """

    def __init__(self, *args, display_settings: DisplaySettings, **kwargs):
        """Create the widget and assign its display settings, passing ``*args`` and ``**kwargs`` to super."""
        super().__init__(*args, **kwargs)

        self.display_settings = display_settings

    def visualize_matrix_transformation(self, matrix: MatrixType) -> None:
        """Transform the grid by the given matrix.

        .. warning:: This method does not call ``update()``. This must be done by the caller.

        .. note::
           This method transforms the background grid, not the basis vectors. This
           means that it cannot be used to compose transformations. Compositions
           should be done by the user.

        :param MatrixType matrix: The matrix to transform by
        """
        self.point_i = (matrix[0][0], matrix[1][0])
        self.point_j = (matrix[0][1], matrix[1][1])

    def paintEvent(self, event: QPaintEvent) -> None:
        """Handle a :class:`QPaintEvent` by drawing the background grid and the transformed grid.

        The transformed grid is defined by the basis vectors i and j, which can
        be controlled with the :meth:`visualize_matrix_transformation` method.
        """
        painter = QPainter()
        painter.begin(self)

        painter.setRenderHint(QPainter.Antialiasing)
        painter.setBrush(Qt.NoBrush)

        self.draw_background(painter, self.display_settings.draw_background_grid)

        if self.display_settings.draw_eigenlines:
            self.draw_eigenlines(painter)

        if self.display_settings.draw_eigenvectors:
            self.draw_eigenvectors(painter)

        if self.display_settings.draw_determinant_parallelogram:
            self.draw_determinant_parallelogram(painter)

            if self.display_settings.show_determinant_value:
                self.draw_determinant_text(painter)

        if self.display_settings.draw_transformed_grid:
            self.draw_transformed_grid(painter)

```

```
80              if self.display_settings.draw_basis_vectors:
81                  self.draw_basis_vectors(painter)
82
83              painter.end()
84              event.accept()
85
86
87      class DefineVisuallyWidget(VisualizeTransformationWidget):
88          """This class is the widget that allows the user to visually define a matrix.
89
90          This is just the widget itself. If you want the dialog, use
91          :class:`lintrans.gui.dialogs.define_new_matrix.DefineVisuallyDialog`.
92          """
93
94          def __init__(self, *args, display_settings: DisplaySettings, **kwargs):
95              """Create the widget and enable mouse tracking. ``*args`` and ``**kwargs`` are passed to ``super()``."""
96              super().__init__(*args, display_settings=display_settings, **kwargs)
97
98              self.dragged_point: Tuple[float, float] | None = None
99
100             # This is the distance that the cursor needs to be from the point to drag it
101             self.epsilon: int = 5
102
103         def mousePressEvent(self, event: QMouseEvent) -> None:
104             """Handle a :class:`QMouseEvent` when the user presses a button."""
105             mx = event.x()
106             my = event.y()
107             button = event.button()
108
109             if button != Qt.LeftButton:
110                 event.ignore()
111                 return
112
113             for point in (self.point_i, self.point_j):
114                 px, py = self.canvas_coords(*point)
115                 if abs(px - mx) <= self.epsilon and abs(py - my) <= self.epsilon:
116                     self.dragged_point = point[0], point[1]
117
118             event.accept()
119
120         def mouseReleaseEvent(self, event: QMouseEvent) -> None:
121             """Handle a :class:`QMouseEvent` when the user releases a button."""
122             if event.button() == Qt.LeftButton:
123                 self.dragged_point = None
124                 event.accept()
125             else:
126                 event.ignore()
127
128         def mouseMoveEvent(self, event: QMouseEvent) -> None:
129             """Handle the mouse moving on the canvas."""
130             mx = event.x()
131             my = event.y()
132
133             if self.dragged_point is None:
134                 event.ignore()
135                 return
136
137             x, y = self.grid_coords(mx, my)
138
139             possible_snaps: List[Tuple[int, int]] = [
140                 (floor(x), floor(y)),
141                 (floor(x), ceil(y)),
142                 (ceil(x), floor(y)),
143                 (ceil(x), ceil(y))
144             ]
145
146             snap_distances: List[Tuple[float, Tuple[int, int]]] = [
147                 (dist((x, y), coord), coord)
148                 for coord in possible_snaps
149             ]
150
151             for snap_dist, coord in snap_distances:
152                 if snap_dist < 0.1:
```

```
                x, y = coord

        if self.dragged_point == self.point_i:
            self.point_i = x, y

        elif self.dragged_point == self.point_j:
            self.point_j = x, y

        self.dragged_point = x, y

        self.update()

        event.accept()
```

A.13 gui/plots/classes.py

```
# lintrans - The linear transformation visualizer
# Copyright (C) 2021-2022 D. Dyson (DoctorDalek1963)

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# <https://www.gnu.org/licenses/gpl-3.0.html>

"""This module provides superclasses for plotting transformations."""

from __future__ import annotations

from abc import abstractmethod
from typing import Iterable, List, Tuple

import numpy as np
from nptyping import Float, NDArray
from PyQt5.QtCore import QPoint, QRectF, Qt
from PyQt5.QtGui import QBrush, QColor, QPainter, QPainterPath, QPaintEvent, QPen, QWheelEvent
from PyQt5.QtWidgets import QWidget

from lintrans.typing_ import MatrixType


class BackgroundPlot(QWidget):
    """This class provides a background for plotting, as well as setup for a Qt widget.

    This class provides a background (untransformed) plane, and all the backend
    details for a Qt application, but does not provide useful functionality. To
    be useful, this class must be subclassed and behaviour must be implemented
    by the subclass.

    .. warning:: This class should never be directly instantiated, only subclassed.

    .. note::
       I would make this class have ``metaclass=abc.ABCMeta``, but I can't because it subclasses :class:`QWidget`,
       and every superclass of a class must have the same metaclass, and :class:`QWidget` is not an abstract class.
    """

    default_grid_spacing: int = 85
    minimum_grid_spacing: int = 5

    def __init__(self, *args, **kwargs):
        """Create the widget and setup backend stuff for rendering.

        .. note:: ``*args`` and ``**kwargs`` are passed the superclass constructor (:class:`QWidget`).
        """
        super().__init__(*args, **kwargs)

        self.setAutoFillBackground(True)

        # Set the background to white
        palette = self.palette()
        palette.setColor(self.backgroundRole(), Qt.white)
        self.setPalette(palette)

```

```
55              # Set the grid colour to grey and the axes colour to black
56              self.colour_background_grid = QColor('#808080')
57              self.colour_background_axes = QColor('#000000')
58
59              self.grid_spacing = BackgroundPlot.default_grid_spacing
60              self.width_background_grid: float = 0.3
61
62          @property
63          def canvas_origin(self) -> Tuple[int, int]:
64              """Return the canvas coords of the grid origin.
65
66              The return value is intended to be unpacked and passed to a :meth:`QPainter.drawLine:iiii` call.
67
68              See :meth:`canvas_coords`.
69
70              :returns: The canvas coordinates of the grid origin
71              :rtype: Tuple[int, int]
72              """
73              return self.width() // 2, self.height() // 2
74
75          def canvas_x(self, x: float) -> int:
76              """Convert an x coordinate from grid coords to canvas coords."""
77              return int(self.canvas_origin[0] + x * self.grid_spacing)
78
79          def canvas_y(self, y: float) -> int:
80              """Convert a y coordinate from grid coords to canvas coords."""
81              return int(self.canvas_origin[1] - y * self.grid_spacing)
82
83          def canvas_coords(self, x: float, y: float) -> Tuple[int, int]:
84              """Convert a coordinate from grid coords to canvas coords.
85
86              This method is intended to be used like
87
88              .. code::
89
90                 painter.drawLine(*self.canvas_coords(x1, y1), *self.canvas_coords(x2, y2))
91
92              or like
93
94              .. code::
95
96                 painter.drawLine(*self.canvas_origin, *self.canvas_coords(x, y))
97
98              See :attr:`canvas_origin`.
99
100             :param float x: The x component of the grid coordinate
101             :param float y: The y component of the grid coordinate
102             :returns: The resultant canvas coordinates
103             :rtype: Tuple[int, int]
104             """
105             return self.canvas_x(x), self.canvas_y(y)
106
107         def grid_corner(self) -> Tuple[float, float]:
108             """Return the grid coords of the top right corner."""
109             return self.width() / (2 * self.grid_spacing), self.height() / (2 * self.grid_spacing)
110
111         def grid_coords(self, x: int, y: int) -> Tuple[float, float]:
112             """Convert a coordinate from canvas coords to grid coords.
113
114             :param int x: The x component of the canvas coordinate
115             :param int y: The y component of the canvas coordinate
116             :returns: The resultant grid coordinates
117             :rtype: Tuple[float, float]
118             """
119             # We get the maximum grid coords and convert them into canvas coords
120             return (x - self.canvas_origin[0]) / self.grid_spacing, (-y + self.canvas_origin[1]) / self.grid_spacing
121
122         @abstractmethod
123         def paintEvent(self, event: QPaintEvent) -> None:
124             """Handle a :class:`QPaintEvent`.
125
126             .. note:: This method is abstract and must be overridden by all subclasses.
127             """
```

```
128
129         def draw_background(self, painter: QPainter, draw_grid: bool) -> None:
130             """Draw the background grid.
131
132             .. note:: This method is just a utility method for subclasses to use to render the background grid.
133
134             :param QPainter painter: The painter to draw the background with
135             :param bool draw_grid: Whether to draw the grid lines
136             """
137             if draw_grid:
138                 painter.setPen(QPen(self.colour_background_grid, self.width_background_grid))
139
140                 # Draw equally spaced vertical lines, starting in the middle and going out
141                 # We loop up to half of the width. This is because we draw a line on each side in each iteration
142                 for x in range(self.width() // 2 + self.grid_spacing, self.width(), self.grid_spacing):
143                     painter.drawLine(x, 0, x, self.height())
144                     painter.drawLine(self.width() - x, 0, self.width() - x, self.height())
145
146                 # Same with the horizontal lines
147                 for y in range(self.height() // 2 + self.grid_spacing, self.height(), self.grid_spacing):
148                     painter.drawLine(0, y, self.width(), y)
149                     painter.drawLine(0, self.height() - y, self.width(), self.height() - y)
150
151             # Now draw the axes
152             painter.setPen(QPen(self.colour_background_axes, self.width_background_grid))
153             painter.drawLine(self.width() // 2, 0, self.width() // 2, self.height())
154             painter.drawLine(0, self.height() // 2, self.width(), self.height() // 2)
155
156         def wheelEvent(self, event: QWheelEvent) -> None:
157             """Handle a :class:`QWheelEvent` by zooming in or our of the grid."""
158             # angleDelta() returns a number of units equal to 8 times the number of degrees rotated
159             degrees = event.angleDelta() / 8
160
161             if degrees is not None:
162                 new_spacing = max(1, self.grid_spacing + degrees.y())
163
164                 if new_spacing >= self.minimum_grid_spacing:
165                     self.grid_spacing = new_spacing
166
167             event.accept()
168             self.update()
169
170
171     class VectorGridPlot(BackgroundPlot):
172         """This class represents a background plot, with vectors and their grid drawn on top.
173
174         This class should be subclassed to be used for visualization and matrix definition widgets.
175         All useful behaviour should be implemented by any subclass.
176
177         .. warning:: This class should never be directly instantiated, only subclassed.
178         """
179
180         def __init__(self, *args, **kwargs):
181             """Create the widget with ``point_i`` and ``point_j`` attributes.
182
183             .. note:: ``*args`` and ``**kwargs`` are passed to the superclass constructor (:class:`BackgroundPlot`).
184             """
185             super().__init__(*args, **kwargs)
186
187             self.point_i: Tuple[float, float] = (1., 0.)
188             self.point_j: Tuple[float, float] = (0., 1.)
189
190             self.colour_i = QColor('#0808d8')
191             self.colour_j = QColor('#e90000')
192             self.colour_eigen = QColor('#13cf00')
193             self.colour_text = QColor('#000000')
194
195             self.width_vector_line = 1.8
196             self.width_transformed_grid = 0.8
197
198             self.arrowhead_length = 0.15
199
200             self.max_parallel_lines = 150
```

```
201
202         @property
203         def matrix(self) -> MatrixType:
204             """Return the assembled matrix of the basis vectors."""
205             return np.array([
206                 [self.point_i[0], self.point_j[0]],
207                 [self.point_i[1], self.point_j[1]]
208             ])
209
210         @property
211         def det(self) -> float:
212             """Return the determinant of the assembled matrix."""
213             return float(np.linalg.det(self.matrix))
214
215         @property
216         def eigs(self) -> Iterable[Tuple[float, NDArray[(1, 2), Float]]]:
217             """Return the eigenvalues and eigenvectors zipped together to be iterated over.
218
219             :rtype: Iterable[Tuple[float, NDArray[(1, 2), Float]]]
220             """
221             values, vectors = np.linalg.eig(self.matrix)
222             return zip(values, vectors.T)
223
224         @abstractmethod
225         def paintEvent(self, event: QPaintEvent) -> None:
226             """Handle a :class:`QPaintEvent`.
227
228             .. note:: This method is abstract and must be overridden by all subclasses.
229             """
230
231         def draw_parallel_lines(self, painter: QPainter, vector: Tuple[float, float], point: Tuple[float, float]) ->
        ↪  None:
232             """Draw a set of evenly spaced grid lines parallel to ``vector`` intersecting ``point``.
233
234             :param QPainter painter: The painter to draw the lines with
235             :param vector: The vector to draw the grid lines parallel to
236             :type vector: Tuple[float, float]
237             :param point: The point for the lines to intersect with
238             :type point: Tuple[float, float]
239             """
240             max_x, max_y = self.grid_corner()
241             vector_x, vector_y = vector
242             point_x, point_y = point
243
244             # If the determinant is 0
245             if abs(vector_x * point_y - vector_y * point_x) < 1e-12:
246                 rank = np.linalg.matrix_rank(
247                     np.array([
248                         [vector_x, point_x],
249                         [vector_y, point_y]
250                     ])
251                 )
252
253                 # If the matrix is rank 1, then we can draw the column space line
254                 if rank == 1:
255                     # If the vector does not have a 0 x or y component, then we can just draw the line
256                     if abs(vector_x) > 1e-12 and abs(vector_y) > 1e-12:
257                         self.draw_oblique_line(painter, vector_y / vector_x, 0)
258
259                     # Otherwise, we have to draw lines along the axes
260                     elif abs(vector_x) > 1e-12 and abs(vector_y) < 1e-12:
261                         painter.drawLine(0, self.height() // 2, self.width(), self.height() // 2)
262
263                     elif abs(vector_x) < 1e-12 and abs(vector_y) > 1e-12:
264                         painter.drawLine(self.width() // 2, 0, self.width() // 2, self.height())
265
266                     # If the vector is (0, 0), then don't draw a line for it
267                     else:
268                         return
269
270                 # If the rank is 0, then we don't draw any lines
271                 else:
272                     return
```

```
273
274             elif abs(vector_x) < 1e-12 and abs(vector_y) < 1e-12:
275                 # If both components of the vector are practically 0, then we can't render any grid lines
276                 return
277
278             # Draw vertical lines
279             elif abs(vector_x) < 1e-12:
280                 painter.drawLine(self.canvas_x(0), 0, self.canvas_x(0), self.height())
281
282                 for i in range(max(abs(int(max_x / point_x)), self.max_parallel_lines)):
283                     painter.drawLine(
284                         self.canvas_x((i + 1) * point_x),
285                         0,
286                         self.canvas_x((i + 1) * point_x),
287                         self.height()
288                     )
289                     painter.drawLine(
290                         self.canvas_x(-1 * (i + 1) * point_x),
291                         0,
292                         self.canvas_x(-1 * (i + 1) * point_x),
293                         self.height()
294                     )
295
296             # Draw horizontal lines
297             elif abs(vector_y) < 1e-12:
298                 painter.drawLine(0, self.canvas_y(0), self.width(), self.canvas_y(0))
299
300                 for i in range(max(abs(int(max_y / point_y)), self.max_parallel_lines)):
301                     painter.drawLine(
302                         0,
303                         self.canvas_y((i + 1) * point_y),
304                         self.width(),
305                         self.canvas_y((i + 1) * point_y)
306                     )
307                     painter.drawLine(
308                         0,
309                         self.canvas_y(-1 * (i + 1) * point_y),
310                         self.width(),
311                         self.canvas_y(-1 * (i + 1) * point_y)
312                     )
313
314             # If the line is oblique, then we can use y = mx + c
315             else:
316                 m = vector_y / vector_x
317                 c = point_y - m * point_x
318
319                 self.draw_oblique_line(painter, m, 0)
320
321                 # We don't want to overshoot the max number of parallel lines,
322                 # but we should also stop looping as soon as we can't draw any more lines
323                 for i in range(1, self.max_parallel_lines + 1):
324                     if not self.draw_pair_of_oblique_lines(painter, m, i * c):
325                         break
326
327         def draw_pair_of_oblique_lines(self, painter: QPainter, m: float, c: float) -> bool:
328             """Draw a pair of oblique lines, using the equation y = mx + c.
329
330             This method just calls :meth:`draw_oblique_line` with ``c`` and ``-c``,
331             and returns True if either call returned True.
332
333             :param QPainter painter: The painter to draw the vectors and grid lines with
334             :param float m: The gradient of the lines to draw
335             :param float c: The y-intercept of the lines to draw. We use the positive and negative versions
336             :returns bool: Whether we were able to draw any lines on the canvas
337             """
338             return any([
339                 self.draw_oblique_line(painter, m, c),
340                 self.draw_oblique_line(painter, m, -c)
341             ])
342
343         def draw_oblique_line(self, painter: QPainter, m: float, c: float) -> bool:
344             """Draw an oblique line, using the equation y = mx + c.
345
```

```
346             We only draw the part of the line that fits within the canvas, returning True if
347             we were able to draw a line within the boundaries, and False if we couldn't draw a line
348
349             :param QPainter painter: The painter to draw the vectors and grid lines with
350             :param float m: The gradient of the line to draw
351             :param float c: The y-intercept of the line to draw
352             :returns bool: Whether we were able to draw a line on the canvas
353             """
354             max_x, max_y = self.grid_corner()
355
356             # These variable names are shortened for convenience
357             # myi is max_y_intersection, mmyi is minus_max_y_intersection, etc.
358             myi = (max_y - c) / m
359             mmyi = (-max_y - c) / m
360             mxi = max_x * m + c
361             mmxi = -max_x * m + c
362
363             # The inner list here is a list of coords, or None
364             # If an intersection fits within the bounds, then we keep its coord,
365             # else it is None, and then gets discarded from the points list
366             # By the end, points is a list of two coords, or an empty list
367             points: List[Tuple[float, float]] = [
368                 x for x in [
369                     (myi, max_y) if -max_x < myi < max_x else None,
370                     (mmyi, -max_y) if -max_x < mmyi < max_x else None,
371                     (max_x, mxi) if -max_y < mxi < max_y else None,
372                     (-max_x, mmxi) if -max_y < mmxi < max_y else None
373                 ] if x is not None
374             ]
375
376             # If no intersections fit on the canvas
377             if len(points) < 2:
378                 return False
379
380             # If we can, then draw the line
381             else:
382                 painter.drawLine(
383                     *self.canvas_coords(*points[0]),
384                     *self.canvas_coords(*points[1])
385                 )
386                 return True
387
388         def draw_transformed_grid(self, painter: QPainter) -> None:
389             """Draw the transformed version of the grid, given by the basis vectors.
390
391             .. note:: This method draws the grid, but not the basis vectors. Use :meth:`draw_basis_vectors` to draw
    ↪ them.
392
393             :param QPainter painter: The painter to draw the grid lines with
394             """
395             # Draw all the parallel lines
396             painter.setPen(QPen(self.colour_i, self.width_transformed_grid))
397             self.draw_parallel_lines(painter, self.point_i, self.point_j)
398             painter.setPen(QPen(self.colour_j, self.width_transformed_grid))
399             self.draw_parallel_lines(painter, self.point_j, self.point_i)
400
401         def draw_arrowhead_away_from_origin(self, painter: QPainter, point: Tuple[float, float]) -> None:
402             """Draw an arrowhead at ``point``, pointing away from the origin.
403
404             :param QPainter painter: The painter to draw the arrowhead with
405             :param point: The point to draw the arrowhead at, given in grid coords
406             :type point: Tuple[float, float]
407             """
408             # This algorithm was adapted from a C# algorithm found at
409             # http://csharphelper.com/blog/2014/12/draw-lines-with-arrowheads-in-c/
410
411             # Get the x and y coords of the point, and then normalize them
412             # We have to normalize them, or else the size of the arrowhead will
413             # scale with the distance of the point from the origin
414             x, y = point
415             vector_length = np.sqrt(x * x + y * y)
416
417             if vector_length < 1e-12:
```

```
418                 return
419
420             nx = x / vector_length
421             ny = y / vector_length
422
423             # We choose a length and find the steps in the x and y directions
424             length = min(
425                 self.arrowhead_length * self.default_grid_spacing / self.grid_spacing,
426                 vector_length
427             )
428             dx = length * (-nx - ny)
429             dy = length * (nx - ny)
430
431             # Then we just plot those lines
432             painter.drawLine(*self.canvas_coords(x, y), *self.canvas_coords(x + dx, y + dy))
433             painter.drawLine(*self.canvas_coords(x, y), *self.canvas_coords(x - dy, y + dx))
434
435         def draw_position_vector(self, painter: QPainter, point: Tuple[float, float], colour: QColor) -> None:
436             """Draw a vector from the origin to the given point.
437
438             :param QPainter painter: The painter to draw the position vector with
439             :param point: The tip of the position vector in grid coords
440             :type point: Tuple[float, float]
441             :param QColor colour: The colour to draw the position vector in
442             """
443             painter.setPen(QPen(colour, self.width_vector_line))
444             painter.drawLine(*self.canvas_origin, *self.canvas_coords(*point))
445             self.draw_arrowhead_away_from_origin(painter, point)
446
447         def draw_basis_vectors(self, painter: QPainter) -> None:
448             """Draw arrowheads at the tips of the basis vectors.
449
450             :param QPainter painter: The painter to draw the basis vectors with
451             """
452             self.draw_position_vector(painter, self.point_i, self.colour_i)
453             self.draw_position_vector(painter, self.point_j, self.colour_j)
454
455         def draw_determinant_parallelogram(self, painter: QPainter) -> None:
456             """Draw the parallelogram of the determinant of the matrix.
457
458             :param QPainter painter: The painter to draw the parallelogram with
459             """
460             if self.det == 0:
461                 return
462
463             path = QPainterPath()
464             path.moveTo(*self.canvas_origin)
465             path.lineTo(*self.canvas_coords(*self.point_i))
466             path.lineTo(*self.canvas_coords(self.point_i[0] + self.point_j[0], self.point_i[1] + self.point_j[1]))
467             path.lineTo(*self.canvas_coords(*self.point_j))
468
469             color = (16, 235, 253) if self.det > 0 else (253, 34, 16)
470             brush = QBrush(QColor(*color, alpha=128), Qt.SolidPattern)
471
472             painter.fillPath(path, brush)
473
474         def draw_determinant_text(self, painter: QPainter) -> None:
475             """Write the string value of the determinant in the middle of the parallelogram.
476
477             :param QPainter painter: The painter to draw the determinant text with
478             """
479             painter.setPen(QPen(self.colour_text, self.width_vector_line))
480
481             # We're building a QRect that encloses the determinant parallelogram
482             # Then we can center the text in this QRect
483             coords: List[Tuple[float, float]] = [
484                 (0, 0),
485                 self.point_i,
486                 self.point_j,
487                 (
488                     self.point_i[0] + self.point_j[0],
489                     self.point_i[1] + self.point_j[1]
490                 )
```

```
            ]

            xs = [t[0] for t in coords]
            ys = [t[1] for t in coords]

            top_left = QPoint(*self.canvas_coords(min(xs), max(ys)))
            bottom_right = QPoint(*self.canvas_coords(max(xs), min(ys)))

            rect = QRectF(top_left, bottom_right)

            painter.drawText(
                rect,
                Qt.AlignHCenter | Qt.AlignVCenter,
                f'{self.det:.2f}'
            )

        def draw_eigenvectors(self, painter: QPainter) -> None:
            """Draw the eigenvectors of the displayed matrix transformation.

            :param QPainter painter: The painter to draw the eigenvectors with
            """
            for value, vector in self.eigs:
                x = value * vector[0]
                y = value * vector[1]

                if x.imag != 0 or y.imag != 0:
                    continue

                self.draw_position_vector(painter, (x, y), self.colour_eigen)

                # Now we need to draw the eigenvalue at the tip of the eigenvector

                offset = 3
                top_left: QPoint
                bottom_right: QPoint
                alignment_flags: int

                if x >= 0 and y >= 0:  # Q1
                    top_left = QPoint(self.canvas_x(x) + offset, 0)
                    bottom_right = QPoint(self.width(), self.canvas_y(y) - offset)
                    alignment_flags = Qt.AlignLeft | Qt.AlignBottom

                elif x < 0 and y >= 0:  # Q2
                    top_left = QPoint(0, 0)
                    bottom_right = QPoint(self.canvas_x(x) - offset, self.canvas_y(y) - offset)
                    alignment_flags = Qt.AlignRight | Qt.AlignBottom

                elif x < 0 and y < 0:  # Q3
                    top_left = QPoint(0, self.canvas_y(y) + offset)
                    bottom_right = QPoint(self.canvas_x(x) - offset, self.height())
                    alignment_flags = Qt.AlignRight | Qt.AlignTop

                else:  # Q4
                    top_left = QPoint(self.canvas_x(x) + offset, self.canvas_y(y) + offset)
                    bottom_right = QPoint(self.width(), self.height())
                    alignment_flags = Qt.AlignLeft | Qt.AlignTop

                painter.setPen(QPen(self.colour_text, self.width_vector_line))
                painter.drawText(QRectF(top_left, bottom_right), alignment_flags, f'{value:.2f}')

        def draw_eigenlines(self, painter: QPainter) -> None:
            """Draw the eigenlines (invariant lines).

            :param QPainter painter: The painter to draw the eigenlines with
            """
            painter.setPen(QPen(self.colour_eigen, self.width_transformed_grid))

            for value, vector in self.eigs:
                if value.imag != 0:
                    continue

                x, y = vector
```

```
564                 if x == 0:
565                     x_mid = int(self.width() / 2)
566                     painter.drawLine(x_mid, 0, x_mid, self.height())
567 
568                 elif y == 0:
569                     y_mid = int(self.height() / 2)
570                     painter.drawLine(0, y_mid, self.width(), y_mid)
571 
572                 else:
573                     self.draw_oblique_line(painter, y / x, 0)
```

A.14 typing_/__init__.py

```
1   # lintrans - The linear transformation visualizer
2   # Copyright (C) 2021-2022 D. Dyson (DoctorDalek1963)
3   
4   # This program is licensed under GNU GPLv3, available here:
5   # <https://www.gnu.org/licenses/gpl-3.0.html>
6   
7   """This package supplies type aliases for linear algebra and transformations.
8   
9   .. note::
10     This package is called ``typing_`` and not ``typing`` to avoid name collisions with the
11     builtin :mod:`typing`. I don't quite know how this collision occurs, but renaming
12     this module fixed the problem.
13  """
14  
15  from __future__ import annotations
16  
17  from sys import version_info
18  from typing import Any, List, Tuple
19  
20  from numpy import ndarray
21  from nptyping import NDArray, Float
22  
23  if version_info >= (3, 10):
24      from typing import TypeGuard
25  
26  __all__ = ['is_matrix_type', 'MatrixType', 'MatrixParseList', 'VectorType']
27  
28  MatrixType = NDArray[(2, 2), Float]
29  """This type represents a 2x2 matrix as a NumPy array."""
30  
31  VectorType = NDArray[(2,), Float]
32  """This type represents a 2D vector as a NumPy array, for use with :attr:`MatrixType`."""
33  
34  MatrixParseList = List[List[Tuple[str, str, str]]]
35  """This is a list containing lists of tuples. Each tuple represents a matrix and is ``(multiplier,
36  matrix_identifier, index)`` where all of them are strings. These matrix-representing tuples are
37  contained in lists which represent multiplication groups. Every matrix in the group should be
38  multiplied together, in order. These multiplication group lists are contained by a top level list,
39  which is this type. Once these multiplication group lists have been evaluated, they should be summed.
40  
41  In the tuples, the multiplier is a string representing a real number, the matrix identifier
42  is a capital letter or ``rot(x)`` where x is a real number angle, and the index is a string
43  representing an integer, or it's the letter ``T`` for transpose.
44  """
45  
46  
47  def is_matrix_type(matrix: Any) -> TypeGuard[NDArray[(2, 2), Float]]:
48      """Check if the given value is a valid matrix type.
49  
50      .. note::
51         This function is a TypeGuard, meaning if it returns True, then the
52         passed value must be a :attr:`lintrans.typing_.MatrixType`.
53      """
54      return isinstance(matrix, ndarray) and matrix.shape == (2, 2)
```

A.15 `matrices/parse.py`

```python
# lintrans - The linear transformation visualizer
# Copyright (C) 2021-2022 D. Dyson (DoctorDalek1963)

# This program is licensed under GNU GPLv3, available here:
# <https://www.gnu.org/licenses/gpl-3.0.html>

"""This module provides functions to parse and validate matrix expressions."""

from __future__ import annotations

import re
from dataclasses import dataclass
from typing import List, Pattern, Tuple

from lintrans.typing_ import MatrixParseList

NAIVE_CHARACTER_CLASS = r'[-+\sA-Z0-9.rot()^{}]'


class MatrixParseError(Exception):
    """A simple exception to be raised when an error is found when parsing."""


def compile_naive_expression_pattern() -> Pattern[str]:
    """Compile the single RegEx pattern that will match a valid matrix expression."""
    digit_no_zero = '[123456789]'
    digits = '\\d+'
    integer_no_zero = digit_no_zero + '(' + digits + ')?'
    real_number = f'({integer_no_zero}(\\.{digits})?|0\\.{digits})'

    index_content = f'(-?{integer_no_zero}|T)'
    index = f'(\\^{{{index_content}}}|\\^{index_content})'
    matrix_identifier = f'([A-Z]|rot\\(-?{real_number}\\)|\\({NAIVE_CHARACTER_CLASS}+\\))'
    matrix = '(' + real_number + '?' + matrix_identifier + index + '?)'
    expression = f'^-?{matrix}+((\\+-?|-){matrix}+)*$'

    return re.compile(expression)


# This is an expensive pattern to compile, so we compile it when this module is initialized
naive_expression_pattern = compile_naive_expression_pattern()


def find_sub_expressions(expression: str) -> List[str]:
    """Find all the sub-expressions in the given expression.

    This function only goes one level deep, so may return strings like ``'A(BC)D'``.

    :raises MatrixParseError: If there are unbalanced parentheses
    """
    sub_expressions: List[str] = []
    string = ''
    paren_depth = 0
    pointer = 0

    while True:
        char = expression[pointer]

        if char == '(' and expression[pointer - 3:pointer] != 'rot':
            paren_depth += 1

            # This is a bit of a manual bodge, but it eliminates extraneous parens
            if paren_depth == 1:
                pointer += 1
                continue

        elif char == ')' and re.match(f'{NAIVE_CHARACTER_CLASS}*?rot\\([-\\d.]+$', expression[:pointer]) is None:
            paren_depth -= 1

        if paren_depth > 0:
```

```
71                  string += char
72
73              if paren_depth == 0 and string:
74                  sub_expressions.append(string)
75                  string = ''
76
77              pointer += 1
78
79              if pointer >= len(expression):
80                  break
81
82          if paren_depth != 0:
83              raise MatrixParseError('Unbalanced parentheses in expression')
84
85          return sub_expressions
86
87
88      def validate_matrix_expression(expression: str) -> bool:
89          """Validate the given matrix expression.
90
91          This function simply checks the expression against the BNF schema documented in
92          :ref:`expression-syntax-docs`. It is not aware of which matrices are actually defined
93          in a wrapper. For an aware version of this function, use the
94          :meth:`lintrans.matrices.wrapper.MatrixWrapper.is_valid_expression` method.
95
96          :param str expression: The expression to be validated
97          :returns bool: Whether the expression is valid according to the schema
98          """
99          # Remove all whitespace
100         expression = re.sub(r'\s', '', expression)
101
102         match = naive_expression_pattern.match(expression)
103
104         if match is None:
105             return False
106
107         # Check that the whole expression was matched against
108         if expression != match.group(0):
109             return False
110
111         try:
112             sub_expressions = find_sub_expressions(expression)
113         except MatrixParseError:
114             return False
115
116         if not sub_expressions:
117             return True
118
119         return all(validate_matrix_expression(m) for m in sub_expressions)
120
121
122     @dataclass
123     class MatrixToken:
124         """A simple dataclass to hold information about a matrix token being parsed."""
125
126         multiplier: str = ''
127         identifier: str = ''
128         exponent: str = ''
129
130         @property
131         def tuple(self) -> Tuple[str, str, str]:
132             """Create a tuple of the token for parsing."""
133             return self.multiplier, self.identifier, self.exponent
134
135
136     class ExpressionParser:
137         """A class to hold state during parsing.
138
139         Most of the methods in this class are class-internal and should not be used from outside.
140
141         This class should be used like this:
142
143         >>> ExpressionParser('3A^-1B').parse()
```

```
        [[('3', 'A', '-1'), ('', 'B', '')]]
        >>> ExpressionParser('4(M^TA^2)^-2').parse()
        [[('4', 'M^{T}A^{2}', '-2')]]
        """

        def __init__(self, expression: str):
            """Create an instance of the parser with the given expression and initialise variables to use during
            ↪  parsing."""
            # Remove all whitespace
            expression = re.sub(r'\s', '', expression)

            # Check if it's valid
            if not validate_matrix_expression(expression):
                raise MatrixParseError('Invalid expression')

            # Wrap all exponents and transposition powers with {}
            expression = re.sub(r'(?<=\^)(-?\d+|T)(?=[^}]|$)', r'{\g<0>}', expression)

            # Remove any standalone minuses
            expression = re.sub(r'-(?=[A-Z])', '-1', expression)

            # Replace subtractions with additions
            expression = re.sub(r'-(?=\d+\.?\d*([A-Z]|rot))', '+-', expression)

            # Get rid of a potential leading + introduced by the last step
            expression = re.sub(r'^\+', '', expression)

            self.expression = expression
            self.pointer: int = 0

            self.current_token = MatrixToken()
            self.current_group: List[Tuple[str, str, str]] = []

            self.final_list: MatrixParseList = []

        def __repr__(self) -> str:
            """Return a simple repr containing the expression."""
            return f'{self.__class__.__module__}.{self.__class__.__name__}("{self.expression}")'

        @property
        def char(self) -> str:
            """Return the char pointed to by the pointer."""
            return self.expression[self.pointer]

        def parse(self) -> MatrixParseList:
            """Fully parse the instance's matrix expression and return the :attr:`lintrans.typing_.MatrixParseList`.

            This method uses all the private methods of this class to parse the
            expression in parts. All private methods mutate the instance variables.

            :returns: The parsed expression
            :rtype: :attr:`lintrans.typing_.MatrixParseList`
            """
            self._parse_multiplication_group()

            while self.pointer < len(self.expression):
                if self.expression[self.pointer] != '+':
                    raise MatrixParseError('Expected "+" between multiplication groups')

                self.pointer += 1
                self._parse_multiplication_group()

            return self.final_list

        def _parse_multiplication_group(self) -> None:
            """Parse a group of matrices to be multiplied together.

            This method just parses matrices until we get to a ``+``.
            """
            # This loop continues to parse matrices until we fail to do so
            while self._parse_matrix():
                # Once we get to the end of the multiplication group, we add it the final list and reset the group list
                if self.pointer >= len(self.expression) or self.char == '+':
```

```
                self.final_list.append(self.current_group)
                self.current_group = []
                self.pointer += 1

    def _parse_matrix(self) -> bool:
        """Parse a full matrix using :meth:`_parse_matrix_part`.

        This method will parse an optional multiplier, an identifier, and an optional exponent. If we
        do this successfully, we return True. If we fail to parse a matrix (maybe we've reached the
        end of the current multiplication group and the next char is ``+``), then we return False.

        :returns bool: Success or failure
        """
        self.current_token = MatrixToken()

        while self._parse_matrix_part():
            pass  # The actual execution is taken care of in the loop condition

        if self.current_token.identifier == '':
            return False

        self.current_group.append(self.current_token.tuple)
        return True

    def _parse_matrix_part(self) -> bool:
        """Parse part of a matrix (multiplier, identifier, or exponent).

        Which part of the matrix we parse is dependent on the current value of the pointer and the expression.
        This method will parse whichever part of matrix token that it can. If it can't parse a part of a matrix,
        or it's reached the next matrix, then we just return False. If we succeeded to parse a matrix part, then
        we return True.

        :returns bool: Success or failure
        :raises MatrixParseError: If we fail to parse this part of the matrix
        """
        if self.pointer >= len(self.expression):
            return False

        if self.char.isdigit() or self.char == '-':
            if self.current_token.multiplier != '' \
                    or (self.current_token.multiplier == '' and self.current_token.identifier != ''):
                return False

            self._parse_multiplier()

        elif self.char.isalpha() and self.char.isupper():
            if self.current_token.identifier != '':
                return False

            self.current_token.identifier = self.char
            self.pointer += 1

        elif self.char == 'r':
            if self.current_token.identifier != '':
                return False

            self._parse_rot_identifier()

        elif self.char == '(':
            if self.current_token.identifier != '':
                return False

            self._parse_sub_expression()

        elif self.char == '^':
            if self.current_token.exponent != '':
                return False

            self._parse_exponent()

        elif self.char == '+':
            return False
```

```
289             else:
290                 raise MatrixParseError(f'Unrecognised character "{self.char}" in matrix expression')
291 
292             return True
293 
294         def _parse_multiplier(self) -> None:
295             """Parse a multiplier from the expression and pointer.
296 
297             This method just parses a numerical multiplier, which can include
298             zero or one ``.`` character and optionally a ``-`` at the start.
299 
300             :raises MatrixParseError: If we fail to parse this part of the matrix
301             """
302             multiplier = ''
303 
304             while self.char.isdigit() or self.char in ('.', '-'):
305                 multiplier += self.char
306                 self.pointer += 1
307 
308             try:
309                 float(multiplier)
310             except ValueError as e:
311                 raise MatrixParseError(f'Invalid multiplier "{multiplier}"') from e
312 
313             self.current_token.multiplier = multiplier
314 
315         def _parse_rot_identifier(self) -> None:
316             """Parse a ``rot()``-style identifier from the expression and pointer.
317 
318             This method will just parse something like ``rot(12.5)``. The angle number must be a real number.
319 
320             :raises MatrixParseError: If we fail to parse this part of the matrix
321             """
322             if match := re.match(r'rot\(([\d.-]+)\)', self.expression[self.pointer:]):
323                 # Ensure that the number in brackets is a valid float
324                 try:
325                     float(match.group(1))
326                 except ValueError as e:
327                     raise MatrixParseError(f'Invalid angle number "{match.group(1)}" in rot-identifier') from e
328 
329                 self.current_token.identifier = match.group(0)
330                 self.pointer += len(match.group(0))
331             else:
332                 raise MatrixParseError(f'Invalid rot-identifier "{self.expression[self.pointer:self.pointer + 15]
    }..."')
333 
334         def _parse_sub_expression(self) -> None:
335             """Parse a parenthesized sub-expression as the identifier.
336 
337             This method will also validate the expression in the parentheses.
338 
339             :raises MatrixParseError: If we fail to parse this part of the matrix
340             """
341             if self.char != '(':
342                 raise MatrixParseError('Sub-expression must start with "("')
343 
344             self.pointer += 1
345             paren_depth = 1
346             identifier = ''
347 
348             while paren_depth > 0:
349                 if self.char == '(':
350                     paren_depth += 1
351                 elif self.char == ')':
352                     paren_depth -= 1
353 
354                 if paren_depth == 0:
355                     self.pointer += 1
356                     break
357 
358                 identifier += self.char
359                 self.pointer += 1
360 
```

```
361             if not validate_matrix_expression(identifier):
362                 raise MatrixParseError(f'Invalid sub-expression identifier "{identifier}"')
363 
364             self.current_token.identifier = identifier
365 
366         def _parse_exponent(self) -> None:
367             """Parse a matrix exponent from the expression and pointer.
368 
369             The exponent must be an integer or ``T`` for transpose.
370 
371             :raises MatrixParseError: If we fail to parse this part of the token
372             """
373             if match := re.match(r'\^\{(-?\d+|T)\}', self.expression[self.pointer:]):
374                 exponent = match.group(1)
375 
376                 try:
377                     if exponent != 'T':
378                         int(exponent)
379                 except ValueError as e:
380                     raise MatrixParseError(f'Invalid exponent "{match.group(1)}"') from e
381 
382                 self.current_token.exponent = exponent
383                 self.pointer += len(match.group(0))
384             else:
385                 raise MatrixParseError(f'Invalid exponent "{self.expression[self.pointer:self.pointer + 10]}..."')
386 
387 
388     def parse_matrix_expression(expression: str) -> MatrixParseList:
389         """Parse the matrix expression and return a :data:`lintrans.typing_.MatrixParseList`.
390 
391         :Example:
392 
393         >>> parse_matrix_expression('A')
394         [[('', 'A', '')]]
395         >>> parse_matrix_expression('-3M^2')
396         [[('-3', 'M', '2')]]
397         >>> parse_matrix_expression('1.2rot(12)^{3}2B^T')
398         [[('1.2', 'rot(12)', '3'), ('2', 'B', 'T')]]
399         >>> parse_matrix_expression('A^2 + 3B')
400         [[('', 'A', '2')], [('3', 'B', '')]]
401         >>> parse_matrix_expression('-3A^{-1}3B^T - 45M^2')
402         [[('-3', 'A', '-1'), ('3', 'B', 'T')], [('-45', 'M', '2')]]
403         >>> parse_matrix_expression('5.3A^{4} 2.6B^{-2} + 4.6D^T 8.9E^{-1}')
404         [[('5.3', 'A', '4'), ('2.6', 'B', '-2')], [('4.6', 'D', 'T'), ('8.9', 'E', '-1')]]
405         >>> parse_matrix_expression('2(A+B^TC)^2D')
406         [[('2', 'A+B^{T}C', '2'), ('', 'D', '')]]
407 
408         :param str expression: The expression to be parsed
409         :returns: A list of parsed components
410         :rtype: :data:`lintrans.typing_.MatrixParseList`
411         """
412         return ExpressionParser(expression).parse()
```

A.16 matrices/__init__.py

```
1   # lintrans - The linear transformation visualizer
2   # Copyright (C) 2021-2022 D. Dyson (DoctorDalek1963)
3 
4   # This program is licensed under GNU GPLv3, available here:
5   # <https://www.gnu.org/licenses/gpl-3.0.html>
6 
7   """This package supplies classes and functions to parse, evaluate, and wrap matrices."""
8 
9   from . import parse, utility
10  from .utility import create_rotation_matrix
11  from .wrapper import MatrixWrapper
12 
13  __all__ = ['create_rotation_matrix', 'MatrixWrapper', 'parse', 'utility']
```

A.17 `matrices/utility.py`

```python
#  lintrans - The linear transformation visualizer
#  Copyright (C) 2022 D. Dyson (DoctorDalek1963)
#
#  This program is licensed under GNU GPLv3, available here:
#  <https://www.gnu.org/licenses/gpl-3.0.html>

"""This module provides simple utility methods for matrix and vector manipulation."""

from __future__ import annotations

import math
from typing import Tuple

import numpy as np

from lintrans.typing_ import MatrixType


def polar_coords(x: float, y: float, *, degrees: bool = False) -> Tuple[float, float]:
    """Return the polar coordinates of a given (x, y) Cartesian coordinate.

    .. note:: We're returning the angle in the range [0, 2pi)
    """
    radius = math.hypot(x, y)

    # PyCharm complains about np.angle taking a complex argument even though that's what it's designed for
    # noinspection PyTypeChecker
    angle = float(np.angle(x + y * 1j, degrees))

    if angle < 0:
        angle += 2 * np.pi

    return radius, angle


def rect_coords(radius: float, angle: float, *, degrees: bool = False) -> Tuple[float, float]:
    """Return the rectilinear coordinates of a given polar coordinate."""
    if degrees:
        angle = np.radians(angle)

    return radius * np.cos(angle), radius * np.sin(angle)


def rotate_coord(x: float, y: float, angle: float, *, degrees: bool = False) -> Tuple[float, float]:
    """Rotate a rectilinear coordinate by the given angle."""
    if degrees:
        angle = np.radians(angle)

    r, theta = polar_coords(x, y, degrees=degrees)
    theta = (theta + angle) % (2 * np.pi)

    return rect_coords(r, theta, degrees=degrees)


def create_rotation_matrix(angle: float, *, degrees: bool = True) -> MatrixType:
    """Create a matrix representing a rotation (anticlockwise) by the given angle.

    :Example:

    >>> create_rotation_matrix(30)
    array([[ 0.8660254, -0.5      ],
           [ 0.5      ,  0.8660254]])
    >>> create_rotation_matrix(45)
    array([[ 0.70710678, -0.70710678],
           [ 0.70710678,  0.70710678]])
    >>> create_rotation_matrix(np.pi / 3, degrees=False)
    array([[ 0.5      , -0.8660254],
           [ 0.8660254,  0.5      ]])

    :param float angle: The angle to rotate anticlockwise by
```

```
        :param bool degrees: Whether to interpret the angle as degrees (True) or radians (False)
        :returns MatrixType: The resultant matrix
        """
        rad = np.deg2rad(angle % 360) if degrees else angle % (2 * np.pi)
        return np.array([
            [np.cos(rad), -1 * np.sin(rad)],
            [np.sin(rad), np.cos(rad)]
        ])


    def is_valid_float(string: str) -> bool:
        """Check if the string is a valid float (or anything that can be cast to a float, such as an int).

        This function simply checks that ``float(string)`` doesn't raise an error.

        .. note:: An empty string is not a valid float, so will return False.

        :param str string: The string to check
        :returns bool: Whether the string is a valid float
        """
        try:
            float(string)
            return True
        except ValueError:
            return False


    def round_float(num: float, precision: int = 5) -> str:
        """Round a floating point number to a given number of decimal places for pretty printing.

        :param float num: The number to round
        :param int precision: The number of decimal places to round to
        :returns str: The rounded number for pretty printing
        """
        # Round to ``precision`` number of decimal places
        string = str(round(num, precision))

        # Cut off the potential final zero
        if string.endswith('.0'):
            return string[:-2]

        elif 'e' in string:  # Scientific notation
            split = string.split('e')
            # The leading 0 only happens when the exponent is negative, so we know there'll be a minus sign
            return split[0] + 'e-' + split[1][1:].lstrip('0')

        else:
            return string
```

A.18 `matrices/wrapper.py`

```
    # lintrans - The linear transformation visualizer
    # Copyright (C) 2021-2022 D. Dyson (DoctorDalek1963)

    # This program is licensed under GNU GPLv3, available here:
    # <https://www.gnu.org/licenses/gpl-3.0.html>

    """This module contains the main :class:`MatrixWrapper` class and a function to create a matrix from an angle."""

    from __future__ import annotations

    import re
    from copy import copy
    from functools import reduce
    from operator import add, matmul
    from typing import Any, Dict, List, Optional, Tuple, Union

    import numpy as np

    from lintrans.typing_ import is_matrix_type, MatrixType
```

```
from .parse import parse_matrix_expression, validate_matrix_expression
from .utility import create_rotation_matrix


class MatrixWrapper:
    """A wrapper class to hold all possible matrices and allow access to them.

    .. note::
       When defining a custom matrix, its name must be a capital letter and cannot be ``I``.

    The contained matrices can be accessed and assigned to using square bracket notation.

    :Example:

    >>> wrapper = MatrixWrapper()
    >>> wrapper['I']
    array([[1., 0.],
           [0., 1.]])
    >>> wrapper['M']  # Returns None
    >>> wrapper['M'] = np.array([[1, 2], [3, 4]])
    >>> wrapper['M']
    array([[1., 2.],
           [3., 4.]])
    """

    def __init__(self):
        """Initialize a :class:`MatrixWrapper` object with a dictionary of matrices which can be accessed."""
        self._matrices: Dict[str, Optional[Union[MatrixType, str]]] = {
            'A': None, 'B': None, 'C': None, 'D': None,
            'E': None, 'F': None, 'G': None, 'H': None,
            'I': np.eye(2),  # I is always defined as the identity matrix
            'J': None, 'K': None, 'L': None, 'M': None,
            'N': None, 'O': None, 'P': None, 'Q': None,
            'R': None, 'S': None, 'T': None, 'U': None,
            'V': None, 'W': None, 'X': None, 'Y': None,
            'Z': None
        }

    def __repr__(self) -> str:
        """Return a nice string repr of the :class:`MatrixWrapper` for debugging."""
        defined_matrices = ''.join([k for k, v in self._matrices.items() if v is not None])
        return f'<{self.__class__.__module__}.{self.__class__.__name__} object with ' \
               f"{len(defined_matrices)} defined matrices: '{defined_matrices}'>"

    def __eq__(self, other: Any) -> bool:
        """Check for equality in wrappers by comparing dictionaries.

        :param Any other: The object to compare this wrapper to
        """
        if not isinstance(other, self.__class__):
            return NotImplemented

        # We loop over every matrix and check if every value is equal in each
        for name in self._matrices:
            s_matrix = self[name]
            o_matrix = other[name]

            if s_matrix is None and o_matrix is None:
                continue

            elif (s_matrix is None and o_matrix is not None) or \
                 (s_matrix is not None and o_matrix is None):
                return False

            # This is mainly to satisfy mypy, because we know these must be matrices
            elif not is_matrix_type(s_matrix) or not is_matrix_type(o_matrix):
                return False

            # Now we know they're both NumPy arrays
            elif np.array_equal(s_matrix, o_matrix):
                continue

            else:
```

```
                return False

        return True

    def __hash__(self) -> int:
        """Return the hash of the matrices dictionary."""
        return hash(self._matrices)

    def __getitem__(self, name: str) -> Optional[MatrixType]:
        """Get the matrix with the given name.

        If it is a simple name, it will just be fetched from the dictionary. If the name is ``rot(x)``, with
        a given angle in degrees, then we return a new matrix representing a rotation by that angle.

        .. note::
           If the named matrix is defined as an expression, then this method will return its evaluation.
           If you want the expression itself, use :meth:`get_expression`.

        :param str name: The name of the matrix to get
        :returns Optional[MatrixType]: The value of the matrix (could be None)

        :raises NameError: If there is no matrix with the given name
        """
        # Return a new rotation matrix
        if (match := re.match(r'^rot\((-?\d*\.?\d*)\)$', name)) is not None:
            return create_rotation_matrix(float(match.group(1)))

        if name not in self._matrices:
            if validate_matrix_expression(name):
                return self.evaluate_expression(name)

            raise NameError(f'Unrecognised matrix name "{name}"')

        # We copy the matrix before we return it so the user can't accidentally mutate the matrix
        matrix = copy(self._matrices[name])

        if isinstance(matrix, str):
            return self.evaluate_expression(matrix)

        return matrix

    def __setitem__(self, name: str, new_matrix: Optional[Union[MatrixType, str]]) -> None:
        """Set the value of matrix ``name`` with the new_matrix.

        The new matrix may be a simple 2x2 NumPy array, or it could be a string, representing an
        expression in terms of other, previously defined matrices.

        :param str name: The name of the matrix to set the value of
        :param Optional[Union[MatrixType, str]] new_matrix: The value of the new matrix (could be None)

        :raises NameError: If the name isn't a legal matrix name
        :raises TypeError: If the matrix isn't a valid 2x2 NumPy array or expression in terms of other defined
↪  matrices
        :raises ValueError: If you attempt to define a matrix in terms of itself
        """
        if not (name in self._matrices and name != 'I'):
            raise NameError('Matrix name is illegal')

        if new_matrix is None:
            self._matrices[name] = None
            return

        if isinstance(new_matrix, str):
            if self.is_valid_expression(new_matrix):
                if name not in new_matrix:
                    self._matrices[name] = new_matrix
                    return
                else:
                    raise ValueError('Cannot define a matrix recursively')

        if not is_matrix_type(new_matrix):
            raise TypeError('Matrix must be a 2x2 NumPy array')
```

```python
        # All matrices must have float entries
        a = float(new_matrix[0][0])
        b = float(new_matrix[0][1])
        c = float(new_matrix[1][0])
        d = float(new_matrix[1][1])

        self._matrices[name] = np.array([[a, b], [c, d]])

    def get_expression(self, name: str) -> Optional[str]:
        """If the named matrix is defined as an expression, return that expression, else return None.

        :param str name: The name of the matrix
        :returns Optional[str]: The expression that the matrix is defined as, or None

        :raises NameError: If the name is invalid
        """
        if name not in self._matrices:
            raise NameError('Matrix must have a legal name')

        matrix = self._matrices[name]
        if isinstance(matrix, str):
            return matrix

        return None

    def is_valid_expression(self, expression: str) -> bool:
        """Check if the given expression is valid, using the context of the wrapper.

        This method calls :func:`lintrans.matrices.parse.validate_matrix_expression`, but also
        ensures that all the matrices in the expression are defined in the wrapper.

        :param str expression: The expression to validate
        :returns bool: Whether the expression is valid in this wrapper

        :raises LinAlgError: If a matrix is defined in terms of the inverse of a singular matrix
        """
        # Get rid of the transposes to check all capital letters
        new_expression = expression.replace('^T', '').replace('^{T}', '')

        # Make sure all the referenced matrices are defined
        for matrix in [x for x in new_expression if re.match('[A-Z]', x)]:
            if self[matrix] is None:
                return False

            if (expr := self.get_expression(matrix)) is not None:
                if not self.is_valid_expression(expr):
                    return False

        return validate_matrix_expression(expression)

    def evaluate_expression(self, expression: str) -> MatrixType:
        """Evaluate a given expression and return the matrix evaluation.

        :param str expression: The expression to be parsed
        :returns MatrixType: The matrix result of the expression

        :raises ValueError: If the expression is invalid
        """
        if not self.is_valid_expression(expression):
            raise ValueError('The expression is invalid')

        parsed_result = parse_matrix_expression(expression)
        final_groups: List[List[MatrixType]] = []

        for group in parsed_result:
            f_group: List[MatrixType] = []

            for multiplier, identifier, index in group:
                if index == 'T':
                    m = self[identifier]

                    # This assertion is just so mypy doesn't complain
                    # We know this won't be None, because we know that this matrix is defined in this wrapper
```

```
                    assert m is not None
                    matrix_value = m.T

                else:
                    matrix_value = np.linalg.matrix_power(self[identifier], 1 if index == '' else int(index))

                matrix_value *= 1 if multiplier == '' else float(multiplier)
                f_group.append(matrix_value)

            final_groups.append(f_group)

        return reduce(add, [reduce(matmul, group) for group in final_groups])

    def get_defined_matrices(self) -> List[Tuple[str, Union[MatrixType, str]]]:
        """Return a list of tuples containing the name and value of all defined matrices in the wrapper.

        :returns: A list of tuples where the first element is the name, and the second element is the value
        :rtype: List[Tuple[str, Union[MatrixType, str]]]
        """
        matrices = []

        for name, value in self._matrices.items():
            if value is not None:
                matrices.append((name, value))

        return matrices
```

B Testing code

B.1 `matrices/test_parse_and_validate_expression.py`

```python
# lintrans - The linear transformation visualizer
# Copyright (C) 2021-2022 D. Dyson (DoctorDalek1963)

# This program is licensed under GNU GPLv3, available here:
# <https://www.gnu.org/licenses/gpl-3.0.html>

"""Test the :mod:`matrices.parse` module validation and parsing."""

from typing import List, Tuple

import pytest

from lintrans.matrices.parse import (
    MatrixParseError, find_sub_expressions, parse_matrix_expression, validate_matrix_expression
)
from lintrans.typing_ import MatrixParseList

expected_sub_expressions: List[Tuple[str, List[str]]] = [
    ('2(AB)^-1', ['AB']),
    ('-3(A+B)^2-C(B^TA)^-1', ['A+B', 'B^TA']),
    ('rot(45)', []),
    ('()', []),
    ((()), ['()']),
    ('2.3A^-1(AB)^-1+(BC)^2', ['AB', 'BC']),
    ('(2.3A^-1(AB)^-1+(BC)^2)', ['2.3A^-1(AB)^-1+(BC)^2']),
]


def test_find_sub_expressions() -> None:
    """Test the :func:`lintrans.matrices.parse.find_sub_expressions` function."""
    for inp, output in expected_sub_expressions:
        assert find_sub_expressions(inp) == output


valid_inputs: List[str] = [
    'A', 'AB', '3A', '1.2A', '-3.4A', 'A^2', 'A^-1', 'A^{-1}',
    'A^12', 'A^T', 'A^{5}', 'A^{T}', '4.3A^7', '9.2A^{18}', '0.1A'

    'rot(45)', 'rot(12.5)', '3rot(90)',
    'rot(135)^3', 'rot(51)^T', 'rot(-34)^-1',

    'A+B', 'A+2B', '4.3A+9B', 'A^2+B^T', '3A^7+0.8B^{16}',
    'A-B', '3A-4B', '3.2A^3-16.79B^T', '4.752A^{17}-3.32B^{36}',
    'A-1B', '-A', '-1A'

    '3A4B', 'A^TB', 'A^{T}B', '4A^6B^3',
    '2A^{3}4B^5', '4rot(90)^3', 'rot(45)rot(13)',
    'Arot(90)', 'AB^2', 'A^2B^2', '8.36A^T3.4B^12',

    '3.5A^{4}5.6rot(19.2)^T-B^{-1}4.1C^5',

    '(A)', '(AB)^-1', '2.3(3B^TA)^2', '-3.4(9D^{2}3F^-1)^T+C', '(AB)(C)',
    '3(rot(34)^-7A)^-1+B', '3A^2B+4A(B+C)^-1D^T-A(C(D+E)B)'
]

invalid_inputs: List[str] = [
    '', 'rot()', 'A^', 'A^1.2', 'A^{3.4}', '1,2A', 'ro(12)', '5', '12^2', '^T', '^{12}', '.1A',
    'A^{13', 'A^3}', 'A^A', '^2', 'A--B', '--A', '+A', '--1A', 'A--B', 'A--1B', '.A', '1.A',
    '2.3AB)^T', '(AB+)', '-4.6(9A', '-2(3.4A^{-1}-C^)^2', '9.2)', '3A^2B+4A(B+C)^-1D^T-A(C(D+EB)',
    '3()^2', '4(your mum)^T', 'rot()', 'rot(10.1.1)', 'rot(--2)',

    'This is 100% a valid matrix expression, I swear'
]


@pytest.mark.parametrize('inputs,output', [(valid_inputs, True), (invalid_inputs, False)])
def test_validate_matrix_expression(inputs: List[str], output: bool) -> None:
```

```
68          """Test the validate_matrix_expression() function."""
69          for inp in inputs:
70              assert validate_matrix_expression(inp) == output
71
72
73      expressions_and_parsed_expressions: List[Tuple[str, MatrixParseList]] = [
74          # Simple expressions
75          ('A', [[('', 'A', '')]]),
76          ('A^2', [[('', 'A', '2')]]),
77          ('A^{2}', [[('', 'A', '2')]]),
78          ('3A', [[('3', 'A', '')]]),
79          ('1.4A^3', [[('1.4', 'A', '3')]]),
80          ('0.1A', [[('0.1', 'A', '')]]),
81          ('0.1A', [[('0.1', 'A', '')]]),
82          ('A^12', [[('', 'A', '12')]]),
83          ('A^234', [[('', 'A', '234')]]),
84
85          # Multiplications
86          ('A 0.1B', [[('', 'A', ''), ('0.1', 'B', '')]]),
87          ('A^2 3B', [[('', 'A', '23'), ('', 'B', '')]]),
88          ('4A^{3} 6B^2', [[('4', 'A', '3'), ('6', 'B', '2')]]),
89          ('4.2A^{T} 6.1B^-1', [[('4.2', 'A', 'T'), ('6.1', 'B', '-1')]]),
90          ('-1.2A^2 rot(45)^2', [[('-1.2', 'A', '2'), ('', 'rot(45)', '2')]]),
91          ('3.2A^T 4.5B^{5} 9.6rot(121.3)', [[('3.2', 'A', 'T'), ('4.5', 'B', '5'), ('9.6', 'rot(121.3)', '')]]),
92          ('-1.18A^{-2} 0.1B^{2} 9rot(-34.6)^-1', [[('-1.18', 'A', '-2'), ('0.1', 'B', '2'), ('9', 'rot(-34.6)', '-1')]]),
93
94          # Additions
95          ('A + B', [[('', 'A', '')], [('', 'B', '')]]),
96          ('A + B - C', [[('', 'A', '')], [('', 'B', '')], [('-1', 'C', '')]]),
97          ('A^2 + 0.5B', [[('', 'A', '2')], [('0.5', 'B', '')]]),
98          ('2A^3 + 8B^T - 3C^-1', [[('2', 'A', '3')], [('8', 'B', 'T')], [('-3', 'C', '-1')]]),
99          ('4.9A^2 - 3rot(134.2)^-1 + 7.6B^8', [[('4.9', 'A', '2')], [('-3', 'rot(134.2)', '-1')], [('7.6', 'B', '8')]]),
100
101         # Additions with multiplication
102         ('2.14A^{3} 4.5rot(14.5)^-1 + 8B^T - 3C^-1', [[('2.14', 'A', '3'), ('4.5', 'rot(14.5)', '-1')],
103                                                       [('8', 'B', 'T')], [('-3', 'C', '-1')]]),
104         ('2.14A^{3} 4.5rot(14.5)^-1 + 8.5B^T 5.97C^14 - 3.14D^{-1} 6.7E^T',
105          [[('2.14', 'A', '3'), ('4.5', 'rot(14.5)', '-1')], [('8.5', 'B', 'T'), ('5.97', 'C', '14')],
106           [('-3.14', 'D', '-1'), ('6.7', 'E', 'T')]]),
107
108         # Parenthesized expressions
109         ('(AB)^-1', [[('', 'AB', '-1')]]),
110         ('-3(A+B)^2-C(B^TA)^-1', [[('-3', 'A+B', '2')], [('-1', 'C', ''), ('', 'B^{T}A', '-1')]]),
111         ('2.3(3B^TA)^2', [[('2.3', '3B^{T}A', '2')]]),
112         ('-3.4(9D^{2}3F^-1)^T+C', [[('-3.4', '9D^{2}3F^{-1}', 'T')], [('', 'C', '')]]),
113         ('2.39(3.1A^{-1}2.3B(CD)^-1)^T + (AB^T)^-1', [[('2.39', '3.1A^{-1}2.3B(CD)^{-1}', 'T')], [('', 'AB^{T}',
        ↪  '-1')]])
114     ]
115
116
117     def test_parse_matrix_expression() -> None:
118         """Test the parse_matrix_expression() function."""
119         for expression, parsed_expression in expressions_and_parsed_expressions:
120             # Test it with and without whitespace
121             assert parse_matrix_expression(expression) == parsed_expression
122             assert parse_matrix_expression(expression.replace(' ', '')) == parsed_expression
123
124
125     def test_parse_error() -> None:
126         """Test that parse_matrix_expression() raises a MatrixParseError."""
127         for expression in invalid_inputs:
128             with pytest.raises(MatrixParseError):
129                 parse_matrix_expression(expression)
```

B.2 `matrices/utility/test_rotation_matrices.py`

```
1       # lintrans - The linear transformation visualizer
2       # Copyright (C) 2021-2022 D. Dyson (DoctorDalek1963)
3
4       # This program is licensed under GNU GPLv3, available here:
```

```python
    # <https://www.gnu.org/licenses/gpl-3.0.html>

    """Test functions for rotation matrices."""

    from typing import List, Tuple

    import numpy as np
    import pytest

    from lintrans.matrices import create_rotation_matrix
    from lintrans.typing_ import MatrixType

    angles_and_matrices: List[Tuple[float, float, MatrixType]] = [
        (0, 0, np.array([[1, 0], [0, 1]])),
        (90, np.pi / 2, np.array([[0, -1], [1, 0]])),
        (180, np.pi, np.array([[-1, 0], [0, -1]])),
        (270, 3 * np.pi / 2, np.array([[0, 1], [-1, 0]])),
        (360, 2 * np.pi, np.array([[1, 0], [0, 1]])),

        (45, np.pi / 4, np.array([
            [np.sqrt(2) / 2, -1 * np.sqrt(2) / 2],
            [np.sqrt(2) / 2, np.sqrt(2) / 2]
        ])),
        (135, 3 * np.pi / 4, np.array([
            [-1 * np.sqrt(2) / 2, -1 * np.sqrt(2) / 2],
            [np.sqrt(2) / 2, -1 * np.sqrt(2) / 2]
        ])),
        (225, 5 * np.pi / 4, np.array([
            [-1 * np.sqrt(2) / 2, np.sqrt(2) / 2],
            [-1 * np.sqrt(2) / 2, -1 * np.sqrt(2) / 2]
        ])),
        (315, 7 * np.pi / 4, np.array([
            [np.sqrt(2) / 2, np.sqrt(2) / 2],
            [-1 * np.sqrt(2) / 2, np.sqrt(2) / 2]
        ])),

        (30, np.pi / 6, np.array([
            [np.sqrt(3) / 2, -1 / 2],
            [1 / 2, np.sqrt(3) / 2]
        ])),
        (60, np.pi / 3, np.array([
            [1 / 2, -1 * np.sqrt(3) / 2],
            [np.sqrt(3) / 2, 1 / 2]
        ])),
        (120, 2 * np.pi / 3, np.array([
            [-1 / 2, -1 * np.sqrt(3) / 2],
            [np.sqrt(3) / 2, -1 / 2]
        ])),
        (150, 5 * np.pi / 6, np.array([
            [-1 * np.sqrt(3) / 2, -1 / 2],
            [1 / 2, -1 * np.sqrt(3) / 2]
        ])),
        (210, 7 * np.pi / 6, np.array([
            [-1 * np.sqrt(3) / 2, 1 / 2],
            [-1 / 2, -1 * np.sqrt(3) / 2]
        ])),
        (240, 4 * np.pi / 3, np.array([
            [-1 / 2, np.sqrt(3) / 2],
            [-1 * np.sqrt(3) / 2, -1 / 2]
        ])),
        (300, 10 * np.pi / 6, np.array([
            [1 / 2, np.sqrt(3) / 2],
            [-1 * np.sqrt(3) / 2, 1 / 2]
        ])),
        (330, 11 * np.pi / 6, np.array([
            [np.sqrt(3) / 2, 1 / 2],
            [-1 / 2, np.sqrt(3) / 2]
        ]))
    ]


    def test_create_rotation_matrix() -> None:
        """Test that create_rotation_matrix() works with given angles and expected matrices."""
```

```
    for degrees, radians, matrix in angles_and_matrices:
        assert create_rotation_matrix(degrees, degrees=True) == pytest.approx(matrix)
        assert create_rotation_matrix(radians, degrees=False) == pytest.approx(matrix)

        assert create_rotation_matrix(-1 * degrees, degrees=True) == pytest.approx(np.linalg.inv(matrix))
        assert create_rotation_matrix(-1 * radians, degrees=False) == pytest.approx(np.linalg.inv(matrix))

    assert (create_rotation_matrix(-90, degrees=True) ==
            create_rotation_matrix(270, degrees=True)).all()
    assert (create_rotation_matrix(-0.5 * np.pi, degrees=False) ==
            create_rotation_matrix(1.5 * np.pi, degrees=False)).all()
```

B.3 matrices/utility/test_coord_conversion.py

```
#  lintrans - The linear transformation visualizer
#  Copyright (C) 2022 D. Dyson (DoctorDalek1963)
#
#  This program is licensed under GNU GPLv3, available here:
#  <https://www.gnu.org/licenses/gpl-3.0.html>

"""Test conversion between polar and rectilinear coordinates in :mod:`lintrans.matrices.utility`."""

from typing import List, Tuple

from numpy import pi, sqrt
from pytest import approx

from lintrans.matrices.utility import polar_coords, rect_coords

expected_coords: List[Tuple[Tuple[float, float], Tuple[float, float]]] = [
    ((0, 0), (0, 0)),
    ((1, 1), (sqrt(2), pi / 4)),
    ((0, 1), (1, pi / 2)),
    ((1, 0), (1, 0)),
    ((sqrt(2), sqrt(2)), (2, pi / 4)),
    ((-3, 4), (5, 2.214297436)),
    ((4, -3), (5, 5.639684198)),
    ((5, -0.2), (sqrt(626) / 5, 6.24320662)),
    ((-1.3, -10), (10.08414597, 4.583113976)),
    ((23.4, 0), (23.4, 0)),
    ((pi, -pi), (4.442882938, 1.75 * pi))
]


def test_polar_coords() -> None:
    """Test that :func:`lintrans.matrices.utility.polar_coords` works as expected."""
    for rect, polar in expected_coords:
        assert polar_coords(*rect) == approx(polar)


def test_rect_coords() -> None:
    """Test that :func:`lintrans.matrices.utility.rect_coords` works as expected."""
    for rect, polar in expected_coords:
        assert rect_coords(*polar) == approx(rect)

    assert rect_coords(1, 0) == approx((1, 0))
    assert rect_coords(1, pi) == approx((-1, 0))
    assert rect_coords(1, 2 * pi) == approx((1, 0))
    assert rect_coords(1, 3 * pi) == approx((-1, 0))
    assert rect_coords(1, 4 * pi) == approx((1, 0))
    assert rect_coords(1, 5 * pi) == approx((-1, 0))
    assert rect_coords(1, 6 * pi) == approx((1, 0))
    assert rect_coords(20, 100) == approx(rect_coords(20, 100 % (2 * pi)))
```

B.4 matrices/utility/test_float_utility_functions.py

```
# lintrans - The linear transformation visualizer
# Copyright (C) 2021-2022 D. Dyson (DoctorDalek1963)
```

```

    # This program is licensed under GNU GPLv3, available here:
    # <https://www.gnu.org/licenses/gpl-3.0.html>

    """Test the utility functions for GUI dialog boxes."""

    from typing import List, Tuple

    import numpy as np
    import pytest

    from lintrans.matrices.utility import is_valid_float, round_float

    valid_floats: List[str] = [
        '0', '1', '3', '-2', '123', '-208', '1.2', '-3.5', '4.252634', '-42362.352325',
        '1e4', '-2.59e3', '4.13e-6', '-5.5244e-12'
    ]

    invalid_floats: List[str] = [
        '', 'pi', 'e', '1.2.3', '1,2', '-', '.', 'None', 'no', 'yes', 'float'
    ]


    @pytest.mark.parametrize('inputs,output', [(valid_floats, True), (invalid_floats, False)])
    def test_is_valid_float(inputs: List[str], output: bool) -> None:
        """Test the is_valid_float() function."""
        for inp in inputs:
            assert is_valid_float(inp) == output


    def test_round_float() -> None:
        """Test the round_float() function."""
        expected_values: List[Tuple[float, int, str]] = [
            (1.0, 4, '1'), (1e-6, 4, '0'), (1e-5, 6, '1e-5'), (6.3e-8, 5, '0'), (3.2e-8, 10, '3.2e-8'),
            (np.sqrt(2) / 2, 5, '0.70711'), (-1 * np.sqrt(2) / 2, 5, '-0.70711'),
            (np.pi, 1, '3.1'), (np.pi, 2, '3.14'), (np.pi, 3, '3.142'), (np.pi, 4, '3.1416'), (np.pi, 5, '3.14159'),
            (1.23456789, 2, '1.23'), (1.23456789, 3, '1.235'), (1.23456789, 4, '1.2346'), (1.23456789, 5, '1.23457'),
            (12345.678, 1, '12345.7'), (12345.678, 2, '12345.68'), (12345.678, 3, '12345.678'),
        ]

        for num, precision, answer in expected_values:
            assert round_float(num, precision) == answer
```

B.5 matrices/matrix_wrapper/test_misc.py

```

    # lintrans - The linear transformation visualizer
    # Copyright (C) 2021-2022 D. Dyson (DoctorDalek1963)

    # This program is licensed under GNU GPLv3, available here:
    # <https://www.gnu.org/licenses/gpl-3.0.html>

    """Test the miscellaneous methods of the MatrixWrapper class."""

    from lintrans.matrices import MatrixWrapper


    def test_get_expression(test_wrapper: MatrixWrapper) -> None:
        """Test the get_expression method of the MatrixWrapper class."""
        test_wrapper['N'] = 'A^2'
        test_wrapper['O'] = '4B'
        test_wrapper['P'] = 'A+C'

        test_wrapper['Q'] = 'N^-1'
        test_wrapper['R'] = 'P-4O'
        test_wrapper['S'] = 'NOP'

        assert test_wrapper.get_expression('A') is None
        assert test_wrapper.get_expression('B') is None
        assert test_wrapper.get_expression('C') is None
        assert test_wrapper.get_expression('D') is None
```

```
        assert test_wrapper.get_expression('E') is None
        assert test_wrapper.get_expression('F') is None
        assert test_wrapper.get_expression('G') is None

        assert test_wrapper.get_expression('N') == 'A^2'
        assert test_wrapper.get_expression('O') == '4B'
        assert test_wrapper.get_expression('P') == 'A+C'

        assert test_wrapper.get_expression('Q') == 'N^-1'
        assert test_wrapper.get_expression('R') == 'P-4O'
        assert test_wrapper.get_expression('S') == 'NOP'
```

B.6 matrices/matrix_wrapper/test_setitem_and_getitem.py

```
# lintrans - The linear transformation visualizer
# Copyright (C) 2021-2022 D. Dyson (DoctorDalek1963)

# This program is licensed under GNU GPLv3, available here:
# <https://www.gnu.org/licenses/gpl-3.0.html>

"""Test the MatrixWrapper __setitem__() and __getitem__() methods."""

from typing import Any, List

import numpy as np
import pytest
from numpy import linalg as la

from lintrans.matrices import MatrixWrapper
from lintrans.typing_ import MatrixType

valid_matrix_names = 'ABCDEFGHJKLMNOPQRSTUVWXYZ'
invalid_matrix_names = ['bad name', '123456', 'Th15 Is an 1nV@l1D n@m3', 'abc', 'a']

test_matrix: MatrixType = np.array([[1, 2], [4, 3]])


def test_basic_get_matrix(new_wrapper: MatrixWrapper) -> None:
    """Test MatrixWrapper().__getitem__()."""
    for name in valid_matrix_names:
        assert new_wrapper[name] is None

    assert (new_wrapper['I'] == np.array([[1, 0], [0, 1]])).all()


def test_get_name_error(new_wrapper: MatrixWrapper) -> None:
    """Test that MatrixWrapper().__getitem__() raises a NameError if called with an invalid name."""
    for name in invalid_matrix_names:
        with pytest.raises(NameError):
            _ = new_wrapper[name]


def test_basic_set_matrix(new_wrapper: MatrixWrapper) -> None:
    """Test MatrixWrapper().__setitem__()."""
    for name in valid_matrix_names:
        new_wrapper[name] = test_matrix
        assert (new_wrapper[name] == test_matrix).all()

        new_wrapper[name] = None
        assert new_wrapper[name] is None


def test_set_expression(test_wrapper: MatrixWrapper) -> None:
    """Test that MatrixWrapper.__setitem__() can accept a valid expression."""
    test_wrapper['N'] = 'A^2'
    test_wrapper['O'] = 'BA+2C'
    test_wrapper['P'] = 'E^T'
    test_wrapper['Q'] = 'C^-1B'
    test_wrapper['R'] = 'A^{2}3B'
    test_wrapper['S'] = 'N^-1'
```

```
        test_wrapper['T'] = 'PQP^-1'

        with pytest.raises(TypeError):
            test_wrapper['U'] = 'A+1'

        with pytest.raises(TypeError):
            test_wrapper['V'] = 'K'

        with pytest.raises(TypeError):
            test_wrapper['W'] = 'L^2'

        with pytest.raises(TypeError):
            test_wrapper['X'] = 'M^-1'


    def test_simple_dynamic_evaluation(test_wrapper: MatrixWrapper) -> None:
        """Test that expression-defined matrices are evaluated dynamically."""
        test_wrapper['N'] = 'A^2'
        test_wrapper['O'] = '4B'
        test_wrapper['P'] = 'A+C'

        assert (test_wrapper['N'] == test_wrapper.evaluate_expression('A^2')).all()
        assert (test_wrapper['O'] == test_wrapper.evaluate_expression('4B')).all()
        assert (test_wrapper['P'] == test_wrapper.evaluate_expression('A+C')).all()

        assert (test_wrapper.evaluate_expression('N^2 + 3O') ==
                la.matrix_power(test_wrapper.evaluate_expression('A^2'), 2) +
                3 * test_wrapper.evaluate_expression('4B')
                ).all()
        assert (test_wrapper.evaluate_expression('P^-1 - 3NO^2') ==
                la.inv(test_wrapper.evaluate_expression('A+C')) -
                (3 * test_wrapper.evaluate_expression('A^2')) @
                la.matrix_power(test_wrapper.evaluate_expression('4B'), 2)
                ).all()

        test_wrapper['A'] = np.array([
            [19, -21.5],
            [84, 96.572]
        ])
        test_wrapper['B'] = np.array([
            [-0.993, 2.52],
            [1e10, 0]
        ])
        test_wrapper['C'] = np.array([
            [0, 19512],
            [1.414, 19]
        ])

        assert (test_wrapper['N'] == test_wrapper.evaluate_expression('A^2')).all()
        assert (test_wrapper['O'] == test_wrapper.evaluate_expression('4B')).all()
        assert (test_wrapper['P'] == test_wrapper.evaluate_expression('A+C')).all()

        assert (test_wrapper.evaluate_expression('N^2 + 3O') ==
                la.matrix_power(test_wrapper.evaluate_expression('A^2'), 2) +
                3 * test_wrapper.evaluate_expression('4B')
                ).all()
        assert (test_wrapper.evaluate_expression('P^-1 - 3NO^2') ==
                la.inv(test_wrapper.evaluate_expression('A+C')) -
                (3 * test_wrapper.evaluate_expression('A^2')) @
                la.matrix_power(test_wrapper.evaluate_expression('4B'), 2)
                ).all()


    def test_recursive_dynamic_evaluation(test_wrapper: MatrixWrapper) -> None:
        """Test that dynamic evaluation works recursively."""
        test_wrapper['N'] = 'A^2'
        test_wrapper['O'] = '4B'
        test_wrapper['P'] = 'A+C'

        test_wrapper['Q'] = 'N^-1'
        test_wrapper['R'] = 'P-4O'
        test_wrapper['S'] = 'NOP'

```

```
130         assert test_wrapper['Q'] == pytest.approx(test_wrapper.evaluate_expression('A^-2'))
131         assert test_wrapper['R'] == pytest.approx(test_wrapper.evaluate_expression('A + C - 16B'))
132         assert test_wrapper['S'] == pytest.approx(test_wrapper.evaluate_expression('A^{2}4BA + A^{2}4BC'))
133
134
135     def test_set_identity_error(new_wrapper: MatrixWrapper) -> None:
136         """Test that MatrixWrapper().__setitem__() raises a NameError when trying to assign to the identity matrix."""
137         with pytest.raises(NameError):
138             new_wrapper['I'] = test_matrix
139
140
141     def test_set_name_error(new_wrapper: MatrixWrapper) -> None:
142         """Test that MatrixWrapper().__setitem__() raises a NameError when trying to assign to an invalid name."""
143         for name in invalid_matrix_names:
144             with pytest.raises(NameError):
145                 new_wrapper[name] = test_matrix
146
147
148     def test_set_type_error(new_wrapper: MatrixWrapper) -> None:
149         """Test that MatrixWrapper().__setitem__() raises a TypeError when trying to set a non-matrix."""
150         invalid_values: List[Any] = [
151                                      12,
152                                      [1, 2, 3, 4, 5],
153                                      [[1, 2], [3, 4]],
154                                      True,
155                                      24.3222,
156                                      'This is totally a matrix, I swear',
157                                      MatrixWrapper,
158                                      MatrixWrapper(),
159                                      np.array([[1, 2, 3], [4, 5, 6], [7, 8, 9]]),
160                                      np.eye(100)
161                                      ]
162
163         for value in invalid_values:
164             with pytest.raises(TypeError):
165                 new_wrapper['M'] = value
```

B.7 `matrices/matrix_wrapper/conftest.py`

```
1   # lintrans - The linear transformation visualizer
2   # Copyright (C) 2021-2022 D. Dyson (DoctorDalek1963)
3
4   # This program is licensed under GNU GPLv3, available here:
5   # <https://www.gnu.org/licenses/gpl-3.0.html>
6
7   """A simple conftest.py containing some re-usable fixtures."""
8
9   import numpy as np
10  import pytest
11
12  from lintrans.matrices import MatrixWrapper
13
14
15  def get_test_wrapper() -> MatrixWrapper:
16      """Return a new MatrixWrapper object with some preset values."""
17      wrapper = MatrixWrapper()
18
19      root_two_over_two = np.sqrt(2) / 2
20
21      wrapper['A'] = np.array([[1, 2], [3, 4]])
22      wrapper['B'] = np.array([[6, 4], [12, 9]])
23      wrapper['C'] = np.array([[-1, -3], [4, -12]])
24      wrapper['D'] = np.array([[13.2, 9.4], [-3.4, -1.8]])
25      wrapper['E'] = np.array([
26          [root_two_over_two, -1 * root_two_over_two],
27          [root_two_over_two, root_two_over_two]
28      ])
29      wrapper['F'] = np.array([[-1, 0], [0, 1]])
30      wrapper['G'] = np.array([[np.pi, np.e], [1729, 743.631]])
31
```

```
32          return wrapper
33
34
35      @pytest.fixture
36      def test_wrapper() -> MatrixWrapper:
37          """Return a new MatrixWrapper object with some preset values."""
38          return get_test_wrapper()
39
40
41      @pytest.fixture
42      def new_wrapper() -> MatrixWrapper:
43          """Return a new MatrixWrapper with no initialized values."""
44          return MatrixWrapper()
```

B.8 matrices/matrix_wrapper/test_evaluate_expression.py

```
1       # lintrans - The linear transformation visualizer
2       # Copyright (C) 2021-2022 D. Dyson (DoctorDalek1963)
3
4       # This program is licensed under GNU GPLv3, available here:
5       # <https://www.gnu.org/licenses/gpl-3.0.html>
6
7       """Test the MatrixWrapper evaluate_expression() method."""
8
9       import numpy as np
10      from numpy import linalg as la
11      import pytest
12      from pytest import approx
13
14      from lintrans.matrices import MatrixWrapper, create_rotation_matrix
15      from lintrans.typing_ import MatrixType
16
17      from conftest import get_test_wrapper
18
19
20      def test_simple_matrix_addition(test_wrapper: MatrixWrapper) -> None:
21          """Test simple addition and subtraction of two matrices."""
22
23          # NOTE: We assert that all of these values are not None just to stop mypy complaining
24          # These values will never actually be None because they're set in the wrapper() fixture
25          # There's probably a better way  do this, because this method is a bit of a bodge, but this works for now
26          assert test_wrapper['A'] is not None and test_wrapper['B'] is not None and test_wrapper['C'] is not None and \
27                 test_wrapper['D'] is not None and test_wrapper['E'] is not None and test_wrapper['F'] is not None and \
28                 test_wrapper['G'] is not None
29
30          assert (test_wrapper.evaluate_expression('A+B') == test_wrapper['A'] + test_wrapper['B']).all()
31          assert (test_wrapper.evaluate_expression('E+F') == test_wrapper['E'] + test_wrapper['F']).all()
32          assert (test_wrapper.evaluate_expression('G+D') == test_wrapper['G'] + test_wrapper['D']).all()
33          assert (test_wrapper.evaluate_expression('C+C') == test_wrapper['C'] + test_wrapper['C']).all()
34          assert (test_wrapper.evaluate_expression('D+A') == test_wrapper['D'] + test_wrapper['A']).all()
35          assert (test_wrapper.evaluate_expression('B+C') == test_wrapper['B'] + test_wrapper['C']).all()
36
37          assert test_wrapper == get_test_wrapper()
38
39
40      def test_simple_two_matrix_multiplication(test_wrapper: MatrixWrapper) -> None:
41          """Test simple multiplication of two matrices."""
42          assert test_wrapper['A'] is not None and test_wrapper['B'] is not None and test_wrapper['C'] is not None and \
43                 test_wrapper['D'] is not None and test_wrapper['E'] is not None and test_wrapper['F'] is not None and \
44                 test_wrapper['G'] is not None
45
46          assert (test_wrapper.evaluate_expression('AB') == test_wrapper['A'] @ test_wrapper['B']).all()
47          assert (test_wrapper.evaluate_expression('BA') == test_wrapper['B'] @ test_wrapper['A']).all()
48          assert (test_wrapper.evaluate_expression('AC') == test_wrapper['A'] @ test_wrapper['C']).all()
49          assert (test_wrapper.evaluate_expression('DA') == test_wrapper['D'] @ test_wrapper['A']).all()
50          assert (test_wrapper.evaluate_expression('ED') == test_wrapper['E'] @ test_wrapper['D']).all()
51          assert (test_wrapper.evaluate_expression('FD') == test_wrapper['F'] @ test_wrapper['D']).all()
52          assert (test_wrapper.evaluate_expression('GA') == test_wrapper['G'] @ test_wrapper['A']).all()
53          assert (test_wrapper.evaluate_expression('CF') == test_wrapper['C'] @ test_wrapper['F']).all()
54          assert (test_wrapper.evaluate_expression('AG') == test_wrapper['A'] @ test_wrapper['G']).all()
```

```
55
56          assert test_wrapper.evaluate_expression('A2B') == approx(test_wrapper['A'] @ (2 * test_wrapper['B']))
57          assert test_wrapper.evaluate_expression('2AB') == approx((2 * test_wrapper['A']) @ test_wrapper['B'])
58          assert test_wrapper.evaluate_expression('C3D') == approx(test_wrapper['C'] @ (3 * test_wrapper['D']))
59          assert test_wrapper.evaluate_expression('4.2E1.2A') == approx((4.2 * test_wrapper['E']) @ (1.2 *
            ↪  test_wrapper['A']))
60
61          assert test_wrapper == get_test_wrapper()
62
63
64      def test_identity_multiplication(test_wrapper: MatrixWrapper) -> None:
65          """Test that multiplying by the identity doesn't change the value of a matrix."""
66          assert test_wrapper['A'] is not None and test_wrapper['B'] is not None and test_wrapper['C'] is not None and \
67                 test_wrapper['D'] is not None and test_wrapper['E'] is not None and test_wrapper['F'] is not None and \
68                 test_wrapper['G'] is not None
69
70          assert (test_wrapper.evaluate_expression('I') == test_wrapper['I']).all()
71          assert (test_wrapper.evaluate_expression('AI') == test_wrapper['A']).all()
72          assert (test_wrapper.evaluate_expression('IA') == test_wrapper['A']).all()
73          assert (test_wrapper.evaluate_expression('GI') == test_wrapper['G']).all()
74          assert (test_wrapper.evaluate_expression('IG') == test_wrapper['G']).all()
75
76          assert (test_wrapper.evaluate_expression('EID') == test_wrapper['E'] @ test_wrapper['D']).all()
77          assert (test_wrapper.evaluate_expression('IED') == test_wrapper['E'] @ test_wrapper['D']).all()
78          assert (test_wrapper.evaluate_expression('EDI') == test_wrapper['E'] @ test_wrapper['D']).all()
79          assert (test_wrapper.evaluate_expression('IEIDI') == test_wrapper['E'] @ test_wrapper['D']).all()
80          assert (test_wrapper.evaluate_expression('EI^3D') == test_wrapper['E'] @ test_wrapper['D']).all()
81
82          assert test_wrapper == get_test_wrapper()
83
84
85      def test_simple_three_matrix_multiplication(test_wrapper: MatrixWrapper) -> None:
86          """Test simple multiplication of two matrices."""
87          assert test_wrapper['A'] is not None and test_wrapper['B'] is not None and test_wrapper['C'] is not None and \
88                 test_wrapper['D'] is not None and test_wrapper['E'] is not None and test_wrapper['F'] is not None and \
89                 test_wrapper['G'] is not None
90
91          assert (test_wrapper.evaluate_expression('ABC') == test_wrapper['A'] @ test_wrapper['B'] @
            ↪  test_wrapper['C']).all()
92          assert (test_wrapper.evaluate_expression('ACB') == test_wrapper['A'] @ test_wrapper['C'] @
            ↪  test_wrapper['B']).all()
93          assert (test_wrapper.evaluate_expression('BAC') == test_wrapper['B'] @ test_wrapper['A'] @
            ↪  test_wrapper['C']).all()
94          assert (test_wrapper.evaluate_expression('EFG') == test_wrapper['E'] @ test_wrapper['F'] @
            ↪  test_wrapper['G']).all()
95          assert (test_wrapper.evaluate_expression('DAC') == test_wrapper['D'] @ test_wrapper['A'] @
            ↪  test_wrapper['C']).all()
96          assert (test_wrapper.evaluate_expression('GAE') == test_wrapper['G'] @ test_wrapper['A'] @
            ↪  test_wrapper['E']).all()
97          assert (test_wrapper.evaluate_expression('FAG') == test_wrapper['F'] @ test_wrapper['A'] @
            ↪  test_wrapper['G']).all()
98          assert (test_wrapper.evaluate_expression('GAF') == test_wrapper['G'] @ test_wrapper['A'] @
            ↪  test_wrapper['F']).all()
99
100         assert test_wrapper == get_test_wrapper()
101
102
103     def test_matrix_inverses(test_wrapper: MatrixWrapper) -> None:
104         """Test the inverses of single matrices."""
105         assert test_wrapper['A'] is not None and test_wrapper['B'] is not None and test_wrapper['C'] is not None and \
106                test_wrapper['D'] is not None and test_wrapper['E'] is not None and test_wrapper['F'] is not None and \
107                test_wrapper['G'] is not None
108
109         assert (test_wrapper.evaluate_expression('A^{-1}') == la.inv(test_wrapper['A'])).all()
110         assert (test_wrapper.evaluate_expression('B^{-1}') == la.inv(test_wrapper['B'])).all()
111         assert (test_wrapper.evaluate_expression('C^{-1}') == la.inv(test_wrapper['C'])).all()
112         assert (test_wrapper.evaluate_expression('D^{-1}') == la.inv(test_wrapper['D'])).all()
113         assert (test_wrapper.evaluate_expression('E^{-1}') == la.inv(test_wrapper['E'])).all()
114         assert (test_wrapper.evaluate_expression('F^{-1}') == la.inv(test_wrapper['F'])).all()
115         assert (test_wrapper.evaluate_expression('G^{-1}') == la.inv(test_wrapper['G'])).all()
116
117         assert (test_wrapper.evaluate_expression('A^-1') == la.inv(test_wrapper['A'])).all()
118         assert (test_wrapper.evaluate_expression('B^-1') == la.inv(test_wrapper['B'])).all()
```

```
        assert (test_wrapper.evaluate_expression('C^-1') == la.inv(test_wrapper['C'])).all()
        assert (test_wrapper.evaluate_expression('D^-1') == la.inv(test_wrapper['D'])).all()
        assert (test_wrapper.evaluate_expression('E^-1') == la.inv(test_wrapper['E'])).all()
        assert (test_wrapper.evaluate_expression('F^-1') == la.inv(test_wrapper['F'])).all()
        assert (test_wrapper.evaluate_expression('G^-1') == la.inv(test_wrapper['G'])).all()

        assert test_wrapper == get_test_wrapper()


    def test_matrix_powers(test_wrapper: MatrixWrapper) -> None:
        """Test that matrices can be raised to integer powers."""
        assert test_wrapper['A'] is not None and test_wrapper['B'] is not None and test_wrapper['C'] is not None and \
               test_wrapper['D'] is not None and test_wrapper['E'] is not None and test_wrapper['F'] is not None and \
               test_wrapper['G'] is not None

        assert (test_wrapper.evaluate_expression('A^2') == la.matrix_power(test_wrapper['A'], 2)).all()
        assert (test_wrapper.evaluate_expression('B^4') == la.matrix_power(test_wrapper['B'], 4)).all()
        assert (test_wrapper.evaluate_expression('C^{12}') == la.matrix_power(test_wrapper['C'], 12)).all()
        assert (test_wrapper.evaluate_expression('D^12') == la.matrix_power(test_wrapper['D'], 12)).all()
        assert (test_wrapper.evaluate_expression('E^8') == la.matrix_power(test_wrapper['E'], 8)).all()
        assert (test_wrapper.evaluate_expression('F^{-6}') == la.matrix_power(test_wrapper['F'], -6)).all()
        assert (test_wrapper.evaluate_expression('G^-2') == la.matrix_power(test_wrapper['G'], -2)).all()

        assert test_wrapper == get_test_wrapper()


    def test_matrix_transpose(test_wrapper: MatrixWrapper) -> None:
        """Test matrix transpositions."""
        assert test_wrapper['A'] is not None and test_wrapper['B'] is not None and test_wrapper['C'] is not None and \
               test_wrapper['D'] is not None and test_wrapper['E'] is not None and test_wrapper['F'] is not None and \
               test_wrapper['G'] is not None

        assert (test_wrapper.evaluate_expression('A^{T}') == test_wrapper['A'].T).all()
        assert (test_wrapper.evaluate_expression('B^{T}') == test_wrapper['B'].T).all()
        assert (test_wrapper.evaluate_expression('C^{T}') == test_wrapper['C'].T).all()
        assert (test_wrapper.evaluate_expression('D^{T}') == test_wrapper['D'].T).all()
        assert (test_wrapper.evaluate_expression('E^{T}') == test_wrapper['E'].T).all()
        assert (test_wrapper.evaluate_expression('F^{T}') == test_wrapper['F'].T).all()
        assert (test_wrapper.evaluate_expression('G^{T}') == test_wrapper['G'].T).all()

        assert (test_wrapper.evaluate_expression('A^T') == test_wrapper['A'].T).all()
        assert (test_wrapper.evaluate_expression('B^T') == test_wrapper['B'].T).all()
        assert (test_wrapper.evaluate_expression('C^T') == test_wrapper['C'].T).all()
        assert (test_wrapper.evaluate_expression('D^T') == test_wrapper['D'].T).all()
        assert (test_wrapper.evaluate_expression('E^T') == test_wrapper['E'].T).all()
        assert (test_wrapper.evaluate_expression('F^T') == test_wrapper['F'].T).all()
        assert (test_wrapper.evaluate_expression('G^T') == test_wrapper['G'].T).all()

        assert test_wrapper == get_test_wrapper()


    def test_rotation_matrices(test_wrapper: MatrixWrapper) -> None:
        """Test that 'rot(angle)' can be used in an expression."""
        assert (test_wrapper.evaluate_expression('rot(90)') == create_rotation_matrix(90)).all()
        assert (test_wrapper.evaluate_expression('rot(180)') == create_rotation_matrix(180)).all()
        assert (test_wrapper.evaluate_expression('rot(270)') == create_rotation_matrix(270)).all()
        assert (test_wrapper.evaluate_expression('rot(360)') == create_rotation_matrix(360)).all()
        assert (test_wrapper.evaluate_expression('rot(45)') == create_rotation_matrix(45)).all()
        assert (test_wrapper.evaluate_expression('rot(30)') == create_rotation_matrix(30)).all()

        assert (test_wrapper.evaluate_expression('rot(13.43)') == create_rotation_matrix(13.43)).all()
        assert (test_wrapper.evaluate_expression('rot(49.4)') == create_rotation_matrix(49.4)).all()
        assert (test_wrapper.evaluate_expression('rot(-123.456)') == create_rotation_matrix(-123.456)).all()
        assert (test_wrapper.evaluate_expression('rot(963.245)') == create_rotation_matrix(963.245)).all()
        assert (test_wrapper.evaluate_expression('rot(-235.24)') == create_rotation_matrix(-235.24)).all()

        assert test_wrapper == get_test_wrapper()


    def test_multiplication_and_addition(test_wrapper: MatrixWrapper) -> None:
        """Test multiplication and addition of matrices together."""
        assert test_wrapper['A'] is not None and test_wrapper['B'] is not None and test_wrapper['C'] is not None and \
               test_wrapper['D'] is not None and test_wrapper['E'] is not None and test_wrapper['F'] is not None and \
```

```
            test_wrapper['G'] is not None

    assert (test_wrapper.evaluate_expression('AB+C') ==
            test_wrapper['A'] @ test_wrapper['B'] + test_wrapper['C']).all()
    assert (test_wrapper.evaluate_expression('DE-D') ==
            test_wrapper['D'] @ test_wrapper['E'] - test_wrapper['D']).all()
    assert (test_wrapper.evaluate_expression('FD+AB') ==
            test_wrapper['F'] @ test_wrapper['D'] + test_wrapper['A'] @ test_wrapper['B']).all()
    assert (test_wrapper.evaluate_expression('BA-DE') ==
            test_wrapper['B'] @ test_wrapper['A'] - test_wrapper['D'] @ test_wrapper['E']).all()

    assert (test_wrapper.evaluate_expression('2AB+3C') ==
            (2 * test_wrapper['A']) @ test_wrapper['B'] + (3 * test_wrapper['C'])).all()
    assert (test_wrapper.evaluate_expression('4D7.9E-1.2A') ==
            (4 * test_wrapper['D']) @ (7.9 * test_wrapper['E']) - (1.2 * test_wrapper['A'])).all()

    assert test_wrapper == get_test_wrapper()


def test_complicated_expressions(test_wrapper: MatrixWrapper) -> None:
    """Test evaluation of complicated expressions."""
    assert test_wrapper['A'] is not None and test_wrapper['B'] is not None and test_wrapper['C'] is not None and \
           test_wrapper['D'] is not None and test_wrapper['E'] is not None and test_wrapper['F'] is not None and \
           test_wrapper['G'] is not None

    assert (test_wrapper.evaluate_expression('-3.2A^T 4B^{-1} 6C^{-1} + 8.1D^{2} 3.2E^4') ==
            (-3.2 * test_wrapper['A'].T) @ (4 * la.inv(test_wrapper['B'])) @ (6 * la.inv(test_wrapper['C']))
            + (8.1 * la.matrix_power(test_wrapper['D'], 2)) @ (3.2 * la.matrix_power(test_wrapper['E'], 4))).all()

    assert (test_wrapper.evaluate_expression('53.6D^{2} 3B^T - 4.9F^{2} 2D + A^3 B^-1') ==
            (53.6 * la.matrix_power(test_wrapper['D'], 2)) @ (3 * test_wrapper['B'].T)
            - (4.9 * la.matrix_power(test_wrapper['F'], 2)) @ (2 * test_wrapper['D'])
            + la.matrix_power(test_wrapper['A'], 3) @ la.inv(test_wrapper['B'])).all()

    assert test_wrapper == get_test_wrapper()


def test_parenthesized_expressions(test_wrapper: MatrixWrapper) -> None:
    """Test evaluation of parenthesized expressions."""
    assert test_wrapper['A'] is not None and test_wrapper['B'] is not None and test_wrapper['C'] is not None and \
           test_wrapper['D'] is not None and test_wrapper['E'] is not None and test_wrapper['F'] is not None and \
           test_wrapper['G'] is not None

    assert (test_wrapper.evaluate_expression('(A^T)^2') == la.matrix_power(test_wrapper['A'].T, 2)).all()
    assert (test_wrapper.evaluate_expression('(B^T)^3') == la.matrix_power(test_wrapper['B'].T, 3)).all()
    assert (test_wrapper.evaluate_expression('(C^T)^4') == la.matrix_power(test_wrapper['C'].T, 4)).all()
    assert (test_wrapper.evaluate_expression('(D^T)^5') == la.matrix_power(test_wrapper['D'].T, 5)).all()
    assert (test_wrapper.evaluate_expression('(E^T)^6') == la.matrix_power(test_wrapper['E'].T, 6)).all()
    assert (test_wrapper.evaluate_expression('(F^T)^7') == la.matrix_power(test_wrapper['F'].T, 7)).all()
    assert (test_wrapper.evaluate_expression('(G^T)^8') == la.matrix_power(test_wrapper['G'].T, 8)).all()

    assert (test_wrapper.evaluate_expression('(rot(45)^1)^T') == create_rotation_matrix(45).T).all()
    assert (test_wrapper.evaluate_expression('(rot(45)^2)^T') == la.matrix_power(create_rotation_matrix(45),
    ↪  2).T).all()
    assert (test_wrapper.evaluate_expression('(rot(45)^3)^T') == la.matrix_power(create_rotation_matrix(45),
    ↪  3).T).all()
    assert (test_wrapper.evaluate_expression('(rot(45)^4)^T') == la.matrix_power(create_rotation_matrix(45),
    ↪  4).T).all()
    assert (test_wrapper.evaluate_expression('(rot(45)^5)^T') == la.matrix_power(create_rotation_matrix(45),
    ↪  5).T).all()

    assert (test_wrapper.evaluate_expression('D^3(A+6.2F-0.397G^TE)^-2+A') ==
            la.matrix_power(test_wrapper['D'], 3) @ la.matrix_power(
                test_wrapper['A'] + 6.2 * test_wrapper['F'] - 0.397 * test_wrapper['G'].T @ test_wrapper['E'],
                -2
            ) + test_wrapper['A']).all()

    assert (test_wrapper.evaluate_expression('-1.2F^{3}4.9D^T(A^2(B+3E^TF)^-1)^2') ==
            -1.2 * la.matrix_power(test_wrapper['F'], 3) @ (4.9 * test_wrapper['D'].T) @
            la.matrix_power(
                la.matrix_power(test_wrapper['A'], 2) @ la.matrix_power(
                    test_wrapper['B'] + 3 * test_wrapper['E'].T @ test_wrapper['F'],
                    -1
```

```
                ),
                2
            )).all()


def test_value_errors(test_wrapper: MatrixWrapper) -> None:
    """Test that evaluate_expression() raises a ValueError for any malformed input."""
    invalid_expressions = ['', '+', '-', 'This is not a valid expression', '3+4',
                           'A+2', 'A^', '^2', 'A^-', 'At', 'A^t', '3^2']

    for expression in invalid_expressions:
        with pytest.raises(ValueError):
            test_wrapper.evaluate_expression(expression)


def test_linalgerror() -> None:
    """Test that certain expressions raise np.linalg.LinAlgError."""
    matrix_a: MatrixType = np.array([
        [0, 0],
        [0, 0]
    ])

    matrix_b: MatrixType = np.array([
        [1, 2],
        [1, 2]
    ])

    wrapper = MatrixWrapper()
    wrapper['A'] = matrix_a
    wrapper['B'] = matrix_b

    assert (wrapper.evaluate_expression('A') == matrix_a).all()
    assert (wrapper.evaluate_expression('B') == matrix_b).all()

    with pytest.raises(np.linalg.LinAlgError):
        wrapper.evaluate_expression('A^-1')

    with pytest.raises(np.linalg.LinAlgError):
        wrapper.evaluate_expression('B^-1')

    assert (wrapper['A'] == matrix_a).all()
    assert (wrapper['B'] == matrix_b).all()
```