Dyson - draft #1

By the time I started Sixth Form, I had already completed AS Maths (Pure) and half of AS Further Maths (Core Pure), since my Year 11 maths teacher gave me these textbooks early because he knew I excelled at the subject. This head start allowed me to gain an even bigger lead on my peers as well as giving me time to start my Computer Science project a year early. This project (called lintrans) is entirely focussed on visualizing mathematics for the benefit of teaching. I watched the 3blue1brown linear algebra series and wanted to create an interactive app inspired by the series. In the course of this development, I faced many mathematical challenges around converting between coordinate systems and drawing lines on the canvas. It has involved solving many small but interesting geometric problems. The project has also proved very useful for explaining matrices to other people, and for investigating certain questions involving 2D transformations.

I attended the Imperial College London Further mA*ths Year 12 online course, which gave me a better understanding of many core principles of the subject, including many A Level ones, and gave me a more significant advantage over my peers since I was becoming competent in certain topics before they'd even heard of them. These topics included things like Maclaurin series, de Moivre's theorem, and hyperbolic functions, all taught to Year 12s. I will also be doing their Year 13 course.

I've found questions in textbooks that have prompted me to write my own papers using LaTeX to explore the ideas these questions present. A simple example would be a question that asked how many regions can be formed by dividing the plane with n lines. It was an interesting question that I enjoyed solving, but I then wanted to produce a nicely formatted paper using TikZ to draw some illustrative graphics, so that's what I did that afternoon. A more interesting example of writing my own papers is the question that asked for polynomials which give pure powers of n when summed from 1 to n. While exploring this problem, I found an interesting pattern in the coefficients that looked like binomial expansions. I explored this pattern and found a nice formula. My proof of this formula was originally several pages long but I then asked StackExchange if my discovery had a name and learned that it was a finite telescoping series. This made the proof far shorter, and I learned a valuable lesson about stepping back to see simpler solutions.

I listened to A Brief History of Mathematics on BBC Sounds, which has episodes on various people in mathematical history. My favourite episode was the one on Galois. The episode details his life and inspired me to look further into Galois Theory. I found two YouTube videos on a channel called Aleph 0 about the topic, and they were both very interesting. The topic is obviously beyond me right now, but I think I've got a grasp of the very basics, and I definitely want to learn more. Abstract algebras like Galois Theory, Group Theory, and even things like the dual numbers are incredibly interesting to me. It's these kind of abstract (and often fundamental) algebras that interest me the most.

Early in Year 12, I joined a MOOC with MIT about using matrices and their eigenstuffs to solve linear differential equations. I didn't know what differential equations were, and I barely understood matrices, but I wanted to jump into the course. It started slow with linear transformations and matrices and their relationship and then introduced eigenvalues and eigenvectors. It was actually a very nice introduction and taught me a lot about the subject. My main takeaways were row echelon form and Gauss-Jordan elimination, which allowed me to easily solve linear simultaneous equations by hand. I also supplemented it with the 3blue1brown linear algebra series. However, when the course started to talk about using these techniques to solve differential equations, I had to give up because I knew nothing about the subject. I tried to learn the topic with another MIT course in the same series, but this course was way over my head. It felt like it was aimed at people who had a much greater knowledge of calculus and already knew some differential equation stuff. I, however, knew nothing about the topic and felt completely out of my depth, so I unfortunately had to give up again, wanting to come back later.

I watched Welch Labs' series on complex numbers, which has a fantastic visualization of multifunctions and 4D Riemann Surfaces. I first watched this series before I had learned about complex numbers in school and only understood the first half of it. I have since rewatched the series and I still don't understand all of it, but I've watched enough of Cliff Stoll talking about Klein Bottles to understand a fake intersection caused by projecting 4D into 3D. The series is fantastic at communicating complex ideas in a visual way, and has greatly increased my understanding of complex numbers, along with other online learning resources.

Episode 2 of the Numberphile podcast is an interview with Ken Ribet about Fermat's Last Theorem. This episode is very interesting and gives a nice overview of the story and touches on Wiles' proof. However, I wanted to investigate further. I watched some YouTube videos from Aleph 0 about elliptic curves and modular forms and got a very surface-level overview of the topic. Upon further research, I continued to learn more about the topic. I still don't understand it in much detail, but I get the gist and I love the connections between number theory and complex analysis. Connecting seemingly disconnected areas of mathematics like this is something I love, just like how I connected polynomials that sum to pure powers with coefficients of binomial expansions.

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