§7. Theorem Environments in LaTeX

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1 Group Theory

I might start my section with a bit of introductory text highlighting the motivation, or signposting towards the end goal of this section. For example, I might want to point out the very important theorem we are going to prove.

Lagrange's Theorem. *Here I state the theorem which I've* chosen *to name rather than number.*

With this under my belt, I will probably want to make some definitions.

Definition 1.1. Here I might define a *subgroup.*

I might like to mention that the definition is often axiom-heavy when it comes to double-checking, so I might provide a useful result.

Proposition 1.1 (The One Step Test)**.** *Here I might give a minimal set of axioms which can be used to guarantee that a subset is in fact a subgroup.*

Proof. I might prove the lemma here. Suppose that it splits into cases.

Case 1. First case proven

Case 2. Second case proven

Case 3. third case proven

And since all cases have been proven we are done. □

To demonstrate that Proposition 1.1 really is useful, I might provide the following examples.

Example 1.1. Here I might show that $\mathbb{Z}[i]$ is a subgroup of $\mathbb{C}$.

Some more intermediate text might lead me to the following definition.

Definition 1.2. Here I would define a *coset* gH.

Remark. Here I might point out some subtlety that the reader might have missed.

I might add text which isn't part of the definition or lemma. This could be quite lengthy in fact because I have attendance to waffle on and on, so it's a good job this will get edited down at a later date. Do we have milk in or should I buy some on the way home? Such questions are almost certainly not answered by the following result.:

Lemma 1.2. *I might add a lemma about disjoint cosets.*

Proof. Let's pretend I have proven my lemma. If the proof ends with displayed maths you might find that the tombstone is in the wrong place, you can change this using "qedhere" as I have here:

$$g^2H = g(gH) = gH. \tag{1}$$

□

Lemma 1.3. *We all know a good lemma about cosets being of equal size.*

Proof. This is left as an exercise for the reader. (Never do this in your work!) □

Theorem 1.4 (Lagrange's Theorem)**.** *A very important theorem in group theory linking the cardinalities of the subgroup, index and group.*

Proof. Proof involving cosets, and probably referring to Lemma 1.2 and Lemma 1.3. If my proof is very complicated, then I might want to introduce a claim.

Claim. *Here is a claim that will help me with my proof.*

Proof of Claim. Here is where I might prove my claim. □

And now that the claim is established, I can finish my big proof. □

2 More Group Theory

Definition 2.1. Now I might define a *normal subgroup.*

I might add text which isn't part of the definition or lemma.

Definition 2.2. Now I might define the operation on cosets of a normal subgroup and call the result a *quotient group.*

Lemma 2.1. *I might add a lemma about the quotient group being a group.*

Proof. Let's pretend I have proven my lemma. I might refer to Theorem 1.4.

□

Some intermediate text might lead me to create the following.

Definition 2.3. Now I might define the terms *homomorphism* and *isomorphism.*

Theorem 2.2 (The First Isomorphism Theorem for Groups)**.** *Another very important theorem in group theory.*

Proof. Proof involving cosets, and probably referring to Lemma 2.1 and Lemma 1.3. It probably doesn't but what if the proof split into cases again.

Case 1. First case.

Case 2. Second case.

Case 3. third case.

□

2.1 Testing Environments

Assumption 2.4. Suppose henceforth that G is an abelian group.

Note. There are of course examples of non-abelian groups.

Conjecture 2.1.1. *The centre $Z(G)$ of G is the whole of G.*

Proof. Suppose that $g \in G$, then given any $h \in G$, we have $gh = hg$ since G is abelian, and since this holds for any $h \in G$, we see that $g \in Z(G)$. □