(a) Yes: choose the route d, e, f, g, i, a, b and the reversing loop b, c, h, b.

(b) Given a graph G, find all 3-cycles in G. If there are no 3-cycles, return No. For each 3-cycle, conduct a depth-first search on each vertex in that cycle to find a Hamiltonian path through the rest of G (not including the vertices in this 3-cycle). If a Hamiltonian path is found at any point, return Yes. If every vertex in every 3-cycle is checked without finding a Hamiltonian path, return No.

(c) To show NP-membership, we need to construct a polynomial-time verifier. Given a graph G and a proposed witness solution w, we first check that w is connected and is a subgraph of G. If either is false, return No. Count all the 3-cycles in w. If this number is not 1, return No.

Checking connectedness, subgraph-ness, and counting 3-cycles are all polynomial-time in the number of vertices, so this verifier is polynomial-time and SANTA-SERVICE $\in$ NP.

SANTA-SERVICE requires finding a Hamiltonian path, which is known to be an NP-hard problem. Therefore SANTA-SERVICE is NP-hard, and therefore SANTA-SERVICE is NP-complete.

(d) FIND-SANTA-SERVICE is not a decision problem, so cannot be in NP and therefore cannot be NP-complete. However, it is NP-hard, using the same logic we used in part **(c)**.

(e) If the graph contains a known Hamiltonian cycle, then we only need to check each vertex for a 3-cycle, which makes the algorithm polynomial time.