MA146 Methods of Mathematical Modelling 1, Assignment 4

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Question 2

$$\frac{\mathrm{d}}{\mathrm{d}t}v(t) = -v(t) + \varepsilon v(t)^2, \quad t > 0, \quad v(0) = 1$$

Q2 (a)

I have no idea what substitution to use, sorry.

Q2 (b)

Consider $v(t) = v_0(t) + \varepsilon v_1(t)$. Then

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}v(t) &= \frac{\mathrm{d}}{\mathrm{d}t}v_0(t) + \varepsilon\frac{\mathrm{d}}{\mathrm{d}t}v_1(t) \\
&= -v_0(t) - \varepsilon v_1(t) + \varepsilon\left(v_0(t)^2 + 2\varepsilon v_0(t)v_1(t) + \varepsilon^2 v_1(t)^2\right) \\
&= -v_0(t) - \varepsilon v_1(t) + \varepsilon v_0(t)^2 + \cancelto{0}{\varepsilon^2(\cdots)} \\
\therefore \frac{\mathrm{d}}{\mathrm{d}t}v_0(t) + \varepsilon\frac{\mathrm{d}}{\mathrm{d}t}v_1(t) &= -v_0(t) - \varepsilon v_1(t) + \varepsilon v_0(t)^2
\end{aligned}$$

And the initial condition becomes $v_0(0) + \varepsilon v_1(0) = 1$. Unfortunately, I have no idea where to go from here.

Question 3

Q3 (a)

$$3\mathrm{K} + 4\mathrm{M} \underset{r_2}{\overset{r_1}{\rightleftharpoons}} 2\mathrm{A} + 6\mathrm{B}$$

Let $k(t)$ be the concentration of K and likewise pairing $m(t)$ with M, $a(t)$ with A, and $b(t)$ with B. Then we get the following equations

$$\frac{\mathrm{d}}{\mathrm{d}t}k(t) = -3r_1k(t)m(t) + 3r_2a(t)b(t)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}m(t) = -4r_1k(t)m(t) + 4r_2a(t)b(t)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}a(t) = 2r_1k(t)m(t) - 2r_2a(t)b(t)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}b(t) = 6r_1k(t)m(t) - 6r_2a(t)b(t)$$

Q3 (b)

$$2\mathrm{NOCl} \xrightarrow{k} 2\mathrm{NO} + \mathrm{Cl}_2$$

Let $s(t)$ be the concentration of nitrosyl chloride, $n(t)$ be the concentration of nitric oxide, and $c(t)$ be the concentration of chlorine. Then the rate equations are

$$\frac{\mathrm{d}}{\mathrm{d}t}s(t) = -2ks(t)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}n(t) = 2ks(t)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}c(t) = ks(t)$$

Q3 (c)

$$S + I \xrightarrow{\beta} I \xrightarrow{\gamma} R$$

Question 4

Q4 (a)

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b, c = -0.2, 8
x_1, x_2 = symbols("x_1 x_2", cls=Function)
t = Symbol('t')

ode_sys4a = [Eq(x_1(t).diff(t), x_2(t)), Eq(x_2(t).diff(t), -c*x_1(t) -b*x_2(t))]

sol_sys1 = dsolve(ode_sys4a, ics={x_1(0): 1.5, x_2(0): 0})
sol_sys2 = dsolve(ode_sys4a, ics={x_1(0): 2.0, x_2(0): 0})
sol_sys3 = dsolve(ode_sys4a, ics={x_1(0): 2.7, x_2(0): 0})

plot_parametric(
    (sol_sys1[0].rhs, sol_sys1[1].rhs),
    (sol_sys2[0].rhs, sol_sys2[1].rhs),
    (sol_sys3[0].rhs, sol_sys3[1].rhs),
    (t, 0, 4*pi)
)
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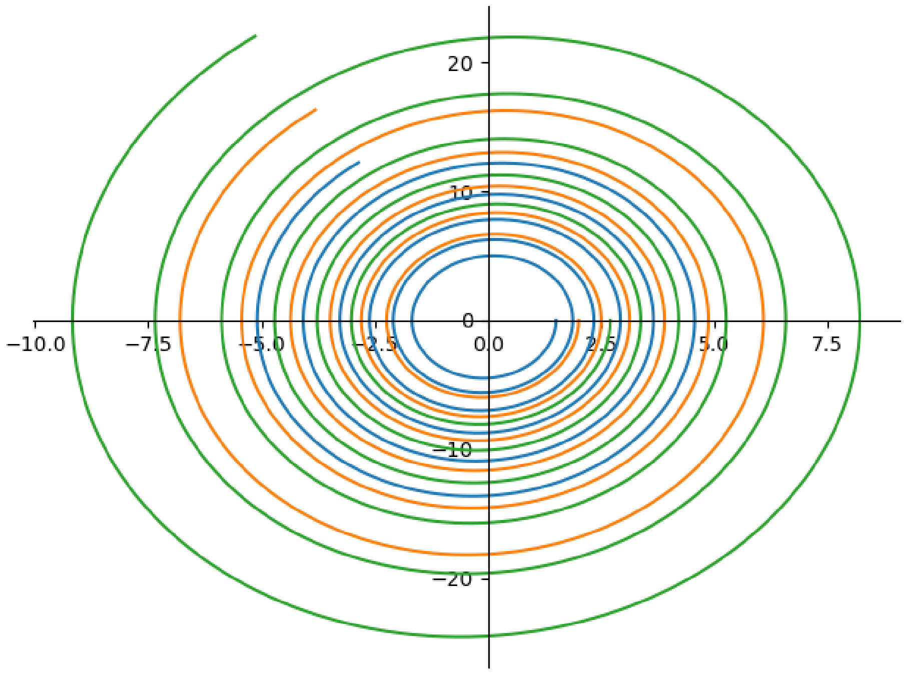

Q4 (b)

Assignment 1 question 2 was about the differential equation

$$a\frac{\mathrm{d}^2}{\mathrm{d}t^2}x(t) + b\frac{\mathrm{d}}{\mathrm{d}t}x(t) + cx(t) = 0$$

The system

$$x_1'(t) = x_2(t)$$
$$x_2'(t) = -cx_1(t) - bx_2(t)$$

is related in that we can imagine $x_1(t) = x(t)$ and $x_2(t) = \frac{\mathrm{d}}{\mathrm{d}t}x(t)$. Then the system becomes

$$\frac{\mathrm{d}}{\mathrm{d}t}x(t) = \frac{\mathrm{d}}{\mathrm{d}t}x(t)$$
$$\frac{\mathrm{d}^2}{\mathrm{d}t^2}x(t) = -cx(t) - b\frac{\mathrm{d}}{\mathrm{d}t}x(t)$$

The first line is just obviously always true, and the second line is equivalent to the differential equation given in assignment 1 question 2, just with b and c scaled to make $a = 1$.