Exercise: Creating your own LATEX commands

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1 Sets of numbers

The sets of natural numbers $\mathbb{N}$, integers $\mathbb{Z}$, rational numbers $\mathbb{Q}$ and complex numbers $\mathbb{C}$ form a chain of sets.

2 Calculus

The Fundamental Theorem of Calculus along with the chain rule for differentiation gives a useful formula for differentiating an integral so long as the integrand is smooth enough.

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_{a(t)}^{b(t)} f(x,t)\mathrm{d}x = \frac{\mathrm{d}}{\mathrm{d}t}b(t) - \frac{\mathrm{d}}{\mathrm{d}t}a(t) + \int_{a(t)}^{b(t)} \frac{\partial}{\partial t}f(x,t)\mathrm{d}x.$$

Vector fields often twist and turn, and we can quantify this effect using a vector called the *curl* of v, defined by

$$\nabla \times v = \left(\frac{\partial v_3}{\partial y} - \frac{\partial v_2}{\partial z}\right)\hat{\imath} + \left(\frac{\partial v_1}{\partial z} - \frac{\partial v_3}{\partial x}\right)\hat{\jmath} + \left(\frac{\partial v_2}{\partial x} - \frac{\partial v_1}{\partial y}\right)\hat{k}.$$

3 Norms

It is utter nonsense to write $\left\|\frac{u}{v}\right\| = \frac{\|u\|}{\|v\|}$ if u, v are vectors in $\mathbb{R}^3$.

4 Miscellaneous

The gradient of a function grad f is the vector of its partial derivatives.

Complex conjugation is additive and multiplicative, so $\overline{w+z} = \overline{w} + \overline{z}$ and $\overline{wz} = \overline{w}\,\overline{z}$. It is now easy to prove that $\overline{(w/z)} = \overline{w}/\overline{z}$.

The adjugate of a matrix is the matrix formed by taking the transpose of the cofactor matrix of a given original matrix. The adjoint of matrix $A = (\alpha_{i,j})$ is often written adj A — not to be confused with the adjoint matrix $A^* = (\overline{\alpha_{j,i}})$ which is the transpose of the matrix of complex conjugate entries.

In Linear Algebra, the space of all linear maps from a vector space U to the vector space V is denoted by $\mathrm{Hom}(U, V)$.

The signum function extracts the sign of a real number. Formally, $\mathrm{sgn}\colon \mathbb{R} \to \{-1, 0, 1\}$ is given by

$$\mathrm{sgn}(x) = \begin{cases} -1 & \text{if } x < 0, \\ 0 & \text{if } x = 0, \\ 1 & \text{if } x > 0. \end{cases}$$

5 Groups

The Dihedral group, D_{2n} of $2n$ elements is given by

$$\left\langle a, b \mid a^n = 1, b^2 = 1, ba = ba^{-1} \right\rangle.$$

6 Functional Analysis

The *arguments of the maxima* are the points of the domain of some function at which the function values are maximized.

Given a function, $f\colon X \to Y$ the $\arg\max$ over some subset, S, of X of f is defined by

$$\arg\max_{x \in S \subset X} f(x) := \{x \in S \mid \text{for each } y \in S, f(y) \leq f(x)\}.$$