Exercise: Mathematical Symbols in LaTeX

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1 Linear Algebra: determinants

Suppose that we have an $n \times n$ matrix $A = \big(a_{i,j}\big)$, so

$$A = \begin{pmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,n} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n,1} & a_{n,2} & \cdots & a_{n,n} \end{pmatrix}.$$

Then we recall the definition of determinant of A to be

$$\det(A) = \sum_{\sigma \in S_n} \operatorname{sgn}(\sigma) \prod_{i=1}^{n} a_{i,\sigma(i)}.$$

But in practice, we often use row & column expansion methods in combination with elementary row & column operations to compute them.

2 Vector Analysis: Change of Variables

One use of the determinant is its ability to determine how area and volume elements change under a transformation of coordinates.

Suppose that we wish to compute the integral $\iint_\Omega f(x,y)\mathrm{d}A$ using the transformation given by

$$\begin{cases} x = g(u,v), \\ y = h(u,v). \end{cases}$$

Assume also that under this transformation, the region Ω is taken to the region $S = \big\{(u,v) \in \mathbb{R}^2 \mid a \leq u \leq b, c \leq v \leq d\big\}$, then

$$\iint_\Omega f(x,y)\mathrm{d}A = \int_a^b \int_c^d f\Big(g(u,v), h(u,v)\Big) \left|\frac{\partial(x,y)}{\partial(u,v)}\right| \mathrm{d}u\mathrm{d}v$$

where

$$\frac{\partial(x,y)}{\partial(u,v)} = \det\begin{pmatrix} \frac{\partial g}{\partial u} & \frac{\partial g}{\partial v} \\ \frac{\partial h}{\partial u} & \frac{\partial h}{\partial v} \end{pmatrix}.$$

Similar methods allow easy computation of integrals such as $\oint\limits_{\partial\Xi} \nabla v \cdot \mathrm{d}\underline{\xi}$.